R22

Code No: 181AN

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, March/April - 2023
MATRICES AND CALCULUS
(Common to CE, ME, ECE, EIE, AE, MIE, CSE(AI&ML), CSE(IOT), AI&DS, AI&ML)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) What is the value of 'k' if the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2? [1]

b) Find the value of 'a' for which the equations have infinite number of solutions: [1]
x+y+z=1; ax-ay+3z = 5; 5x-3y +az =6.

c) If the eigenvalues of A= $\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$ are 3, 6 and 9, then what are the eigenvalues of adj A? [1]

d) What is the nature of the quadratic form $2xy + 6xz - 4yz$? [1]

e) Find the value of the constant in Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ defined on [a, b], $0 < a < b$. [1]

f) Find the Taylor's expansion of $f(x) = e^x$ around x=1. [1]

g) If $u = \frac{y}{x}, v = xy,$ then find $J\left(\frac{u,v}{x,y}\right)$. [1]

h) Find the stationary point's of the function $x^2+2xy+2y^2+2x+2y$. [1]

i) Evaluate $\int_0^1 \int_2^4 xy\,dx\,dy$. [1]

j) In the integral $\int_0^4 \int_x^4 f(x,y)\,dx\,dy$, write the limits after changing the order of integration. [1]

PART-B

(50 Marks)

2.a) Find rank of matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ 2 & 2 & 8 & 0 \end{bmatrix}$ by reducing it into normal form.

b) Solve the system of equations:
x+y+54z=110; 27x +6y –z =85; 6x+15y +2z =72 using Gauss-Seidel method. [5+5]

OR

3.a) Solve the system of equations by Gauss -elimination method
$5x + y + z + w = 4$, $x + 7y + z + w = 12$, $x + y + 6z + w = -5$,
$x + y + z + 4w = -6$.

b) Find the inverse of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ using Gauss-Jordan method. [5+5]

4.a) Find the eigenvalues and the corresponding eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}$.

b) Using Cayley Hamilton theorem for the matrix A= $\begin{bmatrix} 1 & -3 & 1 \\ 6 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix}$, find A^4. [5+5]

OR

5. Reduce the quadratic form $3x^2 + 2y^2 + 3z^2 - 2xy - 2yz$ to the canonical form by orthogonal transformations and find rank, index, signature, nature of the quadratic form. [10]

6.a) Find the region in which f(x) = 1-4x-x^2 is increasing and the region in which it is decreasing using Mean Value Theorem.

b) Find the volume of the solid generated by the revolution of the area bounded by y = x^2 and y = x about y – axis. [5+5]

OR

7.a) Expand $\tan^{-1} x$ in powers of (x – 1) up to the term containing fourth degree.

b) Evaluate $\int_0^1 x^4 \left(\log \frac{1}{x} \right)^3 dx$. [5+5]

8.a) If u = $\sin^{-1}\left(\frac{x^2 + y^2}{x + y} \right)$ find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

b) If x + y + z = u, y + z = uv, z = uvw, then evaluate $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. [5+5]

OR

9.a) If x = $e^r \sec\theta$, y = $e^r \tan\theta$, prove that $\frac{\partial(x, y)}{\partial(r, \theta)} \cdot \frac{\partial(r, \theta)}{\partial(x, y)} = 1$.

b) The Temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature at the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. [5+5]

10.a) By changing the order of integration, evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \sqrt{a^2 - x^2 - y^2}\, dy dx$.

b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the plane $z = 0, y + z = 4$. [5+5]

OR

11.a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dydx$ by changing into polar coordinates.

b) Using spherical polar coordinates, evaluate $\iiint \frac{xyz\ dxdydz}{\sqrt{x^2+y^2+z^2}}$ taken over the volume bounded by the sphere $x^2+y^2+z^2=a^2$ in the first octant. [5+5]

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