

Figure 3.1 Input-output characteristic of a nonlinear system.

approximation, and higher order terms are insignificant. In other words,  $\Delta y = a_1 \Delta x$ , indicating a linear relationship between the *increments* at the input and output. As  $x(t)$  increases in magnitude, higher order terms manifest themselves, leading to nonlinearity and necessitating large-signal analysis. From another point of view, if the slope of the characteristic (the incremental gain) *varies* with the signal level, then the system is nonlinear. These concepts are described in detail in Chapter 13.

What aspects of the performance of an amplifier are important? In addition to gain and speed, such parameters as power dissipation, supply voltage, linearity, noise, or maximum voltage swings may be important. Furthermore, the input and output impedances determine how the circuit interacts with preceding and subsequent stages. In practice, most of these parameters trade with each other, making the design a multi-dimensional optimization problem. Illustrated in the "analog design octagon" of Fig. 3.2, such trade-offs present many challenges in the design of high-performance amplifiers, requiring intuition and experience to arrive at an acceptable compromise.

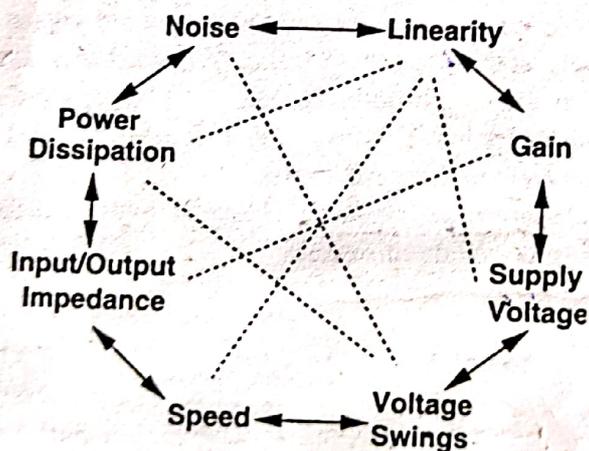
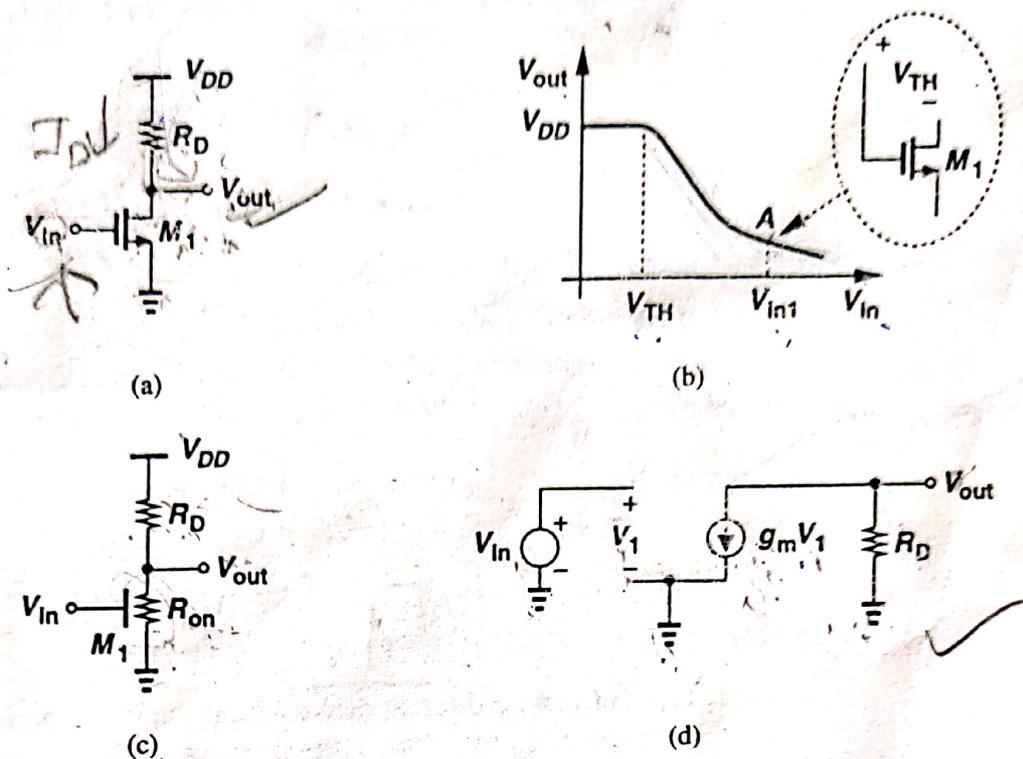


Figure 3.2 Analog design octagon.

## 3.2 Common-Source Stage

### 3.2.1 Common-Source Stage with Resistive Load

By virtue of its transconductance, a MOSFET converts variations in its gate-source voltage to a small-signal drain current, which can pass through a resistor to generate an output voltage. Shown in Fig. 3.3(a), the common-source (CS) stage performs such an operation.



**Figure 3.3** (a) Common-source stage, (b) input-output characteristic, (c) equivalent circuit in deep triode region, (d) small-signal model for the saturation region.

We study both the large-signal and the small-signal behavior of the circuit. Note that the input impedance of the circuit is very high at low frequencies.

If the input voltage increases from zero,  $M_1$  is off and  $V_{out} = V_{DD}$  [Fig. 3.3(b)]. As  $V_{in}$  approaches  $V_{TH}$ ,  $M_1$  begins to turn on, drawing current from  $R_D$  and lowering  $V_{out}$ . If  $V_{DD}$  is not excessively low,  $M_1$  turns on in saturation, and we have

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 \quad (3.3)$$

where channel-length modulation is neglected. With further increase in  $V_{in}$ ,  $V_{out}$  drops more and the transistor continues to operate in saturation until  $V_{in}$  exceeds  $\sqrt{V_{out}}$  by  $V_{TH}$  [point A in Fig. 3.3(b)]. At this point,

$$V_{in1} - V_{TH} = \sqrt{V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})^2} \quad (3.4)$$

from which  $V_{in1} - V_{TH}$  and hence  $V_{out}$  can be calculated.

For  $V_{in} > V_{in1}$ ,  $M_1$  is in the triode region:

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(V_{in} - V_{TH})V_{out} - V_{out}^2] \quad (3.5)$$

*Handwritten note:*  $g_m(V_{in} - V_{TH})V_{out} - V_{out}^2$

If  $V_{in}$  is high enough to drive  $M_1$  into deep triode region,  $V_{out} \ll 2(V_{in} - V_{TH})$ , and, from the equivalent circuit of Fig. 3.3(c),

$$V_{out} = V_{DD} \frac{R_{on}}{R_{on} + R_D} \quad (3.6)$$

$$= \frac{V_{DD}}{1 + \mu_n C_{ox} \frac{W}{L} R_D (V_{in} - V_{TH})} \quad (3.7)$$

Since the transconductance drops in the triode region, we usually ensure that  $V_{out} > V_{in} - V_{TH}$ , operating to the left of point A in Fig. 3.3(b). Using (3.3) as the input-output characteristic and viewing its slope as the small-signal gain, we have:

$$A_v = \frac{\partial V_{out}}{\partial V_{in}} \quad (3.8)$$

$$= -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH}) \quad (3.9)$$

$$A_v = -g_m R_D \quad (3.10)$$

This result can be directly derived from the observation that  $M_1$  converts an input voltage change  $\Delta V_{in}$  to a drain current change  $g_m \Delta V_{in}$ , and hence an output voltage change  $-g_m R_D \Delta V_{in}$ . The small-signal model of Fig. 3.3(d) yields the same result.

Even though derived for small-signal operation, the equation  $A_v = -g_m R_D$  predicts certain effects if the circuit senses a *large* signal swing. Since  $g_m$  itself varies with the input signal according to  $g_m = \mu_n C_{ox} (W/L) (V_{GS} - V_{TH})$ , the gain of the circuit changes substantially if the signal is large. In other words, if the gain of the circuit *varies* significantly with the signal swing, then the circuit operates in the large-signal mode. The dependence of the gain upon the signal level leads to nonlinearity (Chapter 13), usually an undesirable effect.

A key result here is that to minimize the nonlinearity, the gain equation must be a weak function of signal-dependent parameters such as  $g_m$ . We present several examples of this concept in this chapter and in Chapter 13.

### Example 3.1

Sketch the drain current and transconductance of  $M_1$  in Fig. 3.3(a) as a function of the input voltage.

#### Solution

The drain current becomes significant for  $V_{in} > V_{TH}$ , eventually approaching  $V_{DD}/R_D$  if  $R_{on1} \ll R_D$  [Fig. 3.4(a)]. Since in saturation,  $g_m = \mu_n C_{ox} (W/L) (V_{in} - V_{TH})$ , the transconductance begins to rise for  $V_{in} > V_{TH}$ . In the triode region,  $g_m = \mu_n C_{ox} (W/L) V_{DS}$ , falling as  $V_{in}$  exceeds  $V_{in1}$  [Fig. 3.4(b)].

How do we maximize the voltage gain of a common-source stage? Writing (3.10) as

$$A_v = -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \frac{V_{RD}}{I_D} \quad (3.11)$$

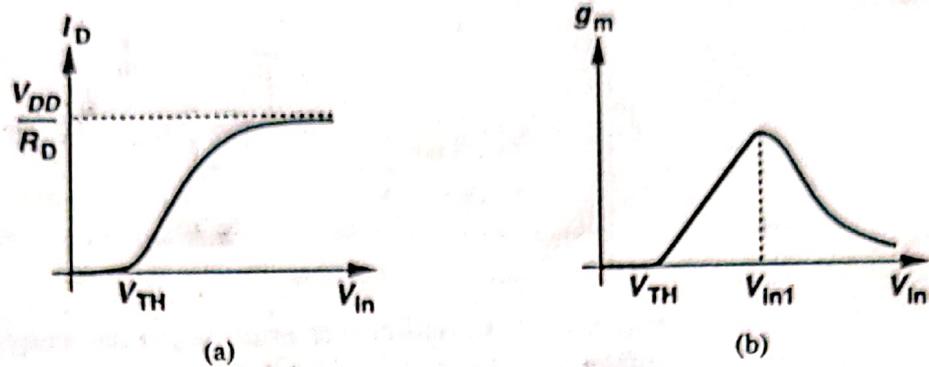


Figure 3.4

where  $V_{RD}$  denotes the voltage drop across  $R_D$ , we have

$$A_v = -\sqrt{2\mu_n C_{ox} \frac{W}{L} \frac{V_{RD}}{I_D}} \quad (3.12)$$

Thus, the magnitude of  $A_v$  can be increased by increasing  $W/L$  or  $V_{RD}$  or decreasing  $I_D$  if other parameters are constant. It is important to understand the trade-offs resulting from this equation. A larger device size leads to greater device capacitances, and a higher  $V_{RD}$  limits the maximum voltage swings. For example, if  $V_{DD} - V_{RD} = V_{in} - V_{TH}$ , then  $M_1$  is at the edge of the triode region, allowing only very small swings at the output (and input). If  $V_{RD}$  remains constant and  $I_D$  is reduced, then  $R_D$  must increase, thereby leading to a greater time constant at the output node. In other words, as noted in the analog design octagon, the circuit exhibits trade-offs between gain, bandwidth, and voltage swings. Lower supply voltages further tighten these trade-offs.

For large values of  $R_D$ , the effect of channel length modulation in  $M_1$  becomes significant. Modifying (3.4) to include this effect,

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 (1 + \lambda V_{out}) \quad (3.13)$$

we have

$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})(1 + \lambda V_{out}) - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 \lambda \frac{\partial V_{out}}{\partial V_{in}} \quad (3.14)$$

Using the approximation  $I_D \approx (1/2)\mu_n C_{ox}(W/L)(V_{in} - V_{TH})^2$ , we obtain:

$$A_v = -R_D g_m - R_D I_D \lambda A_v \quad (3.15)$$

$$A_v + R_D I_D \lambda A_v = -R_D g_m$$

$$A_v [1 + R_D I_D \lambda] = -R_D g_m$$

$$A_v = -g_m R_D / [1 + R_D I_D \lambda]$$

and hence

$$A_v = -\frac{g_m R_D}{1 + R_D \lambda I_D} = -\frac{g_m R_D}{1 + R_D \lambda \left[ \frac{1}{r_o} \right]} \quad (3.16)$$

Since  $\lambda I_D = 1/r_o$ ,

$$A_v = -g_m \frac{r_o R_D}{r_o + R_D} \quad (3.17)$$

The small-signal model of Fig. 3.5 gives the same result with much less effort. That is, since

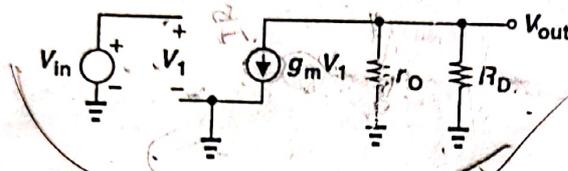


Figure 3.5 Small-signal model of CS stage including the transistor output resistance.

$g_m V_1 (r_o \parallel R_D) = -V_{out}$  and  $V_1 = V_{in}$ , we have  $V_{out}/V_{in} = -g_m (r_o \parallel R_D)$ . Note that, as mentioned in Chapter 1,  $V_{in}$ ,  $V_1$ , and  $V_{out}$  in this figure denote small-signal quantities.

### Example 3.2

Assuming  $M_1$  in Fig. 3.6 is biased in saturation, calculate the small-signal voltage gain of the circuit.

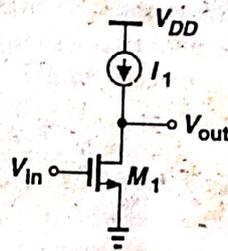


Figure 3.6

### Solution

Since  $I_1$  introduces an infinite impedance, the gain is limited by the output resistance of  $M_1$ :

$$A_v = -g_m r_o. \quad (3.18)$$

Called the "intrinsic gain" of a transistor, this quantity represents the maximum voltage gain that can be achieved using a single device. In today's CMOS technology,  $g_m r_o$  of short-channel devices is between roughly 10 and 30. Thus, we usually assume  $1/g_m \ll r_o$ .

In Fig. 3.6, Kirchhoff's current law (KCL) requires that  $I_{D1} = I_1$ . Then, how can  $V_{in}$  change the current of  $M_1$  if  $I_1$  is constant? Writing the total drain current of  $M_1$  as

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} (V_{in} - V_{TH})^2 (1 + \lambda V_{out}) \quad (3.19)$$

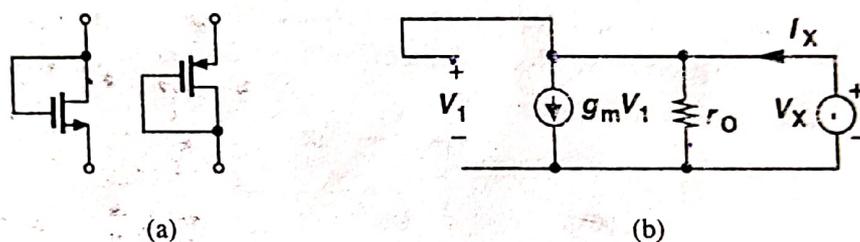
$$= I_1, \quad (3.20)$$

we note that  $V_{in}$  appears in the square term and  $V_{out}$  in the linear term. As  $V_{in}$  increases,  $V_{out}$  must decrease such that the product remains constant. We may nevertheless say " $I_{D1}$  increases as  $V_{in}$  increases." This statement simply refers to the quadratic part of the equation.

### 3.2.2 CS Stage with Diode-Connected Load

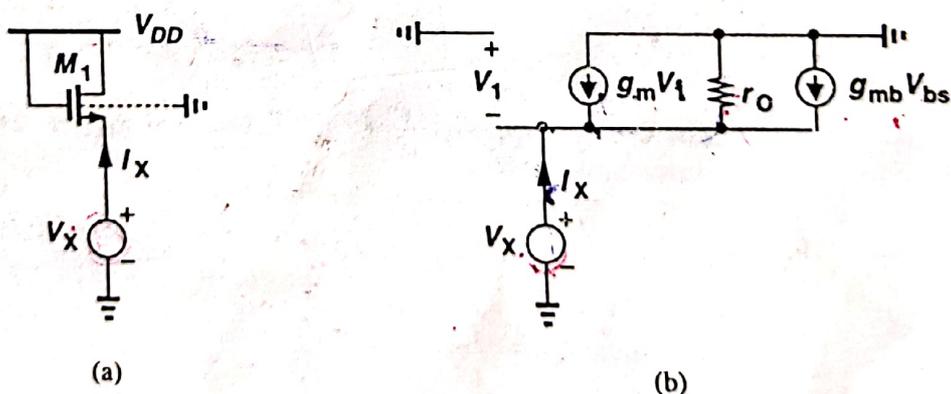
In many CMOS technologies, it is difficult to fabricate resistors with tightly-controlled values or a reasonable physical size (Chapter 17). Consequently, it is desirable to replace  $R_D$  in Fig. 3.3(a) with a MOS transistor.

A MOSFET can operate as a small-signal resistor if its gate and drain are shorted [Fig. 3.7(a)]. Called a "diode-connected" device in analogy with its bipolar counterpart,



**Figure 3.7** (a) Diode-connected NMOS and PMOS devices, (b) small-signal equivalent circuit.

this configuration exhibits a small-signal behavior similar to a two-terminal resistor. Note that the transistor is always in saturation because the drain and the gate have the same potential. Using the small-signal equivalent shown in Fig. 3.7(b) to obtain the impedance of the device, we write  $V_1 = V_X$  and  $I_X = V_X/r_o + g_m V_X$ . That is, the impedance of the diode is simply equal to  $(1/g_m) \parallel r_o \approx 1/g_m$ . If body effect exists, we can use the circuit in Fig. 3.8 to write  $V_1 = -V_X$ ,  $V_{bs} = -V_X$  and



**Figure 3.8** (a) Arrangement for measuring the equivalent resistance of a diode-connected MOSFET, (b) small-signal equivalent circuit.

$$(g_m + g_{mb})V_X + \frac{V_X}{r_o} = I_X. \tag{3.21}$$

It follows that

$$\frac{V_x}{I_x} = \frac{1}{g_m + g_{mb} + r_o^{-1}} \quad (3.22)$$

$$= \frac{1}{g_m + g_{mb}} \parallel r_o \quad (3.23)$$

$$\approx \frac{1}{g_m + g_{mb}} \quad (3.24)$$

Interestingly, the impedance seen at the source of  $M_1$  is *lower* when body effect is included. Intuitive explanation of this effect is left as an exercise for the reader.

We now study a common-source stage with a diode-connected load (Fig. 3.9). For negligible channel-length modulation, (3.24) can be substituted in (3.10) for the load impedance,

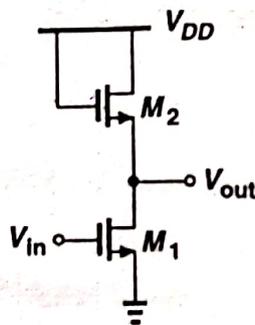


Figure 3.9 CS stage with diode-connected load.

yielding

$$A_v = -g_{m1} \frac{1}{g_{m2} + g_{mb2}} \quad (3.25)$$

$$= -\frac{g_{m1}}{g_{m2}} \frac{1}{1 + \eta} \quad (3.26)$$

where  $\eta = g_{mb2}/g_{m2}$ . Expressing  $g_{m1}$  and  $g_{m2}$  in terms of device dimensions and bias currents, we have

$$A_v = -\frac{\sqrt{2\mu_n C_{ox}(W/L)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox}(W/L)_2 I_{D2}}} \frac{1}{1 + \eta} \quad (3.27)$$

and, since  $I_{D1} = I_{D2}$ ,

$$A_v = -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \frac{1}{1 + \eta} \quad (3.28)$$

This equation reveals an interesting property: if the variation of  $\eta$  with the output voltage is neglected, the gain is independent of the bias currents and voltages (so long as  $M_1$  stays in saturation). In other words, as the input and output signal levels vary, the gain remains relatively constant, indicating that the input-output characteristic is relatively linear.

The linear behavior of the circuit can also be confirmed by large-signal analysis. Neglecting channel-length modulation for simplicity, we have in Fig. 3.9

$$\frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})^2, \quad (3.29)$$

and hence

$$\sqrt{\left(\frac{W}{L}\right)_1} (V_{in} - V_{TH1}) = \sqrt{\left(\frac{W}{L}\right)_2} (V_{DD} - V_{out} - V_{TH2}). \quad (3.30)$$

Thus, if the variation of  $V_{TH2}$  with  $V_{out}$  is small, the circuit exhibits a linear input-output characteristic. The small-signal gain can also be computed by differentiating both sides with respect to  $V_{in}$ :

$$\sqrt{\left(\frac{W}{L}\right)_1} = \sqrt{\left(\frac{W}{L}\right)_2} \left(-\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{in}}\right), \quad (3.31)$$

which, upon application of the chain rule  $\partial V_{TH2}/\partial V_{in} = (\partial V_{TH2}/\partial V_{out})(\partial V_{out}/\partial V_{in}) = \eta(\partial V_{out}/\partial V_{in})$ , reduces to

$$\frac{\partial V_{out}}{\partial V_{in}} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \frac{1}{1 + \eta}. \quad (3.32)$$

It is instructive to study the overall large-signal characteristic of the circuit as well. But let us first consider the circuit shown in Fig. 3.10(a). What is the final value of  $V_{out}$  if  $I_1$  drops to zero? As  $I_1$  decreases, so does the overdrive of  $M_2$ . Thus, for small  $I_1$ ,  $V_{GS2} \approx V_{TH2}$  and  $V_{out} \approx V_{DD} - V_{TH2}$ . In reality, the subthreshold conduction in  $M_2$  eventually brings  $V_{out}$  to  $V_{DD}$  if  $I_D$  approaches zero, but at very low current levels, the finite capacitance at the output node slows down the change from  $V_{DD} - V_{TH2}$  to  $V_{DD}$ . This is illustrated in the time-domain waveforms of Fig. 3.10(b). For this reason, in circuits that have frequent switching activity, we assume  $V_{out}$  remains around  $V_{DD} - V_{TH2}$  when  $I_1$  falls to small values.

Now we return to the circuit of Fig. 3.9. Plotted in Fig. 3.11 versus  $V_{in}$ , the output voltage equals  $V_{DD} - V_{TH2}$  if  $V_{in} < V_{TH1}$ . For  $V_{in} > V_{TH1}$ , Eq. (3.30) holds and  $V_{out}$  follows an approximately straight line. As  $V_{in}$  exceeds  $V_{out} + V_{TH1}$  (beyond point A),  $M_1$  enters the triode region, and the characteristic becomes nonlinear.

The diode-connected load of Fig. 3.9 can be implemented with a PMOS device as well. Shown in Fig. 3.12, the circuit is free from body effect, providing a small-signal voltage gain equal to

$$A_v = -\sqrt{\frac{\mu_n(W/L)_1}{\mu_p(W/L)_2}}, \quad (3.33)$$

where channel-length modulation is neglected.

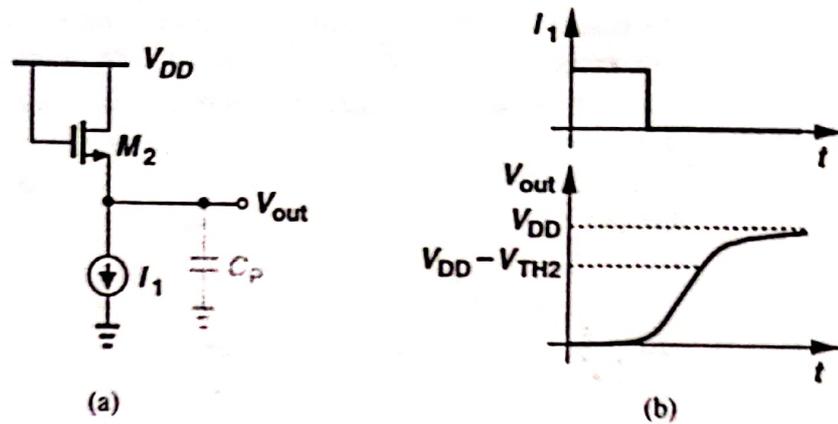


Figure 3.10 (a) Diode-connected device with stepped bias current, (b) variation of source voltage versus time.

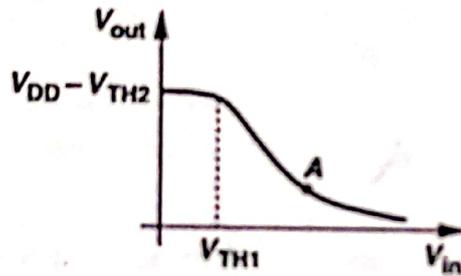


Figure 3.11 Input-output characteristic of a CS stage with diode-connected load.

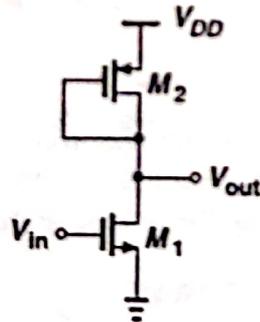


Figure 3.12 CS stage with diode-connected PMOS device.

Equations (3.28) and (3.33) indicate that the gain of a common-source stage with diode-connected load is a relatively weak function of the device dimensions. For example, to achieve a gain of 10,  $\mu_n(W/L)_1/[\mu_p(W/L)_2] = 100$ , implying that, with  $\mu_n \approx 2\mu_p$ , we must have  $(W/L)_1 \approx 50(W/L)_2$ . In a sense, a high gain requires a “strong” input device and a “weak” load device. In addition to disproportionately wide or long transistors (and hence a large input or load capacitance), a high gain translates to another important limitation: reduction in allowable voltage swings. Specifically, since in Fig. 3.12,  $I_{D1} = |I_{D2}|$ ,

$$\mu_n \left( \frac{W}{L} \right)_1 (V_{GS1} - V_{TH1})^2 \approx \mu_p \left( \frac{W}{L} \right)_2 (V_{GS2} - V_{TH2})^2, \quad (3.34)$$

revealing that

$$\frac{|V_{GS2} - V_{TH2}|}{V_{GS1} - V_{TH1}} \approx A_v. \quad (3.35)$$

In the above example, the overdrive voltage of  $M_2$  must be 10 times that of  $M_1$ . For example, with  $V_{GS1} - V_{TH1} = 200$  mV, and  $|V_{TH2}| = 0.7$  V, we have  $|V_{GS2}| = 2.7$  V, severely limiting the output swing. This is another example of the trade-offs suggested by the analog design octagon. Note that, with diode-connected loads, the swing is constrained by both the required overdrive voltage and the threshold voltage. That is, even with a small overdrive, the output level cannot exceed  $V_{DD} - |V_{TH}|$ .

An interesting paradox arises here if we write  $g_m = \mu C_{ox}(W/L)|V_{GS} - V_{TH}|$ . The voltage gain of the circuit is then given by

$$A_v = \frac{g_{m1}}{g_{m2}} \quad (3.36)$$

$$= \frac{\mu_n C_{ox}(W/L)_1 (V_{GS1} - V_{TH1})}{\mu_p C_{ox}(W/L)_2 |V_{GS2} - V_{TH2}|}. \quad (3.37)$$

Equation (3.37) implies that  $A_v$  is *inversely* proportional to  $|V_{GS2} - V_{TH2}|$ . It is left for the reader to resolve the seemingly opposite trends suggested by (3.35) and (3.37).

### Example 3.3

In the circuit of Fig. 3.13,  $M_1$  is biased in saturation with a drain current equal to  $I_1$ . The current source  $I_S = 0.75I_1$  is added to the circuit. How is (3.35) modified for this case?

#### Solution

Since  $|I_{D2}| = I_1/4$ , we have

$$A_v \approx -\frac{g_{m1}}{g_{m2}} \quad (3.38)$$

$$= -\sqrt{\frac{4\mu_n(W/L)_1}{\mu_p(W/L)_2}}. \quad (3.39)$$

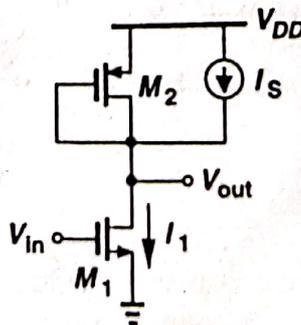


Figure 3.13

Moreover,

$$\mu_n \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH1})^2 \approx 4\mu_p \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH2})^2, \quad (3.40)$$

yielding

$$\frac{|V_{GS2} - V_{TH2}|}{V_{GS1} - V_{TH1}} \approx \frac{A_v}{4}. \quad (3.41)$$

Thus, for a gain of 10, the overdrive of  $M_2$  need be only 2.5 times that of  $M_1$ . Alternatively, for a given overdrive voltage, this circuit achieves a gain four times that of the stage in Fig. 3.12. Intuitively, this is because for a given  $|V_{GS2} - V_{TH2}|$ , if the current decreases by a factor of 4, then  $(W/L)_2$  must decrease proportionally, and  $g_{m2} = \sqrt{2\mu_p C_{ox}(W/L)_2 I_{D2}}$  is lowered by the same factor.

We should also mention that in today's CMOS technology, channel-length modulation is quite significant and, more importantly, the behavior of transistors notably departs from the square law (Chapter 16). Thus, the gain of the stage in Fig. 3.9 must be expressed as

$$A_v = -g_{m1} \left( \frac{1}{g_{m2}} \parallel r_{O1} \parallel r_{O2} \right), \quad (3.42)$$

where  $g_{m1}$  and  $g_{m2}$  must be obtained as described in Chapter 16.

### 3.2.3 CS Stage with Current-Source Load

In applications requiring a large voltage gain in a single stage, the relationship  $A_v = -g_m R_D$  suggests that we increase the load impedance of the CS stage. With a resistor or diode-connected load, however, increasing the load resistance limits the output voltage swing.

A more practical approach is to replace the load with a current source. Described briefly in Example 3.2, the resulting circuit is shown in Fig. 3.14, where both transistors operate in saturation. Since the total impedance seen at the output node is equal to  $r_{O1} \parallel r_{O2}$ , the gain is

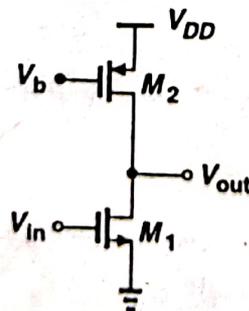


Figure 3.14 CS stage with current-source load.

$$A_v = -g_{m1}(r_{O1} \parallel r_{O2}). \quad (3.43)$$

The key point here is that the output impedance and the minimum required  $|V_{DS}|$  of  $M_2$  are less strongly coupled than the value and voltage drop of a resistor. The voltage

$|V_{DS2,min}| = |V_{GS2} - V_{TH2}|$  can be reduced to even a few hundred millivolts by simply increasing the width of  $M_2$ . If  $r_{O2}$  is not sufficiently high, the length and width of  $M_2$  can be increased to achieve a smaller  $\lambda$  while maintaining the same overdrive voltage. The penalty is the large capacitance introduced by  $M_2$  at the output node.

We should remark that the output bias voltage of the circuit in Fig. 3.14 is not well-defined. Thus, the stage is reliably biased only if a feedback loop forces  $V_{out}$  to a known value (Chapter 8). The large-signal analysis of the circuit is left as an exercise for the reader.

As explained in Chapter 2, the output impedance of MOSFETs at a given drain current can be scaled by changing the channel length, i.e., to the first order,  $\lambda \propto 1/L$  and hence  $r_O \propto L/I_D$ . Since the gain of the stage shown in Fig. 3.14 is proportional to  $r_{O1} \parallel r_{O2}$ , we may surmise that longer transistors yield a higher voltage gain.

Let us consider  $M_1$  and  $M_2$  separately. If  $L_1$  is scaled by a factor  $\alpha (> 1)$ , then  $W_1$  may need to be scaled proportionally as well. This is because, for a given drain current,  $V_{GS1} - V_{TH1} \propto 1/\sqrt{(W/L)_1}$ , i.e., if  $W_1$  is not scaled, the overdrive voltage increases, limiting the output voltage swing. Also, since  $g_{m1} \propto \sqrt{(W/L)_1}$ , scaling up only  $L_1$  lowers  $g_{m1}$ .

In applications where these issues are unimportant,  $W_1$  can remain constant while  $L_1$  increases. Thus, the intrinsic gain of the transistor can be written as

$$g_{m1}r_{O1} = \sqrt{2 \left(\frac{W}{L}\right)_1 \mu_n C_{ox} I_D \frac{1}{\lambda I_D}}, \quad (3.44)$$

indicating that the gain *increases* with  $L$  because  $\lambda$  depends more strongly on  $L$  than  $g_m$  does. Also, note that  $g_m r_O$  *decreases* as  $I_D$  increases.

Increasing  $L_2$  while keeping  $W_2$  constant increases  $r_{O2}$  and hence the voltage gain, but at the cost of higher  $|V_{DS2}|$  required to maintain  $M_2$  in saturation.

### 3.2.4 CS Stage with Triode Load

A MOS device operating in deep triode region behaves as a resistor and can therefore serve as the load in a CS stage. Illustrated in Fig. 3.15, such a circuit biases the gate of  $M_2$  at a sufficiently low level, ensuring the load is in deep triode region for all output voltage swings.

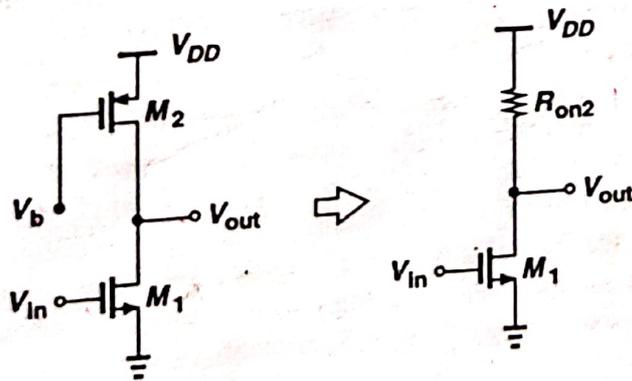


Figure 3.15 CS stage with triode load.

Since

$$R_{on2} = \frac{1}{\mu_p C_{ox} (W/L)_2 (V_{DD} - V_b - |V_{THP}|)} \quad (3.45)$$

the voltage gain can be readily calculated.

The principal drawback of this circuit stems from the dependence of  $R_{on2}$  upon  $\mu_p C_{ox}$ ,  $V_b$ , and  $V_{THP}$ . Since  $\mu_p C_{ox}$  and  $V_{THP}$  vary with process and temperature and since generating a precise value for  $V_b$  requires additional complexity, this circuit is difficult to use. Triode loads, however, consume less voltage headroom than do diode-connected devices because in Fig. 3.15  $V_{out,max} = V_{DD}$  whereas in Fig. 3.12,  $V_{out,max} \approx V_{DD} - |V_{THP}|$ .

### 3.2.5 CS Stage with Source Degeneration

In some applications, the square-law dependence of the drain current upon the overdrive voltage introduces excessive nonlinearity, making it desirable to “soften” the device characteristic. In Section 3.2.2, we noted the linear behavior of a CS stage using a diode-connected load. Alternatively, as depicted in Fig. 3.16, this can be accomplished by placing a “degeneration” resistor in series with the source terminal. Here, as  $V_{in}$  increases, so do  $I_D$  and the

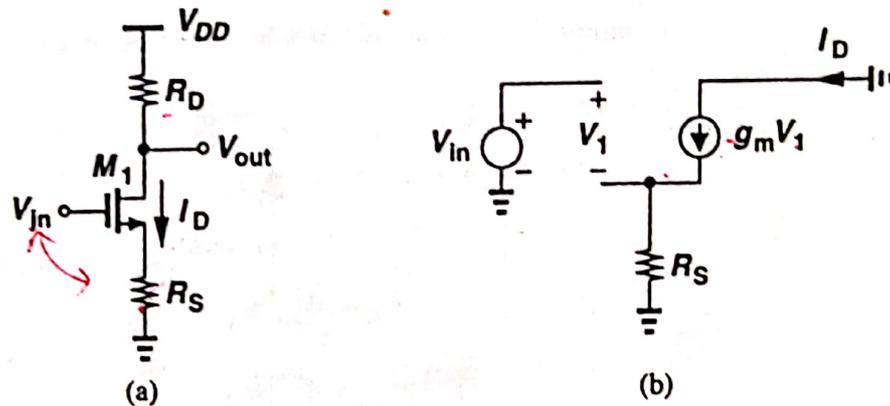


Figure 3.16 CS stage with source degeneration.

voltage drop across  $R_S$ . That is, a fraction of  $V_{in}$  appears across the resistor rather than as the gate-source overdrive, thus leading to a smoother variation of  $I_D$ . From another perspective, we intend to make the gain equation a weaker function of  $g_m$ . Since  $V_{out} = -I_D R_D$ , the nonlinearity of the circuit arises from the nonlinear dependence of  $I_D$  upon  $V_{in}$ . We note that  $\partial V_{out} / \partial V_{in} = -(\partial I_D / \partial V_{in}) R_D$ , and define the equivalent transconductance of the circuit as  $G_m = \partial I_D / \partial V_{in}$ . Now, assuming  $I_D = f(V_{GS})$ , we write

$$G_m = \frac{\partial I_D}{\partial V_{in}} \quad (3.46)$$

$$= \frac{\partial f}{\partial V_{GS}} \frac{\partial V_{GS}}{\partial V_{in}} \quad (3.47)$$

Since  $V_{GS} = V_{in} - I_D R_S$ , we have  $\partial V_{GS} / \partial V_{in} = 1 - R_S \partial I_D / \partial V_{in}$ , obtaining

$$G_m = \left( 1 - R_S \frac{\partial I_D}{\partial V_{in}} \right) \frac{\partial f}{\partial V_{GS}}. \quad (3.48)$$

But,  $\partial f / \partial V_{GS}$  is the transconductance of  $M_1$ , and

$$G_m = \frac{g_m}{1 + g_m R_S}. \quad (3.49)$$

The small-signal voltage gain is thus equal to

$$A_v = -G_m R_D \quad (3.50)$$

$$= \frac{-g_m R_D}{1 + g_m R_S}. \quad (3.51)$$

The same result can be derived using the small-signal model of Fig. 3.16(b). Equation (3.49) implies that as  $R_S$  increases,  $G_m$  becomes a weaker function of  $g_m$  and hence the drain current. In fact, for  $R_S \gg 1/g_m$ , we have  $G_m \approx 1/R_S$ , i.e.,  $\Delta I_D \approx \Delta V_{in} / R_S$ , indicating that most of the change in  $V_{in}$  appears across  $R_S$ . We say the drain current is a "linearized" function of the input voltage. The linearization is obtained at the cost of lower gain [and higher noise (Chapter 7)].

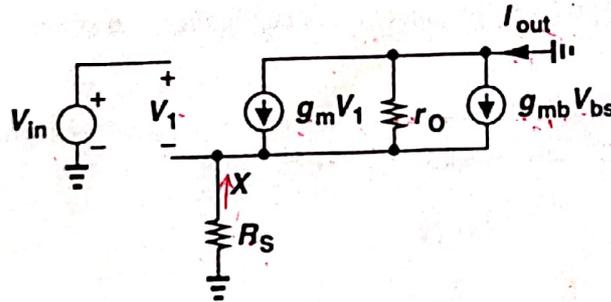


Figure 3.17 Small-signal equivalent circuit of a degenerated CS stage.

For our subsequent calculations, it is useful to determine  $G_m$  in the presence of body effect and channel-length modulation. With the aid of the equivalent circuit shown in Fig. 3.17, we recognize that the current through  $R_S$  equals  $I_{out}$  and, therefore,  $V_{in} = V_1 + I_{out} R_S$ . Summing the currents at node  $X$ , we have

$$I_{out} = g_m V_1 - g_{mb} V_X - \frac{I_{out} R_S}{r_o} \quad (3.52)$$

$$= g_m (V_{in} - I_{out} R_S) + g_{mb} (-I_{out} R_S) - \frac{I_{out} R_S}{r_o}. \quad (3.53)$$

It follows that

$$G_m = \frac{I_{out}}{V_{in}} \quad (3.54)$$

$$= \frac{g_m r_o}{R_S + [1 + (g_m + g_{mb}) R_S] r_o}. \quad (3.55)$$

## 3.3 Source Follower

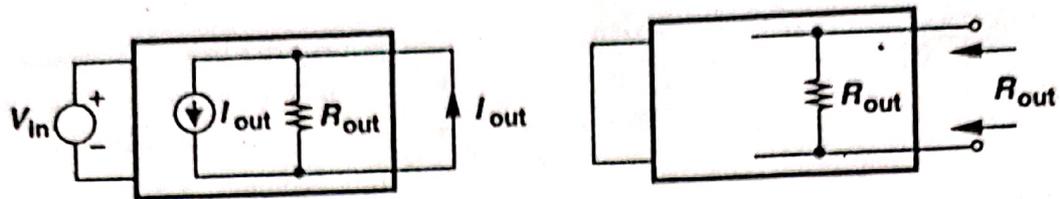


Figure 3.25 Modeling output port of an amplifier by a Norton equivalent.

Defining  $G_m = I_{out}/V_{in}$ , we have  $V_{out} = -G_m V_{in} R_{out}$ . This lemma proves useful if  $G_m$  and  $R_{out}$  can be determined by inspection.

### Example 3.6

Calculate the voltage gain of the circuit shown in Fig. 3.26. Assume  $I_0$  is ideal.

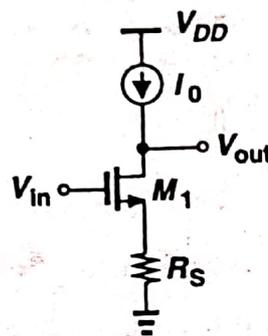


Figure 3.26

### Solution

The transconductance and output resistance of the stage are given by Eqs. (3.55) and (3.60), respectively. Thus,

$$A_v = -\frac{g_m r_o}{R_S + [1 + (g_m + g_{mb})R_S]r_o} \{ [1 + (g_m + g_{mb})r_o]R_S + r_o \} \quad (3.74)$$

$$= -g_m r_o. \quad (3.75)$$

Interestingly, the voltage gain is equal to the intrinsic gain of the transistor and independent of  $R_S$ . This is because, if  $I_0$  is ideal, the current through  $R_S$  cannot change and hence the small-signal voltage drop across  $R_S$  is zero—as if  $R_S$  were zero itself.

### 3 Source Follower

Our analysis of the common-source stage indicates that, to achieve a high voltage gain with limited supply voltage, the load impedance must be as large as possible. If such a stage is to drive a low-impedance load, then a “buffer” must be placed after the amplifier so as to drive the load with negligible loss of the signal level. The source follower (also called the “common-drain” stage) can operate as a voltage buffer.

Illustrated in Fig. 3.27(a), the source follower senses the signal at the gate and drives

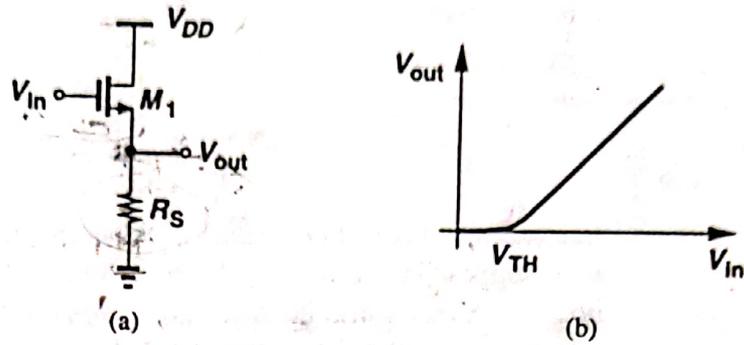


Figure 3.27 (a) Source follower, and (b) its input-output characteristic.

the load at the source, allowing the source potential to “follow” the gate voltage. Beginning with the large-signal behavior, we note that for  $V_{in} < V_{TH}$ ,  $M_1$  is off and  $V_{out} = 0$ . As  $V_{in}$  exceeds  $V_{TH}$ ,  $M_1$  turns on in saturation (for typical values of  $V_{DD}$ ) and  $I_{D1}$  flows through  $R_S$  [Fig. 3.27(b)]. As  $V_{in}$  increases further,  $V_{out}$  follows the input with a difference (level shift) equal to  $V_{GS}$ . We can express the input-output characteristic as:

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out})^2 R_S = V_{out} \quad (3.76)$$

Let us calculate the small-signal gain of the circuit by differentiating both sides of (3.76) with respect to  $V_{in}$ :

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2(V_{in} - V_{TH} - V_{out}) \left( 1 - \frac{\partial V_{TH}}{\partial V_{in}} - \frac{\partial V_{out}}{\partial V_{in}} \right) R_S = \frac{\partial V_{out}}{\partial V_{in}} \quad (3.77)$$

Since  $\partial V_{TH} / \partial V_{in} = \eta \partial V_{out} / \partial V_{in}$ ,

$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out}) R_S}{1 + \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out}) R_S (1 + \eta)} \quad (3.78)$$

Also, note that

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out}) \quad (3.79)$$

Consequently,

$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S} \quad (3.80)$$

The same result is more easily obtained with the aid of a small-signal equivalent circuit. From Fig. 3.28, we have  $V_{in} - V_1 = V_{out}$ ,  $V_{bs} = -V_{out}$ , and  $g_m V_1 - g_{mb} V_{out} = V_{out} / R_S$ .

3.3 Source Follower

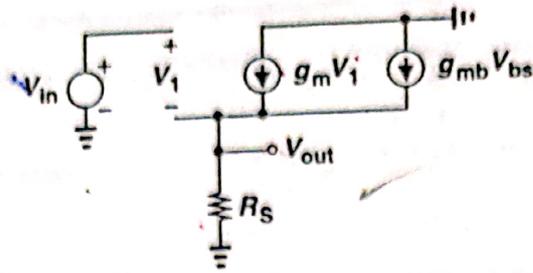


Figure 3.28 Small-signal equivalent circuit of source follower.

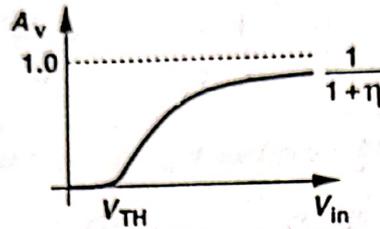


Figure 3.29 Voltage gain of source follower versus input voltage.

Thus,  $V_{out}/V_{in} = g_m R_S / [1 + (g_m + g_{mb}) R_S]$ .

Sketched in Fig. 3.29 vs.  $V_{in}$ , the voltage gain begins from zero for  $V_{in} \approx V_{TH}$  (that is,  $g_m \approx 0$ ) and monotonically increases. As the drain current and  $g_m$  increase,  $A_v$  approaches  $g_m / (g_m + g_{mb}) = 1 / (1 + \eta)$ . Since  $\eta$  itself slowly decreases with  $V_{out}$ ,  $A_v$  would eventually become equal to unity, but for typical allowable source-bulk voltages,  $\eta$  remains greater than roughly 0.2.

An important result of (3.80) is that even if  $R_S = \infty$ , the voltage gain of a source follower is not equal to one. We return to this point later. Note that  $M_1$  in Fig. 3.27 does not enter the triode region if  $V_{in}$  remains below  $V_{DD}$ .

In the source follower of Fig. 3.27, the drain current of  $M_1$  heavily depends on the input dc level. For example, if  $V_{in}$  changes from 1.5 V to 2 V,  $I_D$  may increase by a factor of 2 and hence  $V_{GS} - V_{TH}$  by  $\sqrt{2}$ , thereby introducing substantial nonlinearity in the input-output characteristic. To alleviate this issue, the resistor can be replaced by a current source as shown in Fig. 3.30(a). The current source itself is implemented as an NMOS transistor operating in the saturation region [Fig. 3.30(b)].

*Handwritten notes:*  
 $V_{out} = V_{DD} - I_D R_S$   
 $\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH} - V_{out})^2 R_S = V_{out}$

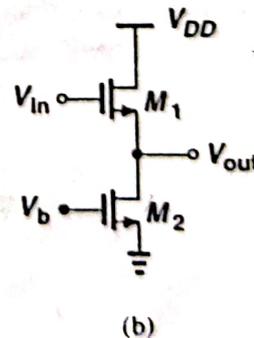
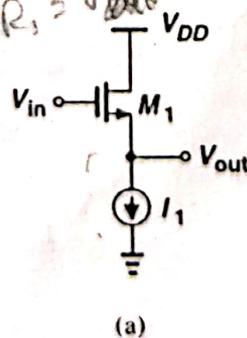


Figure 3.30 Source follower using an NMOS transistor as current source.

**Example 3.7**

Suppose in the source follower of Fig. 3.30(a),  $(W/L)_1 = 20/0.5$ ,  $I_1 = 200 \mu\text{A}$ ,  $V_{TH0} = 0.6 \text{ V}$ ,  $2\Phi_F = 0.7 \text{ V}$ ,  $\mu_n C_{ox} = 50 \mu\text{A/V}^2$ , and  $\gamma = 0.4 \text{ V}^2$ .

(a) Calculate  $V_{out}$  for  $V_{in} = 1.2 \text{ V}$ .

(b) If  $I_1$  is implemented as  $M_2$  in Fig. 3.30(b), find the minimum value of  $(W/L)_2$  for which  $M_2$  remains saturated.

**Solution**

(a) Since the threshold voltage of  $M_1$  depends on  $V_{out}$ , we perform a simple iteration. Noting that

$$(V_{in} - V_{TH} - V_{out})^2 = \frac{2I_D}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1} \quad (3.81)$$

we first assume  $V_{TH} \approx 0.6 \text{ V}$ , obtaining  $V_{out} = 0.153 \text{ V}$ . Now we calculate a new  $V_{TH}$  as

$$V_{TH} = V_{TH0} + \gamma(\sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F}) \quad (3.82)$$

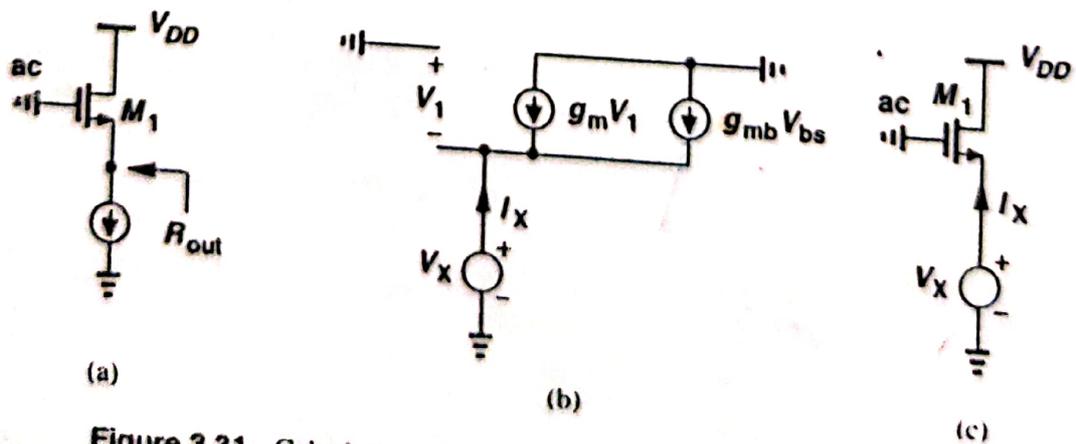
$$= 0.635 \text{ V.} \quad (3.83)$$

This indicates that  $V_{out}$  is approximately 35 mV less than that calculated above, i.e.,  $V_{out} \approx 0.119 \text{ V}$ .

(b) Since the drain-source voltage of  $M_2$  is equal to 0.119 V, the device is saturated only if  $(V_{GS} - V_{TH})_2 \leq 0.119 \text{ V}$ . With  $I_D = 200 \mu\text{A}$ , this gives  $(W/L)_2 \geq 283/0.5$ . Note the substantial drain junction and overlap capacitance contributed by  $M_2$  to the output node.

To gain a better understanding of source followers, let us calculate the small-signal output resistance of the circuit in Fig. 3.31(a). Using the equivalent circuit of Fig. 3.31(b) and noting that  $V_1 = -V_X$ , we write

$$I_X - g_m V_X - g_{mb} V_X = 0. \quad (3.84)$$



**Figure 3.31** Calculation of the output impedance of a source follower.

As explained in Chapter 7, source followers also introduce substantial noise. For this reason, the circuit of Fig. 3.39(b) is ill-suited to low-noise applications.

### 3.4 Common-Gate Stage

In common-source amplifiers and source followers, the input signal is applied to the gate of a MOSFET. It is also possible to apply the signal to the source terminal. Shown in Fig. 3.40(a), a common-gate (CG) stage senses the input at the source and produces the output at the drain. The gate is connected to a dc voltage to establish proper operating conditions. Note that the bias current of  $M_1$  flows through the input signal source. Alternatively, as depicted in Fig. 3.40(b),  $M_1$  can be biased by a constant current source, with the signal capacitively coupled to the circuit.

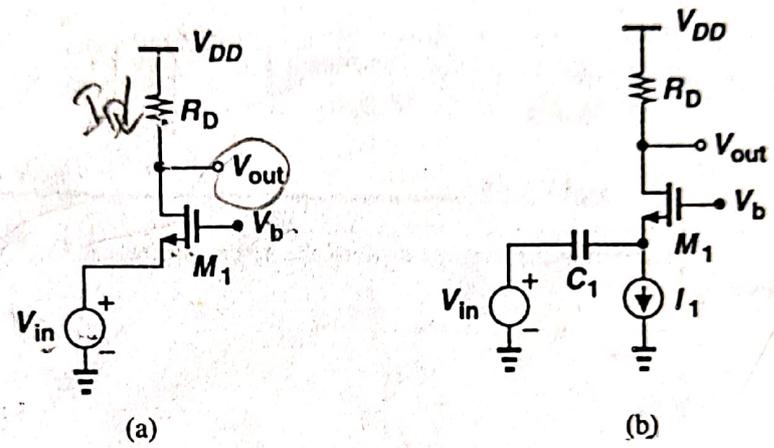


Figure 3.40 (a) Common-gate stage with direct coupling at input, (b) CG stage with capacitive coupling at input.

We first study the large-signal behavior of the circuit in Fig. 3.40(a). For simplicity, let us assume that  $V_{in}$  decreases from a large positive value. For  $V_{in} \geq V_b - V_{TH}$ ,  $M_1$  is off and  $V_{out} = V_{DD}$ . For lower values of  $V_{in}$ , we can write

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2, \tag{3.95}$$

if  $M_1$  is in saturation. As  $V_{in}$  decreases, so does  $V_{out}$ , eventually driving  $M_1$  into the triode region if

$$V_{DD} - I_D R_D = V_b - V_{TH}$$

$$V_{DD} - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2 R_D = V_b - V_{TH}. \tag{3.96}$$

The input-output characteristic is shown in Fig. 3.41. If  $M_1$  is saturated, we can express the output voltage as

$$V_{out} = V_{DD} - I_D R_D = V_{DD} - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2 R_D. \tag{3.97}$$

$$\frac{dV_{out}}{dV_{in}} = 0 - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot 2 (V_b - V_{in} - V_{TH}) R_D \left( -1 - \frac{\partial V_{TH}}{\partial V_{in}} \right)$$

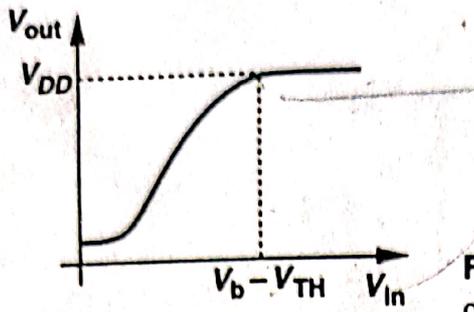


Figure 3.41 Common-gate input-output characteristic.

obtaining a small-signal gain of

$$\frac{\partial V_{out}}{\partial V_{in}} = \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH}) \left( -1 - \frac{\partial V_{TH}}{\partial V_{in}} \right) R_D \quad (3.98)$$

Since  $\partial V_{TH} / \partial V_{in} = \partial V_{TH} / \partial V_{SB} = \eta$ , we have

$$\frac{\partial V_{out}}{\partial V_{in}} = \mu_n C_{ox} \frac{W}{L} R_D (V_b - V_{in} - V_{TH}) (1 + \eta) \quad (3.99)$$

$$A_v = g_m (1 + \eta) R_D \quad g_m = \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH}) \quad (3.100)$$

Note that the gain is positive. Interestingly, body effect increases the equivalent transconductance of the stage.

The input impedance of the circuit is also important. We note that, for  $\lambda = 0$ , the impedance seen at the source of  $M_1$  in Fig. 3.40(a) is the same as that at the source of  $M_1$  in Fig. 3.31, namely,  $1/(g_m + g_{mb}) = 1/[g_m(1 + \eta)]$ . Thus, the body effect decreases the input impedance of the common-gate stage. The relatively low input impedance of the common-gate stage proves useful in some applications.

**Example 3.10**

In Fig. 3.42, transistor  $M_1$  senses  $\Delta V$  and delivers a proportional current to a 50- $\Omega$  transmission line. The other end of the line is terminated by a 50- $\Omega$  resistor in Fig. 3.42(a) and a common-gate stage in Fig. 3.42(b). Assume  $\lambda = \gamma = 0$ .

- (a) Calculate  $V_{out} / V_{in}$  at low frequencies for both arrangements.
- (b) What condition is necessary to minimize wave reflection at node X?

**Solution**

(a) For small signals applied to the gate of  $M_1$ , the drain current experiences a change equal to  $g_{m1} \Delta V_X$ . This current is drawn from  $R_D$  in Fig. 3.42(a) and  $M_2$  in Fig. 3.42(b), producing an output voltage swing equal to  $-g_{m1} \Delta V_X R_D$ . Thus,  $A_v = -g_m R_D$  for both cases.

(b) To minimize reflection at node X, the resistance seen at the source of  $M_2$  must equal 50  $\Omega$  and the reactance must be small. Thus,  $1/(g_m + g_{mb}) = 50 \Omega$ , which can be ensured by proper sizing and biasing of  $M_2$ . To minimize the capacitances of the transistor, it is desirable to use a small device biased at a large current. (Recall that  $g_m = \sqrt{2\mu_n C_{ox} (W/L) I_D}$ .) In addition to higher power dissipation, this remedy also requires a large  $V_{GS}$  for  $M_2$ .

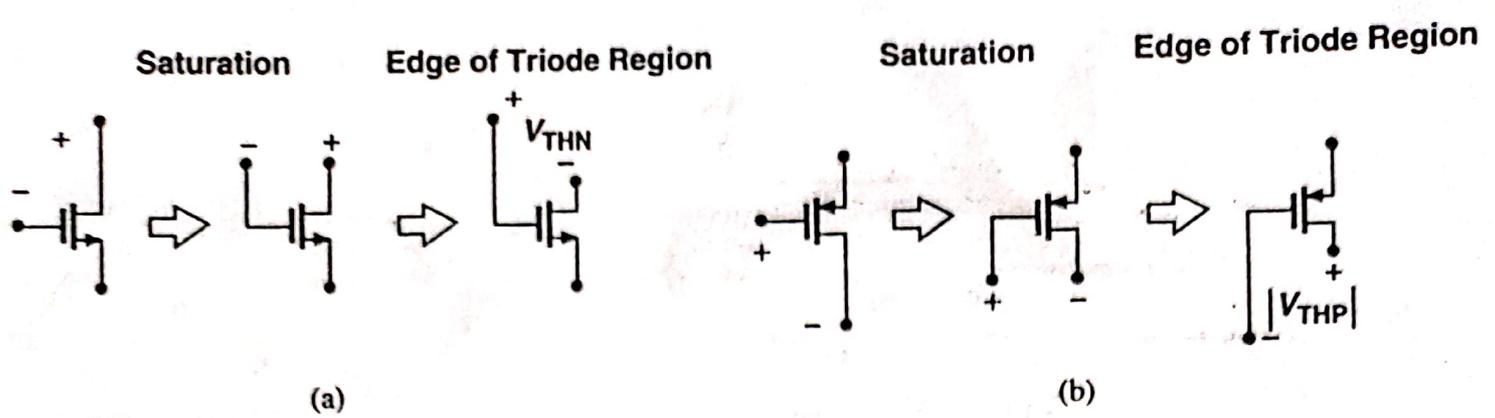


Figure 2.20 Conceptual visualization of saturation and triode regions.

if  $V_D - V_G$  of a PFET is not large enough ( $< |V_{THP}|$ ), the device is saturated. Note that this view does not require knowledge of the source voltage. This means we must know a priori which terminal operates as the drain.

## Second-Order Effects

Our analysis of the MOS structure has thus far entailed various simplifying assumptions, some of which are not valid in many analog circuits. In this section, we describe three second-order effects that are essential in our subsequent circuit analyses. Other phenomena that appear in submicron devices are studied in Chapter 16.

**Body Effect** In the analysis of Fig. 2.10, we tacitly assumed that the bulk and the source of the transistor were tied to ground. What happens if the bulk voltage of an NFET drops below the source voltage (Fig. 2.21)? Since the S and D junctions remain reverse-biased, we surmise that the device continues to operate properly but certain characteristics may

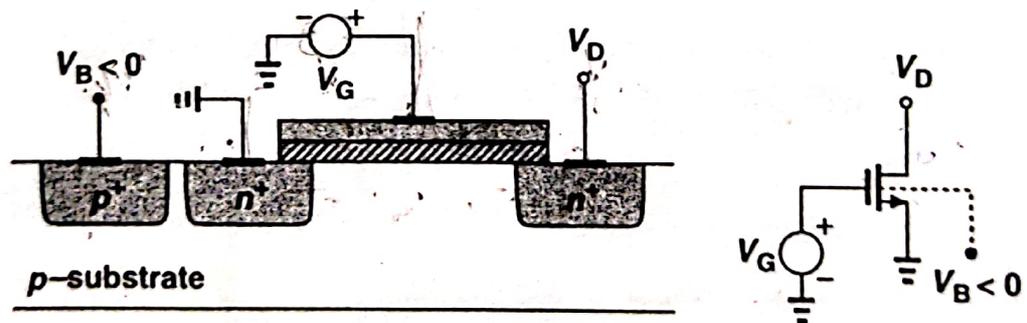


Figure 2.21 NMOS device with negative bulk voltage.

change. To understand the effect, suppose  $V_S = V_D = 0$ , and  $V_G$  is somewhat less than  $V_{TH}$  so that a depletion region is formed under the gate but no inversion layer exists. As  $V_B$  becomes more negative, more holes are attracted to the substrate connection, leaving a larger negative charge behind, i.e., as depicted in Fig. 2.22, the depletion region becomes wider. Now recall from Eq. (2.1) that the threshold voltage is a function of the total charge in the depletion region because the gate charge must mirror  $Q_d$  before an inversion layer is

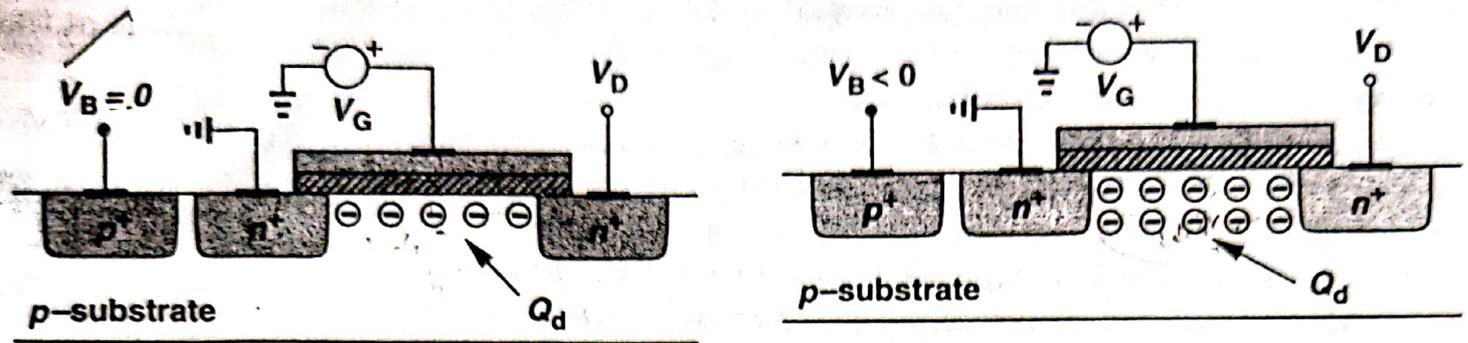


Figure 2.22 Variation of depletion region charge with bulk voltage.

formed. Thus, as  $V_B$  drops and  $Q_d$  increases,  $V_{TH}$  also increases. This is called the “body effect” or the “backgate effect.”

It can be proved that with body effect:

$$V_{TH} = V_{TH0} + \gamma \left( \sqrt{|2\Phi_F + V_{SB}|} - \sqrt{|2\Phi_F|} \right), \quad (2.22)$$

where  $V_{TH0}$  is given by (2.1),  $\gamma = \sqrt{2q\epsilon_{si}N_{sub}}/C_{ox}$  denotes the body effect coefficient, and  $V_{SB}$  is the source-bulk potential difference [1]. The value of  $\gamma$  typically lies in the range of 0.3 to 0.4  $V^{1/2}$ .

### Example 2.3

In Fig. 2.23(a), plot the drain current if  $V_X$  varies from  $-\infty$  to 0. Assume  $V_{TH0} = 0.6$  V,  $\gamma = 0.4$   $V^{1/2}$ , and  $2\Phi_F = 0.7$  V.

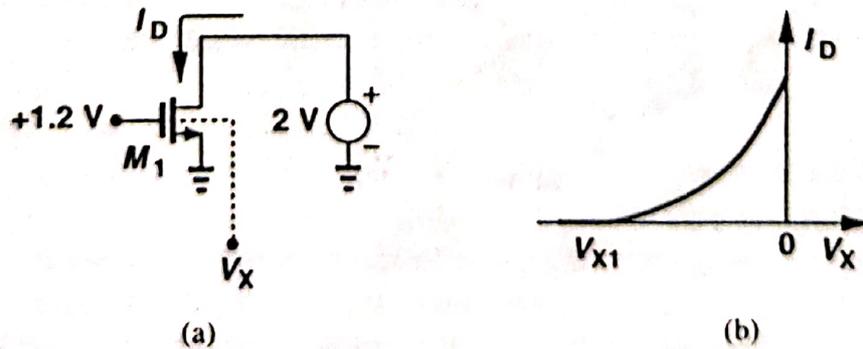


Figure 2.23

### Solution

If  $V_X$  is sufficiently negative, the threshold voltage of  $M_1$  exceeds 1.2 V and the device is off. That is,

$$1.2 \text{ V} = 0.6 + 0.4 \left( \sqrt{0.7 - V_{X1}} - \sqrt{0.7} \right), \quad (2.23)$$

## 6.1 Introduction

The design considerations for a simple inverter circuit were presented in the previous chapter. We now extend this discussion to address the synthesis of arbitrary digital gates, such as NOR, NAND, and XOR. The focus is on *combinational logic* or *nonregenerative* circuits—that is, circuits having the property that at any point in time, the output of the circuit is related to its current input signals by some Boolean expression (assuming that the transients through the logic gates have settled). No intentional connection from outputs back to inputs is present.

This is in contrast to another class of circuits, known as *sequential* or *regenerative*, for which the output is not only a function of the current input data, but also of previous values of the input signals (see Figure 6-1). This can be accomplished by connecting one or more outputs intentionally back to some inputs. Consequently, the circuit “remembers” past events and has a sense of *history*. A sequential circuit includes a combinational logic portion and a module that holds the state. Example circuits are registers, counters, oscillators, and memory. Sequential circuits are the topic of the next chapter.

There are numerous circuit styles to implement a given logic function. As with the inverter, the common design metrics by which a gate is evaluated are area, speed, energy, and power. Depending on the application, the emphasis will be on different metrics. For example, the switching speed of digital circuits is the primary metric in a high-performance processor, while in a battery operated circuit, it is energy dissipation. Recently, power dissipation also has become an important concern and considerable emphasis is placed on understanding the sources of power and approaches to dealing with power. In addition to these metrics, robustness to noise and reliability are also very important considerations. We will see that certain logic styles can significantly improve performance, but they usually are more sensitive to noise.

## 6.2 Static CMOS Design

The most widely used logic style is static complementary CMOS. The static CMOS style is really an extension of the static CMOS inverter to multiple inputs. To review, the primary advantage of the CMOS structure is robustness (i.e., low sensitivity to noise), good performance, and low power consumption with no static power dissipation. Most of those properties are carried over to large fan-in logic gates implemented using a similar circuit topology.

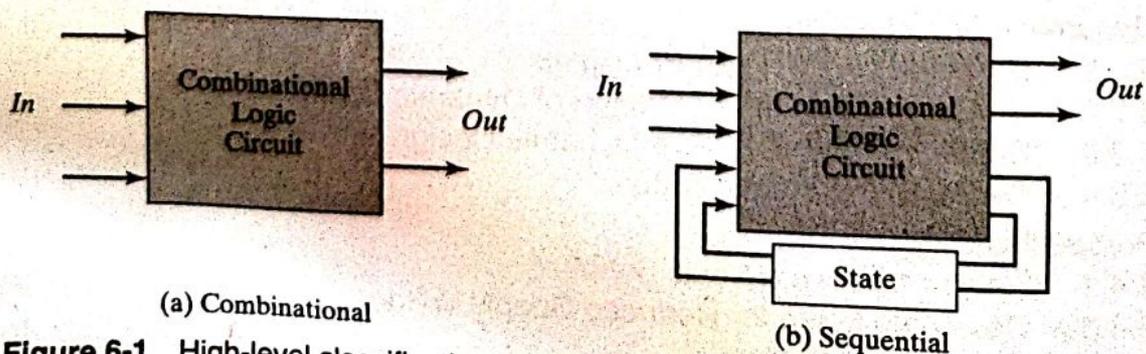


Figure 6-1 High-level classification of logic circuits.