MICROWAVE TRANSMISSION LINES

INTRODUCTION :

Microwaves are electro magnetic waves whose frequencies range from 1GHZ to 1000GHZ. For comparision, the signal from an AM Radio station is 1MHz and the signal from FM radio station is 100MHZ.

Microwaves are so called since they are defined interms of their wavelength in the sense that micro refers to tinyness.

Tinyness referring to the wave length and the period of a cycle of a cm wave. In other words, the wave length (λ) of cm waves at microwave frequencies are very short; typically from a few tens of cm to a fraction of a mm.

In Short, a microwave is a signal that has a wavelength of 1foot or less i.e., $\lambda \leq 30.5$ cms $\cong$ 1 foot. This converts to a frequency of 984 MHZ, approximately 1GHZ.

So, all frequencies above approximately 1000MHZ to about 1000 GHZ are micro wave frequencies.

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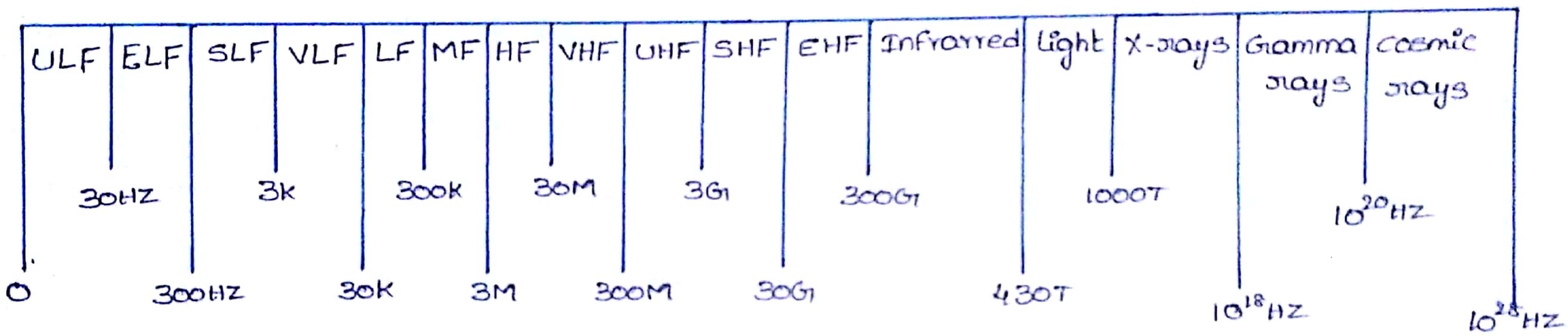
Electro Magnetic Frequency spectrum :



ULF - ultra low frequency
ELF - Extra low frequency
SLF - Super low frequency
VLF - Very low frequency
LF - Low frequency
MF - Medium frequency
HF - High Frequency
VHF - Very high Frequency

UHF - ultra High Frequency
SHF - Super High Frequency
EHF - Extreme high Frequency

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Microwave frequency bands:

Designation	Frequency range in Gigahertz
HF	0.003 - 0.03
VHF	0.03 - 0.3
UHF	0.3 - 1.0
L band	1 - 2
S	2 - 4
C	4 - 8
X	8 - 12
Ku	12 - 18
K	18 - 27
Ka	27 - 40
Millimeter	40 - 300
Submillimeter	> 300

Advantages of Microwaves:

1. **Increased bandwidth availability:**

Microwaves have large bandwidths ($1GHz - 10^3 GHz$) As the bandwidth increases the number of incoming signals also increases. The advantage of large bandwidths is that the frequency range of information channels will be a small percentage of the carrier frequency and more information can be transmitted in microwave frequency ranges.

Bandwidth provides more room for stuff to be packed into the transmission.

2. **Improved directive properties :**

As frequency increases, directivity increases and beamwidth decreases. Hence beamwidth of radiation 'θ' is proportional to λ/D.

At low frequency bands, the size of antenna becomes very large if it is required to get sharp beam of radiation.

At micro frequencies, antenna size of several wavelengths lead to smaller beam widths and an extremely directed beam.

For parabolic antenna,

$$B = \frac{140^\circ}{D/\lambda}$$

At 30GHZ (λ=1cm) for 1° beam width

$$D = \frac{140}{B} \times \lambda \Rightarrow \frac{140}{1} \times 1 \Rightarrow D = 140cm$$

At 300MHZ (λ = 100cm) for 1° beam width

$$D = \frac{140}{B} \times \lambda \Rightarrow \frac{140}{1} \times 100 \Rightarrow D = 140m$$

At micro wave frequencies, the size of antenna is small. As gain is inversely proportional to λ^2, high gain is achievable at microwave frequencies.

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3. Fading effect and reliability :

Fading effect is more at low frequencies and less at high frequencies due to line of sight [so LOS]

i.e., Microwave communication is realiable communication.

4. power requirement :

Transmitter/Receiver power requirements are pretty low at Microwave frequencies compared to that short wave band.

5. Transparency property of Microwaves:

Microwave frequencies upto 10GHZ are capable of freely propagating through the ionized layers surrounding the earth as well as through the atmosphere. The presence of such a transparent window in a microwave band facilitates the study of microwave radiation from the sun and stars in radio astronomical research of space.

Applications of microwaves :

1. Telecommunication :

→ Intercontinental Telephone

→ space communication (earth to space and space to earth)

→ Telemetry communication link for railways

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2. **Radars :**

→ Aircraft Detection

→ Track / Guide supersonic missiles

→ Air traffic control and Burglar alarms

3. **Commercial and industrial applications:**

→ Microwave oven (2.45 GHz)

→ Drying machines - textile, food and paper industry

→ Food processing industry

→ Drying pharmaceuticals

→ Biomedical applications.

4. **Electronic warfare :**

→ Electronic counter measure systems

→ spread Spectrum systems.

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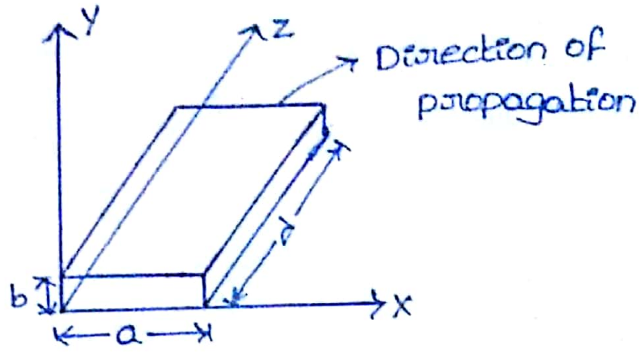
propagation of waves in Rectangular wave guide :



Consider, a rectangular waveguide in which the breadth is along x-direction and width is in the y-direction and then the propagation is in z-direction

The TE and TM wave equations are given as

$$\Delta^2 H_z = -\omega^2 \mu \varepsilon H_z \quad \text{for TE wave } [E_z = 0]$$

$$\Delta^2 E_z = -\omega^2 \mu \varepsilon E_z \quad \text{for TM wave } [H_z = 0]$$

For TE wave,

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \varepsilon H_z$$

consider,

$$\frac{\partial}{\partial z} = -\gamma$$

$$\frac{\partial^2}{\partial z^2} = \gamma^2$$

$$\frac{\partial^2}{\partial x^2} H_z + \frac{\partial^2}{\partial y^2} H_z + \gamma^2 H_z = -\omega^2 \mu \varepsilon H_z$$

$$\frac{\partial^2}{\partial x^2} H_z + \frac{\partial^2}{\partial y^2} H_z + [\gamma^2 + \omega^2 \mu \varepsilon] H_z = 0$$

Let, $[\gamma^2 + \omega^2 \mu \varepsilon] = h^2$ then

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \cancel{\frac{\partial^2 H_z}{}} h^2 H_z = 0 \longrightarrow (1)$$

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similarly TM wave,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \longrightarrow (2)$$

using above two partial differential equations we can evaluate E_z and H_z only.

To determine E_x, E_y and H_x, H_y we need to use maxwell first two equations

From 1st maxwell equation,-

$$\nabla \times H = j\omega\varepsilon E$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right]$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial} - \gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left[\hat{i}E_x + \hat{j}E_y + \hat{k}E_z\right]$$

$$\hat{i}\left[\frac{\partial H_z}{\partial y} + \gamma H_y\right] - \hat{j}\left[\frac{\partial H_z}{\partial x} + \gamma H_x\right] + \hat{k}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] = \hat{i}\, j\omega\varepsilon E_x + \hat{j}\, j\omega\varepsilon E_y + \hat{k}\, j\omega\varepsilon E_z$$

Equate the coefficients of $\hat{i}, \hat{j}, \hat{k}$

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x \longrightarrow (3)$$

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$$\frac{\partial H_z}{\partial x} + \gamma H_x = -j\omega\varepsilon E_y \longrightarrow (4)$$

$$\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \longrightarrow (5)$$

From second maxwell equation,

$$\nabla \times E = -j\omega\mu H$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\left[\hat{i} H_x + \hat{j} H_y + \hat{k} H_z\right]$$

$$\Rightarrow \hat{i}\left[\frac{\partial E_z}{\partial y} + \gamma E_y\right] - \hat{j}\left[\frac{\partial E_z}{\partial x} + \gamma E_x\right] + \hat{k}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right] = -j\omega\mu\left[\hat{i} H_x + \hat{j} H_y + \hat{k} H_z\right]$$

Equate the coefficients of $\hat{i}, \hat{j}, \hat{k}$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \longrightarrow (6)$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = j\omega\mu H_y \longrightarrow (7)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \longrightarrow (8)$$

To calculate E_x, consider eq (3) & eq (7)

From eq (7):

$$H_y = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu} E_x$$

Substitute H_y value in eq (3)

$$\frac{\partial H_z}{\partial y} + \gamma\left[\frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu} E_x\right] = j\omega\varepsilon E_x$$

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$$\Rightarrow \frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\left[\frac{\partial E_z}{\partial x}\right] + \frac{\gamma^2}{j\omega\mu} E_x = j\omega\varepsilon E_x$$

$$\Rightarrow E_x\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right] = \frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow E_x\left[\frac{-\omega^2\mu\varepsilon - \gamma^2}{j\omega\mu}\right] = \frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow -E_x \frac{h^2}{j\omega\mu} = \frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow -E_x = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} + \frac{j\omega\mu}{h^2}\cdot\frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow -E_x = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} + \frac{\gamma\,\partial E_z}{h^2\,\partial x}$$

$$E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} - \frac{\gamma\,\partial E_z}{h^2\,\partial x}$$

Similarly,

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial y}$$

* Why TEM Mode does not exist ??

For transverse electric and magnetic waves [TEM Mode], $E_z = 0$ & $H_z = 0$. If we substitute these values in E_x, E_y, H_x and H_y all field components become zero.

Hence TEM mode does not exist.

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* **Propagation of TM waves in rectangular waveguide**

The magnetic field is always transverse to the direction of propagation is called the Transverse Magnetic or TM wave.

For TM wave, no magnetic line is in direction of propagation

$$E_z \neq 0$$
$$H_z = 0$$

wave equation for TM wave,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0$$

By using the seperation of variable method, we can calculate E_x, E_y, H_x, H_y

Assume $E_z = X \cdot Y$

$X \rightarrow$ pure function of 'x'

$Y \rightarrow$ pure function of 'y'

$$\frac{\partial^2}{\partial x^2} XY + \frac{\partial^2}{\partial y^2} XY + h^2 XY = 0$$

$$Y \frac{\partial^2 X}{\partial x^2} + X \cdot \frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

$$XY\left[\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2\right] = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

Assume,
$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -B^2$$
$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -A^2$$

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$\Rightarrow h^2 = A^2 + B^2$

The solutions of x and y can be obtained by using ordinary second order differential equation

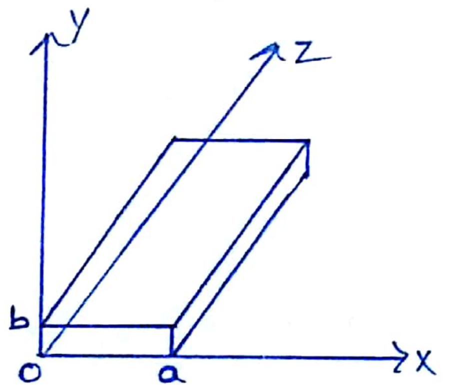
$$X = C_1 \cos BX + C_2 \sin BX$$

$$Y = C_3 \cos AY + C_4 \sin AY$$

where C_1, C_2, C_3, C_4 are constants it can be evaluated using boundary conditions.

$$\therefore E_Z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay] \longrightarrow (1)$$

Boundary conditions :



1st Boundary condition,

$E_z = 0$ at $y = 0$ $\forall$ $x \to 0$ to a

2nd Boundary condition,

$E_z = 0$ at $x = 0$ $\forall$ $y \to 0$ to b

3rd Boundary condition,

$E_z = 0$ at $y = b$ $\forall$ $x \to 0$ to a

4th Boundary condition,

$E_z = 0$ at $x = a$ $\forall$ $y \to 0$ to b

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From 1^{st} Boundary condition :

$$0 = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos A(0) + C_4 \sin A(0)]$$

$$0 = [C_1 \cos Bx + C_2 \sin Bx] C_3$$

$$\therefore C_3 = 0$$

$$[C_1 \cos Bx + C_2 \sin Bx] \neq 0$$

Substitute the C_3 value in eq ①

$$E_z = [C_1 \cos Bx + C_2 \sin Bx] C_4 \sin Ay \longrightarrow ②$$

From 2^{nd} Boundary condition :

$E_z = 0$, $x = 0$ at eq ②

$$E_z = [C_1 \cos B(0) + C_2 \sin B(0)] C_4 \sin Ay$$

$$0 = C_1 [C_4 \sin Ay]$$

$$\therefore C_1 = 0$$

Substitute the C_1 value in eq ②

$$E_z = [C_2 \sin Bx][C_4 \sin Ay] \longrightarrow ③$$

From 3^{rd} Boundary condition :

$E_z = 0$, $y = b$ at eq ③

$$0 = C_2 \sin Bx \; C_4 \sin Ab$$

$$C_4 \sin Ab = 0 \Rightarrow \sin Ab = 0 \Rightarrow Ab = n\pi$$

$$A = \frac{n\pi}{b}$$

$$E_z = [C_2 \sin Bx]\left[C_4 \sin \frac{n\pi y}{b}\right] \longrightarrow ④$$

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From 4th Boundary condition:

$E_z = 0 \, , \; x = a$.

Substitute E_z and x value in eq ④

$$E_z = [C_2 \sin Ba][C_4 \sin(\tfrac{n\pi}{b})y]$$

$$0 = [C_2 \sin Ba][C_4 \sin(\tfrac{n\pi}{b})y]$$

$$C_2 \sin Ba = 0$$

$$\sin Ba = 0$$

$$Ba = m\pi$$

$$E_z = \left[C_2 \sin \frac{m\pi}{a}\right]\left[C_4 . \sin(\tfrac{n\pi}{b})y\right] . e^{-\gamma z} . e^{j\omega t}$$

$e^{-\gamma z}$ represents the direction of propagation of EM waves.

$e^{j\omega t}$ represents sinusoidal variations w.r.t T

Let,

$$C_2 C_4 = C$$

$$E_z = C . \sin\left(\frac{m\pi}{a}\right)x \; \sin\left(\frac{n\pi}{b}\right)y \; . e^{j\omega t - \gamma z}$$

we know that,

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

But for TM wave, $H_z = 0$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x}$$

$$E_x = \frac{-\gamma}{h^2} . \frac{\partial}{\partial x}\left[C . \sin\left(\frac{m\pi}{a}\right)x \; \sin\left(\frac{n\pi}{b}\right)y \; . e^{j\omega t - \gamma z}\right]$$

$$= \frac{-\gamma}{h^2}\left[C . \cos\left(\frac{m\pi}{a}\right)\left(\frac{m\pi}{a}\right)x \; \sin\left(\frac{n\pi}{b}\right)y \; . e^{j\omega t - \gamma z}\right]$$

$$= \frac{-\gamma}{h^2}\left[C . \left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \; \sin\left(\frac{n\pi}{b}\right)y \; . e^{j\omega t - \gamma z}\right]$$

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we know that,

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$H_z = 0$ for TM wave,

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y}$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial}{\partial y}\left[C.\sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\ e^{j\omega t-\gamma z}\right]$$

$$= \frac{-\gamma}{h^2}\left[C.\left(\frac{n\pi}{b}\right)\cos\left(\frac{n\pi}{b}\right)y . \sin\left(\frac{m\pi}{a}\right)x\ e^{j\omega t-\gamma z}\right]$$

we know that,

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

For TM wave, $H_z = 0$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial}{\partial y}\left[C.\sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y . e^{j\omega t-\gamma z}\right]$$

$$= \frac{j\omega\varepsilon}{h^2}\left[C.\left(\frac{n\pi}{b}\right)\cos\left(\frac{n\pi}{b}\right)y . \sin\left(\frac{m\pi}{a}\right)x\ e^{j\omega t-\gamma z}\right]$$

we know that,

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

For TM wave, $H_z = 0$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial}{\partial x}E_z$$

$$= \frac{-j\omega\varepsilon}{h^2}\frac{\partial}{\partial x}\left[C.\sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\ e^{j\omega t-\gamma z}\right]$$

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$$H_y = \frac{-j\omega\varepsilon}{h^2}\left[C\cdot\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x\cdot\sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t-\gamma z}\right]$$

* TM Modes in Rectangular waveguide

The electromagnetic wave inside a waveguide can have an infinite number of patterns which are called modes.

The magnetic field is always parallel to the surface of the conductor and cannot have a component perpendicular to it at the surface.

$$E_x = \frac{-\gamma}{h^2}\left[C\cdot\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x\cdot\sin\left(\frac{n\pi}{b}\right)y\cdot e^{j\omega t-\gamma z}\right]$$

$$E_y = \frac{-\gamma}{h^2}\left[C\left(\frac{n\pi}{b}\right)\cos\left(\frac{n\pi}{b}\right)y\cdot\sin\left(\frac{m\pi}{a}\right)x\cdot e^{j\omega t-\gamma z}\right]$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\left[C\left(\frac{n\pi}{b}\right)\cos\left(\frac{n\pi}{b}\right)y\,\sin\left(\frac{m\pi}{a}\right)x\; e^{j\omega t-\gamma z}\right]$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\left[C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x\cdot\sin\left(\frac{n\pi}{b}\right)y\; e^{j\omega t-\gamma z}\right]$$

For TM_{00} mode :

$m=0$, $n=0$

i.e., $E_x=0$, $E_y=0$, $H_x=0$, $H_y=0$

TM_{00} mode does not exist.

For TM_{01} mode :

$m=0$, $n=1$

$\therefore$ $E_x=0$, $E_y=0$, $H_x=0$, $H_y=0$

i.e., TM_{01} mode does not exist.

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For TM_{10} Mode :

$m=1 , n=0$

$\therefore E_x=0 , E_y=0 , H_x=0 , H_y=0$

i.e., TM_{10} mode does not exist

For TM_{11} mode :

$m=1 , n=1$

Here, all the components E_x, E_y, H_x, H_y are present i.e., TM_{11} mode exist and it is the dominent mode of TM Mode

* propagation of TE waves in rectangular wave guide

The electric field is always transverse to the direction of propagation it is called the transverse Electric (or) TE wave.

For TE wave, no electric line is in direction of propagation.

$E_z=0$

$H_z\neq 0$

wave equation for TE wave :

$$\frac{\partial^2 H_z}{\partial x^2}+\frac{\partial^2 H_z}{\partial y^2}+\frac{\partial^2 H_z}{\partial z^2}=0$$

Assume $H_z = XY$

$$\frac{\partial^2 XY}{\partial x^2}+\frac{\partial^2 XY}{\partial y^2}+\frac{\partial^2 XY}{\partial z^2}=0$$

$$Y\cdot\frac{\partial^2 X}{\partial x^2}+X\cdot\frac{\partial^2 Y}{\partial y^2}+h^2XY=0$$

$$XY\left[\frac{1}{X}\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}+h^2\right]=0$$

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$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

Assume,

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -B^2$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -A^2$$

$$h^2 = A^2 + B^2$$

The solutions of X and Y can be obtained by using ordinary second order differential equation.

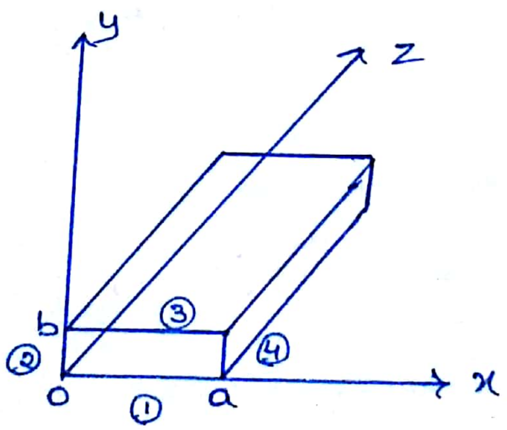
$$X = C_1 \cos BX + C_2 \sin BX$$

$$Y = C_3 \overset{\cos}{\cancel{\sin}} AY + C_4 \sin AY$$

where C_1, C_2, C_3, C_4 are constants can be evaluated by using boundary conditions.

$$H_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay] \longrightarrow (1)$$

Boundary conditions :



From 1st Boundary conditions,

$E_x = 0$ at $y = 0$ $\forall$ $x \longrightarrow 0$ to a

From 2nd Boundary condition,

$E_y = 0$ at $x = 0$ $\forall$ $y \rightarrow 0$ to b

3^{rd} Boundary condition,

$E_x = 0$ at $y = b$ $\forall$ $x \to 0$ to a

4^{th} Boundary condition,

$E_y = 0$ at $x = a$ $\forall$ $y \to 0$ to b

The relation between E_x and H_z is

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \quad [\because E_z = 0]$$

$$E_x = \frac{-j\omega\mu}{h^2}\frac{\partial}{\partial y}\left[C_1\cos Bx + C_2\sin Bx\right] C_3\cos Ay + C_4\sin Ay$$

$$E_x = \frac{-j\omega\mu}{h^2}\left[(C_1\cos Bx + C_2\sin Bx)(-AC_3\sin Ay + AC_4\cos Ay)\right]$$

From 1^{st} Boundary condition:

$E_x = 0$, $y = 0$

$$0 = \frac{-j\omega\mu}{h^2}\left[(C_1\cos Bx + C_2\sin Bx)(-AC_3\sin A(0) + AC_4\cos A(0))\right]$$

$$= \frac{-j\omega\mu}{h^2}\left[(C_1\cos Bx + C_2\sin Bx)(AC_4)\right]$$

$\therefore C_4 = 0$

The reduced equation of H_z is

$$H_z = (C_1\cos Bx + C_2\sin Bx)(C_3\cos Ay) \longrightarrow (2)$$

From 2^{nd} Boundary condition:

$E_y = 0$, $x = 0$ at eq-(2)

The relation between E_y and H_z is

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

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$$E_y = \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial x} \quad [\because E_z = 0]$$

$$= \frac{j\omega\mu}{h^2} \cdot \frac{\partial}{\partial x}\left[(C_1 \cos Bx + C_2 \sin Bx) C_3 \cos Ay\right]$$

$$= \frac{j\omega\mu}{h^2}\left[(-BC_1 \sin Bx + BC_2 \cos Bx) C_3 \cos Ay\right]$$

From the second boundary condition,

$E_y = 0$ at $x = 0 \ \forall \ y \to 0$ to b

$$\Rightarrow 0 = \frac{j\mu\omega}{h^2}\left[BC_2 (C_3 \cos Ay)\right]$$

$\therefore C_2 = 0$

$\therefore$ The reduced equation of H_z is

$$H_z = C_1 \cos Bx \, C_3 \cos Ay \longrightarrow (3)$$

The relation between E_x & H_z

$$E_x = \frac{-j\omega\mu}{h^2} \frac{\partial}{\partial y}\left[C_1 \cos Bx \, C_3 \cos Ay\right]$$

$$= \frac{-j\omega\mu}{h^2}\left[C_1 \cos Bx \, (-AC_3 \sin Ay)\right]$$

From 3rd Boundary condition,

$E_x = 0$, $y = b$

$$0 = \frac{-j\omega\mu}{h^2}\left[-AC_1 C_3 \cos Bx \sin Ab\right]$$

$\sin Ab = 0$

$Ab = n\pi$

$$A = \frac{n\pi}{b}$$

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The relation between E_y & H_z

$$E_y = -\frac{\gamma}{h^2} \cdot \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

From 4th Boundary condition,

$$E_y = 0 \, , \; x = a$$

$$0 = \frac{-\gamma}{h^2} \frac{\partial(0)}{\partial y} + \frac{j\omega\mu}{h^2} \frac{\partial}{\partial x}\left[C_1 \cos Bx \, C_3 \cos Ay\right]$$

$$0 = \frac{j\omega\mu}{h^2}\left[-BC_1C_3 \cos Ay \sin Bx\right]$$

$$= \frac{j\omega\mu}{h^2}\left[-BC_1C_3 \cos Ay \sin Ba\right]$$

$$\sin Ba = 0$$

$$Ba = m\pi$$

$$B = \frac{m\pi}{a}$$

$$H_z = C_1C_3 \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \; e^{-\gamma z} \cdot e^{j\omega t}$$

$e^{-\gamma z}$ = direction of propagation

$e^{j\omega t}$ = variation of EM wave w.r.t time.

$$H_z = C \cdot \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}$$

To determine E_x, E_y, H_x and H_y

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$

Substitute $E_z = 0$ and

$$H_z = C \cdot \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}$$

$$E_x = 0 - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$

$$= -\frac{j\omega\mu}{h^2} \cdot \frac{\partial}{\partial y}\left[C \cdot \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}\right]$$

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$$\boxed{E_x = \frac{-j\omega\mu}{h^2} \, c \cdot \cos\left(\frac{m\pi}{a}\right)x \left(\frac{n\pi}{b}\right) \sin\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}}$$

simillarly,

$$E_y = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

For TE wave, $E_z = 0$, $H_z \neq 0$

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

$$= \frac{j\omega\mu}{h^2} \frac{\partial}{\partial x}\left[c \cdot \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$\boxed{E_y = \frac{-j\omega\mu}{h^2}\left[c \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]}$$

$$H_x = \frac{-\gamma}{h^2} \cdot \frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial y}$$

$E_z = 0$ for TE wave,

$$H_x = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial x}$$

$$= \frac{-\gamma}{h^2} \frac{\partial}{\partial x}\left[c \cdot \left(\frac{m\pi}{a}\right) \cdot -\sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}\right]$$

$$\boxed{H_x = \frac{\gamma}{h^2}\left[c \cdot \frac{m\pi}{a} \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}\right]}$$

$$H_y = \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x}$$

For TE wave, $E_z = 0$

$$= \frac{-\gamma}{h^2} \frac{\partial H_z}{\partial y}$$

$$= \frac{-\gamma}{h^2} \frac{\partial}{\partial y}\left[c \cdot \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

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$$= \frac{-\gamma}{h^2}\left[c.\cos\left(\frac{m\pi}{a}\right)x \cdot \frac{n\pi}{b} \cdot -\sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$\boxed{H_y = \frac{\gamma}{h^2}\left[c.\cos\left(\frac{m\pi}{a}\right)x \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{n\pi}{b}\right)y \; e^{j\omega t - \gamma z}\right]}$$

TE Modes in Rectangular wave guide:

The electro magnetic wave inside a wave guide can have a infinite number of patterns which are called modes.

At the surface of a conductor, the electric field cannot have a component parallel to the surface. This indicates that the electric field must always perpendicular to the surface at a conductor.

For TE_{00} mode:

$m=0$, $n=0$

$\therefore E_x = 0$, $E_y = 0$, $H_x = 0$, $H_y = 0$

i.e., TE_{00} mode cannot exist

For TE_{01} mode,

$m=0$, $n=1$

$\therefore E_x =$ non-zero, $E_y = 0$

$H_x = 0$, $H_y =$ Non-zero

i.e., TE_{01} mode exists

For TE_{10} mode:

$m=1$, $n=0$

$\therefore E_x = 0$, $E_y =$ non-zero, $H_x =$ non-zero, $H_y = 0$

i.e., TE_{10} mode exists.

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TE_{11} mode :

$m=1$, $n=1$

$\therefore$ $E_x =$ Non-zero , $E_y =$ Non-zero

$H_x =$ Non-zero , $H_y =$ Non-zero

i.e., TE_{11} Mode exists

* Calculation of cutoff wave length (λ_c) :

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon$$

$$h^2 = A^2 + B^2$$

$$= \left[\frac{n\pi}{b}\right]^2 + \left[\frac{m\pi}{a}\right]^2$$

$$\gamma^2 + \omega^2\mu\varepsilon = \left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2$$

$$\gamma^2 = \left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} = \alpha + j\beta$$

$\gamma =$ propagation

$\alpha =$ attenuation

$\beta =$ phase

At low frequency,

$$\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$\gamma \rightarrow$ real and positive

At high frequency,

$$\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$\gamma \rightarrow$ Imaginary

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The frequency at which $\gamma = 0$, it is known as cut-off frequency.

At $f = f_c$, $\omega = \omega_c$, $\gamma = 0$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu\varepsilon}$$

$$0 = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2 \mu\varepsilon}$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega_c^2 \mu\varepsilon$$

$$\omega_c^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

$$2\pi f_c = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

we know that, $c = \frac{1}{\sqrt{\mu\varepsilon}}$

$$f_c = \frac{c}{2\pi}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2}$$

$$f_c = \frac{c}{2}\left[\frac{\sqrt{m^2b^2 + n^2a^2}}{ab}\right]$$

$$\lambda_c = \frac{c}{f_c}$$

$$\lambda_{c_{mn}} = \frac{2ab}{\sqrt{m^2b^2 + n^2a^2}}$$

The waves with wave length greater than λ_c will attenuate in the wave guide and the waves with a wavelength less than λ_c are allowed through wave guide.

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* **cut off wavelength for TM mode in rectangular wave guide:**

$$\lambda c_{mn} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

For TM_{11} mode,

$$\lambda c_{11} = \frac{2ab}{\sqrt{a^2+b^2}}$$

For TM_{12} mode,

$$\lambda c_{12} = \frac{2ab}{\sqrt{b^2+4a^2}}$$

For TM_{21} mode,

$$\lambda c_{21} = \frac{2ab}{\sqrt{a^2+4b^2}}$$

Hence, the mode having highest cut off frequency TM_{11} is the dominent mode,

$$\lambda c_{11} > \lambda c_{21}$$

$$\lambda c_{11} > \lambda c_{12}$$

* **cut-off wavelength for TE Mode**

$$\lambda c_{mn} = \frac{2ab}{\sqrt{m^2b^2+n^2a^2}}$$

For TE_{01} mode,

$$\lambda = \frac{2ab}{\sqrt{a^2}} = 2b$$

For TE_{10} mode,

$$\lambda = \frac{2ab}{\sqrt{b^2}} = 2a$$

For TE_{11} mode,

$$\lambda = \frac{2ab}{\sqrt{a^2+b^2}}$$

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TE_{10} is the dominent mode

as a>b , 2a is more

∴ λc_{10} is the highest cut-off wavelength.

Hence,

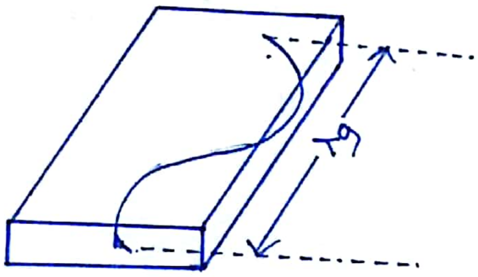
TE_{10} is known as dominent mode.

Guide wavelength (λg) :

It is defined as the distance travelled by the wave inorder to undergo a phase shift of 2π radians in a waveguide.

Guide wavelength related to phase shift β as

$$\lambda g = \frac{2\pi}{\beta}$$



The free space wavelength λ_0 is different from the guide wavelength λg and these two are related as

$$\lambda g = \frac{\lambda_0}{\sqrt{\left(1-\frac{\lambda_0}{\lambda c}\right)^2}}$$

if

$\lambda_0 << \lambda c \Rightarrow \lambda g = \lambda_0$

$\lambda_0 = \lambda c \Rightarrow \lambda g = \infty$

$\lambda_0 >> \lambda c \Rightarrow \lambda g =$ imaginary

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If $\lambda_0 >> \lambda_c$ the wave cannot propagate in the wave guide as λ_g becomes imaginary.

phase velocity :

It is the velocity with which the wave propagates in the wave guide. It is defined as the rate at which the wave changes its phase interms of guide wave length

$$V_p = \lambda_g \cdot f$$

$$= \frac{2\pi\lambda_g}{2\pi} \cdot f$$

$$= \frac{2\pi f}{2\pi/\lambda_g}$$

$$\boxed{V_p = \frac{\omega}{\beta}}$$

Group velocity :

If there is modulation in the carrier, the modulation envelope actually travels at velocity slower than that of carrier alone and of course slower than speed of light. The velocity of modulation envelope is called the group velocity V_g.

It is defined as the rate at which the wave propagates through the waveguide and is given by

$$\boxed{V_g = \frac{d\omega}{d\beta}}$$

Expressions for phase velocity and group velocity

phase velocity :

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$h^2 = A^2 + B^2$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon \longrightarrow (1)$$

Let $\alpha = 0$

$\gamma = j\beta$

at $f = f_c$; $\omega = \omega_c$; $\gamma = 0$

$$\Rightarrow 0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2\mu\varepsilon$$

$$\Rightarrow \omega_c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

From eq (1)

$$(j\beta)^2 = \omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon$$

$$\beta^2 = \omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon$$

$$\beta = \sqrt{\mu\varepsilon(\omega^2 - \omega_c^2)} \longrightarrow (2)$$

$$\beta = \frac{1}{c}\sqrt{\omega^2 - \omega_c^2}$$

$$\beta = \frac{1}{c}\sqrt{(2\pi)^2[f^2 - f_c^2]}$$

$$\beta = \frac{2\pi}{c}\sqrt{f^2 - f_c^2}$$

$$\frac{c}{\sqrt{f^2 - f_c^2}} = \frac{2\pi}{\beta}$$

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$$\frac{c}{f\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{2\pi}{\beta}$$

But we know that $\lambda_g = \frac{2\pi}{\beta}$

$$\lambda_g = \frac{c}{f\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\boxed{\lambda_g \cdot f = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

$$V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Interms of λ,

$$f_c = \frac{c}{\lambda_c} \quad ; \quad f = \frac{c}{\lambda_o}$$

$$\frac{f_c}{f} = \frac{c}{\lambda_c} \times \frac{\lambda_o}{c}$$

$$\frac{f_c}{f} = \frac{\lambda_o}{\lambda_c}$$

$$\boxed{V_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}}}$$

Expression for group velocity (V_g) :

we know that,

$$V_g = \frac{d\omega}{d\beta}$$

From equation (2),

$$\beta = \sqrt{\mu\varepsilon(\omega^2 - \omega_c^2)}$$

Differentiate β w.r.t ω

$$\frac{d\beta}{d\omega} = \frac{d}{d\omega}\left[\sqrt{\mu\varepsilon(\omega^2-\omega_c^2)}\right]$$

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$$= \frac{1}{2\sqrt{\mu\varepsilon(\omega^2-\omega_c^2)}} \times 2\mu\varepsilon\omega$$

$$= \frac{\mu\varepsilon\omega}{\sqrt{\mu\varepsilon(\omega^2-\omega_c^2)}}$$

$$= \frac{\sqrt{\mu\varepsilon}\cdot\omega}{\sqrt{(\omega^2-\omega_c^2)}}$$

$$= \frac{\sqrt{\mu\varepsilon}\cdot\omega}{\omega\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$= \frac{1}{c\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} \qquad \left[\because \sqrt{\mu\varepsilon} = \frac{1}{c}\right]$$

$$\frac{d\beta}{d\omega} = \frac{1}{c\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$V_g = \frac{d\omega}{d\beta}$$

$$\boxed{V_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Group velocity interms of wavelength

$$V_g = c\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}$$

If we multiply phase and group velocities

$$V_p . V_g = \frac{c}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}} \cdot c\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}$$

$$\boxed{V_p . V_g = c^2}$$

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* Relation among λ_o, λ_c and λ_g :

we know that,

$$V_p = \frac{c}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}}$$

$$V_p = \lambda_g \cdot f$$

$$\lambda_g = \frac{V_p}{f}$$

$$= \frac{\frac{c}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}}}{f}$$

$$= \frac{c/f}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}}$$

$$\boxed{\lambda_g = \frac{\lambda_o}{\sqrt{1-\left(\frac{\lambda_o}{\lambda_c}\right)^2}}}$$

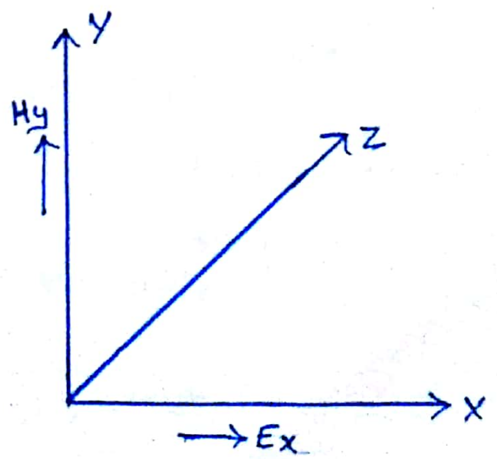
* wave impedance : (Z_z)

where impedance is defined as the ratio of the strength of a field in one transverse direction to the strength of other field in another transverse direction.

i.e., $Z_z = \frac{E_x}{H_y}$

(or)

$Z_z = \frac{H_x}{E_y}$



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For TE wave,

we know that,

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

But for TE wave,

$$E_z = 0$$

$$E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y}$$

$$Z_{TE} = \frac{\frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y}}$$

$$Z_{TE} = \frac{j\omega\mu}{\gamma}$$

$$\gamma = \alpha + j\beta$$

Let $\alpha = 0$

$$\gamma = j\beta$$

$$Z_{TE} = \frac{j\omega\mu}{j\beta}$$

$$Z_{TE} = \frac{\omega\mu}{\beta}$$

we know that,

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}$$

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$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\lambda_c}{\lambda}\right)^2}$$

$$Z_{TE} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$Z_{TE} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

for air,

$$\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

$$= \sqrt{\frac{4\pi\times10^{-7}}{8.25\times10^{-12}}}$$

$$= 120\pi = \eta = 377\,\Omega$$

$$\boxed{Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

$$Z_{TE} > \eta \left[\because \lambda_0 << \lambda_c\right]$$

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For TM waves:

We know that,

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

But for TM waves,

$$H_z = 0$$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$Z_{TM} = \frac{\frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x}}{\frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}} \Rightarrow Z_{TM} = \frac{\gamma}{j\omega\varepsilon}$$

$$\gamma = \alpha + j\beta$$

$$\alpha = 0$$

$$\gamma = j\beta$$

$$Z_{TM} = \frac{j\beta}{j\omega\varepsilon} = \frac{\beta}{\omega\varepsilon}$$

We know that,

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-(\omega_c/\omega)^2}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$Z_{TM} = \frac{\omega\sqrt{\mu\varepsilon}\sqrt{1-(\lambda_0/\lambda_c)^2}}{\omega\varepsilon}$$

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$$Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$= \omega\sqrt{\mu\varepsilon} \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$\boxed{Z_{TM} = \eta\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

where $Z_{TM} < \eta$ $[\because \lambda_0 << \lambda_c]$

For TEM wave, the cut off frequency is zero

i.e., $\lambda_c = \infty$

$$\boxed{Z_{TEM} = \eta}$$

* **power transmission in Rectangular wave guide:**

The power transmitted through a wave guide in the guide walls can be calculated by means of complex poynthing theorem. we assume that the waveguide is terminated in such a way that there is no reflection from the receiving end or that the waveguide is infinitely long as compared with its wavelength.

The power transmitted P_{tr}, through a waveguide is given by

$$P_{tr} = \oint P.ds$$

$$= \frac{1}{2}\int (E \times H) ds$$

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$$Z_z = \frac{E_x}{H_y}$$

$$Z = \frac{E}{H}$$

$$E = H \cdot Z$$

$$H = \frac{E}{Z}$$

For a lossless dielectric, the time average power flow through a rectangular waveguide is

$$P_{tr} = \frac{1}{2Z}\int |E|^2 \, ds$$

$$= \frac{Z}{2}\int |H|^2 \, ds$$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H| = |H_x|^2 + |H_y|^2$$

For TM_{mn} mode, the average power transmitted through a rectangular waveguide of dimensions a and b is

$$P_{tr} = \frac{1}{2Z}\int_0^b \int_0^a |E_x|^2 + |E_y|^2 \, dx \cdot dy$$

Ily,

$$P_{tr} = \frac{Z}{2}\int_0^b \int_0^a |H_x|^2 + |H_y|^2 \, dx \, dy$$

$$P_{tr(TM)} = \frac{1}{2} \frac{1}{\eta} \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \int |E|^2 \, ds$$

$$\boxed{\therefore P_{tr}(TM) = \frac{1}{2\eta} \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \int_0^b \int_0^a |E_x|^2 + |E_y|^2 \, dx \, dy}$$

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For TE wave,

$$\boxed{P_{tr}(TE) = \frac{\frac{1}{2}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\eta} \int_0^b \int_0^a |E_x|^2 + |E_y|^2 \, dx \, dy}$$

Power losses in a Rectangular wave guide:

Losses in a waveguide can be due to the attenuation below cutoff and losses associated with attenuation due to dissipation within the waveguide walls and the dielectric within the waveguide.

At frequency below the cutoff frequency ($f < f_c$) the propagation constant "γ" will have only the attenuation term 'α'. ($\gamma = \alpha + j\beta$) that is to say that the phase constant β itself becomes imaginary implying wave attenuation.

$$\beta = \frac{j2\pi}{\lambda_g}$$

$$\lambda_g = \frac{\lambda}{\cos\theta} = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\beta = j\frac{2\pi}{\lambda}\sqrt{(f_c/f)^2 - 1}$$

$$= j\frac{2\pi f_c}{c}\sqrt{1-(f/f_c)^2} = j\alpha$$

Hence, the cutoff attenuation constant α is given by

$$\alpha = \frac{54.6}{\lambda_c}\sqrt{1-(f/f_c)^2} \text{ dB/length}$$

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In fact this is the stop band attenuation of the waveguide high pass filter. For $f > f_c$, the wave guide exhibits very low loss and for $f < f_c$, the attenuation is high and results in full reflection of the wave i.e., cutoff attenuation is basically the reflection loss

Attenuation constant due to an imperfect, non magnetic dielectric in waveguide is given by

$$\alpha_d = \frac{27.3\sqrt{\epsilon_R}\ \tan\delta}{\lambda_0\sqrt{1-(f_c/f)^2}}\ dB/length$$

The mode which is having highest cut-off wavelength is known as dominent mode.

The dominant modes in TE & TM are TE_{10} & TM_{11} respectively.

If any higher order modes generates a cut-off wavelength = dominant mode, that mode is known as degenerative mode.

In square wave guide TE_{pq}, TE_{qp}, TM_{pq}, TM_{qp} are known as degenerative modes

$\therefore A = B$

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Field patterns:

When a wave travels in a wave guide it exhibits infinite number of patterns. In general there are two modes in a waveguide.

They are:

1] TE Mode
2] TM Mode

TE Mode:

If the electric field is perpendicular to the direction of propagation then the wave is known as TE wave

TM Mode:

If the magnetic field is perpendicular to the direction of propagation.

Each and every mode is disignated by a suffix mn

i.e., TE_{mn}
TM_{mn}

where 'm' indicates no. of half wave variations of electric field in TE Mode [Magnetic field in TM mode]. Across the wider dimension 'a'.

where 'n' indicates the no. of half wave variations of electric field in TE mode [Magnetic field in TM Mode] across the narrow dimension 'b'.

The mode which is having highest cut-off wave length is known as dominent mode.

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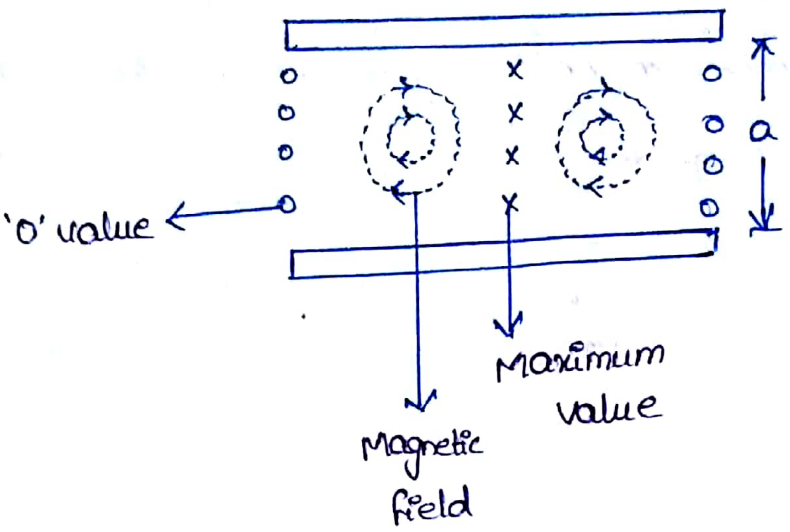
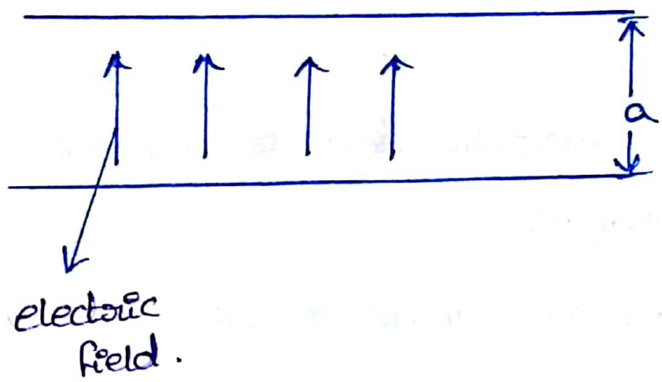
TE_{10} mode is the dominant mode for TE mode.





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