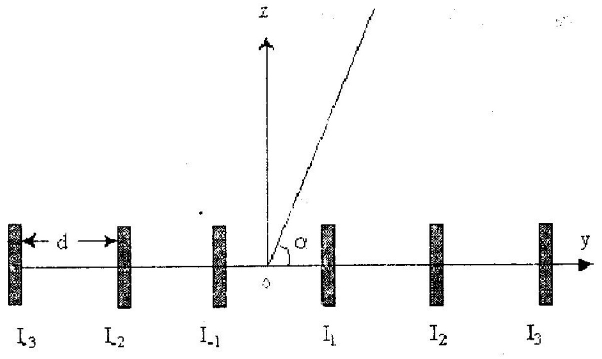
INTRODUCTION TO ANTENNA ARRAYS

For some applications single element antennas are unable to meet the gain or radiation pattern requirements. Combining several single antenna elements in an array can be a possible solution. An antenna Array is a configuration of individual radiating elements that are arranged in space and can be used to produce a directional radiation pattern. Single-element antennas have radiation patterns that are broad and hence have a low directivity that is not suitable for long distance communications. So, antenna arrays are used to increase the directivity of the main lobe in a specified or desired direction.
Arrays usually employ identical antenna elements. This is not necessary, we can use any type of antennas in the antenna array but we have two disadvantages here. They are

i. Fabrication of the array with different antennas is somewhat complex.
ii. Feeding problems for each antenna.



The radiating pattern of the array depends on the configuration, the distance between the elements, the amplitude and phase excitation of the elements, and also the radiation pattern of individual elements.

Array factor

The total field of the array is equal to the field of a single element positioned at the origin multiplied by a factor which is widely referred to as array factor.

The array factor depends on the number of elements, the element spacing, amplitude and phase of the applied signal to each element. The number of elements and the element spacing determine the surface area of the overall radiating structure

Influence of the number of elements on the array factor

The array directivity increases with the number of elements. The number of side lobes and side lobe level increase with the number of elements.

Influence of the spacing between elements on the array factor

The element spacing has a large influence on the array factor as well. A larger element spacing results is a higher directivity. However, the element spacing is generally kept smaller than λ/2 to avoid the occurrence of grating lobes. A grating lobe is another unwanted peak value in the radiation pattern of the array.

Increasing the element spacing towards λ results in an increased directivity and grating lobe effect with maximum grating lobe amplitude equal to the main lobe magnitude. Element spacing beyond λ becomes impractical and results in multiple unwanted grating lobes.

Influence of the radiating element properties

The radiation pattern of an array with isotropic elements is the same as the array factor since an isotropic element radiates the same amount of power in all directions.

The radiation pattern of a dipole, the same array factor without dipoles and the overall radiation pattern of the array with dipoles are different from the array factor i.e. the directivity has increased with the dipole's directivity and the overall radiation pattern is slightly modified due to the dipole's radiation pattern

TYPES OF ARRAYS

The different types of arrays with regard to beam pointing direction are as follows.

1) Broadside array
2) End fire array
3) Collinear array
4) Phased or Scanning array
5) Parasitic array

1 . BROADSIDE ARRAY

Broadside array is one of the most commonly used antenna array in practice. The array in which a number of identical parallel antennas are arranged along a line perpendicular to the line of array axis is known as broadside array, which is shown in figure. In this, the individual antennas are equally spaced along a line and each element is fed with current of equal magnitude, all in the same phase.

Therefore in a broadside array the "**direction of maximum radiation is always perpendicular to the axis of the antenna array**".

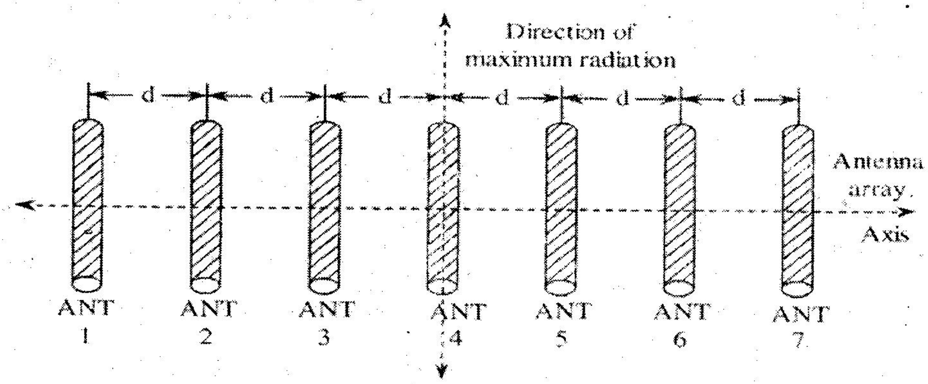
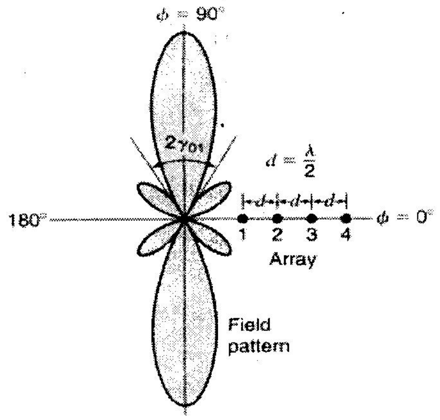


fig 2.1 Board side array

Radiation pattern of the broadside array is of the following form.The radiation pattern of broadside array is **bidirectional**, which radiates equally well in either direction of maximum radiation.

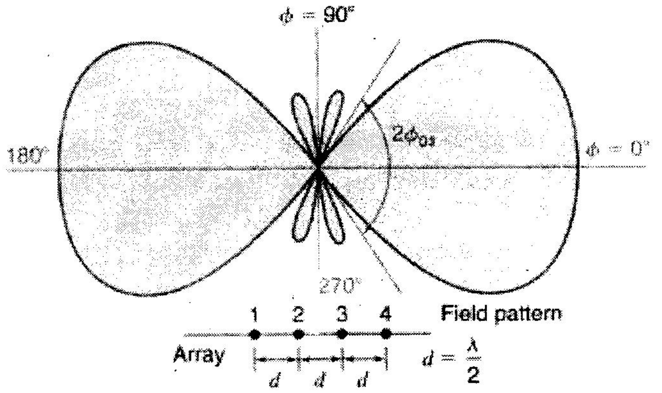


The necessary condition to satisfy the broad side array pattern is **α = 0.**

2.END FIRE ARRAY

The array in which a number of identical antennas are spaced equally along a line and each element is fed with currents of unequal phases (i.e., with a phase shift of 180°) is known as end fire array .This array is similar to that of broadside array except that individual elements are fed in with, a phase shift of 180°.In this, the direction of radiation is coincides with the direction of the array axis, which is shown in figure.So in an end fire array **the direction of maximum radiation is always along the axis of the array i.e. When $\theta = 0^0$ or $\theta = 180^0$.**

The radiation pattern of end fire array is always unidirectional in nature and it is shown in the following figure.

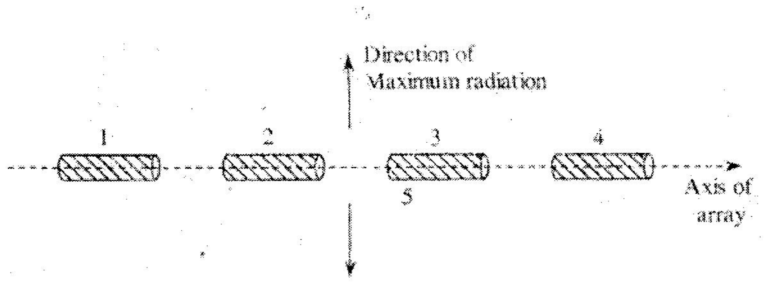


It can also be converted in to bidirectional by providing a phase shift of 180^0 to adjacent elements in the antenna array.

The necessary condition for end fire array pattern is **α = ± βd.**

3. COLLINEAR ARRAY

The array in which antennas are arranged end to end in a single line is known as collinear array. Below figure shows the arrangement of collinear array, in which one antenna is stacked over another antenna. Similar to that of broadside array, the individual elements of the collinear array are fed with equal in phase currents. A collinear array is a broadside radiator, in which the direction of maximum radiation is perpendicular to the line of antenna. The collinear array is sometimes called as broadcast or Omni directional arrays because its radiation pattern has circular symmetry with its main to be everywhere perpendicular to the principal axis.



4. PHASED OR SCANNING ARRAY

In broad side (or) end fire array, the maximum radiation occurs in a specific direction. In Broad side array, the direction of radiation pattern is perpendicular to the array axis whereas in end fire array radiation pattern is normal, to the array axis. It is also possible to change the orientation of maximum radiation in any direction with the help of scanning (or) phased arrays.

Let,

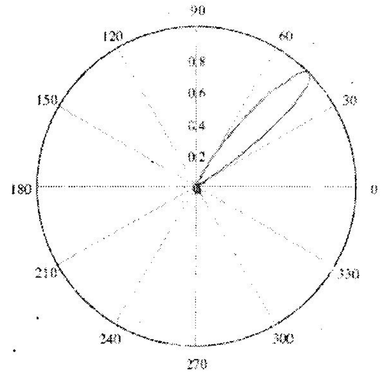
θ_0=Orientation angle

Therefore Phase difference (α) can be calculated by,

$$\psi = (\beta d\cos\theta + \alpha)_{\theta=\theta_0}$$
$$0 = \beta d\cos\theta + \alpha$$
$$\alpha = -\beta d\cos\theta_0$$

From above equation, the phase difference is directly proportional to the orientation angle. By maintaining the proper phase difference between the elements, desired radiation can be obtained in any direction.

The basic principle of scanning and phased array is to get the maximum radiation in any direction like shown in the below figure.

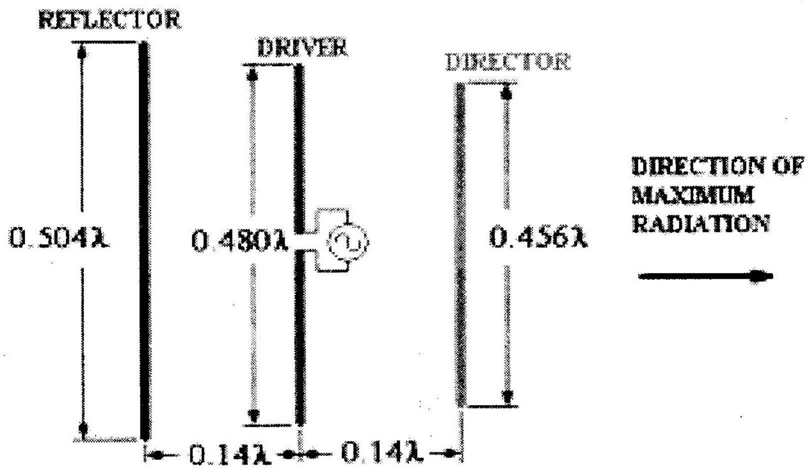


5. PARASITIC ARRAY

The simplest case of a parasitic array is one driven element and one parasitic element and this may be considered as two element array. A parasitic array consists of one or more driven element and a number of parasitic elements. Hence in parasitic arrays there are one or more parasitic elements and at least one driven element to introduce power in the array.

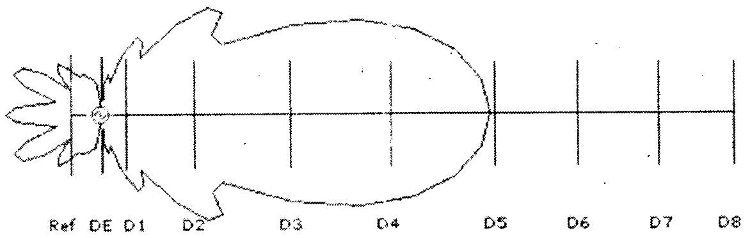
A parasitic array with linear half-wave dipole as elements is normally called as "Yagi-Uda" or simply "Yagi" antenna after the name of inventor S.Uda (Japanese) and H. Yagi.

A simple yagi uda array is shown in the following figure. The reflector and director are called parasitic elements.



Usually it produces maximum radiation when spacing between the elements is varied between 0.3λ to 0.5λ. But this much spacing provides constructive and feeding interference.

The radiation form yagi array is always unidirectional and it is shown in the following figure.



Ten elements Yagi-Uda

MULTIPLICATION OF PATTERNS

The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source pattern and the pattern of an array of isotropic point sources each located at the phase centre of individual source and having the relative amplitude and phase, where as the total phase patterns is the addition of the phase pattern of the individual sources and the array of isotropic point sources. Total field by an array is defined as

$$E = \{ E0(\theta,\phi) \times Ei(\theta,\phi)\} \times \{ Epi(\theta,\phi) + Epa(\theta,\phi) \}$$

= (Multiplication of field patterns) (Addition of phase patterns)

Where

E = Total field

E0 (θ, ϕ) = Field pattern of individual source

Ei (θ, ϕ) = Field pattern of array of isotropic point source

Epi (θ, ϕ) = Phase pattern of individual source

Epi (θ, ϕ) = Phase pattern of array of isotropic point sources.

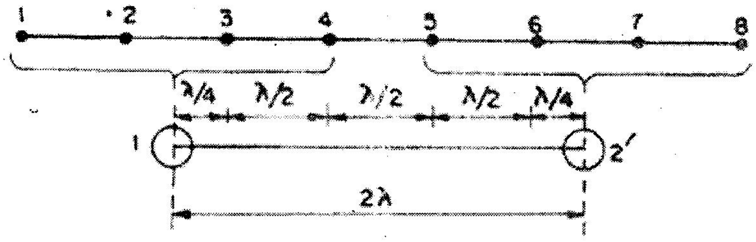
Hence, θ and ϕ are polar and azimuth angles respectively. The principle of multiplication of pattern is best suited for any number of similar sources. Considering a two dimensional case, the resulting pattern is given by the equation,

$$E = 2\ E0 \cos\phi /2$$
$$E = 2\ E1 \sin\theta \cos\phi /2.$$
$$E = E(\theta) \cos\phi /2$$

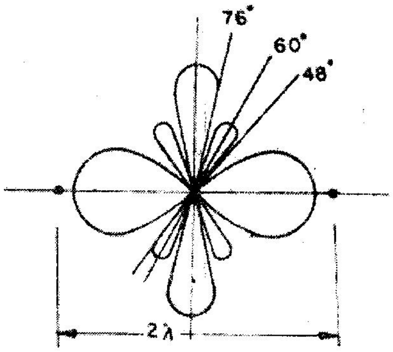
It can be seen that E0 is a function of E (θ). In the above equation the total field pattern is equal to the product of primary pattern E (θ) and a secondary pattern cosϕ/2.

Radiation Pattern of 8-isotropic elements fed in phase, spaced λ/2 apart

As above the principle can be applied to broad-side linear array of 8-isotropic elements also. In this case 4- isotropic elements are assumed to be one unit and then to find the radiation pattern of two such units paced a distance 2λ apart. The radiation pattern of isotropic element is shown in the figure below.

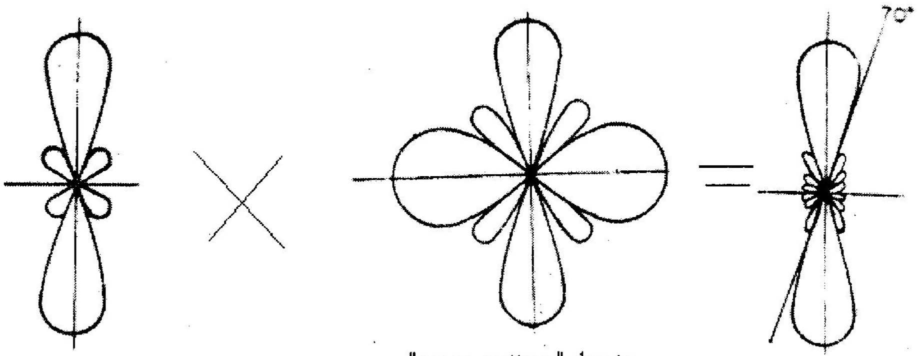


(a)Linear array of 8 isotropic elements spaced λ/2.
(b) equivalent two units array spaced 2λ



Radiation Pattern of isotropic radiators spaced 2λ

Thus the radiation pattern of 8 isotropic elements is obtained by multiplying the unit pattern of 4 individual elements as already obtained in Figure above and Group pattern of two isotropic radiators spaced 2λ is as shown in figure below it and hence the resultant pattern is shown in the following figure.



"unit pattern" due to 4-individual elements

"group pattern" due to twomisotropic elements spaced 2λ apart

Resulting pattern of 8 isotropic elements

TAPERING OF ARRAYS

The bidirectional patterns of antennas contain minor lobes in addition to major lobes. These minor lobes not only waste the amount of power but cause interference thus they are undesirable. The interference is severe in case of radar applications where it may cause improper detection of the target object.

Tapering is a technique in which currents or amplitudes are fed non-uniformly in the sources of a linear array. If the centre source is made to radiate more strongly than the end sources, the level of minor lobes are reduced.

Minor lobes are the lobes other than major lobes in the radiation pattern and the minor lobes adjacent to major lobes are called side lobes. By tapering of arrays from centre to end according to some prescription reduces the side lobe level. I f the tapering amplitudes follow coefficients of binomial series or Tchebyscheff polynomial, then accordingly the arrays are known as binomial arrays or dolph Tchebyscheff arrays respectively.

This technique is primarily intended for broadside arrays and also applicable to end fire arrays because the side lobe ratio in case of broadside arrays is approximately 20 or 13 dB.

Analysis of Two element Arrays

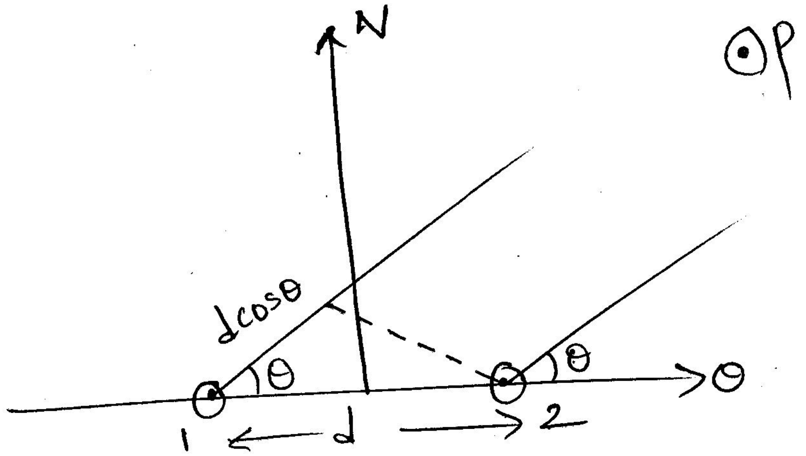
(i) Two elements with equal amplitude and phase.

(ii) Two elements with equal amplitude and opposite phase.

(iii) Two elements with unequal amplitude and any phase.

Case -(i)

Equal Amplitude and phase



consider Two point sources 1 & 2 having current of equal amplitude and phase.

The path difference between two elements is given by

$$\text{Path difference} = d\cos\theta$$

Assume two sources are radiating towards point 'P' in space.

The phase angle difference of the fields of two elements is given by.

$$\psi = 2\pi(\text{path difference})$$

$$= 2\pi\left(\frac{d}{\lambda}\cos\theta\right)$$

$$= \left(\frac{2\pi}{\lambda}\right) d\cos\theta$$

$$\psi = \beta d\cos\theta \quad \text{in rad}$$

Assume that fields from two sources is E_1 & E_2 then total field is given by

$$E_T = E_1 e^{-j\psi/2} + E_2 e^{+j\psi/2}$$

Assume $E_1 = E_2 = E_0$

$$E_T = E_0\left[e^{-j\psi/2} + e^{j\psi/2}\right]$$

$$\boxed{E_T = 2E_0 \cos(\psi/2)}$$

where $2E_0$ = Amplitude

$\cos(\psi/2)$ = phase diff. of array

Normalised field is given by

$$\boxed{(E_T)_n = \cos(\psi/2)}$$

Maximas

$$\cos(\psi/2) = \pm 1$$

$$\cos\left(\frac{\beta d \cos\theta}{2}\right) = \pm 1$$

we have $\beta = \frac{2\pi}{\lambda}$; $d = \lambda/2$

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$

$$\frac{\pi}{2}\cos\theta = \pm n\pi$$

for $n = 0$ $\Rightarrow$ $\cos\theta = 0^\circ$

$$\boxed{\theta_{max} = 90^\circ, 270^\circ}$$

Minimas

$$\cos(\psi/2) = 0$$

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = 0$$

$$\frac{\pi}{2}\cos\theta = \pm(2n+1)\frac{\pi}{2}$$

for $n = 0$ $\Rightarrow$ $\cos\theta = \pm 1$

$$\boxed{\theta_{min} = 0^\circ, 180^\circ}$$

Half power points

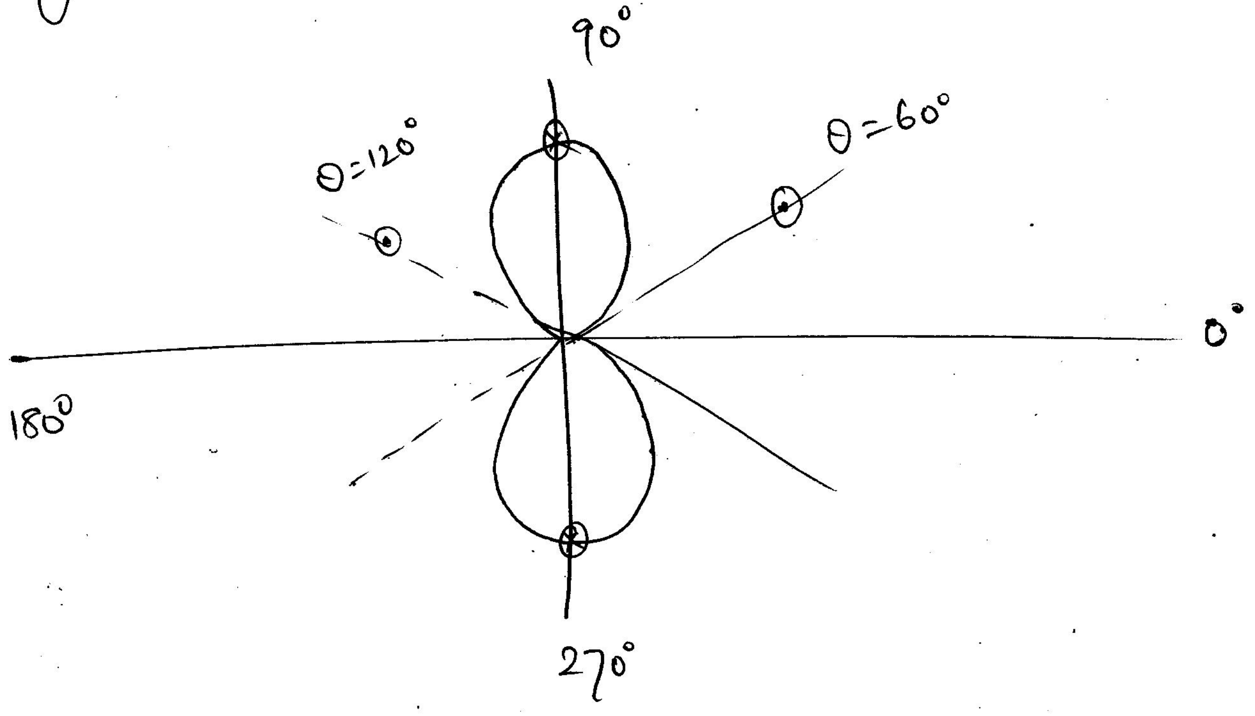
$$\cos(\psi/2) = \pm\frac{1}{\sqrt{2}} = 0.707$$

$$\frac{\pi}{2}\cos\theta = \pm(2n+1)\frac{\pi}{4}$$

for $n=0 \Rightarrow \cos\theta = \pm\frac{1}{2}$

$$\boxed{\theta_{HPP} = 60^\circ, 120^\circ}$$

The Radiation pattern for the above two Element array is drawn as below.



case (ii)

Equal Amplitude and opposite phase

The Two sources are out of phase by 180° Then we can write them as

$$E = -E_1 e^{-j\psi/2} + E_2 e^{j\psi/2}$$

where E_1 = electric field due to source 1

E_2 = electric field due to source 2.

$$\therefore E_T = E_0\left[-e^{-j\psi/2} + e^{j\psi/2}\right]$$

$[\because E_1 = E_2 = E_0]$

$$= E_0 \, 2j \sin(\psi/2)$$

$$\boxed{E_T = (2jE_0)\sin(\psi/2)}$$

where $2jE_0$ = Amplitude

$\sin(\psi/2)$ = phase of array

$$\boxed{(E_T)_n = \sin(\psi/2)}$$

where, $\psi = \beta d \cos\theta$

Maximas

$$\sin(\psi/2) = \pm 1$$

$$\sin\left(\frac{\beta d \cos\theta}{2}\right) = \pm 1$$

$$\frac{\pi}{2}\cos\theta = \pm (2n+1)\,\pi/2$$

for $n=0$

$$\cos\theta = \pm 1$$

$$\boxed{\theta_{max} = 0^\circ, 180^\circ}$$

Minimas

$$\sin(\psi/2) = 0$$

$$\frac{\pi}{2}\cos\theta = \pm n\pi$$

for $n=0$

$$\cos\theta = 0 \Rightarrow \boxed{\theta_{min} = 90^\circ, 270^\circ}$$

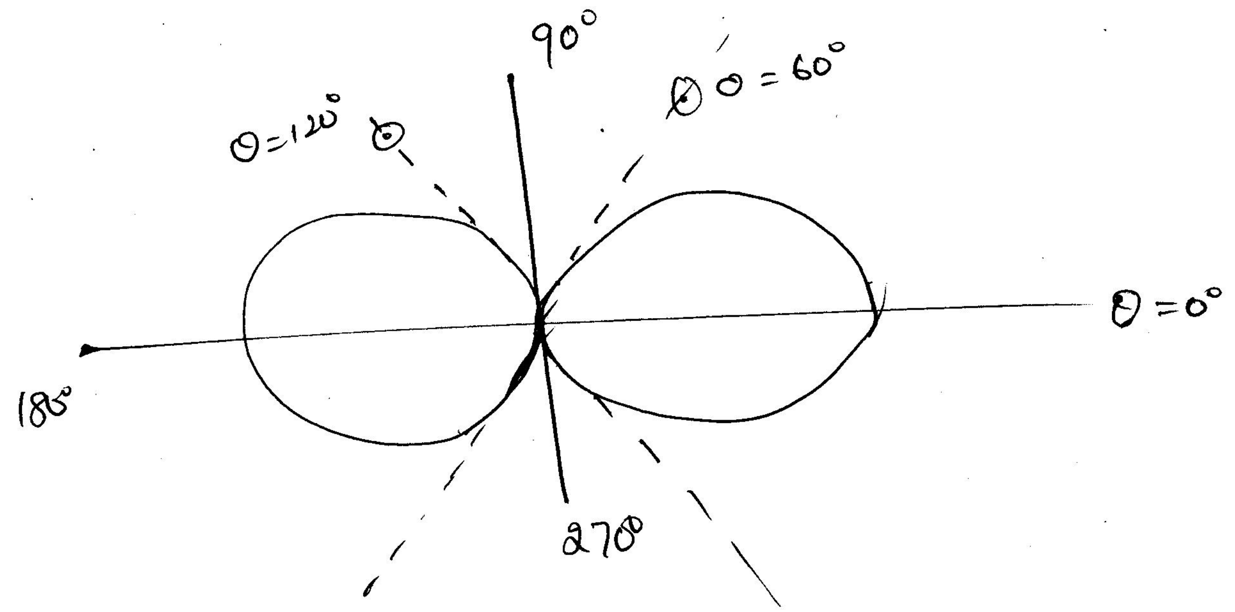
Half power points

$$\sin(\psi/2) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\theta = \pm (2n+1)\,\pi/4$$

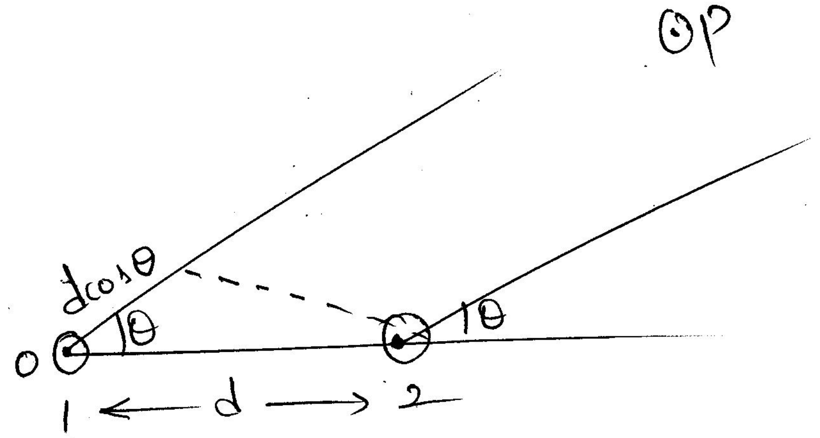
for $n=0$ $\Rightarrow$ $\cos\theta = \pm\frac{1}{2}$

$$\boxed{\theta_{HPP} = 60^\circ, 120^\circ}$$



case-(iii)

consider two point sources, one source placed at origin as a reference antenna for finding amplitude and phase angle.



we have, $\psi = \beta d \cos\theta + \alpha$

where α = progressive phase shift let assume that fields from two sources as E_1 & E_2, then

$$E_T = E_1 e^{(0)j\psi} + E_2 e^{j\psi}$$

$$= E_1\left(1 + \frac{E_2}{E_1} e^{j\psi}\right)$$

$$= E_1\left(1 + K e^{j\psi}\right)$$

$$E_T = E_1\left(1 + K e^{j\psi}\right)$$

where $k = \frac{E_2}{E_1}$

$\because$ $E_1 > E_2$, then $K < 1 \Rightarrow [0 \leq K \leq 1]$

$$E_T = E_1\left(1 + K\cos\varphi + jK\sin\varphi\right)$$

$$|E_T| = |E_1|\sqrt{(1 + K\cos\varphi)^2 + (K\sin\varphi)^2}$$

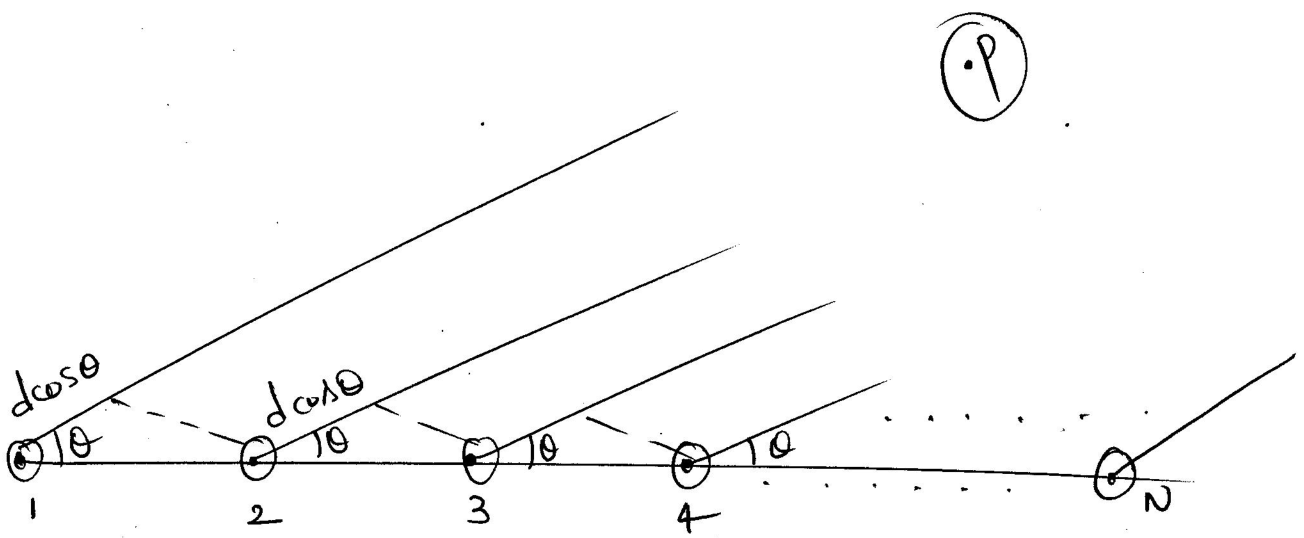
$$\boxed{E_T = E_1\sqrt{(1 + K\cos\varphi)^2 + (K\sin\varphi)^2}}$$

&

phase Angle $\boxed{\varphi = \frac{K\sin\varphi}{1 + K\cos\varphi}}$

N-Element uniform linear Array

consider a N-Element uniform linear arrangement of array in which Each Element is spaced at a distance of 'd' and for Each element we are providing a current of Equal amplitude and phase



Assume all Elements are radiating towards a point in the space and at the same time all Elements are radiating uniform radiation 'E_0'.

Then the resultant field is given by

$$E_T = E_0 e^{j\psi(0)} + E_0 e^{j\psi} + E_0 e^{j2\psi} + \cdots + e^{j(n-1)\psi}$$

$$E_T = E_0 \left[1 + e^{j\psi} + e^{j2\psi} + \cdots \cdots + e^{j(n-1)\psi}\right]$$

multiplying both sides by $e^{j\psi}$, then

$$E_T(e^{j\psi}) = E_0(e^{j\psi} + e^{2j\psi} + \cdots + e^{jn\psi}) \quad \text{---------} \quad (2)$$

subtract eq (2) from eq (1)

$$E_T(1-e^{j\psi}) = E_0(1-e^{jn\psi})$$

$$\frac{E_T}{E_0} = \frac{1-e^{jn\psi}}{1-e^{j\psi}}$$

$$= \frac{e^{jn\psi/2}\left(e^{-jn\psi/2} - e^{j\psi/2}\right)}{e^{j\psi/2}\left(e^{-j\psi/2} - e^{j\psi/2}\right)}$$

$$\frac{E_T}{E_0} = e^{j(n-1)\psi/2}\left[\frac{\cancel{-2j}\sin(n\psi/2)}{\cancel{-2j}\sin(\psi/2)}\right]$$

(or)

$$1-e^{jn\psi} = (1-\cos n\psi) - j\sin\psi$$

$$|1-e^{jn\psi}| = \sqrt{(1-\cos n\psi)^2 + \sin^2 n\psi}$$

$$|1-e^{jn\psi}| = \sqrt{\sin^2(n\psi) + \cos^2(n\psi) + 1 - 2\cos(n\psi)}$$

$$= \sqrt{2(1-\cos n\psi)}$$

$$\left|1-e^{jn\psi}\right| = \sin(n\psi/2)$$

similarly, $\left|1-e^{j\psi}\right| = \sin(\psi/2)$

Then,

$$\boxed{\left|\frac{E_T}{E_0}\right| = \frac{\sin(n\psi/2)}{\sin(\psi/2)}}$$

The ratio is called as Array factor.

$$\boxed{(AF)_n = \frac{\sin(n\psi/2)}{\sin(\psi/2)}}$$

where $\psi = \beta d\cos\theta + \alpha$

If $\alpha = 0°$; it becomes a broadside Array

If $\alpha = 180°$; it becomes a endfire array

The maximum value for the above array factor is given by

$$\boxed{E_T = nE_0}$$

$$\left[\because \lim_{\psi\to 0} \frac{E_T}{E_0} = nE_0\right]$$

Broadside Array characterstics

In a Broadside array, the direction of maximum radiation is always perpendicular to axis, so that we have $\theta = 90^\circ$.

Now we are going to derive the principal maximas, pattern maximas and minimas of the Broadside array.

principal Maximas

since maximum radiation occurs when $\theta = 90^\circ$, so along this direction of principal maximas takes place.

$$\boxed{(\theta_{max})_{major} = 90^\circ, 270^\circ}$$

Pattern Maximas (side lobes)

Along with the main lobe, side lobes will also occur along the pattern. To find their directions, we must take

$$\frac{\sin(n\psi/2)}{\sin(\psi/2)} = \pm 1$$

$\because$ $\sin(n\psi/2)$ has to be maximum, $\sin(\psi/2) \neq 0$

Then $\sin(n\psi/2) = \pm 1$

$$(n\psi/2) = \pm(2N+1)\pi/2$$

$$\psi = \pm(2N+1)\pi/n$$

where $N = 1, 2, 3, \ldots$

$$\beta d\cos\theta + \alpha = \pm(2N+1)\pi/n$$

$[\because \alpha = 0$ for Broadside Array$]$

$$\cos\theta = \pm\frac{(2N+1)\pi}{n\beta d}$$

$$\theta = \cos^{-1}\left[\pm\frac{(2N+1)\pi}{\beta n d}\right]$$

$\because \beta = \frac{2\pi}{\lambda}$

$$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left[\pm\frac{(2N+1)\lambda}{2nd}\right]}$$

Pattern Minima

To find pattern minima, we have to take

$$\sin(n\psi/2) = 0$$

$$\sin\left(n\psi/2\right) = 0$$

$$\frac{n\psi}{2} = \pm N\pi$$

$$\psi = \pm \frac{2N\pi}{n}$$

$$\beta d\cos\theta + \alpha = \pm \frac{2N\pi}{n}$$

$$\left[\because \alpha = 0,\ \beta = \frac{2\pi}{\lambda}\right]$$

$$\cos\theta = \pm \frac{2N\pi}{n\beta d}$$

$$\boxed{\left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\pm \frac{N\lambda}{nd}\right]}$$

Beamwidth of major lobe

Beamwidth is the angle measured between nulls of the major lobe. It is double the angle between the nulls.

It is denoted by complementary angle.

$\gamma = 90 - \theta$, then

$$BWFN = 2\gamma$$

$$(\theta_{min}) = \cos^{-1}\left[\pm \frac{N\lambda}{nd}\right]$$

$$90-\gamma = \pm \cos^{-1}\left(\frac{N\lambda}{nd}\right)$$

$$\cos(90-\gamma) = \pm \frac{N\lambda}{nd}$$

$$\sin\gamma = \pm \frac{N\lambda}{nd} \quad [\text{Assume } \sin\gamma = \gamma]$$

$$\gamma = \pm \frac{N\lambda}{nd}$$

first Null occurs at $N=1$, then

$$\gamma = \pm \frac{\lambda}{nd}$$

But $nd = (n-1)d$ = length of array

$$BWFN = 2\gamma = \pm \frac{2\lambda}{L}$$

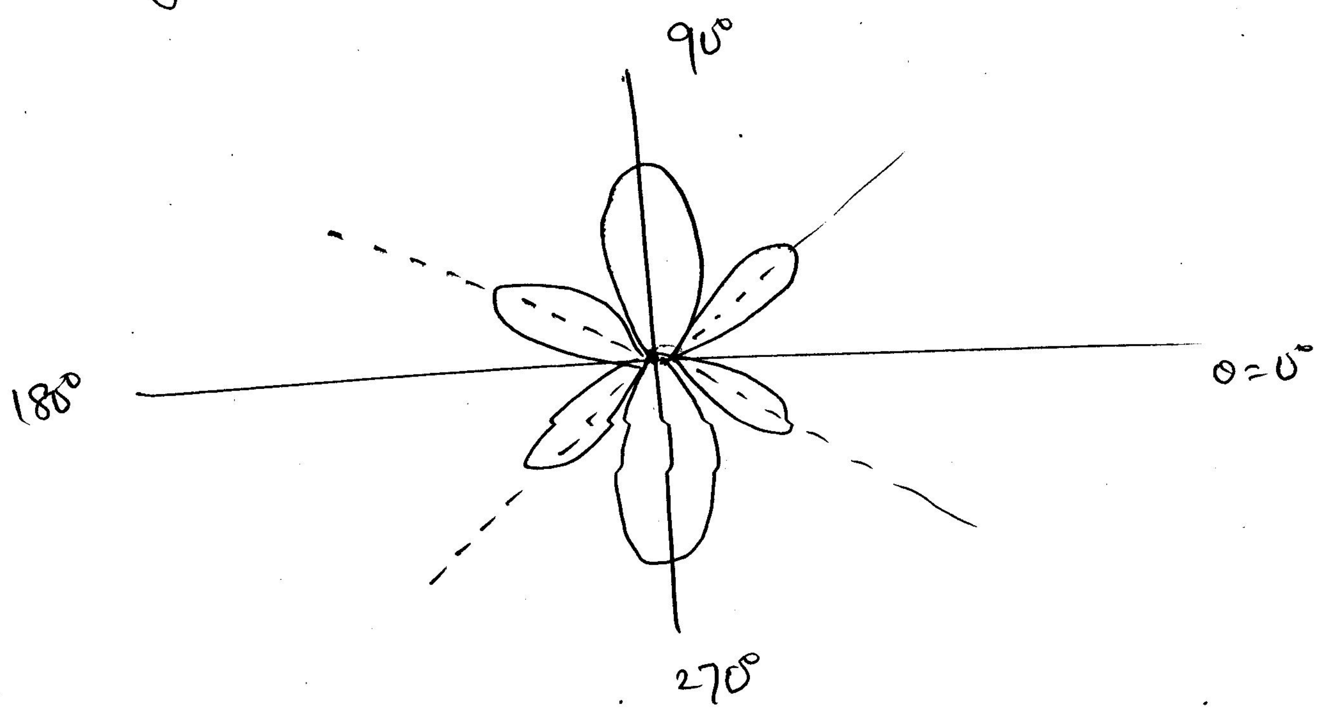
$$= \pm \frac{2}{(L/\lambda)} \quad \text{radians}$$

$$= \pm \frac{2 * 57.3^\circ}{(L/\lambda)}$$

$$\boxed{BWFN = \frac{114.6^\circ}{(L/\lambda)}}$$

$$HPBW = \frac{BWFN}{2} \Rightarrow \boxed{HPBW = \pm \frac{57.3^\circ}{L/\lambda}}$$

For $n=4$, the radiation pattern of Broadside array is as follows.



End-Fire Array characteristics

In an end fire array the direction of maximum radiation is always along the axis of the array.
So, we have $\theta = 0°, 180°$.

principal Maximas

since maximas occur when $\theta=0°, 180°$, the maximum radiation is along axis

$$\boxed{\therefore \theta_{max} = 0^\circ, 180^\circ}$$

<u>pattern Maxima</u> (side lobes)

pattern maximas will occur, when

$$\sin\left(\frac{n\psi}{2}\right) = \pm 1$$

$$\frac{n\psi}{2} = \pm(2N+1)\frac{\pi}{2}$$

$$\psi = \pm\frac{(2N+1)\pi}{n}$$

$$\beta d\cos\theta + \alpha = \pm\frac{(2N+1)\pi}{n}$$

[$\because$ for end fire array $\alpha = -\beta d$]

$$\beta d\cos\theta - \beta d = \beta d(1-\cos\theta) = \pm\frac{(2N+1)\pi}{n}$$

$$\cos\theta = \pm\frac{(2N+1)\pi}{n\beta d} + 1$$

$$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left[\left(\pm\frac{(2N+1)\lambda}{2nd}\right) + 1\right]}$$

Pattern Minima

pattern minimas will occur in endfire array, when

$$\sin\left(\frac{n\psi}{2}\right) = 0$$

$$\frac{n\psi}{2} = \pm N\pi$$

$$\psi = \pm \frac{2N\pi}{n}$$

$$\beta d\cos\theta + \alpha = \beta d\cos\theta - \beta d = \pm \frac{2N\pi}{n}$$

$$(\cos\theta - 1) = \pm \frac{2N\pi}{n\beta d}$$

$$2\sin^2(\theta/2) = \pm \frac{N\lambda}{nd} \qquad \left[\because \beta = \frac{2\pi}{\lambda}\right]$$

$$\sin(\theta/2) = \pm \sqrt{\frac{N\lambda}{2nd}}$$

$$\left(\frac{\theta}{2}\right) = \sin^{-1}\left[\pm\sqrt{\frac{N\lambda}{2nd}}\right]$$

$$\boxed{(\theta_{min})_{minor} = 2\sin^{-1}\left[\pm\sqrt{\frac{N\lambda}{nd}}\right]}$$

Beamwidth of Major lobe

Beamwidth = 2 * angle between first nulls of major lobe.

$B\omega = 2\theta$

since nulls occurs at Minimas,

$$\theta_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{N\lambda}{2nd}}\right]$$

$$\sin\left(\theta_{min}\right) = \pm\sqrt{\frac{4N\lambda}{2nd}}$$

$$\theta_{min} = \pm\sqrt{\frac{2N\lambda}{nd}} \qquad [\because nd = L]$$

$$= \pm\sqrt{\frac{2N\lambda}{L}}$$

for $n=1$ $\Rightarrow$ $BWFN = 2\theta = \pm 2\sqrt{\frac{2N}{(L/\lambda)}}$

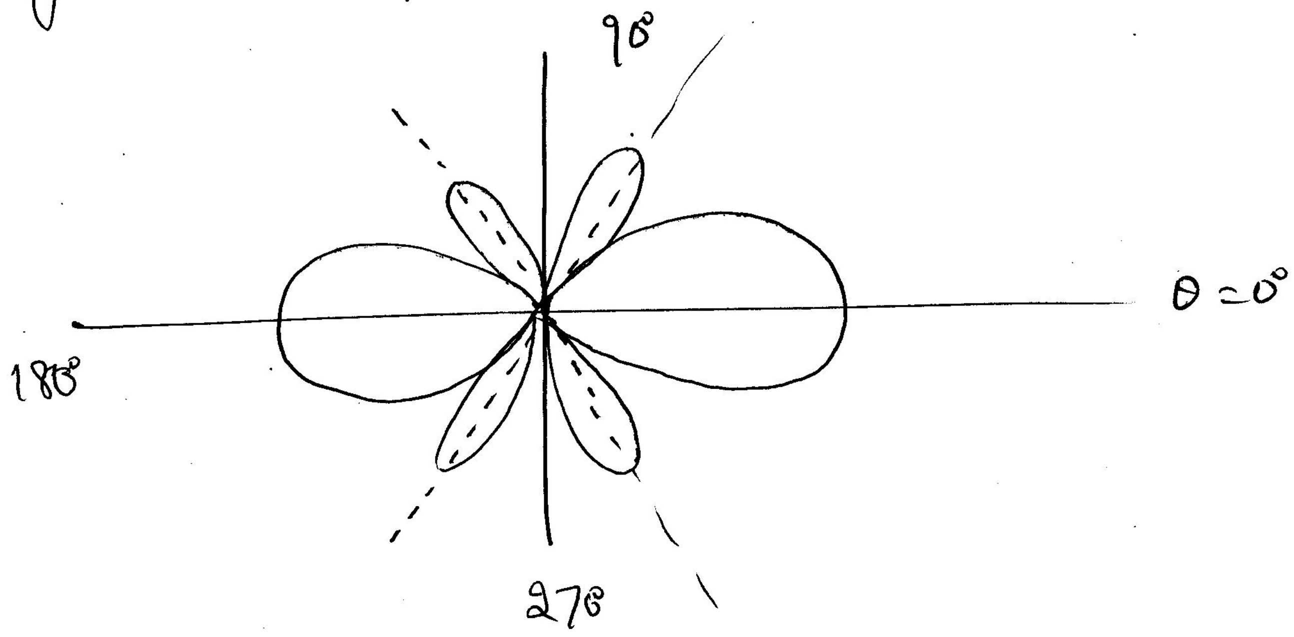
$$= \pm 2\sqrt{\frac{2}{(L/\lambda)}}$$

$$= \pm(2*57.3^\circ)\sqrt{\frac{2}{(L/\lambda)}}$$

$$\boxed{BWFN = \pm 114.6^\circ\sqrt{\frac{2}{(L/\lambda)}}} \text{ degrees}$$

$$\boxed{HPBW = \pm 57.3^\circ\sqrt{\frac{2}{(L/\lambda)}}} \text{ degrees}$$

for $n=4$ elements, the pattern of the end fire array is as follows



Bionomial Arrays

If the amplitudes of the radiating sources are arranged such that according to the co-efficients of the successive terms of Bionomial Series, then the array is called as Bionomial array

we know that,

$$(a+b)^{n-1} = a^{n-1} + \frac{(n-1)}{1!} a^{n-2} \cdot b + \frac{(n-1)(n-2)}{2!} \cdot a^{n-3} \cdot b^2$$

$+ \ldots$

where n = no of elements of array

The purpose of Bionomial array is to supress the side lobes completely. This process should satisfy two conditions.

(i) spacing between the two consecutive sources do not exceed by $\lambda/2$

(ii) The current amplitudes of sources are proportional to the co-efficents of Bionomial series

for example, take $n=5$ sources, the relative amplitudes are as follows.

No of sources	Relative amplitude
$n=1$	1
$n=2$	1 1
$n=3$	1 2 1
$n=4$	1 3 3 1
$n=5$	1 4 6 4 1

The above amplitudes represents Pascals triangle

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Analysis of Bionomial Array

This process utilises pattern multiplication of Arrays.

consider a far field pattern of two point sources of equal amplitude and phase. Then

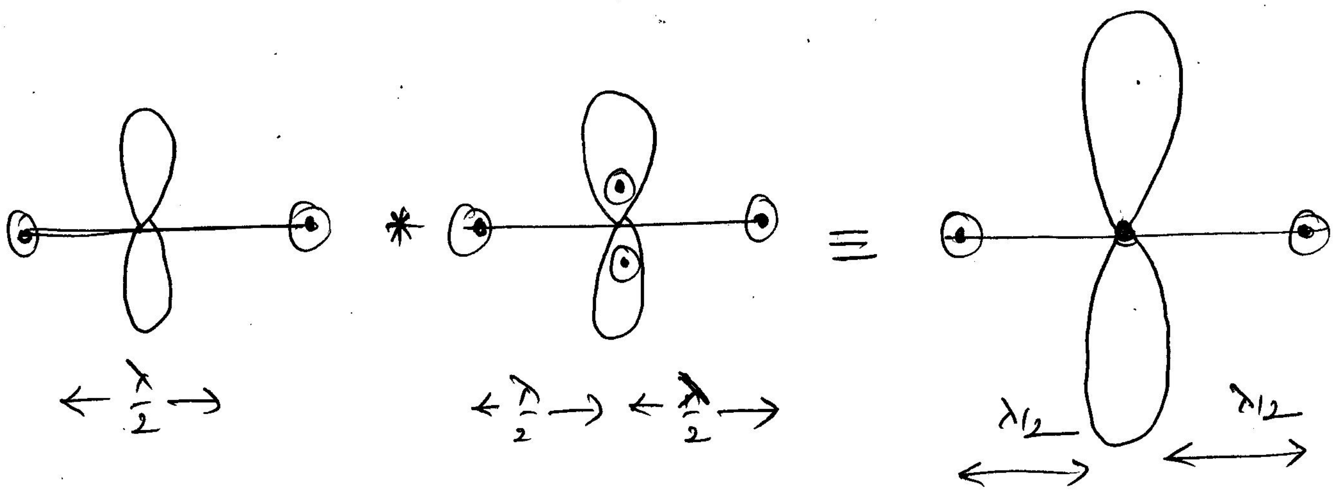
$$E = 2E_0 \cos\left(\frac{\psi}{2}\right)$$

(or)

$$E_n = \cos\left(\frac{\psi}{2}\right)$$

If another identical array pattern is superimposed on above array, then by principle of multiplicity

$$\boxed{E_{nr} = \cos^2(\psi/2) = \cos^2\left(\frac{\pi}{2}\cos\theta\right)}$$



[1 : 2 :]

Therefore, the array has amplitude ratio of 1: 2 :1. similarly it can be applied to any no of sources with a common field of

$$\boxed{E_{nor} = \cos^{n-1}\left(\frac{\pi}{2}\cos\theta\right)}$$

Advantages

complete elimination of side lobes.

Disadvantages

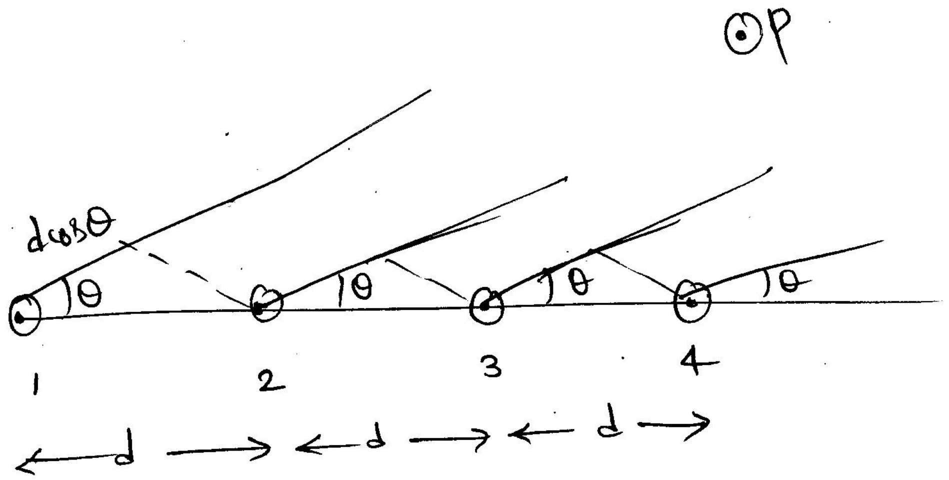
(i) Directivity decreases due to increase in HPBWs.

(ii) Design complexity of array

Pattern multiplication

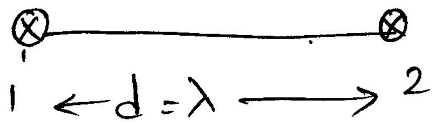
This method is a straight forward method to to obtain radiation pattern of Arrays just by inspe-ction.

To explain this method consider 4 element array of equispaced elements as shown in the figure. Let the spacing be $\lambda/2$. All elements are supplied with equal magnitude currents which are in phase



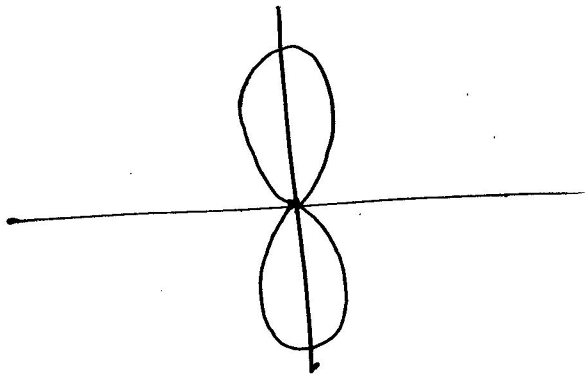
As the point 'p' at which resultant field has to be obtained, the radiation pattern of antennas 1 & 2 treated as single unit.

similarly, 3 & 4 elements assumed as one unit as shown in figure

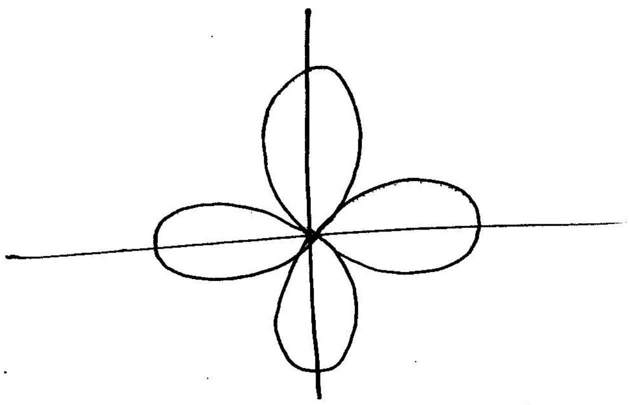


we replaced the entire geometry with two Elements with spacing 'λ'.

Now, the radiation patterns of both antennas have bidirectional patterns as shown in the figure

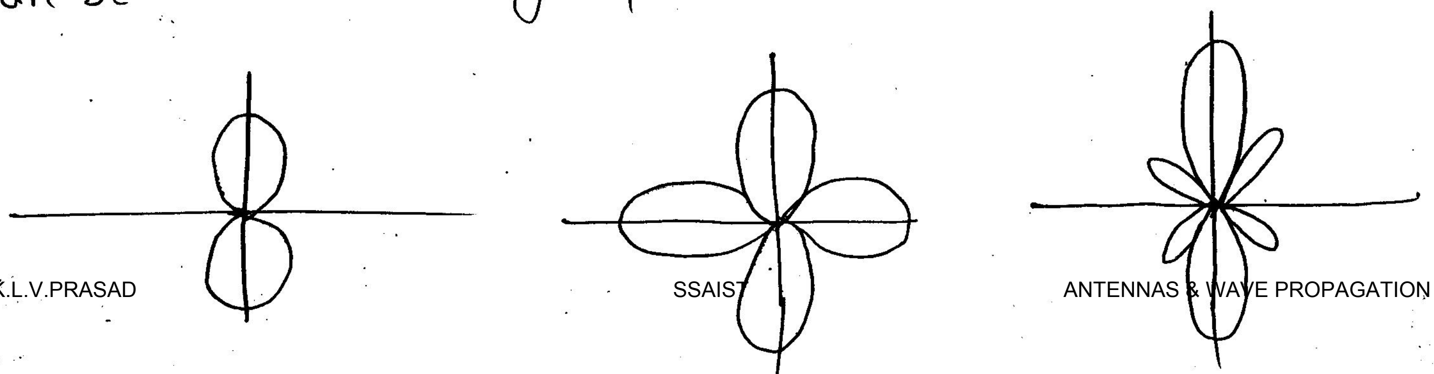


pattern of two Element spaced $\lambda/2$



pattern of 2 Element spaced 'λ'.

Now, the resultant pattern of four Element array can be obtained by pattern multiplication.



End fire Array with increased Directivity

(or)

Hansen – woodward Array

For an end fire array, we know that maximum radiation is along axis of array by allowing the phase shift 'α' between the elements equal to $\pm \beta d$

where $\alpha = -\beta d$ for $\theta = 0°$

$\alpha = +\beta d$ for $\theta = 180°$

This ordinary end fire array gives maximum radiation along $\theta = 0°$ but does not give maximum directivity:

To increase the directivity of end fire array, Hansen & woodyard proposed two conditions.

The required phase shift between closely spaced elements should be

$$\alpha = -\left[\beta d + \frac{\pi}{n}\right] = -\left[\beta d + \frac{2.94}{n}\right]$$

$$\alpha = +\left[\beta d + \frac{\pi}{n}\right] = +\left[\beta d + \frac{2.94}{n}\right]$$

for $\theta = 0^\circ$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta = 0^\circ} = \frac{\pi}{n}$$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta = 180^\circ} \simeq \pi$$

for $\theta = 180^\circ$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta = 180^\circ} = \pi/n$$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta = 0^\circ} = \pi$$

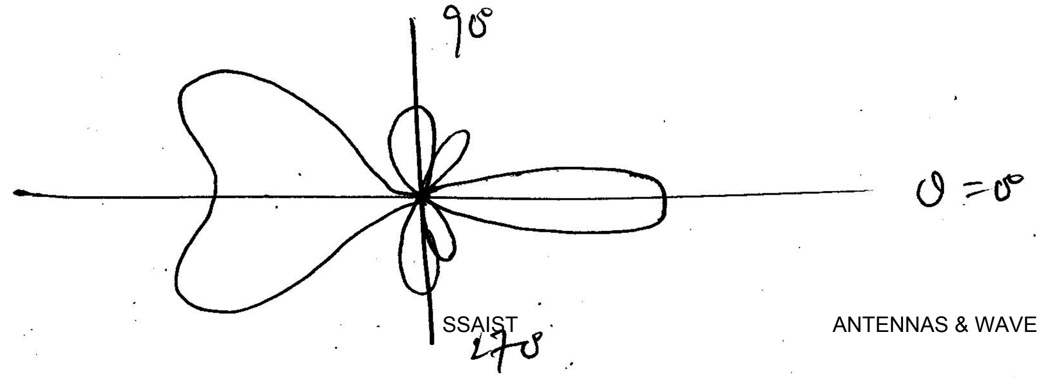
for both cases above, we have satisfied the basic condition $|\psi| = \pi$ for each array.

Example

for $n = 4$; $d = \lambda/2$; $\alpha = -\left[\beta d + \frac{\pi}{n}\right]$

$$\alpha = -\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{2}\right) + \frac{\pi}{4}\right] = -5\pi/4$$

for above characteristics, the pattern is as below

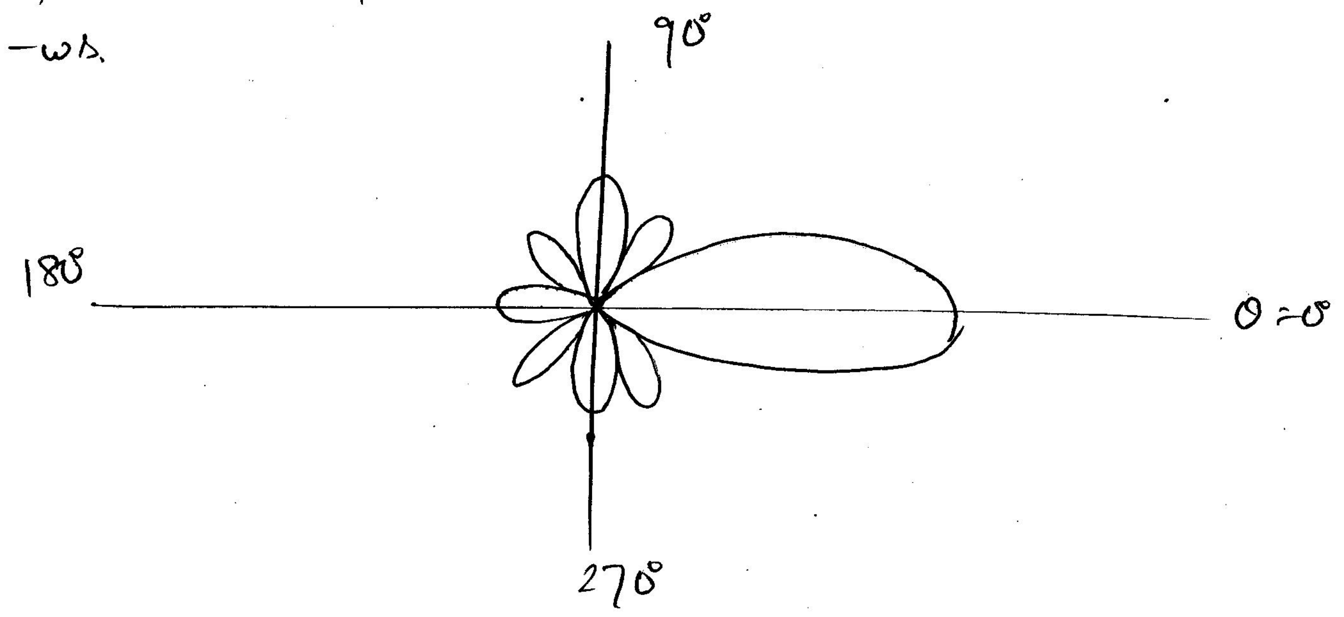


The above pattern suffers a back lobe varying at maximum rate. It is a disadvantage.

To improve this model, let the spacing be $d = \frac{\lambda}{4}$ then

$$\alpha = -\left(\beta d + \frac{\pi}{n}\right) = -\left[\frac{\pi}{2} + \frac{\pi}{4}\right] = -\frac{3\pi}{4}$$

for above phase shift the pattern is as follows.



Therefore, for a large uniform array, the Hansen-woodyard condition can only yield an improved directivity provided that spacing between the elements is approximately

$$\boxed{d = \left[\frac{n-1}{n}\right]\frac{\lambda}{4} \simeq \frac{\lambda}{4}}$$