

B.Tech II Year I Semester (R19) Supplementary Examinations August 2021

COMPLEX VARIABLES & TRANSFORMS

(Common to EEE & ECE)

Time: 3 hours

Max. Marks: 70

PART – A
(Compulsory Question)

- 1 Answer the following: (10 X 02 = 20 Marks)
- Define an analytic function with an example.
 - Define Harmonic function with an example.
 - State Cauchy integral theorem.
 - Find the number of zeros of $\left(\frac{z+1}{z^2+1}\right)^2$.
 - Find $\mathcal{L}(e^{at})$.
 - Find $\mathcal{L}(t^3 + 2t^2 - 4t + 6) =$
 - Write the Fourier coefficients of $f(x)$ in $[\pi, -\pi]$.
 - Define and even and odd function.
 - Write the Fourier cosine transform.
 - Z-transform of $\{\cos n\theta + i \sin n\theta\}$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

- 2 (a) Show that $f(z) = z^3$ is analytic for all z .
- (b) If $f(z)$ is an analytic function of 'z' prove that: $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\text{Rf}(z)|^2 = 2|f'(z)|^2$.

OR

- 3 (a) Prove that $u = y^3 - 3x^2y$ is a harmonic function. Determine its harmonic conjugate.
- (b) Find the bilinear transformation that maps the points $-i, 0, i$ in to points $-1, i, 1$ respectively.

UNIT – II

- 4 (a) Evaluate $\int_c (y + 2xy)dx + (x^2 - 2xy) dy$ where c is the region bounded by $y = x^2$ and $x = y^2$.
- (b) State Maximum and modulus theorem and Liouville's theorem.

OR

- 5 (a) Evaluate using $\int_c \frac{z}{(z^2-6z+5)^2} dz$ where $c: |z - (3 + 4i)| = 4$ by Cauchy's theorem.
- (b) Expand $f(z) = \sin z$ in a Taylor's series about $z = \frac{\pi}{4}$.

UNIT – III

- 6 (a) State and prove First shifting property.
- (b) Find $\mathcal{L}\{e^{-3t} \sin 2t\}$.

OR

- 7 (a) Find $\mathcal{L}\left\{\frac{1-\cos at}{t}\right\}$.
- (b) Using Convolution theorem, find $L^{-1}\left\{\frac{1}{(s+a)(s+b)}\right\}$.

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UNIT - IV

8 Find a Fourier series to represent $f(x) = x^2$ in the interval $(0, 2\pi)$.

OR

9 Find the Half range cosine and sine series for the function $f(x) = x$ in the range $0 < x < \pi$.

UNIT - V

10 (a) Find the Fourier transform of $f(x) = xe^{-x}$.

(b) Find the Fourier sine transform of x .

OR

11 (a) Find the Z- transform of n^p .

(b) Find $y_{n+1} - 2y_n = 0$ using z-transform.
