

HEAT TRANSFER - 80329

MODULE III FORCED CONVECTION



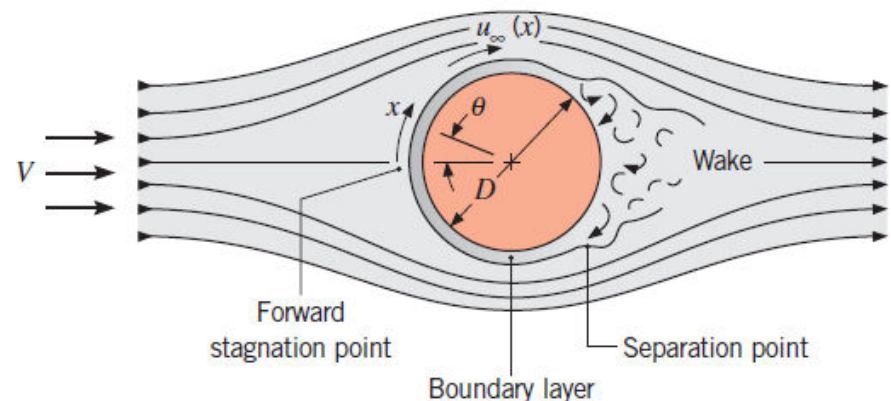
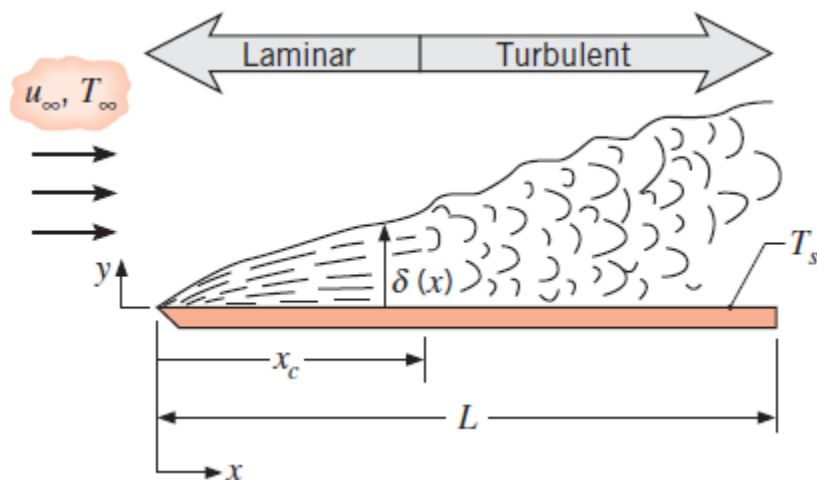
External flows – Solutions of convection equations for Flat plates,

Part 5

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Assumptions involved in deriving numerical relations for external flow

- In this module we focus on the problem of computing heat transfer rates to or from a surface in *external flow*. In such a flow boundary layers develop freely, without constraints imposed by adjacent surfaces
- Examples of external flow include fluid motion over a flat plate (inclined or parallel to the free stream velocity) and flow over curved surfaces such as a sphere, cylinder, airfoil, or turbine blade
- The derivations here are confined to problems of low-speed, forced convection with no phase change occurring within the fluid
- In forced convection, the relative motion between the fluid and the surface is maintained by external means, such as a fan or a pump, and not by buoyancy forces due to temperature gradients in the fluid



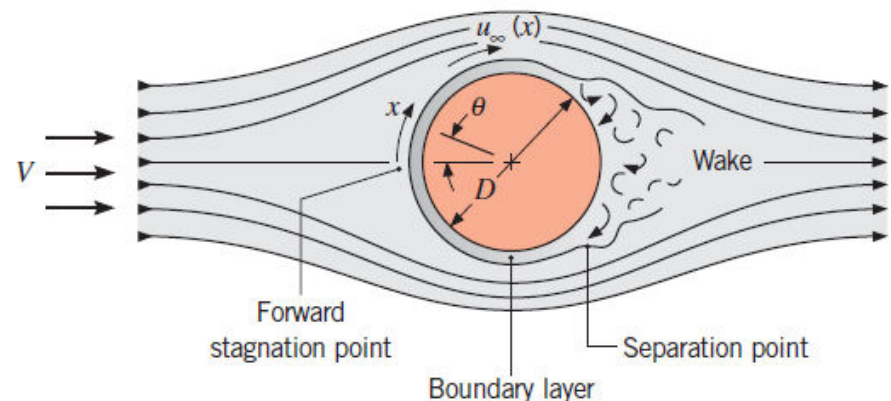
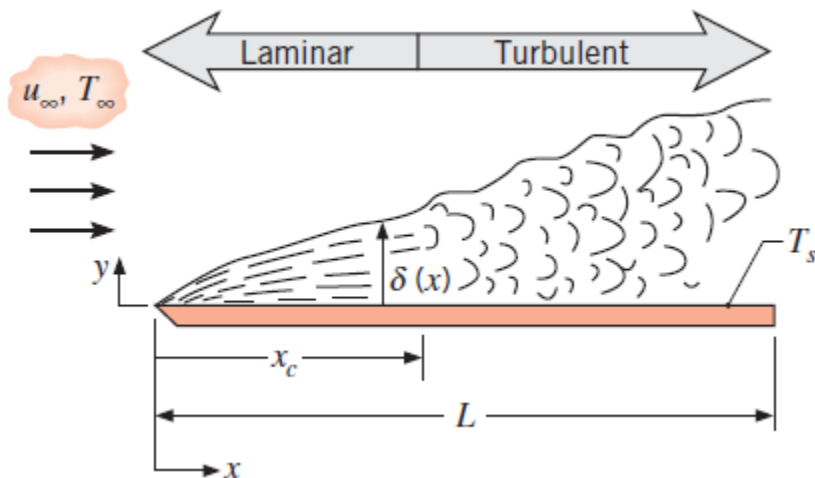
Assumptions involved in deriving numerical relations for external flow

- The primary objective is to determine convection coefficients for different flow geometries. In particular, we wish to obtain specific forms of the functions that represent these coefficients

$$Nu_x = f(x^*, Re_x, Pr)$$

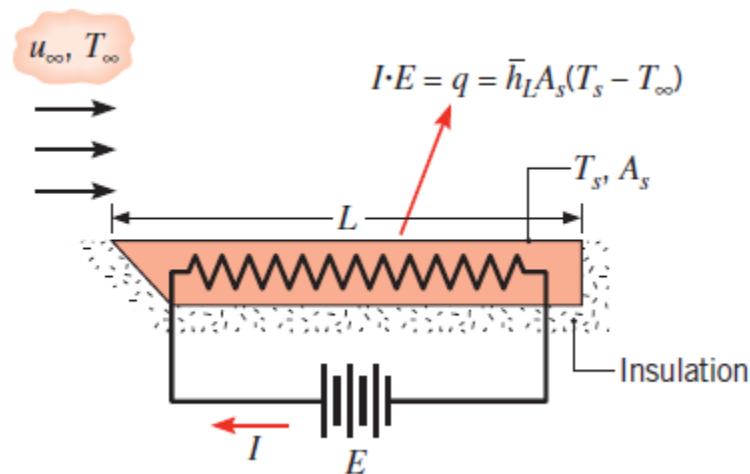
$$\overline{Nu}_x = f(Re_x, Pr)$$

- The subscript x emphasizes conditions at a particular location on the surface. The overbar indicates an average from $x^* = 0$, where the boundary layer begins to develop, to the location of interest
- The theoretical approach involves solving the boundary layer equations for a particular geometry. For example, obtaining the temperature profile T^* from such a solution



Experimental method to determine Nusselt number

- If a prescribed geometry, such as the flat plate in parallel flow, is heated electrically to maintain $T_s > T_\infty$, convection heat transfer occurs from the surface to the fluid
- It would be a simple matter to measure T_s and T_∞ , as well as the electrical power, $E \cdot I$, which is equal to the total heat transfer rate q . The convection coefficient $\bar{h}_L$, which is an average associated with the entire plate, could then be computed from Newton's law of cooling
- From knowledge of the characteristic length L and the fluid properties, the Nusselt, Reynolds, and Prandtl numbers could be computed from their definitions
- *The foregoing procedure could be repeated for a variety of test conditions. We could vary the velocity u_∞ and the plate length L , as well as the nature of the fluid, using, for example, air, water, and engine oil, which have substantially different Prandtl numbers*

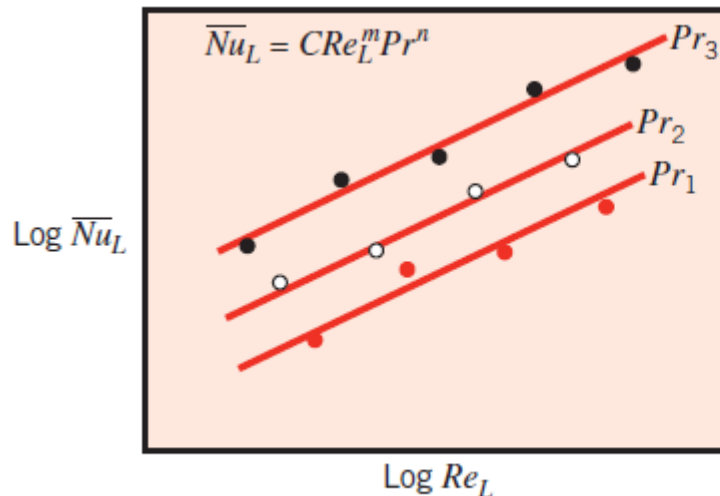


Experimental method to determine Nusselt number

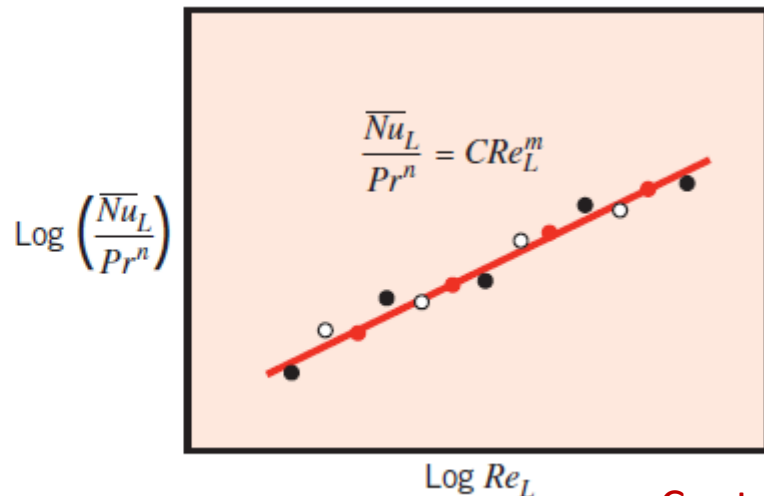
- By experimenting with many different values of the Nusselt numbers corresponding to a wide range of Reynolds and Prandtl numbers, the results could be plotted on a log–log scale, as shown in Fig. a
- The results associated with a given fluid, and hence a fixed Prandtl number, fall close to a straight line, indicating a power law dependence of the Nusselt number on the Reynolds number. Considering all the fluids, the data may then be represented by an algebraic expression of the form:

$$\overline{Nu}_L = C Re_L^m Pr^n$$

- Since the values of C , m , and n are often independent of the nature of the fluid, the family of straight lines corresponding to different Prandtl numbers can be collapsed to a single line by plotting the results in terms of the ratio, $\tilde{Nu}_L / Pr^n$, as shown in Fig. b



(a)



(b)

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Experimental method to determine Nusselt number

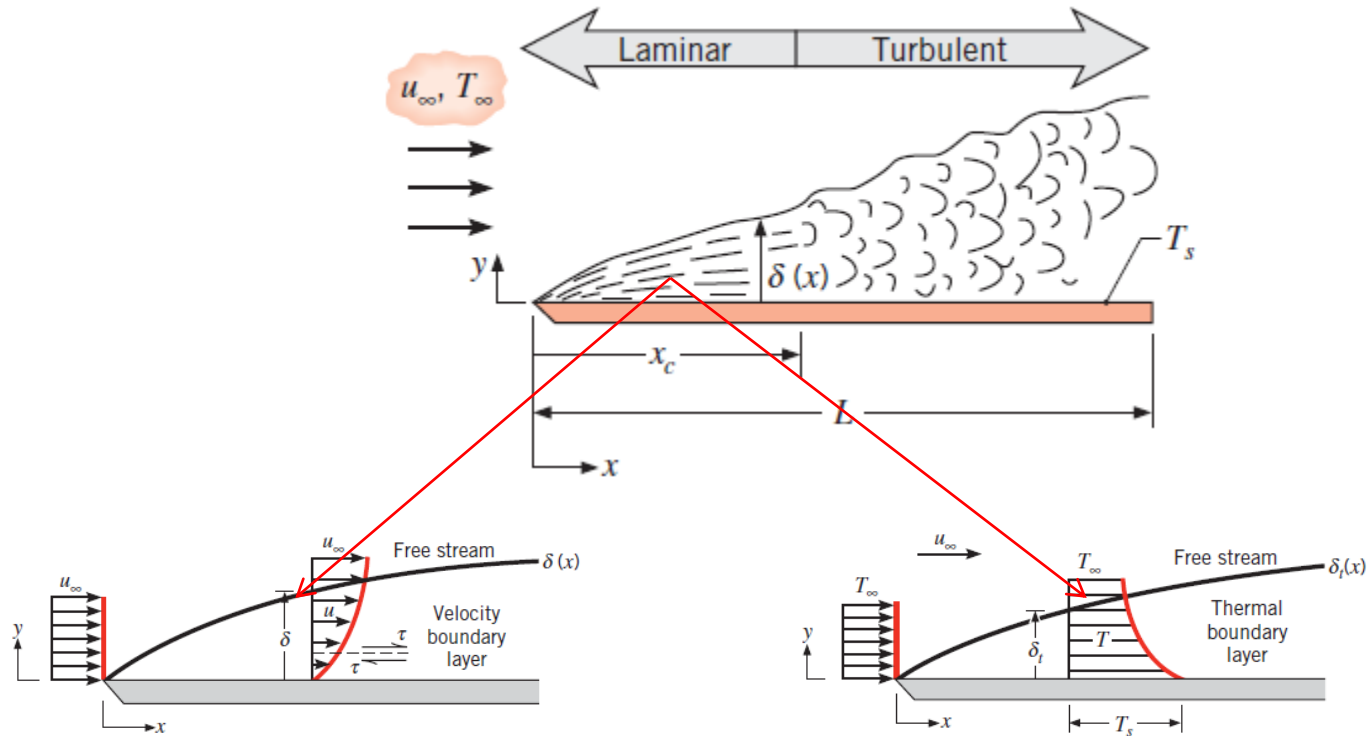
$$\overline{Nu}_L = C Re_L^m Pr^n$$

- The above numerical expression for average Nusselt number will be invoked for many special cases, and it is important to note that the assumption of *constant fluid properties* is often implicit in the results
- As the fluid properties vary with temperature across the boundary layer and this variation can certainly influence the heat transfer rate. Hence such variation is catered for in the above numerical expression in two ways as below:
- In first, average Nusselt number is used with all properties evaluated at a mean boundary layer temperature T_f , termed the *film temperature*

$$T_f \equiv \frac{T_s + T_\infty}{2}$$

- The alternate method is to evaluate all properties at T_∞ and to multiply the right-hand side of the above Nusselt number Equation by an additional parameter to account for the property variations.
- The parameter is commonly of the form $(Pr_\infty/Pr_s)^r$ or $(\mu_\infty/\mu_s)^r$, where the subscripts ∞ and s designate evaluation of the properties at the free stream and surface temperatures, respectively

Derivation for a laminar flow over isothermal flat plate: Methodology



- ❖ The first step is analytical determination of the velocity and temperature distributions in the laminar boundary layers
- ❖ From the knowledge of these distributions, expressions for local, average friction coefficients and Nusselt numbers can be determined

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Derivation for a laminar flow over isothermal flat plate : A similarity solution

- The major convection parameters may be obtained by solving the appropriate form of the boundary layer equations
- *Assuming steady, incompressible, laminar flow with constant fluid properties, negligible viscous dissipation and recognizing that $dp/dx = 0$, the boundary layer equations can be formulated as below:*

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

Energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

- Solution of these equations is simplified by the fact that for constant properties, conditions in the velocity (hydrodynamic) boundary layer are independent of temperature and species concentration
- Once the hydrodynamic problem is solved momentum and energy equations can be solved easily

Hydrodynamic solution to the flat plate

- *Hydrodynamic solution for the flat plate problem is obtained by the method of Blasius, a German Engineer who invented this solution. The first step in this method is to define a stream function $\Psi(x, y)$ such that:*

$$u \equiv \frac{\partial \psi}{\partial y} \quad \text{and} \quad v \equiv -\frac{\partial \psi}{\partial x}$$

- Based on the stream function assumption, continuity equation, is automatically satisfied and no longer needed.
- Next new dependent and independent variables, f and η , respectively, are then defined as follows:

$$f(\eta) \equiv \frac{\psi}{u_{\infty} \sqrt{\nu x / u_{\infty}}}$$
$$\eta \equiv y \sqrt{u_{\infty} / \nu x}$$

- *This introduction of new independent variable, η called similarity variable allows transformation of two partial differential equations (momentum and energy equations) into a single ordinary differential equation*

- **Blasius reasoned that the nondimensional velocity profile u/u_{∞} should remain unchanged when plotted against the nondimensional distance y/δ , where δ is the thickness of the local velocity boundary layer at a given x**

Hydrodynamic solution to the flat plate

- The Blasius solution is termed a *similarity solution*, and η is a *similarity variable*. This terminology is used because, despite growth of the boundary layer with distance x from the leading edge, the velocity profile u/u_∞ remains *geometrically similar*

$$\frac{u}{u_\infty} = \phi\left(\frac{y}{\delta}\right) \quad \Rightarrow \quad \frac{u}{u_\infty} = \phi(\eta)$$

- Since it has been proved by Stokes earlier that boundary layer thickness δ is proportional to $\sqrt{\frac{\nu x}{u_\infty}}$ it leads to expression for similarity variable η

$$f(\eta) \equiv \frac{\psi}{u_\infty \sqrt{\nu x / u_\infty}}$$

$$\eta \equiv y \sqrt{u_\infty / \nu x}$$

- As per definition of stream function the velocity components u and v are:

$$u \equiv \frac{\partial \psi}{\partial y} \quad \text{and} \quad v \equiv -\frac{\partial \psi}{\partial x}$$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \frac{\partial \eta}{\partial y} = u_\infty \sqrt{\frac{\nu x}{u_\infty}} \frac{df}{d\eta} \sqrt{\frac{u_\infty}{\nu x}} = u_\infty \frac{df}{d\eta} \quad v = -\frac{\partial \psi}{\partial x} = -\left(u_\infty \sqrt{\frac{\nu x}{u_\infty}} \frac{\partial f}{\partial x} + \frac{u_\infty}{2} \sqrt{\frac{\nu}{u_\infty x}} f \right)$$

Contd-----

Hydrodynamic solution to the flat plate

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \frac{\partial \eta}{\partial y} = u_{\infty} \sqrt{\frac{\nu x}{u_{\infty}}} \frac{df}{d\eta} \sqrt{\frac{u_{\infty}}{\nu x}} = u_{\infty} \frac{df}{d\eta} \quad v = -\frac{\partial \psi}{\partial x} = -\left(u_{\infty} \sqrt{\frac{\nu x}{u_{\infty}}} \frac{\partial f}{\partial x} + \frac{u_{\infty}}{2} \sqrt{\frac{\nu}{u_{\infty} x}} f \right)$$

Velocity component v can be simplified as:

$$v = \frac{1}{2} \sqrt{\frac{\nu u_{\infty}}{x}} \left(\eta \frac{df}{d\eta} - f \right)$$

By differentiating the velocity components, it may also be shown that:

$$\frac{\partial u}{\partial x} = -\frac{u_{\infty}}{2x} \eta \frac{d^2 f}{d\eta^2}$$

$$\frac{\partial u}{\partial y} = u_{\infty} \sqrt{\frac{u_{\infty}}{\nu x}} \frac{d^2 f}{d\eta^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_{\infty}^2}{\nu x} \frac{d^3 f}{d\eta^3}$$

Substituting the above terms in momentum equation :

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

Hydrodynamic solution to the flat plate

Hence the modified momentum equation can be written as

$$2\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} = 0$$

Hence the hydrodynamic boundary layer problem is reduced to one of solving a nonlinear, third-order ordinary differential equation. The appropriate boundary conditions are:

$$u(x, 0) = v(x, 0) = 0 \quad \text{and} \quad u(x, \infty) = u_\infty$$

In term of similarity variables the boundary conditions are:

$$\left. \frac{df}{d\eta} \right|_{\eta=0} = f(0) = 0 \quad \text{and} \quad \left. \frac{df}{d\eta} \right|_{\eta \rightarrow \infty} = 1$$

The solution to the differential equation, subject to the conditions above, may be obtained by a series expansion or by numerical integration

$$\delta = \frac{5.0}{\sqrt{u_\infty/\nu x}} = \frac{5.0x}{\sqrt{\text{Re}_x}}$$

η	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
∞	∞	1	0

Flat plate laminar
boundary layer functions

Contd----

Hydrodynamic solution to the flat plate

$$\delta = \frac{5.0}{\sqrt{u_\infty/\nu x}} = \frac{5.0x}{\sqrt{\text{Re}_x}}$$

- *The boundary layer thickness increases with increasing kinematic viscosity ν and with increasing distance from the leading edge but decreases with increasing free stream velocity. Hence a large free stream velocity will suppress the boundary layer and causes it to be thinner*
- *The shear stress on the wall can be determined from its definition and the $\partial u/\partial y$ relation:*

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}} \left. \frac{d^2 f}{d\eta^2} \right|_{\eta=0}$$

Substituting the value of the second derivative of f at $\eta = 0$ from the table:

$$\tau_w = 0.332 u_\infty \sqrt{\frac{\rho \mu u_\infty}{x}} = \frac{0.332 \rho u_\infty^2}{\sqrt{\text{Re}_x}}$$

Then the local skin friction coefficient becomes:

$$C_{f,x} = \frac{\tau_w}{\rho V^2/2} = \frac{\tau_w}{\rho u_\infty^2/2} = 0.664 \text{Re}_x^{-1/2}$$

η	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2 f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
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∞	∞	1	0

Flat plate laminar
boundary layer functions

Solution to the thermal energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

- From knowledge of conditions in the velocity boundary layer, the energy equation can now be solved
- By introducing a dimensionless temperature, T^* and assuming a similarity solution of the form: $T^* = T(\eta)$

$$T^* \equiv [(T - T_s)/(T_\infty - T_s)]$$

- The energy equation which originally is a partial differential equation reduces to:

$$\frac{d^2 T^*}{d\eta^2} + \frac{Pr}{2} f \frac{dT^*}{d\eta} = 0$$

- Note the dependence of the thermal solution on hydrodynamic conditions through appearance of the variable f in the above equation. The appropriate boundary conditions are:

$$T^*(0) = 0 \quad \text{and} \quad T^*(\infty) = 1$$

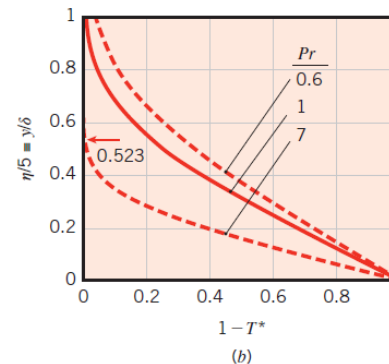
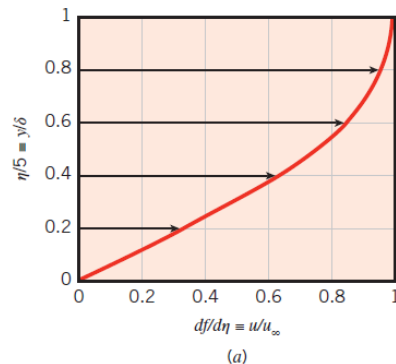
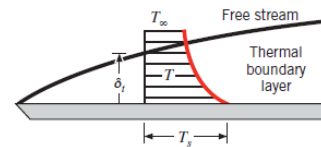
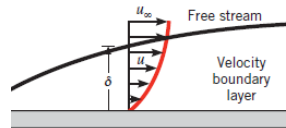
Solution to the thermal energy equation

$$\frac{d^2 T^*}{d\eta^2} + \frac{Pr}{2} f \frac{dT^*}{d\eta} = 0$$

$$T^*(0) = 0 \quad \text{and} \quad T^*(\infty) = 1$$

- Energy equation may be solved by numerical integration for different values of the Prandtl number; representative temperature distributions are shown below:
- The temperature distribution is identical to the velocity distribution for $Pr = 1$. Thermal effects penetrate farther into the velocity boundary layer with decreasing Prandtl number and transcend the velocity boundary layer for $Pr < 1$
- A practical consequence of this solution is that, for $Pr \geq 0.6$, results for the surface temperature gradient $dT^*/d\eta @ \eta = 0$ may be correlated by the following relation

$$\left. \frac{dT^*}{d\eta} \right|_{\eta=0} = 0.332 Pr^{1/3}$$



Similarity solution for laminar flow over an isothermal plate. (a) The x -component of the velocity. (b) Temperature distributions for $Pr = 0.6, 1$, and 7

Solution to the thermal energy equation

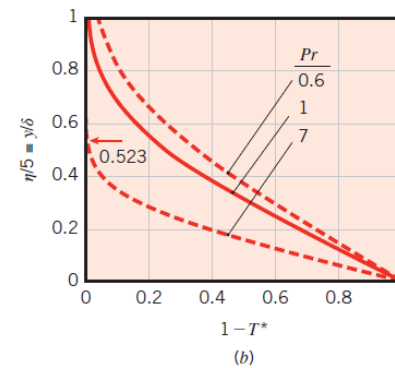
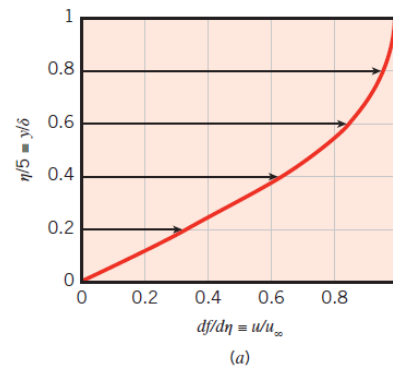
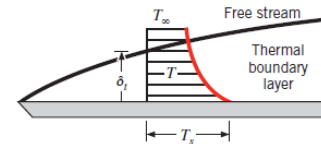
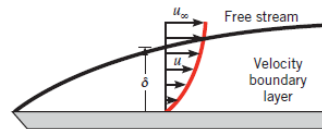
$$h_x = \frac{q_s''}{T_s - T_\infty} = -\frac{T_\infty - T_s}{T_s - T_\infty} k \left. \frac{\partial T^*}{\partial y} \right|_{y=0}$$

$$h_x = k \left(\frac{u_\infty}{\nu x} \right)^{1/2} \left. \frac{dT^*}{d\eta} \right|_{\eta=0}$$

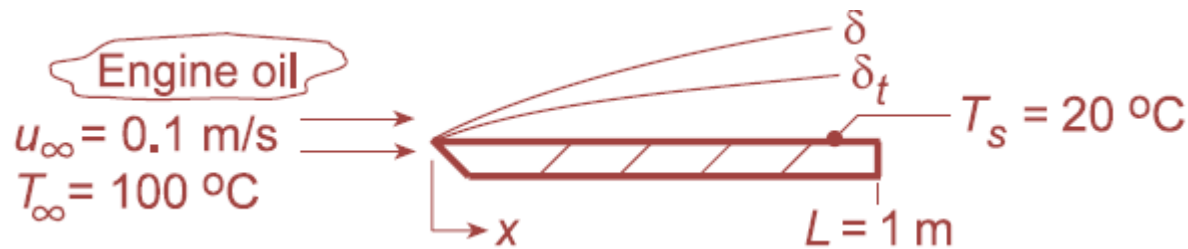
$$Nu_x \equiv \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3} \quad Pr \gtrsim 0.6$$

The ratio of the velocity to thermal boundary layer thickness is

$$\frac{\delta}{\delta_t} \approx Pr^{1/3}$$



Problem: Engine oil at 100°C and a velocity of 0.1 m/s flows over both surfaces of a 1m-long flat plate maintained at 20°C. Determine a) The velocity and thermal boundary layer thicknesses at the trailing edge b) The local heat flux and surface shear stress at the trailing edge



PROPERTIES: Table A.5, Engine Oil ($T_f = 333 \text{ K}$): $\rho = 864 \text{ kg/m}^3$, $\nu = 86.1 \times 10^{-6} \text{ m}^2/\text{s}$, $k = 0.140 \text{ W/m}\cdot\text{K}$, $\text{Pr} = 1081$.

Solution: : Calculate the Reynolds number to determine nature of the flow

$$Re_L = \frac{u_\infty L}{\nu} = \frac{0.1 \text{ m/s} \times 1 \text{ m}}{86.1 \times 10^{-6} \text{ m}^2/\text{s}} = 1161$$

At $x = L$ the flow is laminar, hence boundary layer thickness can be calculated as:

$$\delta = 5L Re_L^{-1/2} = 5(1 \text{ m})(1161)^{-1/2} = 0.147 \text{ m}$$

$$\delta_t = \delta Pr^{-1/3} = 0.147 \text{ m}(1081)^{-1/3} = 0.0143 \text{ m}$$

The velocity and thermal boundary layers are given by expressions:

$$\delta = 5L Re_L^{-1/2} = 5(1 \text{ m})(1161)^{-1/2} = 0.147 \text{ m}$$

$$\delta_t = \delta Pr^{-1/3} = 0.147 \text{ m}(1081)^{-1/3} = 0.0143 \text{ m}$$

The local and convective heat flux are given by:

$$h_L = \frac{k}{L} 0.332 Re_L^{1/2} Pr^{1/3} = \frac{0.140 \text{ W/m} \cdot \text{K}}{1 \text{ m}} 0.332 (1161)^{1/2} (1081)^{1/3} = 16.25 \text{ W/m}^2 \cdot \text{K}$$

$$q''_X = h_L (T_s - T_\infty) = 16.25 \text{ W/m}^2 \cdot \text{K} (20 - 100)^\circ \text{C} = -1300 \text{ W/m}^2$$

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Also, the local shear stress is:

$$\tau_{s,L} = \frac{\rho u_{\infty}^2}{2} 0.664 \text{Re}_L^{-1/2} = \frac{864 \text{kg/m}^3}{2} (0.1 \text{m/s})^2 0.664 (1161)^{-1/2}$$
$$\tau_{s,L} = 0.0842 \text{kg/m} \cdot \text{s}^2 = 0.0842 \text{N/m}^2$$

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MODULE III FORCED CONVECTION



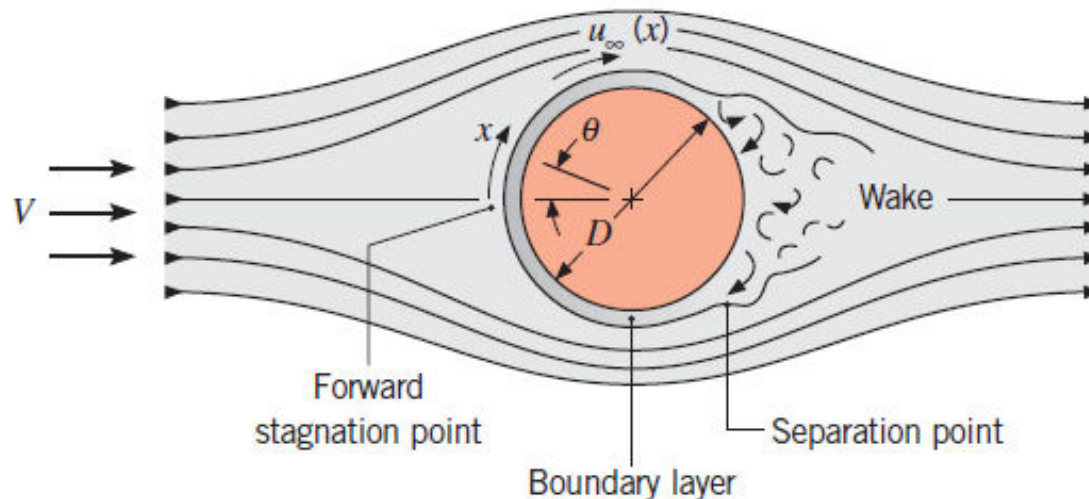
External flows – Convective heat transfer correlations for Cylinders and Spheres

Part 6

Dr. Satish Idury
Associate Professor

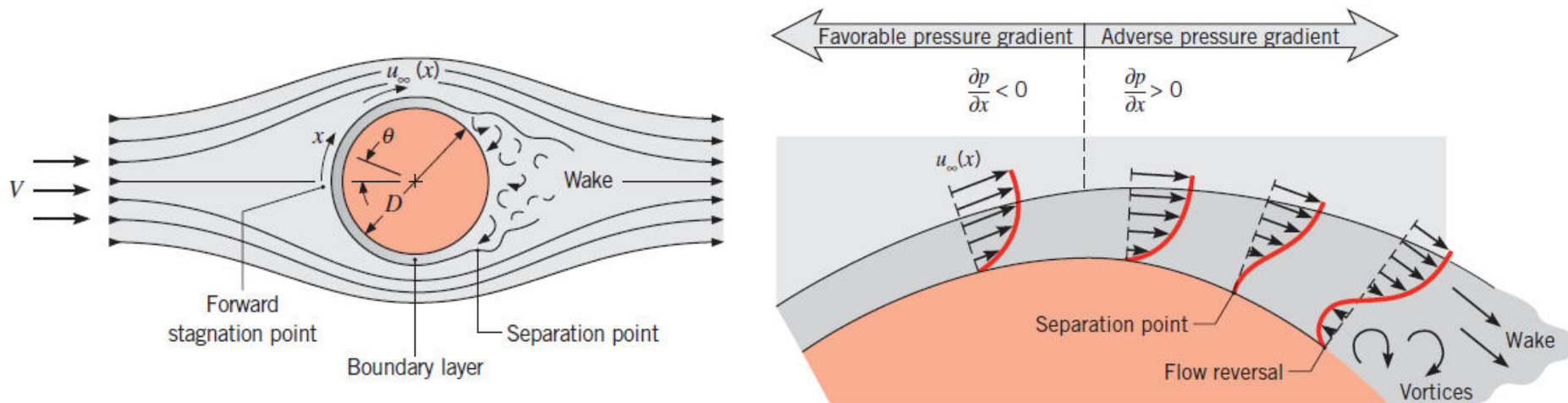
The cylinder in a cross flow

- As shown in Fig. the free stream fluid is brought to rest at the *forward stagnation point*, with an accompanying rise in pressure
- From this forward stagnation point, the pressure decreases with increasing x , the streamline coordinate, and the boundary layer develops under the influence of a *favorable pressure gradient* ($dp/dx < 0$)
- The pressure will eventually reach a minimum, and toward the rear of the cylinder further boundary layer development occurs in the presence of an *adverse pressure gradient* ($dp/dx > 0$)
- Unlike conditions for the flat plate in parallel flow, upstream velocity, V differ, with u_∞ now depending on the distance x from the stagnation point



The cylinder in a cross flow

- From Euler's equation for an Inviscid flow, $u_\infty(x)$ must exhibit behavior opposite to that of $p(x)$. That is, from $u_\infty = 0$ at the stagnation point, the fluid accelerates because of the favorable pressure gradient ($du_\infty/dx > 0$ when $dp/dx < 0$), reaches a maximum velocity when $dp/dx = 0$, and decelerates because of the adverse pressure gradient ($du_\infty/dx < 0$ when $dp/dx > 0$)
- As the fluid decelerates, the velocity gradient at the surface, $\partial u/\partial y$ at $y=0$, eventually becomes zero. At this location, termed the *separation point*, fluid near the surface lacks sufficient momentum to overcome the pressure gradient, and continued downstream movement is impossible
- Since the oncoming fluid also precludes flow back upstream, *boundary layer separation* must occur. This is a condition for which the boundary layer detaches from the surface, and a *wake* is formed in the downstream region. Flow in this region is characterized by vortex formation and is highly irregular



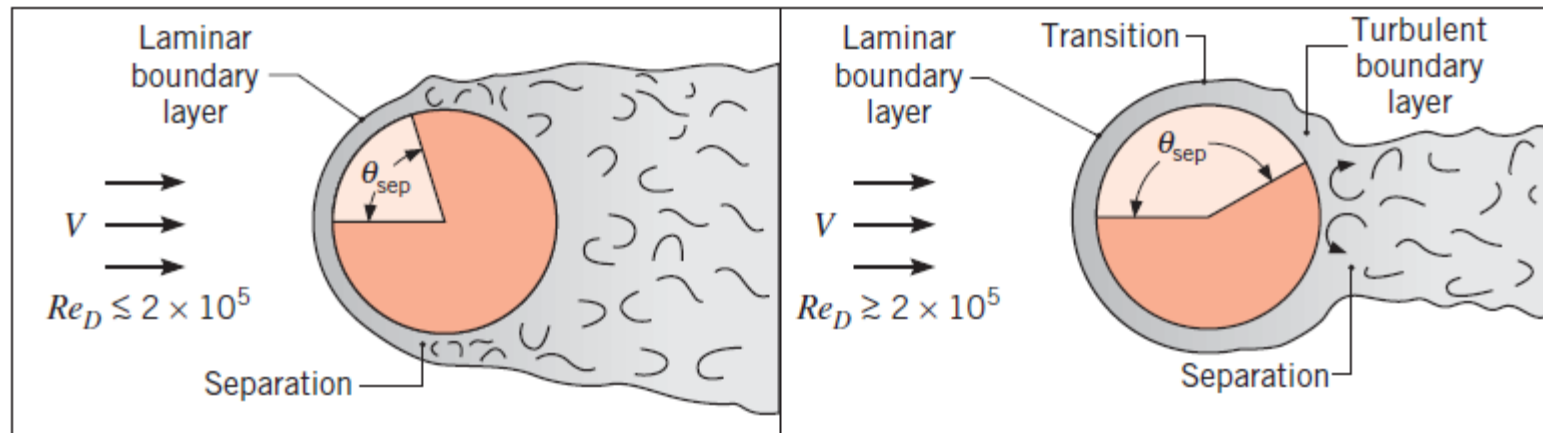
Boundary layer formation and separation on a circular cylinder in cross flow

The cylinder in a cross flow

- The occurrence of *boundary layer transition*, which depends on the Reynolds number, strongly influences the position of the separation point. For the circular cylinder the characteristic length is the diameter, and the Reynolds number is defined as:

$$Re_D \equiv \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

- Since the momentum of fluid in a turbulent boundary layer is larger than in the laminar boundary layer, it is reasonable to expect transition to delay the occurrence of separation
- If $Re_D \leq 2 \times 10^5$, the boundary layer is laminar, and separation occurs at $\theta \approx 80^\circ$). However, if $Re_D \geq 2 \times 10^5$, boundary layer transition occurs, and separation is delayed to $\theta \approx 140^\circ$



The effect of turbulence on separation

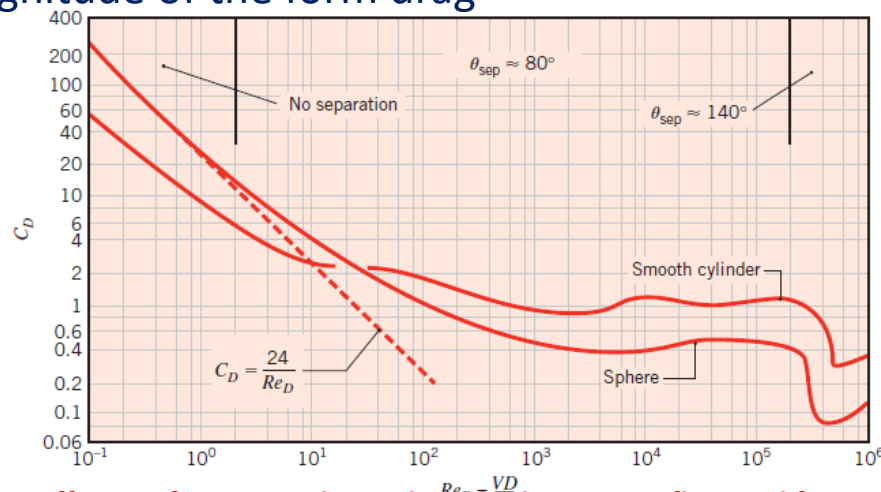
The cylinder in a cross flow

- Boundary layer transition processes strongly influence the drag force, F_D , acting on the cylinder. This force has two components, one of which is due to the boundary layer surface shear stress (*friction drag*). The other component is due to a pressure differential in the flow direction resulting from formation of the wake (*form, or pressure, drag*). A dimensionless *drag coefficient* C_D may be defined as

$$C_D \equiv \frac{F_D}{A_f(\rho V^2/2)}$$

A_f is the cylinder frontal area (the area projected perpendicular to the free stream velocity)

- The large reduction in C_D that occurs for $Re_D \geq 2 \times 10^5$ is due to boundary layer transition, which delays separation, thereby reducing the extent of the wake region and the magnitude of the form drag

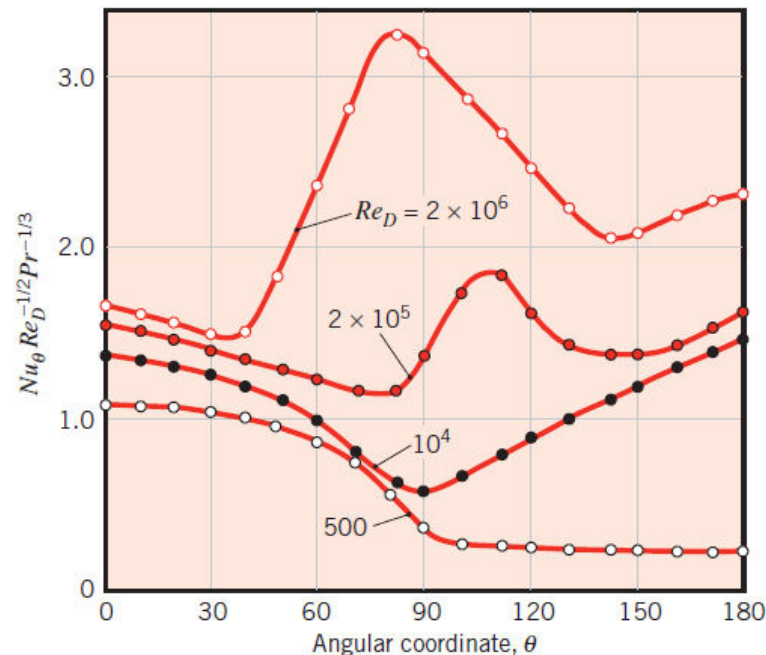


Drag coefficients for a smooth circular cylinder in cross flow and for a sphere.

Boundary layer separation angles are for a cylinder

The cylinder in a cross flow – Convection heat transfer correlation

- Experimental results for the variation of the local Nusselt number with θ are shown in Fig. for the cylinder in a cross flow of air . Nusselt number strongly varies with boundary layer development on the surface
- For $Re_D \leq 10^5$, starting at the stagnation point, Nu_θ decreases with increasing θ as a result of laminar boundary layer development. However, a minimum is reached at $\theta \approx 80^\circ$, where separation occurs and Nu_θ increases with θ due to mixing associated with vortex formation in the wake
- For $Re_D \geq 10^5$, variation of Nusselt number is characterized by two minima. The increase in Nu_θ with increasing Re_D is due to corresponding reduction in boundary layer thickness



Local Nusselt number for airflow normal to circular cylinder

The cylinder in a cross flow – Correlation relations

At the forward stagnation point for $Pr \geq 0.6$, boundary layer analysis yields an expression:

$$Nu_D(\theta = 0) = 1.15 Re_D^{1/2} Pr^{1/3}$$

An empirical correlation due to Hilpert that has been modified to account for fluids of various Prandtl numbers for $Pr \geq 0.7$: The above equation is applicable for cylinders as well as non-circular cross sections. In this equation all the properties are evaluated at film temperature

$$\overline{Nu}_D \equiv \frac{\bar{h}D}{k} = C Re_D^m Pr^{1/3}$$

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

The cylinder in a cross flow – Correlation relations

Other correlations have been suggested for the circular cylinder in cross flow where all properties are evaluated at T_∞ except Pr_s which is evaluated at T_s . If $Pr \leq 10$, $n = 0.37$, If $Pr \geq 10$, $n = 0.36$

$$\overline{Nu}_D = C Re_D^m Pr^n \left(\frac{Pr}{Pr_s} \right)^{1/4}$$

$$\left[\begin{array}{l} 0.7 \lesssim Pr \lesssim 500 \\ 1 \lesssim Re_D \lesssim 10^6 \end{array} \right]$$

Geometry		Re_D	C	m
Square				
$V \rightarrow$ 		6000–60,000	0.304	0.59
$V \rightarrow$ 		5000–60,000	0.158	0.66
Hexagon				
$V \rightarrow$ 		5200–20,400	0.164	0.638
$V \rightarrow$ 		20,400–105,000	0.039	0.78
$V \rightarrow$ 		4500–90,700	0.150	0.638
Thin plate perpendicular to flow				
$V \rightarrow$ 		Front 10,000–50,000	0.667	0.500
		Back 7000–80,000	0.191	0.667

The cylinder in a cross flow – Correlation relations

Churchill and Bernstein have proposed a single comprehensive equation that covers the entire range of Re_D for which data are available, as well as a wide range of Pr . The equation is recommended for all Re_D , $Pr \geq 0.2$ and has the form:

$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282,000} \right)^{5/8} \right]^{4/5}$$

Each correlation is reasonable over a certain range of conditions, but for most engineering calculations one should not expect accuracy to much better than 20%.

Sphere – Correlation relations

Boundary layer effects associated with flow over a sphere are much like those for the circular cylinder, with transition and separation playing prominent roles. All the properties except μ_s are evaluated at T_∞

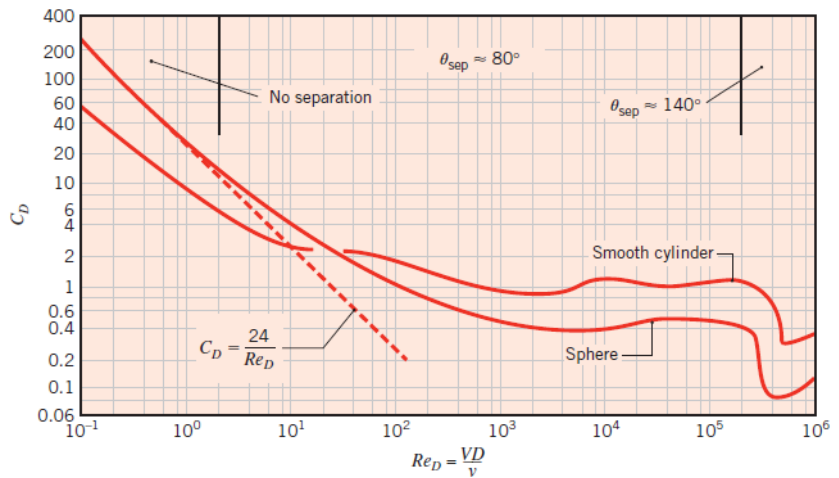
$$\overline{Nu}_D = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}) Pr^{0.4} \left(\frac{\mu}{\mu_s} \right)^{1/4}$$
$$\left[\begin{array}{l} 0.71 \lesssim Pr \lesssim 380 \\ 3.5 \lesssim Re_D \lesssim 7.6 \times 10^4 \\ 1.0 \lesssim (\mu/\mu_s) \lesssim 3.2 \end{array} \right]$$

In the limit of very small Reynolds numbers (creeping flow), the coefficient is inversely proportional to the Reynolds number and the specific relation is termed Stokes law

A special case of convection heat and mass transfer from spheres relates to transport from freely falling liquid drops is given by relation:



$$\overline{Nu}_D = 2 + 0.6 Re_D^{1/2} Pr^{1/3}$$



HEAT TRANSFER - 80329

MODULE III FORCED CONVECTION



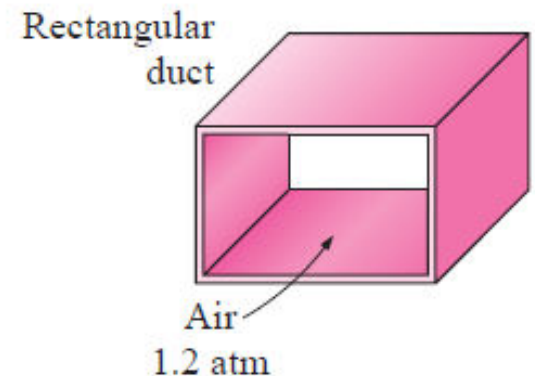
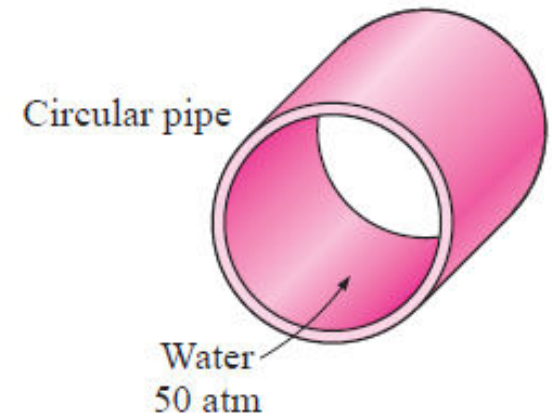
Internal flows – Concepts of hydrodynamic and thermal entry lengths, Flow through pipes and ducts and their empirical relations

Part 7

Dr. Satish Idury
Associate Professor

Introduction- Internal flow

- Most fluids, especially liquids, are transported in circular pipes. This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing any distortion
- Noncircular pipes are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small
- For a fixed surface area, the circular tube gives the most heat transfer for the least pressure drop, which explains the overwhelming popularity of circular tubes in heat transfer equipment
- Frictional heating must be considered for flows that involve highly viscous fluids with large velocity gradients
- In most practical applications, the flow of a fluid through a pipe or duct can be approximated to be *one-dimensional*, and thus the properties can be assumed to vary only in direction of flow



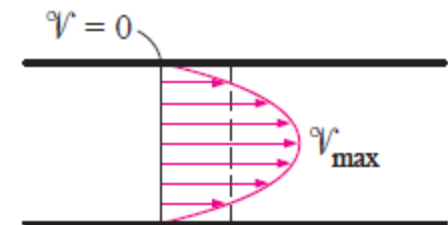
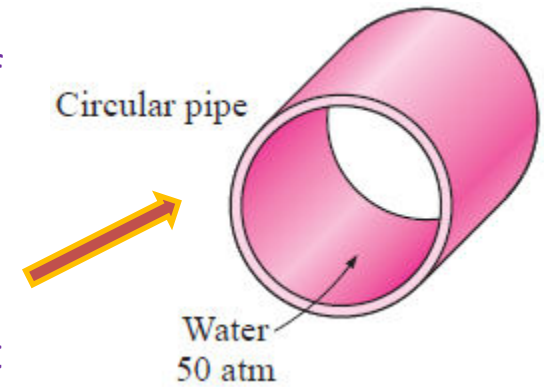
Mean velocity in a circular tube

- In external flow, the free-stream velocity served as a convenient reference velocity for use in the evaluation of the Reynolds number and the friction coefficient
- In internal flow, the fluid velocity in a tube changes from zero at the surface because of the no-slip condition, to a maximum at the tube center. Therefore, it is convenient to represent **average** or **mean velocity** V_m . For incompressible flow V_m is constant when cross sectional area is constant

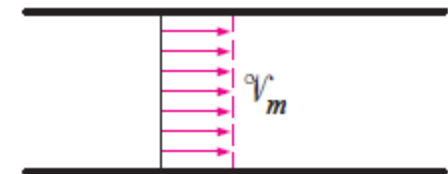
$$\dot{m} = \rho V_m A_c = \int_{A_c} \rho V(r, x) dA_c$$

where $\dot{m}$ is the mass flow rate, ρ is the density, A_c is the cross sectional area, and $V(r, x)$ is the velocity profile. Then the mean velocity for incompressible flow in a circular tube of radius R can be expressed as:

$$V_m = \frac{\int_{A_c} \rho V(r, x) dA_c}{\rho A_c} = \frac{\int_0^R \rho V(r, x) 2\pi r dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R V(r, x) r dr$$



(a) Actual



(b) Idealized

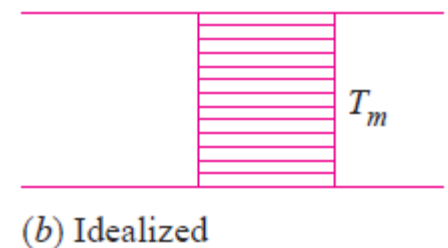
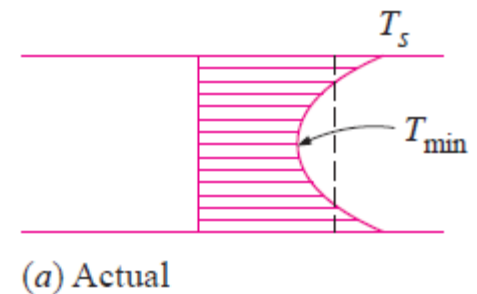
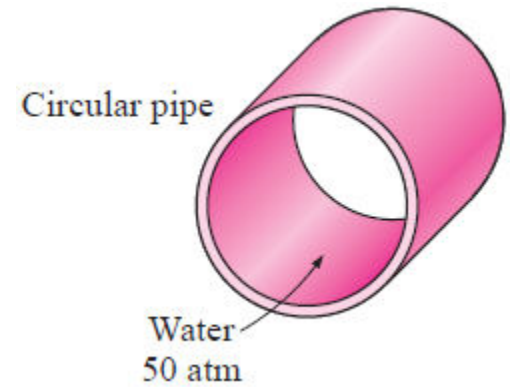
Mean temperature in a circular tube

- When a fluid is heated or cooled as it flows through a tube, the temperature of the fluid at any cross section changes from T_s at the surface of the wall to some maximum (or minimum in the case of heating) at the tube center. Therefore it is convenient **average** or **mean temperature** T_m that remains uniform at a cross section
- As per conservation of energy principle, energy transported by the fluid through a cross section in actual flow must be equal to the energy that would be transported through the same cross section if the fluid were at a constant temperature T_m

$$\dot{E}_{\text{fluid}} = \dot{m} C_p T_m = \int_{\dot{m}} C_p T \delta \dot{m} = \int_{A_c} \rho C_p T \mathcal{V} dA_c$$

where C_p is the specific heat of the fluid. Then the mean temperature of the fluid with constant density and specific heat flowing in circular pipe of radius R is:

$$T_m = \frac{\int_{\dot{m}} C_p T \delta \dot{m}}{\dot{m} C_p} = \frac{\int_0^R C_p T (\rho \mathcal{V} 2\pi r dr)}{\rho \mathcal{V}_m (\pi R^2) C_p} = \frac{2}{\mathcal{V}_m R^2} \int_0^R T(r, x) \mathcal{V}(r, x) r dr$$



Laminar and turbulent flow in the tubes

- Most pipe flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small diameter tubes or narrow passages

In Re calculation, $\bar{V}_m$ is the mean fluid velocity, D is the diameter of the tube, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid

- For flow through noncircular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the **hydraulic diameter** D_h defined as:

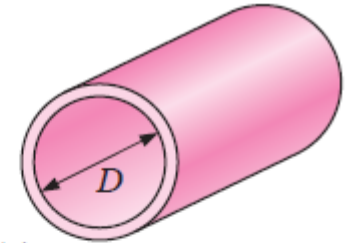
$$D_h = \frac{4A_c}{p}$$

where A_c is the cross sectional area of the tube and p is its perimeter

- Transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations, and the fluctuations in the flow*

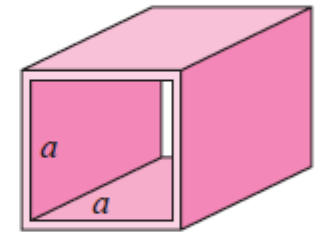
$$Re = \frac{\rho \bar{V}_m D}{\mu} = \frac{\bar{V}_m D}{\nu}$$

Circular tube:



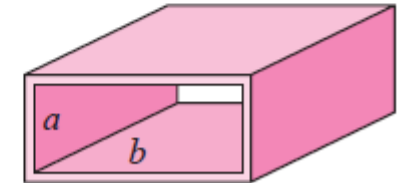
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

$Re < 2300$

laminar flow

$2300 \leq Re \leq 10,000$

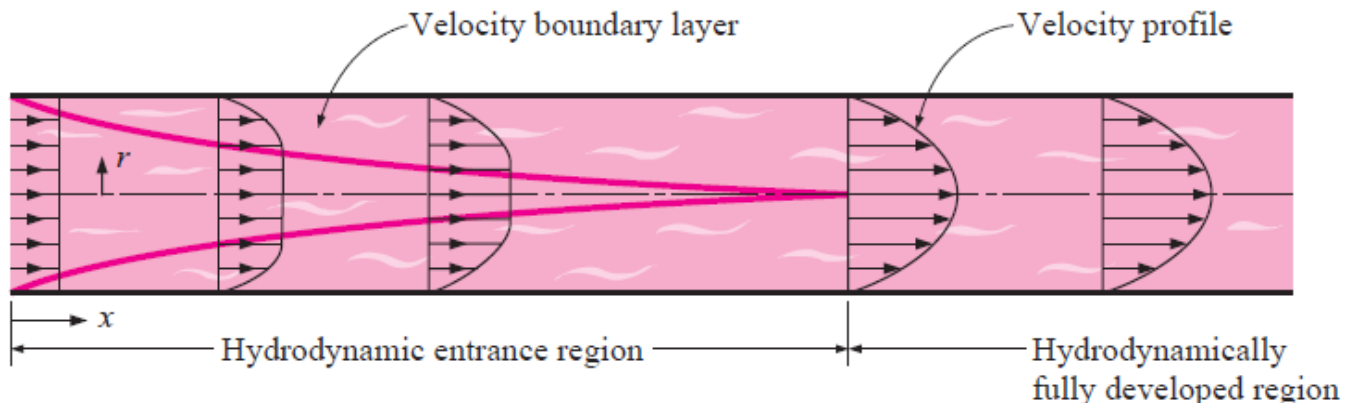
transitional flow

$Re > 10,000$

turbulent flow

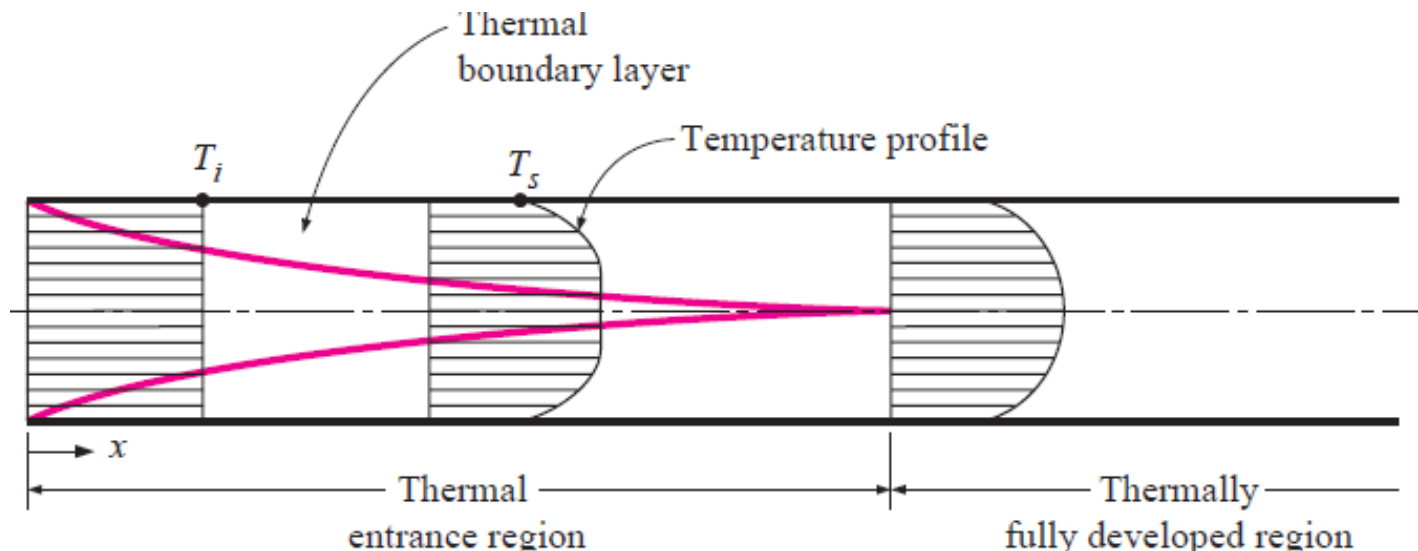
Hydrodynamic entrance region

- For a fluid entering circular tube at uniform velocity, the fluid particles in the layer in contact with the surface of the tube will come to a complete stop. Subsequently adjacent layers will slow down as a result of friction
- To make up for this velocity reduction, the velocity of the fluid at the midsection of the tube will have to increase to keep the mass flow rate through the tube constant. As a result, a *velocity boundary layer* develops along the tube
- The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the tube center and thus fills the entire tube
- The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the **hydrodynamic entrance region**, and the length of this region is called the **hydrodynamic entry length L_h**
- The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the **hydrodynamically fully developed region**. Here the velocity profile is parabolic in laminar flow and flatter in turbulent flow



Thermal entrance region

- Consider a fluid at a uniform temperature entering a circular tube whose surface is maintained at a different temperature. The fluid particles in the layer in contact with the surface of the tube will assume the surface temperature
- Due to convective heat transfer in the tube, thickness of thermal boundary layer formed also increases in the flow direction until the boundary layer reaches the tube center and thus fills the entire tube
- The region of flow over which the thermal boundary layer develops and reaches the tube center is called the **thermal entrance region**, and the length of this region is called the **thermal entry length L_t**
- The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $(T_s - T)/(T_s - T_m)$ remains unchanged is called the **thermally fully developed region**



Characteristics of hydrodynamically and thermally fully developed regions

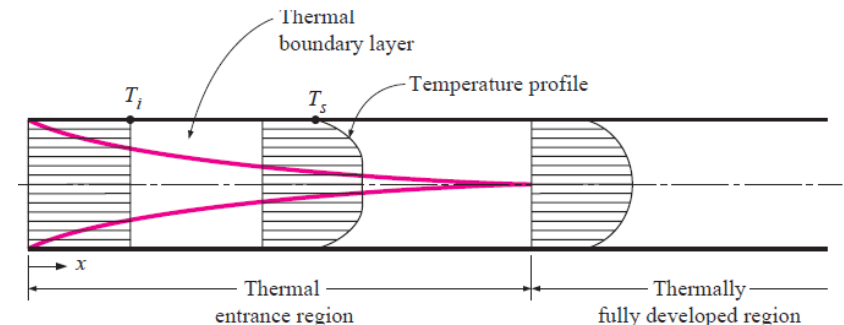
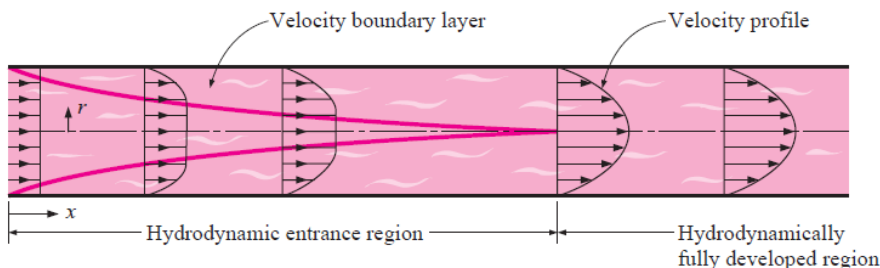
Hydrodynamically fully developed: $\frac{\partial \mathcal{V}(r, x)}{\partial x} = 0 \longrightarrow \mathcal{V} = \mathcal{V}(r)$

Thermally fully developed: $\frac{\partial}{\partial x} \left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)} \right] = 0$

- The friction factor is related to the shear stress at the surface, which is related to the slope of the velocity profile at the surface. Since the velocity profile remains unchanged in fully developed region, friction factor remains unchanged too. A similar argument can be given for heat transfer coefficient in fully developed region
- The derivate $(T_s - T)/(T_s - T_m)$ with respect to r must also be independent of x

$$\left. \frac{\partial}{\partial r} \left(\frac{T_s - T}{T_s - T_m} \right) \right|_{r=R} = \frac{-(\partial T / \partial r)|_{r=R}}{T_s - T_m} \neq f(x) \Rightarrow q_s = h_x(T_s - T_m) = k \left. \frac{\partial T}{\partial r} \right|_{r=R} \longrightarrow h_x = \frac{k(\partial T / \partial r)|_{r=R}}{T_s - T_m}$$

Both the friction and convection coefficients remain constant in the fully developed region of a tube



Characteristics of hydrodynamically and thermally fully developed regions

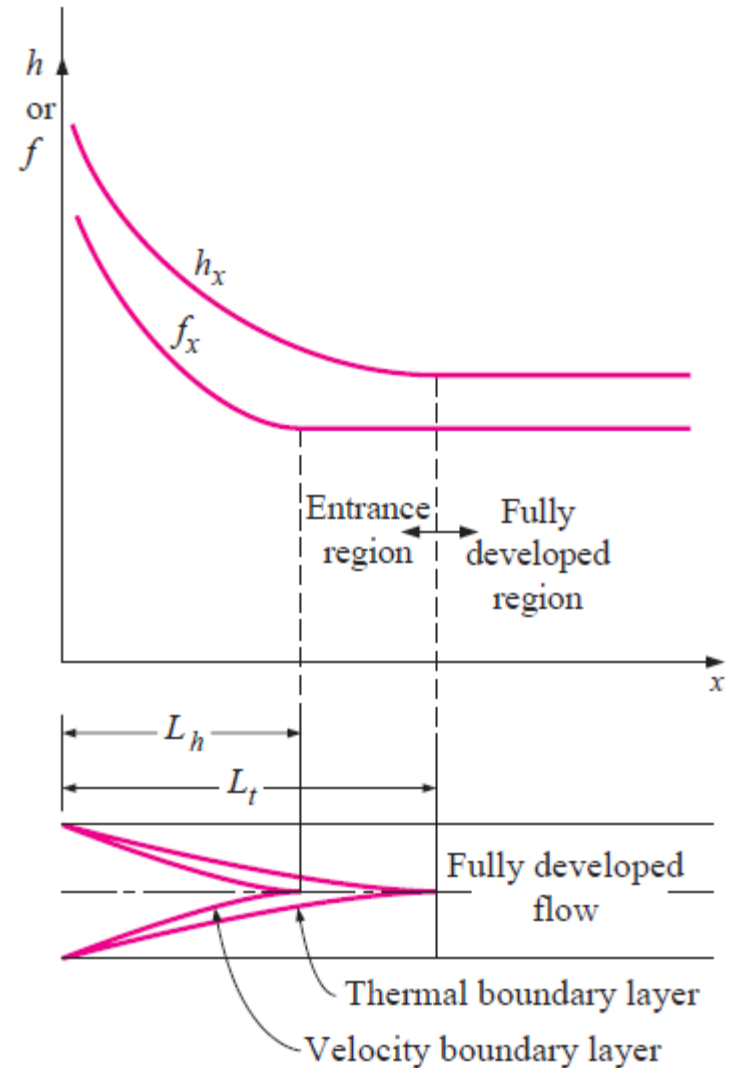
Hydrodynamically fully developed: $\frac{\partial \mathcal{V}(r, x)}{\partial x} = 0 \longrightarrow \mathcal{V} = \mathcal{V}(r)$

Thermally fully developed: $\frac{\partial}{\partial x} \left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)} \right] = 0$

- Unlike the velocity profile, the temperature profile can be different at different cross sections of the tube in the developed region, and it usually is. However, the dimensionless temperature profile defined above remains unchanged in the thermally developed region when the temperature or heat flux at the tube surface remains constant
- For fluids with $Pr \sim 1$, such as gases, the two boundary layers essentially coincide with each other. For fluids with $Pr \gg 1$, such as oils, the velocity boundary layer outgrows the thermal boundary layer. Hence hydrodynamic entry length is smaller than thermal entry length

Characteristics of hydrodynamically and thermally fully developed regions

- Consider a fluid that is being heated (or cooled) in a tube as it flows through it. The friction factor and the heat transfer coefficient are *highest* at the tube inlet where the thickness of the boundary layers is zero, and decrease gradually to the fully developed values
- Pressure drop and heat flux are *higher* in the entrance regions of a tube, and the effect of the entrance region is always to *enhance* the average friction and heat transfer coefficients for the entire tube
- This enhancement effect can be significant for short tubes but negligible for long ones



Magnitude of hydrodynamic and thermal entry lengths

In *laminar flow*, the hydrodynamic and thermal entry lengths are approximately:

$$L_{h, \text{ laminar}} \approx 0.05 \text{ Re } D$$

$$L_{t, \text{ laminar}} \approx 0.05 \text{ Re Pr } D = \text{Pr } L_{h, \text{ laminar}}$$

In the limiting case of $\text{Re} = 2300$, the hydrodynamic entry length is $115D$

- In *turbulent flow*, the intense mixing during random fluctuations usually overshadows the effects of momentum and heat diffusion, and therefore the hydrodynamic and thermal entry lengths are of about the same size and independent of the Prandtl number.
- Also, *the friction factor and the heat transfer coefficient remain constant in fully developed laminar or turbulent flow* since the velocity and normalized temperature profiles do not vary in the flow direction

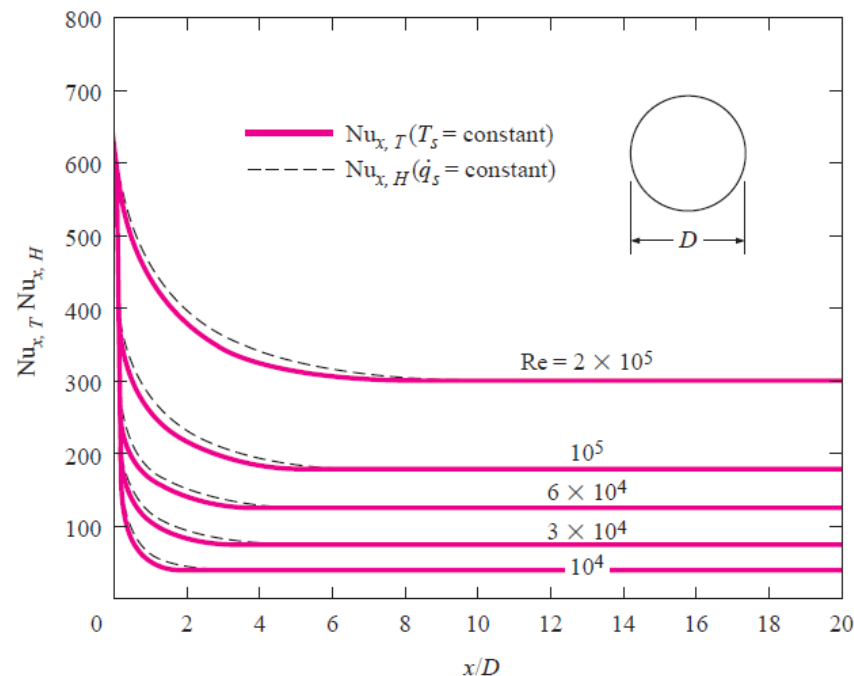
$$L_{h, \text{ turbulent}} = 1.359 \text{ Re}^{1/4}$$

In practice, it is generally agreed that the entrance effects are confined to be:

$$L_{h, \text{ turbulent}} \approx L_{t, \text{ turbulent}} \approx 10D$$

Variation of Nusselt number in a turbulent flow

- The variation of local Nusselt number along a tube in turbulent flow for both uniform surface temperature and uniform surface heat flux is given below for the range of Reynolds numbers encountered in heat transfer equipment
- The Nusselt numbers and thus the convection heat transfer coefficients are much higher in the entrance region
- The Nusselt number reaches a constant value for $x > 10D$
- The Nusselt numbers for the uniform surface temperature and uniform surface heat flux conditions are identical in the fully developed regions, and nearly identical in the entrance regions. Therefore, Nusselt number is insensitive to the type of thermal boundary condition



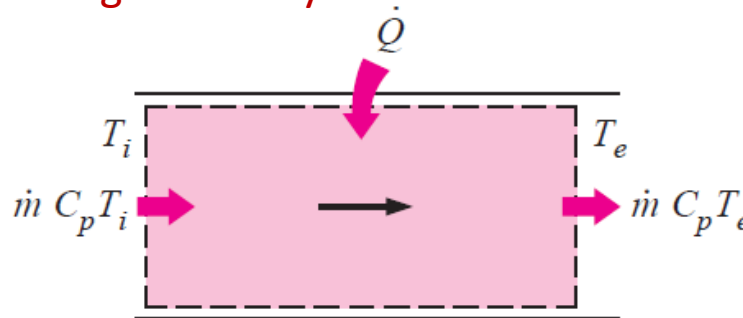
General thermal analysis of heat transfer to a fluid flowing in a tube

- The conservation of energy equation for the steady flow of a fluid in a tube is:

$$\dot{Q} = \dot{m} C_p (T_e - T_i)$$

where T_i and T_e are the mean fluid temperatures at the inlet and exit of the tube, respectively, and $\dot{Q}$ is the rate of heat transfer to or from the fluid. The temperature of a fluid flowing in a tube remains constant in the absence of any energy interactions through the wall of the tube

- The thermal conditions at the surface can usually be approximated with reasonable accuracy to be *constant surface temperature* ($T_s = \text{constant}$) or *constant surface heat flux* $q_s = \text{constant}$
- Constant temperature condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube. The constant surface heat flux condition is realized when the tube is subjected to radiation or electric resistance heating uniformly from all directions



Energy balance:

$$\dot{Q} = \dot{m} C_p (T_e - T_i)$$

Constant Surface Heat Flux ($\dot{q}_s = \text{constant}$)

In the case of $q_s = \text{constant}$, the rate of heat transfer:

$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i)$$

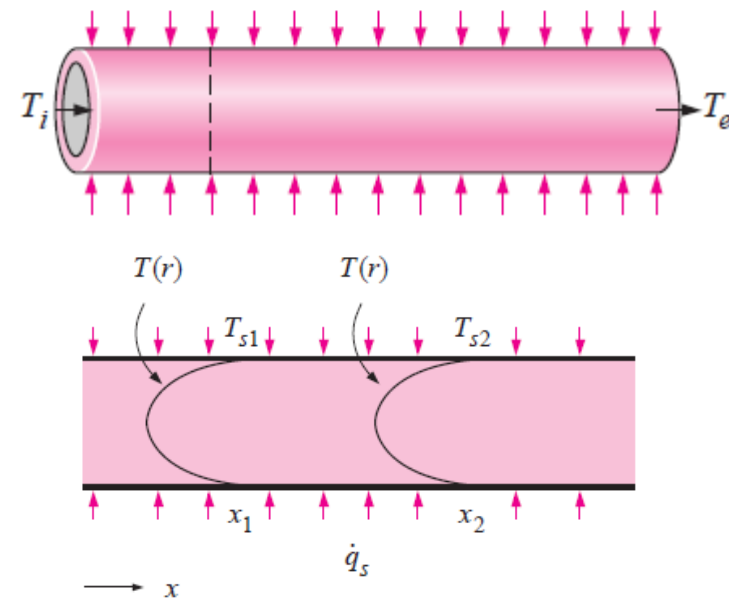
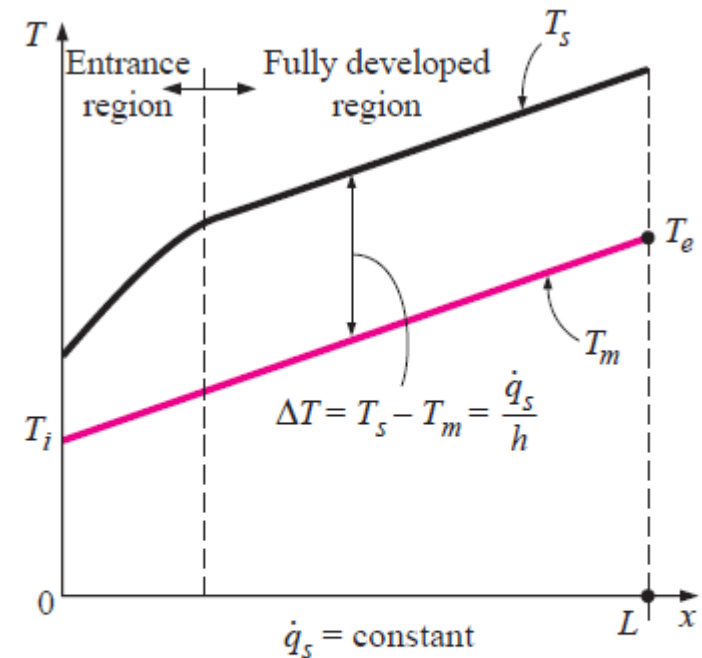
The mean fluid temperature at the tube exit is:

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p}$$

Mean fluid temperature increases *linearly* in the flow direction in the case of constant surface heat flux, since the surface area increases linearly in the flow direction

$$\dot{q}_s = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$

In the fully developed region, the surface temperature T_s will also increase linearly in the flow direction since h is constant and thus $T_s - T_m = \text{constant}$



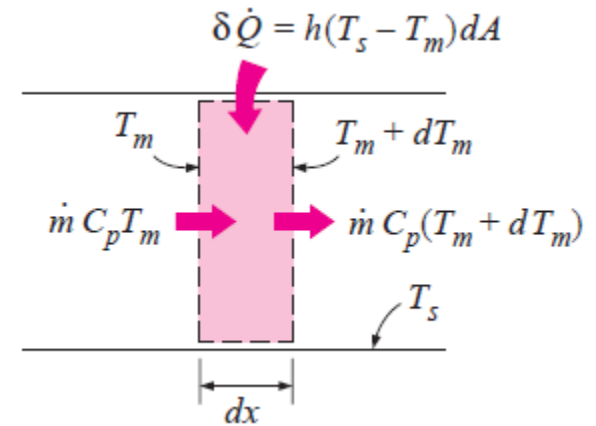
Constant Surface Heat Flux ($\dot{q}_s = \text{constant}$)

The slope of the mean fluid temperature T_m on a T - x diagram can be determined by applying the steady-flow energy balance to a tube slice of thickness dx

$$\dot{m} C_p dT_m = \dot{q}_s (p dx) \longrightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant}$$
$$\frac{dT_m}{dx} = \frac{dT_s}{dx}$$

Also, the requirement that the dimensionless temperature profile remains unchanged in the fully developed region:

$$\frac{\partial}{\partial x} \left(\frac{T_s - T}{T_s - T_m} \right) = 0 \longrightarrow \frac{1}{T_s - T_m} \left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x} \right) = 0 \longrightarrow \frac{\partial T}{\partial x} = \frac{dT_s}{dx}$$



$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} C_p} = \text{constant}$$

For a developed flow in a tube subjected to constant surface heat flux, the temperature gradient is independent of x

Hence for a circular tube:

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_m C_p R} = \text{constant}$$

Constant Surface Temperature ($T_s = \text{constant}$)

From Newton's law of cooling, the rate of heat transfer to or from a fluid flowing in a tube:

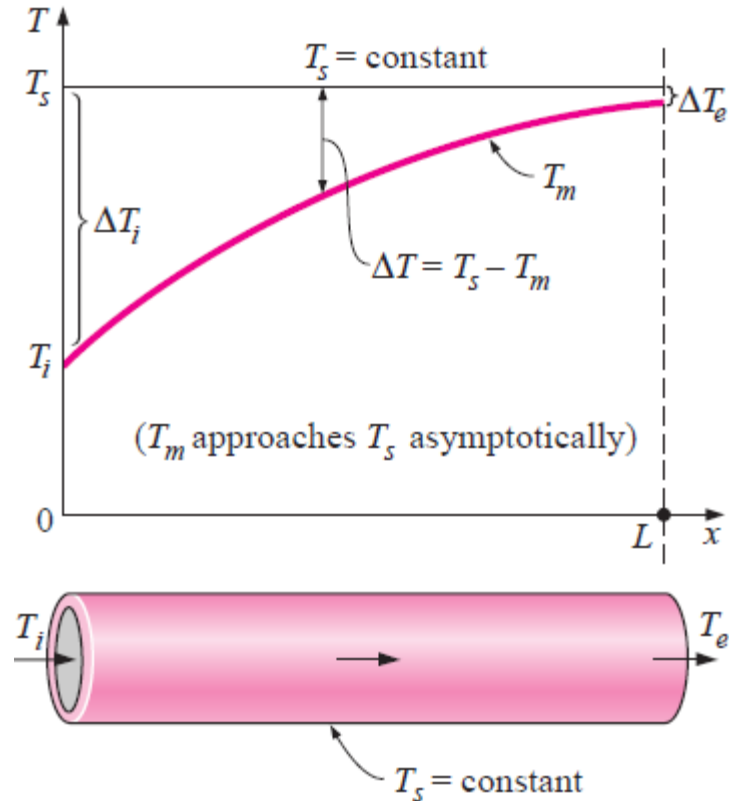
$$\dot{Q} = hA_s\Delta T_{\text{ave}} = hA_s(T_s - T_m)_{\text{ave}}$$

where h is the average convection heat transfer coefficient, $A_s = \pi DL$ for a circular pipe of length L , and T_{ave} is some appropriate *average* temperature difference between the fluid and the surface.

T_{ave} can be approximated by the **arithmetic mean temperature difference** T_{am}

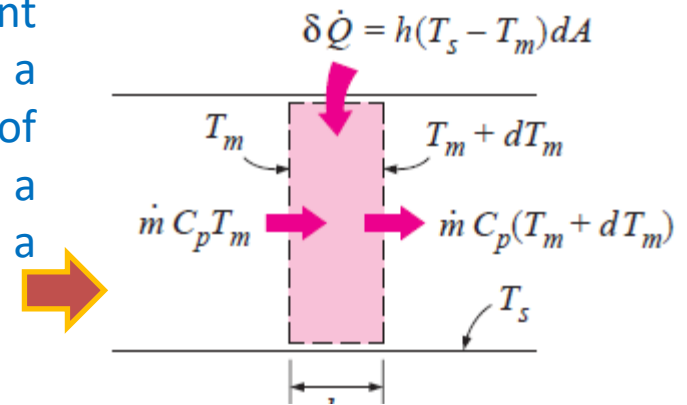
$$\begin{aligned}\Delta T_{\text{ave}} \approx \Delta T_{\text{am}} &= \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2} = T_s - \frac{T_i + T_e}{2} \\ &= T_s - T_b\end{aligned}$$

where $T_b = (T_i + T_e)/2$ is the *bulk mean fluid temperature*, which is the *arithmetic average* of the mean fluid temperatures at the inlet and the exit of the tube.



Constant Surface Temperature ($T_s = \text{constant}$)

Consider the heating of a fluid in a tube of constant cross section whose inner surface is maintained at a constant temperature of T_s . Mean temperature of the fluid T_m will increase in the flow direction as a result of heat transfer. The energy balance on a differential control volume shown in Fig:



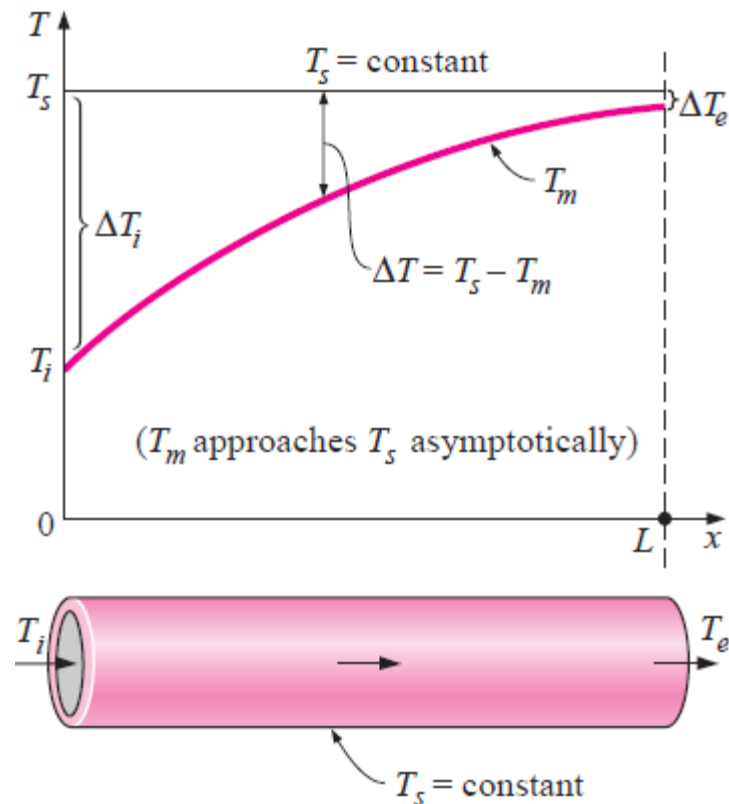
$$\dot{m} C_p dT_m = h(T_s - T_m) dA_s$$

Since differential surface area is $dA_s = p dx$, where p is the perimeter of the tube, and that $dT_m = -d(T_s - T_m)$, since T_s is constant, the relation above can be rearranged as

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m} C_p} dx$$

Integrating from $x = 0$ (tube inlet where $T_m = T_i$) to $x = L$ (tube exit where $T_m = T_e$)

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m} C_p}$$

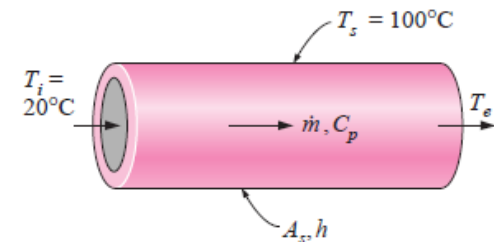
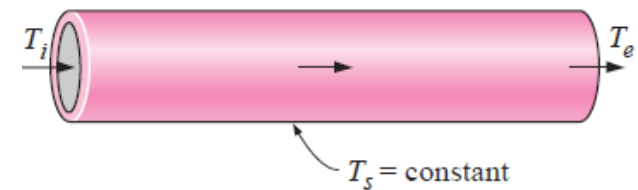
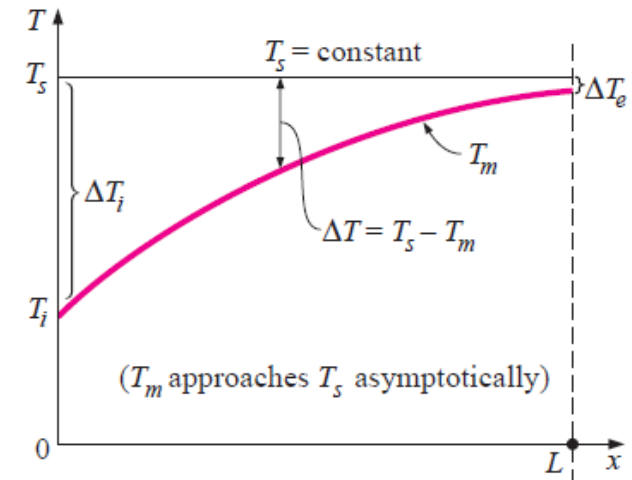


Constant Surface Temperature ($T_s = \text{constant}$)

- $A_s = pL$ is the surface area of the tube and h is the constant *average* convection heat transfer coefficient:

$$T_e = T_s - (T_s - T_i) \exp(-hA_s / \dot{m} C_p)$$

- This relation can also be used to determine the mean fluid temperature $T_m(x)$ at any x by replacing $A_s = pL$ by px
- Temperature difference between the fluid and the surface *decays exponentially* in the flow direction, and the rate of decay depends on the magnitude of the exponent $hA_x / \dot{m} \cdot C_p$, as shown in Fig.
- $hA_x / \dot{m} \cdot C_p$ is a dimensionless parameter and is termed as NTU (number of transfer units) and is a measure of effectiveness of heat transfer systems
- $NTU > 5$ indicates the exit temperature of the fluid is almost equal to surface temperature



$NTU = hA_s / \dot{m} C_p$	$T_e, ^\circ\text{C}$
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

Constant Surface Temperature ($T_s = \text{constant}$)

$$\dot{m}C_p = -\frac{hA_s}{\ln[(T_s - T_e)/(T_s - T_i)]}$$

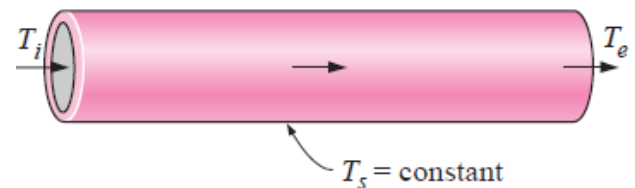
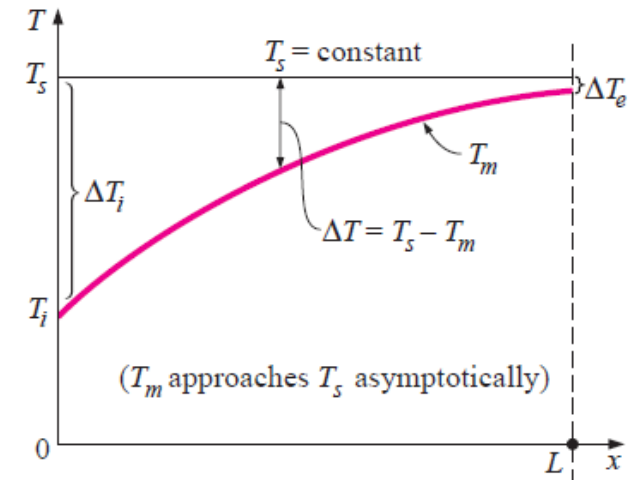
But it is known that:

$$\dot{Q} = \dot{q}_s A_s = \dot{m}C_p(T_e - T_i)$$

Therefore heat rate in constant surface temperature condition can be expressed as:

$$\dot{Q} = hA_s \Delta T_{\ln}$$

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$



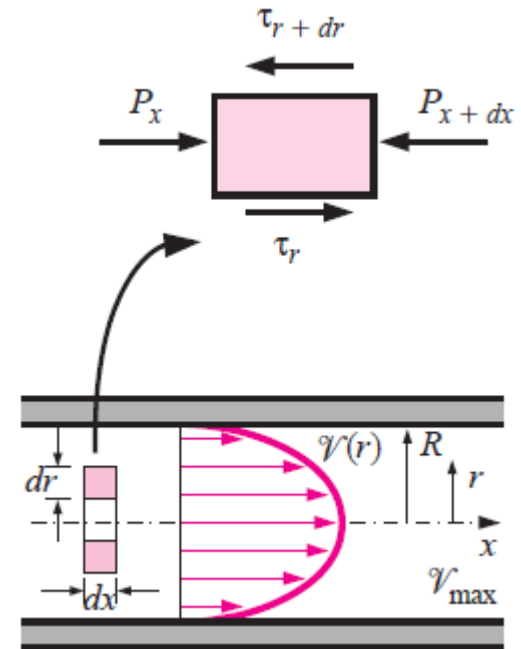
$\Delta T_{\ln}$ is termed as logarithmic mean temperature difference and is obtained by tracing actual temperature profile of the fluid along the tube and is the exact representation of average temperature difference between surface and the fluid at inlet

Laminar flow in the tubes

- Consider steady laminar flow of an incompressible fluid with constant properties in the fully developed region of a straight circular tube
- We obtain the momentum and energy equations by applying momentum and energy balances to a differential volume element, and acquire the velocity and temperature profiles by solving them
- Velocity and temperature profiles will be further used to obtain relations for the friction factor and the Nusselt number

Assumptions in the derivations:

- In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $V(r)$ remains unchanged in the flow direction
- There is no motion in the radial direction, and thus the velocity component v in the direction normal to flow is everywhere zero. There is no acceleration since the flow is steady



Laminar flow in the tubes

- Consider a ring-shaped differential volume element of radius r , thickness dr and length dx oriented coaxially with the tube, as shown in Fig.
- The volume element involves only pressure and viscous effects, and thus the pressure and shear forces must balance each other. A force balance on the volume element in the flow direction gives :

$$(2\pi r dr P)_x - (2\pi r dr P)_{x+dx} + (2\pi r dx \tau)_r - (2\pi r dx \tau)_{r+dr} = 0$$

Dividing by $2\pi r dr dx$ and rearranging:

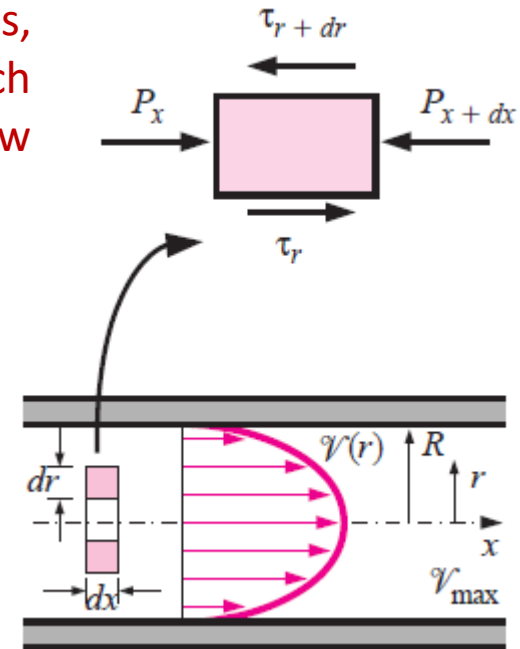
$$r \frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

Taking the limit as $dr, dx \rightarrow 0$ gives

$$r \frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

Substituting $\tau = -\mu(dV/dr)$ and rearranging the desired equation

$$\frac{\mu}{r} \frac{d}{dr} \left(r \frac{dV}{dr} \right) = \frac{dP}{dx}$$



Contd-----

Laminar flow in the tubes

$$\frac{\mu}{r} \frac{d}{dr} \left(r \frac{dV}{dr} \right) = \frac{dP}{dx}$$

The above equality must hold for any value of r and x , and an equality of the form $f(r) = g(x)$ can happen only if both $f(r)$ and $g(x)$ are constants. Thus we conclude that dP/dx is constant

The above equation can be solved by rearranging and integrating it twice to give:

$$V(r) = \frac{r^2}{4\mu} \left(\frac{dP}{dx} \right) + C_1 \ln r + C_2$$

The velocity profile $V(r)$ is obtained by applying the boundary conditions $\partial V / \partial r = 0$ at $r = 0$ (because of symmetry about the centerline) and $V = 0$ at $r = R$ (the no-slip condition at the tube surface)

$$V(r) = -\frac{R^2}{4\mu} \left(\frac{dP}{dx} \right) \left(1 - \frac{r^2}{R^2} \right)$$

Therefore, the velocity profile in fully developed laminar flow in a tube is *parabolic* with a maximum at the centerline and minimum at the tube surface

Contd-----

Laminar flow in the tubes

The mean velocity is determined from its definition:

$$V_m = \frac{2}{R^2} \int_0^R V_r r dr = \frac{-2}{R^2} \int_0^R \frac{R^2}{4\mu} \left(\frac{dP}{dx} \right) \left(1 - \frac{r^2}{R^2} \right) r dr = -\frac{R^2}{8\mu} \left(\frac{dP}{dx} \right)$$

The mathematical relation between $V(r)$ and V_m can be established by:

$$V(r) = 2V_m \left(1 - \frac{r^2}{R^2} \right)$$

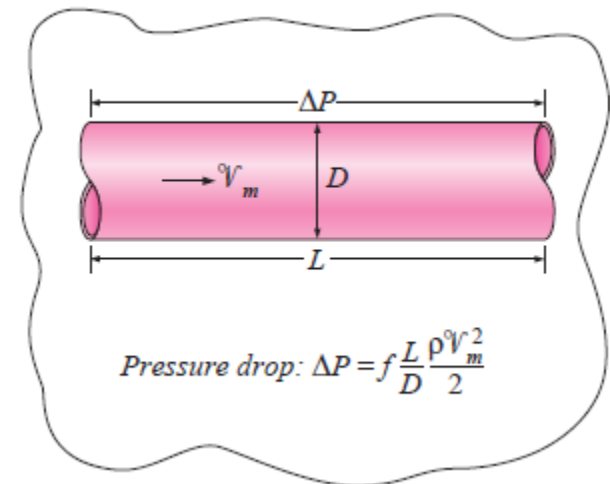
The maximum velocity occurs at the centerline, and is determined from the above equation by substituting $r = 0$

$$V_{\max} = 2V_m$$

Pressure Drop:

We note that dP/dx constant, and integrate it from $x = 0$ where the pressure is P_1 to $x = L$ where the pressure is P_2

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} = -\frac{\Delta P}{L}$$



Laminar flow in the tubes

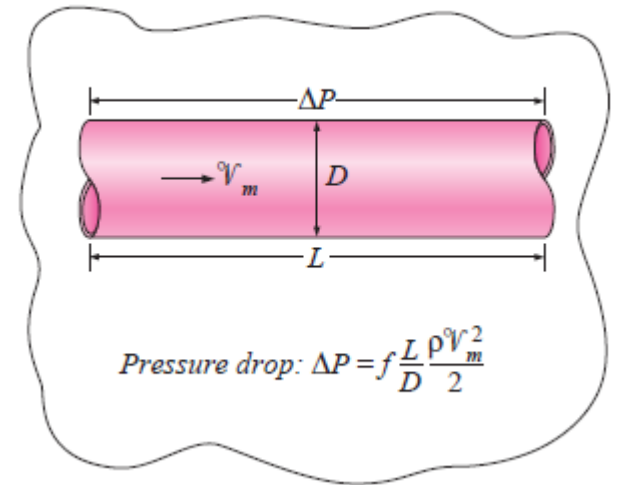
Pressure Drop:

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} = -\frac{\Delta P}{L}$$

$$\text{But } V_m = -\frac{R^2}{8\mu} \left(\frac{dP}{dx} \right)$$

Substituting $-\frac{\Delta P}{L}$ for $\frac{dP}{dx}$ in the above equation:

$$\Delta P = \frac{8\mu L V_m}{R^2} = \frac{32\mu L V_m}{D^2}$$



In practice, it is found convenient to express the pressure drop for all types of internal flows (laminar or turbulent flows, circular or noncircular tubes, smooth or rough surfaces) by assuming: (1) the flow section is horizontal so that there are no hydrostatic or gravity effects, (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure, and (3) the cross sectional area of the flow section is constant and thus the mean flow velocity is constant

$$\Delta P = f \frac{L}{D} \frac{\mu V_m^2}{2} \quad \text{where the dimensionless quantity } f \text{ is the } \mathbf{friction factor} \text{ also called Darcy friction factor}$$

For a circular tube subjected to laminar flow, friction factor is a function of Re

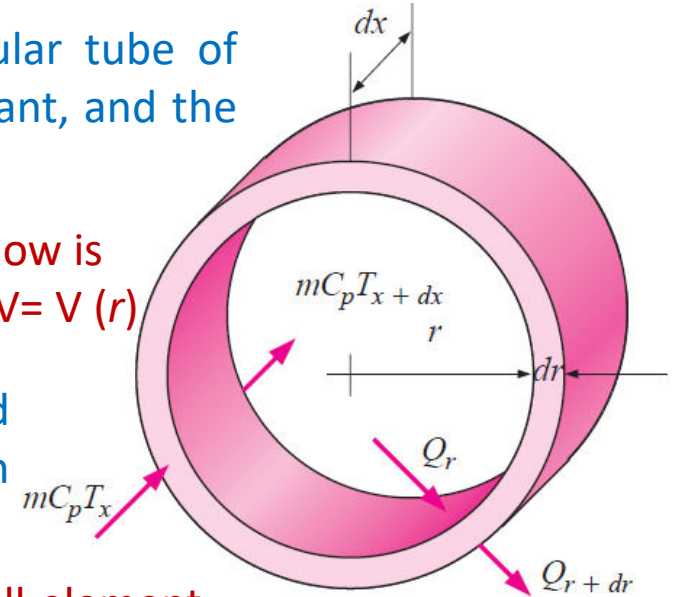
$$f = \frac{64\mu}{\rho D V_m} = \frac{64}{\text{Re}}$$

Laminar flow in the tubes

Temperature Profile and the Nusselt Number

Assumptions in the derivation:

- Consider steady laminar flow of a fluid in a circular tube of radius R . The fluid properties ρ , k , and C_p are constant, and the work done by viscous stresses is negligible
- The fluid flows along the x -axis with velocity, V . The flow is fully developed so that is independent of x and thus $V = V(r)$
- Energy is transferred by mass in the x -direction, and by conduction in the r -direction (heat conduction in the x -direction is assumed to be negligible)
- The steady-flow energy balance for a cylindrical shell element of thickness dr and length dx can be expressed as:



$$\dot{m} C_p T_x - \dot{m} C_p T_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0$$

$$\text{where } \dot{m} = \rho V A_c = \rho V (2\pi r dr)$$

Substituting and dividing by $2\pi r dr dx$ and rearranging gives:

$$\rho C_p V \frac{T_{x+dx} - T_x}{dx} = - \frac{1}{2\pi r dx} \frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr}$$

Laminar flow in the tubes

Temperature Profile and the Nusselt Number

$$\rho C_p \mathcal{V} \frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r dx} \frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr}$$

By taking limits on L.H.S and R.H.S w.r.t $dx, dr \rightarrow 0$ gives

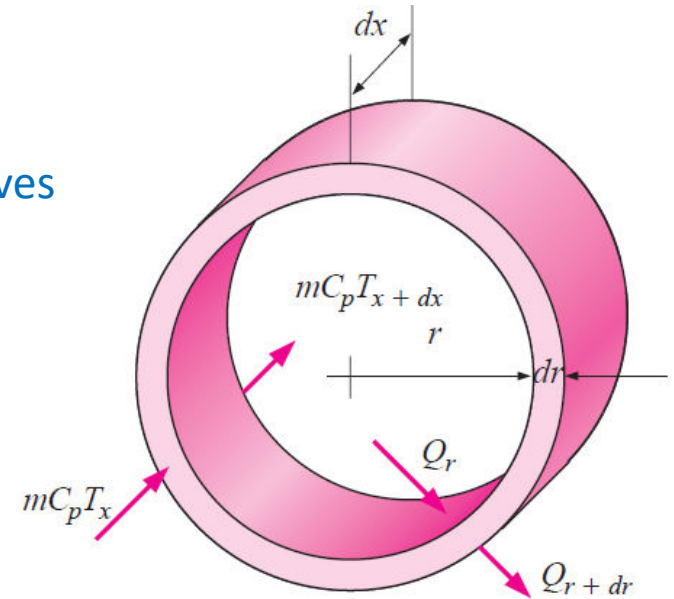
$$\mathcal{V} \frac{\partial T}{\partial x} = -\frac{1}{2\rho C_p \pi r dx} \frac{\partial \dot{Q}}{\partial r}$$

But conduction heat transfer $\dot{Q}$ is given by Fourier's law of heat conduction:

$$\frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r} \left(-k 2\pi r dx \frac{\partial T}{\partial r} \right) = -2\pi k dx \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right)$$

Substituting and using $\alpha = k/\rho C_p$ gives

$$\mathcal{V} \frac{\partial T}{\partial x} = \frac{\alpha}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right)$$



The rate of net energy transfer to the control volume by mass flow is equal to the net rate of heat conduction in the radial direction

Contd----

Laminar flow in the tubes

Temperature Profile and the Nusselt Number - Constant Surface Heat Flux condition

For fully developed flow in a circular pipe subjected to constant surface heat flux:

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_m C_p R} = \text{constant} \quad \text{Equation -i}$$

The governing differential equation for temperature field in circular tube is:

$$V \frac{\partial T}{\partial x} = \frac{\alpha}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \quad \text{Equation -ii}$$

But the velocity at a radius and mean velocity are related by:

$$V(r) = 2V_m \left(1 - \frac{r^2}{R^2} \right) \quad \text{Equation -iii}$$

Substituting Eqn.iii and Eq.i in Eqn.ii gives :

$$\frac{4\dot{q}_s}{kR} \left(1 - \frac{r^2}{R^2} \right) = \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right)$$

The above equation is a second order differential equation , the general solution is obtained by separating variables and integrating twice:

Contd----

Laminar flow in the tubes

Temperature Profile and the Nusselt Number - Constant Surface Heat Flux condition

By integrating twice :

$$\frac{4\dot{q}_s}{kR} \left(1 - \frac{r^2}{R^2} \right) = \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right)$$

$$T(r) = \frac{\dot{q}_s}{kR} \left(r^2 - \frac{r^4}{4R^2} \right) + C_1 \ln r + C_2$$

The above equation is subject to boundary conditions ,at $r=0$, $\frac{\partial T}{\partial r} = 0$ and at $r=R$, $T = T_s$

$$T = T_s - \frac{\dot{q}_s R}{k} \left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4} \right)$$

But we know the mean temperature and mean velocity in a circular tube is given by:

$$T_m = \frac{\int_m C_p T \delta \dot{m}}{\dot{m} C_p} = \frac{\int_0^R C_p T (\rho V 2\pi r dr)}{\rho V_m (\pi R^2) C_p} = \frac{2}{V_m R^2} \int_0^R T(r, x) V(r, x) r dr \quad V(r) = 2V_m \left(1 - \frac{r^2}{R^2} \right)$$

Substituting mean temperature and mean velocity in temperature field equation we get:

$$T_m = T_s - \frac{11}{24} \frac{\dot{q}_s R}{k} \Rightarrow h = \frac{24}{11} \frac{k}{R} = \frac{48}{11} \frac{k}{D} = 4.36 \frac{k}{D} \Rightarrow Nu = \frac{hD}{k} = 4.36$$

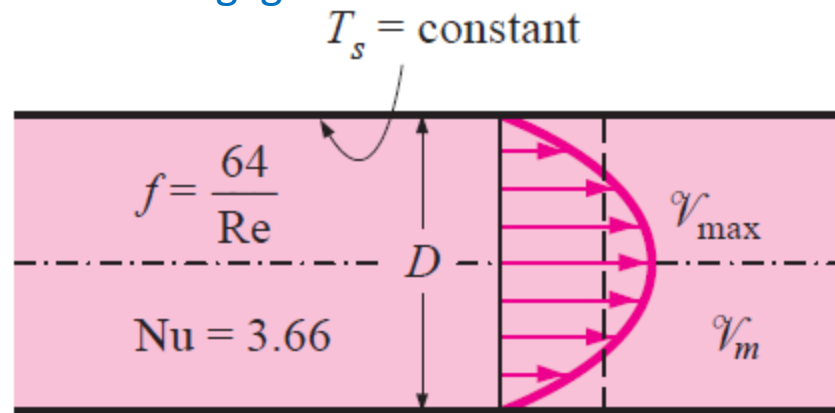
Laminar flow in the tubes

Temperature Profile and the Nusselt Number - Constant Surface temperature condition

- The solution procedure for constant surface temperature is more complex as it requires iterations, but the final Nusselt number relation obtained is equally simple as that of constant surface heat flux condition:

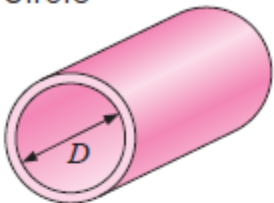
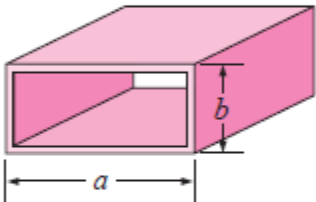
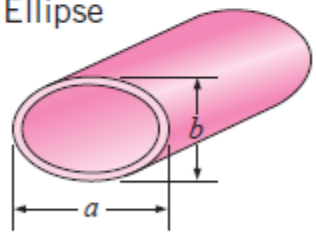
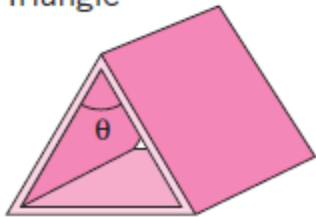
$$\text{Nu} = \frac{hD}{k} = 3.66$$

- The thermal conductivity k for use in the Nu relations above should be evaluated at the bulk mean fluid temperature, which is the arithmetic average of the mean fluid temperatures at the inlet and the exit of the tube
- For laminar flow, the effect of *surface roughness* on the friction factor and the heat transfer coefficient is negligible



Fully developed
laminar flow

Laminar Flow in Noncircular Tubes

Tube Geometry	a/b or θ°	Nusselt Number		Friction Factor f
		$T_s = \text{Const.}$	$\dot{q}_s = \text{Const.}$	
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b 1 2 3 4 6 8 ∞	2.98 3.39 3.96 4.44 5.14 5.60 7.54	3.61 4.12 4.79 5.33 6.05 6.49 8.24	56.92/Re 62.20/Re 68.36/Re 72.92/Re 78.80/Re 82.32/Re 96.00/Re
Ellipse 	a/b 1 2 4 8 16	3.66 3.74 3.79 3.72 3.65	4.36 4.56 4.88 5.09 5.18	64.00/Re 67.28/Re 72.96/Re 76.60/Re 78.16/Re
Triangle 	θ 10° 30° 60° 90° 120°	1.61 2.26 2.47 2.34 2.00	2.45 2.91 3.11 2.98 2.68	50.80/Re 52.28/Re 53.32/Re 52.60/Re 50.96/Re

Convection correlations for developing laminar flow in entrance region

- For a circular tube of length L subjected to constant surface temperature, the average Nusselt number for the *thermal entrance region* is:

$$\text{Nu} = 3.66 + \frac{0.065 (D/L) \text{Re Pr}}{1 + 0.04[(D/L) \text{Re Pr}]^{2/3}}$$

- Nusselt number is larger at the entrance region, and it approaches asymptotically to the fully developed value of 3.66 as $L \rightarrow \infty$
- When the difference between the surface and the fluid temperatures is large, it may be necessary to account for the variation of viscosity with temperature
- The average Nusselt number for developing laminar flow in a circular tube in that case can be determined as:

$$\text{Nu} = 1.86 \left(\frac{\text{Re Pr } D}{L} \right)^{1/3} \left(\frac{\mu_b}{\mu_s} \right)^{0.14}$$

- All properties are evaluated at the bulk mean fluid temperature, except for μ_s , which is evaluated at the surface temperature

Turbulent flow in the tubes

- Turbulent flow is commonly utilized in practice because of the higher heat transfer coefficients associated with it. Most correlations for the friction and heat transfer coefficients in turbulent flow are based on experimental studies because of the difficulty in dealing with turbulent flow theoretically
- For *smooth* tubes, the friction factor in turbulent flow can be determined from the explicit *first Petukhov equation*:

$$f = (0.790 \ln \text{Re} - 1.64)^{-2} \quad 10^4 < \text{Re} < 10^6$$

- Once the friction factor is available, this equation can be used conveniently to evaluate the Nusselt number for both smooth and rough tubes

$$\text{Nu} = 0.125 f \text{Re} \text{Pr}^{1/3}$$

- For fully developed turbulent flow in *smooth tubes*, a simple relation for the Nusselt number can be obtained by substituting the simple power law relation $f = 0.184 \text{Re}^{-0.2}$ for the friction factor in the above equation:

$$\text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{1/3} \quad \left(\begin{array}{l} 0.7 \leq \text{Pr} \leq 160 \\ \text{Re} > 10,000 \end{array} \right)$$

Turbulent flow in the tubes

$$\text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^n$$

where $n = 0.4$ for heating and 0.3 for cooling of the fluid flowing through the tube. This equation is known as the Dittus–Boelter equation

- The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$
- When the temperature difference between the fluid and the wall is very large, it may be necessary to use a correction factor to account for the different viscosities near the wall and at the tube center
- Gnielinski's equation should be preferred in calculations because it greatly eliminates error. Again properties should be evaluated at the bulk mean fluid temperature

$$\text{Nu} = \frac{(f/8)(\text{Re} - 1000) \text{Pr}}{1 + 12.7(f/8)^{0.5} (\text{Pr}^{2/3} - 1)} \quad \left(\begin{array}{l} 0.5 \leq \text{Pr} \leq 2000 \\ 3 \times 10^3 < \text{Re} < 5 \times 10^6 \end{array} \right)$$

- The relations above are not very sensitive to the *thermal conditions* at the tube surfaces and can be used for both $\dot{T}_s = \text{constant}$ and $\dot{q}_s = \text{constant}$ cases

Turbulent flow in the tubes – Role of rough surfaces

- Any irregularity or roughness on the surface disturbs the laminar sublayer, and affects the flow. Therefore, unlike laminar flow, the friction factor and the convection coefficient in turbulent flow are strong functions of surface roughness
- The friction factor in fully developed turbulent flow depends on the Reynolds number and the *relative roughness* ε/D and is given by *Colebrook implicit equation*:

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right) \quad (\text{turbulent flow})$$

Standard sizes for Schedule 40 steel pipes

Nominal Size, in.	Actual Inside Diameter, in.
1/8	0.269
1/4	0.364
3/8	0.493
1/2	0.622
3/4	0.824
1	1.049
1 1/2	1.610
2	2.067
2 1/2	2.469
3	3.068
5	5.047
10	10.02

Relative Roughness, ε/L	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for $\text{Re} = 10^6$,

Turbulent flow in the tubes – Role of rough surfaces

- Commercially available pipes differ from those used in the experiments in that the roughness of pipes in the market is not uniform, and it is difficult to give a precise description of it.
- The relative roughness of pipes may increase with use as a result of corrosion, scale buildup, and precipitation. As a result, the friction factor may increase by a factor of 5 to 10
- A simplified explicit equation for f is given by:

$$\frac{1}{\sqrt{f}} \approx -1.8 \log \left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7} \right)^{1.11} \right]$$

- In turbulent flow, wall roughness increases the heat transfer coefficient h by a factor of 2 or more

Equivalent roughness values for new commercial pipes*

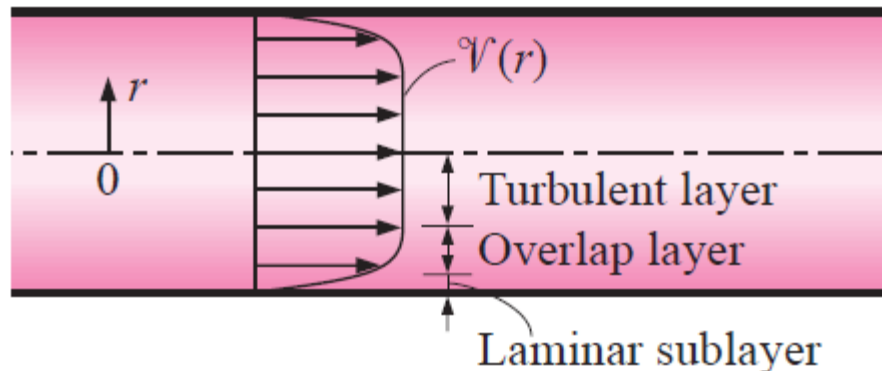
Material	<i>Roughness, ε</i>	
	ft	mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

*The uncertainty in these values can be as much as ± 60 percent.

Turbulent Flow in Noncircular Tubes

- The velocity and temperature profiles in turbulent flow are nearly straight lines in the core region, and any significant velocity and temperature gradients occur in the viscous sublayer
- Despite the small thickness of laminar sublayer (usually much less than 1 percent of the pipe diameter), the characteristics of the flow in this layer are very important since they set the stage for flow in the rest of the pipe
- Consequently, the turbulent flow relations given above for circular tubes can also be used for noncircular tubes with reasonable accuracy by replacing the diameter D in the evaluation of the Reynolds number by the hydraulic diameter, D_h

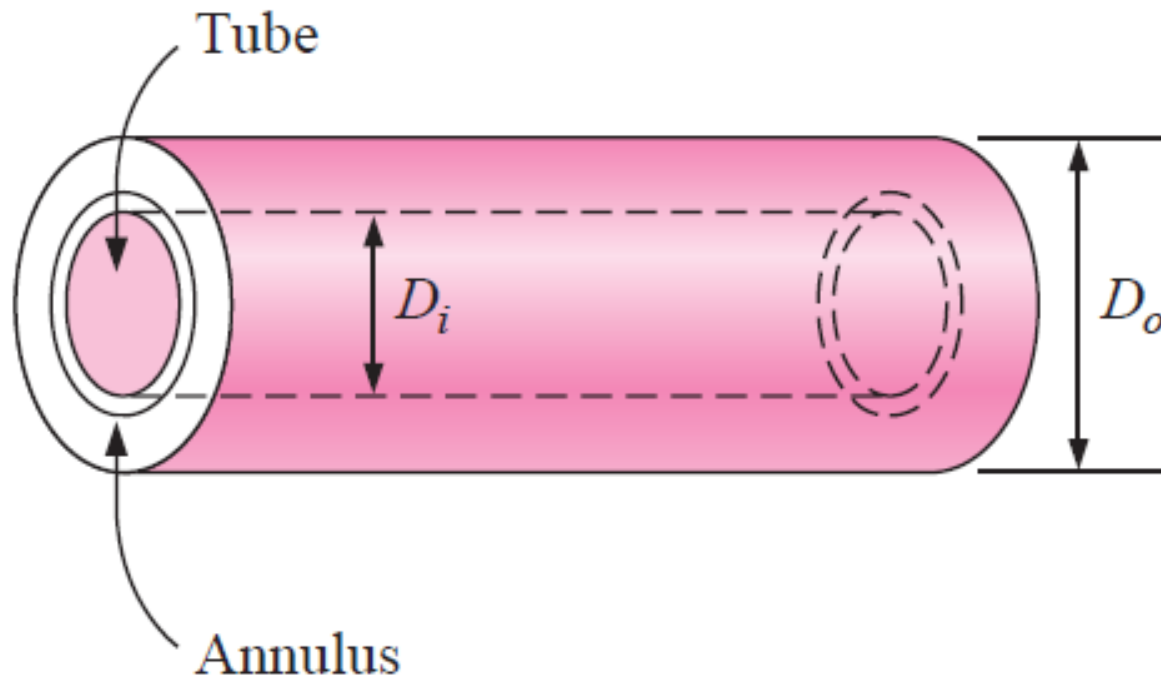
$$D_h = \frac{4A_c}{p}$$



Flow through Tube Annulus

- Some simple heat transfer equipment consist of two concentric tubes, and are called double-tube heat exchangers
- In these devices, one fluid flows through the tube while the other flows through the annular space. The governing differential equations for both flows are identical
- Consider a concentric annulus of inner diameter D_i and outer diameter D_o . The hydraulic diameter of annulus is:

$$D_h = \frac{4A_c}{p} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_o + D_i)} = D_o - D_i$$



Flow through Tube Annulus

- Annular flow is associated with two Nusselt numbers— Nu_i on the inner tube surface and Nu_o on the outer tube surface—since it may involve heat transfer on both surfaces

$$Nu_i = \frac{h_i D_h}{k} \quad \text{and} \quad Nu_o = \frac{h_o D_h}{k}$$

- The Nusselt numbers for fully developed laminar flow with one surface isothermal and the other adiabatic are given as:



Nusselt number for fully developed laminar flow in an annulus with one surface isothermal and the other adiabatic

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

- For fully developed turbulent flow, the inner and outer convection coefficients are approximately equal to each other, and the tube annulus can be treated as a noncircular duct with a hydraulic diameter of $D_h = D_o - D_i$.
- The Nusselt number in this case can be determined from a suitable turbulent flow relation such as the Gnielinski's equation

$$Nu = \frac{(f/8)(Re - 1000) Pr}{1 + 12.7(f/8)^{0.5} (Pr^{2/3} - 1)} \quad \left(\begin{array}{l} 0.5 \leq Pr \leq 2000 \\ 3 \times 10^3 < Re < 5 \times 10^6 \end{array} \right)$$

- To improve the accuracy of Nusselt numbers obtained from these relations for annular flow, Petukhov and Roizen recommended correction factors as:



$$\begin{aligned} F_i &= 0.86 \left(\frac{D_i}{D_o} \right)^{-0.16} && \text{(outer wall adiabatic)} \\ F_o &= 0.86 \left(\frac{D_i}{D_o} \right)^{-0.16} && \text{(inner wall adiabatic)} \end{aligned}$$