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C20-M-CHOT-RAC–102

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BOARD DIPLOMA EXAMINATION, (C-20)

SEPTEMBER/OCTOBER—2021

DME - FIRST YEAR EXAMINATION

ENGINEERING MATHEMATICS – I

Time : 3 hours] [*Total Marks* : 80

PART—A

3×10=30

Instructions : (1) Answer **all** questions.

(2) Each question carries **three** marks.

1. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3x - 2$ is a bijective function, then find $f^{-1}(x)$.

2. Resolve $\frac{1}{(x-3)(x+2)}$ into partial fractions.

3. If $A = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 7 \\ -3 & x & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the value of x.

4. If $A + B = \frac{\pi}{4}$, then prove that $(1 + \tan A)(1 + \tan B) = 2$.

5. Prove that $\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$

6. Find the modulus-amplitude form of the complex number $1+i\sqrt{3}$.

7. Find the equation of the line passing through the point(0, –1) having the inclination 45° with the positive direction of X-axis.

8. Evaluate $\lim_{x\to 0}\frac{\sin 77x}{\sin 11x}$

9. Find the derivative of $\sqrt{x}-\sec x+\log x$ w.r.t. 'x'

10. Find $\frac{dy}{dx}$, if $y = xe^x$

PART—B

8×5=40

Instructions : (1) Answer **all** questions.

(2) Each question carries **eight** marks.

11. *(a)* Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

OR

(b) Solve the following system of linear equations using matrix inversion method :

$x + 2y - z = -1$, $2x + 3y + z = 3$ and $3x - y + 2z = 0$

12. *(a)* If $\cos x+\cos y=\frac{3}{5}$ and $\cos x-\cos y=\frac{2}{7}$, then show that $21\tan\left(\frac{x-y}{2}\right)+10\cot\left(\frac{x+y}{2}\right)=0$.

OR

(b) Find 'x', if $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\tan^{-1}\left(\frac{1}{2}\right)$.

13. *(a)* Solve : $\sqrt{3}\tan x + 2\sec x = 1$

OR

(b) In any ΔABC, show that $\sum bc\cos^2\left(\frac{A}{2}\right) = S^2$.

14. *(a)* Find the equation of the circle with (4, 2) and (1, 5) as the two ends of its diameter and also find its centre and radius.

OR

(b) Find the centre, vertices, equations of axes, lengths of axes, eccentricity, foci, equations of directrices, latera-recta and length of latus-rectum of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

15. *(a)* If $y = a\cos(\log x) + b\sin(\log x)$, then prove that $x^2y_2 + xy_1 + y = 0$.

OR

(b) If $x = t + \sin t$ and $y = t - \cos t$, then find that $\frac{dy}{dx}$ at $t = 1$.

PART—C

10×1=10

Instructions : (1) Answer the following question.

(2) It carries **ten** marks.

16. Show that the maximum rectangle that can be inscribed in a circle is a square.

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