Code No: R1921031 **R19** SET - 1

II B. Tech I Semester Supplementary Examinations, September - 2021
VECTOR CALCULUS & FOURIER TRANSFORMS
(Mechanical Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks
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1 a) Prove that div $curl\,\bar{f} = 0$ where $\bar{f} = f_1\bar{i} + f_2\bar{j} + f_3\bar{k}$. [8M]

b) Find the directional derivative of $\phi = xyz$ in the direction of $\bar{i} + \bar{j} + \bar{k}$ at $(1,2,3)$. [7M]

Or

2 a) Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = (2y+3)\,\bar{i} + xz\,\bar{j} + (yz - x)\bar{k}$ along the curve C give (0,0,0) to (0,0,1), then to (01,1,) and then to (2,1,1) [8M]

b) Using Divergence theorem evaluate $\iint_S \bar{F}.\bar{n}ds$ where s is the surface of the sphere $x^2 + y^2 + z^2 = b^2$ in the first octant where $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$. [7M]

3 a) If $f(t) = \begin{cases} 1, & 0 \le t < 1 \\ -1, & 1 \le t < 2 \end{cases}$ is a periodic function with period 2. Then find its Laplace transform. [7M]

b) Evaluate $L\left\{\frac{sin^2 t}{t}\right\}$ [8M]

Or

4 a) Find inverse Laplace transform of $\frac{1}{s} sin\frac{1}{s}$ [7M]

b) Using Convolution theorem find the $L^{-1}\left\{\frac{s}{(s^2+9)(s^2+16)}\right\}$ [8M]

5 a) Find the Fourier series of $f(x) = e^{-x}$ in (0,2) [8M]

b) Find the half range cosine series of $f(x) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} < x < \pi \end{cases}$. [7M]

Or

6 a) Using Fourier integral, show that $e^{-x}\cos x = \frac{2}{\pi}\int_0^\infty \frac{\lambda^2+2}{\lambda^2+4}\cos\lambda x d\lambda$. [8M]

b) Find the Fourier sine transform of the function $f(x) = \begin{cases} \cos x & if\ \ x < a \\ 0 & if\ \ x > a \end{cases}$. [7M]

7 a) Form a partial differential equation by eliminating arbitrary constants from $z = \left(\sqrt{x+a}\right)\left(\sqrt{y+b}\right)$ [7M]

b) Form a partial differential equation by eliminating arbitrary function from $z = xy + f(xy)$ [8M]

Or

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8 a) Solve the partial differential equation $p - q = x + y + z$. [8M]

b) Solve the partial differential equation $z = p^2 + q^2$. [7M]

9 a) Solve the partial differential equation $\left(D^2 + DD^1 - 6D^{1^2}\right) z = \cos(x + y)$. [8M]

b) Solve the PDE by the method of variation of parameters $\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} = 0$. [7M]

Or

10 Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to [15M]

$(i)\ u(x,0) = 0\ \ for\ \ all\ \ x$

$(ii)\ u(x,l) = 0\ \ for\ \ all\ \ x$

$(iii)\ u(\infty, y) = 0, 0 \le y \le l$

$(iv)\ u(0, y) = y, 0 \le y \le l$

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