Unit - IV

Stress Strain Measurement

Stress $\sigma_a = \frac{L}{A}$ (Uni axial condition.

strain $\epsilon_a = \frac{\Delta L}{L}$ (uni axial condition)

Young's Modulus $E = \frac{\sigma_a}{\epsilon_a}$ (uni axial condition)

Poission's ratio $\nu = -\frac{\epsilon_{Lat}}{\epsilon_{long}}$ (uni axial condition)

Bi-axial Stress Condition :-

Stress - strain relations for a bi-axially loaded member are

Strain in x- direction $\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} \rightarrow (1)$

y - direction $\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} \longrightarrow (2)$

where σ_x, σ_y stress in x,y directions

$(1) + (2) \times \nu \rightarrow$

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\cdot\frac{\sigma_y}{E}$$

$$\nu\cdot\epsilon_y = \nu\frac{\sigma_y}{E} - \nu^2\frac{\sigma_x}{E}$$

$$\epsilon_x + \nu\epsilon_y = \frac{\sigma_x}{E} - \nu^2\frac{\sigma_x}{E}$$

$$\frac{\sigma_x}{E}(1-\nu^2) = \epsilon_x + \epsilon_y$$

$$\sigma_x = \frac{(\epsilon_x + \epsilon_y \nu)E}{(1-\nu^2)}$$

To find σ_y

(1) $\times \nu$ + (2)

$$\nu \cdot \epsilon_x = \nu \cdot \frac{\sigma_x}{E} - \frac{\nu^2 \sigma_y}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu \sigma_x}{E}$$

$$\nu\epsilon_x + \epsilon_y = \frac{\sigma_y}{E}(1-\nu^2)$$

$$\sigma_y = \frac{E}{1-\nu^2}(\epsilon_y + \nu\epsilon_x)$$

Tri-Axial Stress Condition :-

strain in x direction

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E} \longrightarrow (1)$$

strain in y-direction

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} - \frac{\nu \cdot \sigma_z}{E} \longrightarrow (2)$$

strain in z-direction

$$\epsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_y}{E} - \frac{\nu\sigma_x}{E} \longrightarrow (3)$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_x + \nu(\epsilon_y+\epsilon_z)\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_y + \nu(\epsilon_z+\epsilon_x)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\epsilon_z + \nu(\epsilon_x+\epsilon_y)\right]$$

Types of Strain ~~gauges~~ Measurement Techniques :-

Different types of techniques available for the measurement of strain are

(i) strain gauges
(ii) Grid Method
(iii) Brittle laquer Method
(iv) photoelasticity

Types of Strain Gauges :-

Based on the principle of operation & their constructional features, strain gauges may be classified as

1. mechanical gauges
2. optical gauges
3. Electrical Strain gauges.

Mechanical strain gauges :

In mechanical strain gauges the deformation in length is magnified mechanically through lever & gear mechanism.

The most commonly used these gauges are huggen berger type & Berry type extensometers.

Advantages :

* It has self-contained magnification system
* It does not require auxiliary power supply

Disadvantages :-

* Response is slow
* high inertia, these are not suitable for dynamic measurements
* They can be used only when the additional mass of the mechanical gauge does no contribute to any error.

Optical Strain Gauges :

These are very similar to the mechanical gauges except that the magnification is acheived with multiple reflectors using mirrors & prims. Optical strain gauges include L.B Tokeman & Martin mirror type extensometer.

Advantages :

* It is good at static measurements.

* independent of temperature variations

Dis Advantages :-

* It cannot be used for dynamic measurements
* large gauge lengths are required.

Electrical Strain Gauges :-

Electrical strain gauges work on the principle of change in resistance, capacitance or inductance of an element due to the strain. most common electrical strain gauges are use the principle of change in resistance of an element when subjected to strain. Hence resistance strain gauges are discussed in detail in the following sections.

Resistance Strain Gauges :

These are widely used in the experimental stress analysis of machine components and structures. They are also used in the measurement of force, torque, pressure & acceleration.

The resistance of a conductor is given by

$$R = \frac{\rho L}{A}$$

where R = length of the conductor

A = Area of cross section.

ρ = Resistivity of the material

Applying logarithm on both sides

$$\log R = \log\left(\frac{\rho L}{A}\right)$$

$$\log R = \log \rho L - \log A$$

differentiate on both sides

$$\frac{1}{R}dR = \frac{1}{\rho}d\rho + \frac{1}{L}dL - \frac{dA}{A} \longrightarrow ①$$

A = Circular section of conductor.

$$A = \frac{\pi}{4}D^2$$

$$\log A = \log\frac{\pi}{4} + 2\log D$$

differentiating on both sides

$$\frac{dA}{A} = 2\frac{dD}{D} \longrightarrow ②$$

From ① & ②

$$\frac{1}{R}dR = \frac{1}{\rho}d\rho + \frac{1}{L}dL \; \frac{dD}{D}$$

$$\frac{dD}{D} = (\text{lateral strain}) = -\nu\frac{dL}{L}$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} + 2\nu\frac{dL}{L}$$

but $\epsilon_a = \frac{dL}{L}$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \epsilon_a(1+L_\nu)$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \epsilon_a (1+2v)$$

$$\frac{dR}{R\epsilon_a} = \frac{d\rho}{\rho\epsilon_a} + (1+2v)$$

but gauge factor $F = \frac{dR}{R\epsilon_a}$

$$F = (1+2v) + \frac{d\rho}{\rho\epsilon_a}$$

F = gauge factor

if the resistivity of the material does not vary with the strain then $d\rho$ becomes zero

$$F = 1+2v$$

then if $v = 0.3$ then

$$F = 0.6 + 1$$
$$= 1.6.$$

The value of gauge factor F will be the less than of 1.6 then resistivity decreases with strain

However the value of F will be greater than 1.6 then resistivity increases with strain.

Classification of Resistance Strain Gauges :-

Electrical resistance strain gauges may be classified as

1. Bonded Strain gauges
   (a) Wire gauge
   (b) Foil gauge
   (c) Semi-conductor gauge
2. Un-bonded Strain gauge

Bonded Strain Gauges :-

These are fixed or cemented on the surface of the specimen so that the gauge forms the part of the surface & faithfully follows both compressive & tensile strains in the specimen.

(a) Bonded Wire Resistance Strain gauge :-

It consists of a fine wire of size varying from 0.0125 mm to 0.025 mm. The wire is arranged in the form of grid shown in fig. & is bonded to a backing material such as paper, Bakelite or epoxy with an adhesive that can hold the wire element firmly on the base whose strain to be measured. The performance of the strain gauge mainly depends on the strength of the bonding b/w the wire and the base. leads and connecting terminals are also provided to facilitate the measurement

fig :- Resistance with gauge

Advantages :-

* They have high accuracy
* They exhibit linearity

Limitations :

* The gauge cannot be detached and used again.
* These are costly gauges.

Bonded Metal Foil Gauge :-

It consists of a foil of thickness less than 0.025 mm which is formed in to the grid shape & provided with a plastic backing as shown in fig. The desired grid pattern is first printed on a thin sheet of metal alloy foil with an acid resistant ink and then the unprinted portion is etched way

Two ways leads are provided to facilitate measurement of resistance of the gauge. The materials which are used for the wire gauge can also be used for this gauge. Since the grid element in this gauge is wider than its cross section, a larger ratio of the bonding area to the cross sectional area is achieved compared to that of the wire gauge

Hence higher heat dissipation & better bonding can be achieved. In this gauge stress concentration at the terminals is negligible due to the absence of connections (Joints).

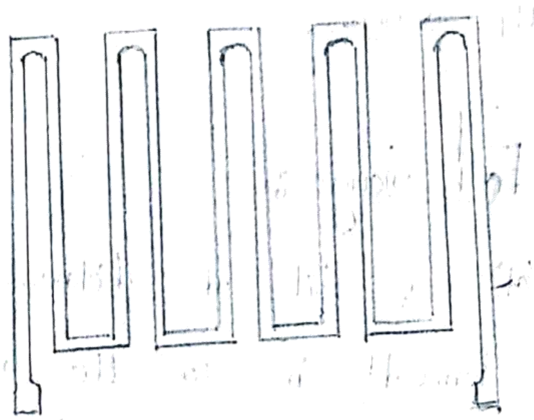

fig:- Foil gauge.

Advantages:

* These gauges can be manufactured in any shape
* perfect bonding is achieved.

* These gauges have better fatigue life
* good sensitivity

Dis advantages :
* the gauge cannot be detached and used again
* they are costly.

Piezo Resistive (or) Semiconductor Strain Gauge :

It consists of a doped silicon or germanium semiconductor material which acts as the gauge. the material is usually produced in brittle wafer having thickness. The sensing element is provided with plastic or stainless steel backing. leads are also incorporated into the gauge to facilitate the measurement. A simple Semiconductor as shown in below fig:

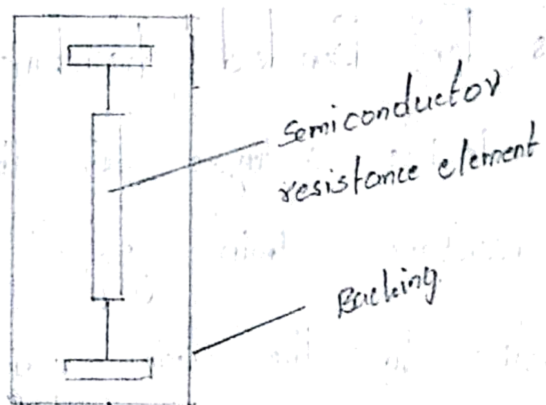

fig: semi conductor strain gauge

The semiconductor element may be either P-type or N-type.

In P type gauge the resistance increases with tensile strain, while in N-type gauge the resistance decreases.

Advantages :

* They are very high gauge i.e., 100 to 150
* they have low hysteresis
* compact in size
* They have good frequency response
* they have good fatigue life.

Limitations :

* These gauges show non-linearity
* Gauge factor is not stable. It changes with temperature.

Materials for Bonded Resistance Strain Gauges :-

The materials employed for the fabrication of bonded resistance strain gauge must impart desirable characteristics to the strain gauge. Desirable characteristics of good strain gauge are.

* high Accuracy
* excellent reproducibility
* good sensitivity
* long life

Different Materials used in the fabrication of a resistance gauge are :

(1) Resistance wire or grid materials
(2) Backing materials
(3) Bonding materials / adhesives.

Resistance wire or Grid Material :-

Desirable properties of resistance wire used in strain gauge are

* high specific resistance
* low temperature coefficient of resistance
* high ~~a~~ corrosion gauge factor
* good corrosion resistance
* high yield point.

Backing Materials :-

Desirable characteristics of a backing material are

* high mechanical strength to transmit forces on the body to strain the bonded gauge
* Excellent transmissibility
* High insulating resistance
* must be immune to moisture.
* High thermal stability

Bonding Materials / Adhesives :-

These are used to attach the strain gauges on the specimen underatest.

Desirable propeties of a bonding material are.

* must be immune to the moisture,
* high creep resistance
* high insulation.
* Ability to dryup in a short time

Mounting of the Gauges :-

The performance of the strain gauge mainly depends on the type of bonding material chosen for the bonding and the procedure followed in fixing the gauge at the location. Hence care must be taken in mounting the gauge & subsequent bonding.

Faulty Mounting & Backing cause the following.

1. Hysteresis
2. creep
3. Fatigue life
4. Humidity & Moisture.
5. Temperature Effects.

Unbonded Resistance Strain Gauges :-

An un-bonded resistance strain gauge is basically a free filament sensing element whose strain is directly transferred to the resistance wire without any backing.

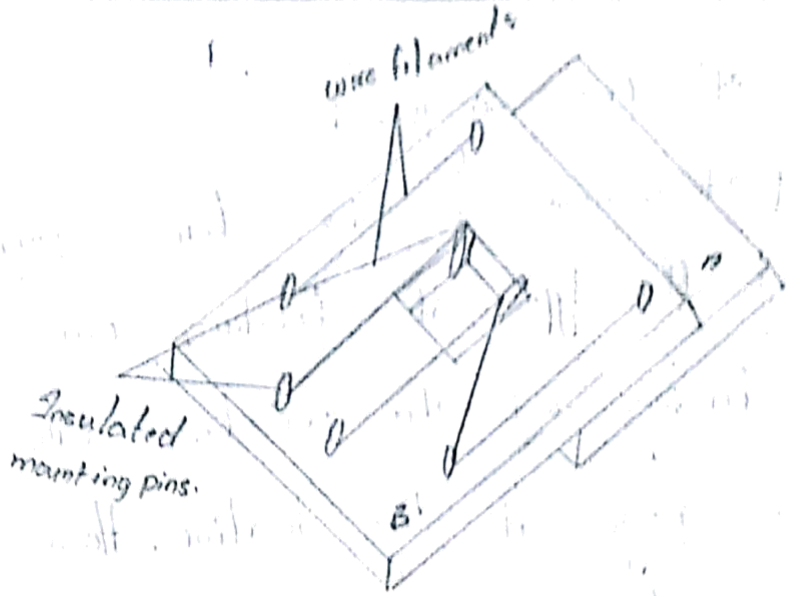

fig:- unbonded resistance strain gauge

Advantages :-

* low hysteresis
* low creep
* Freedom from faulty insulation
* This can be used repeatedly.

Limitations :

* high cost
* more prone to wire breakage.
* poor heat dissipation

Applications :

* Displacement transducers
* pressure transducers.

9

Strain Gauge Circuits :

The measurements of change in resistance of a strain gauge is accomplished by the following circuits.

1. potentiometer / Ballast circuit
2. wheatstone bridge : { Null balance bridge circuit / Deflection bridge circuit.

Potentiometer (Ballast Circuit) :-

The o/p of a strain gauge can also be measured by a potentiometer. This circuit is generally employed for dynamic strain measurement. A simple Ballast Circuit is shown in below

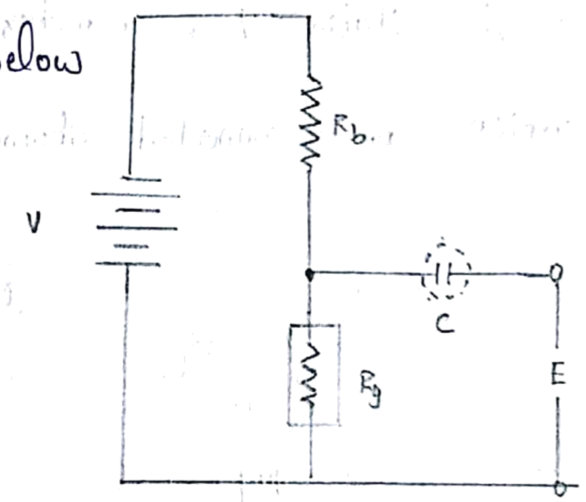

fig:- Strain gauge potentiometer

open circuit o/p voltage is given by

$$E = \left(\frac{R_g}{R_b + R_g} \right) V$$

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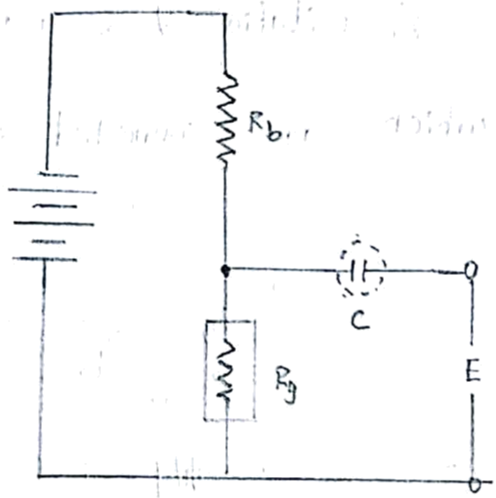


fig:- Strain gauge potentiometer

open circuit o/p voltage is given by

$$E = \left(\frac{R_g}{R_b + R_g}\right) V$$

when the gauge is strained, the resistance of the gauge changes to $R_g + \Delta R_g$ & output voltage changes by

$$\Delta E = V \frac{R_b \Delta R_g}{(R_b + R_g)^2}$$

$$= V \frac{R_b R_g}{(R_b + R_g)^2} \cdot \frac{\Delta R_g}{R_g}$$

if $R_b = R_g$

then $\Delta E = \frac{4V}{4} F\epsilon$

Wheatstone Bridge Circuit :-

It consists of a 4 resistance arms (R_1, R_2 & R_3, R_4) & a voltage source for exitation V & a meter or detector for o/p voltage 'E' which are connected shown in fig.

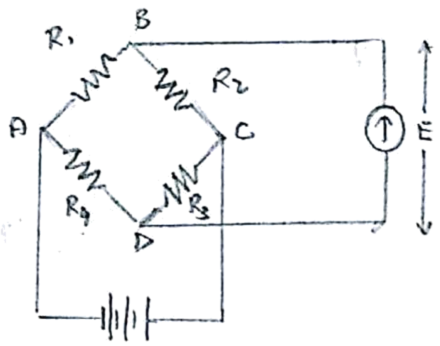


fig:- wheatstone Bridge Circuit.

The o/p voltage across the points B & D is zero when the bridge is balanced. At this condition no current flows through the meter. Then

$$R_1 \cdot R_3 = R_2 \cdot R_4$$

(a) Null Balance Bridge Circuit :

When the specimen under test is not strained, the bridge is balanced by adjusting the arm resistance. In the balanced condition $R_1 R_3 = R_2 R_4$ & o/p voltage is zero

where R_1 = resistance of the strain gauge.

R_2 = resistance of the variable resistor,

$R_3 = R_4$ resistances of the fixed resistors

$$(R_1 + \Delta R_1) R_3 = (R_2 + \Delta R_2) R_4$$

$$R_1 + \Delta R_1 = (R_2 + \Delta R_2) \frac{R_4}{R_3}$$

$$= R_2 \frac{R_4}{R_3} + \frac{\Delta R_2 R_4}{R_3}$$

$$\Delta R_1 = \Delta R_2 \frac{R_4}{R_3} \quad \left[\because \frac{R_2 R_4}{R_3} = R_1 \right]$$

The change in gauge resistance ΔR_1 in terms of strain is given by

$$\Delta R_1 = F \epsilon R$$

$$= \Delta R_2$$

where

F = gauge Factor

ϵ = strain

R = Resistance

(b) Deflection Type Bridge Circuit :-

The fig shows simple deflection type bridge circuit. In this it is assumed that the impedance of voltage measuring device is very high & this voltage can be termed as open circuit voltage

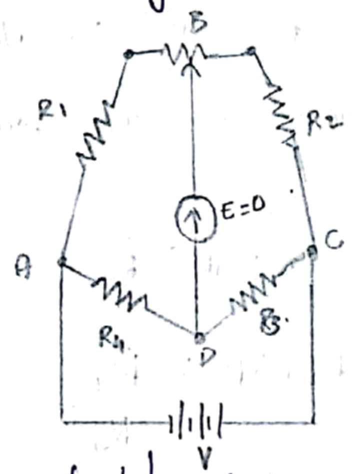


fig:- Deflection wheatstone bridge circuit.

where there is no strain, bridge resistance of the circuit are so adjusted that the bridge is balanced. when the gauge is strained, the equilibrium of the bridge circuit gets disturbed & the deflection in the meter is observed

voltage drop across the points A & B is given by

$$V_{AB} = R_1 \frac{V}{R_1+R_2}$$

voltage drop across the Resistor R_4

$$V_{AD} = R_4 \frac{V}{R_3+R_4}$$

output voltage 'E' across BD is

$$V_{BD} = E = V_{AB} - V_{AD}$$

$$= \frac{R_1}{R_1+R_2} V - \frac{R_4}{R_3+R_4} V$$

$$= \frac{V(R_1R_3 - R_2R_2)}{(R_1+R_2)(R_3+R_4)}$$

As initially the bridge is balanced

$$R_1R_3 = R_2R_4$$

output voltage $E=0$

Let the amount of change in the resistance of the resistors are ΔR_1, ΔR_2, ΔR_3, ΔR_4 respectively

the change output voltage ΔE is given by

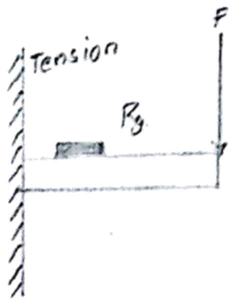
$$\Delta E = \frac{VR_1R_2}{(R_1+R_2)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right]$$

Measurement of Strains in Bending by Un-Balanced wheatstone Bridge :

In strain instrument, the arms of the bridge circuit can be replaced by the strain gauges.

(a) Quarter Bridge (One Active Gage) :-

Consider a cantilever subjected to a load, F to which a single strain gauge is mounted as shown in fig



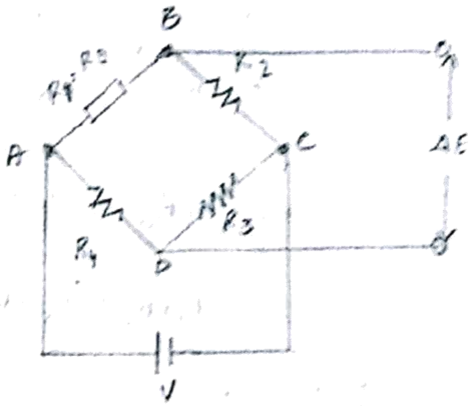


fig:- Quarter Bridge

The bridge used to measure the strain of the cantilever.

$$\Delta E = \frac{VR_1R_2}{(R_1+R_2)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right]$$

where $R_1 = R_g = R_2 = R_3 = R_4$ &

$\Delta R_2 = \Delta R_3 = \Delta R_4 = 0$

$$\Delta E = \frac{V}{4} \cdot \frac{\Delta R_1}{R_1}$$

$$\Delta E = \frac{V}{4} F\epsilon$$

where F = gauge factor

$= \Delta R / R / \epsilon$

ϵ = strain induced in the member

(b) Half Bridge (Two Active Strain Gauges):-

In half bridge two strain gauges are mounted on the beam as shown in fig. the two gauges are placed in such a way that they experience the same strain but opposite in sign.

Let the gauge 1 (R_{g1}) is attached on the tensile side & the gauge 2 (R_{g2}) is attached on the compressive side of the beam & are connected in adjacent arms of the bridge as shown in fig. Known fixed resistances are connected in the remaining arms of the bridge.

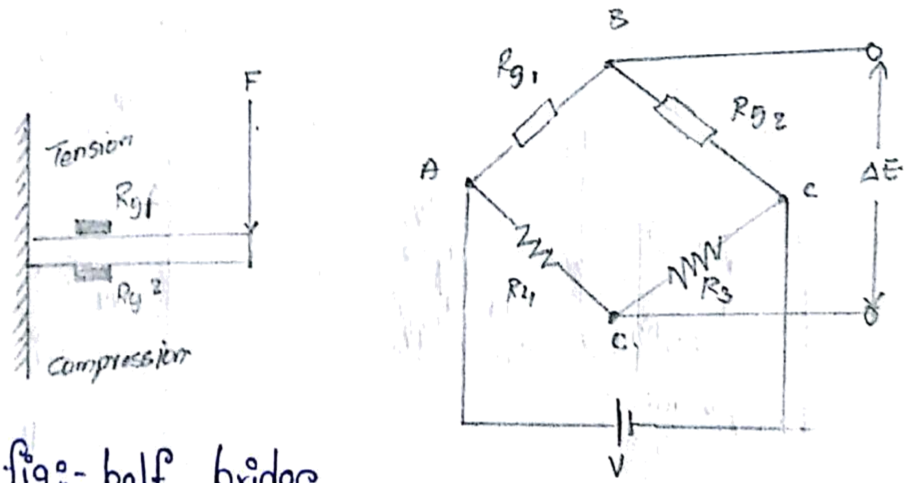


fig:- half bridge

then $R_1 = R_2 = R_3 = R_4 = R$

$\Delta R_3 = \Delta R_4 = 0$

$$\Delta E = \frac{VR_1R_2}{(R_1+R_2)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right]$$

$$= \frac{VR_1^2}{4R_1}\left[\frac{\Delta R}{R} - \left(\frac{-\Delta R}{R}\right)\right]$$

$$= \frac{V\Delta R}{2R} \quad [\because \Delta R_1 = -\Delta R_2]$$

$$= \frac{V}{2} F\epsilon$$

(c) <u>Full Bridge (For Active strain gauges)</u> :-

In this, four strain gauges are mounted on the beam whose strain is being measured. Two gauges are connected on the tension side of the beam & remaining two are connected on the compresed side of the beam as shown in fig

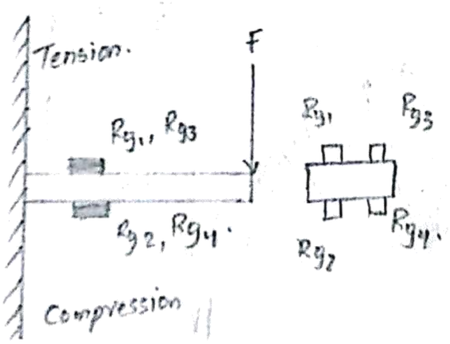


fig:- Cantilever subjected to bending

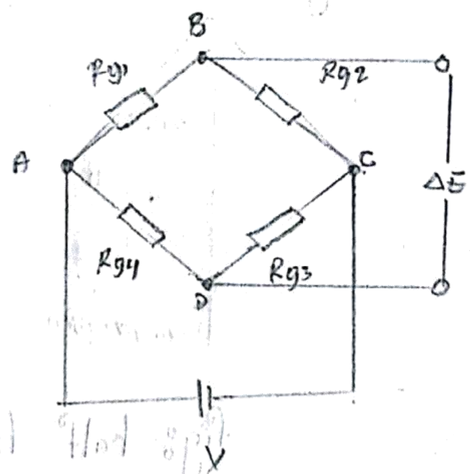


fig:- Bridge Circuit to measure strain.

The output voltage of the bridge is given by

$$\Delta E = \frac{V R_1 R_2}{(R_1+R_2)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{3} - \frac{\Delta R_4}{R_4}\right] \quad \delta_1$$

$$R_{g1} = R_{g2} = R_{g3} = R_{g4} = R$$

$$\Delta R_{g1} = -\Delta R_{g2} = \Delta R_{g3} = -\Delta R_{g4} = \Delta R$$

Then output voltage is

$$\Delta E = \frac{VR^2}{4R}\left[\frac{\Delta R}{R} - \frac{(-\Delta R)}{R} + \frac{\Delta R}{R} - \frac{(-\Delta R)}{R}\right]$$

$$= \frac{VR^2}{4R^2}\left[\frac{4\Delta R}{R}\right]$$

$$= \frac{V\Delta R}{R}$$

$$= VF\epsilon$$

Measurement of Torsional Strain (Torque-meter):

As electrical resistance strain gauges are in-senstive to the shear strains, they must be positioned on the shaft where principle strains occurs as shown in fig.

fig:- Arrangement of strain gauges to measure torsional strains

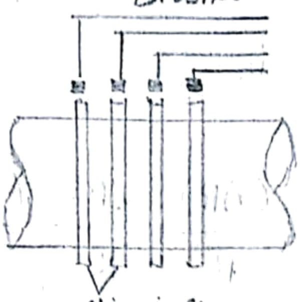


fig:- slip ring arrangement on a shaft.

Strain Gauge Rosettes :

when the loading on the specimen is unidirectional & uniaxial, a single strain gauge is sufficient to perform stress analysis.

For a uniaxially loaded member

stress (σ) = Young's modulus $(E) \times$ Strain (ϵ)

Three Element Rectangular Rosette :

The three strain gauges are arranged as shown in fig. to form a rectangular rossette. The strain gauges are

The strain gauges are placed at 0°, 45°, 90° positions.

principal strains $\epsilon_{max}, \epsilon_{min}$

$$= \frac{1}{2}\left[(\epsilon_1+\epsilon_3) \pm \sqrt{2(\epsilon_1-\epsilon_2)^2+2(\epsilon_2-\epsilon_3)^2}\right]$$

principal stresses $\sigma_{max}, \sigma_{min}$

$$= \frac{E}{2}\left[\frac{\epsilon_1+\epsilon_3}{1-v} \pm \frac{1}{1+v}\left(\sqrt{2(\epsilon_1-\epsilon_2)^2+2(\epsilon_2-\epsilon_3)^2}\right)\right]$$

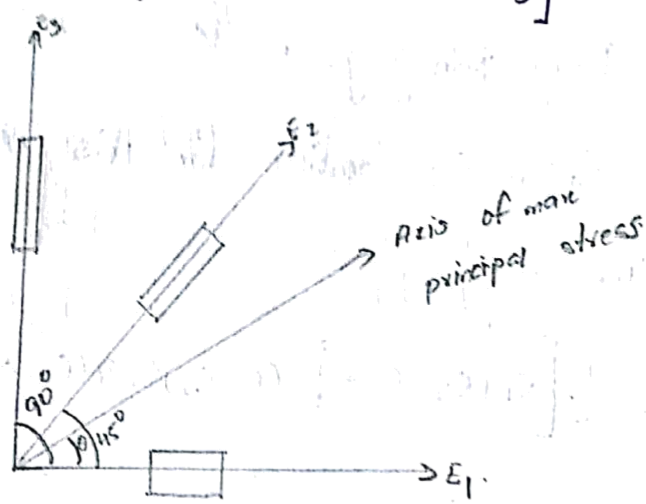


fig:- Rectangular rosette

maximum shear stress is

$$\tau_{max} = \frac{E}{2(1+v)}\sqrt{2(\epsilon_1-\epsilon_2)^2+2(\epsilon_2-\epsilon_3)^2}$$

Orientation of the principal stresses are

$$\tan 2\theta = \frac{2\epsilon_2-\epsilon_1-\epsilon_3}{\epsilon_1-\epsilon_3} \cdot \quad 0<\theta<+90^\circ \text{ when } \epsilon_2 > \frac{\epsilon_1+\epsilon_2}{2}$$

Three Element Delta Rosette :-

The arrangement of strain gauges for a Delta rosette is shown in fig.

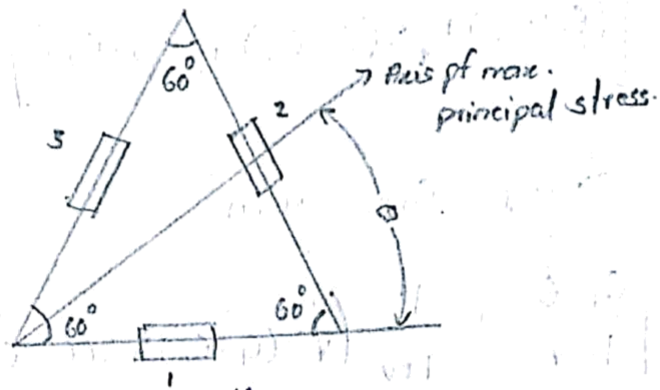


fig:- Delta strain gauge rosette

The stress - strain the location for this arrangement are

principal strains

$$\epsilon_{max}, \epsilon_{min} = \frac{1}{3}\left[\epsilon_1 + \epsilon_2 + \epsilon_3 \pm \sqrt{2(\epsilon_2-\epsilon_3)^2 + 2(\epsilon_2-\epsilon_3)^2 + 2(\epsilon_3-\epsilon_1)^2}\right.$$

principal stresses

$$\sigma_{max}, \sigma_{min} = \frac{E}{3}\left[\frac{\epsilon_1+\epsilon_2+\epsilon_3}{1-\nu} \pm \frac{1}{1-\nu}\sqrt{2(\epsilon_1-\epsilon_2)^2 + 2(\epsilon_2-\epsilon_3)^2 + 2(\epsilon_3-\epsilon_1)^2}\right.$$

maximum shear stresses

$$\tau_{max} = \frac{E}{3(1+\nu)}\left[\sqrt{2(\epsilon_1-\epsilon_2)^2 + 2(\epsilon_2-\epsilon_3)^2 + 2(\epsilon_3-\epsilon_1)^2}\right.$$

Orientation of principal stress is

$$\tan 2\theta = \frac{\sqrt{3}(\epsilon_3-\epsilon_2)}{2\epsilon_1-\epsilon_2-\epsilon_3} \quad 0 < \theta < 90^\circ \text{ when } \epsilon_3 > \epsilon_2$$

Strain Gauge Temperature Compensation :-

It is not possible to estimate corrections to be made for temperature effects in strain gauges. however compensation is done by means of an experimental setup as shown in fig. In this a Compensation gauge is attached to the circuit.

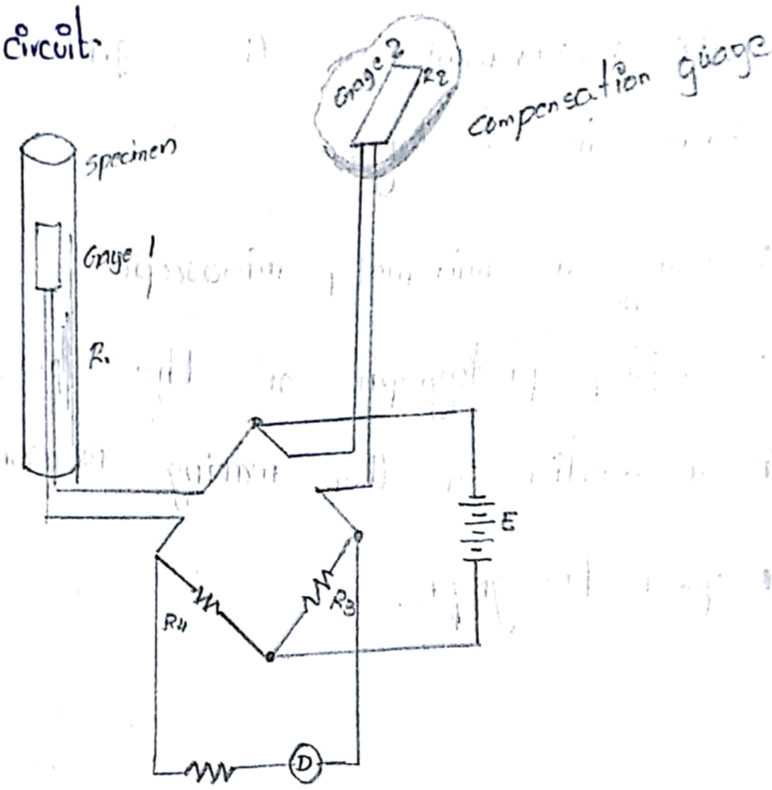


fig:- temperature compensation arrangement for a resistance strain gauge

Strain Measurement by Grid Method :

It is a simple method of strain measurement. In this method some type of grid marking are placed on the surface of an unstrained specimen whose strain is to be measured.

The measurement of the grid deformations may be done in 2 ways

1. using a micrometer microscope
2. Taking photograph of the grid before & after the deformation & then making measurements on the developed photographs.

Unit-IV

Stress strain measurements

Types of strain gauges

→ photo elasticity

→ Brittle lac~~que~~ gauges

→ strain gauges

Non electrical [static] loads

→ mechanical

→ optical

→ photo elasticity

electrical (Dynamic) loads

→ resistence type

→ capacitance / Inductance

used for Large mass & size

retained for long period of time

Requirements of strain gauges:-

→ Small size and negligible mass

→ simple and easy to attachment to the specimen under test

principle:-

→ It is based on when a Conductor is subjected to mechanical deformation its length and diameter are change, also change in its resistence

W.K.T $R = \frac{\rho L}{A}$

Here ρ - specific resistance

L - length

A - Area

apply 'log' on both sides

$$\log R = \log\rho + \log l - \log A$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{dA}{A}$$

Area $A = \frac{\pi}{4} d^2 = cd^2$

$$A = cD^2$$

$$\frac{dA}{A} = \frac{2cdD}{D}$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{2cdD}{D}$$

$$\frac{(dR/R)}{(dl/l)} = \frac{(d\rho/\rho)}{(dl/l)} + 1 - \left[\frac{2cdD}{D}\right] \frac{dl}{l}$$

$$F = \frac{(dR/R)}{(dl/l)} = \text{gauge factor}$$

$$F = \frac{(d\rho/\rho)}{(dl/l)} + 1 - 2\mu$$

$$\mu = \left[\left[\frac{cdD}{D}\right] \left(\frac{dl}{l}\right)\right]$$

Metal Resistence strain gauges :-

(i) un bounded strain gauges

(ii) bounded strain gauges

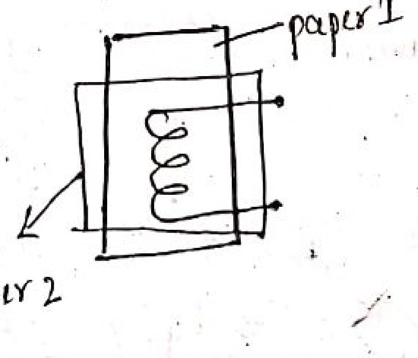
(i) un bounded strain gauges :- (change in length)

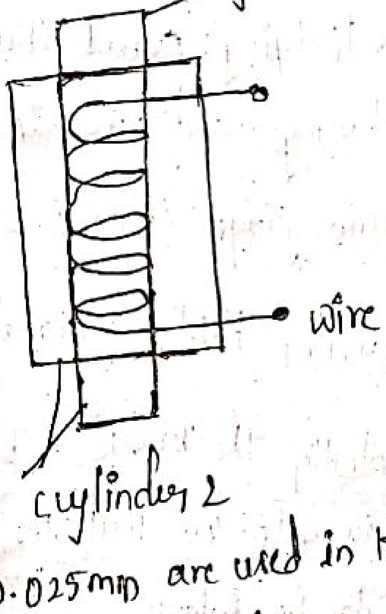
the resistence wire are streached around rigid and electrically insulated fins on two frames A & B which can relatively moved. each other the strain can be detected through change in resistence of wire in electrical circuit the wire and pins are not permanently fitted, simply by screwed.

→ unbounded gauges are specially used for pressure, force

(ii) Bounded strain gauges [change in dia] :-

(a) Wire type straingauges :-





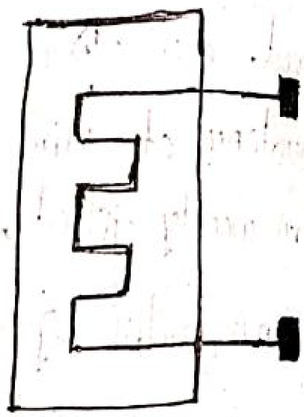
A ~~wire~~ very fine wire of diameter 0.025mm are used in the form of grid shape consists of A series of a long parallel loops with leads.

→ leads and connections terminals provided for the above loops for satisfying performance The bond b/w the resistence elements and cement joints are arranged.

→ Along with grid wire backlite (or) paper used as a cover plates.

(b) metal foil type gauges :-

→ the gauge is provided by a printed circuit board with design grid shapes.

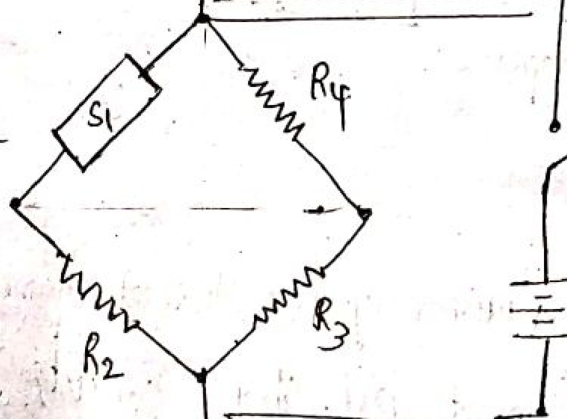


→ the Designed grid pattern is first provided on the Thin sheet of metal alloy with acid resistance link, chemical These can be successfully employed to fill & shapeing of the metal foil.

the satisfying curved shapes are achevied due to chemical machining and accurate construction.

Strain gauge circuits :-

(i) quarter bridge gauges :-



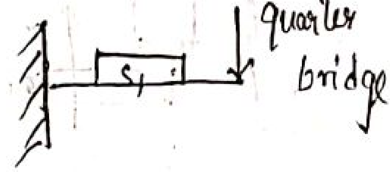
→ Intially the bridge resistence are so adjusted That The bridge is balanced

→ After gauges are strained the equilibrium gets disturbed.

→ Here 'R1' is strain gauge is under Tension load and R_2, R_3, R_4 are fixed resistors when gauge is strained the change in voltage in the circuit is

$$dV_0 = \frac{V_s}{4}\left(\frac{dR}{R}\right)$$

V_s – starting voltage

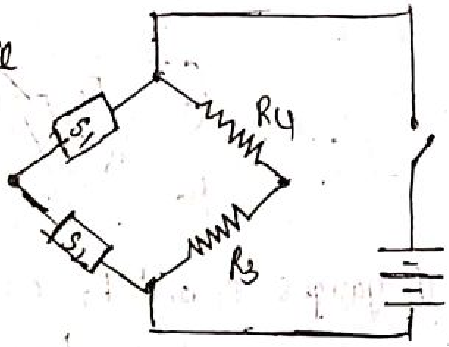


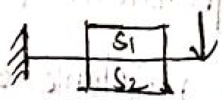
(ii) Half Bridge gauge :-

→ Two of the bridge elements are strain gauges and other two are fixed resistors

→ the gauge 'S1' surface is upper surface of cantilever beam, the gauge 'S2' bounded to lower surface of beam

$$dV_0 = \frac{V_s}{2}\left(\frac{dR}{R}\right)$$





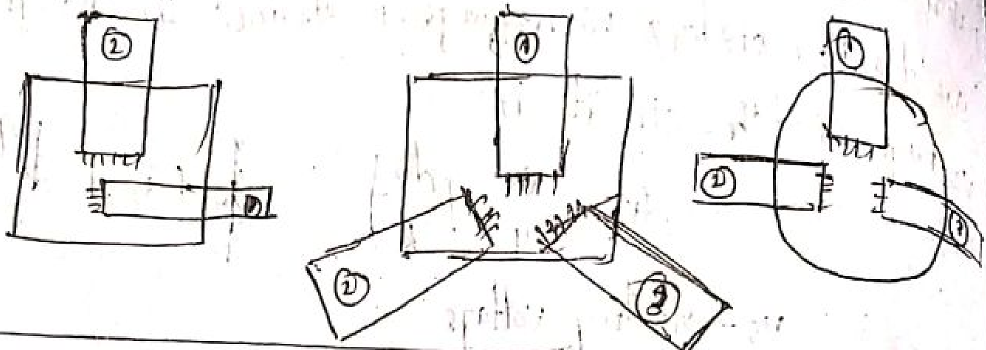
Rosettes of strain [strain Rosettes]:-

→ single gauges are used for uniaxial loads That is when a member loaded either in Tension (or) Compression

→ For biaxial stress feeds two (or) more gauges used in different directions.

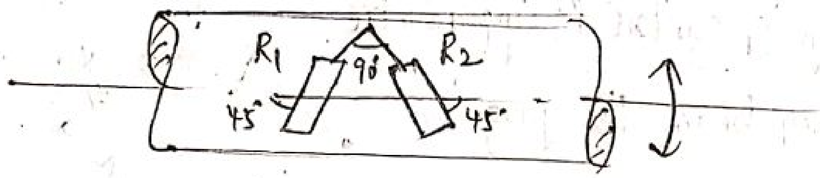
→ A group of gauges bounded to same supporting material in different relative positions depending upon arrangement of gride Those names as multi grid rosettes (or) gauge rosettes

Ex: Rectangular, Delta, T-delta rosettes



strain gauges are used for Torsion:-

→ when a bar subjected to pure tension the maxi. Tensile and Compressive strains occurs at 45° to axis of the bar



→ the gauges R_1 and R_2 are located along the principle direction the gauges are at 90° to each other

→ Here the resistence of strain gauges R_1 increases due to Tensile strain the resistence of strain gauge R_2 decreases due to Compressive strain

→ These strain gauges are connected electrically in adjustment sides of half bridge circuit

→ the strain sensed by each gauge will be

strain in gauge $R_1 = \epsilon_d + \epsilon_s + \epsilon_l$

" " " $R_2 = \epsilon_d - \epsilon_s + \epsilon_l$

change in output voltage

$$dV_0 = \frac{V_s}{4}\left[\frac{dR_d}{R_1} + \frac{dR_s}{R_1} + \frac{dR_l}{R_1}\right] - \left[\frac{dR_R}{R_2} - \frac{dR_s}{R_2} + \frac{dR_l}{R_2}\right]$$

$$dv_0 = \frac{V_s}{4}\left[\frac{dR_s}{R}\right]$$

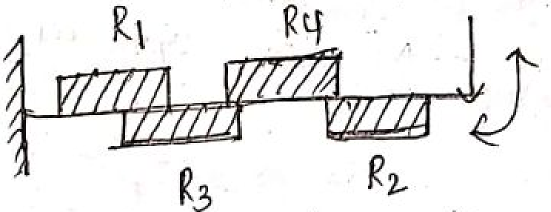
$$\text{if } R_1 = R_2 \Rightarrow dv_0 = \frac{V_s}{4}\left[\frac{dR_s}{R}\right]$$

Arrangement for axial Thrust:-

→ circular bar subjected to axil Thrust and bearing moment

→ The measurement of stress under Conditions of axial loading by mounting four gauges on Test specimen two gauges R_1, R_3 are placed axially and other two gauges R_2, R_4 are placed circumferentially These effect eliminate bending effects.

→ For measurement of bending moment two gauges R_1, R_4 are mounted on top of the Cantilever specimen R_2, R_3 are the bottom of the Cantiliver Specimen



→ the measure of stress under bending moment is acheived by two gauges R_1, R_4 are placed at the top of the test specimen another two gauges R_2, R_3 are Placed at The bottom of the test specimen.

Strain gauge alloys and materials:-

→ The selection of materials used for strain gauges is based on the following parameters

1. High gauge factor
2. resistence change is simple and it is a function of mechanical gauge
3. It is low Temp sensitivity
4. min. Thermo electric tendency at connections
5. High yield point and endurance limit
6. Good work ability and yield ability
7. economical cost

material name:- advance Cu-57%, Ni-43%.

Iridium-platinum - Ir-5%, Pt-95%.

Iso electric - 36% Ni, 52% Fe, 8%-chromium

magnesium - Ni-4%, 84% -Cu.

Force Measurement:

A measure of the unknown force may be accomplished by the methods incorporating following principles:-

(i) Balancing the force against a known gravitational force on a standard mass (scales balances)

(ii) Translating the force to a fluid pressure and then measuring the resulting pressure and pneumatic load cells)

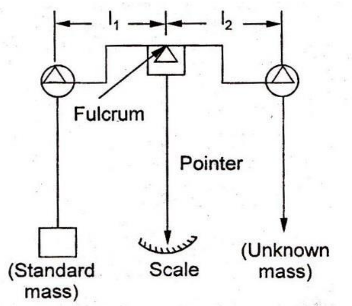
(iii) Applying the force to some elastic member and then measuring the resulting (proving ring)

(iv) Applying the force to a known mass and then measuring the resulting acceleration

(v) Balancing the force against a magnetic force developed by interaction of a magnet current carrying coil.

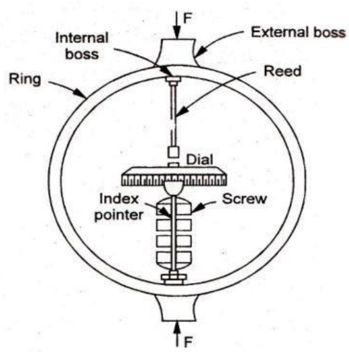
Scales and balances:

Force or weight is indicated by making a comparison the force due to gravity acting on a standard mass and the force due to gravity on the unknown mass.



An equal-arm beam balance of a beam pivoted on a knife-edge fulcrum the centre. Attached to the centre of the beam a pointer which points vertically downwards the beam is in equilibrium. The equilibrium exist when the clockwise rotating equals the anti-clockwise rotating moment i.e., $m_1\, l_1 = m_2 l_2$. Since the two arms of the beam equal; the beam would be in equilibrium again $m_1 = m_2$. Further for a given location, the attraction acts equally on both the masses therefore at the equilibrium conditions $W_1 = W_2$, i. e., the unknown force or weights equal known force or weights.

PROVING (STRESS) RING:



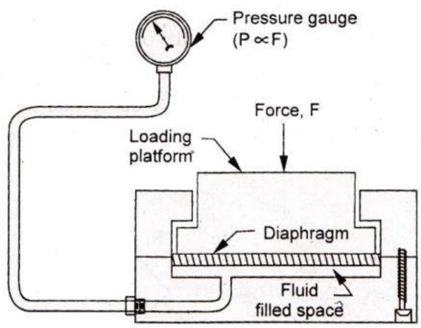
Proving ring

The proving (stress) ring is a ring of known physical dimensions and mechanical properties. When an external compressive or tensile load is applied to the lugs or external bosses, the ring changes in its diameter; the change being proportional to the applied force. The amount of ring deflection is measured by means of a micrometer screw and a vibrating reed which are attached to the internal bosses. During use, the micrometer tip is advanced and its contact with the reed is indicated by considerable damping of the reed vibration. The difference in the micrometer reading taken before and after the application of load is the measure of the amount of the elongation or compression of the ring. The proving ring deflection can also be picked by LVDT, resulting in a proportional voltage change. The device gives precise results when properly calibrated and corrected for temperature variations.

Instead of deflection, strain in an elastic member may be measured by a strain gauge, and then correlated to the applied force.

Mechanical load cells: The term load cell is used to describe a variety of force transducers which may utilize the deflection or strain of elastic member, or the increase in pressure of enclosed fluids. The resulting fluid pressure is transmitted to some form of pressure sensing device such as a manometer or a bourdon tube pressure gauge. The gauge reading is identified and calibrated in units of force.

Hydraulic load cell:

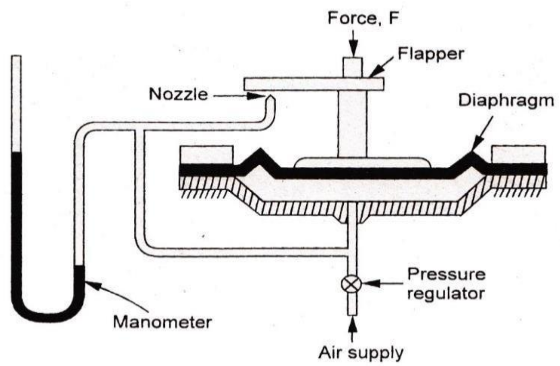


Hydraulic load cell

In a hydraulic load cell the force variable is impressed upon a diaphragm which deflects and there by transmits the force to a liquid. The liquid medium, contained in a confined space, has a preload pressure of the order of 2 bar. Application of force increases the liquid pressure; it equals the force magnitude divided by the effective area of the diaphragm. The pressure is transmitted to and read on an accurate pressure gauge calibrated directly in force units. The system has a good dynamic response; the diaphragm deflection being less than 0.05 mm under full load. This is because the diaphragm has a low modulus and substantially all the force is transmitted to the liquid. These cells have been to measure loads up to about 2.5 MN with an accuracy of the order of 0. I percent of full scale; resolution is about 0.02 percent.

pneumatic load cell:

A pneumatic load cell operates on the force-balance principle and employs a nozzle-flapper transducer similar to the conventional relay system. A variable downward force is balanced by an upward force of air pressure against the effective area of a diaphragm. Application of force causes the flapper to come closer to the nozzle, and the diaphragm to deflect downwards. The nozzle opening is nearly shut-off and this results into an increased back pressure in the system. The increased pressure acts on the diaphragm, produces an effective upward force which tends to return the diaphragm to its preload position.

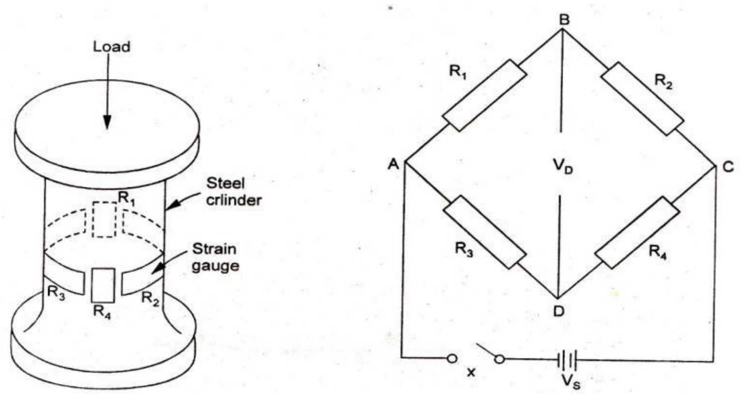


Pneumatic load cell

For any constant applied force, the system attains equilibrium at a specific nozzle opening and a corresponding pressure is indicated by the height of mercury column in a manometer. Since the maximum pressure in the system is limited to the air-supply pressure, the range of 'M unit can be extended only by using a larger diameter diaphragm. The commercially available load cells operating on this principle can measure loads up to 250 KN with an accuracy of 0.5 percent of full scale. The air consumption is of the order of 0.17 m^3/hr of free air.

Strain gauge load cell:

The strain gauge load cell converts weight or force into electrical outputs which are provided by the strain gauges; these outputs can be connected to various measuring instruments for indicating, recording and controlling the weight or force.



A simple load cell consists of a steel cylinder which has four identical strain gauges mounted upon it; the gauges R_1 and R_4 are along the direction of applied load and the gauges R_2 and R_3 are attached circumferentially at right angles to gauges R_1 and R_4. These four gauges are connected electrically to the four limbs of a Wheatstone bridge circuit. When there is no load on the cell, all the four gauges have the same resistance. Evidently then the terminals B and D are at the same potential; the bridge is balanced and the output voltage is zero

$$V_{ab} = V_{ad} = \frac{V_s}{2}$$

$$V_0 = V_{ab} - V_{ad} = 0$$

When a compressive load is applied to the unit, the vertical gauges (R_1 and R_4) undergo compression and so decrease in resistance. Simultaneously the circumferential gauges R_2 and R_3 undergo tension and so increase in resistance. In the Poisson's arrangement, the positive and negative strains (and so changes in resistance) are related to each other by the Poisson's ratio. Thus when strained, the resistance of the various gauges are :

$$R_1 \text{ and } R_4 = R - dR \qquad \text{(compression)}$$

$$R_2 \text{ and } R_3 = R + \mu\, dR \qquad \text{(tension)}$$

Potential at terminal B is,

$$V_{ab} = \frac{R_1}{R_1 + R_2} V_s$$

$$= \frac{R - dR}{(R - dR) + (R + \mu\, dR)} V_s = \frac{R - \mu\, dR}{2R - dR(1 - \mu)} V_s$$

Potential at terminal D is,

$$V_{ad} = \frac{R_3}{R_3 + R_4} V_s$$

$$= \frac{R + \mu\, dR}{(R + \mu\, dR) + (R - dR)} V_s = \frac{R + \mu\, dR}{2R - dR(1 - \mu)} V_s$$

The changed output voltage is,

$$V_0 + dV_0 = \frac{R - dR}{2R - dR(1 - \mu)} V_s = \frac{R + \mu\, dR}{2R - dR(1 - \mu)} V_s$$

$$= \frac{dR(1 + \mu)}{2R} V_s = 2(1 + \mu)\left(\frac{dR}{R}\frac{V_s}{4}\right)$$

The output voltage $V_0 = 0$ under unloaded conditions, and therefore change in output voltage due to applied load becomes.

$$dV_0 = 2(1 + \mu)\left(\frac{dR}{R}\frac{V_s}{4}\right) \qquad ...(13.14)$$

Apparently this output voltage is a measure of the applied load. The use of four identical strain gauges each arm of the bridge provides full temperature compensation and also increases the bridge sensitivity $2(1 + \mu)$ times.

The strain gauge load cells are excellent force measuring devices, particularly when the force is not steady. They are generally stable, accurate and find extensive use in industrial applications such as draw-bar and tool-force dynamometers, crane load monitoring, and road vehicle weighing device etc.

Problem:

A strain gauge load cell consists of a solid steel cylinder which has 4-identical strain gauges mounted upon it in the Poisson's arrangement (Fig. 13.10). For each gauge the nominal resistance $R = 100\ \Omega$, gauge factor $F = 2.0$ and the gauges are connected electrically to the four arms of a wheatstone bridge circuit which is energised with a supply voltage of 6 volts. Make calculations for the sensitivity of the load cell. The steel cylinder is 50 mm in diameter and for steel the modulus of elasticity $E = 200$ GN/m² and the Poisson's ratio $\mu = 0.3$.

Solution : Consider a load of 1 kN applied to the load cell.

$$\text{stress} = \frac{\text{load}}{\text{cross} - \text{sectional area}} = \frac{1 \times 10^3}{\frac{\pi}{4}\left(50 \times 10^{-3}\right)^2}$$

$$= 0.5095 \times 10^6 \text{ N/m}^2$$

$$\text{strain} = \frac{\text{stress}}{\text{modulus of elasticity}} = \frac{0.5095 \times 10^6}{200 \times 10^{-9}}$$

$$= 2.5475 \times 10^{-6}$$

Fractional change in resistance,

$$\frac{dR}{R} = F \in = 2.0 \times 2.5475 \times 10^{-6}$$

$$= 5.095 \times 10^{-6}$$

$$\text{Output voltage } dV_0 = 2\,(1 + \mu)\left(\frac{dR}{R}\frac{V_s}{4}\right)$$

$$= 2\,(1 + 0.3)\left(5.095 \times 10^{-6} \times \frac{6}{4}\right)$$

$$= 19.87 \times 10^{-6} = 19.87\ \mu\text{V}$$

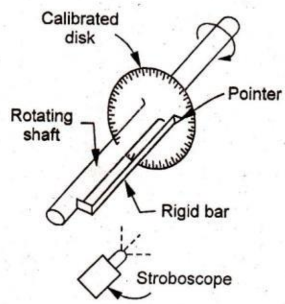
Hence the gauge sensitivity is 19.87 μV/kN

Torques measurement:

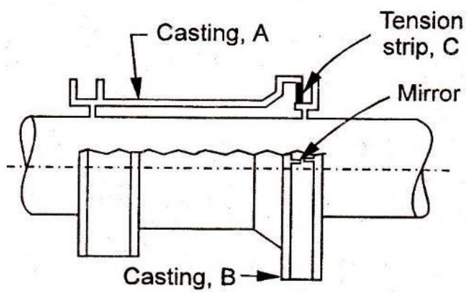
The main purpose of torque measurement is to determine the mechanical power required or developed by a machine. Torque measurement also helps in obtaining load information necessary for stress or strain analysis. In some cases other variables are determined by measuring torque. For example, in the case of rotating cylinder viscometer, measurement of torque developed at the fixed end of the stationary cylinder help in determining the viscosity of the fluid between the movable and stationary cylinder.

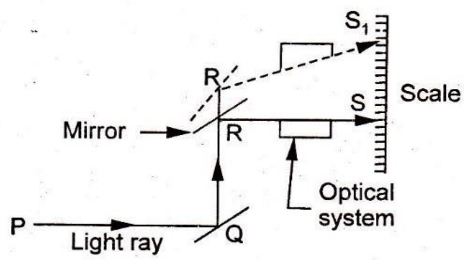
Mechanical torsion meter:

Figure shows the schematics of an elastic torsion bar meter wherein angular deflection of a parallel length of shaft is used to measure torque. The angular twist over fixed length of the bar is observed on a calibrated disk (attached to the rotating shaft) by using the stroboscope effect of intermittent viewing and the persistence of vision. The system gives a varying angle of twist between the driving engine and the driven load as the torque changes.



Optical torsion meter: The meter uses an optical method to detect the angular twist of a rotating shaft.

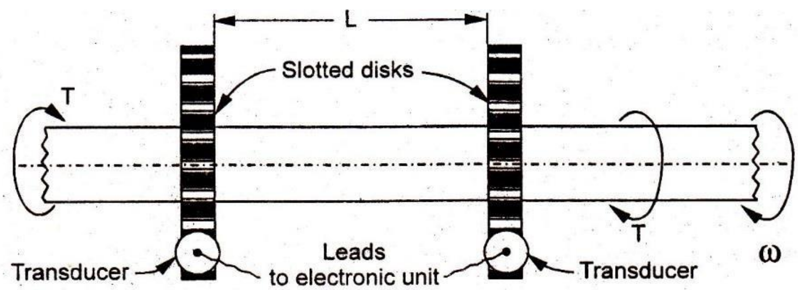




The unit comprises two castings A and B which are fitted to the shaft at a known distance apart. These castings are attached to each other by a tension strip C which transmits torsion but has little resistance to bending. When the shaft is transmitting a torque, there occurs a relative movement between the castings which results in partial inclination between the two mirrors attached to the castings. The mirrors are made to reflect a light beam onto a graduated scale; angular deflection of the light ray is then proportional to the twist of, and hence the torque in, the shaft. For constant torque measurements from a steam turbine, the two mirrors are arranged back to back and there occurs a reflection from each mirror during every half revolution. A second system of mirrors giving four reflections per revolution is desirable when used with a reciprocating engine whose torque varies during a revolution.

Electrical torsion meter: A system using two magnetic or photoelectric transducers, as shown in Fig, involves two sets of measurements.

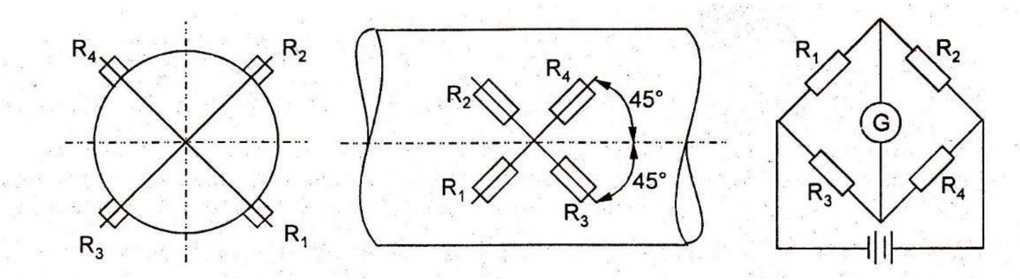
(i) a count of the impulse from either slotted wheel. This count gives the frequency or shaft speed.



(ii) a measure of the time between pulses from the two wheels. This signal is proportional to the twist 6 of, and hence torque T in the shaft.

These two signals, T and ω, can be combined to estimate the power being transmitted by the shaft.

Strain- gauge torsion meter: A general configuration of a strain gauge bridge circuit widely employed for torque measurement from a rotating shaft is shown in Fig.



Four bonded-wire strain gauges are mounted on a 45' helix with the axis of the rotation and are placed in pairs diametrically opposite, If the gauges are accurately placed and have matched characteristics, the system is temperature compensated and insensitive to bending and thrust or pull effects. Any .change in the gauge circuit then results only from torsional deflection. When the shaft is under torsion, gauges I and 4 will elongate as a result of the tensile component of a pure shear stress on one diagonal axis, while gauges 2 and 3 will contract owing to compressive component on the other diagonal axis. These tensile and compressive principal strains can be measured, and the shaft torque can be calculated

A main problem of the system is carrying connections from the strain gauges (mounted on the rotating shaft) to a bridge circuit which is stationary. For slow shaft rotations, the connecting wires are simply wrapped around the shaft. For continuous and fast shaft rotations, leads from the four junctions of the gauges are led along the shaft to the slip rings. Contact with the slip rings is made with the brushes through which connections can be made to the measuring instrument.

Commercial-strain-gauge torque sensors are available with built-in slip rings and speed sensors. A family of such devices covers the range 6 Nm to 1000 kNm with full-scale output of about 40 mV.

SHAFT POWER MEASUREMENT (DYNAMOMETERS):-

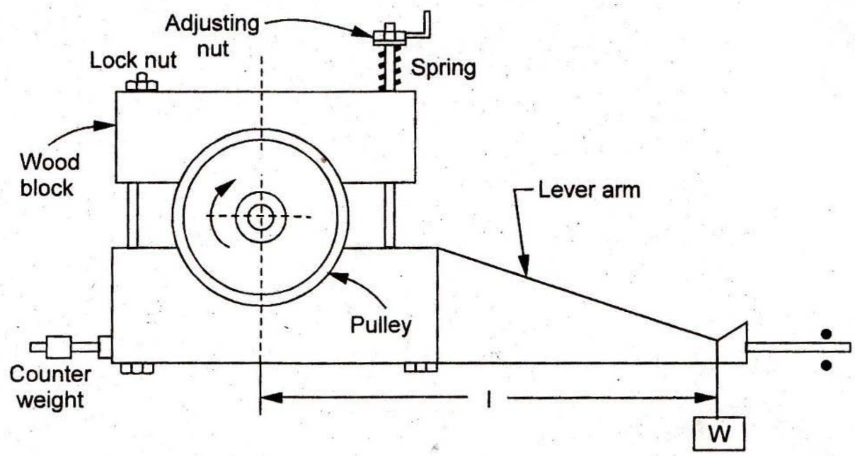
The dynamometer is a device used to measure the torque being exerted along a rotating shaft so as to determine the shaft power input or output of power-generating, transmitting, and absorbing machinery. Dynamometers are generally classified into:

(i) Absorption dynamometers in which the energy is converted into heat by friction whilst being measured. The heat is dissipated to the surroundings where it generally serves no useful purpose. Absorption dynamometers are used when the test-machine is a power generator such as an engine, turbine and an electric motor. The types commonly used include Prony brakes, hydraulic or fluid friction brakes, fan brakes and eddy current dynamometers.

(ii) Transmission dynamometers in which the energy being transmitted either to or from dynamometer is not absorbed or dissipated. After measurement, the energy is conveyed to the surroundings in a useful mechanical or electrical form. A small amount of power ma). However, be lost by friction at the joints of the dynamometer. The type includes torsion and belt dynamometers, and strain gauge dynamometers.

(iii) Driving dynamometer which may be coupled to either power-absorbing or power generating devices since it may operate either a motor or a generator. These instruments measure power and also supply energy to operate the tested devices. They are essentially useful in determining performance characteristics of such machines as pumps and compressors. Electric cradled dynamometer is a typical example of the driving dynamometer.

Mechanical brakes: The Prony and the rope brakes are the two types of mechanical brakes chiefly employed for power measurement. The prony brake has two common arrangements in the block type and the band type. Whereas the block type is employed to high speed shafts with a small pulley, the band type measures the power of low speed shafts having a relatively large pulley.



The block type prony brake consists of two blocks of wood each of which embraces rather less than one half of the pulley rim. One block carries a lever arm to the end of which a pull can be applied by means of a dead weight or spring balance. A second arm projects from the block in the opposite direction and carries a counter-weight to balance the brake when unloaded. When operating, friction between the blocks and the pulley tends to rotate the blocks in the direction of the rotation of the shaft. This tendency is prevented by adding weights at the extremity of the lever arm so that it remains horizontal in a position of equilibrium.

Let W be the weight in newton, I be the effective length of the lever arm in meter, and N be the revolutions of the crankshalt per minute. Then:

$$\text{Torque } T = W\,l \qquad \text{in Nm}$$

and

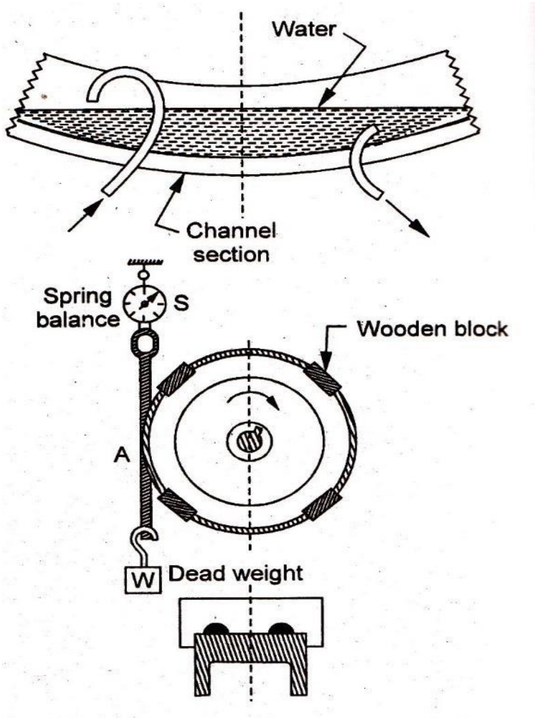
$$\text{Power } P = \frac{2\pi N}{60} \times T \qquad \text{in Nm/s}$$

$$= \frac{2\pi N}{60 \times 1000} \times W\,l \qquad \text{in kW}$$

It may be noted that the power absorbed by this type of dynamometer is independent of the size of the brake and the coefficient of friction.

Rope brake dynamometers:

A rope brake dynamometers consists of one or more ropes wrapped around the fly wheel of an engine whose power is to be measured. The ropes are spaced evenly across the width of the rim by means of U-shaped wooden blocks' located at different points of the rim of the flywheel. The upward ends of the rope are connected together and attached to a spring balance, and the downward ends are kept in place by a dead weight. The rotation of flywheel produces frictional force and the rope tightens. Consequently a force is induced in the spring balance. Generation of heat is enormous and that necessitates a cooling arrangement for the brake. The rim is made trough shaped internally. Water is run into the trough and kept in place by the centrifugal force.



Let W be the dead weight, S be the spring balance reading ; D be the brake drum diameter, and d be the rope diameter. Then effective radius of the brake drum $R_{eff} = (D + d)/ 2$.

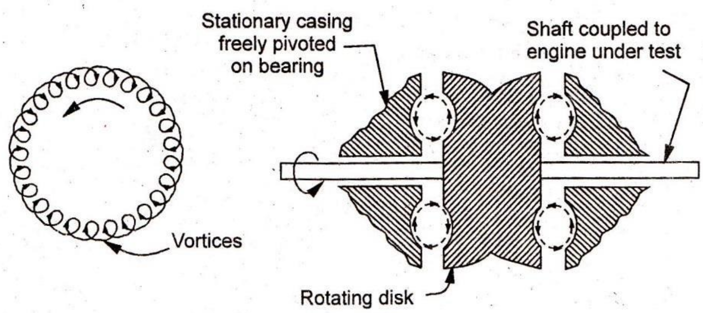
Brake load or net load $= (W - S)$ in N

Braking troque $= (W - S)\, R_{eff}$ in Nm

$$\text{Brake power} = \frac{2\pi N}{60} \times T \quad \text{in Nm/s}$$

$$= \frac{2\pi N}{60 \times 1000} \times (W - S)\, R_{eff} \quad \text{in kW}$$

Hydraulic dynamometer:-



A hydraulic dynamometer uses fluid-friction rather than dry friction for dissipating -the input energy. The unit consists essentially of two elements namely a rotating disk and stationary casing. The rotating disk is keyed to the driving shaft of the prime-mover and it resolves inside the stationary casing. The casing is mounted on antifriction (trunnion) bearings and has a brake arm and a balance system attached to it. Such bearings allow the casing to rotate freely except for the restraint imposed by the brake arm. Further, the casing is in two-halves; one of which is placed on either side of the rotating disk. Semi-elliptical recesses in the casing match with the corresponding grooves inside the rotating disk to form chambers through which a stream of water flow is maintained. When brake is operating, the water follows a helical path in the chamber. Vortices and eddy-currents are set-up in the water and these tend to turn the dynamometer casing .in the direction of rotation of the engine shaft. This tendency is resisted by the brake arm balance system that measure the torque.

Eddy current dynamometer **:** This electrical absorption dynamometer operates on the principle that when an isolated conductor moves through a magnetic field, voltage is induced and local currents flow in a short circular path within the conductor. These currents, called *eddy currents* get dissipated in the form of heat.

Fig. 13.22 shows the basic components comprising and illustrating the principle of operation of an eddy current dynamometer. A toothed steel rotor is mounted onto the shaft of the test-engine and it rotates insides a smooth bored cast iron stator ; the clearance between the rotor tooth and stator being very smally. The stator carries an exciting coil which is energised with a direct current supplied from an external source. Further, the stator is cradled on antifriction trunnions and is provided with a brake arm to which a scale pan or spring balance is attached. The term 'cradled' means that the stator is mounted so as to permit it to swing freely about the axis of the shaft.

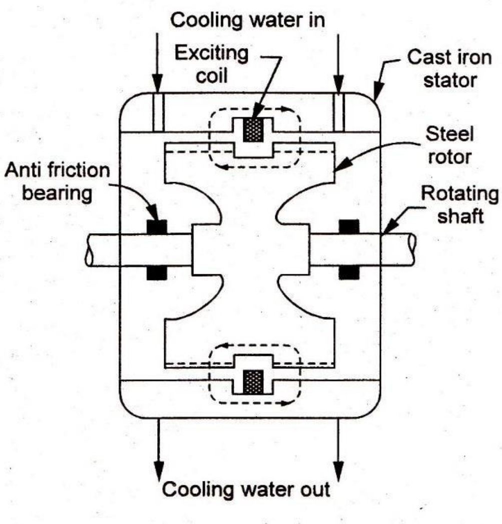


Fig. 13.22 Eddy current dynamometer

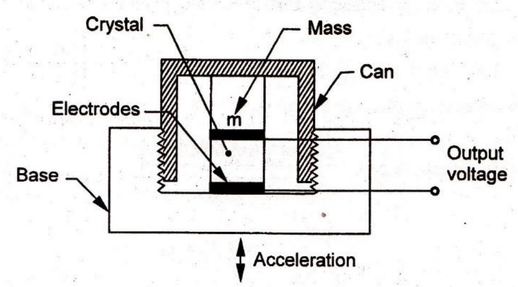
Measurement of acceleration:

There are two types of accelerometers generally used for measurement of acceleration:

(i) Piezo-eletric type, and (ii) seismic type.

(i)Piezo-electric accelerometer: The unit is perhaps the simplest and most commonly used transducer employed for measuring acceleration. The sensor consists of a piezo-electric crystal sandwitched, between two electrodes and has a mass placed on it. The unit is fastened to the base whose acceleration characteristics are to be obtained. The can threaded to the base acts as a 'spring and squeezes the mass against the crystal. Mass exerts a force on the crystal and a certain output voltage is generated. If the base is now accelerated downward, inertial reaction force on the base acts upward against the top of the can. This relieves stress on the crystal. From Newton's second law

$$\text{force} = \text{mass} \times \text{acceleration}$$



Advantages and limitations

* Rugged and inexpensive device

* High output impedance ’

* High frequency response from 010 Hz to 50 kHz

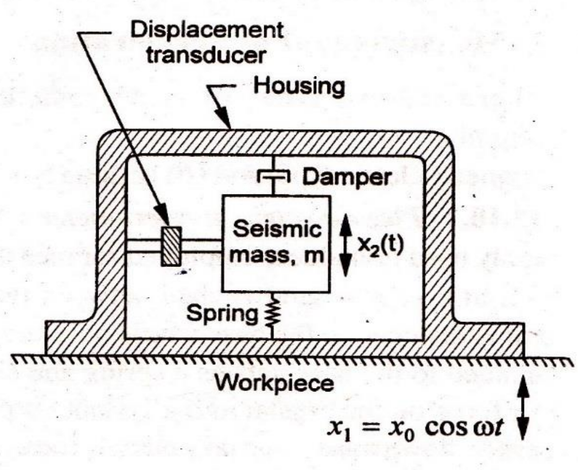
* High sensitivity varies from 10 to 100 mv/g where g = 9.807 m/s^2

* Capability to measure acceleration from a fraction of g to thousands of g

* Somewhat sensitive to changes in temperature

* Subject to hysteresis errors.

Displacement sensing (seismic) accelerometer: In a seismic accelerometer the displacement of a mass resulting from an applied forced is measured and correlated to the acceleration. Fig shows the schematics of a common spring mass damper system which accomplishes this task. The mass is supported by a spring and damper is connected to the housing frame. The frame is rigidly attached to the machine whose acceleration characteristics are to be determined. When an acceleration is imparted by the machine to the housing frame, the mass moves relative to the frame, and this relative displacement between the mass and frame is sensed and indicated by an electrical displacement transducer.



Seismic accelerometer

Theory of seismic accelerometer : The spring-mass-damper system of the seismic accelerometer can be represented by an equilibrium equation obtained through Newton's second law :

$$m\frac{d^2x_2}{dt^2}+c\frac{dx_2}{dt}+kx_2=c\frac{dx_1}{dt}+kx_1 \qquad ...(12.14)$$

where the damping force has been assumed to be proportional to the velocity. For a simple harmonic vibratory motion applied to the housing frame,

$$\text{displacement } x_1 = A\cos\omega t$$

$$\text{velocity } v=\frac{dx_1}{dt}=-\omega A\sin\omega t$$

$$\text{acceleration } a=\frac{dv}{dt}=-\omega^2 A\cos\omega t \qquad ...(12.15)$$

where $\omega = 2\pi f$ rad/s and f is the frequency of vibration in Hz. From these expressions for the instantaneous values of different parameters we have :

$$\begin{aligned}\text{displacement amplitude} &= A\\ \text{velocity amplitude} &= \omega A\\ \text{acceleration amplitude} &= \omega^2 A\end{aligned} \qquad ..(12.16)$$

A solution to equation 12.14 would show that the relative displacement (x_2-x_1) between the mass and housing is given by :

$$(x_2-x_1)=\frac{\omega^2 A}{\omega_n^2\left[\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2+\left\{2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)\right\}^2\right]^{\frac{1}{2}}} \qquad ...(12.17)$$

where the natural frequency ω_n and critical damping coefficient c_c are given by

$$\omega_n=\sqrt{\frac{k}{m}} \quad ; \quad c_c=2\sqrt{mk} \qquad ...(12.18)$$

The seismic instrument may be used either for displacement measurement by proper selection of the mass-spring-damper combination. Since velocity is rate of change of displacement and acceleration is rate of change of velocity, each quantity can be obtained by differentiating or integrating one of the quantity which has been measured. Since the process of integration is more common and easily done in electrical systems, it is a common practice to measure acceleration and then deduce the velocity or displacement by successive integration.

Displacement measurement **: Let the frequency (ω) applied to the base be much higher than the natural frequency (ω_n), then the term $\{2\,(c/c_c)\,(\omega/\omega_n)\}^2$ can be neglected in comparison with $\left[(\omega/\omega_n)^2\right]^2$ and the approximate expression for $(x_2 - x_1)$ becomes :**

$$(x_2 - x_1) = \frac{\omega^2 A}{\omega_n^2\left[\left\{\left(\frac{\omega}{\omega_n}\right)^2\right\}^2\right]^{\frac{1}{2}}} = \frac{\omega^2 A}{\omega_n^2\left(\frac{\omega}{\omega_n}\right)^2} = A \qquad \text{...(12.19)}$$

Thus the output is very nearly equal to the input amplitude *A*. This relation is valid for ω/ω_n ratios greater than 2. Thus for vibration pick-ups, the system is to be operated at frequencies higher than the natural frequency. The task is accomplished by keeping the natural frequency $\left(\omega_n = \sqrt{k/m}\right)$ low by employing soft spring and large mass.

Acceleration measurement : Let the input frequency ω be much smaller than the natural frequency ω_n, then

$$(x_2 - x_1) = \frac{\omega^2 A}{\omega_n^2} = \frac{1}{\omega_n^2} \times \text{maximum acceleration} \qquad \text{...(12.20)}$$

and this relation remains valid for $\omega/\omega_n \le 0.4$ .This if the pick-up is to be used for acceleration measurement, ω_n should be large, *i.e.*, the system should have a stiff spring and small mass. That would enable to operate the system over a wide range of frequencies and still keep the response linear.

In a ***strain gauge accelerometer*** (Fig. 12.25), the sensing mass is mounted on a cantilever beam. A viscous liquid fills the housing and provides the necessary damping. Two strain gauges are attached to the beam, one on each and these sense the strain in the

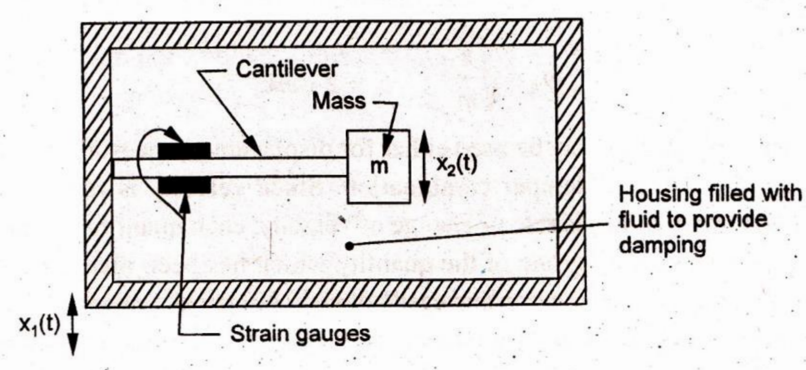


Fig. 12.25 Strain gauge accelerometer

beam which results from vibratory displacement. The leads from the strain gauges are taken to a wheatstone bridge whose output indicates the relative displacement between the mass and the housing form.

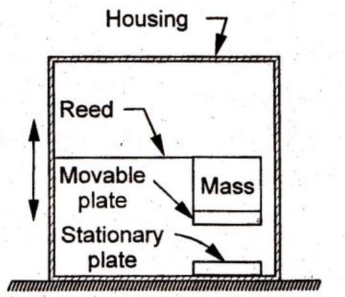


Fig. 12.26 (a)
Capacitance vibration sensor

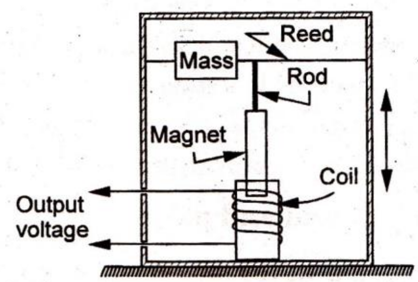


Fig. 12.26 (b)
Inductive vibration sensor

Quite often, vibration amplitudes are translated into an inductance and capacitance change of the system. Magnitude of the output voltage or capacitance is then taken as a measure of the amplitude of vibration. The schematics of such vibration pick-ups are shown in Fig. 12.26 which are self-explanatory.

A suitable estimate of frequency and amplitude of vibrations in light systems (where it is not possible to attach an electrical transducer) is best made by using either a stroboscope or a reed vibrometer.

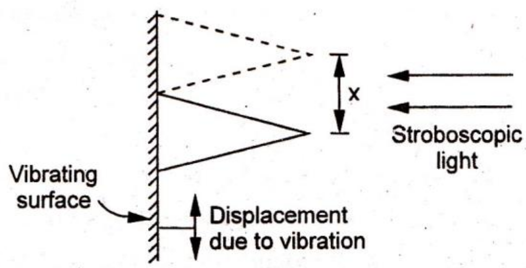


Fig. 12.27 Stroboscopic method for vibration measurement

A fixed pointer is attached to the vibrating surface (Fig. 12.27), flashes from a stroboscope are directed onto the pointer and frequency of light flashes is adjusted until a

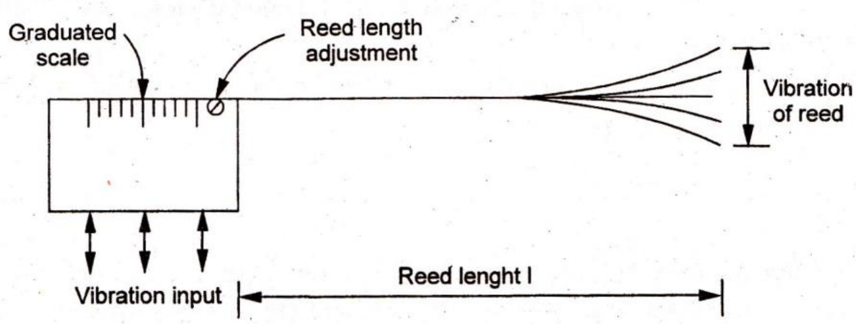


Fig. 12.28 Reed vibrometer

stationary image or a slowly moving image of the pointer is obtained. The flash frequency is then related to the amplitude or frequency of vibration. The stroboscope method is quite suitable for small-amplitude vibrations having an upper frequency range of about 500 Hz.

The reed vibrometer employs a reed which is mounted onto the vibrating structure or mechanism. The length of the reed is adjusted so that its natural frequency is equal to the frequency of the vibrating surface. Under this resonance condition, the reed vibrates with maximum amplitude. The reed length is calibrated directly in frequency units ; typical range of frequency measurement is 5 Hz to 10000 Hz.

Vibration amplitude and acceleration

Vibration refers to the repeated cyclic oscillations of a system; the oscillatory motions may be simple harmonic (sinusoidal) or complex (non-sinusoidal). The oscillations are caused when acceleration is applied to the machine alternately in two directions

The excessive vibration level in a machine is an indication of the following troubles it can cause:

* Catastrophic failure as a result of stress caused by induced resonance and fatigue

* Excessive wear because of failure to compensate for vibration to which a product is subjected or which is created by the product

* Faulty production

* Incorrect operation of precision equipment and machinery because of failure to compensate for vibration and shock encountered in use

* human discomfort leading to adverse effects such as motion sickness, breathing and speech disturbance, loss of touch of sensitivity etc.

Characteristics and units of vibrations: Vibration is generally characterized by

(i) The frequency in Hz, or

(ii) The amplitude of the measured parameter which may be displacement, velocity or acceleration.

Further, the units of vibration depend on the vibration parameter as follows:

(a) Displacement, measured in m, (b) velocity, measured in m/s and (c) acceleration, measured in m/s^2.

Vibrating motions may be simple harmonic or complex. Assuming it to be simple harmonic,

$$\text{displacement } x = A \sin \omega t$$

$$\text{velocity } v = \frac{dx}{dt} = A\,\omega \cos \omega t$$

$$\text{acceleration } a = \frac{dv}{dt} = -A\,\omega^2 \sin \omega t$$

where $\omega = 2\pi f$ rad/s and f is the frequency of vibration in Hz. Obviously, the amplitude of the different parameters are :

$$\text{displacement amplitude} = A$$
$$\text{velocity amplitude} = A\,\omega$$
$$\text{acceleration amplitude} = -A\,\omega^2$$

The measured amplitude is normally expressed in decibels with reference to a fixed value. Let A_1 be the measured amplitude and A_0 be the reference amplitude. Then the vibration level expressed in decibels is

$$\text{vibration level} = 20 \log_{10} \frac{A_1}{A_0} \text{ dB}$$

The internationalyy accepted reference values are:

(a) for velocity, the reference value is 10^{-3} m/s, and

(b) for acceleration, the reference value is 10^{-5} m/s^2