UNIT 4

SPUR AND HELICAL GEAR DRIVES

Course objectives:

To apply principles of design and Analyze the forces in mechanical power transmission elements such gears.

Course Outcomes:

Select appropriate gears for power transmission on the basis of given load and speed Design gears based on the given conditions Apply the design concepts to estimate the strength of the gear

INTRODUCTION:

Mechanical drives may be categorized into two groups;

1. Drives that transmit power by means of friction: eg: belt drives and rope drives.
2. Drives that transmit power by means of engagement: eg: chain drives and gear drives.

However, the selection of a proper mechanical drive for a given application depends upon number of factors such as centre distance, velocity ratio, shifting arrangement, Maintenance and cost.

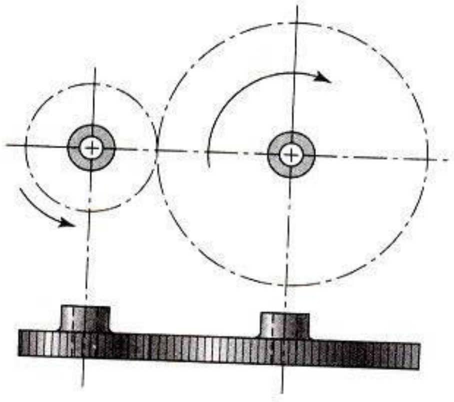
GEAR DRIVES

Gears are defined as toothed wheels, which transmit power and motion from one shaft to another by means of successive engagement of teeth.

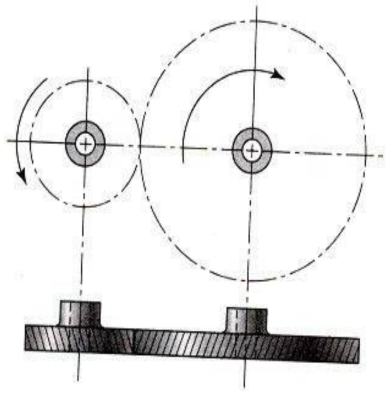
1. The centre distance between the shafts is relatively small.
2. It can transmit very large power
3. It is a positive, and the velocity ratio remains constant.
4. It can transmit motion at a very low velocity.

CLASSIFICATION OF GEARS:

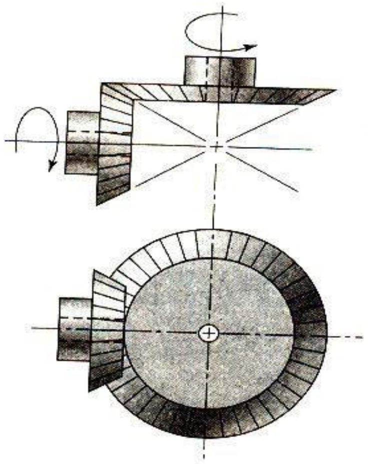
1. Spur Gears
2. Helical gears
3. Bevel gears and
4. Worm Gears



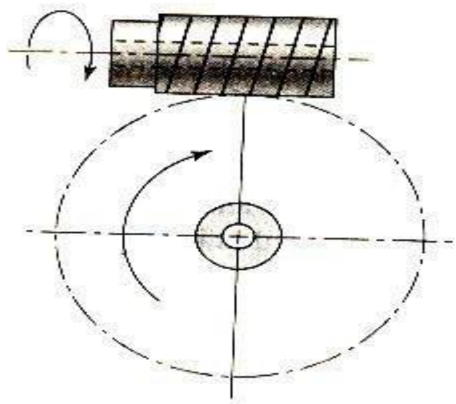
Spur Gear



Helical Gear



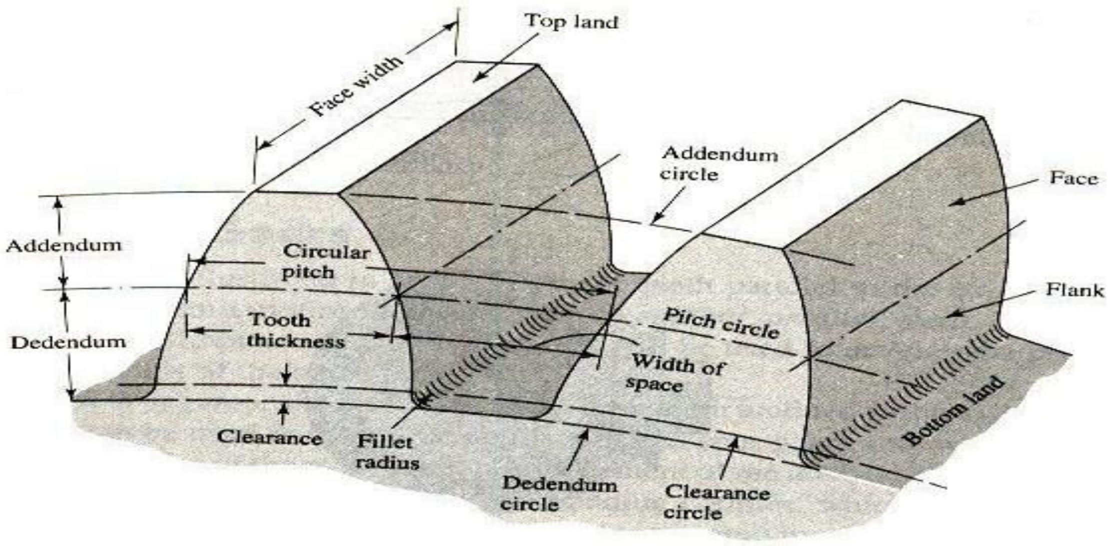
Bevel Gear



Worm Gear Set

NOMEN CLATURE

Spur gears are used to transmit rotary motion between parallel shafts. They are usually cylindrical is shape and the teeth are straight and parallel to the axis of rotation. In a pair of gears, the larger is often called the GEAR and, the smaller one is called the PINION



Nomenclature of Spur Gear

1. **Pitch Surface:** The pitch surfaces of the gears are imaginary planes, cylinders or cones that roll together without slipping.

2. **Pitch circle:** It is a theoretical circle upon which all calculations are usually based. It is an imaginary

circle that rolls without slipping with the pitch circle of a mating gear. Further, pitch circles of a mating gear are tangent to each other.

3. **Pitch circle diameter:** The pitch circle diameter is the diameter of pitch circle. Normally, the size of the gear is usually specified by pitch circle diameter. This is denoted by “d”

4. **Top land:** The top land is the surface of the top of the gear tooth

5. **Base circle:** The base circle is an imaginary circle from which the involute curve of the tooth profile is generated (the base circles of two mating gears are tangent to the pressure line)

6. **Addendum**: The Addendum is the radial distance between the pitch and addendum circles. Addendum indicates the height of tooth above the pitch circle.

6. **Dedendum**: The dedendum is the radial distance between pitch and the dedendum circles. Dedendum indicates the depth of the tooth below the pitch circle.

7. **Whole Depth**: The whole depth is the total depth of the tooth space that is the sum of addendum and Dedendum.

1. **Working depth:** The working depth is the depth of engagement of two gear teeth that is the sum of their addendums

2. **Clearance:** The clearance is the amount by which the Dedendum of a given gear exceeds the addendum of its mating tooth.

3. **Face:** The surface of the gear tooth between the pitch cylinder and the addendum cylinder is called face of the tooth.

4. **Flank:** The surface of the gear tooth between the pitch cylinder and the root cylinder is called flank of the tooth.

5. **Face Width:** is the width of the tooth measured parallel to the axis.

6. **Fillet radius:** The radius that connects the root circle to the profile of the tooth is called fillet radius.

7. **Circular pitch:** is the distance measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth.

8. **Circular tooth thickness:** The length of the arc on pitch circle subtending a single gear tooth is called circular tooth thickness. Theoretically circular tooth thickness is half of circular pitch.

9. **Width of space:** (tooth space) The width of the space between two adjacent teeth measured along the pitch circle. Theoretically, tooth space is equal to circular tooth thickness or half of circular pitch

10. **Working depth:** The working depth is the depth of engagement of two gear teeth, that is the sum of their addendums

11. **Whole depth:** The whole depth is the total depth of the tooth space, that is the sum of addendum and dedendum and (this is also equal to whole depth + clearance)

12. **Centre distance:** it is the distance between centres of pitch circles of mating gears. (it is also equal to the distance between centres of base circles of mating gears)

13. **Line of action:** The line of action is the common tangent to the base circles of mating gears. The contact between the involute surfaces of mating teeth must be on this line to give smooth operation. The force is transmitted from the driving gear to the driven gear on this line.

14. **Pressure angle:** It is the angle that the line of action makes with the common tangent to the pitch circles.

15. **Arc of contact:** Is the arc of the pitch circle through which a tooth moves from the beginning to the end of contact with mating tooth.

16. **Arc of approach:** it is the arc of the pitch circle through which a tooth moves from its beginning of contact until the point of contact arrives at the pitch point.

17. **Arc of recess:** It is the arc of the pitch circle through witch a tooth moves from the contact at the pitch point until the contact ends.

18. **Contact Ratio? Velocity ratio:** if the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

19. **Module:** It is the ratio of pitch circle diameter in mith meters to the number of teeth. it is usually denoted by 'm' Mathematically

$m = D/ Z$

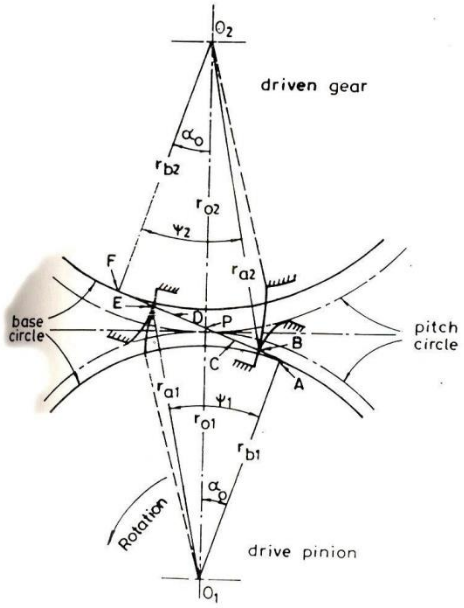
20. **Back lash**: It is the difference between the tooth space and the tooth thickness as measured on the pitch circle.

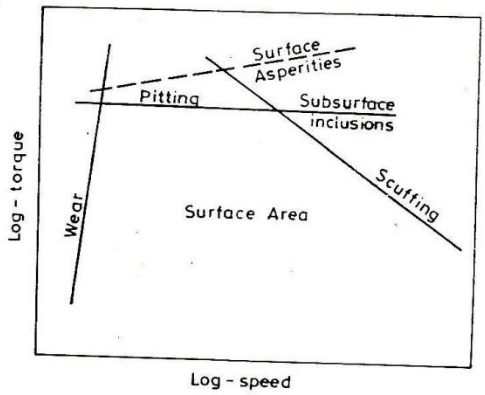
21. **Velocity Ratio**: Is the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

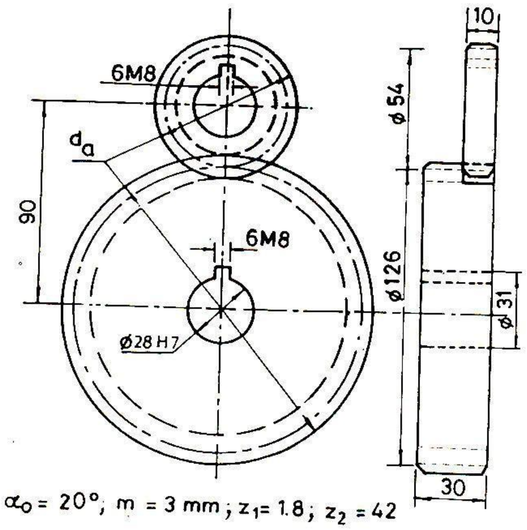
Specification of Test Pinions and Gears

Variable	Symbol	Unit	Values of variables used in the experiments: Pinion		Gears
Module	m	(mm)		3.0	
Pressure angle	α_o	(deg)		20°	
Number of teeth	z	(--)	18		42
Pitch circle diameter	d	(mm)	54.0		126.0
Centre distance	a_o	(mm)		90.0	
Addendum circle diameter	d_a	(mm)	60.0		132.0
Root circle diameter	d_r	(mm)	46.5		118.5
Face width	B	(mm)	10.0		30.0

Failure Map of Involutes Gears







Design consideration for a Gear drive

In the design of gear drive, the following data is usually given

i. The power to be transmitted

ii. The speed of the driving gear

iii. The speed of the driven gear or velocity ratio

iv. The centre distance

The following requirements must be met in the design of a gear drive

(a) The gear teeth should have sufficient strength so that they will not fail under static loading or dynamic loading during normal running conditions

(b) The gear teeth should have wear characteristics so that their life is satisfactory.

(c) The use of space and material should be recommended The alignment of the gears and deflections of the shaft must be considered because they effect on the performance of the gears

(d) The lubrication of the gears must be satisfactory

Selection of Gears:

The first step in the design of the gear drive is selection of a proper type of gear for a given application. The factors to be considered for deciding the type of the gear are

General layout of shafts
Speed ratio
Power to be transmitted
Input speed and
Cost

Spur & Helical Gears – When the shaft are parallel

Bevel Gears – When the shafts intersect at right angles, and,

Worm & Worm Gears – When the axes of the shaft are perpendicular and not intersecting. As a special case, when the axes of the two shafts are neither intersecting nor perpendicular crossed helical gears are employed.

The speed reduction or velocity ratio for a single pair of spur or helical gears is normally taken as 6: 1. On rare occasions this can be raised to 10: 1. When the velocity ratio increases, the size of the gear wheel increases. This results in an increase in the size of the gear box and the material cost increases. For high speed reduction two stage or three stage construction are used.

The normal velocity ratio for a pair of bend gears is 1: 1 which can be increased to 3: 1 under certain circumstances.

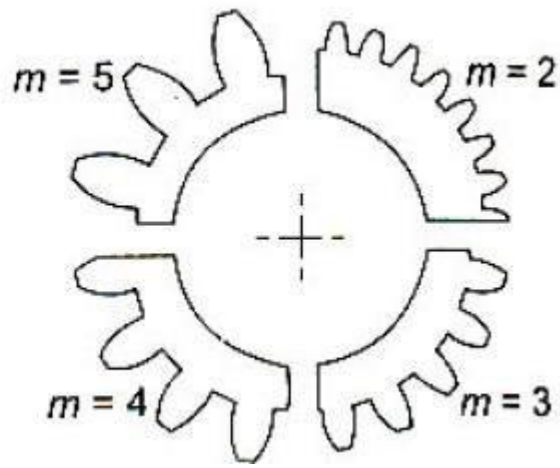
For high-speed reduction worm gears offers the best choice. The velocity ratio in their case is 60: 1, which can be increased to 100: 1. They are widely used in materials handling equipment due to this advantage.

Further, spur gears generate noise in high-speed applications due to sudden contact over the entire face with between two meeting teeth. Whereas, in helical gears the contact between the two meshing teeth begins with a point and gradually extends along the tooth, resulting in guide operations.

From considerations spur gears are the cheapest. They are not only easy to manufacture but there exists a number of methods to manufacture them. The manufacturing of helical, bevel and worm gears is a specialized and costly operation.

Low of Gearing: The fundamental law of gearing states "The common normal to the both profile at the point of contact should always pass through a fixed point called the pitch point, in order to obtain a constant velocity ratio.

MODULE: The module specifies the size of gear tooth. Figure shows the actual sizes of gear tooth with four different modules. It is observed that as the modules increases, the size of the gear tooth also increases. It can be said that module is the index of the size of gear tooth.



Standard values of module are as shown.

Recommended Series of Modules (mm)

Preferred (1)	Choice 2 (2)	Choice 3 (3)	Preferred (1)	Choice 2 (2)	Choice 3 (3)
1			8	7	(6.5)
1.25	1.125		10	9	
1.5	1.375		12	11	
2	1.75		16	14	
2.5	2.25		20	18	
3	2.75	(3.25)	25	22	
4	3.5		32	28	
5	4.5	(3.75)	40	36	
6	5.5		50	45	

Note: The modules given in the above table apply to spur and helical gears. In case of helical gears and double helical gears, the modules represent normal modules

The module given under choice 1, is always preferred. If that is not possible under certain circumstances module under choice 2, can be selected.

Standard proportions of gear tooth in terms of module m, for 20º full depth system.

Addendum = m

Dedendum = 1.25 m

Cl earance (c) = 0.25 m

Working depth = 2 m

Whole depth = 2.25 m

Tooth thickness = 1.5708 m=1.5708 m

Tooth space = 1.5708 m

Fillet radius = 0.4 m

Standard Tooth proportions of involutes spur gear

Gear Terms	Circular pitch p	Diametral pitch P	Module m
Addendum	0.3183 p	1/P	m
Dedendum	0.3977 p	1.25/P	1.25 m
Tooth thickness	0.5 p	1.5708/P	1.5708 m
Tooth space	0.5 p	1.5708/P	1.5708 m
Working depth	0.6366 p	2/P	2 m
Whole depth	0.7160 p	2.25/P	2.25 m
Clearance	0.0794 p	0.25/P	0.25 m
Pitch diameter	zp/π	z/P	zm
Outside diameter	$(z+2)p/\pi$	(z+2)/P	(z+2) m
Root diameter	$(z - 2.5)p/\pi$	(z − 2.5)/P	(z − 2.5) m
Fillet radius	0.1273p	0.4/P	0.4 m

Selection of Material:

- The load carrying capacity of the gear tooth depends upon the ultimate tensile strength or yield strength of the material.
- When the gear tooth is subjected to fluctuating forces, the endurance strength of the tooth is the

deciding Factor.

- The gear material should have sufficient strength to resist failure due to breakage of tte tooth.
- In many cases, it is wear rating rather than strength rating which decides the dimensions of gear
- tooth.
- The resistance to wear depends upon alloying elements, grain size, percentage of carbon and surface hardness.
- The gear material should have sufficient surface endurance strength to avoid failure due to destructive pitting.
- For high-speed power transmission, the sliding velocities are very high and the material should have a low co-efficient of friction to avoid failure due to scoring.
- The amount of thermal distortion or wrapping during the heat treatment process is a major problem on gear application.
- Due to wrapping the load gets concentrated at one corner of the gear tooth.
- Alloy steels are superior to plain carbon steel in this respect (Thermal distortion)

Load-Distribution Factor Km (KH)

The load-distribution factor modified the stress equations to reflect non uniform distribution of load across the line of contact. The idea is to locate the gear "mid span" between two bearings at the zero slope places when the load is applied. However, this is not always possible. The following procedure is applicable to

- Net face width to pinion pitch diameter ratio $F/d \leq 2$
- Gear elements mounted between the bearings
- Face widths up to 40 in
- Contact, when loaded, across the full width of the narrowest member

The load-distribution factor under these conditions is currently given by the *face load* distribution factor, C_{mf} , where

$K_m = C_{mf} = 1 + C_{mc} (C_{pf} C_{pm} + C_{ma} C_e)$

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases}$$

Note that for values of $F/(10d) < 0.05$, $F/(10d) = 0.05$ is used.

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \ge 0.175 \end{cases}$$

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C)$$

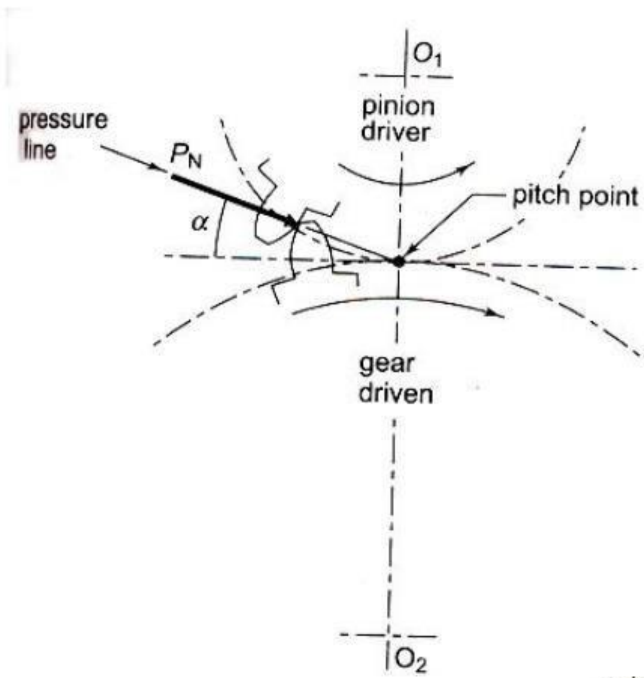
$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases}$$

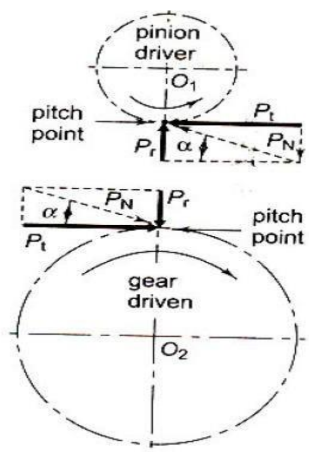
Force analysis – Spur gearing:

We know that, the reaction between the mating teeth occur along the pressure line, and the power is transmitted by means of a force exerted by the tooth of the driving gear on the meshing tooth of the driven gear. (i.e. driving pinion exerting force PN on the tooth of driven gear).

According to fundamental law of gear this resultant force PN always acts along the pressure line.

This resultant force P_N, can be resolved into two components – tangential component Pt and radial components Pr at the pitch point.





The tangential component Pt is a useful component (load) because it determines the magnitude of the torque and consequently the power, which is transmitted.

The radial component Pr services no useful purpose (it is a separating force) and it is always directed towards the centre of the gear.

The torque transmitted by the gear is given by

$$M_t = \frac{P \times 60}{2\,\pi N_1}\ N - m$$

Mt = Torque transmitted gears (N- m) PKW = Power

transmitted by gears N1 = Speed of rotation (rev / mn)

The tangential component Ft acts at the pitch circle radius.

$$\therefore \quad M_t = F_t \frac{d}{2}$$

OR

$$F_t = \frac{2M_t}{d}$$

Where,

Mt = Torque transmitted gears N- mm d = Pitch

Circle diameter, mm

Further, we know,

Power transmitted by gears = 2 πN M_t /60 (KW)

Where $F_r = F_t \tan\alpha$

Resultant force,

$$FN = \frac{F_t}{Cos\alpha}$$

The above analysis of gear tooth force is based on the following assumptions.

i) As the point of contact moves the magnitude of resultant force PN changes. This effect is neglected.

ii) It is assumed that only one pair of teeth takes the entire load. At times, there are two pairs that are simultaneously in contact and share the load. This aspects is also neglected.

iii) This analysis is valid under static conditions for example, when the gears are running at very low velocities. In practice there are dynamic forces in addition to force due to power transmission.

For gear tooth forces, It is always required to find out the magnitude and direction of two components. The magnitudes are determined by using equations

$$M_t = \frac{P \times 60}{2\pi N_1}$$

$$F_t = \frac{2M_t}{d_1}$$

Further, the direction of two components Ft and Fr are decided by constructing the free body diagram. The minimum number of teeth on pinion to avoid interference is given by

$$Z_{\min} = \frac{2}{\sin^2 \alpha}$$

For 20°full depth involutes system, it is always safe to assume the number of teeth as 18 or 20 Once the number of teeth on the pinion is decided, the number of teeth on the gear is calculated by the velocity ratio

Face Width:

$$i = \frac{Z_2}{Z_1}$$

In designing gears, it is required to express the face width in terms of module.

In practice, the optimum range of face width is 9.5 m ≤ b ≤ 12.5m

Generally, face width is assumed as ten times module

$$\therefore \quad \boxed{b = 12.5m}$$

Systems of Gear Teeth

The following four systems of gear teeth are commonly used in practice.

1. $14^1/2°$ Composite system, **2.** $14^1/2°$ Full depth involute system, **3.** 20° Full depth involute system, and **4.** 20° Stub involute system.

The 14 /2° ***composite system*** is used for general purpose gears. It is stronger but has no

Interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. The teeth are produced by formed milling cutters or hobs. The tooth profile of the 14^{1} /2° ***full depth involute system*** was developed for use with gear hobs for spur and helical gears.

The tooth profile of the **20°** ***full depth involute system*** may be cut by hobs. The increase of the pressure angle from $14^{1}/2°$ to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. The **20°** ***stub involute system*** has a strong tooth to take heavy loads.

Standard Proportions of Gear Systems

The following table shows the standard proportions in module (*m*) for the four gear systems as discussed in the previous article.

Table Standard proportions of gear systems.

S. No.	*Particulars*	*$14\frac{1}{2}°$ composite or full depth involute system*	*20° full depth involute system*	*20° stub involute system*
1.	Addendum	1*m*	1*m*	0.8 *m*
2.	Dedendum	1.25 *m*	1.25 *m*	1 *m*
3.	Working depth	2 *m*	2 *m*	1.60 *m*
4.	Minimum total depth	2.25 *m*	2.25 *m*	1.80 *m*
5.	Tooth thickness	1.5708 *m*	1.5708 *m*	1.5708 *m*
6.	Minimum clearance	0.25 *m*	0.25 *m*	0.2 *m*
7.	Fillet radius at root	0.4 *m*	0.4 *m*	0.4 *m*

Causes of Gear Tooth Failure

The different modes of failure of gear teeth and their possible remedies to avoid the failure are as follows:

1. ***Bending failure*****.** Every gear tooth acts as a cantilever. If the total repetitive dynamic load acting on the gear tooth is greater than the beam strength of the gear tooth, then the gear tooth will fail in bending, *i.e*. the gear tooth will break.

In order to avoid such failure, the module and face width of the gear is adjusted so that the beam strength is greater than the dynamic load.

2. ***Pitting.*** It is the surface fatigue failure which occurs due to much repetition of Hertz contact stresses. The failure occurs when the surface contact stresses are higher than the endurance limit of the material. The failure starts with the formation of pits which continue to grow resulting in the rupture of the tooth surface.

In order to avoid the pitting, the dynamic load between the gear tooth should be less than the wear strength of the gear tooth.

3. ***Scoring.*** The excessive heat is generated when there is an excessive surface pressure, high speed or supply of lubricant fails. It is a stick-slip phenomenon in which alternate shearing and weldingtakes place rapidly at high spots.

This type of failure can be avoided by properly designing the parameters such as speed, pressure and proper flow of the lubricant, so that the temperature at the rubbing faces is within the permissible limits.

4. ***Abrasive wear.*** The foreign particles in the lubricants such as dirt, dust or burr enter between the tooth and damage the form of tooth. This type of failure can be avoided by providing filters for the lubricating oil or by using high viscosity lubricant oil which enables the formation of thicker oil film and hence permits easy passage of such particles without damaging the gear surface.

5. ***Corrosive wear.*** The corrosion of the tooth surfaces is mainly caused due to the presence of corrosive elements such as additives present in the lubricating oils. In order to avoid this type of wear, proper anti-corrosive additives should be used.

Design Procedure for Spur Gears

1. First of all, the design tangential tooth load is obtained from the power transmitted and the pitch line velocity by using the following relation :

$$W_T = \frac{P}{v} \times C_S \quad ...(i)$$

where W_T = Permissible tangential tooth load in newtons,

P = Power transmitted in watts,

*v = Pitch line velocity in m / s $= \frac{\pi D N}{60}$,

D = Pitch circle diameter in metres,

* We know that circular pitch,

$$p_c = \pi D / T = \pi m \quad ...(\because m = D / T)$$

$$\therefore \quad D = m.T$$

Thus, the pitch line velocity may also be obtained by using the following relation, *i.e.*

$$v = \frac{\pi D.N}{60} = \frac{\pi m.T.N}{60} = \frac{p_c.T.N}{60}$$

where m = Module in metres, and

T = Number of teeth.

N = Speed in r.p.m., and

*C*S = Service factor.

The following table shows the values of service factor for different types of loads:

Table Values of service factor.

Type of load	*Type of service*		
	Intermittent or 3 hours per day	*8-10 hours per day*	*Continuous 24 hours per day*
Steady	0.8	1.00	1.25
Light shock	1.00	1.25	1.54
Medium shock	1.25	1.54	1.80
Heavy shock	1.54	1.80	2.00

Note : The above values for service factor are for enclosed well lubricated gears. In case of non- enclosed and grease lubricated gears, the values given in the above table should be divided by 0.65.

2. Apply the Lewis equation as follows :

$$W_T = \sigma_w . b . p_c . y = \sigma_w . b . \pi \, m . y$$

$$= (\sigma_o . C_v) \, b . \pi \, m . y \qquad ...(\because \sigma_w = \sigma_o . C_v)$$

Notes : (*i*) The Lewis equation is applied only to the weaker of the two wheels (*i.e*. pinion or gear).

(ii) When both the pinion and the gear are made of the same material, then pinion is the weaker.

(iii) When the pinion and the gear are made of different materials, then the product of ($\sigma_w \times y$) or ($\sigma_o \times y$) is the *deciding factor. The Lewis equation is used to that wheel for which ($\sigma_w \times y$) or ($\sigma_o \times y$) is less.

* We see from the Lewis equation that for a pair of mating gears, the quantities like W_T, *b*, *m*

and C_v are constant. Therefore ($\sigma_w \times y$) or ($\sigma_o \times y$) is the only deciding factor.

(iv) The product ($\sigma_w \times y$) is called ***strength factor*** of the gear.

(v) The face width (*b*) may be taken as 3 p_c to 4 p_c (or 9.5 *m* to 12.5 *m*) for cut teeth and 2 p_c to 3 p_c (or 6.5 *m* to 9.5 *m*) for cast teeth.

Calculate the dynamic load (W_D) on the tooth by using Buckingham equation, *i.e*

$$W_D = W_T + W_I$$

$$= W_T + \frac{21v\,(b.C + W_T)}{21v + \sqrt{b.C + W_T}}$$

In calculating the dynamic load (W_D), the value of tangential load (W_T) may be calculated by neglecting the service factor (C_S) *i.e.*

$W_T = P / v$, where P is in watts and v in m / s.

Find the static tooth load (*i.e*. beam strength or the endurance strength of the tooth) by using

the relation,

$W_S = \sigma_e.b.p_c.y = \sigma_e.b.\pi\, m.y$

For safety against breakage, W_S should be greater than W_D.

3. Finally, find the wear tooth load by using the relation,

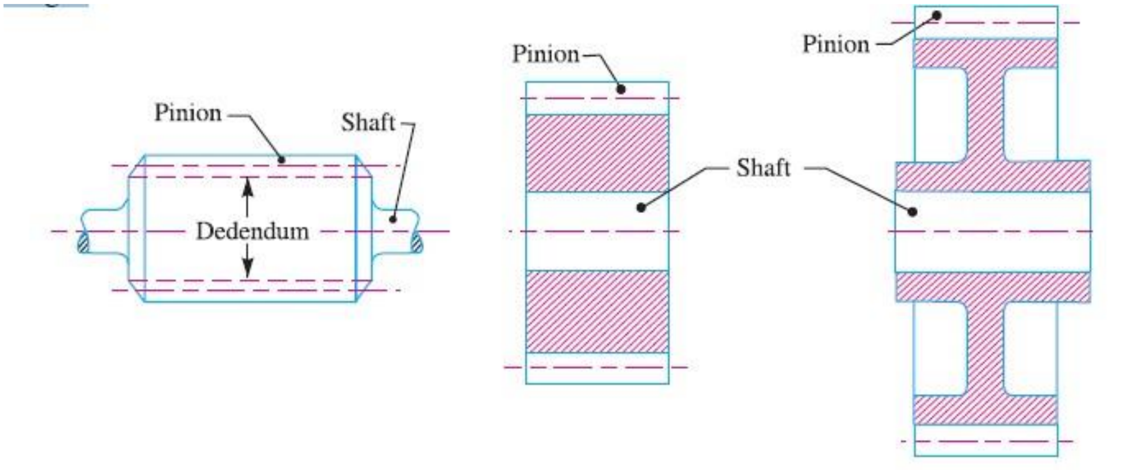
$W_w = D_P.b.Q.K$

The wear load *(W_w)* should not be less than the dynamic load (W_D).

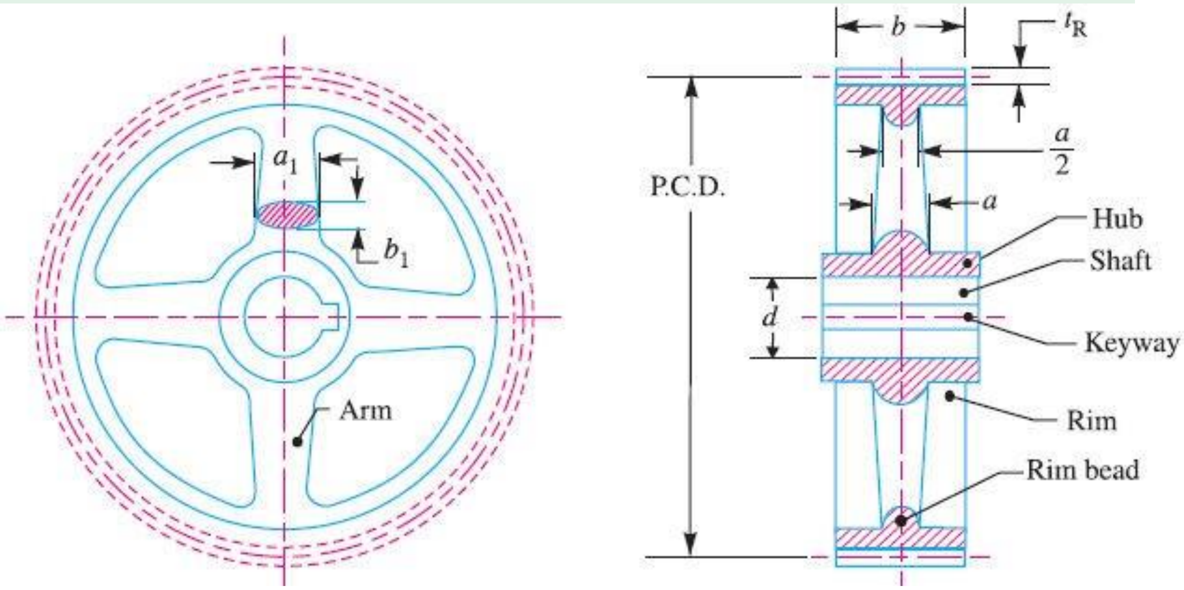
1. The nearest standard module if no interference is to occur;
2. The number of teeth on each wheel;
3. The necessary width of the pinion; and
4. The load on the bearings of the wheels due to power transmitted.

Spur Gear Construction

The gear construction may have different designs depending upon the size and its application. When the dedendum circle diameter is slightly greater than the shaft diameter, then the pinion teeth are cut integral with the shaft as shown in Fig. 28.13 (*a*). If the pitch circle diameter of the pinion is less than or equal to 14.75 m + 60 mm (where m is the module in mm), then the pinion is made solid with uniform thickness equal to the face width, as shown in Fig. 28.13 (*b*). Small gears upto 250 mm pitch circle diameter are built with a web, which joins the hub and the rim. The web thickness is generally equal to half the circular pitch or it may be taken as 1.6 m to 1.9 m, where m is the module. The web may be made solid as shown in Fig. 28.13 (*c*) or may have recesses in order to reduce its weight.



(a) (b) (c)

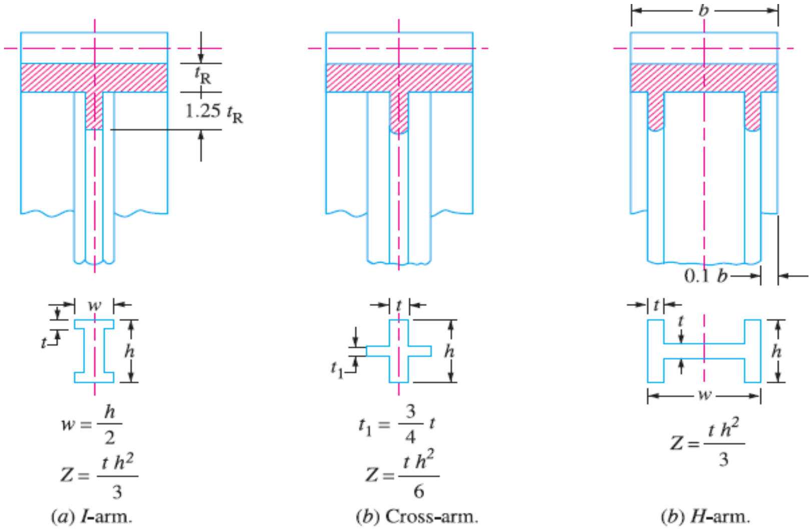


Large gears are provided with arms to join the hub and the rim, as shown in Fig.The number of arms depends upon the pitch circle diameter of the gear. The number of arms may be selected from the following table.

Number of arms for the gears.

S. No.	*Pitch circle diameter*	*Number of arms*
1.	Up to 0.5 m	4 or 5
2.	0.5 – 1.5 m	6
3.	1.5 – 2.0 m	8
4.	Above 2.0 m	10

The cross-section of the arms is most often elliptical, but other sections as shown in Fig.15 may also be used.



(a) I-arm. (b) Cross-arm. (b) H-arm.

Cross-section of the arms.

The hub diameter is kept as 1.8 times the shaft diameter for steel gears, twice the shaft diameter for cast iron gears and 1.65 times the shaft diameter for forged steel gears used for light service. The length of the hub is kept as 1.25 times the shaft diameter for light service and should not be less than the face width of the gear.

The thickness of the gear rim should be as small as possible, but to facilitate casting and to avoid sharp changes of section, the minimum thickness of the rim is generally kept as half of the

circular pitch (or it may be taken as 1.6 *m* to 1.9 *m*, where *m* is the module). The thickness of rim (t_R) may

$$t_R = m\sqrt{\frac{T}{n}}$$

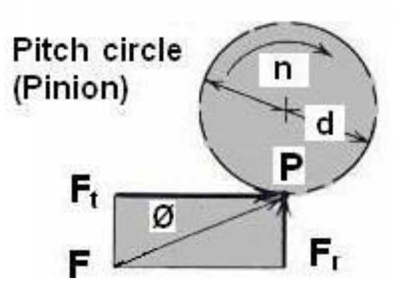
also be calculated by using the following relation, *i.e.*

Where *T* = Number of teeth, and
n = Number of arms.

The rim should be provided with a circumferential rib of thickness equal to the rim thickness.

SPUR GEAR – TOOTH FORCE ANALYSIS

As shown in Fig, the normal force F can be resolved into two components; a tangential force F which



does transmit the power and radial component F which does no work but tends to push the gears apart. They can hence be written as,

F_t = F cos Ø

F_t = F sin Ø

F_t = F tan Ø

The pitch line velocity V, in meters per second, is given as

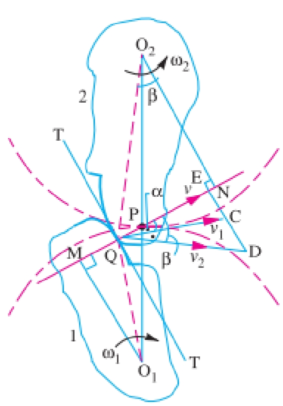
$$V = \frac{\pi d n}{6000}$$

$$W = \frac{F_t V}{1000}$$

1000 where d is the pitch diameter of the gear in millimeters and n is the rotating speed in rpm and power in kW.

Condition for Constant Velocity Ratio of Gears–Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 28.7. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure. Let T be the common tangent and MN be the common normal to the curves at point of contact Q. From the centres O1 and O2, draw O1M and O2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2. Let v1 and v2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.



$$\therefore \quad v_1 \cos\alpha = v_2 \cos\beta$$

or
$$(\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$

$$(\omega_1 \times O_1Q)\frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\frac{O_2N}{O_2Q}$$

$$\therefore \quad \omega_1 . O_1M = \omega_2 . O_2N$$

or
$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \quad ...(i)$$

Also from similar triangles O_1MP and O_2NP,

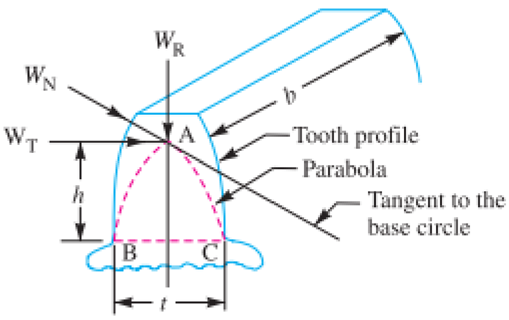
$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \quad ...(ii)$$

Combining equations (*i*) and (*ii*), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \quad ...(iii)$$

We see that the angular velocity ratio is inversely proportional to the ratio of the distance of P from the centers O1 and O2, or the common normal to the two surfaces at the point of contact Q intersects the line of centers at point P which divides the centre distance inversely as the ratio of angular velocities. Therefore, in order to have a constant angular velocity ratio for all positions of the wheels, P must be the fixed point (called pitch point) for the two wheels. In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point. This is fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing.

Beam Strength of Gear Teeth – Lewis Equation



The beam strength of gear teeth is determined from an equation (known as *Lewis equation) and the load carrying ability of the toothed gears as determined by this equation gives satisfactory results. In the investigation, Lewis assumed that as the load is being transmitted from one gear to another, it is all given and taken by one tooth, because it is not always safe to assume that the load is distributed among several teeth. When contact begins, the load is assumed to be at the end of the driven teeth and as contact ceases, it is at the end of the driving teeth. This may not be true when the number of teeth in a pair of mating gears is large, because the load may be distributed among several n teeth. But it is almost certain that at some time during the contact of teeth, the proper distribution of load does not exist and that one tooth must transmit the full load. In any pair of gears having unlike number of teeth, the gear which have the fewer teeth (i.e. pinion) will be the weaker, because the tendency toward undercutting of the teeth becomes more pronounced in gears as the number of teeth becomes smaller. Consider each tooth as a cantilever beam loaded by a normal load (WN) as shown in Fig. It is resolved into two components i.e. tangential component (WT) and radial component (WR) acting perpendicular and parallel to the centre line of the tooth respectively. The tangential component (WT) induces a bending stress which tends to break the tooth. The radial component (WR) induces a compressive stress of relatively small magnitude; therefore its effect on the tooth may be neglected. Hence, the bending stress is used as the basis for design calculations. The critical section or the section of maximum bending stress may be obtained by drawing a parabola through A and tangential to the tooth curves at B and C. This parabola, as shown dotted in Fig. Outlines a beam of uniform strength, i.e. if the teeth are shaped like a parabola, it will have the same stress at all the sections. But the tooth is larger than the parabola at every section except BC. We therefore, conclude that the section BC is the section of maximum stress or the critical section. The maximum value of the bending stress (or the permissible working stress), at the section BC is given by

$$\sigma_w = M.y / I \qquad \text{...}(i)$$

where M = Maximum bending moment at the critical section $BC = W_T \times h$,

W_T = Tangential load acting at the tooth,

h = Length of the tooth,

y = Half the thickness of the tooth (t) at critical section $BC = t/2$,

I = Moment of inertia about the centre line of the tooth $= b.t^3/12$,

b = Width of gear face.

Substituting the values for M, y and I in equation (i), we get

$$\sigma_w = \frac{(W_T \times h)\, t/2}{b.t^3/12} = \frac{(W_T \times h) \times 6}{b.t^2}$$

or

$$W_T = \sigma_w \times b \times t^2 / 6\,h$$

Let $t = x \times p_c$, and $h = k \times p_c$; where x and k are constants.

$$\therefore \quad W_T = \sigma_w \times b \times \frac{x^2 . p_c^2}{6k.p_c} = \sigma_w \times b \times p_c \times \frac{x^2}{6k}$$

Substituting $x^2 / 6k = y$, another constant, we have

$$W_T = \sigma_w . b . p_c . y = \sigma_w . b . \pi m . y \qquad \text{...}(\because p_c = \pi m)$$

The quantity y is known as ***Lewis form factor*** or ***tooth form factor*** and W_T (which is the tangential load acting at the tooth) is called the ***beam strength of the tooth.***

Since $y = \dfrac{x^2}{6k} = \dfrac{t^2}{(p_c)^2} \times \dfrac{p_c}{6h} = \dfrac{t^2}{6h.p_c}$, therefore in order to find the value of y, the

Dynamic Tooth Load

In the previous article, the velocity factor was used to make approximate allowance for the effect of dynamic loading. The dynamic loads are due to the following reasons:

1. Inaccuracies of tooth spacing,

2. Irregularities in tooth profiles, and

3. Deflections of teeth under load.

A closer approximation to the actual conditions may be made by the use of equations based on

extensive series of tests, as follows :

$W_D = W_T + W_I$

Where W_D = Total dynamic load,

W_T = Steady load due to transmitted torque, and

W_I = Increment load due to dynamic action. The increment load (W_I) depends upon the pitch line velocity, the face width, material of the gears, the accuracy of cut and the tangential load. For average conditions, the **dynamic load is determined by using the following Buckingham equation, i.e.**

$$W_D = W_T + W_I = W_T + \frac{21\,v\,(b.C + W_T)}{21\,v + \sqrt{b.C + W_T}}$$

where W_D = Total dynamic load in newtons,

W_T = Steady transmitted load in newtons,

v = Pitch line velocity in m/s,

b = Face width of gears in mm, and

C = A deformation or dynamic factor in N/mm.

$$C = \frac{K.e}{\frac{1}{E_P} + \frac{1}{E_G}}$$

K = A factor depending upon the form of the teeth.

= 0.107, for $14\frac{1}{2}°$ full depth involute system.

= 0.111, for 20° full depth involute system.

= 0.115 for 20° stub system.

E_P = Young's modulus for the material of the pinion in N/mm^2.

E_G = Young's modulus for the material of gear in N/mm^2.

e = Tooth error action in mm.

Static Tooth Load

The static tooth load (also called beam strength or endurance strength of the tooth) is obtained by Lewis formula by substituting flexural endurance limit or elastic limit stress (σ_e) in place of permissible working stress (σ_w).

∴ Static tooth load or beam strength of the tooth,

$$W_S = \sigma_e.b.p_c.y = \sigma_e.b.\pi\,m.y$$

Wear Tooth Load

$W_w = D_P.b.Q.K$

W_w = Maximum or limiting load for wear in newtons,

D_P = Pitch circle diameter of the pinion in mm,

b = Face width of the pinion in mm,

Q = Ratio factor

$= \dfrac{2 \times V.R.}{V.R. + 1} = \dfrac{2T_G}{T_G + T_P}$, for external gears

$= \dfrac{2 \times V.R.}{V.R. - 1} = \dfrac{2T_G}{T_G - T_P}$, for internal gears.

$V.R.$ = Velocity ratio $= T_G / T_P$,

K = Load-stress factor (also known as material combination factor) in N/mm^2.

The load stress factor depends upon the maximum fatigue limit of compressive stress, the pressure angle and the modulus of elasticity of the materials of the gears. According to Buckingham, the load stress factor is given by the following relation :

$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

where σ_{es} = Surface endurance limit in MPa or N/mm^2,

ϕ = Pressure angle,

E_P = Young's modulus for the material of the pinion in N/mm^2, and

E_G = Young's modulus for the material of the gear in N/mm^2.

The values of surface endurance limit (σ_{es}) are given in the following table.

1. The following particulars of a single reduction spur gear are given: Gear ratio = 10 : 1; Distance between centres = 660 mm approximately; Pinion transmits 500 kW at 1800 r.p.m.; Involute teeth of standard proportions (addendum = m) with pressure angle of 22.5°; Permissible normal pressure between teeth = 175 N per mm of width.
   Find: 1. The nearest standard module if no interference is to occur; 2. The number of teeth on each wheel; 3. The necessary width of the pinion; and 4. The load on the bearings of the wheels due to power transmitted.

Solution : Given : $G = T_G / T_P = D_G / D_P = 10$; $L = 660$ mm ; $P = 500$ kW $= 500 \times 10^3$ W ; $N_P = 1800$ r.p.m. ; $\phi = 22.5°$; $W_N = 175$ N/mm width

1. Nearest standard module if no interference is to occur

Let m = Required module,

T_P = Number of teeth on the pinion,

T_G = Number of teeth on the gear,

D_P = Pitch circle diameter of the pinion, and

D_G = Pitch circle diameter of the gear.

We know that minimum number of teeth on the pinion in order to avoid interference,

$$T_P = \frac{2\,A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

$$= \frac{2 \times 1}{10\left[\sqrt{1 + \frac{1}{10}\left(\frac{1}{10} + 2\right)\sin^2 22.5°} - 1\right]} = \frac{2}{0.15} = 13.3 \text{ say } 14$$

... ($\because A_W$ = 1 module)

$\therefore \quad T_G = G \times T_P = 10 \times 14 = 140$ ...($\because T_G / T_P = 10$)

We know that $L = \frac{D_G}{2} + \frac{D_P}{2} = \frac{D_G}{2} + \frac{10\,D_P}{2} = 5.5\,D_P$ ...($\because D_G / D_P = 10$)

$\therefore \quad 660 = 5.5\,D_P$ or $D_P = 660 / 5.5 = 120$ mm

We also know that $D_P = m \,.\, T_P$

$\therefore \quad m = D_P / T_P = 120 / 14 = 8.6$ mm

Since the nearest standard value of the module is 8 mm, therefore we shall take

$m = 8$ mm **Ans.**

2. Number of teeth on each wheel

We know that number of teeth on the pinion,

$T_P = D_P / m = 120 / 8 = 15$ **Ans.**

and number of teeth on the gear,

$T_G = G \times T_P = 10 \times 15 = 150$ **Ans.**

3. Necessary width of the pinion

We know that the torque acting on the pinion,

$$T = \frac{P \times 60}{2\pi N_P} = \frac{500 \times 10^3 \times 60}{2\pi \times 1800} = 2652 \text{ N-m}$$

$\therefore$ Tangential load, $W_T = \frac{T}{D_P / 2} = \frac{2652}{0.12/2} = 44\,200$ N ...($\because D_P$ is taken in metres)

and normal load on the tooth,

$$W_N = \frac{W_T}{\cos\phi} = \frac{44\,200}{\cos 22.5°} = 47\,840 \text{ N}$$

Since the normal pressure between teeth is 175 N per mm of width, therefore necessary width of the pinion,

$$b = \frac{47\,840}{175} = 273.4 \text{ mm}$$ **Ans.**

4. Load on the bearings of the wheels

We know that the radial load on the bearings due to the power transmitted,

$W_R = W_N \,.\, \sin\phi = 47\,840 \times \sin 22.5° = 18\,308$ N $= 18.308$ kN **Ans.**

INDUSTRIAL APPLICATIONS

1. Spur gear in Metal cutting machine

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2. Spur gear in marine engine

3. Spur gear used in fuel pump

4. spur gear in Automobile Gear box

5. Helical gear in fertilize industry

6. Helical gear for food Industries

TUTORIAL QUESTIONS

1. Discuss the design procedure of spur gears?
2. Derive Lewis equation for beam strength of gear tooth on spur gears
3. Write expressions for static limiting wear load, dynamic load for gear tooth of spur gear explain various terms used.
4. Explain the following terms used in helical gears. (i). Helix angle (ii) Normal Pitch (iii) Axial Pitch.
5. How shaft and arms for spur gears are designed?

6. Mention four important types of gears and discuss their applications and their materials used.

7. Terms used in helical gears.

8. Advantages and disadvantages of Gears?

9. Define a) Addendum b) Dedendum C)Module ?

10. Define a) Diametral pitch b) Clearance C) Pitch circle?

11. A pair of helical gears consist of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 r.p.m. The normal pressure angle is 20° while the helix angle is 25°. The face width is 40 mm and the normal module is 4 mm. The pinion as well as gear are made of steel having ultimate strength of 600 MPa and heat treated to a surface hardness of 300 B.H.N. The service factor and factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of the gears.

12. 1. Calculate the power that can be transmitted safely by a pair of spur gears with the data given below. Calculate also the bending stresses induced in the two wheels when the pair transmits this power. Number of teeth in the pinion = 20 Number of teeth in the gear = 80 Spur Gears „ Module = 4 mm Width of teeth = 60 mm ,Tooth profile = 20° involute ,Allowable bending strength of the material = 200 MPa, for pinion = 160 MPa, for gear Speed of the pinion = 400 r.p.m. Service factor = 0.8
Lewis form factor = 0.154 –0.192/ T Velocity factor = 3 /3 + v.

13. A pair of helical gears is to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10 000 r.p.m. and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 MPa; determine a suitable module and face width from static strength considerations and check the gears for wear, given σes = 618 MPa.

14. A pair of helical gears consists of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 r.p.m. The normal pressure angle is 20° while the helix angle is 25°. The face width is 40 mm and the normal module is 4 mm. The pinion as well as gear is made of steel having ultimate strength of 600 MPa and heat treated to a surface hardness of 300 B.H.N. The service factor and factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of the gears.

15. Explain the different causes of gear tooth failures and suggest possible remedies to avoid such failures. And Write the expressions for static, limiting wear load and dynamic load for spur gears and explain the various terms used there in.

ASSIGNMENT QUESTIONS

1. The following particulars of a single reduction spur gear are given : Gear ratio = 10 : 1; Distance between centres = 660 mm approximately; Pinion transmits 500 kW at 1800 r.p.m. Involute teeth of standard proportions (addendum = m) with pressure angle of 22.5°; Permissible normal pressure between teeth = 175 N per mm of width.

Find: 1. the nearest standard module if no interference is to occur;

2. The number of teeth on each wheel;

3. The necessary width of the pinion; and

4. The load on the bearings of the wheels due to power transmitted.

2. A pair of straight teeth spur gears is to transmit 20 kW when the pinion rotates at 300 r.p.m. The velocity ratio is 1 : 3. The allowable static stresses for the pinion and gear materials are 120 MPa and 100 MPa respectively. The pinion has 15 teeth and its face width is 14 times the module. Determine: 1. module; 2. face width; and 3. pitch circle diameters of both the pinion and the gear from the standpoint of strength only, taking into consideration the effect of the dynamic loading. The tooth form factor y can be taken as y = 0.514 –. 0 912 /No of teeth and the velocity factor Cv = , 3 /3+v where v is expressed in m / s.

3. A motor shaft rotating at 1500 r.p.m. has to transmit 15 kW to a low speed shaft with a speed reduction of 3:1. The teeth are 1 ° 14 /2 involute with 25 teeth on the pinion. Both the pinion and gear are made of steel with a maximum safe stress of 200 MPa. A safe stress of 40 MPa may be taken for the shaft on which the gear is mounted and for the key. Design a spur gear drive to suit the above conditions. Also sketch the spur gear drive. Assume starting torque to be 25% higher than the running torque.

4. A pair of helical gears with 30° helix angle is used to transmit 15 kW at 10 000 r.p.m. of the pinion. The velocity ratio is 4 : 1. Both the gears are to be made of hardened steel of static strength 100 N/mm2. The gears are 20° stub and the pinion is to have 24 teeth. The face width may be taken as 14 times the module. Find the module and face width from the standpoint of strength and check the gears for wear.

5. a) Design Considerations for a Gear Drive?

   b) Beam Strength of Gear Teeth – Lewis Equation?

UNIT-4

SPUR AND HELICAL GEARS

MRCET CAMPUS

UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

INTRODUCTION

A gear is a kind of machine element in which teeth are cut around cylindrical or cone shaped surfaces with equal spacing. By meshing a pair of these elements, they are used to transmit rotations and forces from the driving shaft to the driven shaft. Gears can be classified by shape as involutes, cycloidal and trochoidal gears. Also, they can be classified by shaft positions as parallel shaft gears, intersecting shaft gears, and non-parallel and non-intersecting shaft gears. The history of gears is old and the use of gears already appears in ancient Greece in B.C. in the writing of Archimedes.

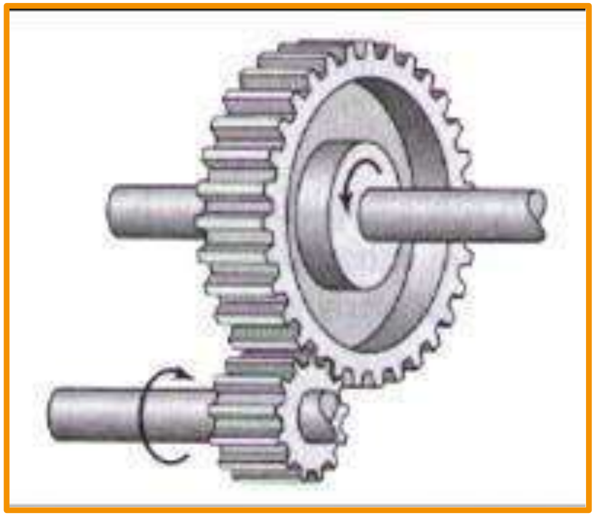
INTRODUCTION

Applications of Gears

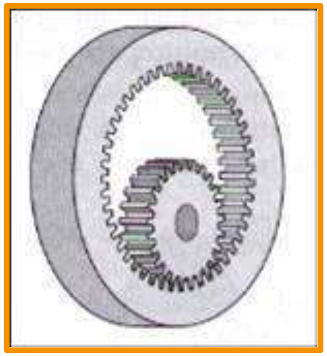
- *Toys and Small Mechanisms* – small, low load, low cost
  kinematic analysis
- *Appliance gears* – long life, low noise & cost, low to moderate load
  kinematic & some stress analysis
- *Power transmission* – long life, high load and speed
  kinematic & stress analysis
- *Aerospace gears* – light weight, moderate to high load
  kinematic & stress analysis
- *Control gears* – long life, low noise, precision gears
  kinematic & stress analysis

GEARS

Spur gears – tooth profile is parallel to the axis of rotation, transmits motion between parallel shafts.



Internal gears



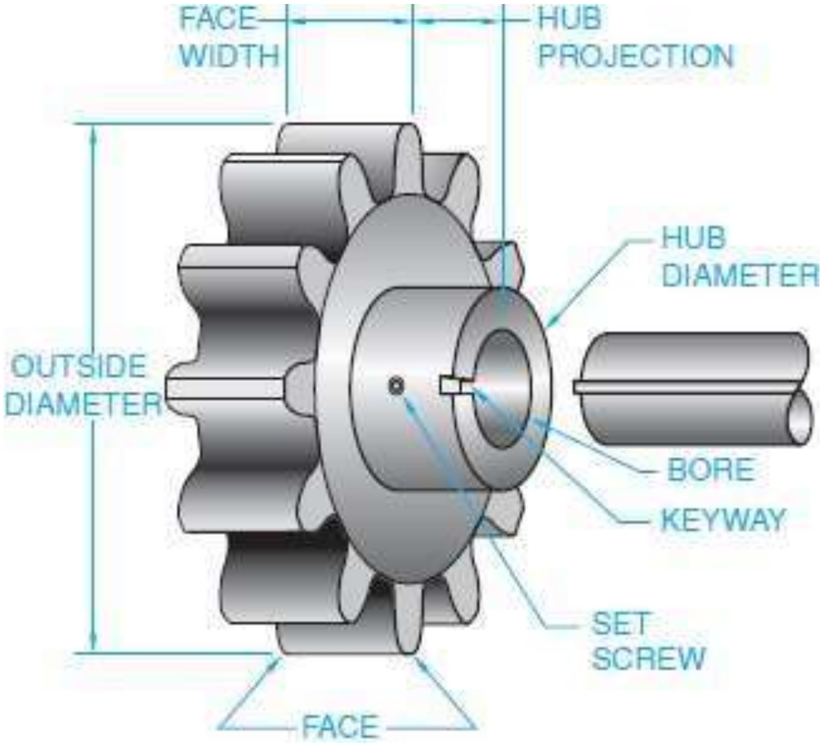
Helical gears– teeth are inclined to the axis of rotation, the angle provides more gradual engagement of the teeth during meshing, transmits motion between parallel shafts.

GEARS

GEAR MATERIALS

- Cast iron
- Steel
- Brass
- Bronze alloys
- Plastic

GEAR STRUCTURE



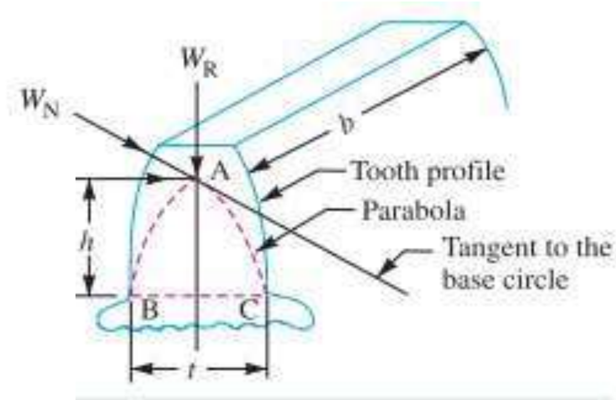
GEARS

Beam Strength of Gear Teeth – Lewis Equation \

∴Static tooth load or beam strength of the tooth,

$$W_S = \sigma_e . b . p_c . y = \sigma_e . b . \pi \, m . y$$

- **Dynamic tooth load or beam strength**



$$W_D = W_T + W_I = W_T + \frac{21\, v\, (b.C + W_T)}{21\, v + \sqrt{b.C + W_T}}$$

where

W_D = Total dynamic load in newtons,

W_T = Steady transmitted load in newtons,

v = Pitch line velocity in m/s,

b = Face width of gears in mm, and

C = A deformation or dynamic factor in N/mm.