CHAPTER

28

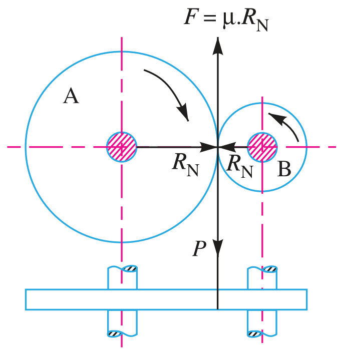
Spur Gears

28.1 Introduction

We have discussed earlier that the slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by ***gears*** or ***toothed wheels***. A gear drive is also provided, when the distance between the driver and the follower is very small.

28.2 Friction Wheels

The motion and power transmitted by gears is kinematically equivalent to that transmitted by frictional wheels or discs. In order to understand how the motion can be transmitted by two toothed wheels, consider two plain circular wheels *A* and *B* mounted on shafts. The wheels have sufficient rough surfaces and press against each other as shown in Fig. 28.1.



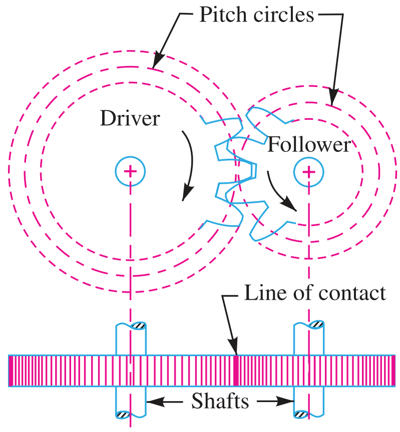
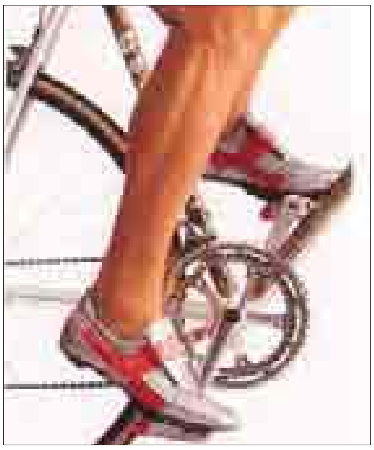


Fig. 28.1. Friction wheels. **Fig. 28.2.** Gear or toothed wheel.

Let the wheel *A* is keyed to the rotating shaft and the wheel *B* to the shaft to be rotated. A little consideration will show that when the wheel *A* is rotated by a rotating shaft, it will rotate the wheel *B* in the opposite direction as shown in Fig. 28.1. The wheel *B* will be rotated by the wheel *A* so long as the tangential force exerted by the wheel *A* does not exceed the maximum frictional resistance between the two wheels. But when the tangential force (*P*) exceeds the *frictional resistance (*F*), slipping will take place between the two wheels.

In order to avoid the slipping, a number of projections (called teeth) as shown in Fig. 28.2 are provided on the periphery of the wheel *A* which will fit into the corresponding recesses on the periphery of the wheel *B*. A friction wheel with the teeth cut on it is known as ***gear*** or ***toothed wheel.*** The usual connection to show the toothed wheels is by their pitch circles.

Note : Kinematically, the friction wheels running without slip and toothed gearing are identical. But due to the possibility of slipping of wheels, the friction wheels can only be used for transmission of small powers.



In bicycle gears are used to transmit motion. Mechanical advantage can be changed by changing gears.

28.3 Advantages and Disadvantages of Gear Drives

The following are the advantages and disadvantages of the gear drive as compared to other drives, *i.e.* belt, rope and chain drives :

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It may be used for small centre distances of shafts.
4. It has high efficiency.
5. It has reliable service.
6. It has compact layout.

Disadvantages

1. Since the manufacture of gears require special tools and equipment, therefore it is costlier than other drives.

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* We know that frictional resistance, $F = \mu \cdot R_N$

where μ = Coefficient of friction between the rubbing surfaces of the two wheels, and

R_N = Normal reaction between the two rubbing surfaces.

2. The error in cutting teeth may cause vibrations and noise during operation.
3. It requires suitable lubricant and reliable method of applying it, for the proper operation of gear drives.

28.4 Classification of Gears

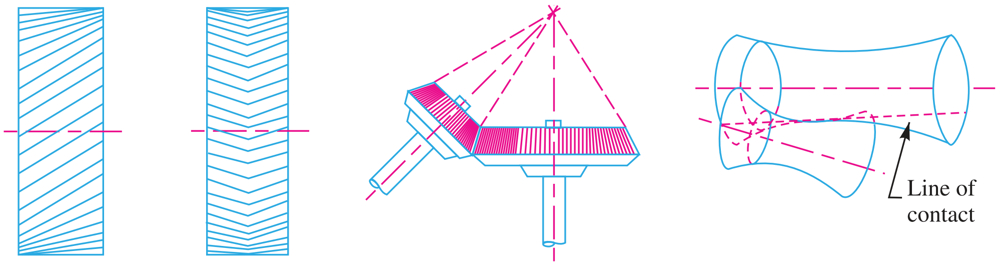
The gears or toothed wheels may be classified as follows :

1. ***According to the position of axes of the shafts.*** The axes of the two shafts between which the motion is to be transmitted, may be

(*a*) Parallel, (*b*) Intersecting, and (*c*) Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. 28.2. These gears are called ***spur gears*** and the arrangement is known as ***spur gearing.*** These gears have teeth parallel to the axis of the wheel as shown in Fig. 28.2. Another name given to the spur gearing is ***helical gearing,*** in which the teeth are inclined to the axis. The ***single*** and ***double helical gears*** connecting parallel shafts are shown in Fig. 28.3 (*a*) and (*b*) respectively. The object of the double helical gear is to balance out the end thrusts that are induced in single helical gears when transmitting load. The double helical gears are known as ***herringbone gears.*** A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to a parallel shaft having line contact.

The two non-parallel or intersecting, but coplaner shafts connected by gears is shown in Fig. 28.3 (*c*). These gears are called ***bevel gears*** and the arrangement is known as ***bevel gearing.*** The ***bevel gears,*** like spur gears may also have their teeth inclined to the face of the bevel, in which case they are known as ***helical bevel gears.***



(*a*) Single helical gear. (*b*) Double helical gear. (*c*) Bevel gear. (*d*) Spiral gear.

Fig. 28.3

The two non-intersecting and non-parallel *i.e.* non-coplanar shafts connected by gears is shown in Fig. 28.3 (*d*). These gears are called ***skew bevel gears*** or ***spiral gears*** and the arrangement is known as ***skew bevel gearing*** or ***spiral gearing.*** This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as ***hyperboloids.***

Notes : (*i*) When equal bevel gears (having equal teeth) connect two shafts whose axes are mutually perpendicular, then the bevel gears are known as ***mitres.***

(*ii*) A hyperboloid is the solid formed by revolving a straight line about an axis (not in the same plane), such that every point on the line remains at a constant distance from the axis.

(*iii*) The worm gearing is essentially a form of spiral gearing in which the shafts are usually at right angles.

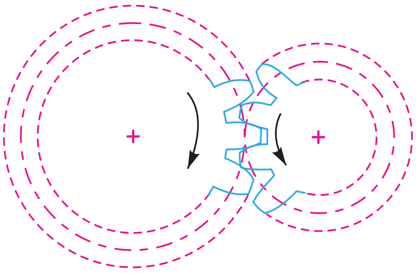
2. ***According to the peripheral velocity of the gears.*** The gears, according to the peripheral velocity of the gears, may be classified as :

(*a*) Low velocity, (*b*) Medium velocity, and (*c*) High velocity.

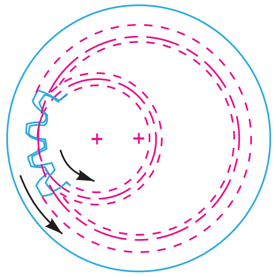
The gears having velocity less than 3 m/s are termed as ***low velocity gears*** and gears having velocity between 3 and 15 m / s are known as ***medium velocity gears.*** If the velocity of gears is more than 15 m / s, then these are called ***high speed gears.***

3. ***According to the type of gearing.*** The gears, according to the type of gearing, may be classified as :

(*a*) External gearing, **(*b*)** Internal gearing, and **(*c*)** Rack and pinion.



(*a*) External gearing.



(*b*) Internal gearing.

Fig. 28.4

In ***external gearing,*** the gears of the two shafts mesh externally with each other as shown in Fig. 28.4 (*a*). The larger of these two wheels is called ***spur wheel*** or ***gear*** and the smaller wheel is called ***pinion.*** In an external gearing, the motion of the two wheels is always unlike, as shown in Fig. 28.4 (*a*).

In ***internal gearing,*** the gears of the two shafts mesh internally with each other as shown in Fig. 28.4 (*b*). The larger of these two wheels is called ***annular wheel*** and the smaller wheel is called ***pinion***. In an internal gearing, the motion of the wheels is always like as shown in Fig. 28.4 (*b*).

Sometimes, the gear of a shaft meshes externally and internally with the gears in a *straight line, as shown in Fig. 28.5. Such a type of gear is called ***rack*** and ***pinion.*** The straight line gear is called ***rack*** and the circular wheel is called ***pinion.*** A little consideration will show that with the help of a rack and pinion, we can convert linear motion into rotary motion and *vice-versa* as shown in Fig. 28.5.

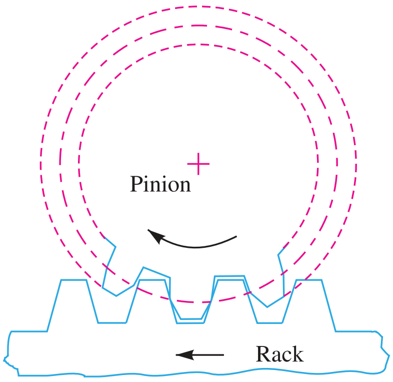


Fig. 28.5. Rack and pinion.

4. ***According to the position of teeth on the gear surface.*** The teeth on the gear surface may be

(*a*) Straight, **(*b*)** Inclined, and **(*c*)** Curved.

We have discussed earlier that the spur gears have straight teeth whereas helical gears have their teeth inclined to the wheel rim. In case of spiral gears, the teeth are curved over the rim surface.

28.5 Terms used in Gears

The following terms, which will be mostly used in this chapter, should be clearly understood at this stage. These terms are illustrated in Fig. 28.6.

1. ***Pitch circle.*** It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

* A straight line may also be defined as a wheel of infinite radius.

2. *Pitch circle diameter.* It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called as *pitch diameter*.

3. *Pitch point.* It is a common point of contact between two pitch circles.

4. *Pitch surface.* It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. *Pressure angle or angle of obliquity.* It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are $14\frac{1}{2}°$ and 20°.

6. *Addendum.* It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. *Dedendum.* It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. *Addendum circle.* It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. *Dedendum circle.* It is the circle drawn through the bottom of the teeth. It is also called ***root circle.***

Note : Root circle diameter = Pitch circle diameter × cos ϕ, where ϕ is the pressure angle.

10. *Circular pitch.* It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively; then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

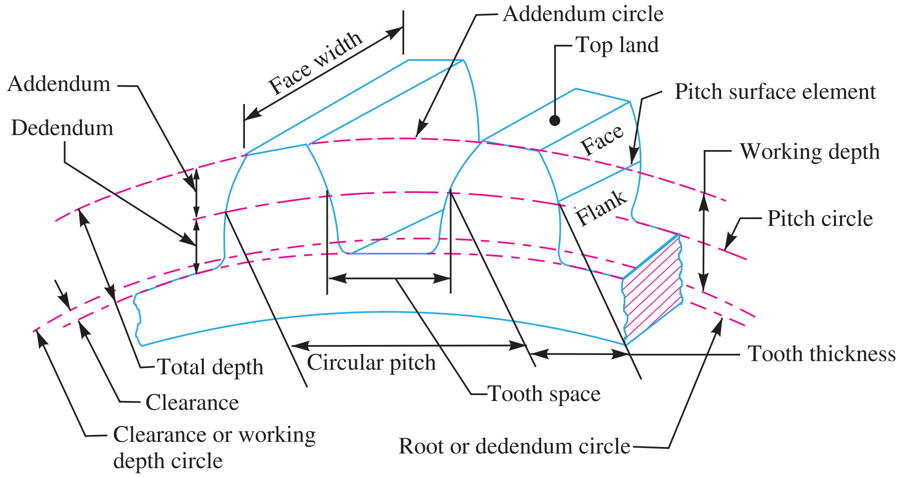


Fig. 28.6. Terms used in gears.

Spur gears

11. *Diametral pitch.* It is the ratio of number of teeth to the pitch circle diameter in millimetres. It denoted by p_d. Mathematically,

$$\text{Diametral pitch, } p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left(\because p_c = \frac{\pi D}{T}\right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

12. *Module.* It is the ratio of the pitch circle diameter in millimetres to the number of teeth. It is usually denoted by m. Mathematically,

$$\text{Module, } m = D / T$$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40 and 50.

The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5,5.5, 7, 9, 11, 14, 18, 22, 28, 36 and 45 are of second choice.

13. *Clearance.* It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle.***

14. *Total depth.* It is the radial distance between the addendum and the dedendum circle of a gear. It is equal to the sum of the addendum and dedendum.

15. *Working depth.* It is radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. *Tooth thickness.* It is the width of the tooth measured along the pitch circle.

17. *Tooth space.* It is the width of space between the two adjacent teeth measured along the pitch circle.

18. *Backlash.* It is the difference between the tooth space and the tooth thickness, as measured on the pitch circle.

19. *Face of the tooth.* It is surface of the tooth above the pitch surface.

20. *Top land.* It is the surface of the top of the tooth.

21. *Flank of the tooth.* It is the surface of the tooth below the pitch surface.

22. *Face width.* It is the width of the gear tooth measured parallel to its axis.

23. *Profile.* It is the curve formed by the face and flank of the tooth.

24. *Fillet radius.* It is the radius that connects the root circle to the profile of the tooth.

25. *Path of contact.* It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. *Length of the path of contact.* It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. *Arc of contact.* It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

(*a*) *Arc of approach.* It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) *Arc of recess.* It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note : The ratio of the length of arc of contact to the circular pitch is known as ***contact ratio*** *i.e.* number of pairs of teeth in contact.

28.6 Condition for Constant Velocity Ratio of Gears–Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 28.7. Let the two teeth come in contact at point *Q*, and the wheels rotate in the directions as shown in the figure.

Let *T T* be the common tangent and *MN* be the common normal to the curves at point of contact *Q*. From the centres O_1 and O_2, draw O_1M and O_2N perpendicular to *MN*. A little consideration will show that the point *Q* moves in the direction *QC*, when considered as a point on wheel 1, and in the direction *QD* when considered as a point on wheel 2.

Let v_1 and v_2 be the velocities of the point *Q* on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal *MN* must be equal.

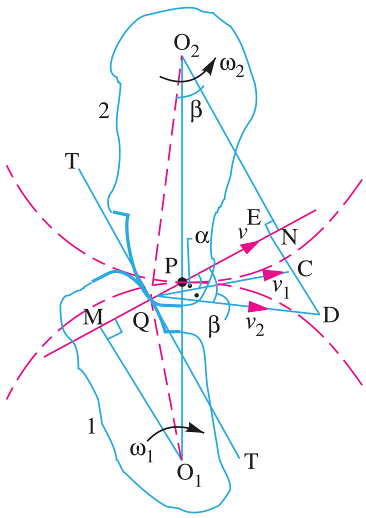


Fig. 28.7. Law of gearing.

$$\therefore \quad v_1 \cos \alpha = v_2 \cos \beta$$

or $$(\omega_1 \times O_1Q) \cos \alpha = (\omega_2 \times O_2Q) \cos \beta$$

$$(\omega_1 \times O_1Q) \frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q) \frac{O_2N}{O_2Q}$$

$$\therefore \quad \omega_1 . O_1M = \omega_2 . O_2N$$

or $$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \qquad \text{...}(i)$$

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \text{...}(ii)$$

Combining equations (*i*) and (*ii*), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \qquad \text{...}(iii)$$

We see that the angular velocity ratio is inversely proportional to the ratio of the distance of *P* from the centres

O_1 and O_2, or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.

Aircraft landing gear is especially designed to absorb shock and energy when an aircraft lands, and then release gradually.

Therefore, in order to have a constant angular velocity ratio for all positions of the wheels, P must be the fixed point (called pitch point) for the two wheels. In other words, ***the common normal at the point of contact between a pair of teeth must always pass through the pitch point.*** This is fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as ***law of gearing.***

Gear trains inside a mechanical watch

Notes : 1. The above condition is fulfilled by teeth of involute form, provided that the root circles from which the profiles are generated are tangential to the common normal.

2. If the shape of one tooth profile is arbitrary chosen and another tooth is designed to satisfy the above condition, then the second tooth is said to be **conjugate** to the first. The conjugate teeth are not in common use because of difficulty in manufacture and cost of production.

3. If D_1 and D_2 are pitch circle diameters of wheel 1 and 2 having teeth T_1 and T_2 respectively, then velocity ratio,

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

28.7 Forms of Teeth

We have discussed in Art. 28.6 (Note 2) that conjugate teeth are not in common use. Therefore, in actual practice, following are the two types of teeth commonly used.

1. Cycloidal teeth ; and **2.** Involute teeth.

We shall discuss both the above mentioned types of teeth in the following articles. Both these forms of teeth satisfy the condition as explained in Art. 28.6.

28.8 Cycloidal Teeth

A ***cycloid*** is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as ***epicycloid.*** On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called ***hypocycloid.***

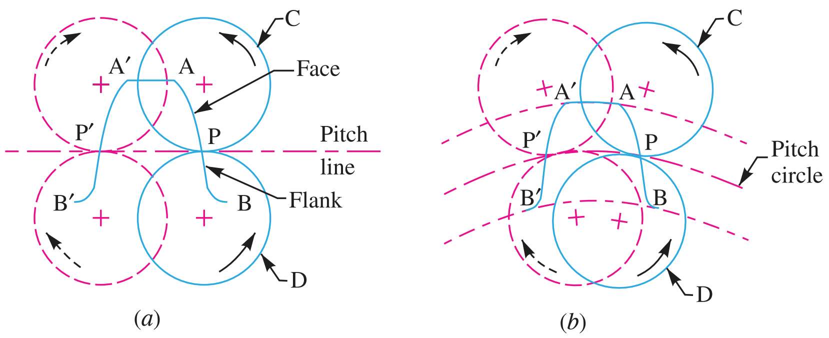


Fig. 28.8. Construction of cycloidal teeth of a gear.

In Fig. 28.8 (*a*), the fixed line or pitch line of a rack is shown. When the circle *C* rolls without slipping above the pitch line in the direction as indicated in Fig. 28.8 (*a*), then the point *P* on the circle traces the epicycloid *PA*. This represents the face of the cycloidal tooth profile. When the circle *D* rolls without slipping below the pitch line, then the point *P* on the circle *D* traces hypocycloid *PB* which represents the flank of the cycloidal tooth. The profile *BPA* is one side of the cycloidal rack tooth. Similarly, the two curves *P' A'* and *P' B'* forming the opposite side of the tooth profile are traced by the point *P'* when the circles *C* and *D* roll in the opposite directions.

In the similar way, the cycloidal teeth of a gear may be constructed as shown in Fig. 28.8 (*b*). The circle *C* is rolled without slipping on the outside of the pitch circle and the point *P* on the circle *C* traces epicycloid *PA*, which represents the face of the cycloidal tooth. The circle *D* is rolled on the inside of pitch circle and the point *P* on the circle *D* traces hypocycloid *PB*, which represents the flank of the tooth profile. The profile *BPA* is one side of the cycloidal tooth. The opposite side of the tooth is traced as explained above.

The construction of the two mating cycloidal teeth is shown in Fig. 28.9. A point on the circle *D* will trace the flank of the tooth T_1 when circle *D* rolls without slipping on the inside of pitch circle of wheel 1 and face of tooth T_2 when the circle *D* rolls without slipping on the outside of pitch circle of wheel 2. Similarly, a point on the circle *C* will trace the face of tooth T_1 and flank of tooth T_2. The rolling circles *C* and *D* may have unequal diameters, but if several wheels are to be interchangeable, they must have rolling circles of equal diameters.

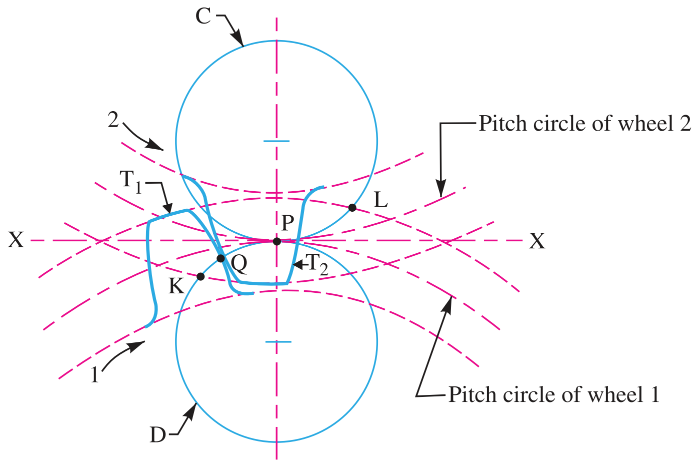


Fig. 28.9. Construction of two mating cycloidal teeth.

A little consideration will show that the common normal *XX* at the point of contact between two cycloidal teeth always passes through the pitch point, which is the fundamental condition for a constant velocity ratio.

28.9 Involute Teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which is unwrapped from a reel as shown in Fig. 28.10 (*a*). In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows :

Let *A* be the starting point of the involute. The base circle is divided into equal number of parts *e.g.* AP_1, $P_1\,P_2$, $P_2\,P_3$ etc.The tangents at P_1, P_2, P_3 etc., are drawn and the lenghts P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc., we obtain the involute curve *AR*. A little consideration will show that at any instant A_3, the tangent A_3T to the involute is perpendicular to P_3A_3 and P_3A_3 is the normal to the involute. In other words, normal at any point of an involute is a tangent to the circle.

Now, let O_1 and O_2 be the fixed centres of the two base circles as shown in Fig. 28.10(*b*). Let the corresponding involutes *AB* and *A'B'* be in contact at point *Q*. *MQ* and *NQ* are normals to the involute at *Q* and are tangents to base circles. Since the normal for an involute at a given point is the tangent drawn from that point to the base circle, therefore the common normal *MN* at *Q* is also the common tangent to the two base circles. We see that the common normal *MN* intersects the line of centres O_1O_2 at the fixed point *P* (called pitch point). Therefore the involute teeth satisfy the fundamental condition of constant velocity ratio.

The clock built by Galelio used gears.

From similar triangles O_2 *NP* and O_1 *MP*,

$$\frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\omega_2}{\omega_1} \qquad \text{...}(i)$$

which determines the ratio of the radii of the two base circles. The radii of the base circles is given by

$$O_1M = O_1 P \cos\phi, \quad \text{and} \quad O_2N = O_2 P \cos\phi$$

where ϕ is the pressure angle or the angle of obliquity.

Also the centre distance between the base circles

$$= O_1P + O_2P = \frac{O_1M}{\cos\phi} + \frac{O_2N}{\cos\phi} = \frac{O_1M + O_2N}{\cos\phi}$$

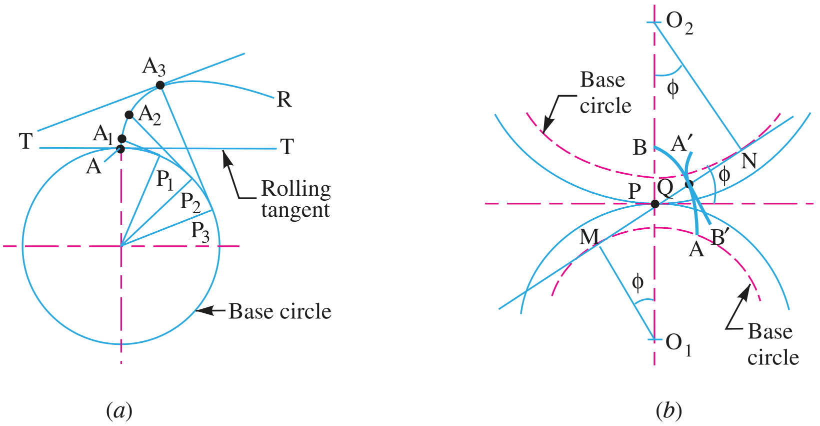


Fig. 28.10. Construction of involute teeth.

A little consideration will show, that if the centre distance is changed, then the radii of pitch circles also changes. But their ratio remains unchanged, because it is equal to the ratio of the two radii of the base circles [See equation (*i*)]. The common normal, at the point of contact, still passes through the pitch point. As a result of this, the wheel continues to work correctly*. However, the pressure angle increases with the increase in centre distance.

28.10 Comparison Between Involute and Cycloidal Gears

In actual practice, the involute gears are more commonly used as compared to cycloidal gears, due to the following advantages :

Advantages of involute gears

Following are the advantages of involute gears :

1. The most important advantage of the involute gears is that the centre distance for a pair of involute gears can be varied within limits without changing the velocity ratio. This is not true for cycloidal gears which requires exact centre distance to be maintained.

2. In involute gears, the pressure angle, from the start of the engagement of teeth to the end of the engagement, remains constant. It is necessary for smooth running and less wear of gears. But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts increasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.

3. The face and flank of involute teeth are generated by a single curve whereas in cycloidal gears, double curves (*i.e.* epicycloid and hypocycloid) are required for the face and flank respectively.

* It is not the case with cycloidal teeth.

Thus the involute teeth are easy to manufacture than cycloidal teeth. In involute system, the basic rack has straight teeth and the same can be cut with simple tools.

Note : The only disadvantage of the involute teeth is that the interference occurs (Refer Art. 28.13) with pinions having smaller number of teeth. This may be avoided by altering the heights of addendum and dedendum of the mating teeth or the angle of obliquity of the teeth.

Advantages of cycloidal gears

Following are the advantages of cycloidal gears :

1. Since the cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears for the same pitch. Due to this reason, the cycloidal teeth are preferred specially for cast teeth.

2. In cycloidal gears, the contact takes place between a convex flank and concave surface, whereas in involute gears, the convex surfaces are in contact. This condition results in less wear in cycloidal gears as compared to involute gears. However the difference in wear is negligible.

3. In cycloidal gears, the interference does not occur at all. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

28.11 Systems of Gear Teeth

The following four systems of gear teeth are commonly used in practice.

1. $14\frac{1}{2}°$ Composite system, **2.** $14\frac{1}{2}°$ Full depth involute system, **3.** 20° Full depth involute system, and **4.** 20° Stub involute system.

The $14\frac{1}{2}°$ ***composite system*** is used for general purpose gears. It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. The teeth are produced by formed milling cutters or hobs. The tooth profile of the $14\frac{1}{2}°$ ***full depth involute system*** was developed for use with gear hobs for spur and helical gears.

The tooth profile of the **20° *full depth involute system*** may be cut by hobs. The increase of the pressure angle from $14\frac{1}{2}°$ to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. The **20° *stub involute system*** has a strong tooth to take heavy loads.

28.12 Standard Proportions of Gear Systems

The following table shows the standard proportions in module (*m*) for the four gear systems as discussed in the previous article.

Table 28.1. Standard proportions of gear systems.

S. No.	*Particulars*	*$14\frac{1}{2}°$ composite or full depth involute system*	*20° full depth involute system*	*20° stub involute system*
1.	Addendum	1*m*	1*m*	0.8 *m*
2.	Dedendum	1.25 *m*	1.25 *m*	1 *m*
3.	Working depth	2 *m*	2 *m*	1.60 *m*
4.	Minimum total depth	2.25 *m*	2.25 *m*	1.80 *m*
5.	Tooth thickness	1.5708 *m*	1.5708 *m*	1.5708 *m*
6.	Minimum clearance	0.25 *m*	0.25 *m*	0.2 *m*
7.	Fillet radius at root	0.4 *m*	0.4 *m*	0.4 *m*

28.13 Interference in Involute Gears

A pinion gearing with a wheel is shown in Fig. 28.11. *MN* is the common tangent to the base circles and *KL* is the path of contact between the two mating teeth. A little consideration will show, that if the radius of the addendum circle of pinion is increased to O_1N, the point of contact *L* will move from *L* to *N*. When this radius is further increased, the point of contact *L* will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as ***interference*** and occurs when the teeth are being cut. In brief, ***the phenomenon when the tip of a tooth undercuts the root on its mating gear is known as interference.***

A drilling machine drilling holes for lamp retaining screws

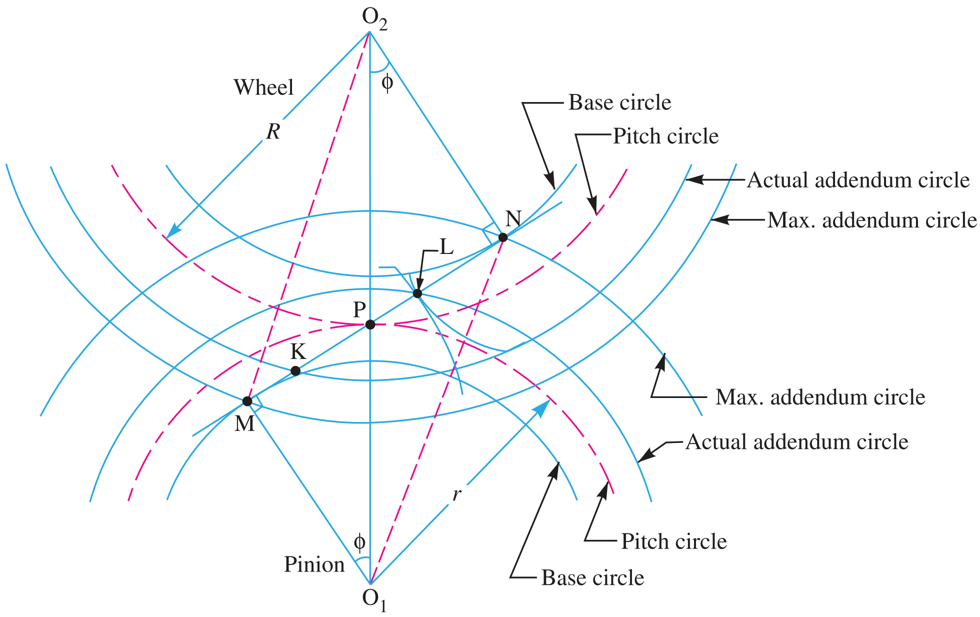


Fig. 28.11. Interference in involute gears.

Similarly, if the radius of the addendum circle of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points *M* and *N* are called ***interference points.*** Obviously interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O_1N and of the wheel is O_2M.

From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other words, ***interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.***

Note : In order to avoid interference, the limiting value of the radius of the addendum circle of the pinion ($O_1 N$) and of the wheel ($O_2 M$), may be obtained as follows :

From Fig. 28.11, we see that

$$O_1N = \sqrt{(O_1M)^2 + (MN)^2} = \sqrt{(r_b)^2 + [(r + R)\sin\phi]^2}$$

where r_b = Radius of base circle of the pinion = $O_1P\cos\phi = r\cos\phi$

Similarly

$$O_2M = \sqrt{(O_2N)^2 + (MN)^2} = \sqrt{(R_b)^2 + [(r + R)\sin\phi]^2}$$

where R_b = Radius of base circle of the wheel = $O_2P\cos\phi = R\cos\phi$

28.14 Minimum Number of Teeth on the Pinion in Order to Avoid Interference

We have seen in the previous article that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. The minimum number of teeth on the pinion which will mesh with any gear (also rack) without interference are given in the following table.

Table 28.2. Minimum number of teeth on the pinion in order to avoid interference.

S. No.	*Systems of gear teeth*	*Minimum number of teeth on the pinion*
1.	$14\frac{1}{2}°$ Composite	12
2.	$14\frac{1}{2}°$ Full depth involute	32
3.	20° Full depth involute	18
4.	20° Stub involute	14

The number of teeth on the pinion (T_P) in order to avoid interference may be obtained from the following relation :

$$T_P = \frac{2A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

where A_W = Fraction by which the standard addendum for the wheel should be multiplied,

G = Gear ratio or velocity ratio = $T_G / T_P = D_G / D_P$,

ϕ = Pressure angle or angle of obliquity.

28.15 Gear Materials

The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc. The gears may be manufactured from metallic or non-metallic materials. The metallic gears with cut teeth are commercially obtainable in cast iron, steel and bronze. The non-metallic materials like wood, rawhide, compressed paper and synthetic resins like nylon are used for gears, especially for reducing noise.

The cast iron is widely used for the manufacture of gears due to its good wearing properties, excellent machinability and ease of producing complicated shapes by casting method. The cast iron gears with cut teeth may be employed, where smooth action is not important.

The steel is used for high strength gears and steel may be plain carbon steel or alloy steel. The steel gears are usually heat treated in order to combine properly the toughness and tooth hardness.

The phosphor bronze is widely used for worm gears in order to reduce wear of the worms which will be excessive with cast iron or steel. The following table shows the properties of commonly used gear materials.

Table 28.3. Properties of commonly used gear materials.

Material (1)	*Condition* (2)	*Brinell hardness number* (3)	*Minimum tensile strength* (N/mm^2) (4)
Malleable cast iron			
(*a*) White heart castings, Grade B	—	217 max.	280
(*b*) Black heart castings, Grade B	—	149 max.	320
Cast iron			
(*a*) Grade 20	As cast	179 min.	200
(*b*) Grade 25	As cast	197 min.	250
(*c*) Grade 35	As cast	207 min.	250
(*d*) Grade 35	Heat treated	300 min.	350
Cast steel	—	145	550
Carbon steel			
(*a*) 0.3% carbon	Normalised	143	500
(*b*) 0.3% carbon	Hardened and tempered	152	600
(*c*) 0.4% carbon	Normalised	152	580
(*d*) 0.4% carbon	Hardened and tempered	179	600
(*e*) 0.35% carbon	Normalised	201	720
(*f*) 0.55% carbon	Hardened and tempered	223	700
Carbon chromium steel			
(*a*) 0.4% carbon	Hardened and tempered	229	800
(*b*) 0.55% carbon	"	225	900
Carbon manganese steel			
(*a*) 0.27% carbon	Hardened and tempered	170	600
(*b*) 0.37% carbon	"	201	700
Manganese molybdenum steel			
(*a*) 35 Mn 2 Mo 28	Hardened and tempered	201	700
(*b*) 35 Mn 2 Mo 45	"	229	800
Chromium molybdenum steel			
(*a*) 40 Cr 1 Mo 28	Hardened and tempered	201	700
(*b*) 40 Cr 1 Mo 60	"	248	900

(1)	(2)	(3)	(4)
Nickel steel			
40 Ni 3	”	229	800
Nickel chromium steel			
30 Ni 4 Cr 1	”	444	1540
Nickel chromium molybdenum steel	Hardness and		
40 Ni 2 Cr 1 Mo 28	tempered	255	900
Surface hardened steel			
(*a*) 0.4% carbon steel	—	145 (core) 460 (case)	551
(*b*) 0.55% carbon steel	—	200 (core) 520 (case)	708
(*c*) 0.55% carbon chromium steel	—	250 (core) 500 (case)	866
(*d*) 1% chromium steel	—	500 (case)	708
(*e*) 3% nickel steel	—	200 (core) 300 (case)	708
Case hardened steel			
(*a*) 0.12 to 0.22% carbon	—	650 (case)	504
(*b*) 3% nickel	—	200 (core) 600 (case)	708
(*c*) 5% nickel steel	—	250 (core) 600 (case)	866
Phosphor bronze castings	Sand cast	60 min.	160
	Chill cast	70 min.	240
	Centrifugal cast	90	260

28.16 Design Considerations for a Gear Drive

In the design of a gear drive, the following data is usually given :

1. The power to be transmitted.

2. The speed of the driving gear,

3. The speed of the driven gear or the velocity ratio, and

4. The centre distance.

The following requirements must be met in the design of a gear drive :

(*a*) The gear teeth should have sufficient strength so that they will not fail under static loading or dynamic loading during normal running conditions.

(*b*) The gear teeth should have wear characteristics so that their life is satisfactory.

(*c*) The use of space and material should be economical.

(*d*) The alignment of the gears and deflections of the shafts must be considered because they effect on the performance of the gears.

(*e*) The lubrication of the gears must be satisfactory.

28.17 Beam Strength of Gear Teeth – Lewis Equation

The beam strength of gear teeth is determined from an equation (known as *Lewis equation) and the load carrying ability of the toothed gears as determined by this equation gives satisfactory results. In the investigation, Lewis assumed that as the load is being transmitted from one gear to another, it is all given and taken by one tooth, because it is not always safe to assume that the load is distributed among several teeth. When contact begins, the load is assumed to be at the end of the driven teeth and as contact ceases, it is at the end of the driving teeth. This may not be true when the number of teeth in a pair of mating gears is large, because the load may be distributed among several teeth. But it is almost certain that at some time during the contact of teeth, the proper distribution of load does not exist and that one tooth must transmit the full load. In any pair of gears having unlike number of teeth, the gear which have the fewer teeth (*i.e.* pinion) will be the weaker, because the tendency toward undercutting of the teeth becomes more pronounced in gears as the number of teeth becomes smaller.

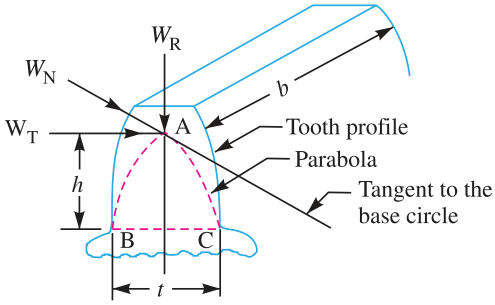


Fig. 28.12. Tooth of a gear.

Consider each tooth as a cantilever beam loaded by a normal load (W_N) as shown in Fig. 28.12. It is resolved into two components *i.e.* tangential component (W_T) and radial component (W_R) acting perpendicular and parallel to the centre line of the tooth respectively. The tangential component (W_T) induces a bending stress which tends to break the tooth. The radial component (W_R) induces a compressive stress of relatively small magnitude, therefore its effect on the tooth may be neglected. Hence, the bending stress is used as the basis for design calculations. The critical section or the section of maximum bending stress may be obtained by drawing a parabola through *A* and tangential to the tooth curves at *B* and *C*. This parabola, as shown dotted in Fig. 28.12, outlines a beam of uniform strength, *i.e.* if the teeth are shaped like a parabola, it will have the same stress at all the sections. But the tooth is larger than the parabola at every section except *BC*. We therefore, conclude that the section *BC* is the section of maximum stress or the critical section. The maximum value of the bending stress (or the permissible working stress), at the section *BC* is given by

$$\sigma_w = M.y / I \qquad \text{...}(i)$$

where M = Maximum bending moment at the critical section $BC = W_T \times h$,

W_T = Tangential load acting at the tooth,

h = Length of the tooth,

y = Half the thickness of the tooth (t) at critical section $BC = t/2$,

I = Moment of inertia about the centre line of the tooth $= b.t^3/12$,

b = Width of gear face.

Substituting the values for M, y and I in equation (*i*), we get

$$\sigma_w = \frac{(W_T \times h)\, t/2}{b.t^3/12} = \frac{(W_T \times h) \times 6}{b.t^2}$$

or

$$W_T = \sigma_w \times b \times t^2 / 6\,h$$

In this expression, t and h are variables depending upon the size of the tooth (*i.e.* the circular pitch) and its profile.

* In 1892, Wilfred Lewis investigated for the strength of gear teeth. He derived an equation which is now extensively used by industry in determining the size and proportions of the gear.

Let $t = x \times p_c$, and $h = k \times p_c$; where x and k are constants.

$$\therefore \quad W_T = \sigma_w \times b \times \frac{x^2 \cdot p_c^2}{6k \cdot p_c} = \sigma_w \times b \times p_c \times \frac{x^2}{6k}$$

Substituting $x^2 / 6k = y$, another constant, we have

$$W_T = \sigma_w \cdot b \cdot p_c \cdot y = \sigma_w \cdot b \cdot \pi m \cdot y \qquad ...(\because p_c = \pi m)$$

The quantity y is known as ***Lewis form factor*** or ***tooth form factor*** and W_T (which is the tangential load acting at the tooth) is called the ***beam strength of the tooth.***

Since $y = \frac{x^2}{6k} = \frac{t^2}{(p_c)^2} \times \frac{p_c}{6h} = \frac{t^2}{6h \cdot p_c}$, therefore in order to find the value of y, the quantities t, h and p_c may be determined analytically or measured from the drawing similar to Fig. 28.12. It may be noted that if the gear is enlarged, the distances t, h and p_c will each increase proportionately. Therefore the value of y will remain unchanged. A little consideration will show that the value of y is independent of the size of the tooth and depends only on the number of teeth on a gear and the system of teeth. The value of y in terms of the number of teeth may be expressed as follows :

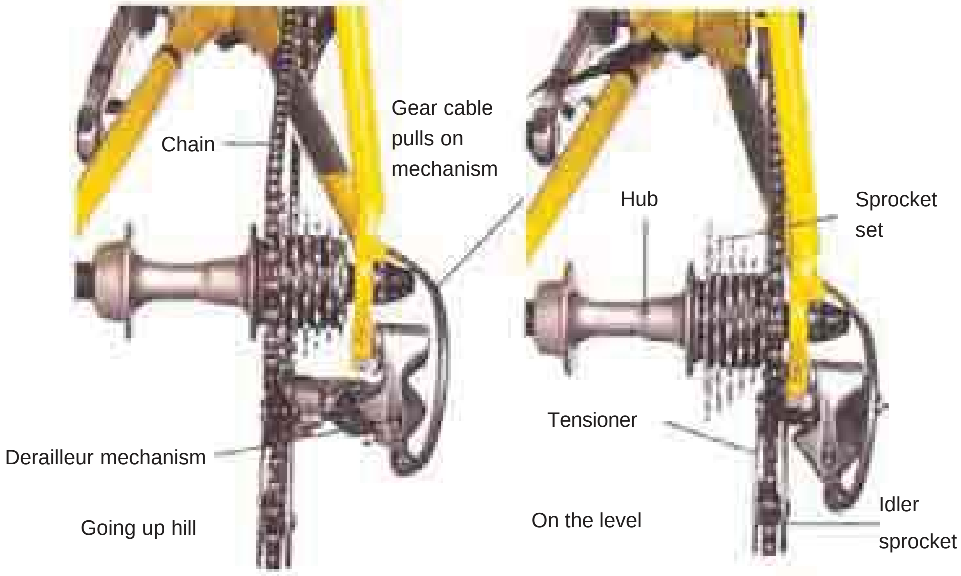
$$y = 0.124 - \frac{0.684}{T}, \text{ for } 14\tfrac{1}{2}^\circ \text{ composite and full depth involute system.}$$

$$= 0.154 - \frac{0.912}{T}, \text{ for } 20^\circ \text{ full depth involute system.}$$

$$= 0.175 - \frac{0.841}{T}, \text{ for } 20^\circ \text{ stub system.}$$

28.18 Permissible Working Stress for Gear Teeth in the Lewis Equation

The permissible working stress (σ_w) in the Lewis equation depends upon the material for which an allowable static stress (σ_o) may be determined. The ***allowable static stress*** is the stress at the



Bicycle gear mechanism switches the chain between different sized sprockets at the pedals and on the back wheel. Going up hill, a small front and a large rear sprocket are selected to reduce the push required for the rider. On the level, a large front and small rear. sprocket are used to prevent the rider having to pedal too fast.

elastic limit of the material. It is also called the ***basic stress.*** In order to account for the dynamic effects which become more severe as the pitch line velocity increases, the value of σ_w is reduced. According to the Barth formula, the permissible working stress,

$$\sigma_w = \sigma_o \times C_v$$

where σ_o = Allowable static stress, and

C_v = Velocity factor.

The values of the velocity factor (C_v) are given as follows :

$$C_v = \frac{3}{3 + v}, \text{ for ordinary cut gears operating at velocities upto 12.5 m / s.}$$

$$= \frac{4.5}{4.5 + v}, \text{ for carefully cut gears operating at velocities upto 12.5 m/s.}$$

$$= \frac{6}{6 + v}, \text{ for very accurately cut and ground metallic gears operating at velocities upto 20 m / s.}$$

$$= \frac{0.75}{0.75 + \sqrt{v}}, \text{ for precision gears cut with high accuracy and operating at velocities upto 20 m / s.}$$

$$= \left(\frac{0.75}{1 + v}\right) + 0.25, \text{ for non-metallic gears.}$$

In the above expressions, v is the pitch line velocity in metres per second.

The following table shows the values of allowable static stresses for the different gear materials.

Table 28.4. Values of allowable static stress.

Material	*Allowable static stress* (σ_o) *MPa or N/mm*^{2}
Cast iron, ordinary	56
Cast iron, medium grade	70
Cast iron, highest grade	105
Cast steel, untreated	140
Cast steel, heat treated	196
Forged carbon steel-case hardened	126
Forged carbon steel-untreated	140 to 210
Forged carbon steel-heat treated	210 to 245
Alloy steel-case hardened	350
Alloy steel-heat treated	455 to 472
Phosphor bronze	84
Non-metallic materials	
Rawhide, fabroil	42
Bakellite, Micarta, Celoron	56

Note : The allowable static stress (σ_o) for steel gears is approximately one-third of the ultimate tensile stregth (σ_u) *i.e.* $\sigma_o = \sigma_u / 3$.

28.19 Dynamic Tooth Load

In the previous article, the velocity factor was used to make approximate allowance for the effect of dynamic loading. The dynamic loads are due to the following reasons :

1. Inaccuracies of tooth spacing,
2. Irregularities in tooth profiles, and
3. Deflections of teeth under load.

A closer approximation to the actual conditions may be made by the use of equations based on extensive series of tests, as follows :

$$W_D = W_T + W_I$$

where W_D = Total dynamic load,

W_T = Steady load due to transmitted torque, and

W_I = Increment load due to dynamic action.

The increment load (W_I) depends upon the pitch line velocity, the face width, material of the gears, the accuracy of cut and the tangential load. For average conditions, the dynamic load is determined by using the following Buckingham equation, *i.e.*

$$W_D = W_T + W_I = W_T + \frac{21\, v\,(b.C + W_T)}{21\, v + \sqrt{b.C + W_T}} \quad \text{...}(i)$$

where W_D = Total dynamic load in newtons,

W_T = Steady transmitted load in newtons,

v = Pitch line velocity in m/s,

b = Face width of gears in mm, and

C = A deformation or dynamic factor in N/mm.

A deformation factor (C) depends upon the error in action between teeth, the class of cut of the gears, the tooth form and the material of the gears. The following table shows the values of deformation factor (C) for checking the dynamic load on gears.

Table 28.5. Values of deformation factor (C).

Material		*Involute tooth form*	*Values of deformation factor (C) in N-mm*				
			Tooth error in action (e) in mm				
Pinion	*Gear*		0.01	0.02	0.04	0.06	0.08
Cast iron	Cast iron	$14\frac{1}{2}°$	55	110	220	330	440
Steel	Cast iron		76	152	304	456	608
Steel	Steel		110	220	440	660	880
Cast iron	Cast iron	20° full depth	57	114	228	342	456
Steel	Cast iron		79	158	316	474	632
Steel	Steel		114	228	456	684	912
Cast iron	Cast iron	20° stub	59	118	236	354	472
Steel	Cast iron		81	162	324	486	648
Steel	Steel		119	238	476	714	952

The value of C in N/mm may be determined by using the following relation :

$$C = \frac{K.e}{\dfrac{1}{E_P} + \dfrac{1}{E_G}} \quad \text{...}(ii)$$

where K = A factor depending upon the form of the teeth.

= 0.107, for $14\frac{1}{2}°$ full depth involute system.

= 0.111, for 20° full depth involute system.

= 0.115 for 20° stub system.

E_P = Young's modulus for the material of the pinion in N/mm^2.

E_G = Young's modulus for the material of gear in N/mm^2.

e = Tooth error action in mm.

The maximum allowable tooth error in action (e) depends upon the pitch line velocity (v) and the class of cut of the gears. The following tables show the values of tooth errors in action (e) for the different values of pitch line velocities and modules.

Table 28.6. Values of maximum allowable tooth error in action (*e*) verses pitch line velocity, for well cut commercial gears.

Pitch line velocity (v) m/s	*Tooth error in action (e) mm*	*Pitch line velocity (v) m/s*	*Tooth error in action (e) mm*	*Pitch line velocity (v) m/s*	*Tooth error in action (e) mm*
1.25	0.0925	8.75	0.0425	16.25	0.0200
2.5	0.0800	10	0.0375	17.5	0.0175
3.75	0.0700	11.25	0.0325	20	0.0150
5	0.0600	12.5	0.0300	22.5	0.0150
6.25	0.0525	13.75	0.0250	25 and over	0.0125
7.5	0.0475	15	0.0225		

Table 28.7. Values of tooth error in action (*e*) verses module.

Module (m) in mm	*Tooth error in action (e) in mm*		
	First class commercial gears	*Carefully cut gears*	*Precision gears*
Upto 4	0.051	0.025	0.0125
5	0.055	0.028	0.015
6	0.065	0.032	0.017
7	0.071	0.035	0.0186
8	0.078	0.0386	0.0198
9	0.085	0.042	0.021
10	0.089	0.0445	0.023
12	0.097	0.0487	0.0243
14	0.104	0.052	0.028
16	0.110	0.055	0.030
18	0.114	0.058	0.032
20	0.117	0.059	0.033

28.20 Static Tooth Load

The ***static tooth load*** (also called ***beam strength*** or ***endurance strength*** of the tooth) is obtained by Lewis formula by substituting flexural endurance limit or elastic limit stress (σ_e) in place of permissible working stress (σ_w).

∴ Static tooth load or beam strength of the tooth,

$$W_S = \sigma_e . b . p_c . y = \sigma_e . b . \pi\, m . y$$

The following table shows the values of flexural endurance limit (σ_e) for different materials.

Table 28.8. Values of flexural endurance limit.

Material of pinion and gear	*Brinell hardness number (B.H.N.)*	*Flexural endurance limit (σ_e) in MPa*
Grey cast iron	160	84
Semi-steel	200	126
Phosphor bronze	100	168
Steel	150	252
	200	350
	240	420
	280	490
	300	525
	320	560
	350	595
	360	630
	400 and above	700

For safety, against tooth breakage, the static tooth load (W_S) should be greater than the dynamic load (W_D). Buckingham suggests the following relationship between W_S and W_D.

For steady loads, $W_S \geq 1.25\, W_D$

For pulsating loads, $W_S \geq 1.35\, W_D$

For shock loads, $W_S \geq 1.5\, W_D$

Note : For steel, the flexural endurance limit (σ_e) may be obtained by using the following relation :

$$\sigma_e = 1.75 \times \text{B.H.N. (in MPa)}$$

28.21 Wear Tooth Load

The maximum load that gear teeth can carry, without premature wear, depends upon the radii of curvature of the tooth profiles and on the elasticity and surface fatigue limits of the materials. The maximum or the limiting load for satisfactory wear of gear teeth, is obtained by using the following Buckingham equation, *i.e.*

$$W_w = D_P . b . Q . K$$

where W_w = Maximum or limiting load for wear in newtons,

D_P = Pitch circle diameter of the pinion in mm,

b = Face width of the pinion in mm,

Q = Ratio factor

$$= \frac{2 \times V.R.}{V.R. + 1} = \frac{2 T_G}{T_G + T_P}, \text{ for external gears}$$

$$= \frac{2 \times V.R.}{V.R. - 1} = \frac{2 T_G}{T_G - T_P}, \text{ for internal gears.}$$

$V.R.$ = Velocity ratio = T_G / T_P,

K = Load-stress factor (also known as material combination factor) in N/mm^2.

The load stress factor depends upon the maximum fatigue limit of compressive stress, the pressure angle and the modulus of elasticity of the materials of the gears. According to Buckingham, the load stress factor is given by the following relation :

$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

where σ_{es} = Surface endurance limit in MPa or N/mm^2,

ϕ = Pressure angle,

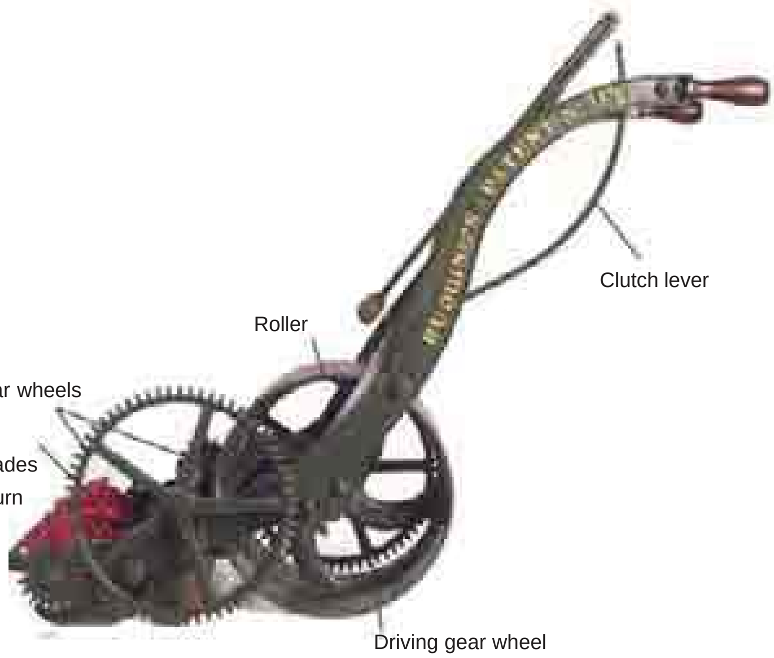
E_P = Young's modulus for the material of the pinion in N/mm^2, and

E_G = Young's modulus for the material of the gear in N/mm^2.

The values of surface endurance limit (σ_{es}) are given in the following table.

Table 28.9. Values of surface endurance limit.

Material of pinion and gear	*Brinell hardness number (B.H.N.)*	*Surface endurance limit (σ_{es}) in N/mm^2*
Grey cast iron	160	630
Semi-steel	200	630
Phosphor bronze	100	630
Steel	150	350
	200	490
	240	616
	280	721
	300	770
	320	826
	350	910
	400	1050



An old model of a lawn-mower

Notes : 1. The surface endurance limit for steel may be obtained from the following equation :

$$\sigma_{es} = (2.8 \times \text{B.H.N.} - 70) \text{ N/mm}^2$$

2. The maximum limiting wear load (W_w) must be greater than the dynamic load (W_D).

28.22 Causes of Gear Tooth Failure

The different modes of failure of gear teeth and their possible remedies to avoid the failure, are as follows :

1. *Bending failure.* Every gear tooth acts as a cantilever. If the total repetitive dynamic load acting on the gear tooth is greater than the beam strength of the gear tooth, then the gear tooth will fail in bending, *i.e.* the gear tooth will break.

In order to avoid such failure, the module and face width of the gear is adjusted so that the beam strength is greater than the dynamic load.

2. *Pitting.* It is the surface fatigue failure which occurs due to many repetition of Hertz contact stresses. The failure occurs when the surface contact stresses are higher than the endurance limit of the material. The failure starts with the formation of pits which continue to grow resulting in the rupture of the tooth surface.

In order to avoid the pitting, the dynamic load between the gear tooth should be less than the wear strength of the gear tooth.

3. *Scoring.* The excessive heat is generated when there is an excessive surface pressure, high speed or supply of lubricant fails. It is a stick-slip phenomenon in which alternate shearing and welding takes place rapidly at high spots.

This type of failure can be avoided by properly designing the parameters such as speed, pressure and proper flow of the lubricant, so that the temperature at the rubbing faces is within the permissible limits.

4. *Abrasive wear.* The foreign particles in the lubricants such as dirt, dust or burr enter between the tooth and damage the form of tooth. This type of failure can be avoided by providing filters for the lubricating oil or by using high viscosity lubricant oil which enables the formation of thicker oil film and hence permits easy passage of such particles without damaging the gear surface.

5. *Corrosive wear.* The corrosion of the tooth surfaces is mainly caused due to the presence of corrosive elements such as additives present in the lubricating oils. In order to avoid this type of wear, proper anti-corrosive additives should be used.

28.23 Design Procedure for Spur Gears

In order to design spur gears, the following procedure may be followed :

1. First of all, the design tangential tooth load is obtained from the power transmitted and the pitch line velocity by using the following relation :

$$W_T = \frac{P}{v} \times C_S \qquad \text{...}(i)$$

where W_T = Permissible tangential tooth load in newtons,

P = Power transmitted in watts,

*v = Pitch line velocity in m / s $= \dfrac{\pi D N}{60}$,

D = Pitch circle diameter in metres,

---

* We know that circular pitch,

$$p_c = \pi D / T = \pi m \qquad \text{...}(\because m = D / T)$$

$$\therefore \quad D = m.T$$

Thus, the pitch line velocity may also be obtained by using the following relation, *i.e.*

$$v = \frac{\pi D.N}{60} = \frac{\pi\, m.T.N}{60} = \frac{p_c.T.N}{60}$$

where m = Module in metres, and

T = Number of teeth.

N = Speed in r.p.m., and

C_S = Service factor.

The following table shows the values of service factor for different types of loads :

Table 28.10. Values of service factor.

Type of load	*Type of service*		
	Intermittent or 3 hours per day	*8-10 hours per day*	*Continuous 24 hours per day*
Steady	0.8	1.00	1.25
Light shock	1.00	1.25	1.54
Medium shock	1.25	1.54	1.80
Heavy shock	1.54	1.80	2.00

Note : The above values for service factor are for enclosed well lubricated gears. In case of non-enclosed and grease lubricated gears, the values given in the above table should be divided by 0.65.

2. Apply the Lewis equation as follows :

$$W_T = \sigma_w.b.p_c.y = \sigma_w.b.\pi\, m.y$$

$$= (\sigma_o.C_v)\, b.\pi\, m.y \qquad ...(\because\ \sigma_w = \sigma_o.C_v)$$

Notes : (*i*) The Lewis equation is applied only to the weaker of the two wheels (*i.e.* pinion or gear).

(*ii*) When both the pinion and the gear are made of the same material, then pinion is the weaker.

(*iii*) When the pinion and the gear are made of different materials, then the product of $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is the *deciding factor. The Lewis equation is used to that wheel for which $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is less.

A bicycle with changeable gears.

* We see from the Lewis equation that for a pair of mating gears, the quantities like W_T, b, m and C_v are constant. Therefore $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is the only deciding factor.

(iv) The product ($\sigma_w \times y$) is called ***strength factor*** of the gear.

(v) The face width (b) may be taken as 3 p_c to 4 p_c (or 9.5 m to 12.5 m) for cut teeth and 2 p_c to 3 p_c (or 6.5 m to 9.5 m) for cast teeth.

3. Calculate the dynamic load (W_D) on the tooth by using Buckingham equation, *i.e.*

$$W_D = W_T + W_I$$

$$= W_T + \frac{21v\,(b.C + W_T)}{21v + \sqrt{b.C + W_T}}$$

In calculating the dynamic load (W_D), the value of tangential load (W_T) may be calculated by neglecting the service factor (C_S) *i.e.*

$$W_T = P / v, \text{ where } P \text{ is in watts and } v \text{ in m / s.}$$

4. Find the static tooth load (*i.e.* beam strength or the endurance strength of the tooth) by using the relation,

$$W_S = \sigma_e.b.p_c.y = \sigma_e.b.\pi\, m.y$$

For safety against breakage, W_S should be greater than W_D.

5. Finally, find the wear tooth load by using the relation,

$$W_w = D_P.b.Q.K$$

The wear load (W_w) should not be less than the dynamic load (W_D).

Example 28.1. *The following particulars of a single reduction spur gear are given :*

Gear ratio = 10 : 1; Distance between centres = 660 mm approximately; Pinion transmits 500 kW at 1800 r.p.m.; Involute teeth of standard proportions (addendum = m) with pressure angle of 22.5°; Permissible normal pressure between teeth = 175 N per mm of width. Find :

1. The nearest standard module if no interference is to occur;

2. The number of teeth on each wheel;

3. The necessary width of the pinion; and

4. The load on the bearings of the wheels due to power transmitted.

Solution : Given : $G = T_G / T_P = D_G / D_P = 10$; $L = 660$ mm ; $P = 500$ kW $= 500 \times 10^3$ W ; $N_P = 1800$ r.p.m. ; $\phi = 22.5°$; $W_N = 175$ N/mm width

1. Nearest standard module if no interference is to occur

Let m = Required module,

T_P = Number of teeth on the pinion,

T_G = Number of teeth on the gear,

D_P = Pitch circle diameter of the pinion, and

D_G = Pitch circle diameter of the gear.

We know that minimum number of teeth on the pinion in order to avoid interference,

$$T_P = \frac{2\,A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

$$= \frac{2 \times 1}{10\left[\sqrt{1 + \frac{1}{10}\left(\frac{1}{10} + 2\right)\sin^2 22.5°} - 1\right]} = \frac{2}{0.15} = 13.3 \text{ say } 14$$

... ($\because A_W$ = 1 module)

$\therefore \quad T_G = G \times T_P = 10 \times 14 = 140$ ...($\because T_G / T_P = 10$)

We know that $L = \frac{D_G}{2} + \frac{D_P}{2} = \frac{D_G}{2} + \frac{10\,D_P}{2} = 5.5\,D_P$ ...($\because D_G / D_P = 10$)

$\therefore$ $660 = 5.5\,D_P$ or $D_P = 660 / 5.5 = 120$ mm

We also know that $D_P = m \,.\, T_P$

$\therefore$ $m = D_P / T_P = 120 / 14 = 8.6$ mm

Since the nearest standard value of the module is 8 mm, therefore we shall take

$m = 8$ mm **Ans.**

2. Number of teeth on each wheel

We know that number of teeth on the pinion,

$T_P = D_P / m = 120 / 8 = 15$ **Ans.**

and number of teeth on the gear,

$T_G = G \times T_P = 10 \times 15 = 150$ **Ans.**

3. Necessary width of the pinion

We know that the torque acting on the pinion,

$$T = \frac{P \times 60}{2\pi N_P} = \frac{500 \times 10^3 \times 60}{2\pi \times 1800} = 2652 \text{ N-m}$$

$\therefore$ Tangential load, $W_T = \frac{T}{D_P / 2} = \frac{2652}{0.12/2} = 44\ 200$ N ...($\because D_P$ is taken in metres)

and normal load on the tooth,

$$W_N = \frac{W_T}{\cos\phi} = \frac{44\ 200}{\cos 22.5°} = 47\ 840 \text{ N}$$

Since the normal pressure between teeth is 175 N per mm of width, therefore necessary width of the pinion,

$$b = \frac{47\ 840}{175} = 273.4 \text{ mm}$$ **Ans.**

4. Load on the bearings of the wheels

We know that the radial load on the bearings due to the power transmitted,

$W_R = W_N \,.\, \sin\phi = 47\ 840 \times \sin 22.5° = 18\ 308$ N $= 18.308$ kN **Ans.**

Example 28.2. *A bronze spur pinion rotating at 600 r.p.m. drives a cast iron spur gear at a transmission ratio of 4 : 1. The allowable static stresses for the bronze pinion and cast iron gear are 84 MPa and 105 MPa respectively.*

The pinion has 16 standard 20° full depth involute teeth of module 8 mm. The face width of both the gears is 90 mm. Find the power that can be transmitted from the standpoint of strength.

Solution. Given : $N_P = 600$ r.p.m. ; $V.R. = T_G / T_P = 4$; $\sigma_{OP} = 84$ MPa $= 84$ N / mm^2 ; $\sigma_{OG} = 105$ MPa $= 105$ N/mm^2 ; $T_P = 16$; $m = 8$ mm ; $b = 90$ mm

We know that pitch circle diameter of the pinion,

$D_P = m.T_P = 8 \times 16 = 128$ mm $= 0.128$ m

$\therefore$ Pitch line velocity,

$$v = \frac{\pi D_P \,.\, N_P}{60} = \frac{\pi \times 0.128 \times 600}{60} = 4.02 \text{ m/s}$$

Since the pitch line velocity (v) is less than 12.5 m/s, therefore velocity factor,

$$C_v = \frac{3}{3 + v} = \frac{3}{3 + 4.02} = 0.427$$

We know that for 20° full depth involute teeth, tooth form factor for the pinion,

$$y_P = 0.154 - \frac{0.912}{T_P} = 0.154 - \frac{0.912}{16} = 0.097$$

and tooth form factor for the gear,

$$y_G = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{4 \times 16} = 0.14 \qquad \text{... } (\because T_G / T_P = 4)$$

$\therefore \qquad \sigma_{OP} \times y_P = 84 \times 0.097 = 8.148$

and $\qquad \sigma_{OG} \times y_G = 105 \times 0.14 = 14.7$

Since ($\sigma_{OP} \times y_P$) is less than ($\sigma_{OG} \times y_G$), therefore the pinion is weaker. Now using the Lewis equation for the pinion, we have tangential load on the tooth (or beam strength of the tooth),

$$W_T = \sigma_{wP}.b.\pi\, m.y_P = (\sigma_{OP} \times C_v)\, b.\pi\, m.y_P \qquad (\because \sigma_{WP} = \sigma_{OP}.C_v)$$

$$= 84 \times 0.427 \times 90 \times \pi \times 8 \times 0.097 = 7870 \text{ N}$$

∴ Power that can be transmitted

$$= W_T \times v = 7870 \times 4.02 = 31\ 640 \text{ W} = 31.64 \text{ kW}$$ **Ans.**

Example 28.3. *A pair of straight teeth spur gears is to transmit 20 kW when the pinion rotates at 300 r.p.m. The velocity ratio is 1 : 3. The allowable static stresses for the pinion and gear materials are 120 MPa and 100 MPa respectively.*

The pinion has 15 teeth and its face width is 14 times the module. Determine : 1. module; 2. face width; and 3. pitch circle diameters of both the pinion and the gear from the standpoint of strength only, taking into consideration the effect of the dynamic loading.

The tooth form factor y can be taken as

$$y = 0.154 - \frac{0.912}{\text{No. of teeth}}$$

and the velocity factor C_v as

$$C_v = \frac{3}{3 + v}, \text{ where } v \text{ is expressed in m / s.}$$

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N_P = 300$ r.p.m. ; $V.R. = T_G / T_P = 3$; $\sigma_{OP} = 120$ MPa $= 120$ N/mm^2 ; $\sigma_{OG} = 100$ MPa $= 100$ N/mm^2 ; $T_P = 15$; $b = 14$ module $= 14\,m$

1. *Module*

Let $\qquad m$ = Module in mm, and

D_P = Pitch circle diameter of the pinion in mm.

We know that pitch line velocity,

$$v = \frac{\pi D_P\, N_P}{60} = \frac{\pi\, m.T_P\, .N_P}{60} \qquad \text{... } (\because D_P = m.T_P)$$

$$= \frac{\pi\, m \times 15 \times 300}{60} = 236\ m \text{ mm/s} = 0.236\ m \text{ m/s}$$

Assuming steady load conditions and 8-10 hours of service per day, the service factor (C_S) from Table 28.10 is given by

$$C_S = 1$$

We know that design tangential tooth load,

$$W_T = \frac{P}{v} \times C_S = \frac{20 \times 10^3}{0.236\ m} \times 1 = \frac{84\ 746}{m} \text{ N}$$

and velocity factor,

$$C_v = \frac{3}{3 + v} = \frac{3}{3 + 0.236\ m}$$

We know that tooth form factor for the pinion,

$$y_P = 0.154 - \frac{0.912}{T_P} = 0.154 - \frac{0.912}{15}$$

$$= 0.154 - 0.0608 = 0.0932$$

and tooth form factor for the gear,

$$y_G = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{3 \times 15}$$

$$= 0.154 - 0.203 = 0.1337 \qquad \text{... } (\because T_G = 3T_P)$$

$$\therefore \quad \sigma_{OP} \times y_P = 120 \times 0.0932 = 11.184$$

and $\sigma_{OG} \times y_G = 100 \times 0.1337 = 13.37$

Since $(\sigma_{OP} \times y_P)$ is less than $(\sigma_{OG} \times y_G)$, therefore the pinion is weaker. Now using the Lewis equation to the pinion, we have

$$W_T = \sigma_{wP}.b.\pi\, m.y_P = (\sigma_{OP} \times C_v)\, b.\pi\, m \,.\, y_P$$

$$\therefore \quad \frac{84\,746}{m} = 120\left(\frac{3}{3 + 0.236\, m}\right) 14\, m \times \pi\, m \times 0.0932 = \frac{1476\, m^2}{3 + 0.236\, m}$$

or $3 + 0.236\, m = 0.0174\, m^3$

Solving this equation by hit and trial method, we find that

$$m = 6.4 \text{ mm}$$

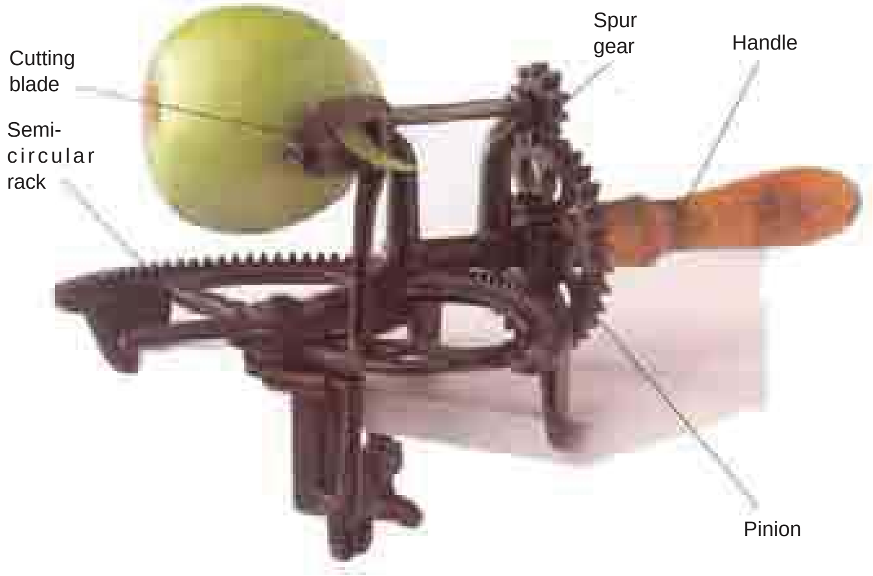
The standard module is 8 mm. Therefore let us take

$m = 8$ mm **Ans.**

2. *Face width*

We know that the face width,

$b = 14\, m = 14 \times 8 = 112$ mm **Ans.**



Kitchen Gear : *This 1863 fruit and vegetable peeling machine uses a rack and pinion to drive spur gears that turn an apple against a cutting blade. As the handle is pushed round the semi-circular base, the peel is removed from the apple in a single sweep.*

3. *Pitch circle diameter of the pinion and gear*

We know that pitch circle diameter of the pinion,

$$D_P = m.T_P = 8 \times 15 = 120 \text{ mm } \textbf{Ans.}$$

and pitch circle diameter of the gear,

$$D_G = m.T_G = 8 \times 45 = 360 \text{ mm } \textbf{Ans.} \qquad \text{... } (\because T_G = 3\, T_P)$$

Example 28.4. *A gear drive is required to transmit a maximum power of 22.5 kW. The velocity ratio is 1:2 and r.p.m. of the pinion is 200. The approximate centre distance between the shafts may be taken as 600 mm. The teeth has 20° stub involute profiles. The static stress for the gear material (which is cast iron) may be taken as 60 MPa and face width as 10 times the module. Find the module, face width and number of teeth on each gear.*

Check the design for dynamic and wear loads. The deformation or dynamic factor in the Buckingham equation may be taken as 80 and the material combination factor for the wear as 1.4.

Solution. Given : P = 22.5 kW = 22 500 W ; $V.R. = D_G/D_P = 2$; N_P = 200 r.p.m. ; L = 600 mm ; $\sigma_{OP} = \sigma_{OG}$ = 60 MPa = 60 N/mm^2 ; $b = 10\, m$; C = 80 ; K = 1.4

Module

Let m = Module in mm,

D_P = Pitch circle diameter of the pinion, and

D_G = Pitch circle diameter of the gear.

We know that centre distance between the shafts (L),

$$600 = \frac{D_P}{2} + \frac{D_G}{2} = \frac{D_P}{2} + \frac{2\,D_P}{2} = 1.5\, D_P \qquad \text{... } (\because D_G = V.R. \times D_P)$$

Arm of a material handler In addition to gears, hydraulic rams as shown above, play important role in transmitting force and energy.

$\therefore \quad D_P = 600 / 1.5 = 400 \text{ mm} = 0.4 \text{ m}$

and $\quad D_G = 2\, D_P = 2 \times 400 = 800 \text{ mm} = 0.8 \text{ m}$

Since both the gears are made of the same material, therefore pinion is the weaker. Thus the design will be based upon the pinion.

We know that pitch line velocity of the pinion,

$$v = \frac{\pi D_P . N_P}{60} = \frac{\pi \times 0.4 \times 200}{60} = 4.2 \text{ m/s}$$

Since v is less than 12 m / s, therefore velocity factor,

$$C_v = \frac{3}{3 + v} = \frac{3}{3 + 4.2} = 0.417$$

We know that number of teeth on the pinion,

$$T_P = D_P / m = 400 / m$$

$\therefore$ Tooth form factor for the pinion,

$$y_P = 0.175 - \frac{0.841}{T_P} = 0.175 - \frac{0.841 \times m}{400} \quad \text{... (For 20° stub system)}$$

$$= 0.175 - 0.0021\, m \quad \text{...}(i)$$

Assuming steady load conditions and 8–10 hours of service per day, the service factor (C_S) from Table 28.10 is given by

$$C_S = 1$$

We know that design tangential tooth load,

$$W_T = \frac{P}{v} \times C_S = \frac{22\,500}{4.2} \times 1 = 5357 \text{ N}$$

We also know that tangential tooth load (W_T),

$$5357 = \sigma_{wP} \,.\, b.\, \pi\, m.y_P = (\sigma_{OP} \times C_v)\, b.\pi\, m.y_P$$

$$= (60 \times 0.417)\, 10\, m \times \pi\, m\, (0.175 - 0.0021\, m)$$

$$= 137.6\, m^2 - 1.65\, m^3$$

Solving this equation by hit and trial method, we find that

$$m = 0.65 \text{ say } 8 \text{ mm} \quad \textbf{Ans.}$$

Face width

We know that face width,

$$b = 10\, m = 10 \times 8 = 80 \text{ mm} \quad \textbf{Ans.}$$

Number of teeth on the gears

We know that number of teeth on the pinion,

$$T_P = D_P / m = 400 / 8 = 50 \quad \textbf{Ans.}$$

and number of teeth on the gear,

$$T_G = D_G / m = 800 / 8 = 100 \quad \textbf{Ans.}$$

Checking the gears for dynamic and wear load

We know that the dynamic load,

$$W_D = W_T + \frac{21v\,(b.C + W_T)}{21v + \sqrt{b.C + W_T}}$$

$$= 5357 + \frac{21 \times 4.2\,(80 \times 80 + 5357)}{21 \times 4.2 + \sqrt{80 \times 80 + 5357}}$$

$$= 5357 + \frac{1.037 \times 10^6}{196.63} = 5357 + 5273 = 10\ 630 \text{ N}$$

From equation *(i)*, we find that tooth form factor for the pinion,

$$y_P = 0.175 - 0.0021\ m = 0.175 - 0.0021 \times 8 = 0.1582$$

From Table 28.8, we find that flexural endurance limit (σ_e) for cast iron is 84 MPa or 84 N/mm^2.

∴ Static tooth load or endurance strength of the tooth,

$$W_S = \sigma_e \cdot b.\ \pi\ m.\ y_P = 84 \times 80 \times \pi \times 8 \times 0.1582 = 26\ 722 \text{ N}$$

We know that ratio factor,

$$Q = \frac{2 \times V.R.}{V.R. + 1} = \frac{2 \times 2}{2 + 1} = 1.33$$

∴ Maximum or limiting load for wear,

$$W_w = D_P \cdot b \cdot Q \cdot K = 400 \times 80 \times 1.33 \times 1.4 = 59\ 584 \text{ N}$$

Since both W_S and W_w are greater than W_D, therefore the design is safe.

Example 28.5. *A pair of straight teeth spur gears, having 20° involute full depth teeth is to transmit 12 kW at 300 r.p.m. of the pinion. The speed ratio is 3 : 1. The allowable static stresses for gear of cast iron and pinion of steel are 60 MPa and 105 MPa respectively. Assume the following:*

Number of teeth of pinion = 16; Face width = 14 times module; Velocity factor $(C_v) = \dfrac{4.5}{4.5 + v}$,

v being the pitch line velocity in m / s; and tooth form factor (y) = $0.154 - \dfrac{0.912}{\text{No. of teeth}}$

Determine the module, face width and pitch diameter of gears. Check the gears for wear; given σ_{es} *= 600 MPa;* E_P *= 200 kN/mm*^{2} *and* E_G *= 100 kN/mm*^{2}*. Sketch the gears.*

Solution : Given : $\phi = 20°$; $P = 12$ kW $= 12 \times 10^3$ W ; $N_P = 300$ r.p.m ; $V.R. = T_G / T_P = 3$; $\sigma_{OG} = 60$ MPa $= 60$ N/mm^2 ; $\sigma_{OP} = 105$ MPa $= 105$ N/mm^2 ; $T_P = 16$; $b = 14$ module $= 14\ m$; $\sigma_{es} = 600$ MPa $= 600$ N/mm^2 ; $E_P = 200$ kN/mm^2 $= 200 \times 10^3$ N/mm^2 ; $E_G = 100$ kN/mm^2 $= 100 \times 10^3$ N/mm^2

Module

Let m = Module in mm, and

D_P = Pitch circle diameter of the pinion in mm.

We know that pitch line velocity,

$$v = \frac{\pi D_P \cdot N_P}{60} = \frac{\pi m . T_P . N_P}{60} \qquad \ldots (\because D_P = m.T_P)$$

$$= \frac{\pi m \times 16 \times 300}{60} = 251\ m \text{ mm/s} = 0.251\ m \text{ m/s}$$

Assuming steady load conditions and 8–10 hours of service per day, the service factor (C_S) from Table 28.10 is given by $C_S = 1$.

We know that the design tangential tooth load,

$$W_T = \frac{P}{v} \times C_S = \frac{12 \times 10^3}{0.251\ m} \times 1 = \frac{47.8 \times 10^3}{m} \text{ N}$$

and velocity factor,

$$C_v = \frac{4.5}{4.5 + v} = \frac{4.5}{4.5 + 0.251\ m}$$

We know that tooth form factor for pinion,

$$y_P = 0.154 - \frac{0.912}{T_P} = 0.154 - \frac{0.912}{16} = 0.097$$

and tooth form factor for gear,

$$y_G = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{3 \times 16} = 0.135 \qquad \text{... } (\because T_G = 3\, T_P)$$

$$\therefore \qquad \sigma_{OP} \times y_P = 105 \times 0.097 = 10.185$$

and $$\sigma_{OG} \times y_G = 60 \times 0.135 = 8.1$$

Since ($\sigma_{OG} \times y_G$) is less than ($\sigma_{OP} \times y_P$), therefore the gear is weaker. Now using the Lewis equation to the gear, we have

$$W_T = \sigma_{wG} \,.\, b.\, \pi\, m.y_G = (\sigma_{OG} \times C_v)\, b.\pi\, m.y_G \qquad \text{... } (\because \sigma_{wG} = \sigma_{OG}.C_v)$$

$$\frac{47.8 \times 10^3}{m} = 60 \left(\frac{4.5}{4.5 + 0.251\, m} \right) 14\, m \times \pi m \times 0.135 = \frac{1603.4\, m^2}{4.5 + 0.251\, m}$$

or $$4.5 + 0.251\, m = 0.0335\, m^3$$

Solving this equation by hit and trial method, we find that

$$m = 5.6 \text{ say } 6 \text{ mm}$$ **Ans.**

Face width

We know that face width,

$$b = 14\, m = 14 \times 6 = 84 \text{ mm}$$ **Ans.**

Pitch diameter of gears

We know that pitch diameter of the pinion,

$$D_P = m.T_P = 6 \times 16 = 96 \text{ mm}$$ **Ans.**

and pitch diameter of the gear,

$$D_G = m.T_G = 6 \times 48 = 288 \text{ mm} \qquad \text{...} (\because T_G = 3\, T_P)$$ **Ans.**

This is a close-up photo (magnified 200 times) of a micromotor's gear cogs. Micromotors have been developed for use in space missions and microsurgery.

Checking the gears for wear

We know that the ratio factor,

$$Q = \frac{2 \times V.R.}{V.R. + 1} = \frac{2 \times 3}{3 + 1} = 1.5$$

and load stress factor,

$$K = \frac{(\sigma_{es})^2 \sin \phi}{1.4} \left(\frac{1}{E_P} + \frac{1}{E_G} \right)$$

$$= \frac{(600)^2 \sin 20°}{1.4} \left[\frac{1}{200 \times 10^3} + \frac{1}{100 \times 10^3} \right]$$

$$= 0.44 + 0.88 = 1.32 \text{ N/mm}^2$$

We know that the maximum or limiting load for wear,

$$W_w = D_P.b.Q.K = 96 \times 84 \times 1.5 \times 1.32 = 15\ 967 \text{ N}$$

and tangential load on the tooth (or beam strength of the tooth),

$$W_T = \frac{47.8 \times 10^3}{m} = \frac{47.8 \times 10^3}{6} = 7967 \text{ N}$$

Since the maximum wear load is much more than the tangential load on the tooth, therefore the design is satisfactory from the standpoint of wear. **Ans.**

Example 28.6. *A reciprocating compressor is to be connected to an electric motor with the help of spur gears. The distance between the shafts is to be 500 mm. The speed of the electric motor is 900 r.p.m. and the speed of the compressor shaft is desired to be 200 r.p.m. The torque, to be transmitted is 5000 N-m. Taking starting torque as 25% more than the normal torque, determine : 1. Module and face width of the gears using 20 degrees stub teeth, and 2. Number of teeth and pitch circle diameter of each gear. Assume suitable values of velocity factor and Lewis factor.*

Solution. Given : L = 500 mm ; N_M = 900 r.p.m. ; N_C = 200 r.p.m. ; T = 5000 N-m ; T_{max} = 1.25 T

1. Module and face width of the gears

Let m = Module in mm, and

b = Face width in mm.

Since the starting torque is 25% more than the normal torque, therefore the maximum torque,

$$T_{max} = 1.25\ T = 1.25 \times 5000 = 6250 \text{ N-m} = 6250 \times 10^3 \text{ N-mm}$$

We know that velocity ratio,

$$V.R. = \frac{N_M}{N_C} = \frac{900}{200} = 4.5$$

Let D_P = Pitch circle diameter of the pinion on the motor shaft, and

D_G = Pitch circle diameter of the gear on the compressor shaft.

We know that distance between the shafts (L),

$$500 = \frac{D_P}{2} + \frac{D_G}{2} \quad \text{or} \quad D_P + D_G = 500 \times 2 = 1000 \qquad ...(i)$$

and velocity ratio,

$$V.R. = \frac{D_G}{D_P} = 4.5 \quad \text{or} \quad D_G = 4.5\ D_P \qquad ...(ii)$$

Substituting the value of D_G in equation (*i*), we have

$$D_P + 4.5\ D_P = 1000 \quad \text{or} \quad D_P = 1000 / 5.5 = 182 \text{ mm}$$

and

$$D_G = 4.5\ D_P = 4.5 \times 182 = 820 \text{ mm} = 0.82 \text{ m}$$

We know that pitch line velocity of the drive,

$$v = \frac{\pi D_G . N_C}{60} = \frac{\pi \times 0.82 \times 200}{60} = 8.6 \text{ m/s}$$

∴ Velocity factor,

$$C_v = \frac{3}{3+v} = \frac{3}{3+8.6} = 0.26 \qquad \text{...(}\because v \text{ is less than 12.5 m/s)}$$

Let us assume than motor pinion is made of forged steel and the compressor gear of cast steel. Since the allowable static stress for the cast steel is less than the forged steel, therefore the design should be based upon the gear. Let us take the allowable static stress for the gear material as

$$\sigma_{OG} = 140 \text{ MPa} = 140 \text{ N/mm}^2$$

We know that for 20° stub teeth, Lewis factor for the gear,

$$y_G = 0.175 - \frac{0.841}{T_G} = 0.175 - \frac{0.841 \times m}{D_G} \qquad \text{...}\left(\because T_G = \frac{D_G}{m}\right)$$

$$= 0.175 - \frac{0.841\, m}{820} = 0.175 - 0.001\, m$$

and maximum tangential force on the gear,

$$W_T = \frac{2\, T_{max}}{D_G} = \frac{2 \times 6250 \times 10^3}{820} = 15\,244 \text{ N}$$

We also know that maximum tangential force on the gear,

$$W_T = \sigma_{wG}.b.\pi\, m.y_G = (\sigma_{OG} \times C_v)\, b \times \pi\, m \times y_G \qquad \text{...(}\because \sigma_{wG} = \sigma_{OG}.\, C_v\text{)}$$

$$15\,244 = (140 \times 0.26) \times 10\, m \times \pi\, m\, (0.175 - 0.001\, m)$$

$$= 200\, m^2 - 1.144\, m^3 \qquad \text{...(Assuming } b = 10\, m\text{)}$$

Solving this equation by hit and trial method, we find that

m = 8.95 say 10 mm **Ans.**

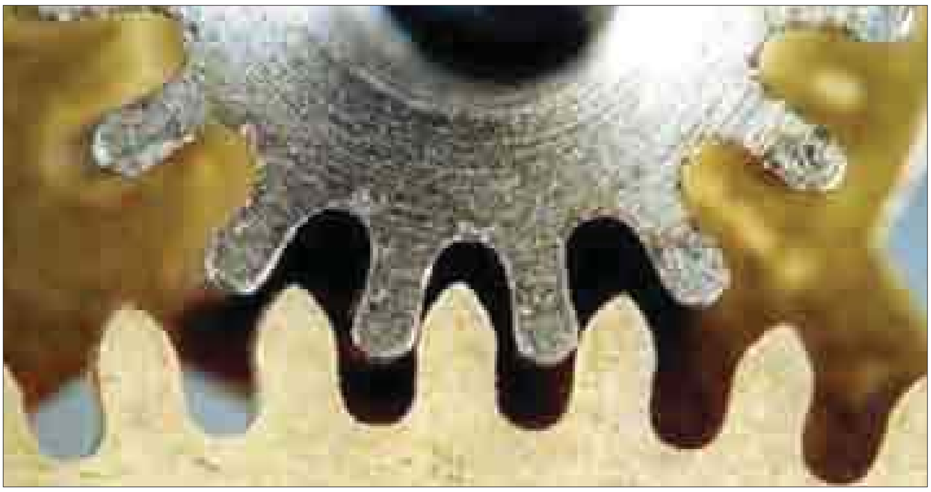
and b = 10 m = 10 × 10 = 100 mm **Ans.**

2. Number of teeth and pitch circle diameter of each gear

We know that number of teeth on the pinion,

$$T_P = \frac{D_P}{m} = \frac{182}{10} = 18.2$$

$$T_G = \frac{D_G}{m} = \frac{820}{10} = 82$$



In order to have the exact velocity ratio of 4.5, we shall take

$$T_P = 18 \text{ and } T_G = 81 \textbf{ Ans.}$$

∴ Pitch circle diameter of the pinion,

$$D_P = m \times T_P = 10 \times 18 = 180 \text{ mm } \textbf{Ans.}$$

and pitch circle diameter of the gear,

$$D_G = m \times T_G = 10 \times 81 = 810 \text{ mm } \textbf{Ans.}$$

28.24 Spur Gear Construction

The gear construction may have different designs depending upon the size and its application. When the dedendum circle diameter is slightly greater than the shaft diameter, then the pinion teeth are cut integral with the shaft as shown in Fig. 28.13 (*a*). If the pitch circle diameter of the pinion is less than or equal to 14.75 m + 60 mm (where m is the module in mm), then the pinion is made solid with uniform thickness equal to the face width, as shown in Fig. 28.13 (*b*). Small gears upto 250 mm pitch circle diameter are built with a web, which joins the hub and the rim. The web thickness is generally equal to half the circular pitch or it may be taken as 1.6 m to 1.9 m, where m is the module. The web may be made solid as shown in Fig. 28.13 (*c*) or may have recesses in order to reduce its weight.

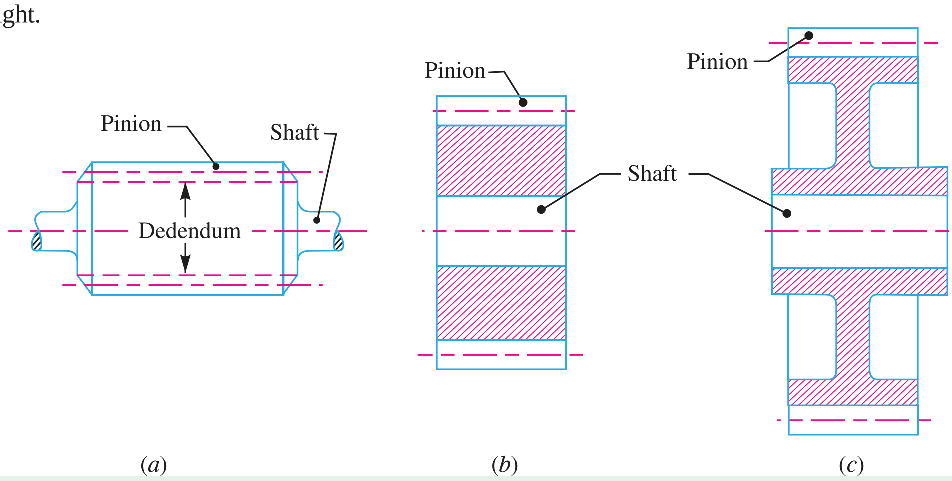


Fig. 28.13. Construction of spur gears.

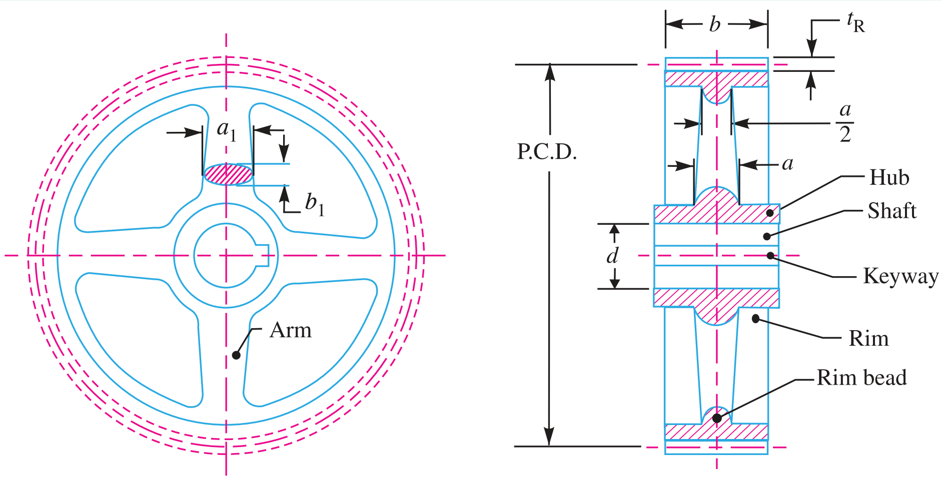


Fig. 28.14. Gear with arms.

Large gears are provided with arms to join the hub and the rim, as shown in Fig. 28.14. The number of arms depends upon the pitch circle diameter of the gear. The number of arms may be selected from the following table.

Table 28.11. Number of arms for the gears.

S. No.	*Pitch circle diameter*	*Number of arms*
1.	Up to 0.5 m	4 or 5
2.	0.5 – 1.5 m	6
3.	1.5 – 2.0 m	8
4.	Above 2.0 m	10

The cross-section of the arms is most often elliptical, but other sections as shown in Fig. 28.15 may also be used.

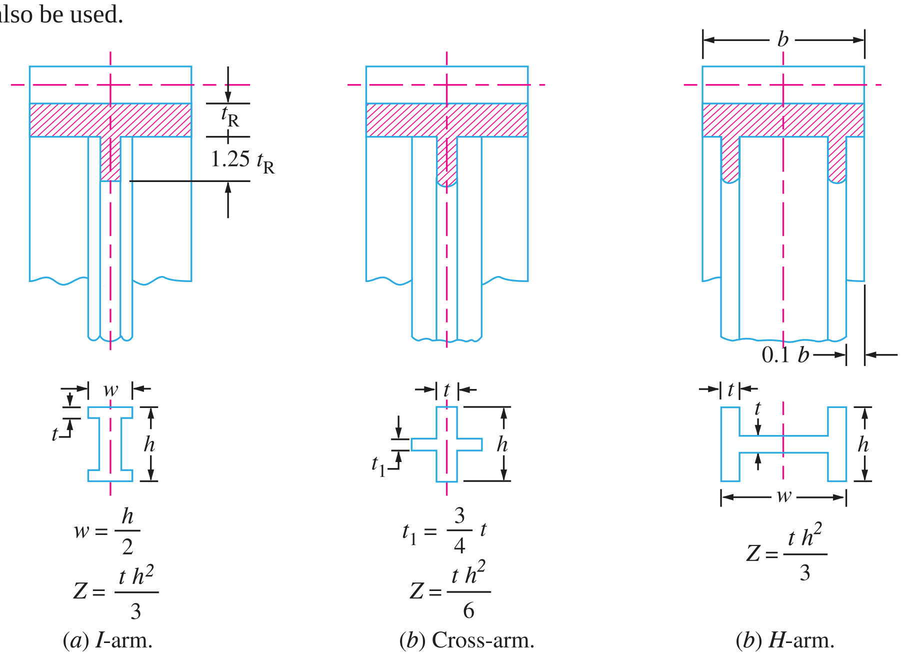


Fig. 28.15. Cross-section of the arms.

The hub diameter is kept as 1.8 times the shaft diameter for steel gears, twice the shaft diameter for cast iron gears and 1.65 times the shaft diameter for forged steel gears used for light service. The length of the hub is kept as 1.25 times the shaft diameter for light service and should not be less than the face width of the gear.

The thickness of the gear rim should be as small as possible, but to facilitate casting and to avoid sharp changes of section, the minimum thickness of the rim is generally kept as half of the circular pitch (or it may be taken as 1.6 *m* to 1.9 *m*, where *m* is the module). The thickness of rim (t_R) may also be calculated by using the following relation, *i.e.*

$$t_R = m\sqrt{\frac{T}{n}}$$

where T = Number of teeth, and

n = Number of arms.

The rim should be provided with a circumferential rib of thickness equal to the rim thickness.

28.25 Design of Shaft for Spur Gears

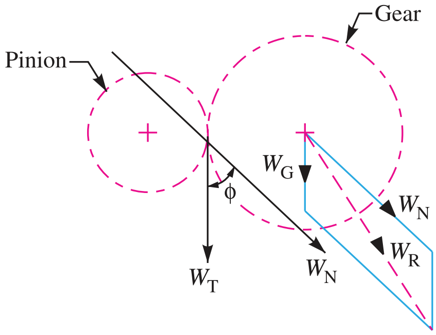


Fig. 28.16. Load acting on the gear.

In order to find the diameter of shaft for spur gears, the following procedure may be followed.

1. First of all, find the normal load (W_N), acting between the tooth surfaces. It is given by

$$W_N = W_T / \cos \phi$$

where W_T = Tangential load, and

ϕ = Pressure angle.

A thrust parallel and equal to W_N will act at the gear centre as shown in Fig. 28.16.

2. The weight of the gear is given by

$$W_G = 0.001\ 18\ T_G.b.m^2 \text{ (in N)}$$

where T_G = No. of teeth on the gear,

b = Face width in mm, and

m = Module in mm.

3. Now the resultant load acting on the gear,

$$W_R = \sqrt{(W_N)^2 + (W_G)^2 + 2\, W_N \times W_G \cos \phi}$$

4. If the gear is overhung on the shaft, then bending moment on the shaft due to the resultant load,

$$M = W_R \times x$$

where x = Overhang *i.e.* the distance between the centre of gear and the centre of bearing.

5. Since the shaft is under the combined effect of torsion and bending, therefore we shall determine the equivalent torque. We know that equivalent torque,

$$T_e = \sqrt{M^2 + T^2}$$

where T = Twisting moment = $W_T \times D_G / 2$

6. Now the diameter of the gear shaft (d) is determined by using the following relation, *i.e.*

$$T_e = \frac{\pi}{16} \times \tau \times d^3$$

where τ = Shear stress for the material of the gear shaft.

Note : Proceeding in the similar way as discussed above, we may calculate the diameter of the pinion shaft.

28.26 Design of Arms for Spur Gears

The cross-section of the arms is calculated by assuming them as a cantilever beam fixed at the hub and loaded at the pitch circle. It is also assumed that the load is equally distributed to all the arms. It may be noted that the arms are designed for the stalling load. The ***stalling load*** is a load that will develop the maximum stress in the arms and in the teeth. This happens at zero velocity, when the drive just starts operating.

The stalling load may be taken as the design tangential load divided by the velocity factor.

Let W_S = Stalling load = $\dfrac{\text{Design tangential load}}{\text{Velocity factor}} = \dfrac{W_T}{C_v}$,

D_G = Pitch circle diameter of the gear,

n = Number of arms, and

σ_b = Allowable bending stress for the material of the arms.

Now, maximum bending moment on each arm,

$$M = \frac{W_S \times D_G / 2}{n} = \frac{W_S \times D_G}{2n}$$

and the section modulus of arms for elliptical cross-section,

$$Z = \frac{\pi (a_1)^2 b_1}{32}$$

where a_1 = Major axis, and b_1 = Minor axis.

The major axis is usually taken as twice the minor axis. Now, using the relation, $\sigma_b = M / Z$, we can calculate the dimensions a_1 and b_1 for the gear arm at the hub end.

Note : The arms are usually tapered towards the rim about 1/16 per unit length of the arm (or radius of the gear).

∴ Major axis of the section at the rim end

$$= a_1 - \text{Taper} = a_1 - \frac{1}{16} \times \text{Length of the arm} = a_1 - \frac{1}{16} \times \frac{D_G}{2} = a_1 - \frac{D_G}{32}$$

Example 28.7. *A motor shaft rotating at 1500 r.p.m. has to transmit 15 kW to a low speed shaft with a speed reduction of 3:1. The teeth are $14\frac{1}{2}°$ involute with 25 teeth on the pinion. Both the pinion and gear are made of steel with a maximum safe stress of 200 MPa. A safe stress of 40 MPa may be taken for the shaft on which the gear is mounted and for the key.*

Design a spur gear drive to suit the above conditions. Also sketch the spur gear drive. Assume starting torque to be 25% higher than the running torque.

Solution : Given : N_P = 1500 r.p.m. ; P = 15 kW = 15 × 10^3 W ; $V.R.$ = T_G/T_P = 3 ; $\phi = 14\frac{1}{2}°$; T_P = 25 ; $\sigma_{OP} = \sigma_{OG}$ = 200 MPa = 200 N/mm^2 ; τ = 40 MPa = 40 N/mm^2

Design for spur gears

Since the starting torque is 25% higher than the running torque, therefore the spur gears should be designed for power,

$$P_1 = 1.25\,P = 1.25 \times 15 \times 10^3 = 18\ 750 \text{ W}$$

We know that the gear reduction ratio (T_G / T_P) is 3. Therefore the number of teeth on the gear,

$$T_G = 3\,T_P = 3 \times 25 = 75$$

Let us assume that the module (m) for the pinion and gear is 6 mm.

∴ Pitch circle diameter of the pinion,

$$D_P = m.T_P = 6 \times 25 = 150 \text{ mm} = 0.15 \text{ m}$$

and pitch circle diameter of the gear,

$$D_G = m.\ T_G = 6 \times 75 = 450 \text{ mm}$$

We know that pitch line velocity,

$$v = \frac{\pi D_P . N_P}{60} = \frac{\pi \times 0.15 \times 1500}{60} = 11.8 \text{ m/s}$$

Assuming steady load conditions and 8–10 hours of service per day, the service factor (C_S) from Table 28.10 is given by

$$C_S = 1$$

∴ Design tangential tooth load,

$$W_T = \frac{P_1}{v} \times C_S = \frac{18\ 750}{11.8} \times 1 = 1590 \text{ N}$$

We know that for ordinary cut gears and operating at velocities upto 12.5 m/s, the velocity factor,

$$C_v = \frac{3}{3 + v} = \frac{3}{3 + 11.8} = 0.203$$

Since both the pinion and the gear are made of the same material, therefore the pinion is the weaker.

We know that for $14\frac{1}{2}°$ involute teeth, tooth form factor for the pinion,

$$y_P = 0.124 - \frac{0.684}{T_P} = 0.124 - \frac{0.684}{25} = 0.0966$$

Let b =Face width for both the pinion and gear.

We know that the design tangential tooth load (W_T),

$$1590 = \sigma_{wP}.b.\pi\, m.y_P = (\sigma_{OP}.C_v)\, b.\pi\, m.y_P$$

$$= (200 \times 0.203)\, b \times \pi \times 6 \times 0.0966 = 74\, b$$

$\therefore \quad b = 1590 / 74 = 21.5$ mm

In actual practice, the face width (b) is taken as 9.5 m to 12.5 m, but in certain cases, due to space limitations, it may be taken as 6 m. Therefore let us take the face width,

$b = 6\,m = 6 \times 6 = 36$ mm **Ans.**

From Table 28.1, the other proportions, for the pinion and the gear having $14\frac{1}{2}°$ involute teeth, are as follows :

This mathematical machine called difference engine, assembled in 1832, used 2,000 levers, cams and gears.

Addendum	=	1 m = 6 mm **Ans.**
Dedendum	=	1.25 m = 1.25 × 6 = 7.5 mm **Ans.**
Working depth	=	2 m = 2 × 6 = 12 mm **Ans.**
Minimum total depth	=	2.25 m = 2.25 × 6 = 13.5 mm **Ans.**
Tooth thickness	=	1.5708 m = 1.5708 × 6 = 9.4248 mm **Ans.**
Minimum clearance	=	0.25 m = 0.25 × 6 = 1.5 mm **Ans.**

Design for the pinion shaft

We know that the normal load acting between the tooth surfaces,

$$W_N = \frac{W_T}{\cos\phi} = \frac{1590}{\cos 14^1/_2°} = \frac{1590}{0.9681} = 1643 \text{ N}$$

and weight of the pinion,

$$W_P = 0.00118\, T_P.b.m^2 = 0.001\,18 \times 25 \times 36 \times 6^2 = 38 \text{ N}$$

$\therefore$ Resultant load acting on the pinion,

$$*W_R = \sqrt{(W_N)^2 + (W_P)^2 + 2W_N.W_P.\cos\phi}$$

$$= \sqrt{(1643)^2 + (38)^2 + 2 \times 1643 \times 38 \times \cos 14^1/_2°} = 1680 \text{ N}$$

Assuming that the pinion is overhung on the shaft and taking overhang as 100 mm, therefore

Bending moment on the shaft due to the resultant load,

$$M = W_R \times 100 = 1680 \times 100 = 168\,000 \text{ N-mm}$$

* Since the weight of the pinion (W_P) is very small as compared to the normal load (W_N), therefore it may be neglected. Thus the resultant load acting on the pinion (W_R) may be taken equal to W_N.

and twisting moment on the shaft,

$$T = W_T \times \frac{D_P}{2} = 1590 \times \frac{150}{2} = 119\ 250 \text{ N-mm}$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(168\ 000)^2 + (119\ 250)^2} = 206 \times 10^3 \text{ N-mm}$$

Let d_P = Diameter of the pinion shaft.

We know that equivalent twisting moment (T_e),

$$206 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_P)^2 = \frac{\pi}{16} \times 40\, (d_P)^3 = 7.855\, (d_P)^3$$

∴ $(d_P)^3 = 206 \times 10^3 / 7.855 = 26.2 \times 10^3$ or $d_P = 29.7$ say 30 mm **Ans.**

We know that the diameter of the pinion hub

$= 1.8\, d_P = 1.8 \times 30 = 54$ mm **Ans.**

and length of the hub $= 1.25\, d_P = 1.25 \times 30 = 37.5$ mm

Since the length of the hub should not be less than that of the face width *i.e.* 36 mm, therefore let us take length of the hub as 36 mm. **Ans.**

Note : Since the pitch circle diameter of the pinion is 150 mm, therefore the pinion should be provided with a web and not arms. Let us take thickness of the web as 1.8 *m*, where *m* is the module.

∴ Thickness of the web $= 1.8\, m = 1.8 \times 6 = 10.8$ mm **Ans.**

Design for the gear shaft

We have calculated above that the normal load acting between the tooth surfaces,

$$W_N = 1643 \text{ N}$$

We know that weight of the gear,

$$W_G = 0.001\ 18\ T_G.b.m^2 = 0.001\ 18 \times 75 \times 36 \times 6^2 = 115 \text{ N}$$

∴ Resulting load acting on the gear,

$$W_R = \sqrt{(W_N)^2 + (W_G)^2 + 2W_N \times W_G \cos\phi}$$

$$= \sqrt{(1643)^2 + (115)^2 + 2 \times 1643 \times 115 \cos 14^1\!/\!_2{}^\circ} = 1755 \text{ N}$$

Assuming that the gear is overhung on the shaft and taking the overhang as 100 mm, therefore bending moment on the shaft due to the resultant load,

$$M = W_R \times 100 = 1755 \times 100 = 175\ 500 \text{ N-mm}$$

and twisting moment on the shaft,

$$T = W_T \times \frac{D_G}{2} = 1590 \times \frac{450}{2} = 357\ 750 \text{ N-mm}$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(175\ 500)^2 + (357\ 750)^2} = 398 \times 10^3 \text{ N-mm}$$

Let d_G = Diameter of the gear shaft.

We know that equivalent twisting moment (T_e),

$$398 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_G)^3 = \frac{\pi}{16} \times 40\, (d_G)^3 = 7.855\, (d_G)^3$$

∴ $(d_G)^3 = 398 \times 10^3 / 7.855 = 50.7 \times 10^3$ or $d_G = 37$ say 40 mm **Ans.**

We know that diameter of the gear hub

$$= 1.8\, d_G = 1.8 \times 40 = 72 \text{ mm } \textbf{Ans.}$$

and length of the hub $= 1.25\, d_G = 1.25 \times 40 = 50$ mm **Ans.**

Design for the gear arms

Since the pitch circle diameter of the gear is 450 mm, therefore the gear should be provided with four arms. Let us assume the cross-section of the arms as elliptical with major axis (a_1) equal to twice the minor axis (b_1).

∴ Section modulus of arms,

$$Z = \frac{\pi (a_1)^2 b_1}{32} = \frac{\pi (a_1)^2}{32} \times \frac{a_1}{2} = 0.05 (a_1)^3 \quad \text{... } (\because b_1 = a_1/2)$$

Since the arms are designed for the stalling load and stalling load is taken as the design tangential load divided by the velocity factor, therefore stalling load,

$$W_S = \frac{W_T}{C_v} = \frac{1590}{0.203} = 7830 \text{ N} \quad \text{... } (\because C_v = 0.203)$$

∴ Maximum bending moment on each arm,

$$M = \frac{W_S}{n} \times \frac{D_G}{2} = \frac{7830}{4} \times \frac{450}{2} = 440\ 440 \text{ N-mm}$$

We know that bending stress (σ_b),

$$42 = \frac{M}{Z} = \frac{440\ 440}{0.05 (a_1)^3} = \frac{9 \times 10^6}{(a_1)^3} \quad \text{... (Taking } \sigma_b = 42 \text{ N/mm}^2\text{)}$$

∴ $(a_1)^3 = 9 \times 10^6 / 42 = 0.214 \times 10^6$ or $a_1 = 60$ mm **Ans.**

and $b_1 = a_1 / 2 = 60 / 2 = 30$ mm **Ans.**

These dimensions refer to the hub end. Since the arms are tapered towards the rim and the taper is 1 / 16 per unit length of the arm (or radius of the gear), therefore

Major axis of the arm at the rim end,

$$a_2 = a_1 - \text{Taper} = a_1 - \frac{1}{16} \times \frac{D_G}{2}$$

$$= 60 - \frac{1}{16} \times \frac{450}{2} = 46 \text{ mm } \textbf{Ans.}$$

and minor axis of the arm at the rim end,

$$b_2 = \frac{\text{Major axis}}{2} = \frac{46}{2} = 23 \text{ mm } \textbf{Ans.}$$

Design for the rim

The thickness of the rim for the pinion (t_{RP}) may be taken as 1.6 m to 1.9 m, where m is the module. Let us take thickness of the rim for the pinion,

$$t_{RP} = 1.6\, m = 1.6 \times 6 = 9.6 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

The thickness of the rim for the gear (t_{RG}) may be obtained by using the relation,

$$t_{RG} = m \sqrt{\frac{T_G}{n}} = 6 \sqrt{\frac{45}{4}} = 20 \text{ mm } \textbf{Ans.}$$

EXERCISES

1. Calculate the power that can be transmitted safely by a pair of spur gears with the data given below. Calculate also the bending stresses induced in the two wheels when the pair transmits this power.

   Number of teeth in the pinion = 20

   Number of teeth in the gear = 80

Module	= 4 mm
Width of teeth	= 60 mm
Tooth profile	= 20° involute
Allowable bending strength of the material	
	= 200 MPa, for pinion
	= 160 MPa, for gear
Speed of the pinion	= 400 r.p.m.
Service factor	= 0.8
Lewis form factor	$= 0.154 - \frac{0.912}{T}$
Velocity factor	$= \frac{3}{3 + v}$

[Ans. 13.978 kW ; 102.4 MPa ; 77.34 MPa]

2. A spur gear made of bronze drives a mid steel pinion with angular velocity ratio of $3^1/_2$: 1. The pressure angle is $14\frac{1}{2}^{\circ}$. It transmits 5 kW at 1800 r.p.m. of pinion. Considering only strength, design the smallest diameter gears and find also necessary face width. The number of teeth should not be less than 15 teeth on either gear. The elastic strength of bronze may be taken as 84 MPa and of steel as 105 MPa. Lewis factor for $14\frac{1}{2}^{\circ}$ pressure angle may be taken as

$$y = 0.124 - \frac{0.684}{\text{No. of teeth}}$$

[Ans. m = 3 mm ; b = 35 mm ; D_P = 48 mm ; D_G = 168 mm]

3. A pair of 20° full-depth involute tooth spur gears is to transmit 30 kW at a speed of 250 r.p.m. of the pinion. The velocity ratio is 1 : 4. The pinion is made of cast steel having an allowable static stress, σ_o = 100 MPa, while the gear is made of cast iron having allowable static stress, σ_o = 55 MPa.

The pinion has 20 teeth and its face width is 12.5 times the module. Determine the module, face width and pitch diameters of both the pinion and gear from the standpoint of strength only taking velocity factor into consideration. The tooth form factor is given by the expression

$$y = 0.154 - \frac{0.912}{\text{No. of teeth}}$$

and velocity factor is given by

$$C_v = \frac{3}{3 + v},$$ where v is the peripheral speed of the gear in m/s.

[Ans. m = 20 mm ; b = 250 mm ; D_P = 400 mm ; D_G = 1600 mm]

4. A micarta pinion rotating at 1200 r.p.m. is to transmit 1 kW to a cast iron gear at a speed of 192 r.p.m. Assuming a starting overload of 20% and using 20° full depth involute teeth, determine the module, number of teeth on the pinion and gear and face width. Take allowable static strength for micarta as 40 MPa and for cast iron as 53 MPa. Check the pair in wear.

5. A 15 kW and 1200 r.p.m. motor drives a compressor at 300 r.p.m. through a pair of spur gears having 20° stub teeth. The centre to centre distance between the shafts is 400 mm. The motor pinion is made of forged steel having an allowable static stress as 210 MPa, while the gear is made of cast steel having allowable static stress as 140 MPa. Assuming that the drive operates 8 to 10 hours per day under light shock conditions, find from the standpoint of strength,

1. Module; 2. Face width and 3. Number of teeth and pitch circle diameter of each gear.

Check the gears thus designed from the consideration of wear. The surface endurance limit may be taken as 700 MPa.**[Ans. m = 6 mm ; b = 60 mm ; T_P = 24 ; T_G = 96 ; D_P = 144mm ; D_G = 576 mm]**

6. A two stage reduction drive is to be designed to transmit 2 kW; the input speed being 960 r.p.m. and overall reduction ratio being 9. The drive consists of straight tooth spur gears only, the shafts being spaced 200 mm apart, the input and output shafts being co-axial.

(*a*) Draw a layout of a suitable system to meet the above specifications, indicating the speeds of all rotating components.

(*b*) Calculate the module, pitch diameter, number of teeth, blank diameter and face width of the gears for medium heavy duty conditions, the gears being of medium grades of accuracy.

(*c*) Draw to scale one of the gears and specify on the drawing the calculated dimensions and other data complete in every respect for manufacturing purposes.

7. A motor shaft rotating at 1440 r.p.m. has to transmit 15 kW to a low speed shaft rotating at 500 r.p.m. The teeth are 20° involute with 25 teeth on the pinion. Both the pinion and gear are made of cast iron with a maximum safe stress of 56 MPa. A safe stress of 35 MPa may be taken for the shaft on which the gear is mounted. Design and sketch the spur gear drive to suit the above conditions. The starting torque may be assumed as 1.25 times the running torque.

8. Design and draw a spur gear drive transmitting 30 kW at 400 r.p.m. to another shaft running approximately at 100 r.p.m. The load is steady and continuous. The materials for the pinion and gear are cast steel and cast iron respectively. Take module as 10 mm. Also check the design for dynamic load and wear.

[**Hint :** Assume : σ_{OP} = 140 MPa ; σ_{OG} = 56 MPa ; T_P = 24 ; $y = 0.154 - \dfrac{0.912}{\text{No. of teeth}}$;

$C_v = \dfrac{3}{3 + v}$; σ_e = 84 MPa ; e = 0.023 mm ; σ_{es} = 630 MPa ; E_P = 210 kN/mm^2 ; E_G = 100 kN/mm^2]

9. Design a spur gear drive required to transmit 45 kW at a pinion speed of 800 r.p.m. The velocity ratio is 3.5 : 1. The teeth are 20° full-depth involute with 18 teeth on the pinion. Both the pinion and gear are made of steel with a maximum safe static stress of 180 MPa. Assume a safe stress of 40 MPa for the material of the shaft and key.

10. Design a pair of spur gears with stub teeth to transmit 55 kW from a 175 mm pinion running at 2500 r.p.m. to a gear running at 1500 r.p.m. Both the gears are made of steel having B.H.N. 260. Approximate the pitch by means of Lewis equation and then adjust the dimensions to keep within the limits set by the dynamic load and wear equation.

QUESTIONS

1. Write a short note on gear drives giving their merits and demerits.

2. How are the gears classified and what are the various terms used in spur gear terminology ?

3. Mention four important types of gears and discuss their applications, the materials used for them and their construction.

4. What condition must be satisfied in order that a pair of spur gears may have a constant velcoity ratio?

5. State the two most important reasons for adopting involute curves for a gear tooth profile.

6. Explain the phenomenon of interference in involute gears. What are the conditions to be satisfied in order to avoid interference ?

7. Explain the different causes of gear tooth failures and suggest possible remedies to avoid such failures.

8. Write the expressions for static, limiting wear load and dynamic load for spur gears and explain the various terms used there in.

9. Discuss the design procedure of spur gears.

10. How the shaft and arms for spur gears are designed ?

OBJECTIVE TYPE QUESTIONS

1. The gears are termed as medium velocity gears, if their peripheral velocity is

(*a*) 1–3 m / s (*b*) 3–15 m / s

(*c*) 15–30 m / s (*d*) 30–50 m / s

2. The size of gear is usually specified by
   (*a*) pressure angle (*b*) pitch circle diameter
   (*c*) circular pitch (*d*) diametral pitch
3. A spur gear with pitch circle diameter *D* has number of teeth *T*. The module *m* is defined as
   (*a*) $m = d / T$ (*b*) $m = T / D$
   (*c*) $m = \pi D / T$ (*d*) $m = D.T$
4. In a rack and pinion arrangement, the rack has teeth of .... shape.
   (*a*) square (*b*) trepazoidal
5. The radial distance from the .... to the clearance circle is called working depth.
   (*a*) addendum circle (*b*) dedendum circle
6. The product of the diametral pitch and circular pitch is equal to
   (*a*) 1 (*b*) $1/\pi$
   (*c*) π (*d*) $\pi \times$ No. of teeth
7. The backlash for spur gears depends upon
   (*a*) module (*b*) pitch line velocity
   (*c*) tooth profile (*d*) both (*a*) and (*b*)
8. The contact ratio for gears is
   (*a*) zero (*b*) less than one
   (*c*) greater than one (*d*) none of these
9. If the centre distance of the mating gears having involute teeth is increased, then the pressure angle
   (*a*) increases (*b*) decreases
   (*c*) remains unchanged (*d*) none of these
10. The form factor of a spur gear tooth depends upon
   (*a*) circular pitch only (*b*) pressure angle only
   (*c*) number of teeth and circular pitch (*d*) number of teeth and the system of teeth
11. Lewis equation in spur gears is used to find the
   (*a*) tensile stress in bending (*b*) shear stress
   (*c*) compressive stress in bending (*d*) fatigue stress
12. The minimum number of teeth on the pinion in order to avoid interference for 20° stub system is
   (*a*) 12 (*b*) 14
   (*c*) 18 (*d*) 32
13. The allowable static stress for steel gears is approximately .... of the ultimate tensile stress.
   (*a*) one-fourth (*b*) one-third
   (*c*) one-half (*d*) double
14. Lewis equation in spur gears is applied
   (*a*) only to the pinion (*b*) only to the gear
   (*c*) to stronger of the pinion or gear (*d*) to weaker of the pinion or gear
15. The static tooth load should be .... the dynamic load.
   (*a*) less than (*b*) greater than
   (*c*) equal to

ANSWERS

1. (*b*)	**2.** (*b*)	**3.** (*a*)	**4.** (*b*)	**5.** (*a*)
6. (*c*)	**7.** (*d*)	**8.** (*c*)	**9.** (*a*)	**10.** (*d*)
11. (*c*)	**12.** (*b*)	**13.** (*b*)	**14.** (*d*)	**15.** (*b*)

C H A P T E R

29

Helical Gears

29.1 Introduction

A helical gear has teeth in form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears. The helixes may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission.

We have already discussed in Art. 28.4 that the helical gears may be of ***single helical type*** or ***double helical type.*** In case of single helical gears there is some axial thrust between the teeth, which is a disadvantage. In order to eliminate this axial thrust, double helical gears (*i.e.*

herringbone gears) are used. It is equivalent to two single helical gears, in which equal and opposite thrusts are provided on each gear and the resulting axial thrust is zero.

29.2 Terms used in Helical Gears

The following terms in connection with helical gears, as shown in Fig. 29.1, are important from the subject point of view.

1. *Helix angle.* It is a constant angle made by the helices with the axis of rotation.

2. *Axial pitch*. It is the distance, parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by p_c. The axial pitch may also be defined as the circular pitch in the plane of rotation or the diametral plane.

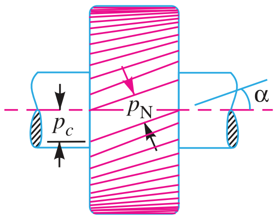


Fig. 29.1. Helical gear (nomenclature).

3. *Normal pitch.* It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by p_N. The normal pitch may also be defined as the circular pitch in the normal plane which is a plane perpendicular to the teeth. Mathematically, normal pitch,

$$p_N = p_c \cos \alpha$$

Note : If the gears are cut by standard hobs, then the pitch (or module) and the pressure angle of the hob will apply in the normal plane. On the other hand, if the gears are cut by the Fellows gear-shaper method, the pitch and pressure angle of the cutter will apply to the plane of rotation. The relation between the normal pressure angle (ϕ_N) in the normal plane and the pressure angle (ϕ) in the diametral plane (or plane of rotation) is given by

$$\tan \phi_N = \tan \phi \times \cos \alpha$$

29.3 Face Width of Helical Gears

In order to have more than one pair of teeth in contact, the tooth displacement (*i.e.* the advancement of one end of tooth over the other end) or overlap should be atleast equal to the axial pitch, such that

$$\text{Overlap} = p_c = b \tan \alpha \quad \text{...(i)}$$

The normal tooth load (W_N) has two components ; one is tangential component (W_T) and the other axial component (W_A), as shown in Fig. 29.2. The axial or end thrust is given by

$$W_A = W_N \sin \alpha = W_T \tan \alpha \quad \text{...(ii)}$$

From equation ***(i)***, we see that as the helix angle increases, then the tooth overlap increases. But at the same time, the end thrust as given by equation ***(ii)***, also increases, which is undesirable. It is usually recommended that the overlap should be 15 percent of the circular pitch.

$$\therefore \quad \text{Overlap} = b \tan \alpha = 1.15\, p_c$$

or
$$b = \frac{1.15\, p_c}{\tan \alpha} = \frac{1.15 \times \pi m}{\tan \alpha} \quad \text{... } (\because p_c = \pi m)$$

where
b = Minimum face width, and
m = Module.

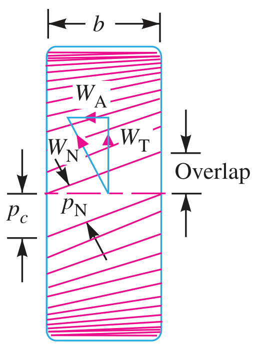


Fig. 29.2. Face width of helical gear.

Notes : 1. The maximum face width may be taken as 12.5 *m* to 20 *m*, where *m* is the module. In terms of pinion diameter (D_P), the face width should be 1.5 D_P to 2 D_P, although 2.5 D_P may be used.

2. In case of double helical or herringbone gears, the minimum face width is given by

$$b = \frac{2.3\, p_c}{\tan \alpha} = \frac{2.3 \times \pi m}{\tan \alpha}$$

The maximum face width ranges from 20 *m* to 30 *m*.

3. In single helical gears, the helix angle ranges from 20° to 35°, while for double helical gears, it may be made upto 45°.

29.4 Formative or Equivalent Number of Teeth for Helical Gears

The formative or equivalent number of teeth for a helical gear may be defined as the number of teeth that can be generated on the surface of a cylinder having a radius equal to the radius of curvature at a point at the tip of the minor axis of an ellipse obtained by taking a section of the gear in the normal plane. Mathematically, formative or equivalent number of teeth on a helical gear,

$$T_E = T / \cos^3 \alpha$$

where T = Actual number of teeth on a helical gear, and

α = Helix angle.

29.5 Proportions for Helical Gears

Though the proportions for helical gears are not standardised, yet the following are recommended by American Gear Manufacturer's Association (AGMA).

Pressure angle in the plane of rotation,

ϕ = 15° to 25°

Helix angle, α = 20° to 45°

Addendum = 0.8 m (Maximum)

Dedendum = 1 m (Minimum)

Minimum total depth = 1.8 m

Minimum clearance = 0.2 m

Thickness of tooth = 1.5708 m

In helical gears, the teeth are inclined to the axis of the gear.

29.6 Strength of Helical Gears

In helical gears, the contact between mating teeth is gradual, starting at one end and moving along the teeth so that at any instant the line of contact runs diagonally across the teeth. Therefore in order to find the strength of helical gears, a modified Lewis equation is used. It is given by

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y'$$

where W_T = Tangential tooth load,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y' = Tooth form factor or Lewis factor corresponding to the formative or virtual or equivalent number of teeth.

Notes : 1. The value of velocity factor (C_v) may be taken as follows :

$$C_v = \frac{6}{6+v}, \text{ for peripheral velocities from 5 m / s to 10 m / s.}$$

$$= \frac{15}{15+v}, \text{ for peripheral velocities from 10 m / s to 20 m / s.}$$

$$= \frac{0.75}{0.75+\sqrt{v}}, \text{ for peripheral velocities greater than 20 m / s.}$$

$$= \frac{0.75}{1+v} + 0.25, \text{ for non-metallic gears.}$$

2. The dynamic tooth load on the helical gears is given by

$$W_D = W_T + \frac{21\, v\, (b.C \cos^2 \alpha + W_T) \cos \alpha}{21\, v + \sqrt{b.C \cos^2 \alpha + W_T}}$$

where v, b and C have usual meanings as discussed in spur gears.

3. The static tooth load or endurance strength of the tooth is given by

$$W_S = \sigma_e.b.\pi\, m.y'$$

4. The maximum or limiting wear tooth load for helical gears is given by

$$W_w = \frac{D_P.b.Q.K}{\cos^2 \alpha}$$

where D_P, b, Q and K have usual meanings as discussed in spur gears.

In this case, $$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

where ϕ_N = Normal pressure angle.

Example 29.1. *A pair of helical gears are to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10 000 r.p.m. and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 MPa; determine a suitable module and face width from static strength considerations and check the gears for wear, given σ_{es} = 618 MPa.*

Solution. Given : P = 15 kW = 15 × 10^3 W; ϕ = 20° ; α = 45° ; N_P = 10 000 r.p.m. ; D_P = 80 mm = 0.08 m ; D_G = 320 mm = 0.32 m ; $\sigma_{OP} = \sigma_{OG}$ = 100 MPa = 100 N/mm^2 ; σ_{es} = 618 MPa = 618 N/mm^2

Module and face width

Let m = Module in mm, and

b = Face width in mm.

Since both the pinion and gear are made of the same material (*i.e.* cast steel), therefore the pinion is weaker. Thus the design will be based upon the pinion.

We know that the torque transmitted by the pinion,

$$T = \frac{P \times 60}{2\,\pi\,N_P} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 10\,000} = 14.32 \text{ N-m}$$

$\therefore$ *Tangential tooth load on the pinion,

$$W_T = \frac{T}{D_P / 2} = \frac{14.32}{0.08/2} = 358 \text{ N}$$

We know that number of teeth on the pinion,

$$T_P = D_P / m = 80 / m$$

and formative or equivalent number of teeth for the pinion,

$$T_E = \frac{T_P}{\cos^3 \alpha} = \frac{80/m}{\cos^3 45°} = \frac{80/m}{(0.707)^3} = \frac{226.4}{m}$$

$\therefore$ Tooth form factor for the pinion for 20° stub teeth,

$$y'_P = 0.175 - \frac{0.841}{T_E} = 0.175 - \frac{0.841}{226.4/m} = 0.175 - 0.0037\,m$$

We know that peripheral velocity,

$$v = \frac{\pi\,D_P\,.N_P}{60} = \frac{\pi \times 0.08 \times 10\,000}{60} = 42 \text{ m/s}$$

$\therefore$ Velocity factor,

$$C_v = \frac{0.75}{0.75 + \sqrt{v}} = \frac{0.75}{0.75 + \sqrt{42}} = 0.104 \qquad \text{...}(\because v \text{ is greater than 20 m/s})$$

Since the maximum face width (b) for helical gears may be taken as 12.5 m to 20 m, where m is the module, therefore let us take

$$b = 12.5\,m$$

We know that the tangential tooth load (W_T),

$$358 = (\sigma_{OP}\,.\,C_v)\,b.\pi\,m.y'_P$$

$$= (100 \times 0.104)\,12.5\,m \times \pi\,m\,(0.175 - 0.0037\,m)$$

$$= 409\,m^2\,(0.175 - 0.0037\,m) = 72\,m^2 - 1.5\,m^3$$

Solving this expression by hit and trial method, we find that

$$m = 2.3 \text{ say } 2.5 \text{ mm} \quad \textbf{Ans.}$$

and face width, $\quad b = 12.5\,m = 12.5 \times 2.5 = 31.25$ say 32 mm **Ans.**

Checking the gears for wear

We know that velocity ratio,

$$V.R. = \frac{D_G}{D_P} = \frac{320}{80} = 4$$

$\therefore$ Ratio factor,

$$Q = \frac{2 \times V.R.}{V.R. + 1} = \frac{2 \times 4}{4 + 1} = 1.6$$

We know that $\quad \tan \phi_N = \tan\phi \cos\alpha = \tan 20° \times \cos 45° = 0.2573$

$\therefore \quad \phi_N = 14.4°$

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* The tangential tooth load on the pinion may also be obtained by using the relation,

$$W_T = \frac{P}{v}, \text{ where } v = \frac{\pi\,D_P\,.N_P}{60} \text{ (in m/s)}$$

The picture shows double helical gears which are also called herringbone gears.

Since both the gears are made of the same material (*i.e.* cast steel), therefore let us take

$$E_P = E_G = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$$

∴ Load stress factor,

$$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

$$= \frac{(618)^2 \sin 14.4°}{1.4}\left(\frac{1}{200 \times 10^3} + \frac{1}{200 \times 10^3}\right) = 0.678 \text{ N/mm}^2$$

We know that the maximum or limiting load for wear,

$$W_w = \frac{D_P . b . Q . K}{\cos^2 \alpha} = \frac{80 \times 32 \times 1.6 \times 0.678}{\cos^2 45°} = 5554 \text{ N}$$

Since the maximum load for wear is much more than the tangential load on the tooth, therefore the design is satisfactory from consideration of wear.

Example 29.2. *A helical cast steel gear with 30° helix angle has to transmit 35 kW at 1500 r.p.m. If the gear has 24 teeth, determine the necessary module, pitch diameter and face width for 20° full depth teeth. The static stress for cast steel may be taken as 56 MPa. The width of face may be taken as 3 times the normal pitch. What would be the end thrust on the gear? The tooth factor for 20° full depth involute gear may be taken as* $0.154 - \frac{0.912}{T_E}$, *where* T_E *represents the equivalent number of teeth.*

Solution. Given : $\alpha = 30°$; $P = 35$ kW $= 35 \times 10^3$ W ; $N = 1500$ r.p.m. ; $T_G = 24$; $\phi = 20°$; $\sigma_o = 56$ MPa $= 56$ N/mm^2 ; $b = 3 \times$ Normal pitch $= 3\, p_N$

Module

Let m = Module in mm, and

D_G = Pitch circle diameter of the gear in mm.

We know that torque transmitted by the gear,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{35 \times 10^3 \times 60}{2\,\pi \times 1500} = 223 \text{ N-m} = 223 \times 10^3 \text{ N-mm}$$

Formative or equivalent number of teeth,

$$T'_E = \frac{T_G}{\cos^3 \alpha} = \frac{24}{\cos^3 30°} = \frac{24}{(0.866)^3} = 37$$

∴ Tooth factor, $y' = 0.154 - \frac{0.912}{T_E} = 0.154 - \frac{0.912}{37} = 0.129$

We know that the tangential tooth load,

$$W_T = \frac{T}{D_G / 2} = \frac{2T}{D_G} = \frac{2T}{m \times T_G} \quad \text{... } (\because D_G = m.T_G)$$

$$= \frac{2 \times 223 \times 10^3}{m \times 24} = \frac{18\ 600}{m} \text{ N}$$

and peripheral velocity,

$$v = \frac{\pi D_G . N}{60} = \frac{\pi . m. T_G . N}{60} \text{ mm / s} \quad \text{...}(D_G \text{ and } m \text{ are in mm})$$

$$= \frac{\pi \times m \times 24 \times 1500}{60} = 1885\ m \text{ mm / s} = 1.885\ m \text{ m / s}$$

Let us take velocity factor,

$$C_v = \frac{15}{15 + v} = \frac{15}{15 + 1.885\ m}$$

We know that tangential tooth load,

$$W_T = (\sigma_o \times C_v)\ b.\pi\ m.y' = (\sigma_o \times C_v)\ 3p_N \times \pi\ m \times y' \quad \text{...}(\because b = 3\ p_N)$$

$$= (\sigma_o \times C_v)\ 3 \times p_c \cos \alpha \times \pi\ m \times y' \quad \text{...}(\because p_N = p_c \cos \alpha)$$

$$= (\sigma_o \times C_v)\ 3\ \pi\ m \cos \alpha \times \pi\ m \times y' \quad \text{...}(\because p_c = \pi\ m)$$

$$\therefore \quad \frac{18\ 600}{m} = 56 \left(\frac{15}{15 + 1.885\ m}\right) 3\ \pi\ m \times \cos 30° \times \pi\ m \times 0.129$$

$$= \frac{2780\ m^2}{15 + 1.885\ m}$$

or $279\ 000 + 35\ 061\ m = 2780\ m^3$

Solving this equation by hit and trial method, we find that

$m = 5.5$ say 6 mm **Ans.**

Pitch diameter of the gear

We know that the pitch diameter of the gear,

$D_G = m \times T_G = 6 \times 24 = 144$ mm **Ans.**

Face width

It is given that the face width,

$$b = 3\ p_N = 3\ p_c \cos \alpha = 3 \times \pi\ m \cos \alpha$$

$= 3 \times \pi \times 6 \cos 30° = 48.98$ say 50 mm **Ans.**

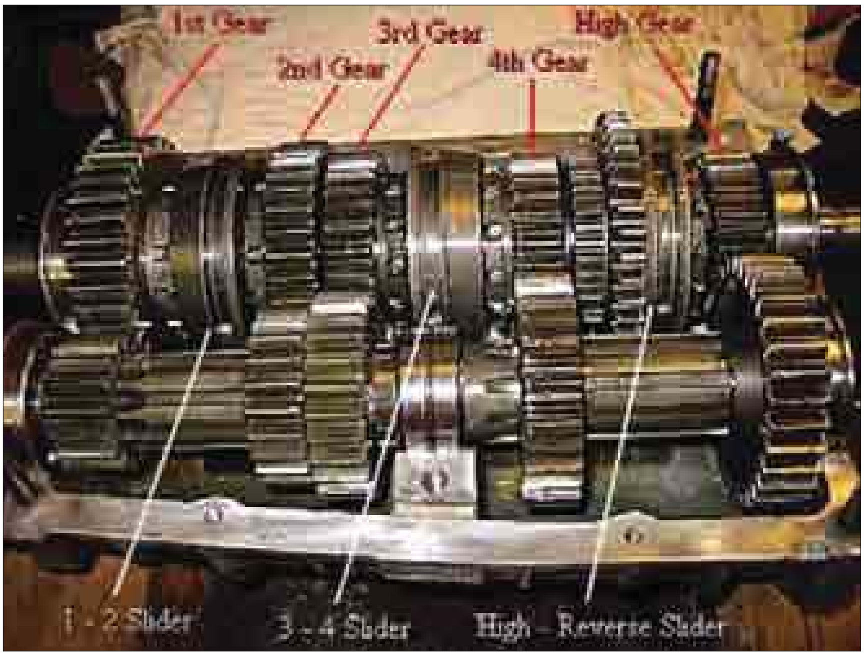
End thrust on the gear

We know that end thrust or axial load on the gear,

$$W_A = W_T \tan \alpha = \frac{18\ 600}{m} \times \tan 30° = \frac{18\ 600}{6} \times 0.577 = 1790 \text{ N}$$ **Ans.**

Example 29.3. *Design a pair of helical gears for transmitting 22 kW. The speed of the driver gear is 1800 r.p.m. and that of driven gear is 600 r.p.m. The helix angle is 30° and profile is corresponding to 20° full depth system. The driver gear has 24 teeth. Both the gears are made of cast steel with allowable static stress as 50 MPa. Assume the face width parallel to axis as 4 times the circular pitch and the overhang for each gear as 150 mm. The allowable shear stress for the shaft material may be taken as 50 MPa. The form factor may be taken as 0.154 – 0.912 /* T_E*, where* T_E *is the equivalent number of teeth. The velocity factor may be taken as* $\frac{350}{350+v}$*, where v is pitch line velocity in m / min. The gears are required to be designed only against bending failure of the teeth under dynamic condition.*

Solution. Given : P = 22 kW = 22 × 10^3 W ; N_P = 1800 r.p.m.; N_G = 600 r.p.m. ; α = 30° ; φ = 20° ; T_P = 24 ; σ_o = 50 MPa = 50 N/mm² ; $b = 4\,p_c$; Overhang = 150 mm ; τ = 50 MPa = 50 N/mm²



Gears inside a car

Design for the pinion and gear

We know that the torque transmitted by the pinion,

$$T = \frac{P \times 60}{2\,\pi\,N_P} = \frac{22 \times 10^3 \times 60}{2\,\pi \times 1800} = 116.7 \text{ N-m} = 116\,700 \text{ N-mm}$$

Since both the pinion and gear are made of the same material (*i.e.* cast steel), therefore the pinion is weaker. Thus the design will be based upon the pinion. We know that formative or equivalent number of teeth,

$$T_E = \frac{T_P}{\cos^3 \alpha} = \frac{24}{\cos^3 30°} = \frac{24}{(0.866)^3} = 37$$

∴ Form factor,

$$y' = 0.154 - \frac{0.912}{T_E} = 0.154 - \frac{0.912}{37} = 0.129$$

First of all let us find the module of teeth.

Let m = Module in mm, and

D_P = Pitch circle diameter of the pinion in mm.

We know that the tangential tooth load on the pinion,

$$W_T = \frac{T}{D_P/2} = \frac{2T}{D_P} = \frac{2T}{m \times T_P} \quad ...(\because D_P = m.T_P)$$

$$= \frac{2 \times 116\ 700}{m \times 24} = \frac{9725}{m} \text{ N}$$

and peripheral velocity, $v = \pi D_P.N_P = \pi m.T_P.N_P$

$= \pi m \times 24 \times 1800 = 135\ 735\ m$ mm / min = 135.735 m m / min

$$\therefore \text{ Velocity factor, } C_v = \frac{350}{350 + v} = \frac{350}{350 + 135.735\ m}$$

We also know that the tangential tooth load on the pinion,

$$W_T = (\sigma_o.C_v)\ b.\pi\ m.y' = (\sigma_o.C_v)\ 4\ p_c \times \pi\ m \times y' \quad ...(\because b = 4\ p_c)$$

$$= (\sigma_o.C_v)\ 4 \times \pi\ m \times \pi\ m \times y' \quad ...(\because p_c = \pi\ m)$$

$$\therefore \quad \frac{9725}{m} = 50\left(\frac{350}{350 + 135.735\,m}\right) 4 \times \pi^2\ m^2 \times 0.129 = \frac{89\ 126\ m^2}{350 + 135.735m}$$

$3.4 \times 10^6 + 1.32 \times 10^6\ m = 89\ 126\ m^3$

Solving this expression by hit and trial method, we find that

m = 4.75 mm say 6 mm **Ans.**

Helical gears.

We know that face width,

$b = 4\ p_c = 4\ \pi\ m = 4\ \pi \times 6 = 75.4$ say 76 mm **Ans.**

and pitch circle diameter of the pinion,

$D_P = m \times T_P = 6 \times 24 = 144$ mm **Ans.**

Since the velocity ratio is 1800 / 600 = 3, therefore number of teeth on the gear,

$T_G = 3\ T_P = 3 \times 24 = 72$

and pitch circle diameter of the gear,

$D_G = m \times T_G = 6 \times 72 = 432$ mm **Ans.**

Design for the pinion shaft

Let d_P = Diameter of the pinion shaft.

We know that the tangential load on the pinion,

$$W_T = \frac{9725}{m} = \frac{9725}{6} = 1621 \text{ N}$$

and the axial load of the pinion,

$$W_A = W_T \tan \alpha = 1621 \tan 30°$$
$$= 1621 \times 0.577 = 935 \text{ N}$$

Since the overhang for each gear is 150 mm, therefore bending moment on the pinion shaft due to the tangential load,

$$M_1 = W_T \times \text{Overhang} = 1621 \times 150 = 243\,150 \text{ N-mm}$$

and bending moment on the pinion shaft due to the axial load,

$$M_2 = W_A \times \frac{D_P}{2} = 935 \times \frac{144}{2} = 67\,320 \text{ N-mm}$$

Since the bending moment due to the tangential load (*i.e.* M_1) and bending moment due to the axial load (*i.e.* M_2) are at right angles, therefore resultant bending moment on the pinion shaft,

$$M = \sqrt{(M_1)^2 + (M_2)^2} = \sqrt{(243\,150)^2 + (67\,320)^2} = 252\,293 \text{ N-mm}$$

The pinion shaft is also subjected to a torque T = 116 700 N-mm, therefore equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(252\,293)^2 + (116\,700)^2} = 277\,975 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$277\,975 = \frac{\pi}{16} \times \tau\,(d_P)^3 = \frac{\pi}{16} \times 50\,(d_P)^3 = 9.82\,(d_P)^3$$

∴ $(d_P)^3 = 277\,975 / 9.82 = 28\,307$ or $d_P = 30.5$ say 35 mm **Ans.**

Let us now check for the principal shear stress.

We know that the shear stress induced,

$$\tau = \frac{16\,T_e}{\pi\,(d_P)^3} = \frac{16 \times 277\,975}{\pi (35)^3} = 33 \text{ N/mm}^2 = 33 \text{ MPa}$$

and direct stress due to axial load,

$$\sigma = \frac{W_A}{\frac{\pi}{4}\,(d_P)^2} = \frac{935}{\frac{\pi}{4}\,(35)^2} = 0.97 \text{ N/mm}^2 = 0.97 \text{ MPa}$$

Helical gears

∴ Principal shear stress,

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(0.97)^2 + 4(33)^2}\right] = 33 \text{ MPa}$$

Since the principal shear stress is less than the permissible shear stress of 50 MPa, therefore the design is satisfactory.

We know that the diameter of the pinion hub

$$= 1.8\, d_P = 1.8 \times 35 = 63 \text{ mm } \textbf{Ans.}$$

and length of the hub $= 1.25\, d_P = 1.25 \times 35 = 43.75$ say 44 mm

Since the length of the hub should not be less than the face width, therefore let us take length of the hub as 76 mm. **Ans.**

Note : Since the pitch circle diameter of the pinion is 144 mm, therefore the pinion should be provided with a web. Let us take the thickness of the web as 1.8 *m*, where *m* is the module.

∴ Thickness of the web $= 1.8\, m = 1.8 \times 6 = 10.8$ say 12 mm **Ans.**

Design for the gear shaft

Let d_G = Diameter of the gear shaft.

We have already calculated that the tangential load,

$$W_T = 1621 \text{ N}$$

and the axial load, $W_A = 935$ N

∴ Bending moment due to the tangential load,

$$M_1 = W_T \times \text{Overhang} = 1621 \times 150 = 243\ 150 \text{ N-mm}$$

and bending moment due to the axial load,

$$M_2 = W_A \times \frac{D_G}{2} = 935 \times \frac{432}{2} = 201\ 960 \text{ N-mm}$$

∴ Resultant bending moment on the gear shaft,

$$M = \sqrt{(M_1)^2 + (M_2)^2} = \sqrt{(243\ 150)^2 + (201\ 960)^2} = 316\ 000 \text{ N-mm}$$

Since the velocity ratio is 3, therefore the gear shaft is subjected to a torque equal to 3 times the torque on the pinion shaft.

∴ Torque on the gear shaft,

$$T = \text{Torque on the pinion shaft} \times V.R.$$

$$= 116\ 700 \times 3 = 350\ 100 \text{ N-mm}$$

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(316\ 000)^2 + (350\ 100)^2} = 472\ 000 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$472\ 000 = \frac{\pi}{16} \times \tau \times (d_G)^3 = \frac{\pi}{16} \times 50\ (d_G)^3 = 9.82\ (d_G)^3$$

∴ $(d_G)^3 = 472\ 000 / 9.82 = 48\ 065$ or $d_G = 36.3$ say 40 mm **Ans.**

Let us now check for the principal shear stress.

We know that the shear stress induced,

$$\tau = \frac{16\, T_e}{\pi\, (d_G)^3} = \frac{16 \times 472\,000}{\pi\, (40)^3} = 37.6 \text{ N/mm}^2 = 37.6 \text{ MPa}$$

and direct stress due to axial load,

$$\sigma = \frac{W_A}{\frac{\pi}{4}(d_G)^2} = \frac{935}{\frac{\pi}{4}(40)^2} = 0.744 \text{ N/mm}^2 = 0.744 \text{ MPa}$$

∴ Principal shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(0.744)^2 + 4\,(37.6)^2}\right] = 37.6 \text{ MPa}$$

Since the principal shear stress is less than the permissible shear stress of 50 MPa, therefore the design is satisfactory.

We know that the diameter of the gear hub

$$= 1.8\, d_G = 1.8 \times 40 = 72 \text{ mm } \textbf{Ans.}$$

and length of the hub $= 1.25\, d_G = 1.25 \times 40 = 50$ mm

We shall take the length of the hub equal to the face width, *i.e.* 76 mm. **Ans.**

Since the pitch circle diameter of the gear is 432 mm, therefore the gear should be provided with four arms. The arms are designed in the similar way as discussed for spur gears.

Design for the gear arms

Let us assume that the cross-section of the arms is elliptical with major axis (a_1) equal to twice the minor axis (b_1). These dimensions refer to hub end.

∴ Section modulus of arms,

$$Z = \frac{\pi b_1 (a_1)^2}{32} = \frac{\pi\,(a_1)^3}{64} = 0.05\,(a_1)^3 \qquad \left(\because b_1 = \frac{a_1}{2}\right)$$

Since the arms are designed for the stalling load and it is taken as the design tangential load divided by the velocity factor, therefore

Stalling load,

$$W_S = \frac{W_T}{C_v} = 1621\left(\frac{350 + 135.735m}{350}\right)$$

$$= 1621\left(\frac{350 + 135.735 \times 6}{350}\right) = 5393 \text{ N}$$

∴ Maximum bending moment on each arm,

$$M = \frac{W_S}{n} \times \frac{D_G}{2} = \frac{5393}{4} \times \frac{432}{2} = 291\ 222 \text{ N-mm}$$

We know that bending stress (σ_b),

$$42 = \frac{M}{Z} = \frac{291\ 222}{0.05\,(a_1)^3} = \frac{5824 \times 10^3}{(a_1)^3} \qquad \text{... (Taking } \sigma_b = 42 \text{ N/mm}^2\text{)}$$

∴ $(a_1)^3 = 5824 \times 10^3 / 42 = 138.7 \times 10^3$ or $a_1 = 51.7$ say 54 mm **Ans.**

and $b_1 = a_1 / 2 = 54 / 2 = 27$ mm **Ans.**

Since the arms are tapered towards the rim and the taper is 1/16 mm per mm length of the arm (or radius of the gear), therefore

Major axis of the arm at the rim end,

$$a_2 = a_1 - \text{Taper} = a_1 - \frac{1}{16} \times \frac{D_G}{2}$$

$$= 54 - \frac{1}{16} \times \frac{432}{2} = 40 \text{ mm } \textbf{Ans.}$$

and minor axis of the arm at the rim end,

$$b_2 = a_2 / 2 = 40 / 2 = 20 \text{ mm } \textbf{Ans.}$$

Design for the rim

The thickness of the rim for the pinion may be taken as 1.6 *m* to 1.9 *m*, where *m* is the module. Let us take thickness of the rim for pinion,

$$t_{RP} = 1.6\, m = 1.6 \times 6 = 9.6 \text{ say } 10 \text{ mm} \quad \textbf{Ans.}$$

The thickness of the rim for the gear (t_{RG}) is given by

$$t_{RG} = m\sqrt{\frac{T_G}{n}} = 6\sqrt{\frac{72}{4}} = 25.4 \text{ say } 26 \text{ mm} \quad \textbf{Ans.}$$

EXERCISES

1. A helical cast steel gear with 30° helix angle has to transmit 35 kW at 2000 r.p.m. If the gear has 25 teeth, find the necessary module, pitch diameters and face width for 20° full depth involute teeth. The static stress for cast steel may be taken as 100 MPa. The face width may be taken as 3 times the normal pitch. The tooth form factor is given by the expression $y' = 0.154 - 0.912/T_E$, where T_E represents the equivalent number of teeth. The velocity factor is given by $C_v = \dfrac{6}{6+v}$, where v is the peripheral speed of the gear in m/s. **[Ans. 6 mm ; 150 mm ; 50 mm]**

2. A pair of helical gears with 30° helix angle is used to transmit 15 kW at 10 000 r.p.m. of the pinion. The velocity ratio is 4 : 1. Both the gears are to be made of hardened steel of static strength 100 N/mm². The gears are 20° stub and the pinion is to have 24 teeth. The face width may be taken as 14 times the module. Find the module and face width from the standpoint of strength and check the gears for wear. **[Ans. 2 mm ; 28 mm]**

Gears inside a car engine.

3. A pair of helical gears consist of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 r.p.m. The normal pressure angle is 20° while the helix angle is 25°. The face width is 40 mm and the normal module is 4 mm. The pinion as well as gear are made of steel having ultimate strength of 600 MPa and heat treated to a surface hardness of 300 B.H.N. The service factor and factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of the gears. **[Ans. 8.6 kW]**

4. A single stage helical gear reducer is to receive power from a 1440 r.p.m., 25 kW induction motor. The gear tooth profile is involute full depth with 20° normal pressure angle. The helix angle is 23°, number of teeth on pinion is 20 and the gear ratio is 3. Both the gears are made of steel with allowable beam stress of 90 MPa and hardness 250 B.H.N.

   (*a*) Design the gears for 20% overload carrying capacity from standpoint of bending strength and wear.

   (*b*) If the incremental dynamic load of 8 kN is estimated in tangential plane, what will be the safe power transmitted by the pair at the same speed?

QUESTIONS

1. What is a herringbone gear? Where they are used?
2. Explain the following terms used in helical gears :

   (*a*) Helix angle; (*b*) normal pitch; and

   (*c*) axial pitch.
3. Define formative or virtual number of teeth on a helical gear. Derive the expression used to obtain its value.
4. Write the expressions for static strength, limiting wear load and dynamic load for helical gears and explain the various terms used therein.

OBJECTIVE TYPE QUESTIONS

1. If T is the actual number of teeth on a helical gear and ϕ is the helix angle for the teeth, the formative number of teeth is written as

   (*a*) $T \sec^3 \phi$ (*b*) $T \sec^2 \phi$

   (*c*) $T/\sec^3\phi$ (*d*) $T \operatorname{cosec} \phi$
2. In helical gears, the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth, is called

   (*a*) normal pitch (*b*) axial pitch

   (*c*) diametral pitch (*d*) module
3. In helical gears, the right hand helices on one gear will mesh .... helices on the other gear.

   (*a*) right hand (*b*) left hand
4. The helix angle for single helical gears ranges from

   (*a*) 10° to 15° (*b*) 15° to 20°

   (*c*) 20° to 35° (*d*) 35° to 50°
5. The helix angle for double helical gears may be made up to

   (*a*) 45° (*b*) 60°

   (*c*) 75° (*d*) 90°

ANSWERS

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*c*) **5.** (*a*)