UNIT 3

POWER TRANSMISSION SYSTEM & PULLEYS

Course objectives:

1. To introduce the concept, procedures, and data to analyze machine elements in power transmission systems.

Course Outcomes:

1. To understand the types belt drives and Select suitable belt drives and associated elements from manufacturers catalogues under given loading conditions to design the springs for different loading conditions

Introduction

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

The amount of power transmitted depends upon the following factors:

1. The velocity of the belt.

2. The tension under which the belt is placed on the pulleys.

3. The arc of contact between the belt and the smaller pulley.

4. The conditions under which the belt is used. It may be noted that

(a) The shafts should be properly in line to insure uniform tension across the belt section.

(b) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.

(c) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings.

(d) A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt.

(e) The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.

(f) In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 meters and the minimum should not be less than 3.5 times the diameter of the larger pulley. Selection of a Belt Drive

Various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,

2. Speed reduction ratio,

3. Power to be transmitted,

4. Centre distance between the shafts,

5. Positive drive requirements,

6. Shafts layout,

7. Space available, and 8. Service conditions.

Types of Belt Drives

The belt drives are usually classified into the following three groups:

1. Light drives. These are used to transmit small powers at belt speeds up to about 10 m/s as in agricultural machines and small machine tools.

2. Medium drives. These are used to transmit medium powers at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

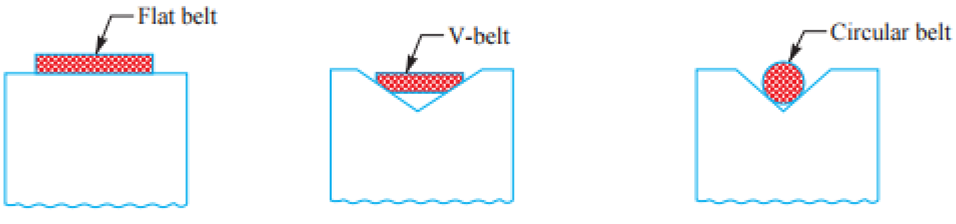
3. Heavy drives. These are used to transmit large powers at belt speeds above 22 m/s as in Compressors and generators.

Types of Belts

Though there are many types of belts used these days, yet the following are important from the

1. Flat belt.

The flat belt as shown in Fig. 18.1 (a), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.



(a) Flat belt. (b) V-belt. (c) Circular belt.

2. V- belt. The V-belt as shown in Fig. (b) is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. Circular belt or rope. The circular belt or rope as shown in Fig. (c) Is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart. If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

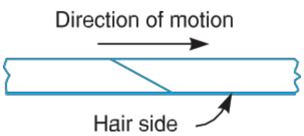
MATERIAL USED FORBELTS

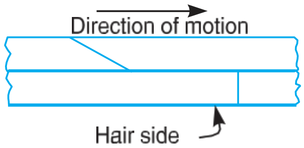
The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of friction. The belts, according to the material used, are classified as follows

LEATHER BELTS.

The most important material for the belt is leather. The best leather belts are made from 1.2 metres to 1.5 metres long strips cut from either side of the back bone of the top grade steer hides. The hair side of the leather is smoother and harder than the flesh side, but the flesh side is stronger. The fibres on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. Therefore for these reasons, the hair side of a belt should be in contact with the pulley surface, as shown in Fig. This gives a more intimate contact between the belt and the pulley and places the greatest tensile strength of the belt section on the outside, where the tension is maximum as the belt passes over the pulley.

(a) Single layer belt. (b) Double layer belt





2. The leather may be either oak-tanned or mineral salt tanned e.g. chrome tanned. In order to increase the thickness of belt, the strips are cemented together. The belts are specified according to the number of layers e.g. single, double or triple ply and according to the thickness of hides used e.g. light, medium or heavy.

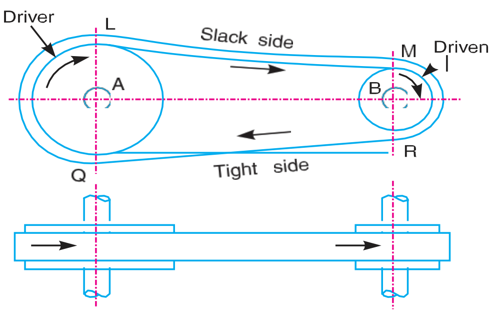
The leather belts must be periodically cleaned and dressed or treated with a compound or dressing containing neat foot or other suitable oils so that the belt will remain soft and flexible.

3. **COTTON OR FABRIC BELTS**: Most of the fabric belts are made by folding canvass or cotton duck to three or more layers (depending upon the thickness desired) and stitching together. These belts are woven also into a strip of the desired width and thickness. They are impregnated with some filler like linseed oil in order to make the belts water proof and to prevent injury to the fibres. The cotton belts are cheaper and suitable in warm climates, in damp atmospheres and in exposed positions. Since the cotton belts require little attention, therefore these belts are mostly used in farm machinery, belt conveyor etc.
4. **RUBBER BELT.** The rubber belts are made of layers of fabric impregnated with rubber com- position and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. One of the principal advantages of these belts is that they may be easily made endless. These belts are found suitable for saw mills, paper mills where they are exposed to moisture.
5. **BALATA BELTS.** These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and water proof and it is not effected animal oils or alkalies. The balata belts should not be at temperatures above 40° C because at this temperature the balata begins to soften and becomes sticky. The strength of balata belts is 25 per cent higher than rubber belts.

TYPES OF FLAT BELT DRIVES

The power from one pulley to another may be transmitted by any of the following types of belt drives:

OPEN BELT DRIVE. The open belt drive, as shown in Fig. 3.3, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver A pulls the belt from one side (i.e. lower



side RQ) and delivers it to the other side (i.e. upper side L M). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as tight side whereas the upper side belt (because of less tension) is known as slack side, as shown in Fig.

CROSSED OR TWIST BELT DRIVE : The crossed or twist belt drive, as shown in Fig. is used with shafts arranged parallel and rotating in the opposite directions.

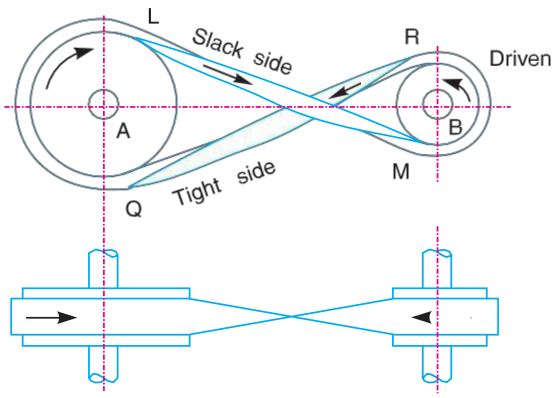
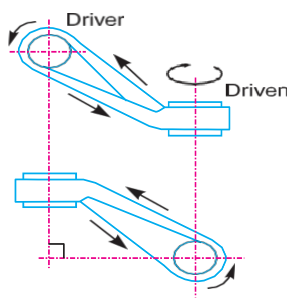


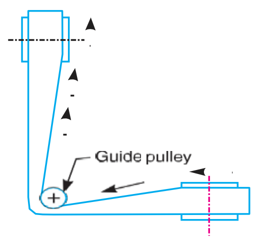
Fig. Crossed or twist belt drive.

In this case, the driver pulls the belt from one side (i.e. RQ) and delivers it to the other side (i.e. L M). Thus the tension in the belt RQ will be more than that in the belt L M. The belt RQ (because of more tension) is known as tight side, whereas the belt LM (because of less tension) is known as slack side, as shown in Fig. A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a maximum distance of 20 b, where b is the width of belt and the speed of the belt should be less than 15m/s.

QUARTER TURN BELT DRIVE. The quarter turn belt drives also known as right angle belt drive, as shown in Fig. (a), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to b, where b is the width of belt.

In case the pulleys cannot be arranged, as shown in Fig.(a), or when the reversible motion is desired, then a quarter turn belt drive with guide pulley, as shown in Fig.(b), may be used.

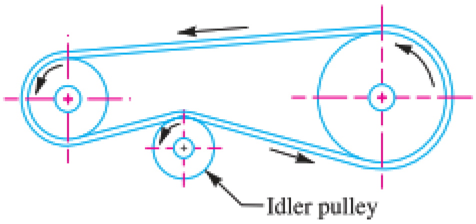


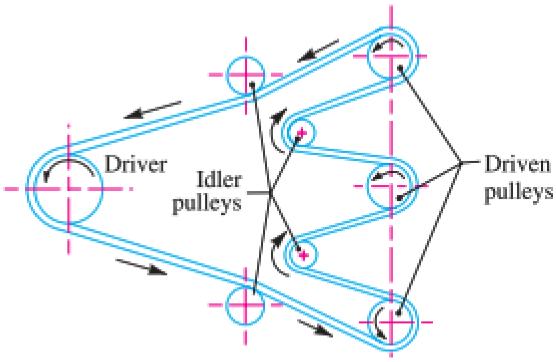


(a) Quarter turns belt drive. (b) Quarter turn belt drive with guide pulley

4. BELT DRIVE WITH IDLER PULLEYS.

A belt drive with an idler pulley (also known as jockey pulley drive) as shown in Fig. 18.7, is used with shafts arranged parallel and when an open belt drive can't be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension can't be obtained by other means. When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig.





COMPOUND BELT DRIVE.

A compound belt drive, as shown in Fig. is used when power is transmitted from one shaft to another through a number of pulleys.

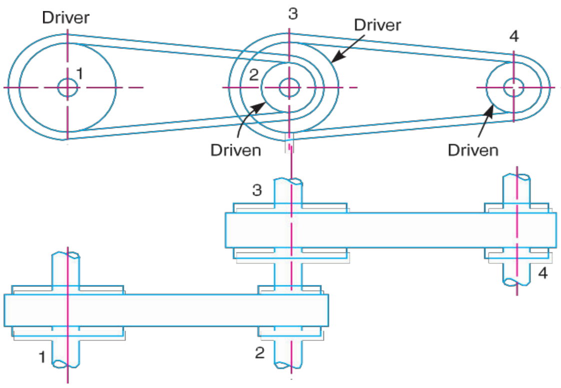
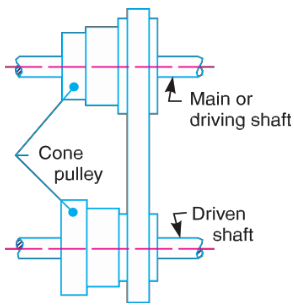


Fig: Compound belt drive.

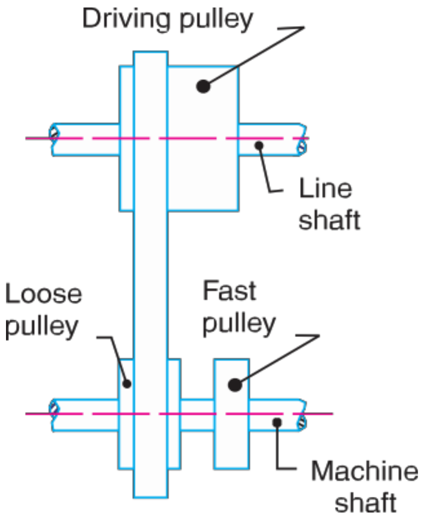
STEPPED OR CONE PULLEY DRIVE.

A stepped or cone pulley drive, as shown in Fig. is used for changing the speed of the driven shaft while the



main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

7. Fast and loose pulley drive. A fast and loose pulley drive, as shown in Fig. is used when the driven or



machine shaft is to be started or stopped whenever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called fast pulley and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.

VELOCITY RATIO OF A BELT DRIVE:

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let d_1 = Diameter of the driver,

d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m.,

N_2 = Speed of the follower in r.p.m.,

∴ Length of the belt that passes over the driver, in one minute

$$= \pi d_1 N_1$$

Similarly, length of the belt that passes over the follower, in one minute

$$= \pi d_2 N_2$$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\because \quad \pi d_1 N_1 = \pi d_2 N_2$$

and velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

When thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Notes : 1. The velocity ratio of a belt drive may also be obtained as discussed below:

We know that the peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi d_1 N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven pulley,

$$v_2 = \frac{\pi d_2 N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \quad \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} \text{ or } \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

2. In case of a compound belt drive as shown in Fig. 18.7, the velocity ratio is given by

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \text{ or } \frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

SLIP OF THE BELT

In the previous articles we have discussed the motion of belts and pulleys assuming a firm Frictional grip between the belts and the pulleys. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called slip of the belt and is generally expressed as a percentage.

s1 % = Slip between the driver and the belt, and

s2 % = Slip between the belt and follower,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

CREEP OF BELT:

When the belt passes from slack side to the tight side, certain of the belt extends and it contracts again when the belt passes from the tight side to the slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep. The total effect of creep is reducing slightly the speed of the driven pulley or follower. Considering creep, velocity ratio is given by

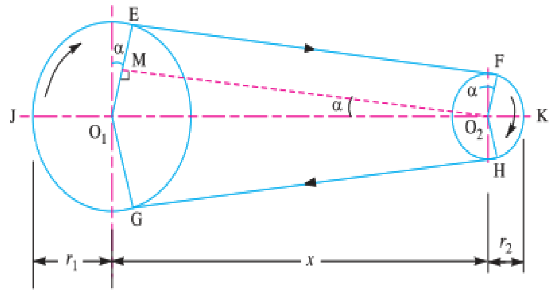
$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

Where σ_1 & σ_2 =stress in the belt on the tight and slack side

E = young's modulus for the material of the belt

Note: since the effect of creep is very small, therefore it is generally neglected.

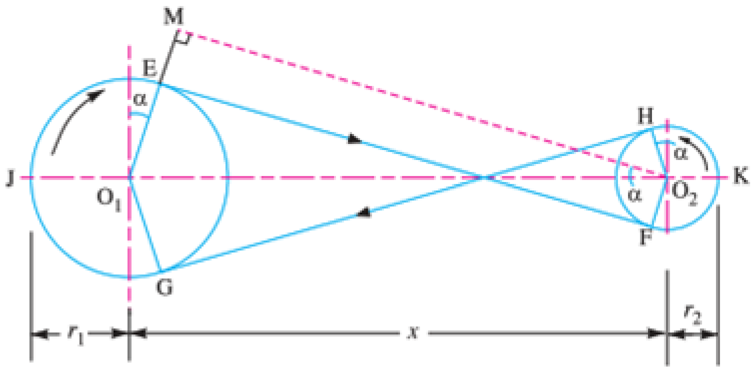
Length of Open Belt Drive:



$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

Length of a Cross Belt Drive



$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

Power Transmitted by a Belt:

T1 and T2 = Tensions in the tight side and slack side of the belt respectively in Newton's,

r1 and r2 = Radii of the driving and driven pulleys respectively in meters,

v = Velocity of the belt in m/s.

$$P = (T_1 - T_2)\, V \ \frac{N-m}{sec}$$

CENTRIFUGAL TENSION:

When the belt runs at lower speed, the initial tension given to the belt will be sufficient to keep the belt on the pulley with required grip, on the other hand, if the belt speed increases, due to centrifugal action, the belt will try to fly off from the pulley. At the same time, the tensions at the tight side and slack side will increase. The force applied on the shaft due to centrifugal action is called as centrifugal tension.

Let T_1 = Tension in the tight side

T_2 = Tension in the slack side

Centrifugal tension

$$T_c = mv^2$$

Note: It is known that, the total tensions at tight side and slack side are given by

$T_{t1} = T_1 + T_c$ and $T_{t2} = T_2 + T_c$

Since the centrifugal tension depends on the belt velocity, at low speeds the centrifugal action and its tension may be neglected. But for the higher speeds, the centrifugal tension will be taken into account.

$T_{t1} = T_1$ and $T_{t2} = T_2$ at low speeds, and $T_{t1} = T_1 + T_c$ and $T_{t2} = T_2 + T_c$ high speeds.

Also since the centrifugal force tries to pull the belt away from the pulley resulting the decrease of power transmitting capacity, the linear velocity of the belt is limited to 17.5 to 22.5 m/s, in order to control the centrifugal tension. If μ is the coefficient of friction between the belt and pulley and θ is the angle of contact for driving pulley in radians, then it is found that the ratio of driving tensions is

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\left(\frac{T_1}{T_2}\right) = e^{\mu\theta}$$

when the centrifugal tension (T_c) is neglected.

$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta}$$

When the centrifugal tension (T_c) is considered.

Maximum Tension in the Belt

σ = Maximum safe stress,

b = Width of the belt, and

t = Thickness of the belt.

T = Maximum stress × Cross-sectional area of belt **= σ.b.t**

When centrifugal tension is neglected, then

T (or Tt1) =T1, i.e. Tension in the tight side of the belt.

When centrifugal tension is considered, then

T (or Tt1) =T1 + TC

Condition for the Transmission of Maximum Power

1. We know that $T_1 = T - T_C$ and for maximum power, $T_C = \frac{T}{3}$.

$$T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

From equation (*iv*), we find that the velocity of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}}$$

Initial Tension in the Belt

the belt is subjected to some tension, called initial tension

T_0= Initial tension in the belt,

T_1= Tension in the tight side of the belt,

T_2= Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

$$T_0 = \frac{T_1 - T_2}{2} \text{ (Neglecting centrifugal tension)}$$

$$T_0 = \frac{T_1 + T_2 + 2T_C}{2} \text{ (Considering centrifugal tension)}$$

Problems:

1. In a horizontal belt drive for a centrifugal blower, the blower is belt driven At 600 r.p.m. by a 15 kW, 1750 r.p.m. electric motor. The centre distance is twice the diameter of the larger pulley. The density of the belt material = 1500 kg/m; maximum allowable stress = 4 MPa;µ1 = 0.5 (motor pulley); µ2= 0.4 (blower pulley); peripheral velocity of the belt = 20 m/s. Determine the following:

1. Pulley diameters, 2. Belt length, 3. Cross-sectional area of the belt;

4. Minimum initial tension for operation without slip; and 5. Resultant force in the plane of the blower when operating with an initial tension 50 per cent greater than the minimum value.

Solution.

Solution.

N2 = 600 r.p.m. ;

P = 15 kW = 15 × 10 W;

N1 = 1750 r.p.m . ;ρ = 1500 kg/m^3

σ = 4 MPa = 4 × 10^6 N/m^2 ,

µ1 = 0.5 ; µ2 = 0.4 ;

v = 20 m/s

Fig. Shows a horizontal belt drive. Suffix 1 refers to a motor pulley and suffix 2 refers to a blower pulley.

1. *Pulley diameters*

Let d_1 = Diameter of the motor pulley, and

d_2 = Diameter of the blower pulley.

We know that peripheral velocity of the belt (v),

$$20 = \frac{\pi\, d_1\, N_1}{60} = \frac{\pi\, d_1 \times 1750}{60} = 91.64\, d_1$$

$\therefore$ $d_1 = 20 / 91.64 = 0.218 \text{ m} = 218 \text{ mm}$ **Ans.**

We also know that $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

$$\therefore \quad d_2 = \frac{d_1 \times N_1}{N_2} = \frac{218 \times 1750}{600} = 636 \text{ mm}$$ **Ans.**

2. *Belt length*

Since the centre distance (x) between the two pulleys is twice the diameter of the larger pulley (*i.e.* $2\, d_2$), therefore centre distance,

$$x = 2\, d_2 = 2 \times 636 = 1272 \text{ mm}$$

We know that length of belt,

$$L = \frac{\pi}{2}(d_1 + d_2) + 2\, x + \frac{(d_1 - d_2)^2}{4x}$$

$$= \frac{\pi}{2}(218 + 636) + 2 \times 1272 + \frac{(218 - 636)^2}{4 \times 1272}$$

$$= 1342 + 2544 + 34 = 3920 \text{ mm} = 3.92 \text{ m}$$ **Ans.**

3. *Cross-sectional area of the belt*

Let a = Cross-sectional area of the belt.

First of all, let us find the angle of contact for both the pulleys. From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2\,x} = \frac{636 - 218}{2 \times 1272} = 0.1643$$

$\therefore$ $\alpha = 9.46°$

We know that angle of contact on the motor pulley,

$$\theta_1 = 180° - 2\alpha = 180 - 2 \times 9.46 = 161.08°$$

$$= 161.08 \times \pi / 180 = 2.8 \text{ rad}$$

and angle of contact on the blower pulley,

$$\theta_2 = 180° + 2\alpha = 180 + 2 \times 9.46 = 198.92°$$

$$= 198.92 \times \pi / 180 = 3.47 \text{ rad}$$

Since both the pulleys have different coefficient of friction (μ), therefore the design will refer to a pulley for which $\mu.\theta$ is small.

$\therefore$ For motor pulley,

$$\mu_1.\theta_1 = 0.5 \times 2.8 = 1.4$$

and for blower pulley, $\mu_2.\theta_2 = 0.4 \times 3.47 = 1.388$

Since $\mu_2.\theta_2$ for the blower pulley is less then $\mu_1.\theta_1$, therefore the design is based on the blower pulley.

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that power transmitted (P),

$$15 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 20$$

$\therefore$ $T_1 - T_2 = 15 \times 10^3 / 20 = 750 \text{ N}$ ...(*i*)

We also know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu_2.\theta_2 = 0.4 \times 3.47 = 1.388$$

$\therefore$ $\log\left(\dfrac{T_1}{T_2}\right) = \dfrac{1.388}{2.3} = 0.6035$ or $\dfrac{T_1}{T_2} = 4$ ...(*ii*)

... (Taking antilog of 0.6035)

From equations (*i*) and (*ii*),

$$T_1 = 1000 \text{ N} \text{ ; and } T_2 = 250 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = a \times l \times \rho$$
$$= a \times 1 \times 1500 = 1500\, a \text{ kg / m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1500\, a\, (20)^2 = 0.6 \times 10^6\, a \text{ N}$$

We know that maximum or total tension in the belt,

$$T = T_1 + T_C = 1000 + 0.6 \times 10^6\, a \text{ N} \quad ...(iii)$$

We also know that maximum tension in the belt,

$$T = \text{Stress} \times \text{area} = \sigma \times a = 4 \times 10^6\, a \text{ N} \quad ...(iv)$$

4. *Minimum initial tension for operation without slip*

We know that centrifugal tension,

$$T_C = 0.6 \times 10^6\, a = 0.6 \times 10^6 \times 294 \times 10^{-6} = 176.4 \text{ N}$$

∴ Minimum initial tension for operation without slip,

$$T_0 = \frac{T_1 + T_2 + 2T_C}{2} = \frac{1000 + 250 + 2 \times 176.4}{2} = 801.4 \text{ N} \quad \textbf{Ans.}$$

5. *Resultant force in the plane of the blower when operating with an initial tension 50 per cent greater than the minimum value*

We have calculated above that the minimum initial tension,

$$T_0 = 801.4 \text{ N}$$

∴ Increased initial tension,

$$T_0' = 801.4 + 801.4 \times \frac{50}{100} = 1202 \text{ N}$$

Let T_1' and T_2' be the corresponding tensions in the tight side and slack side of the belt respectively.

We know that increased initial tension (T_0'),

$$1202 = \frac{T_1' + T_2' + 2T_C}{2} = \frac{T_1' + T_2' + 2 \times 176.4}{2}$$

∴ $$T_1' + T_2' = 1202 \times 2 - 2 \times 176.4 = 2051.2 \text{ N} \quad ...(v)$$

Since the ratio of tensions will be constant, *i.e.* $\frac{T_1'}{T_2'} = \frac{T_1}{T_2} = 4$, therefore from equation (*v*), we have

$$4T_2' + T_2' = 2051.2 \text{ or } T_2' = 2051.2 / 5 = 410.24 \text{ N}$$

and $$T_1' = 4\, T_2' = 4 \times 410.24 = 1640.96 \text{ N}$$

∴ Resultant force in the plane of the blower

$$= T_1' - T_2' = 1640.96 - 410.24 = 1230.72 \text{ N} \quad \textbf{Ans.}$$

2. A belt 100 mm wide and 10 mm thick is transmitting power at 1000 meters/min. The net driving tension is 1.8 times the tension on the slack side. If the safe permissible stress on the belt section in 1.6 MPa, calculate the maximum power that can be transmitted at this speed. Assume density of the leather as 1000 kg/m^3.Calculate the absolute maximum power that can be transmitted by this belt and the speed at which this can be transmitted.

Solution. Given : $b = 100$ mm = 0.1 m ; $t = 10$ mm = 0.01 m ; $v = 1000$ m/min = 16.67 m/s ; $T_1 - T_2 = 1.8\, T_2$; $\sigma = 1.6$ MPa = 1.6 N/mm^2 ; $\rho = 1000$ kg/m^3

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the maximum tension in the belt,

$$T = \sigma.b.t = 1.6 \times 100 \times 10 = 1600 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$
$$= 0.1 \times 0.01 \times 1 \times 1000 = 1 \text{ kg/m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1\,(16.67)^2 = 278 \text{ N}$$

We know that

$$T_1 = T - T_C = 1600 - 278 = 1322 \text{ N}$$

and

$$T_1 - T_2 = 1.8\, T_2$$

∴

$$T_2 = \frac{T_1}{2.8} = \frac{1322}{2.8} = 472 \text{ N}$$

We know that the power transmitted.

$$P = (T_1 - T_2)\, v = (1322 - 472)\, 16.67 = 14\,170 \text{ W} = 14.17 \text{ kW} \textbf{ Ans.}$$

Speed at which absolute maximum power can be transmitted

We know that the speed of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{1600}{3 \times 1}} = 23.1 \text{ m/s} \textbf{ Ans.}$$

Absolute maximum power

We know that for absolute maximum power, the centrifugal tension,

$$T_C = T/3 = 1600/3 = 533 \text{ N}$$

∴ Tension in the tight side,

$$T_1 = T - T_C = 1600 - 533 = 1067 \text{ N}$$

and tension in the slack side,

$$T_2 = \frac{T_1}{2.8} = \frac{1067}{2.8} = 381 \text{ N}$$

∴ Absolute maximum power transmitted,

$$P = (T_1 - T_2)\, v = (1067 - 381)\, 23.1 = 15\,850 \text{ W} = 15.85 \text{ kW} \textbf{ Ans.}$$

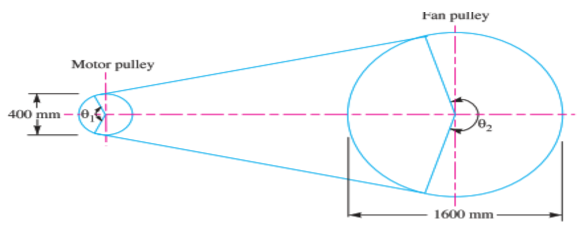
An electric motor drives an exhaust fan. Following data are provided :

	Motor pulley	*Fan pulley*
Diameter	*400 mm*	*1600 mm*
Angle of warp	*2.5 radians*	*3.78 radians*
Coefficient of friction	*0.3*	*0.25*
Speed	*700 r.p.m.*	—
Power transmitted	*22.5 kW*	—

Calculate the width of 5 mm thick flat belt. Take permissible stress for the belt material as 2.3 MPa.

Solution. Given : $d_1 = 400$ mm or $r_1 = 200$ mm ; $d_2 = 1600$ mm or $r_2 = 800$ mm ; $\theta_1 = 2.5$ rad ; $\theta_2 = 3.78$ rad ; $\mu_1 = 0.3$; $\mu_2 = 0.25$; $N_1 = 700$ r.p.m. ; $P = 22.5$ kW $= 22.5 \times 10^3$ W ; $t = 5$ mm $= 0.005$ m ; $\sigma = 2.3$ MPa $= 2.3 \times 10^6$ N/m^2

Fig. 18.19 shows a system of flat belt drive. Suffix 1 refers to motor pulley and suffix 2 refers to fan pulley.



We have discussed in Art. 18.19 (Note 2) that when the pulleys are made of different material [*i.e.* when the pulleys have different coefficient of friction (μ) or different angle of contact (θ), then the design will refer to a pulley for which $\mu.\theta$ is small.

$\therefore$ For motor pulley, $\mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$

and for fan pulley, $\mu_2.\theta_2 = 0.25 \times 3.78 = 0.945$

Since $\mu_1.\theta_1$ for the motor pulley is small, therefore the design is based on the motor pulley.

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the velocity of the belt,

$$v = \frac{\pi\, d_1.N_1}{60} = \frac{\pi \times 0.4 \times 700}{60} = 14.7 \text{ m/s} \quad \text{... } (d_1 \text{ is taken in metres})$$

and the power transmitted (P),

$$22.5 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 14.7$$

$$\therefore \quad T_1 - T_2 = 22.5 \times 10^3 / 14.7 = 1530 \text{ N} \quad \text{...}(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.75}{2.3} = 0.3261 \quad \text{or} \quad \frac{T_1}{T_2} = 2.12 \quad \text{...}(ii)$$

... (Taking antilog of 0.3261)

From equations (*i*) and (*ii*), we find that

$T_1 = 2896$ N ; and $T_2 = 1366$ N

Let b = Width of the belt in metres.

Since the velocity of the belt is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg / m^3.

$\therefore$ Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= b \times 0.005 \times 1 \times 1000 = 5\, b \text{ kg/m}$$

and centrifugal tension, $T_C = m.v^2 = 5\, b\, (14.7)^2 = 1080\, b$ N

We know that the maximum (or total) tension in the belt,

$$T = T_1 + T_C = \text{Stress} \times \text{Area} = \sigma.b.t$$

or $\quad 2896 + 1080\, b = 2.3 \times 10^6\, b \times 0.005 = 11\,500\, b$

$\therefore \quad 11\,500\, b - 1080\, b = 2896 \quad$ or $\quad b = 0.278$ say 0.28 m or 280 mm **Ans.**

3. A belt is required to transmit 18.5 kW from a pulley of 1.2 m diameter running at 250rpm to another pulley which runs at 500 rpm. The distance between the centers of pulleys is 2.7 m. The following data refer to an open belt drive, = 0.25. Safe working stress for leather is 1.75 N/mm2. Thickness of belt = 10mm. Determine the width and length of belt taking centrifugal tension into account. Also find the initial tension in the belt and absolute power that can be transmitted by this belt and the speed at which this can be transmitted.

Data:

Open belt drive; N = 18.5 kW; n_1 = 500 rpm = Speed of smaller pulley;

d_2 = 1.2 m = 1200 mm = D = Diameter of larger pulley; n_2 = 250 rpm = Speed of larger pulley;

C = 2.7 m = 2700 mm; μ = 0.25; σ_1 = 1.75 N/mm^2; t = 10 mm

(i) **Diameter of smaller pulley**

$$n_1 d_1 = n_2 d_2$$
$$500 \times d_1 = 250 \times 1200$$
$$\therefore \text{Diameter of smaller pulley } \mathbf{d_1} = \mathbf{600\ mm = d}$$

(ii) **Velocity**

$$v = \frac{\pi(D+t)n_2}{60{,}000} = \frac{\pi(1200+10)250}{60{,}000} = 15.839 \text{ m/sec.}$$

(iii) Centrifugal stress

$$\sigma_C = \frac{wv^2}{g} \times 10^6$$

Assume specific weight of leather as 10×10^{-6} N/mm³

$$\therefore \sigma_c = \frac{10 \times 10^{-6}}{9810} \times 15.839^2 \times 10^6 = 0.25573 \text{ N/mm}^2$$

(iv) **Capacity**

Since coefficient of friction is same for both smaller and larger pulleys, capacity = $e^{\mu\theta_s}$

$$\text{i.e., } e^{\mu\theta} = e^{\mu\theta_s}$$

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180}$$

$$= \pi - \left\{2\sin^{-1}\left(\frac{1200-600}{2 \times 2700}\right)\right\}\frac{\pi}{180} = 2.92 \text{ radians}$$

$$\therefore e^{\mu\theta} = e^{0.25 \times 2.92} = 2.075$$

(v) Constant

$$k = \frac{e^{\mu\theta}-1}{e^{\mu\theta}} = \frac{2.075-1}{2.075} = 0.52$$

(vi) Width of belt

$$\text{Power transmitted per mm}^2 \text{ area} = \frac{(\sigma_1 - \sigma_c)kv}{1000}$$

$$= \frac{(1.75 - 0.25573)0.52 \times 15.839}{1000} = 0.01231 \text{ kW}$$

(ix) Absolute power

For maximum power transmission

$$\sigma_C = \frac{\sigma_1}{3} = \frac{1.75}{3} = 0.5833 \text{ N/mm}^2$$

$$\text{Also } \sigma_C = \frac{w}{g}v^2 \times 10^6$$

$$\therefore 0.5833 = \frac{10 \times 10^{-6}}{9810} \times v^2 \times 10^6$$

$$\therefore \mathbf{v = 23.92\ m/sec}$$

$$\therefore \text{Power transmitted} \setminus \text{mm}^2 = \frac{(\sigma_1 - \sigma_C)kv}{1000}$$

$$= \frac{(1.75 - 0.5833)0.52 \times 23.92}{1000}$$

$$= 0.0145 \text{ kW}$$

$$\therefore \text{Total absolute power} = \text{Area of c/s of belt} \times \text{power per mm}^2$$

$$= 1503.18 \times 0.0145 = 21.7961 \text{ kW}$$

$$\therefore \text{Absolute power} = 21.8 \text{ kW.}$$

4. Select a V-belt drive to transmit 10 kW of power from a pulley of 200 mm diameter mounted on an electric motor running at 720 rpm to another pulley mounted on compressor running at 200 rpm. The service is heavy duty varying from 10 hours to 14 hours per day and centre distance between centres of pulleys is 600 mm.

Data:

$$N = 10\,kW;\ d_1 = 200\ mm = d;\ n_1 = 720\ rpm;\ n_2 = 200\ rpm;\ C = 600\ mm$$

Heavy duty 10 hours to 14 hours per day.

Solution :

i. Diameter of larger pulley

$$n_1 d_1 = n_2 d_2$$

$$720 \times 200 = 200 \times d_2$$

$$\therefore\ d_2 = 720\ mm = D = \text{diameter of larger pulley}$$

ii. Select the cross-section of belt

Equivalent Pitch diameter of smaller pulley $d_e = d_p F_b$ where $d_p = d_1 = 200$ mm

$$\frac{n_1}{n_2} = \frac{720}{200} = 3.6$$

From Table when $\frac{n_1}{n_2} = 3.6$

Smaller diameter factor $F_b = 1.14$

$$\therefore\ d_e = 200 \times 1.14 = 228\ mm.$$

iii. Velocity

$$v = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 200 \times 720}{60000} = 7.54\ m/sec$$

iv. Power capacity

For 'C' cross-section belt

$$N^* = v\left[\frac{1.47}{V^{0.09}} - \frac{143.27}{d_e} - \frac{2.34 v^2}{10^4}\right]$$

$$= 7.54\left[\frac{1.47}{7.54^{0.09}} - \frac{143.27}{228} - \frac{2.34 \times 7.54^2}{10^4}\right]$$

$$N^* = 4.4\ kW$$

Number of bolts:

$$i = \frac{NF_a}{N^* F_c . F_d}$$

for heavy duty 10 – 14 hours/day correction factor for service $F_a = 1.3$

$$L = 2C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C}$$

$$= 2 \times 600 + \frac{\pi}{2}(720 + 200) + \frac{(720 - 200)^2}{4 \times 600} = 2757.8\ mm$$

The nearest standard value of nominal pitch length for the selected C- cross section belt L = 2723 mm ,Nominal inside length = 2667 mm, For nominal inside length = 2667 mm, and C-cross section belt, correction factor for length Fe = 0.94

$$\text{Angle of contact } \theta = 2\cos^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 2\cos^{-1}\left(\frac{720-200}{2\times 600}\right) = 128.64°$$

From Table when $\theta = 128.64°$

Correction factor for angle of contact $F_d = 0.86$ (Assume V-V belt)

$$\therefore\ i = \frac{10\times 1.3}{4.4\times 0.94\times 0.86} = 3.655$$

$\therefore$ Number of V belts **i = 4**

Types of Pulleys for Flat Belts:

Following are the various types of pulleys for flat belts:

1. Cast iron pulleys, 2. Steel pulleys, 3. Wooden pulleys, 4. Paper pulleys and 5. Fast and loose pulleys.

Design of Cast Iron Pulleys

1. Dimensions of pulley

(i) The diameter of the pulley (D) may be obtained either from velocity ratio consideration or centrifugal stress consideration. We know that the centrifugal stress induced in the rim of the pulley,

$$\sigma_t = \rho . v^2$$

where

ρ = Density of the rim material

= 7200 kg/m^3 for cast iron

v = Velocity of the rim = πDN / 60, D being the diameter of pulley and

N is speed of the pulley.

The following are the diameter of pulleys in mm for flat and V-belts.

20, 22, 25, 28, 32, 36, 40, 45, 50, 56, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224,250, 280, 315, 355, 400, 450, 500, 560, 630, 710, 800, 900, 1000, 1120, 1250, 1400, 1600, 1800,2000, 2240, 2500, 2800, 3150, 3550, 4000, 5000, 5400.

The first six sizes (20 to 36 mm) are used for V-belts only.

The first six sizes (20 to 36 mm) are used for V-belts only.

B = 1.25 b ; where b = Width of belt.

(iii) The thickness of the pulley rim (t) varies from

$\frac{D}{300} + 2$ mm to $\frac{D}{300} + 3$ for single belt

$\frac{D}{300} + 6$ mm for double belt.

The diameter of the pulley (D) is in mm.

2. Dimensions of arms

(i) The number of arms may be taken as 4 for pulley diameter from 200 mm to 600 mm and 6 for diameter from 600 mm to 1500 mm.

(ii) The cross-section of the arms is usually elliptical with major axis (a1) equal to twice the minor axis (b1). The cross-section of the arm is obtained by considering the arm as cantilever i.e. fixed at the hub end and carrying a concentrated load at the rim end. The length of the cantilever is taken equal to the radius of the pulley. It is further assumed that at any given time, the power is transmitted from the hub to the rim or vice versa, through only half the total number of arms.

T = Torque transmitted,

R = Radius of pulley, and

n = Number of arms,

∴ Tangential load per arm,

$$W_T = \frac{T}{R \times n / 2} = \frac{2T}{R \cdot n}$$

Maximum bending moment on the arm at the hub end,

$$M = \frac{2T}{R \times n} \times R = \frac{2T}{n}$$

and section modulus,

$$Z = \frac{\pi}{32} \times b_1 \, (a_1)^2$$

Now using the relation,

σ_b or σ_t = M / Z, the cross-section of the arms is

(iii) The arms are tapered from hub to rim. The taper is usually

/48 to 1/32.

(iv) When the width of the pulley exceeds the diameter of the pulley, then two rows of arms are provided, as shown in Fig. 19.4. This is done to avoid heavy arms in one row.

3. Dimensions of hub

(i) The diameter of the hub (d1) in terms of shaft diameter (d) may be fixed by the following relation :

d1= 1.5 d + 25 mm

The diameter of the hub should not be greater than 2 d.

(ii) The length of the hub,

$$\mathbf{L} = \frac{\pi}{2} \times d$$

The minimum length of the hub is $\frac{2}{3}$ B but it should not be more than width of the pulley (B).

Advantages and Disadvantages of V-belt Drive over Flat Belt Drive

Advantages

1. The V-belt drive gives compactness due to the small distance between centres of pulleys.

2. The drive is positive, because the slip between the belt and the pulley groove is negligible.

3. Since the V-belts are made endless and there is no joint trouble, therefore the drive is smooth.

4. It provides longer life, 3 to 5 years.

5. It can be easily installed and removed.

6. The operation of the belt and pulley is quiet.

7. The belts have the ability to cushion the shock when machines are started.

8. The high velocity ratio (maximum 10) may be obtained.

9. The wedging action of the belt in the groove gives high value of limiting *ratio of tensions. Therefore the power transmitted by V-belts is more than flat belts for the same coefficient of friction, arc of contact and allowable tension in the belts.

10. The V-belt may be operated in either direction, with tight side of the belt at the top or bottom. The centre line may be horizontal, vertical or inclined.

Disadvantages

1. The V-belt drive cannot be used with large centre distances, because of larger weight per unit length.

2. The V-belts are not so durable as flat belts.

3. The construction of pulleys for V-belts is more complicated than pulleys of flat belts.

4. Since the V-belts are subjected to certain amount of creep, therefore these are not suitable for constant speed applications such as synchronous machines and timing devices.

5. The belt life is greatly influenced with temperature changes, improper belt tension and mismatching of belt lengths.

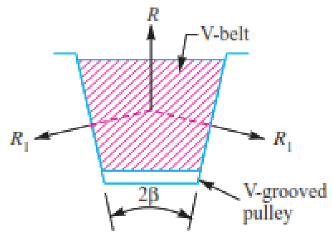
6. The centrifugal tension prevents the use of V-belts at speeds below 5 m / s and above 50 m / s.

Ratio of Driving Tensions for V-belt

R1= Normal reactions between belts and sides of the groove.

R = Total reaction in the plane of the groove.

μ = Coefficient of friction between the belt and sides of the groove.



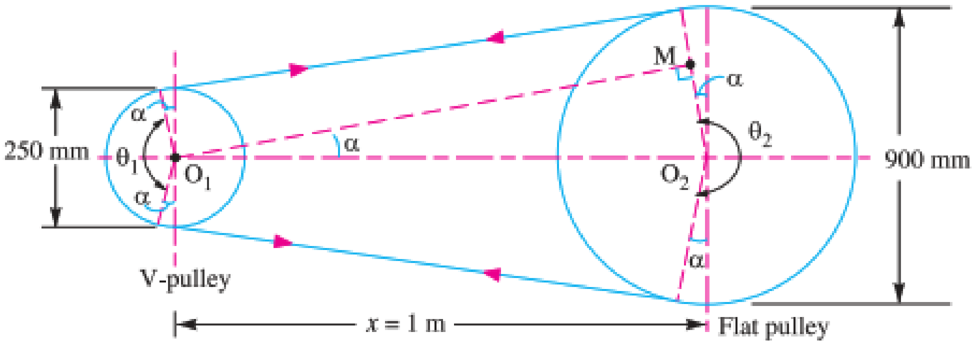
$$2.3 \log (T_1 / T_2) = \mu.\theta \operatorname{cosec} \beta$$

5. A V-belt is driven on a flat pulley and a V-pulley. The drive transmits 20 kW from a 250 mm diameter V-pulley operating at 1800 r.p.m. to a 900 mm diameter flat pulley. The centre distance is 1 m, the angle of groove 40° and μ = 0.2. If density of belting is 1110 kg / m and allowable stress is 2.1 MPa for belt material, what will be the number of belts required if C-size V-belts having 230 mm³ cross-sectional areas are used.

Solution. Given : $P = 20$ kW ; $d_1 = 250$ mm $= 0.25$ m ; $N_1 = 1800$ r.p.m. ; $d_2 = 900$ mm $= 0.9$ m ; $x = 1$ m $= 1000$ mm ; $2\beta = 40°$ or $\beta = 20°$; $\mu = 0.2$; $\rho = 1110$ kg/m^3 ; $\sigma = 2.1$ MPa $= 2.1$ N/mm^2 ; $a = 230$ mm$^2 = 230 \times 10^{-6}$ m^2

$$\sin\alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{900 - 250}{2 \times 1000} = 0.325$$

$$\alpha = 18.96°$$



We know that angle of contact on the smaller or *V*-pulley,

$$\theta_1 = 180° - 2\alpha = 180° - 2 \times 18.96 = 142.08°$$
$$= 142.08 \times \pi/180 = 2.48 \text{ rad}$$

and angle of contact on the larger or flat pulley,

$$\theta_2 = 180° + 2\alpha = 180° + 2 \times 18.96 = 217.92°$$
$$= 217.92 \times \pi / 180 = 3.8 \text{ rad}$$

We have already discussed that when the pulleys have different angle of contact (θ), then the design will refer to a pulley for which μ.θ is small.

We know that for a smaller or *V*-pulley,

$$\mu.\theta = \mu.\theta_1 \operatorname{cosec} \beta = 0.2 \times 2.48 \times \operatorname{cosec} 20° = 1.45$$

and for larger or flat pulley,

$$\mu.\theta = \mu.\theta_2 = 0.2 \times 3.8 = 0.76$$

Since (μ.θ) for the larger or flat pulley is small, therefore the design is based on the larger or flat pulley.

We know that peripheral velocity of the belt,

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.25 \times 1800}{60} = 23.56 \text{ m/s}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = a \times l \times \rho$$
$$= 230 \times 10^{-6} \times 1 \times 1100 = 0.253 \text{ kg / m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 0.253\ (23.56)^2 = 140.4 \text{ N}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that maximum tension in the belt,

$$T = \text{Stress} \times \text{area} = \sigma \times a = 2.1 \times 230 = 483 \text{ N}$$

We also know that maximum or total tension in the belt,

$$T = T_1 + T_C$$

$$\therefore \quad T_1 = T - T_C = 483 - 140.4 = 342.6 \text{ N}$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta_2 = 0.2 \times 3.8 = 0.76$$

$$\log \left(\frac{T_1}{T_2}\right) = 0.76 / 2.3 = 0.3304 \quad \text{or} \quad \frac{T_1}{T_2} = 2.14 \qquad \text{...(Taking antilog of 0.3304)}$$

and $T_2 = T_1 / 2.14 = 342.6 / 2.14 = 160$ N

∴ Power transmitted per belt

$$= (T_1 - T_2)\, v = (342.6 - 160)\ 23.56 = 4302 \text{ W} = 4.302 \text{ kW}$$

We know that number of belts required

$$= \frac{\text{Total power transmitted}}{\text{Power transmitted per belt}} = \frac{20}{4.302} = 4.65 \text{ say } 5 \quad \textbf{Ans.}$$

Rope Drives:

The ropes drives use the following two types of ropes :

1. Fibre ropes, and 2. *Wire ropes.

The fibre ropes operate successfully when the pulleys are about 60 metres apart, while the wire ropes are used when the pulleys are upto 150 metres apart.

Advantages of Fibre Rope Drives

The fibre rope drives have the following advantages :

1. They give smooth, steady and quiet service.

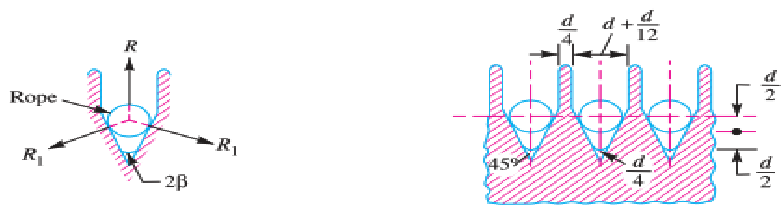
2. They are little affected by out door conditions.

3. The shafts may be out of strict alignment.

4. The power may be taken off in any direction and in fractional parts of the whole amount.

5. They give high mechanical efficiency.

Sheave for Fibre Ropes



(*a*) Cross-section of a rope. (*b*) Sheave (grooved pulley) for ropes.

Ratio of Driving Tensions for Fibre Rope

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta$$

where μ, θ and β have usual meanings..

6. A pulley used to transmit power by means of ropes has a diameter of 3.6 metres and has 15 grooves of 45° angle. The angle of contact is 170° and the coefficient of friction between the ropes and the groove sides is 0.28. The maximum possible tension in the ropes is 960 N and the mass of the rope is 1.5 kg per metre length. Determine the speed of the pulley in r.p.m. and the power transmitted if the condition of maximum power prevail.

Solution. Given : d = 3.6 m ; n = 15 ; 2 β = 45° or β = 22.5° ; θ = 170° = 170 × π / 180 = 2.967 rad ; μ = 0.28 ; T = 960 N ; m = 1.5 kg / m

Solution. Given : d = 3.6 m ; n = 15 ; 2 β = 45° or β = 22.5° ; θ = 170° = 170 × π / 180 = 2.967 rad ; μ = 0.28 ; T = 960 N ; m = 1.5 kg / m

Speed of the pulley

Let N = Speed of the pulley in r.p.m.

We know that for maximum power, speed of the pulley,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{960}{3 \times 1.5}} = 14.6 \text{ m/s}$$

We also know that speed of the pulley (v),

$$14.6 = \frac{\pi d . N}{60} = \frac{\pi \times 3.6 \times N}{60} = 0.19\, N$$

∴ N = 14.6 / 0.19 = 76.8 r.p.m. **Ans.**

Power transmitted

We know that for maximum power, centrifugal tension,

$$T_C = T / 3 = 960 / 3 = 320 \text{ N}$$

∴ Tension in the tight side of the rope,

$$T_1 = T - T_C = 960 - 320 = 640 \text{ N}$$

Let T_2 = Tension in the slack side of the rope.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta = 0.28 \times 2.967 \times \operatorname{cosec} 22.5° = 2.17$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{2.17}{2.3} = 0.9435 \quad \text{or} \quad \frac{T_1}{T_2} = 8.78 \qquad \text{...(Taking antilog of 0.9435)}$$

and $T_2 = T_1 / 8.78 = 640 / 8.78 = 73$ N

∴ Power transmitted,

$$P = (T_1 - T_2)\, v \times n = (640 - 73)\, 14.6 \times 15 = 124\,173 \text{ W}$$
$$= 124.173 \text{ kW}$$ **Ans.**

Wire Ropes:

When a large amount of power is to be transmitted over long distances from one pulley to

another (i.e. when the pulleys are upto 150 metres apart), then wire ropes are used. The wire ropes are widely used in elevators, mine hoists, cranes, conveyors, hauling devices and suspension bridges. The wire ropes run on grooved pulleys but they rest on the bottom of the *grooves and are not wedged between the sides of the grooves

Advantages of Wire Ropes.

1. These are lighter in weight,

2. These offer silent operation,

3. These can withstand shock loads,

4. These are more reliable,

5. These are more durable,

6. They do not fail suddenly

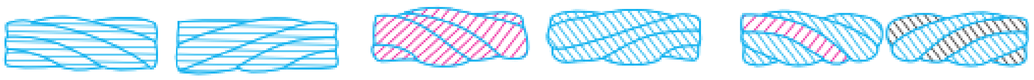
7. The efficiency is high, and

8. The cost is low.

Classification of Wire Ropes:

1. Cross or regular lay ropes. In these types of ropes, the direction of twist of wires in the strands is opposite to the direction of twist of the stands, as shown in Fig. (a). Such type of ropes are most popular.

2. Parallel or lang lay ropes. In these type of ropes, the direction of twist of the wires in the strands is same as that of strands in the rope, as shown in Fig. (b). These ropes have better bearing surface but is harder to splice and twists more easily when loaded. These ropes are more flexible and resists wear more effectively. Since such ropes have the tendency to spin, therefore these are used in lifts and hoists with guide ways and also as haulage ropes.



(a) Cross or regular lay ropes. (b) Parallel or lang lay ropes. (c) Composite or reverse laid ropes.

3. Composite or reverse laid ropes. In these types of ropes, the wires in the two adjacent strands are twisted in the opposite direction, as shown in Fig.

Springs

A spring is defined as an elastic bodyis to distort when loaded and to recover its original shapewhen the load is removed.

The various important applications of springs are as follows :

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

2. To apply forces, as in brakes, clutches and springloaded valves.

3.To control motion by maintaining contact between two elements as in cams and followers.

4. To measure forces, as in spring balances and engine indicators.

5. To store energy, as in watches, toys, etc.

Types of Springs : 1. Helical springs. The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are compression helical spring as shown in Fig. (a) and tension helical spring as shown in Fig.(b).

UNIT-3

POWER TRANSMISSION SYSTEM & PULLEYS AND SPRINGS

LIVE TO LEARN & LEARN TO SHARE

MRCET CAMPUS

UGC Autonomous

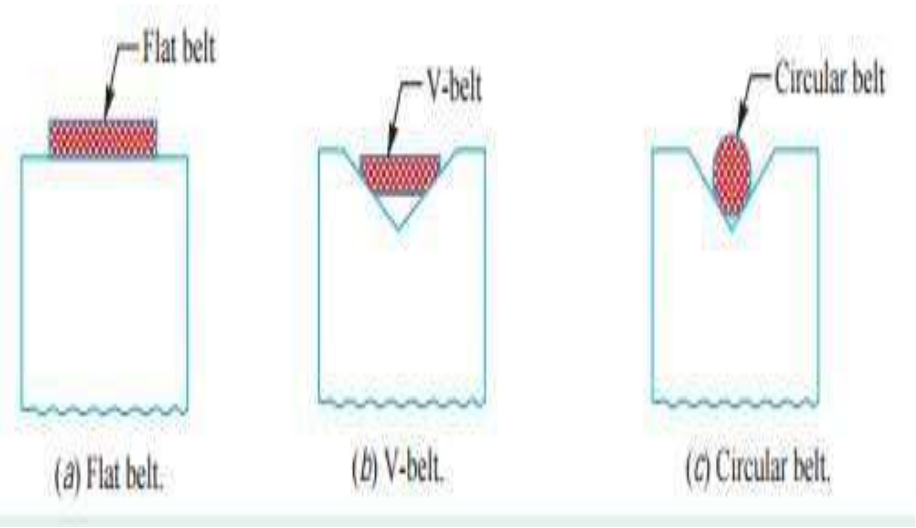
DEPARTMENT OF MECHANICAL ENGINEERING

INTRODUCTION

- The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

BELTS

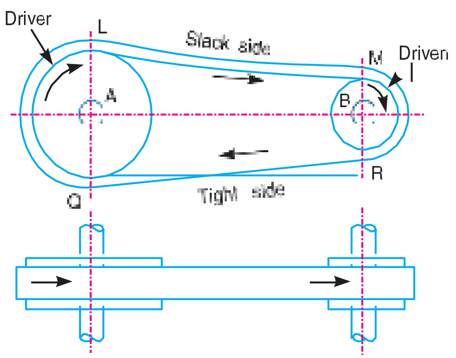
TYPES OF BELTS



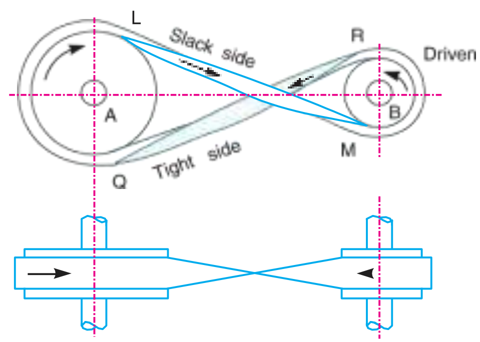
MATERIALS

- Leather Belts
- Cotton or fabric
- Rubber Belts
- Balata Belts

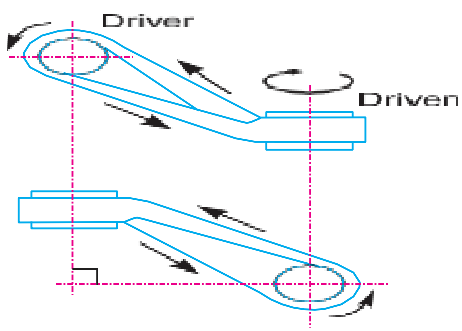
TYPES OF FLAT BELT DRIVE



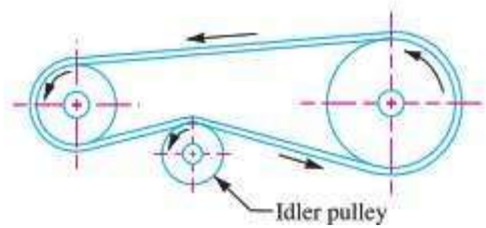
OPEN BELT DRIVE



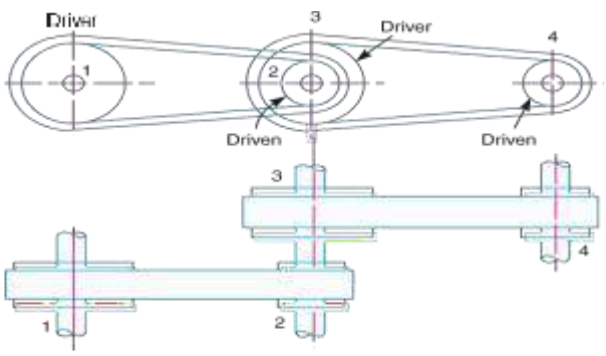
CROSS BELT DRIVE



QUARTER BELT DRIVE



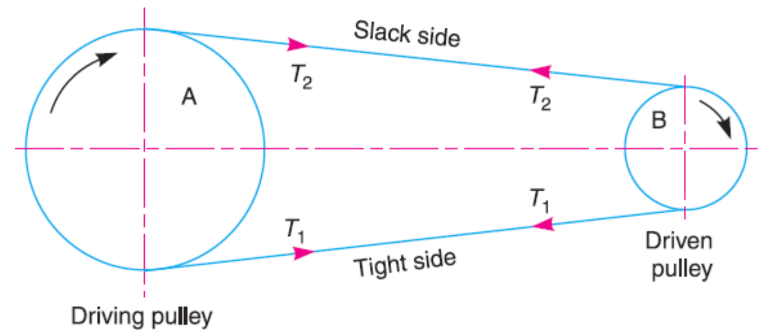
BELT DRIVE WITH IDLER



STEPPED OR CONE PULLEY DRIVE

BELTS

$$\therefore \text{ Velocity ratio, } \frac{N_2}{N_1} = \frac{d_1}{d_2}$$



Power Transmitted by a Belt

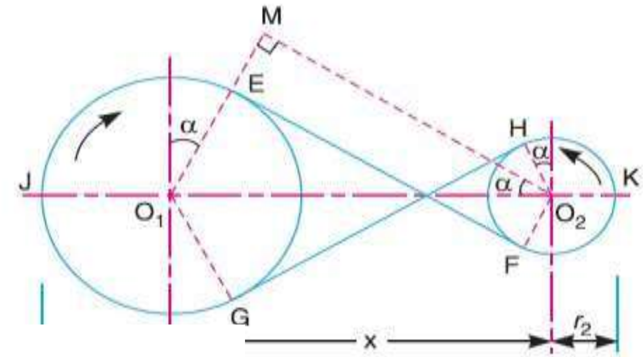
$$\therefore \text{ Work done per second} = (T_1 - T_2)\, v \text{ N-m/s}$$

power transmitted, $P = (T_1 - T_2)\, v$ W

Length of Open belt drive

$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$ ...(In tern

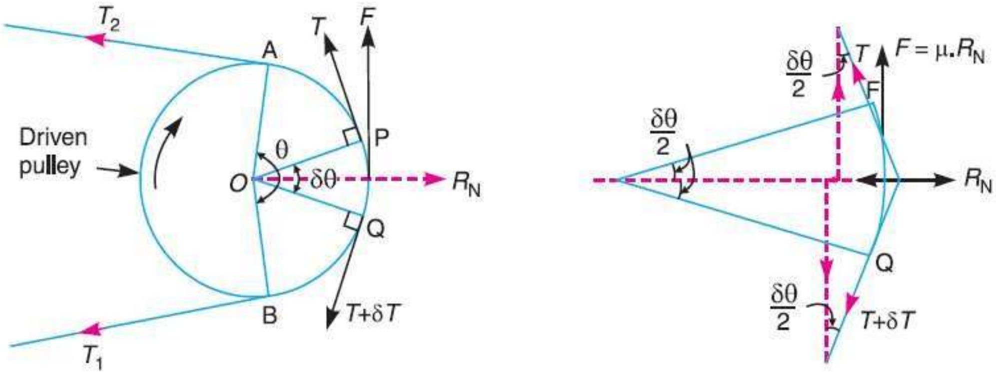
Length of cross belt drive



$$L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$ ...(In terms of pulley diameters)

BELT

Ratio of Driving Tensions For Flat Belt Drive



T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc *A B*, along which the belt touches the pulley at the centre).

$$\frac{T_1}{T_2} = e^{\mu.\theta}$$

BELT

Maximum Tension in the Belt

σ = Maximum safe stress in N/mm^2,

b = Width of the belt in mm, and

t = Thickness of the belt in mm.

T = Maximum stress × cross-sectional area of belt = $\sigma . b . t$

Initial Tension in the Belt

$$T_0 = \frac{T_1 + T_2}{2} \quad \text{...(Neglecting centrifugal tension)}$$

$$= \frac{T_1 + T_2 + 2T_C}{2} \quad \text{...(Considering centrifugal tension)}$$

UNIT 3

POWER SCREWS

Course objectives:

Implement basic principles for the design of power screws And the forces, couples, torques etc,

Course Outcomes:

Analyze power screws subjected to loading

Introduction

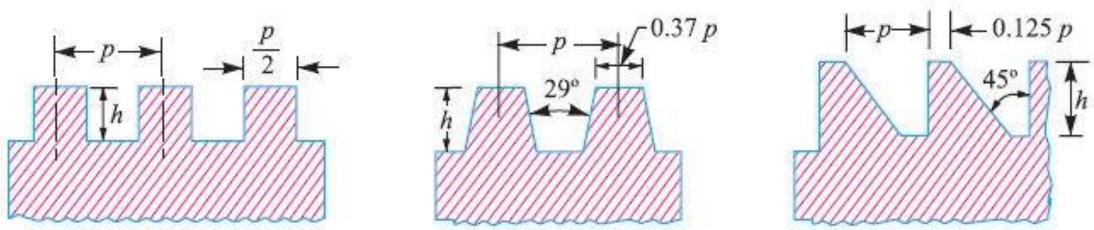
The power screws (also known as ***translation screws***) are used to convert rotary motion into translator motion. For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material. In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc.

In most of the power screws, the nut has axial motion against the resisting axial force while the screw rotates in its bearings. In some screws, the screw rotates and moves axially against the resisting force while the nut is stationary and in others the nut rotates while the screw moves axially with no rotation.

Types of Screw Threads used for Power Screws

Following are the three types of screw threads mostly used for power screws:

1. ***Square thread.*** A square thread, as shown in Fig. 17.1 (*a*), is adapted for the transmission of power in either direction. This thread results in maximum efficiency and minimum radial or bursting



$h = 0.5\,p$ (*a*) Square thread. $h = 0.5\,p + 0.25$ mm (*b*) Acme thread. $h = 0.75\,p$ (*c*) Buttress thread.

Pressure on the nut. It is difficult to cut with taps and dies. It is usually cut on a lathe with a

single point tool and it cannot be easily compensated for wear. The square threads are employed in screw jacks, presses and clamping devices. The standard dimensions for square threads according to IS: 4694 – 1968 (Reaffirmed 1996), are shown in Table 17.1 to 17.3.

2. ***Acme or trapezoidal thread.*** An acme or trapezoidal thread, as shown in Fig. 17.1 (*b*), is a modification of square thread. The slight slope given to its sides lowers the efficiency slightly than square thread and it also introduce some bursting pressure on the nut, but increases its area in shear. It is used where a split nut is required and where provision is made to take up wear as in the lead screw of a lathe.

Wear may be taken up by means of an adjustable split nut. An acme thread may be cut by means of dies and hence it is more easily manufactured than square thread. The standard dimensions for acme or trapezoidal threads are shown in Table 17.4 (Page 630).

3. ***Buttress thread*.** A buttress thread, as shown in Fig. 5.1 (*c*), is used when large forces act along the screw axis in one direction only. This thread combines the higher efficiency of square thread and the ease of cutting and the adaptability to a split nut of acme thread. It is stronger than other threads because of greater thickness at the base of the thread. The buttress thread has limited use for power transmission. It is employed as the thread for light jack screws and vices.

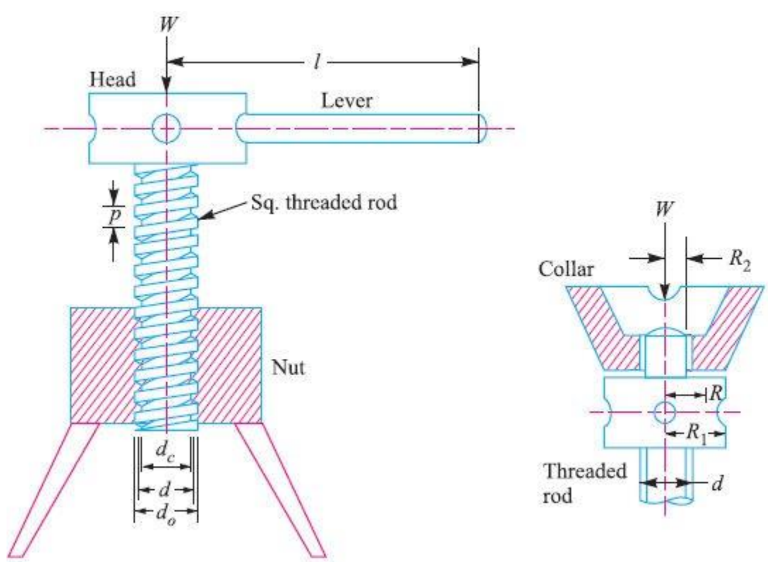
Multiple Threads

The power screws with multiple threads such as double, triple etc. are employed when it is desired to secure a large lead with fine threads or high efficiency. Such type of threads is usually found in high speed actuators.

Torque Required to Raise Load by Square Threaded Screws

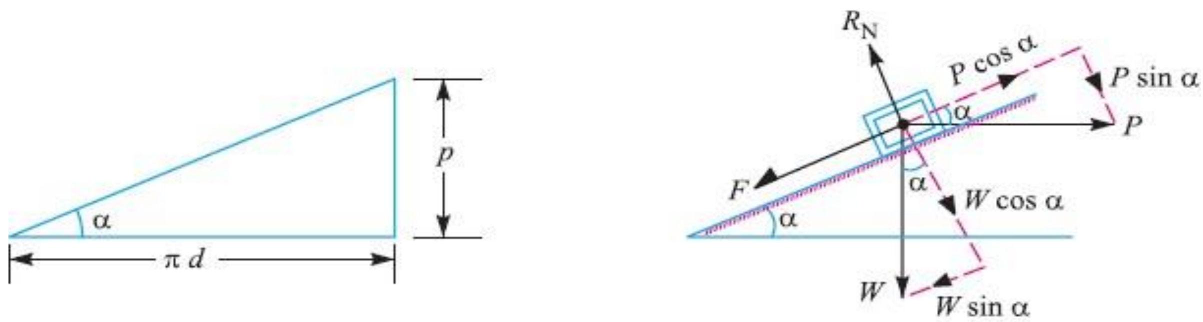
The torque required to raise a load by means of square threaded screw may be determined by considering a screw jack as shown in Fig. 5.2 (*a*). The load to be raised or lowered is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of lever for lifting or lowering the load.

A little consideration will show that if one complete turn of a screw thread be imagined to be unwound,



(*a*) Screw jack. (*b*) Thrust collar.

from the body of the screw and developed, it will form an inclined plane as shown in Fig. (*a*).



(*a*) Development of a screw. (*b*) Forces acting on the screw.

Let p = Pitch of the screw,

d = Mean diameter of the screw,

$\alpha = Helix\ angle$

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

$\mu = co\ efficient\ of\ friction$

= tan ∅ , is the friction angle

From the geometry of the Fig. (*a*), we find that

$\tan \alpha = p / \pi d$

Since the principle, on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the circumference of a screw jack may be considered to be horizontal as shown in Fig. 5.3 (*b*).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the body are shown in Fig. 5.3 (*b*).

Resolving the forces along the plane,

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu.R_N \qquad \text{.....(i)}$$

and resolving the forces perpendicular to the plane

$$R_N = P \sin \alpha + W \cos \alpha \qquad \text{.....(ii)}$$

Substituting this value of *R*N in equation ***(i)***, we have

$$P\cos\alpha = W\sin\alpha + \mu(P\sin\alpha + W\cos\alpha)$$
$$= W\sin\alpha + \mu P\sin\alpha + \mu W\cos\alpha$$

or $$P\cos\alpha - \mu P\sin\alpha = W\sin\alpha + \mu W\cos\alpha$$

or $$P(\cos\alpha - \mu\sin\alpha) = W(\sin\alpha + \mu\cos\alpha)$$

$$\therefore \quad P = W \times \frac{(\sin\alpha + \mu\cos\alpha)}{(\cos\alpha - \mu\sin\alpha)}$$

Substituting the value of $\mu = \tan\phi$ in the above equation, we get

or $$P = W \times \frac{\sin\alpha + \tan\phi\cos\alpha}{\cos\alpha - \tan\phi\sin\alpha}$$

Multiplying the numerator and denominator by $\cos\phi$, we have

$$P = W \times \frac{\sin\alpha\cos\phi + \sin\phi\cos\alpha}{\cos\alpha\cos\phi - \sin\alpha\sin\phi}$$

$$= W \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)} = W\tan(\alpha+\phi)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W\tan(\alpha+\phi)\frac{d}{2}$$

When the axial load is taken up by a thrust collar as shown in Fig. 5.2 (*b*), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \frac{2}{3} \times \mu_1 \times W\left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2}\right]$$

... (Assuming uniform pressure conditions)

$$= \mu_1 \times W\left(\frac{R_1 + R_2}{2}\right) = \mu_1 W R$$

....(Assuming uniform wear conditions)

where R_1 and R_2 = Outside and inside radii of collar,

R = Mean radius of collar $= \frac{R_1 + R_2}{2}$, and

μ_1 = Coefficient of friction for the collar.

Total torque required to overcome friction (*i.e.* to rotate the screw),

$T = T_1 + T_2$

If an effort *P*1 is applied at the end of a lever of arm length *l*, then the total torque required to overcome friction must be equal to the torque applied at the end of lever, *i.e.*

$$T = P \times \frac{d}{2} = P_1 \times l$$

$$T = P \times \frac{d}{2} = P_1 \times l$$

Notes: 1. When the *nominal diameter (*do*) and the **core diameter (*dc*) of the screw is given, then

Mean diameter of screw,

$$d = \frac{d_o + d_c}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2}$$

2. Since the mechanical advantage is the ratio of the load lifted (*W*) to the effort applied (P_1) at the end of the lever, therefore mechanical advantage,

$$\text{M.A.} = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} \qquad \ldots\left(\because P \times \frac{d}{2} = P_1 \times l \text{ or } P_1 = \frac{P \times d}{2l}\right)$$

$$= \frac{W \times 2l}{W \tan(\alpha + \phi)\, d} = \frac{2l}{d \tan(\alpha + \phi)}$$

Efficiency of Square Threaded Screws

The efficiency of square threaded screws may be defined as the ratio between the ideal effort (*i.e*. the effort required to move the load, neglecting friction) to the actual effort (*i.e*. the effort required to move the load taking friction into account).

We have seen in fig. that the effort applied at the circumference of the screw to lift the load is

$$P = W \tan(\alpha + \emptyset) \qquad \ldots(i)$$

where *W* = Load to be lifted,

α= Helix angle,

The value of effort *P*0 necessary to raise the load will then be given by the equation,

$$P_0 = W \tan(\alpha) \qquad \text{substituting } \emptyset = 0 \text{ in eqn (i)}$$

$$\therefore \text{Efficiency}, \quad \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W \tan \alpha}{W \tan(\alpha + \phi)} = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

This shows that the efficiency of a screw jack, is independent of the load raised.

In the above expression for efficiency, only the screw friction is considered. However, if the screw friction

$$\eta = \frac{\text{Torque required to move the load, neglecting friction}}{\text{Torque required to move the load, including screw and collar friction}}$$

$$= \frac{T_0}{T} = \frac{P_0 \times d/2}{P \times d/2 + \mu_1 . W . R}$$

and collar friction is taken into account, then

Note: The efficiency may also be defined as the ratio of mechanical advantage to the velocity ratio. We know that mechanical advantage,

$$M.A. = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} = \frac{W \times 2l}{W \tan(\alpha + \phi)\, d} = \frac{2l}{d \tan(\alpha + \phi)}$$

and velocity ratio, $V.R. = \dfrac{\text{Distance moved by the effort } (P_1) \text{ in one revolution}}{\text{Distance moved by the load } (W) \text{ in one revolution}}$

$$= \frac{2\pi l}{p} = \frac{2\pi l}{\tan\alpha \times \pi d} = \frac{2l}{d \tan\alpha} \qquad \ldots (\because \tan\alpha = p/\pi d)$$

$$\therefore \text{Efficiency,} \quad \eta = \frac{M.A.}{V.R.} = \frac{2l}{d \tan(\alpha + \phi)} \times \frac{d \tan\alpha}{2l} = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

Maximum Efficiency of a Square Threaded Screw

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)} = \frac{\sin\alpha / \cos\alpha}{\sin(\alpha + \phi) / \cos(\alpha + \phi)} = \frac{\sin\alpha \times \cos(\alpha + \phi)}{\cos\alpha \times \sin(\alpha + \phi)} \qquad \ldots(i)$$

Multiplying the numerator and denominator by 2, we have,

$$\eta = \frac{2\sin\alpha \times \cos(\alpha + \phi)}{2\cos\alpha \times \sin(\alpha + \phi)} = \frac{\sin(2\alpha + \phi) - \sin\phi}{\sin(2\alpha + \phi) + \sin\phi} \qquad \ldots(ii)$$

$$\ldots \begin{bmatrix} \because 2\sin A \cos B = \sin(A + B) + \sin(A - B) \\ 2\cos A \sin B = \sin(A + B) - \sin(A - B) \end{bmatrix}$$

The efficiency given by equation *(ii)* will be maximum when $\sin(2\alpha + \phi)$ is maximum, *i.e.* when

$$\sin(2\alpha + \phi) = 1 \quad \text{or} \quad \text{when} \quad 2\alpha + \phi = 90^\circ$$

$$\therefore \quad 2\alpha = 90^\circ - \phi \quad \text{or} \quad \alpha = 45^\circ - \phi/2$$

Substituting the value of 2α in equation *(ii)*, we have maximum efficiency,

$$\eta_{max} = \frac{\sin(90^\circ - \phi + \phi) - \sin\phi}{\sin(90^\circ - \phi + \phi) + \sin\phi} = \frac{\sin 90^\circ - \sin\phi}{\sin 90^\circ + \sin\phi} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

1. A vertical screw with single start square threads of 50 mm mean diameter and

12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 60 mm. The coefficient of friction is 0.15 for the screw and 0.18 for the collar. If the tangential force applied by each hand to the wheel is 100 N, find suitable diameter of the hand wheel.

Solution. Given : d = 50 mm ; p = 12.5 mm ; W = 10 kN = 10 × 10^3N ; D = 60 mm or

R = 30 mm ; $\mu = \tan \phi = 0.15$; $\mu_1 = 0.18$; P_1 = 100 N

We know that $\tan \alpha = \dfrac{p}{\pi d} = \dfrac{12.5}{\pi \times 50} = 0.08$

and the tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right)$$

$$= 10 \times 10^3\left[\frac{0.08 + 0.15}{1 - 0.08 \times 0.15}\right] = 2328 \text{ N}$$

We also know that the total torque required to turn the hand wheel,

$$T = P \times \frac{d}{2} + \mu_1 W R = 2328 \times \frac{50}{2} + 0.18 \times 10 \times 10^3 \times 30 \text{ N-mm}$$

= 58 200 + 54 000 = 112 200 N-mm ...*(i)*

Let $D1$ = Diameter of the hand wheel in mm.

$$T = 2 P_1 \times \frac{D_1}{2} = 2 \times 100 \times \frac{D_1}{2} = 100 D_1 \text{ N-mm} \quad ...(ii)$$

We know that the torque applied to the hand wheel,

Equating equations **(*i*)** and **(*ii*)**,

D_1 = 112 200 / 100 = 1122 mm = 1.122 m **Ans.**

Acme or Trapezoidal Threads

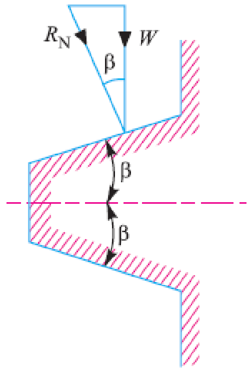
We know that the normal reaction in case of a square threaded screw is
$R_N = W \cos \alpha$

where $\alpha = Helix\ angle$

But in case of Acme or trapezoidal thread, the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load (W).

Consider an Acme or trapezoidal thread as shown in Fig. 5.6.

Let 2β= angle of the ACME Thread=29^0



$$\therefore \qquad R_N = \frac{W}{\cos\beta}$$

and frictional force, $F = \mu.R_N = \mu \times \frac{W}{\cos\beta} = \mu_1.W$

where $\mu / \cos\beta = \mu_1$, known as *virtual coefficient of friction.*

Notes : 1. When coefficient of friction, $\mu_1 = \frac{\mu}{\cos\beta}$ is considered, then the Acme thread is equivalent to a square thread.

2. All equations of square threaded screw also hold good for Acme threads. In case of Acme threads, μ_1 (*i.e.* $\tan\phi_1$) may be substituted in place of μ (*i.e.* $\tan\phi$). Thus for Acme threads,

$$P = W\tan(\alpha + \phi_1)$$

where ϕ_1 = Virtual friction angle, and $\tan\phi_1 = \mu_1$.

2. The lead screw of a lathe has Acme threads of 50 mm outside diameter and 8 mm pitch. The screw must exert an axial pressure of 2500 N in order to drive the tool carriage. The thrust is carried on a collar 110 mm outside diameter and 55 mm inside diameter and the lead screw rotates at 30 r.p.m. Determine (a) the power required to drive the screw; and (b) the efficiency of the lead screw. Assume a coefficient of friction of 0.15 for the screw and 0.12 for the collar.

Solution. Given : do = 50 mm ; p = 8 mm ; W = 2500 N ; $D1$ = 110 mm or $R1$ = 55 mm ;

$D2$ = 55 mm or $R2$ = 27.5 mm ;

(a) *Power required to drive the screw*

We know that mean diameter of the screw,

$$d = do - p / 2 = 50 - 8 / 2 = 46 \text{ mm}$$

$$\therefore \quad \tan \alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 46} = 0.055$$

Since the angle for Acme threads is $2\beta = 29°$ or $\beta = 14.5°$, therefore virtual coefficient of

fricti

$$\therefore \quad \tan \alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 46} = 0.055$$

Since the angle for Acme threads is $2\beta = 29°$ or $\beta = 14.5°$, therefore virtual coefficient of

friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.15}{\cos 14.5°} = \frac{0.15}{0.9681} = 0.155$$

We know that the force required to overcome friction at the screw,

$$P = W \tan (\alpha + \phi_1) = W \left[\frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right]$$

$$= 2500 \left[\frac{0.055 + 0.155}{1 - 0.055 \times 0.155} \right] = 530 \text{ N}$$

and t

and torque required to overcome friction at the screw.

$$T_1 = P \times d / 2 = 530 \times 46 / 2 = 12\,190 \text{ N-mm}$$

We know that mean radius of collar,

$$R = \frac{R_1 + R_2}{2} = \frac{55 + 27.5}{2} = 41.25 \text{ mm}$$

Assuming uniform wear, the torque required to overcome friction at collars,

$$T_2 = \mu_2 W R = 0.12 \times 2500 \times 41.25 = 12\,375 \text{ N-mm}$$

Total torque required to overcome the friction

$T = T1 + T2 = 12\,190 + 12\,375 = 24\,565$ N-mm $= 24.565$ N-m

$$= T.\omega = \frac{T \times 2\pi N}{60} = \frac{24.565 \times 2\pi \times 30}{60} = 77 \text{ W} = 0.077 \text{ kW} \qquad \textbf{Ans.}$$

... $(\because \ \omega = 2\pi N / 60)$

We know that power required to drive the screw

$= T$ ... $T \times 2\pi N$ ... $24.565 \times 2\pi \times 30$ ...

$$T_o = W \tan \alpha \times \frac{d}{2} = 2500 \times 0.055 \times \frac{46}{2} = 3163 \text{ N-mm} = 3.163 \text{ N-m}$$

$\therefore$ Efficiency of the lead screw,

$$\eta = \frac{T_o}{T} = \frac{3.163}{24.565} = 0.13 \text{ or } 13\% \qquad \textbf{Ans.}$$

(b) Efficiency of the lead screw We know that the torque required to drive the screw with no friction,

Stresses in Power Screws

A power screw must have adequate strength to withstand axial load and the applied torque. Following types of stresses are induced in the screw.

1. ***Direct tensile or compressive stress due to an axial load.*** The direct stress due to the axial load may be determined by dividing the axial load (W) by the minimum cross-sectional area of the screw (Ac) *i.e.* area corresponding to minor or core diameter (dc). Direct stress (tensile or compressive)

$=W/A$

This is only applicable when the axial load is compressive and the unsupported length of the screw between the load and the nut is short. But when the screw is axially loaded in compression and the unsupported length of the screw between the load and the nut is too great, then the design must be based on column theory assuming suitable end conditions. In such cases, the cross- sectional area corresponding to core diameter may be obtained by using Rankine-Gordon formula or J.B. Johnson's formula. According to this,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4\,C\,\pi^2\,E}\left(\frac{L}{k}\right)^2\right]$$

$$\therefore \qquad \sigma_c = \frac{W}{A_c}\left[\frac{1}{1 - \frac{\sigma_y}{4\,C\,\pi^2\,E}\left(\frac{L}{k}\right)^2}\right]$$

W_{cr} = Critical load,

σ_y = Yield stress,
L = Length of screw,

k = Least radius of gyration,

C = End-fixity coefficient,

E = Modulus of elasticity, and

σ_c = Stress induced due to load W.

Note : In actual practice, the core diameter is first obtained by considering the screw under simple compression and then checked for critical load or buckling load for stability of the screw.

2. ***Torsional shear stress.*** Since the screw is subjected to a twisting moment, therefore torsional shear stress is induced. This is obtained by considering the minimum cross-section of the screw. We know that torque transmitted by the screw,

$$T = \frac{\pi}{16} \times \tau\,(d_c)^3$$

or shear stress induced,

$$\tau = \frac{16\,T}{\pi (d_c)^3}$$

When the screw is subjected to both direct stress and torsional shear stress, then the design must be based on maximum shear stress theory, according to which maximum shear stress on the minor

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t \text{ or } \sigma_c)^2 + 4\,\tau^2}$$

diameter section,

It may be noted that when the unsupported length of the screw is short, then failure will take place when the maximum shear stress is equal to the shear yield strength of the material. In this case,

$$\tau_y = \tau_{max} \times \text{Factor of safety}$$

Shear yield strength,

3. ***Shear stress due to axial load.*** The threads of the screw at the core or root diameter and the threads of the nut at the major diameter may shear due to the axial load. Assuming that the load is

$$\tau_{(screw)} = \frac{W}{\pi\, n\, .d_c\,.\,t}$$

uniformly distributed over the threads in contact, we have Shear stress for screw,

and shear stress for nut,

$$\tau_{(nut)} = \frac{W}{\pi\, n\, .d_o\,.\,t}$$

Where W = Axial load on the screw,

n = Number of threads in engagement,

dc = Core or root diameter of the screw,

do = Outside or major diameter of nut or screw, and

t = Thickness or width of thread.

4. ***Bearing pressure.*** In order to reduce wear of the screw and nut, the bearing pressure on the thread surfaces must be within limits. In the design of power screws, the bearing pressure depends upon the materials of the screw and nut, relative velocity between the nut and screw and the nature of lubrication. Assuming that the load is uniformly distributed over the threads in contact, the bearing pressure on the threads is given by

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{{}^{*}W}{\pi\, d.t.n}$$

where d = Mean diameter of screw,

t = Thickness or width of screw = $p / 2$, and

n = Number of threads in contact with the nut

$$= \frac{\text{Height of the nut}}{\text{Pitch of threads}} = \frac{h}{p}$$

Therefore, from the above expression, the height of nut or the length of thread engagement of the screw and nut may be obtained.

* We know that $\frac{(d_o)^2 - (d_c)^2}{4} = \frac{d_o + d_c}{2} \times \frac{d_o - d_c}{2} = d \times \frac{p}{2} = d.t$

The following table shows some limiting values of bearing pressures.

3 . A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN. The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and collar friction may be assumed as 0.2 and 0.15 respectively. The screw rotates at 12 r.p.m. Assuming uniform wear condition at the collar and allowable thread bearing pressure of 5.8 N/mm2, find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw.

Solution. Given : *do* = 25 mm ; *p* = 5 mm ; *W* = 10 kN = 10 × 103 N ; *D*1 = 50 mm or

R_1 = 25 mm ; D_2 = 20 mm or R_2 = 10 mm; μ_1 = 0.15 ; *N* = 12 r.p.m. ; p_b =5.8 N/mm2

1. ***Torque required to rotate the screw*** We

know that mean diameter of the screw, *d* = *do* –

p / 2 = 25 – 5 / 2 = 22.5 mm

Since the screw is a double start square threaded screw, therefore lead of the screw,

= 2 *p* = 2 × 5 = 10 mm

$\therefore \quad \tan\alpha = \frac{\text{Lead}}{\pi d} = \frac{10}{\pi \times 22.5} = 0.1414$

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W \left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi} \right]$$

$$= 10 \times 10^3 \left[\frac{0.1414 + 0.2}{1 - 0.1414 \times 0.2} \right] = 3513 \text{ N}$$

and mean radius of the screw collar,

$$R = \frac{R_1 + R_2}{2} = \frac{25 + 10}{2} = 17.5$$

$\therefore$ Total torque required to rotate the screw,

$$T = P \times \frac{d}{2} + \mu_1 W R = 3513 \times \frac{22.5}{2} + 0.15 \times 10 \times 10^3 \times 17.5 \text{ N-mm}$$

$$= 65\,771 \text{ N-mm} = 65.771 \text{ N-m}$$ **Ans.**

2. *Stress in the screw*

We know that the inner diameter or core diameter of the screw,

$dc = do - p = 25 - 5 = 20$ mm

-sectional area of the screw,

Corresponding cross

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(20)^2 = 314.2 \text{ mm}^2$$

We know that direct stress,

$$\sigma_c = \frac{W}{A_c} = \frac{10 \times 10^3}{314.2} = 31.83 \text{ N/mm}^2$$

and shear stress,

$$\tau = \frac{16\,T}{\pi (d_c)^3} = \frac{16 \times 65\,771}{\pi (20)^3} = 41.86 \text{ N/mm}^2$$

We know that maximum shear stress in the screw,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(31.83)^2 + 4(41.86)^2}$$

$$= 44.8 \text{ N/mm}^2 = 44.8 \text{ MPa}$$ **Ans.**

3. *Number of threads of nut in engagement with screw*

Let n = Number of threads of nut in engagement with screw, and

t = Thickness of threads = $p / 2 = 5 / 2 = 2.5$ mm

We know that bearing pressure on the threads (p_b),

$$5.8 = \frac{W}{\pi d \times t \times n} = \frac{10 \times 10^3}{\pi \times 22.5 \times 2.5 \times n} = \frac{56.6}{n}$$

$$\therefore \quad n = 56.6 / 5.8 = 9.76 \text{ say } 10 \quad \textbf{Ans.}$$

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$
$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

or $$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

or $$P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$$\therefore \quad P = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

or $$P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$, we have

$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi}$$

$$= W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} = W \tan (\alpha + \phi)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

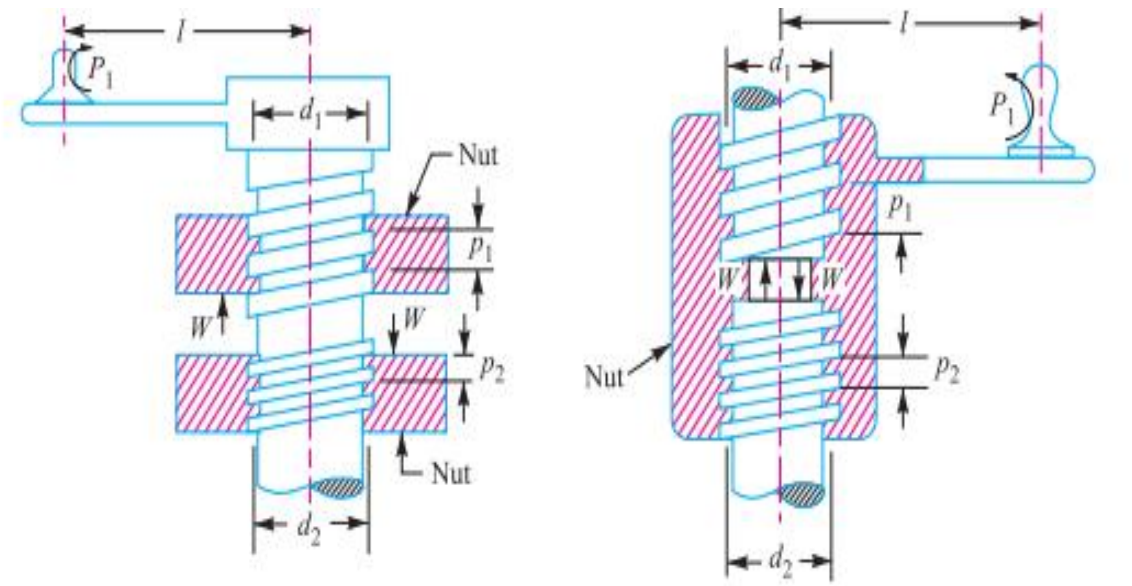
$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

Differential and Compound Screws

There are certain cases in which a very slow movement of the screw is required whereas in other cases, a very fast movement of the screw is needed. The slow movement of the screw may be obtained by using a small pitch of the threads, but it results in weak threads. The fast movement of the screw may be obtained by using multiple-start threads, but this method requires expensive machning and the loss of self-locking property. In order to overcome these difficulties, differential or compound screws, as discussed below, are used.

1. Differential screw. When a slow movement or fine adjustment is desired in precision

equipments, then a differential screw is used. It consists of two threads of the same hand (i.e. right handed or left handed) but of different pitches, wound on the same cylinder or different cylinders as shown in Fig. It may be noted that when the threads are wound on the same cylinder, then two nuts are employed as shown in Fig. (a) and when the threads are wound on different cylinders, then only one nut is employed as shown in Fig. (b).



(a) Threads wound on the same cylinder. (b) Threads wound on the different cylinders.

p_1 = Pitch of the upper screw,

d_1 = Mean diameter of the upper screw,

α_1 = Helix angle of the upper screw, and

μ_1 = Coefficient of friction between the upper screw and the upper nut

= tan ϕ_1, where ϕ_1 is the friction angle.

p_2, d_2, α_2 and μ_2 = Corresponding values for the lower screw.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan(\alpha_1 + \phi_1)\frac{d_1}{2} = W\left[\frac{\tan\alpha_1 + \tan\phi_1}{1 - \tan\alpha_1 \tan\phi_1}\right]\frac{d_1}{2} \qquad ...(i)$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan(\alpha_2 + \phi_2)\frac{d_2}{2} = W\left[\frac{\tan\alpha_2 + \tan\phi_2}{1 - \tan\alpha_2 \tan\phi_2}\right]\frac{d_2}{2} \qquad ...(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1 \times l = T_1 - T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan\phi_1 = 0$ and $\mu_2 = \tan\phi_2 = 0$. Substituting these values in the above expressions, we have

$$\therefore \quad T_1' = W \tan\alpha_1 \times \frac{d_1}{2}$$

and

$$T_2' = W \tan\alpha_2 \times \frac{d_2}{2}$$

∴ Total torque required when there is no friction,

$$T_0 = T_1' - T_2'$$

$$= W \tan\alpha_1 \times \frac{d_1}{2} - W \tan\alpha_2 \times \frac{d_2}{2}$$

$$= W\left[\frac{p_1}{\pi d_1} \times \frac{d_1}{2} - \frac{p_2}{\pi d_2} \times \frac{d_2}{2}\right] = \frac{W}{2\pi}(p_1 - p_2)$$

$$\left[\because \tan\alpha_1 = \frac{p_1}{\pi d_1};\ \text{and} \tan\alpha_2 = \frac{p_2}{\pi d_2}\right]$$

We know that efficiency of the differential screw,

$$\eta = \frac{T_0}{T}$$

2. Compound screw. When a fast movement is desired, then a compound screw is employed. It consists of two threads of opposite hands (i.e. one right handed and the other left handed) wound on the same cylinder or different cylinders, as shown in Fig. (a) and (b) respectively. In this case, each revolution of the screw causes the nuts to move towards one another equal to the sum of the pitches of the threads. Usually the pitch of both the threads are made equal.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan(\alpha_1 + \phi_1)\frac{d_1}{2} = W\left[\frac{\tan\alpha_1 + \tan\phi_1}{1 - \tan\alpha_1 \tan\phi_1}\right]\frac{d_1}{2} \qquad ...(i)$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan(\alpha_2 + \phi_2)\frac{d_2}{2} = W\left[\frac{\tan\alpha_2 + \tan\phi_2}{1 - \tan\alpha_2 \tan\phi_2}\right]\frac{d_2}{2} \qquad \text{...}(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1 \times l = T_1 + T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan\phi_1 = 0$ and $\mu_2 = \tan\phi_2 = 0$. Substituting these values in the above expressions, we have

$$T_1' = W \tan\alpha_1 \times \frac{d_1}{2}$$

$$T_2' = W \tan\alpha_2 \times \frac{d_2}{2}$$

∴ Total torque required when there is no friction,

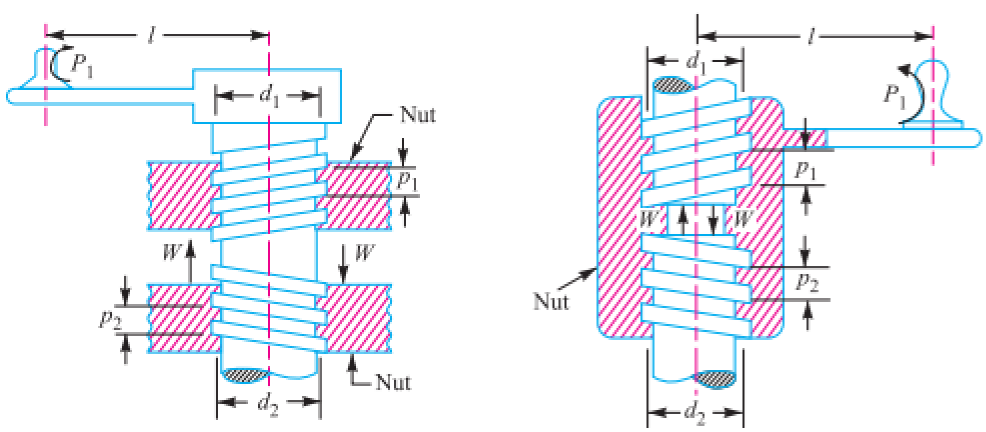
$$T_0 = T_1' + T_2'$$

$$= W \tan\alpha_1 \times \frac{d_2}{2} + W \tan\alpha_2 \times \frac{d_2}{2}$$

$$= W\left[\frac{p_1}{\pi d_1} \times \frac{d_1}{2} + \frac{p_2}{\pi d_2} \times \frac{d_2}{2}\right] = \frac{W}{2\pi}(p_1 + p_2)$$

We know that efficiency of the compound screw,

$$\eta = \frac{T_0}{T}$$



(*a*) Threads wound on the same cylinder. (*b*) Threads wound on the different cylinders.

4. A differential screw jack is to be made as shown in Fig. Neither screw rotates. The outside screw diameter is 50 mm. The screw threads are of square form single start and the coefficient of thread friction is 0.15.

Determine : 1. Efficiency of the screw jack; 2. Load

that can be lifted if the shear stress in the body of the screw is limited to 28 MPa.
Solution. Given : d_o = 50 mm ; $\mu = \tan\phi = 0.15$;

p_1 = 16 mm ; p_2 = 12 mm ; τ_{max} = 28 MPa = 28 N/mm2

1. Efficiency of the screw jack

We know that the mean diameter of the upper screw,

$$d_1 = d_o - p_1/2 = 50 - 16/2 = 42 \text{ mm}$$

and mean diameter of the lower screw,

$$d_2 = d_o - p_2/2 = 50 - 12/2 = 44 \text{ mm}$$

$$\therefore \quad \tan\alpha_1 = \frac{p_1}{\pi d_1} = \frac{16}{\pi \times 42} = 0.1212$$

and

$$\tan\alpha_2 = \frac{p_2}{\pi d_2} = \frac{12}{\pi \times 44} = 0.0868$$

Let W = Load that can be lifted in N.

$$T_1 = W\tan(\alpha_1 + \phi)\frac{d_1}{2} = W\left[\frac{\tan\alpha_1 + \tan\phi}{1 - \tan\alpha_1 \tan\phi}\right]\frac{d_1}{2}$$

$$= W\left[\frac{0.1212 + 0.15}{1 - 0.1212 \times 0.15}\right]\frac{42}{2} = 5.8\,W \text{ N-mm}$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W\tan(\alpha_2 - \phi)\frac{d_2}{2} = W\left[\frac{\tan\alpha_2 - \tan\phi}{1 + \tan\alpha_2 \tan\phi}\right]\frac{d_2}{2}$$

$$= W\left[\frac{0.0868 - 0.15}{1 + 0.0868 \times 0.15}\right]\frac{44}{2} = -1.37\,W \text{ N-mm}$$

$\therefore$ Total torque required to overcome friction,

$$T = T_1 - T_2 = 5.8\,W - (-1.37\,W) = 7.17\,W \text{ N-mm}$$

We know that the torque required when there is no friction,

$$T_0 = \frac{W}{2\pi}(p_1 - p_2) = \frac{W}{2\pi}(16 - 12) = 0.636\,W \text{ N-mm}$$

$\therefore$ Efficiency of the screw jack,

$$\eta = \frac{T_0}{T} = \frac{0.636\,W}{7.17\,W} = 0.0887 \text{ or } 8.87\% \quad \textbf{Ans.}$$

2. Load that can be lifted

Since the upper screw is subjected to a larger torque, therefore the load to be lifted (W) will be calculated on the basis of larger torque (T1).

We know that core diameter of the upper screw,

$$d_{c1} = d_o - p_1 = 50 - 16 = 34 \text{ mm}$$

Since the screw is subjected to direct compressive stress due to load W and shear stress due to torque T_1, therefore

Direct compressive stress,

$$\sigma_c = \frac{W}{A_{c1}} = \frac{W}{\frac{\pi}{4}(d_{c1})^2} = \frac{W}{\frac{\pi}{4}(34)^2} = \frac{W}{908} \text{ N/mm}^2$$

and shear stress,

$$\tau = \frac{16\,T_1}{\pi\,(d_{c1})^3} = \frac{16 \times 5.8\,W}{\pi(34)^3} = \frac{W}{1331} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

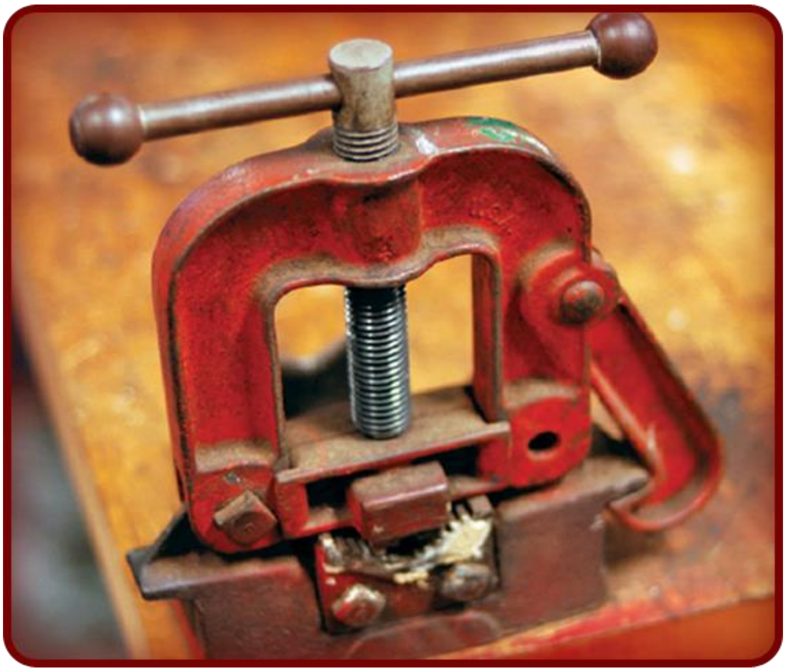
$$28 = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{W}{908}\right)^2 + 4\left(\frac{W}{1331}\right)^2}$$

$$= \frac{1}{2}\sqrt{1.213 \times 10^{-6}\,W^2 + 2.258 \times 10^{-6}\,W^2} = \frac{1}{2}\,1.863 \times 10^{-3}\,W$$

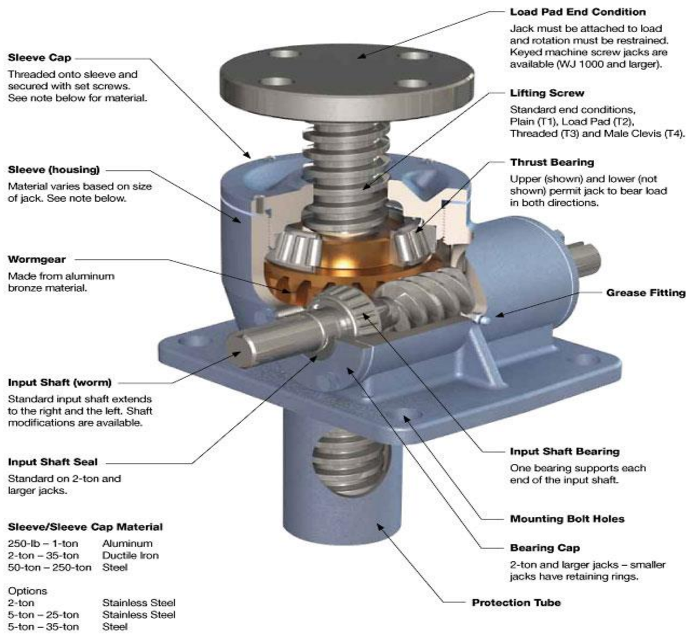
$$\therefore \quad W = \frac{28 \times 2}{1.863 \times 10^{-3}} = 30\,060 \text{ N} = 30.06 \text{ kN} \quad \textbf{Ans.}$$

Industriual Applications:

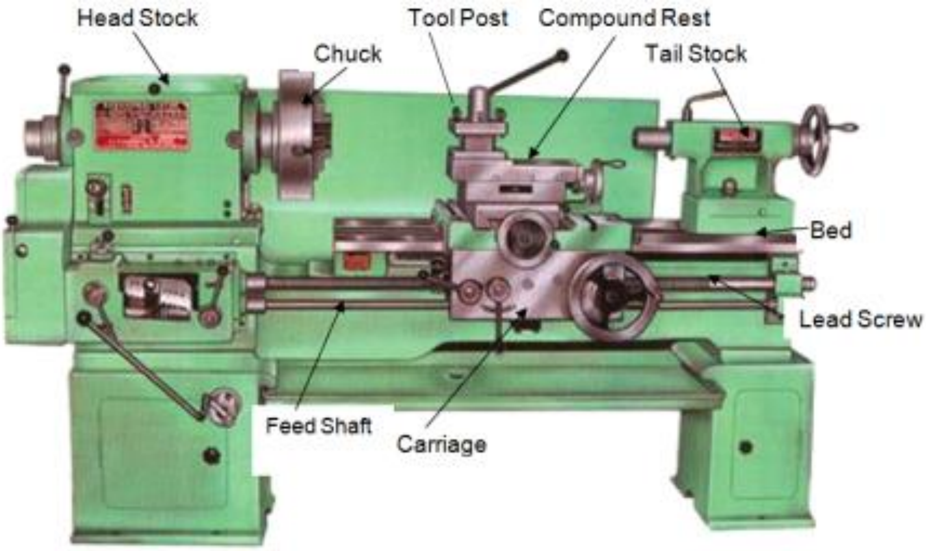
1. Carpentary vices



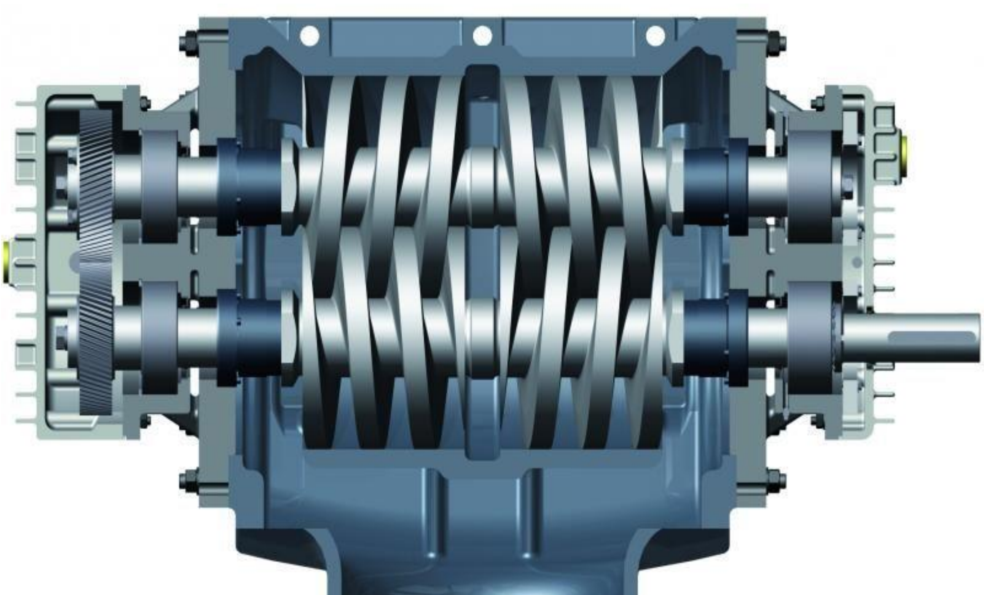
2. Lifting tool accessories



3. Lathe machines



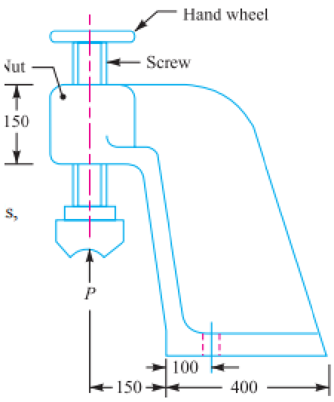
4. Differential screw in marine applications



TUTORIAL QUESTIONS

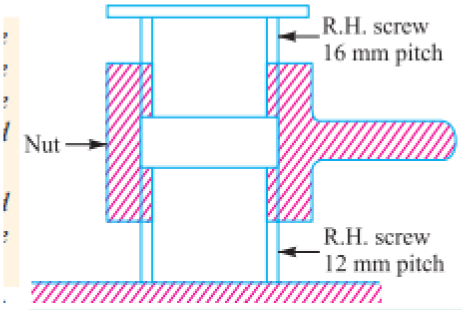
1. Discuss the various types of power threads. Give atleast two practical applications for each type.Discuss their relative advantages and disadvantages.
2. Why are square threads preferable to V-threads for power transmission?
3. What is self locking property of threads and where it is necessary?
4. A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN. The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and collar friction maybe assumed as 0.2 and 0.15 respectively. The screw rotates at 12 r.p.m. Assuming uniform wear condition at the collar and allowable thread bearing pressure of 5.8 N/mm2, find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw. The screw of a shaft straightener exerts a load of 30 kN as shown in Fig. The screw is square threaded of outside diameter 75 mm and 6 mm
5. pitch. Determine:1. Force required at the rim of a 300 mm diameter hand wheel, assuming the coefficient of friction for the threads as 0.12;

3. Maximum compressive stress in the screw, bearing pressure on the threads and maximum shear stress in threads; and 3. Efficiency of the straightner



4. A screw jack is to lift a load of 80 kN through a height of 400 mm. The elastic strength of screw material in tension and compression is 200 MPa and in shear 120 MPa. The material for nut is phosphor-bronze for which the elastic limit may be taken as 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18 N/mm2. Design and draw the screw jack. The design should include the design of 1.screw, 2. nut, 3. handle and cup,and 4. body.

5. A differential screw jack is to be made as shown in Fig. Neither screw rotates. The outside screw diameter is 50 mm. The screw threads are of square form single start and the coefficient of thread friction is 0.15.Determine : 1. Efficiency of the screw jack; 2. Loadthat can be lifted if the shear stress in the body of the screw is limited to 28 MPa.



ASSIGNMENT QUESTIONS

1. A screw jack is to lift a load of 80 kN through aheight of 400 mm. The elastic strength of screw material in tension and compression is 200 MPa and in shear 120 MPa. The material for nut is phosphor-bronze for which the elastic limit may be taken as 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18 N/mm2. Design and draw the screw jack. The design should include the design of 1. screw, 2. nut, 3. handle and cup,and 4. body.

2. A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN. The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and collar friction may be assumed as 0.2 and 0.15 respectively. The screw rotates at 12 r.p.m. Assuming uniform wear condition at the collar and allowable thread bearing pressure of 5.8 N/mm2, find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw.

3. A power transmission screw of a screw press is required to transmit maximum load of 100 kN and rotates at 60 r.p.m. Trapezoidal threads are as under :

Nominal dia, mm	40	50	60	70
Core dia, mm	32.5	41.5	50.5	59.5
Mean dia, mm	36.5	46	55.5	65
Core area, mm^2	830	1353	2003	2781
Pitch, mm	7	8	9	10

The screw thread friction coefficient is 0.12. Torque required for collar friction and journal bearing is about 10% of the torque to drive the load considering screw friction. Determine screw dimensions and its efficiency. Also determine motor power required to drive the screw. Maximum permissible compressive stress in screw is 100 MPa.

4. A vertical two start square threaded screw of 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The nut of the screw is fitted in the hub of a gear wheel having 80 teeth which meshes with a pinion of 20 teeth. The mechanical efficiency of the pinion and gear wheel drive is 90 percent. The axial thrust on the screw is taken by a collar bearing 250 mm outside diameter and 100 mm inside diameter. Assuming uniform pressure conditions, find, minimum diameter of pinion shaft and height of nut, when coefficient of friction for the vertical screw and nut is 0.15 and that for the collar bearing is 0.20. The permissible shear stress in the shaft material is 56 MPa and allowable bearing pressure is 1.4 N/mm^2

5. A square threaded bolt of mean diameter 24 mm and pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50 mm. If the coefficient of friction for the nut and bolt is 0.1 and for the nut and bearing surfaces 0.16, find the force required at the end of a spanner 0.5 m long when the load on the bolt is 10 kN.

UNIT-5

POWER SCREWS

MRCET CAMPUS

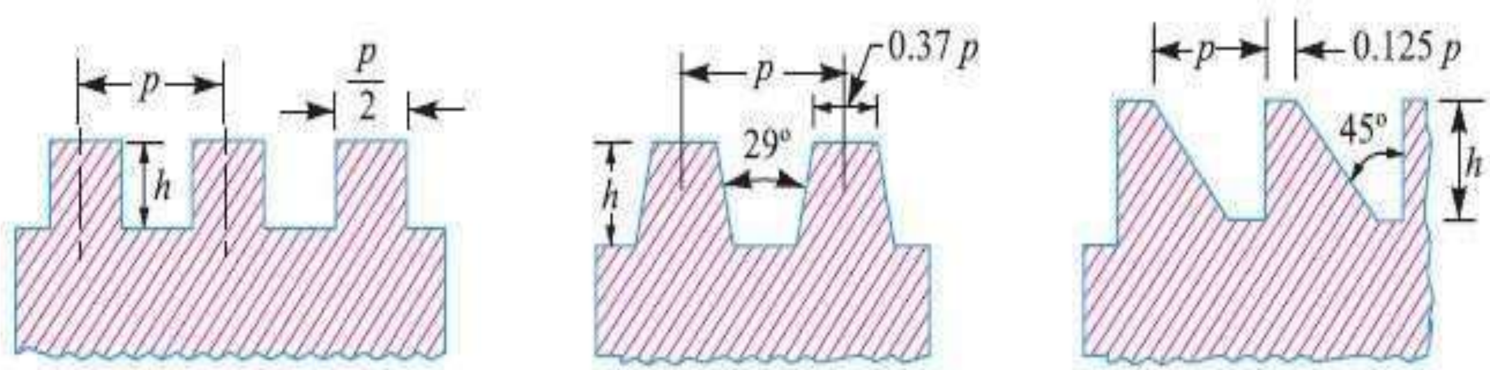
UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

INTRODUCTION

- rotates and moves axially against the resisting force while the nut is stationary and in others the nut rotates while the screw The power screws (also known as ***translation screws***) are used to convert rotary motion into translator motion. For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material. In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc.

TYPES OF SCREW THREADS



$h = 0.5\,p$
(*a*) Square thread.

$h = 0.5\,p + 0.25$ mm
(*b*) Acme thread.

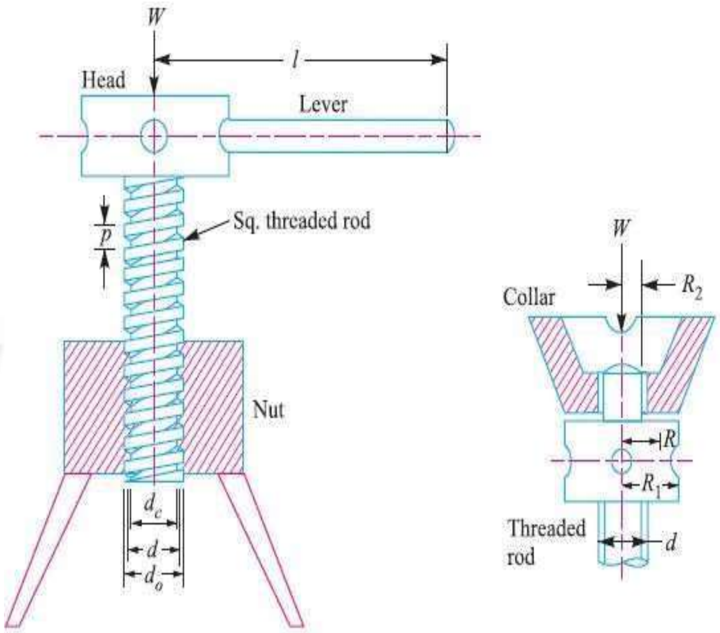
$h = 0.75\,p$
(*c*) Buttress thread.

Torque Required to Raise Load by Square Threaded Screws

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi) \frac{d}{2}$$

$$T_2 = \frac{2}{3} \times \mu_1 \times W \left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2} \right]$$ ... (Assuming uniform pressure conditions)

$$= \mu_1 \times W \left(\frac{R_1 + R_2}{2} \right) = \mu_1 W R$$ ....(Assuming uniform wear conditions)



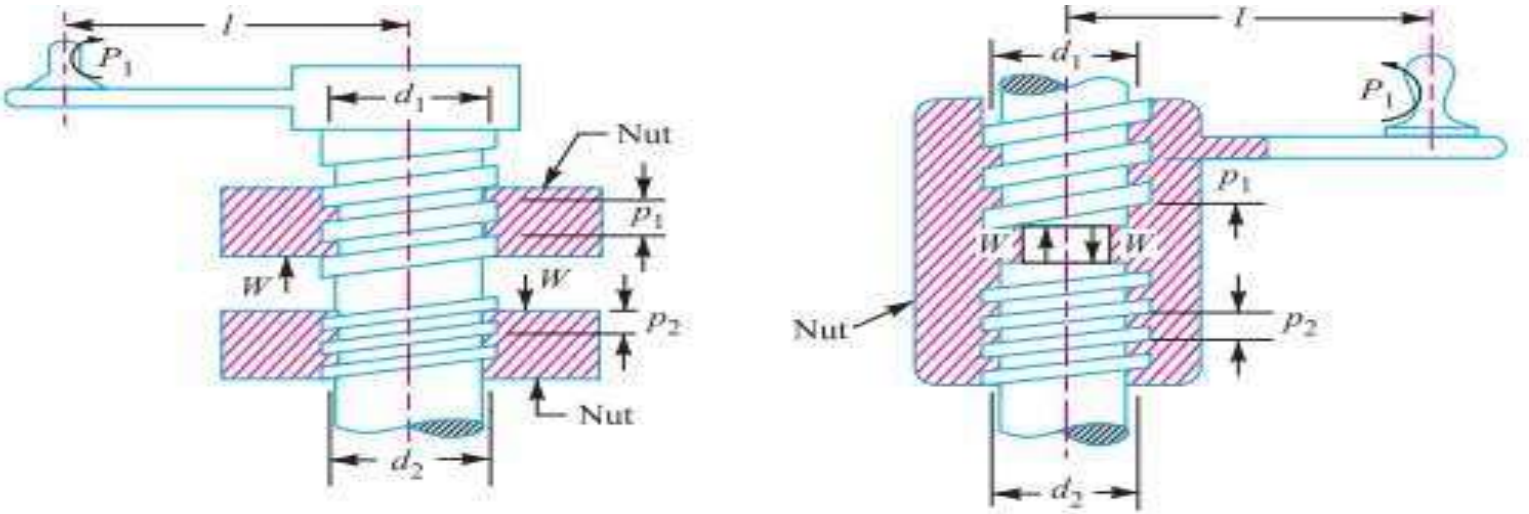
(a) Screw jack. (b) Thrust collar.

Total torque required to overcome friction (*i.e.* to rotate the screw),

$T = T_1 + T_2$

$$T = P \times \frac{d}{2} = P_1 \times l$$

DIFFERENTIAL SCREWS



inders.

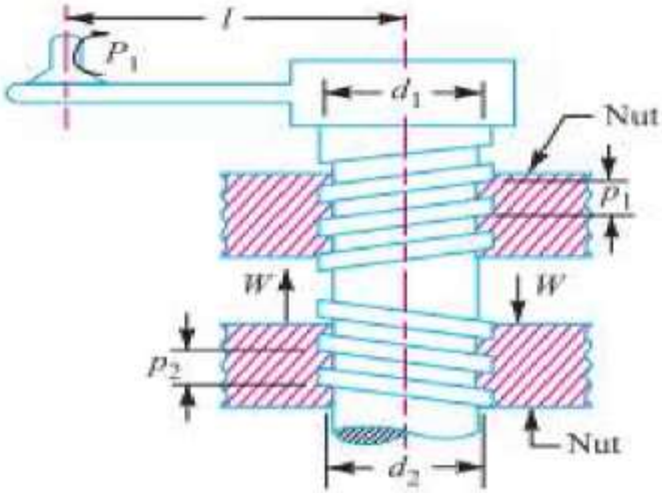
$$= W\left[\frac{p_1}{\pi d_1}\times\frac{d_1}{2} - \frac{p_2}{\pi d_2}\times\frac{d_2}{2}\right] = \frac{W}{2\pi}(p_1 - p_2)$$

$$\left[\because \tan\alpha_1 = \frac{p_1}{\pi d_1}; \text{ and } \tan\alpha_2 = \frac{p_2}{\pi d_2}\right]$$

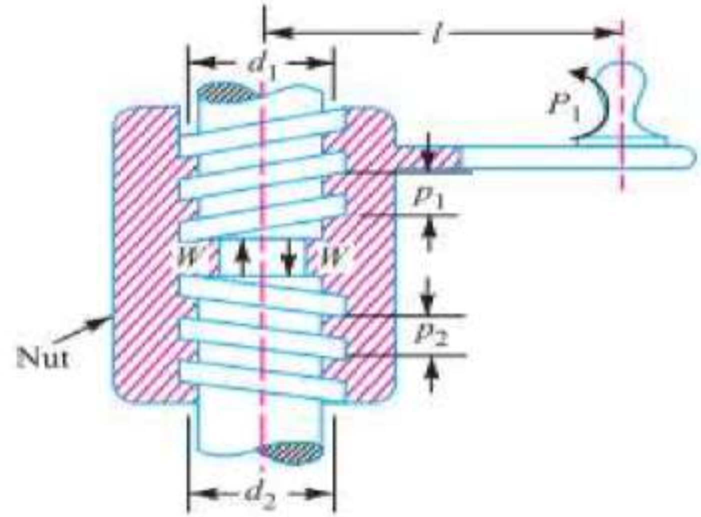
We know that efficiency of the differential screw,

$$\eta = \frac{T_0}{T}$$

COMPOUND SCREWS



(a) Threads wound on the same cylinder.



(b) Threads wound on the different cylinders.

$$= W\left[\frac{p_1}{\pi d_1}\times\frac{d_1}{2}+\frac{p_2}{\pi d_2}\times\frac{d_2}{2}\right]=\frac{W}{2\pi}(p_1+p_2)$$

We know that efficiency of the compound screw,

$$\eta = \frac{T_0}{T}$$