



UNIT 1

PRECESSION



COURSE OBJECTIVES

To study about gyroscope and its effects during precession motion of moving vehicles.

COURSE OUTCOMES

Knowledge acquired about Gyroscope and its precession motion.



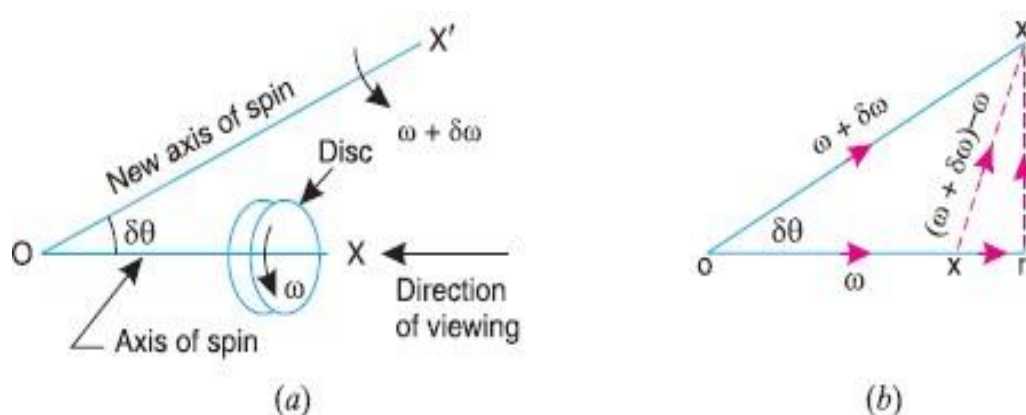
Introduction

When a body moves along a curved path with a uniform linear velocity, a force in the direction of centripetal acceleration (known as centripetal force) has to be applied externally over the body, so that it moves along the required curved path. This external force applied is known as **active force**.

When a body, itself, is moving with uniform linear velocity along a circular path, it is subjected to the centrifugal force* radially outwards. This centrifugal force is called **reactive force**. The action of the reactive or centrifugal force is to tilt or move the body along radially outward direction.

Precessional Angular Motion

We have already discussed that the angular acceleration is the rate of change of angular velocity with respect to time. It is a vector quantity and may be represented by drawing a vector diagram with the help



of right hand screw rule.

Consider a disc, as shown in Fig (a), revolving or spinning about the axis OX (known as **axis of spin**) in anticlockwise when seen from the front, with an angular velocity ω in a plane at right angles to the paper.

After a short interval of time δt , let the disc be spinning about the new axis of spin OX' (at an angle $\delta\theta$) with an angular velocity $(\omega + \delta\omega)$. Using the right hand screw rule, initial angular velocity of the disc (ω) is represented by vector ox ; and the final angular velocity of the disc $(\omega + \delta\omega)$ is represented by vector ox' as shown in Fig. 14.1 (b). The vector xx' represents the change of angular velocity in time δt i.e. the angular

Component of angular acceleration in the direction of ox ,

$$\begin{aligned}\alpha_t &= \frac{xr}{\delta t} = \frac{or - ox}{\delta t} = \frac{ox' \cos \delta\theta - ox}{\delta t} \\ &= \frac{(\omega + \delta\omega) \cos \delta\theta - \omega}{\delta t} = \frac{\omega \cos \delta\theta + \delta\omega \cos \delta\theta - \omega}{\delta t}\end{aligned}$$

Since $\delta\theta$ is very small, therefore substituting $\cos \delta\theta = 1$, we have

$$\alpha_t = \frac{\omega + \delta\omega - \omega}{\delta t} = \frac{\delta\omega}{\delta t}$$

acceleration of the disc. This may be resolved into two components.

One parallel to ox and the other perpendicular to ox .



In the limit, when $\delta t \rightarrow 0$,

$$\alpha_t = \lim_{\delta t \rightarrow 0} \left(\frac{\delta \omega}{\delta t} \right) = \frac{d\omega}{dt}$$

Component of angular acceleration in the direction perpendicular to ox ,

$$\alpha_c = \frac{rx'}{\delta t} = \frac{ox' \sin \delta \theta}{\delta t} = \frac{(\omega + \delta \omega) \sin \delta \theta}{\delta t} = \frac{\omega \sin \delta \theta + \delta \omega \sin \delta \theta}{\delta t}$$

Since $\delta \theta$ is very small, therefore substituting $\sin \delta \theta = \delta \theta$, we have

$$\alpha_c = \frac{\omega \cdot \delta \theta + \delta \omega \cdot \delta \theta}{\delta t} = \frac{\omega \cdot \delta \theta}{\delta t}$$

...(Neglecting $\delta \omega \cdot \delta \theta$, being very small)

In the limit when $\delta t \rightarrow 0$,

$$\alpha_c = \lim_{\delta t \rightarrow 0} \frac{\omega \cdot \delta \theta}{\delta t} = \omega \times \frac{d\theta}{dt} = \omega \cdot \omega_p \quad \dots \left(\text{Substituting } \frac{d\theta}{dt} = \omega_p \right)$$

$\therefore$ Total angular acceleration of the disc

= vector xx' = vector sum of α_t and α_c

$$= \frac{d\omega}{dt} + \omega \times \frac{d\theta}{dt} = \frac{d\omega}{dt} + \omega \cdot \omega_p$$

Where $d\theta/dt$ is the angular velocity of the axis of spin about a certain axis, which is perpendicular to the plane in which the axis of spin is going to rotate. This angular velocity of the axis of spin (i.e. $d\theta/dt$) is known as angular velocity of precession and is denoted by ω_p . The axis, about which the axis of spin is to turn, is known as axis of precession. The angular motion of the axis of spin about the axis of precession is known as precessional angular motion.

Gyroscopic Couple

Consider a disc spinning with an angular velocity ω rad/s about the axis of

spin OX , in anticlockwise direction when seen from the front, as shown in Fig. 14.2 (a). Since the plane in which the disc is rotating is parallel to the plane YOZ , therefore it is called plane of spinning. The plane XOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis OY . In other words, the axis of spin is said to be rotating or processing about an axis OY . In other words, the axis of spin is said to be rotating or processing about an axis OY (which is perpendicular to both the axes OX and OZ) at an angular velocity ω_p rad/s. This horizontal plane XOZ is called plane of precession and OY is the axis of precession.

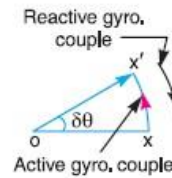
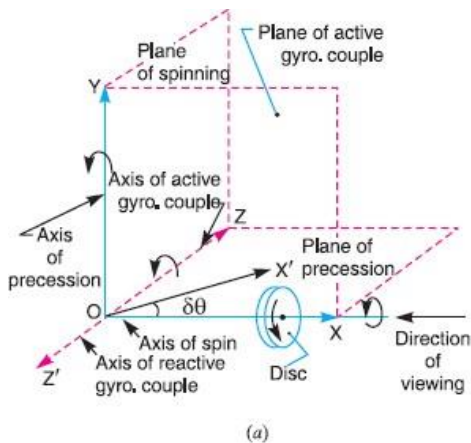
Let I = Mass moment of inertia of the disc about OX , and ω = Angular velocity of the disc.

Angular momentum of the disc = $I \cdot \omega$

Since the angular momentum is a vector quantity, therefore it may be represented by the vector ox' , as shown in Fig. 14.2 (b). The axis of spin OX is also rotating anticlockwise when seen from the top about the axis OY . Let the axis OX is turned in the plane XOZ through a small angle $\delta \theta$ radians to the position OX' , in



time δt seconds. Assuming the angular velocity ω to be constant, the angular momentum will now be represented by vector ox' .



$$= \vec{ox'} - \vec{ox} = \vec{xx'} = \vec{ox} \cdot \delta\theta \quad \dots (\text{in the direction of } \vec{xx'})$$

$$= I \cdot \omega \cdot \delta\theta$$

Change in angular momentum and rate of change of angular momentum

$$= I \cdot \omega \times \frac{\delta\theta}{dt}$$

$$C = \lim_{\delta t \rightarrow 0} I \cdot \omega \times \frac{\delta\theta}{\delta t} = I \cdot \omega \times \frac{d\theta}{dt} = I \cdot \omega \cdot \omega_p \quad \dots \left(\because \frac{d\theta}{dt} = \omega_p \right)$$

Since the rate of change of angular momentum will result by the application of a couple to the disc,

Therefore the couple applied to the disc causing precession,

Where ω_p = Angular velocity of precession of the axis of spin or the speed of rotation of the axis of spin about the axis of precession OY.

The couple $I \cdot \omega \cdot \omega_p$, in the direction of the vector xx' (representing the change in angular momentum) is the *active gyroscopic couple*, which has to be applied over the disc when the axis of spin is made to rotate with angular velocity ω_p about the axis of precession. The vector xx' lies in the plane XOZ or the horizontal plane. In case of a very small displacement $\delta\theta$, the vector xx' will be perpendicular to the vertical plane XOY. Therefore the couple causing this change in the angular momentum will lie in the plane XOY. The vector xx' , as shown in Fig(b), represents an anticlockwise couple in the plane XOY. Therefore, the plane XOY is called the *plane of active gyroscopic couple* and the axis OZ perpendicular to the plane XOY, about which the couple acts, is called the axis of active gyroscopic couple.

When the axis of spin itself moves with angular velocity ω_p , the disc is subjected to *reactive couple* whose magnitude is same (i.e. $I \cdot \omega \cdot \omega_p$) but opposite in direction to that of active couple. This reactive couple to which the disc is subjected when the axis of spin



rotates about the axis of precession is known as *reactive gyroscopic couple*. The axis of the reactive gyroscopic couple is represented by OZ' in Fig(a).

The gyroscopic couple is usually applied through the bearings which support the shaft. The bearings will resist equal and opposite couple.

The gyroscopic principle is used in an instrument or toy known as gyroscope. The gyroscopes are installed in ships in order to minimize the rolling and pitching effects of waves. They are also used in aero planes, monorail cars, gyrocompasses etc.

Effect of the Gyroscopic Couple on an aero plane:

The top and front view of an aero plane is shown in Fig (a). Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aero plane takes a turn to the left.

Let ω = Angular velocity of the engine in rad/sec,

m = Mass of the engine and the propeller in kg,

k = Its radius of gyration in metre

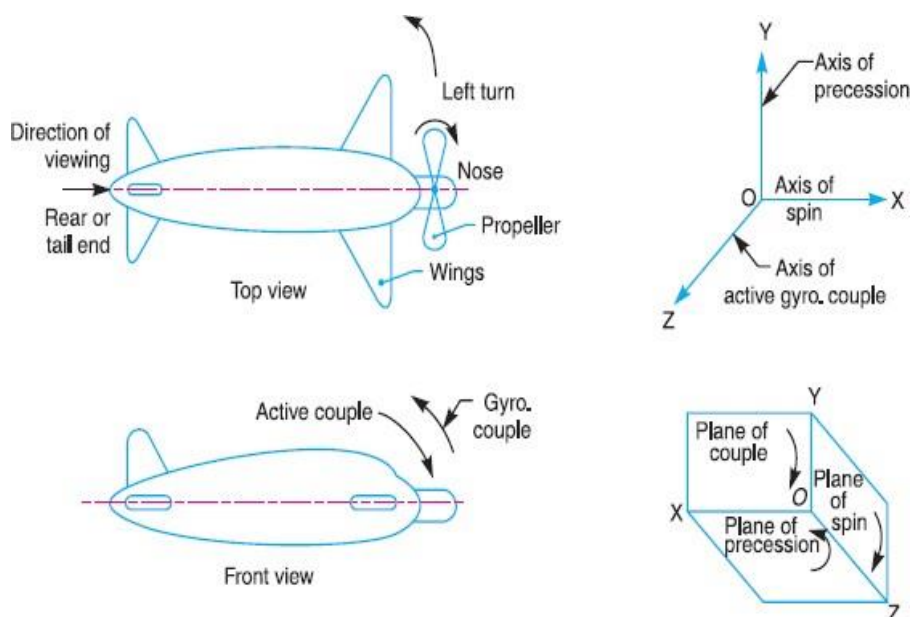
I = Mass moment of inertia of the engine and the propeller in kg-m^2

$$= mk^2$$

v = Linear velocity of the aero plane in m/s,

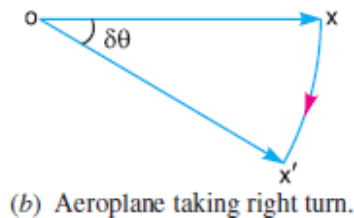
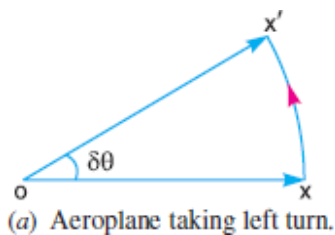
R = Radius of curvature in metres, and

ω_p = Angular velocity of precession = v/R



Before taking the left turn, the angular momentum vector is represented by ox . When it takes left turn, the active gyroscopic couple will change the direction of the angular momentum vector from ox to ox' as shown in Fig(a). The vector xx' , in the limit, represents the change of angular momentum or the active gyroscopic couple and is perpendicular to ox . Thus the plane of active gyroscopic couple XOY will be perpendicular to xx' , i.e. vertical in this case, as shown in Fig (b). By applying right hand screw rule to vector xx' , we find that the direction of active gyroscopic couple is clockwise as shown in the front view of Fig (a).

In other words, for left hand turning, the active gyroscopic couple on the aero plane in the axis OZ will be clockwise as shown in Fig (b). The reactive gyroscopic couple (equal in magnitude of active gyroscopic



couple) will act in the opposite direction (i.e. in the anticlockwise direction) and the effect of this couple is, therefore, to raise the nose and dip the tail of the aero plane.

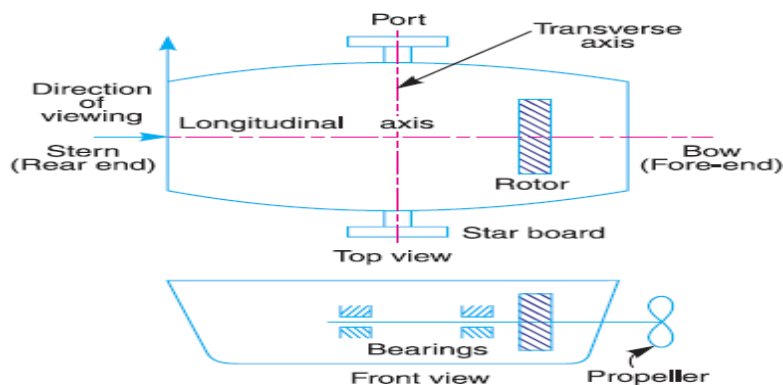
Terms Used in a Naval Ship.

The top and front views of a naval ship are shown in Fig 14.7. The fore end of the ship is called **bow** and the rear end is known as **stern** or **aft**. The left hand and right hand sides of the ship, when viewed from the stern are called **port** and **star-board** respectively. We shall now discuss the effect of gyroscopic couple on the naval ship in the following three cases:

Steering,

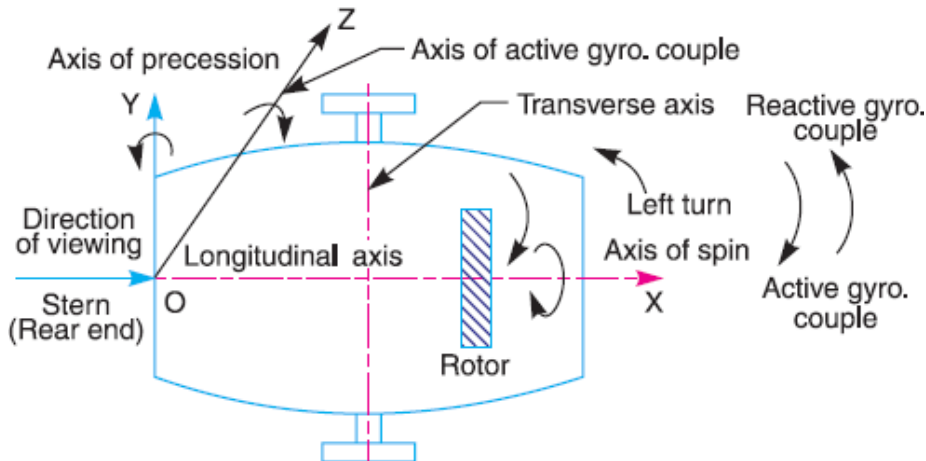
Pitching,

Rolling.

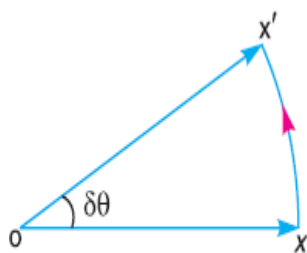


Effect of Gyroscopic Couple on a Naval Ship during Steering

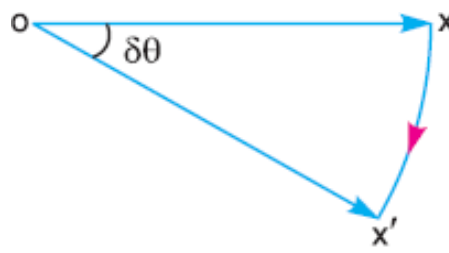
Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aeroplane.



When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction ox as shown in Fig(a). As the ship steers to the left, the active gyroscopic couple will change the angular momentum vector from ox to ox' . The vector xx' now represents the active gyroscopic couple and is perpendicular to ox . Thus the plane of active gyroscopic couple is perpendicular to xx' and its direction in the axis OZ for left hand turn is clockwise as shown in Fig. The reactive gyroscopic couple of the same magnitude will act in the opposite direction (i.e. in anticlockwise direction). The effect of this reactive gyroscopic couple is to raise the bow and lower the stern.



(a) Steering to the left

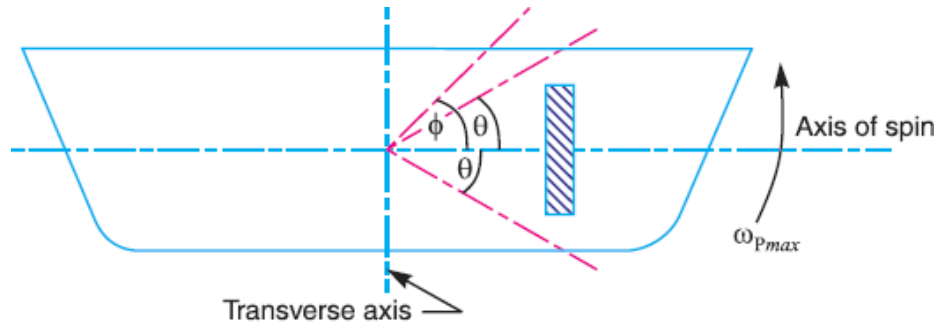


(b) Steering to the right



Effect of Gyroscopic Couple on a Naval Ship during Pitching

Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis, as shown in Fig(a). In this case, the transverse axis is the axis of precession. The pitching of the ship is assumed to take place with simple harmonic motion i.e. the motion of the axis of spin about transverse



(a) Pitching of a naval ship

axis is simple harmonic.

Let I = Moment of inertia of the rotor in kg-m^2 , and ω = Angular velocity of the rotor in rad/s .

Minimum gyroscopic couple,

$$C_{\max} = I \cdot \omega \cdot \omega_{p\max}$$

When the pitching is upward, the effect of the reactive gyroscopic couple, as shown in Fig.(b), will try to move the ship toward star-board. On the other hand, if the pitching is downward, the effect of the reactive gyroscopic couple, as shown in Fig(c), is to turn the ship towards port side.

Effect of Gyroscopic Couple on a Naval Ship during Rolling

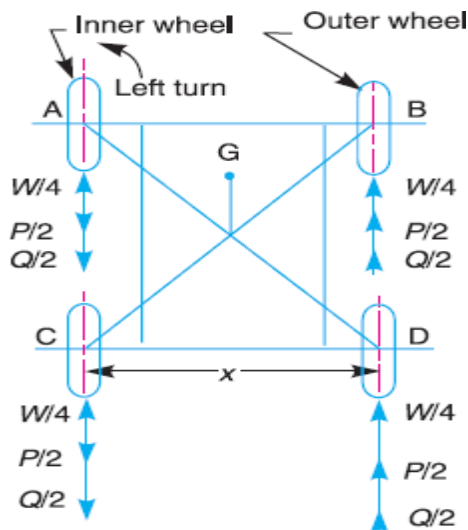
We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship.

In case of rolling of a ship, the axis of precession (i.e. longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

Stability of a Four Wheel Drive Moving in a Curved Path



Consider the four wheels A , B , C and D of an automobile locomotive taking a turn towards left as shown in Fig. The wheels A and C are inner wheels, whereas B and D are outer wheels. The centre of gravity (C.G.) of the vehicle lies vertically above the road surface.



Let m = Mass of the vehicle in kg,

W = Weight of the vehicle in newtons = $m \cdot g$, r_w = Radius of the wheels in metres,

R = Radius of curvature in metres ($R > r_w$),

h = Distance of centre of gravity, vertically above the road surface in metres,

x = Width of track in metres,

I_w = Mass moment of inertia of one of the wheels in kg-m^2 ,

ω_w = Angular velocity of the wheels or velocity of spin in rad/s,

I_E = Mass moment of inertia of the rotating parts of the engine in kg-m^2

ω_E = Angular velocity of the rotating parts of the engine in rad/s,

G = Gear ratio = ω_E / ω_w

v = Linear velocity of the vehicle in m/s = $\omega_w \cdot r_w$

A little consideration will show that the weight of the vehicle (W) will be equally distributed over the four wheels which will act downwards. The reaction between each wheel and the road surface of the same magnitude will act upwards.



Therefore Road reaction over each wheel = $W/4 = m.g/4$ newtons

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle.

∴ Angular displacement of the axis of spin from mean position after time t seconds,

$$\theta = \phi \sin \omega_1 \cdot t$$

ϕ = Amplitude of swing *i.e.* maximum angle turned from the mean position in radians, and

ω_1 = Angular velocity of S.H.M.

$$= \frac{2\pi}{\text{Time period of S.H.M. in seconds}} = \frac{2\pi}{t_p} \text{ rad/s}$$

Angular velocity of precession,

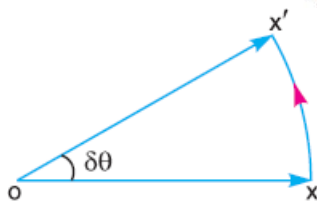
$$\omega_p = \frac{d\theta}{dt} = \frac{d}{dt}(\phi \sin \omega_1 \cdot t) = \phi \omega_1 \cos \omega_1 t$$

The angular velocity of precession will be maximum, if $\cos \omega_1 t = 1$.

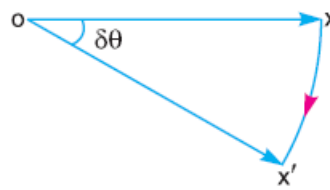
∴ Maximum angular velocity of precession,

$$\omega_{p_{max}} = \phi \cdot \omega_1 = \phi \times 2\pi / t_p$$

...(Substituting $\cos \omega_1 t = 1$)



(b) Pitching upward



(c) Pitching downward

Effect of the gyroscopic couple

Since the vehicle takes a turn towards left due to the precession and other rotating parts, therefore a gyroscopic couple will act.

We know that velocity of precession, $\omega_p = v/R$

Gyroscopic couple due to 4 wheels,

$$C_W = 4 I_W \cdot \omega_W \cdot \omega_p$$

and gyroscopic couple due to the rotating parts of the engine,

$$C_E = I_E \cdot \omega_E \cdot \omega_p = I_E \cdot G \cdot \omega_W \cdot \omega_p$$

Net gyroscopic couple,

$$C = C_W \pm C_E = 4 I_W \cdot \omega_W \cdot \omega_p \pm I_E \cdot G \cdot \omega_W \cdot \omega_p$$



$$= \omega_W \cdot \omega_P (4 I_W \pm G \cdot I_E)$$

The **positive** sign is used when the wheels and rotating parts of the engine rotate in the same direction. If the rotating parts of the engine revolve in opposite direction, then **negative** sign is used.

Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels.

Let the magnitude of this reaction at the two outer or inner wheels be P new tons. Then

$$P \times x = C \text{ or } P = C/x$$

Vertical reaction at each of the outer or inner wheels,

$$P/2 = C/2x$$

Effect of the centrifugal couple

Since the vehicle moves along a curved path, therefore centrifugal force will act outwardly at the centre of gravity of the vehicle. The effect of this centrifugal force is also to overturn the vehicle.

$$F_C = \frac{m \times v^2}{R}$$

We know that centrifugal force,

The couple tending to overturn the vehicle or overturning couple,

$$C_O = F_C \times h = \frac{m \cdot v^2}{R} \times h$$

This overturning couple is balanced by vertical reactions, which are vertically up wards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer

$$Q \times x = C_O \text{ or } Q = \frac{C_O}{x} = \frac{m \cdot v^2 \cdot h}{R \cdot x}$$

or inner wheels be Q . Then

Vertical reaction at each of the outer or inner wheels,

Total vertical reaction at each of the outer wheel,

$$\frac{Q}{2} = \frac{m \cdot v^2 \cdot h}{2R \cdot x}$$



$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

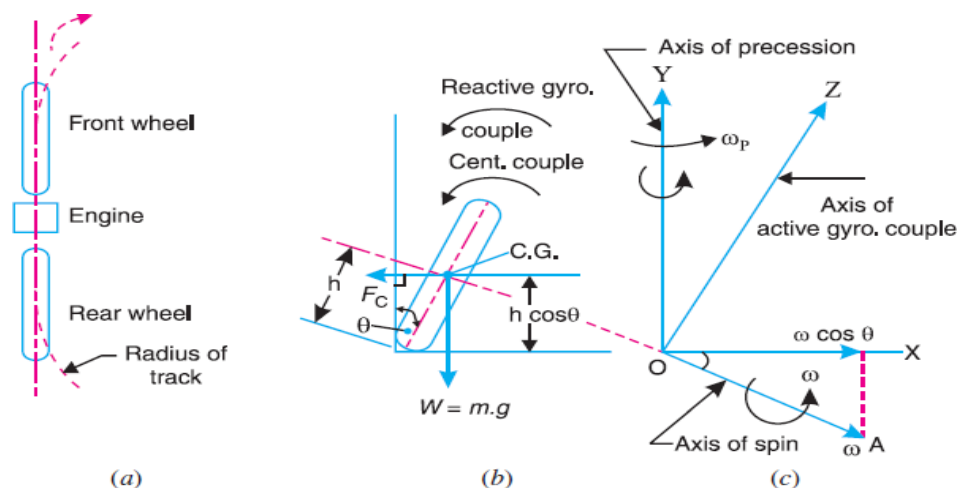
Total vertical reaction at each of the inner wheel

$$P_1 = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

A little consideration will show that when the vehicle is running at high speeds, P_1 may be zero or even negative. This will cause the inner wheels to leave the ground thus tending to overturn the automobile. In order to have the contact between the inner wheels and the ground, the sum of $P/2$ and $Q/2$ must be less than $W/4$.

Stability of a Two Wheel Vehicle Taking a Turn

Consider a two wheel vehicle (say a scooter or motor cycle) taking a right turn as shown in fig.



Let

m = Mass of the vehicle and its rider in kg,

W = Weight of the vehicle and its rider in Newton's = $m.g$, h = Height of the centre of gravity of the vehicle and rider, r_w = Radius of the wheels,

R = Radius of track or curvature,

I_w = Mass moment of inertia of each wheel,

I_E = Mass moment of inertia of the rotating parts of the engine, ω_w = Angular velocity of the wheels,

ω_E = Angular velocity of the engine,

G = Gear ratio = ω_E / ω_w ,



v = Linear velocity of the vehicle = $\omega_W \times r_W$,

θ = Angle of wheel. It is inclination of the vehicle to the vertical for equilibrium.

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle,

Effect of gyroscopic couple

$$\omega_E = G \cdot \omega_W = G \times \frac{v}{r_W}$$

$$\begin{aligned} \therefore \text{Total } (I \times \omega) &= 2 I_W \times \omega_W \pm I_E \times \omega_E \\ &= 2 I_W \times \frac{v}{r_W} \pm I_E \times G \times \frac{v}{r_W} = \frac{v}{r_W} (2 I_W \pm G I_E) \end{aligned}$$

We know that $v = \omega_W \times r_W$ or $\omega_W = v / r_W$

Velocity of precession, $\omega_P = v / R$

A little consideration will show that when the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown in Fig (b). This angle is known as **angle of heel**. In other words, the axis of spin is inclined to the horizontal at an angle θ , as shown in Fig (c). Thus the angular

Momentum vector $I\omega$ due to spin is represented by OA inclined to OX at an angle θ . But the precession

Gyroscopic couple,

$$\begin{aligned} C_1 &= I \cdot \omega \cos \theta \times \omega_P = \frac{v}{r_W} (2 I_W \pm G I_E) \cos \theta \times \frac{v}{R} \\ &= \frac{v^2}{R \cdot r_W} (2 I_W \pm G I_E) \cos \theta \end{aligned}$$

axis is vertical. Therefore the spin vector is resolved along OX .

Effect of centrifugal couple

We know that centrifugal force,

$$F_C = \frac{m \cdot v^2}{R}$$

This force acts horizontally through the centre of gravity (C.G.) along the outward direction.

$$C_2 = F_C \times h \cos \theta = \left(\frac{m \cdot v^2}{R} \right) h \cos \theta$$



Centrifugal couple,

Since the centrifugal couple has a tendency to overturn the vehicle, therefore Total overturning couple,

$$C_O = \text{Gyroscopic couple} + \text{Centrifugal couple}$$

$$\begin{aligned} &= \frac{v^2}{R.r_W} (2 I_W + G.I_E) \cos \theta + \frac{m.v^2}{R} \times h \cos \theta \\ &= \frac{v^2}{R} \left[\frac{2 I_W + G.I_E}{r_W} + m.h \right] \cos \theta \end{aligned}$$

We know that balancing couple = $m.g.h \sin \theta$

The balancing couple acts in clockwise direction when seen from the front of the vehicle. Therefore for stability, the overturning couple must be equal to the balancing couple, *i.e.*

$$\frac{v^2}{R} \left(\frac{2 I_W + G.I_E}{r_W} + m.h \right) \cos \theta = m.g.h \sin \theta$$

From this expression, the value of the angle of heel (θ) may be determined, so that the vehicle does not skid.

PROBLEMS

Example 1.

A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis?

Solution. Given: $d = 300$ mm or $r = 150$ mm = 0.15 m ; $m = 5$ kg ; $l = 600$ mm = 0.6 m
 $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

We know that the mass moment of inertia of the disc, about an axis through its centre of gravity and perpendicular to the plane of disc,

$$I = m.r^2/2 = 5(0.15)^2/2 = 0.056 \text{ kg-m}^2$$

couple due to mass of disc,

$$C = m.g.l = 5 \times 9.81 \times 0.6 = 29.43 \text{ N-m}$$

Let ω_P = Speed of precession.

We know that couple (C),



$$29.43 = I \cdot \omega \cdot \omega_p = 0.056 \times 31.42 \times \omega_p = 1.76 \omega_p$$

$$\omega_p = 29.43/1.76 = 16.7 \text{ rad/s}$$

Example 2.

An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.

Solution: Given : $R = 50 \text{ m}$; $v = 200 \text{ km/hr} = 55.6 \text{ m/s}$; $m = 400 \text{ kg}$; $k = 0.3 \text{ m}$; $N =$

2400 r.p.m. or $\omega = 2\pi \times 2400/60 = 251 \text{ rad/s}$

We know that mass moment of inertia of the engine and the propeller,

$$I = m \cdot k^2 = 400(0.3)^2 = 36 \text{ kg-m}^2$$

Angular velocity of precession, $\omega_p = v/R = 55.6/50 = 1.11 \text{ rad/s}$

We know that gyroscopic couple acting on the aircraft,

$$C = I \cdot \omega \cdot \omega_p = 36 \times 251.4 \times 1.11 = 10046 \text{ N-m} = 10.046 \text{ kN-m}$$

When the aeroplane turns towards left, the effect of the gyroscopic couple is to lift the nose upwards and tail downwards.

Example 3 : The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.

Solution. Given: $m = 8 \text{ t} = 8000 \text{ kg}$; $k = 0.6 \text{ m}$; $N = 1800 \text{ r.p.m. or } \omega = 2\pi \times 1800/60 = 188.5 \text{ rad/s}$;
 $v = 100 \text{ km/h} = 27.8 \text{ m/s}$; $R = 75 \text{ m}$

We know that mass moment of inertia of the rotor,

$$I = m \cdot k^2 = 8000 (0.6)^2 = 2880 \text{ kg-m}^2$$

Angular velocity of precession,

$$\omega_p = v / R = 27.8 / 75 = 0.37 \text{ rad/s}$$

We know that gyroscopic

$$C = I \cdot \omega \cdot \omega_p = 2880 \times 188.5 \times 0.37 = 200866 \text{ N-m}$$

$$= 200.866 \text{ kN-m}$$



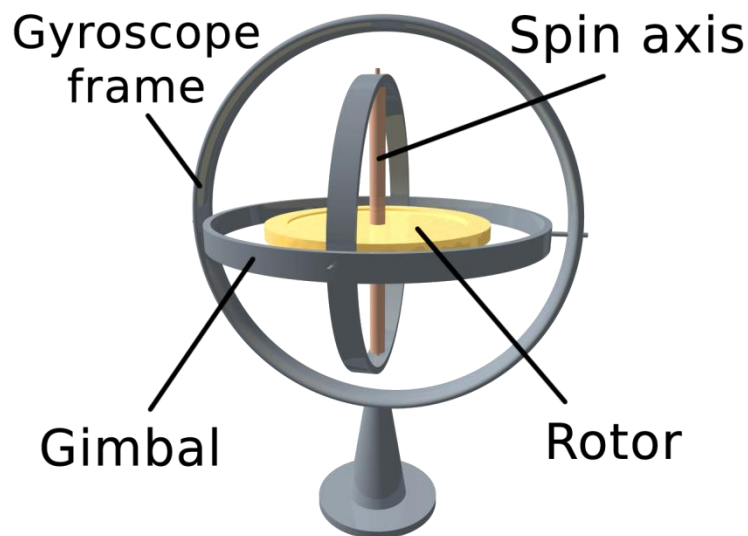
When the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.



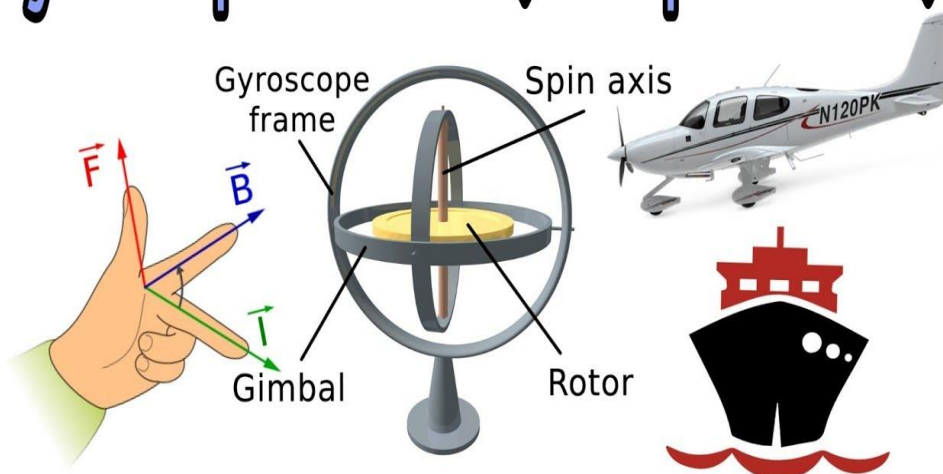
INDUSTRIAL APPLICATIONS OF GYROSCOPIC

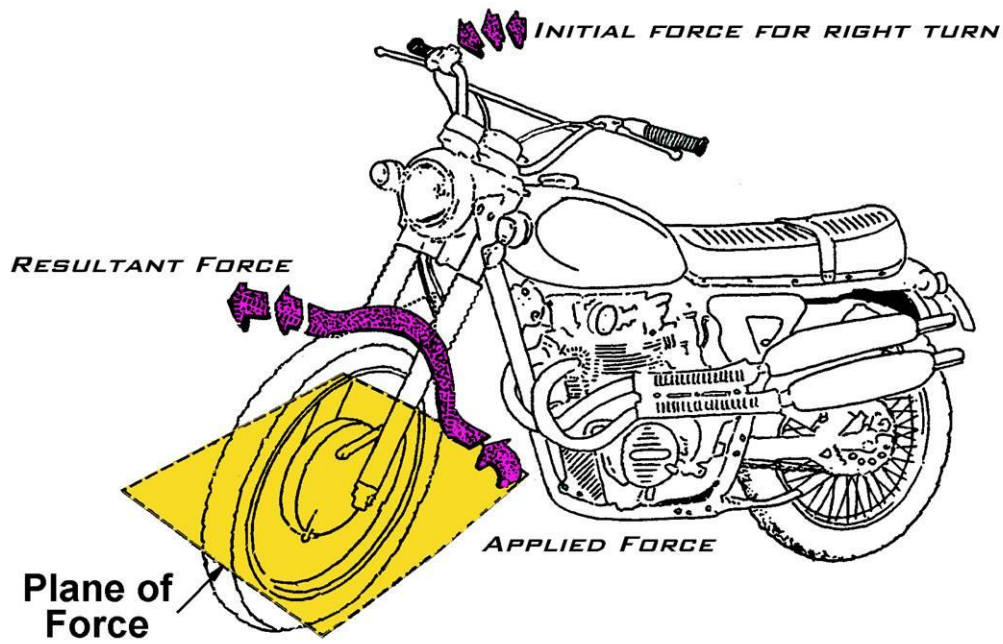
1. Motor car
2. Motor cycle
3. Aero planes
4. Ships

5. Applications of gyroscopes include inertial navigation systems, such as in the Hubble Telescope, or inside the steel hull of a submerged submarine. Due to their precision, gyroscopes are also used in gyrotheodolites to maintain direction in tunnel mining.



Gyroscopic Effect (Simple Trick)





TUTORIAL QUESTIONS

1. What do you understand by gyroscopic couple? Derive a formula for its magnitude.
2. Explain the effect of the gyroscopic couple on the reaction of the four wheels of a vehicle negotiating a curve.
3. Discuss the effect of the gyroscopic couple on a two wheeled vehicle when taking a turn.
4. What will be the effect of the gyroscopic couple on a disc fixed at a certain angle to a rotating shaft?
5. A flywheel of mass 10 kg and radius of gyration 200 mm is spinning about its axis, which is horizontal and is suspended at a point distant 150 mm from the plane of rotation of the flywheel. Determine the angular velocity of precession of the flywheel. The spin speed of flywheel is 900 r.p.m.
6. Find the angle of inclination with respect to the vertical of a two wheeler negotiating a turn. Given : combined mass of the vehicle with its rider 250 kg ; moment of inertia of the engine flywheel 0.3 kg-m² ; moment of inertia of each road wheel 1 kg-m² ; speed of engine flywheel 5 times that of road wheels and in the same direction ; height of centre of gravity of rider with vehicle 0.6 m ; two wheeler speed 90 km/h ; wheel radius 300 mm ; radius of turn 50 m.
7. A pair of locomotive driving wheels with the axle, have a moment of inertia of 180 kg-m². The diameter of the wheel treads is 1.8 m and the distance between wheel centres is 1.5 m. When the locomotive is travelling on a level track at 95 km/h, defective ballasting causes one wheel to fall 6 mm and to rise again in a total time of 0.1 s. If the displacement of the wheel takes place with simple harmonic motion, find: 1. The gyroscopic couple set up, and 2. The reaction between the wheel and rail due to this couple.
8. A four-wheeled trolley car of total mass 2000 kg running on rails of 1.6 m gauge, rounds a curve of 30 m radius at 54 km/h. The track is banked at 8°. The wheels have an external diameter of 0.7 m and each pair with axle has a mass of 200 kg. The radius of gyration for each pair is 0.3 m. The height of centre of gravity of the car above the wheel base is 1 m. Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail.
9. A horizontal axle AB, 1 m long, is pivoted at the midpoint C. It carries a weight of 20 N at A and a wheel weighing 50 N at B. The wheel is made to spin at a speed of 600 r.p.m in a clockwise direction looking from its front. Assuming that the weight of the flywheel is uniformly distributed around the rim whose mean diameter is 0.6 m; calculate the angular velocity of precession of the system around the vertical axis through C.



ASSIGNMENT QUESTIONS

1. Explain the effect of the gyroscopic couple on the reaction of the four wheels of a vehicle negotiating a curve
2. Discuss the effect of the gyroscopic couple on a two wheeled vehicle when taking a turn.
3. What will be the effect of the gyroscopic couple on a disc fixed at a certain angle to a rotating shaft?
4. A uniform disc of 150 mm diameter has a mass of 5 kg. It is mounted centrally in bearings which maintain its axle in a horizontal plane. The disc spins about its axle with a constant speed of 1000 r.p.m. while the axle precesses uniformly about the vertical at 60 r.p.m. The directions of rotation are as shown in Fig. 14.3. If the distance between the bearings is 100 mm, find the resultant reaction at each bearing due to the mass and gyroscopic effects.
5. An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.
6. The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steers to the left in a curve of 75 m radius.
7. The heavy turbine rotor of a sea vessel rotates at 1500 r.p.m. clockwise looking from the stern, its mass being 750 kg. The vessel pitches with an angular velocity of 1 rad/s. Determine the gyroscopic couple transmitted to the hull when bow is rising, if the radius of gyration for the rotor is 250 mm. Also show in what direction the couple acts on the hull?
8. The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:
 1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.
 2. when the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.
9. A ship propelled by a turbine rotor which has a mass of 5 tonnes and a speed of 2100 r.p.m. The rotor has a radius of gyration of 0.5 m and rotates in a clockwise direction when viewed from the stern. Find the gyroscopic effects in the following conditions:



1. The ship sails at a speed of 30 km/h and steers to the left in a curve having 60 m radius.
2. The ship pitches 6 degree above and 6 degree below the horizontal position. The bow is descending with its maximum velocity. The motion due to pitching is simple harmonic and the periodic time is 20 seconds.
3. The ship rolls and at a certain instant it has an angular velocity of 0.03 rad/s clockwise when viewed from stern.

Determine also the maximum angular acceleration during pitching. Explain how the direction of motion due to gyroscopic effect is determined in each case.

