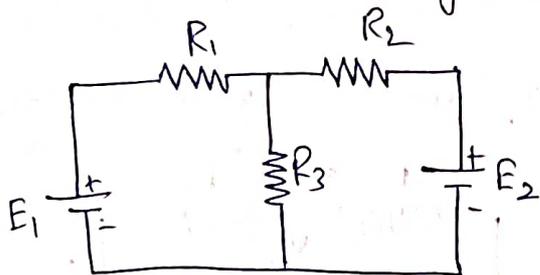


#### IV. Network Theorems

##### Superposition Theorem:-

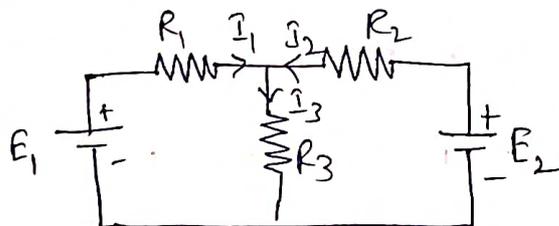
Statement:- In a linear bilateral resistive network containing two or more voltage sources, the resultant current in any branch is the algebraic sum of the currents that would be produced by each voltage source acting alone, all other voltage sources are being replaced by their internal resistances.

Proof:- Consider the circuit shown below to find out the current flowing through various branches.



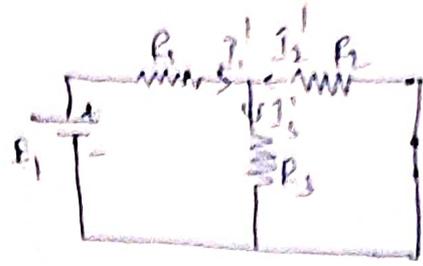
Step 1:- Identify the number of branches in the given circuit and mark the resultant current directions in each branch.

Let  $I_1$ ,  $I_2$  and  $I_3$  are the resultant currents shown below.



Step 2:- Consider voltage source  $E_1$  and replacing the voltage source  $E_2$  by its internal resistance or short circuit the voltage source  $E_2$ .

then determine the currents in each branch as  $I_1'$ ,  $I_2'$  and  $I_3'$  respectively



Total Resistance  $R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3}$

Total current produced by source  $E_1 = I_1' = \frac{E_1}{R_T}$

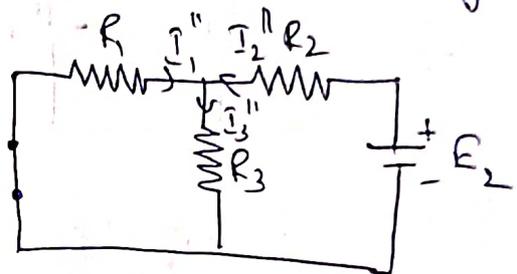
By the current divider Rule

current through  $R_2$ ,  $I_2' = I_1' \frac{R_3}{R_2 + R_3}$

current through  $R_3 = I_3' = I_1' \frac{R_2}{R_2 + R_3}$

Step 3: - consider the voltage source  $E_2$  and short circuit the voltage source  $E_1$ , then determine the current in each branch consider as  $I_1''$ ,  $I_2''$  and  $I_3''$  respectively.

Total resistance  $R_{T2} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$



Total current produced by

source  $E_2$ ,  $I_2'' = \frac{E_2}{R_{T2}}$

By current divider rule,

current through  $R_1$ ,  $I_1'' = I_2'' \frac{R_3}{R_1 + R_3}$

current through  $R_3$ ,  $I_3'' = I_2'' \frac{R_1}{R_1 + R_3}$

Steps:- now applying superposition theorem to combine the results in order to find out the resultant current in various branches.

Resultant current in Resistor  $R_1$ ,  $I_1 = I_1' - I_1''$

Resultant current in Resistor  $R_2$ ,  $I_2 = I_2' - I_2''$

Resultant current in Resistor  $R_3$ ,  $I_3 = I_3' + I_3''$

Applications:-

- 1. This Theorem is applicable to all linear networks i.e. circuit consisting of resistors or impedances in which ohm's law is valid
- 2. This Theorem is used to determine the current in any branch of a network containing two or more than two voltage sources or current sources or both voltage and current sources.

Steps for solving super position Theorem:-

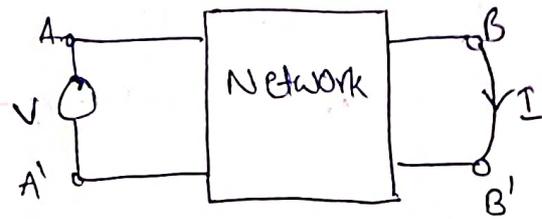
- 1. Identify the number of branches in given ckt, and mark the current directions.
- 2. Take only one independent source of voltage/current and deactivate the other independent voltage/current sources.  
i.e short circuit the voltage source and open circuit the current source.
- 3. Then calculate the each branch current
- 3. Repeat the above step for each of the independent source.

4. To determine the net branch current- using superposition Theorem, by adding currents obtained in step 1 and step 2 for each branch.

### Reciprocity Theorem:-

Statement:- The Reciprocity Theorem states that in a linear, bilateral single source networks the ratio of excitation to response is constant- even when their positions are interchanged.

Explanation:- consider the networks shown below, AA' denotes input terminals and BB' denotes output terminals.



\* The application of voltage  $V$  across AA' produces current  $I$  at BB'.

\* Now if the positions of the source and responses are interchanged, by connecting the voltage source across BB', the resultant current  $I$  will be at terminals AA'.

\* According to the reciprocity theorem, the ratio of response to excitation is the same in both cases.

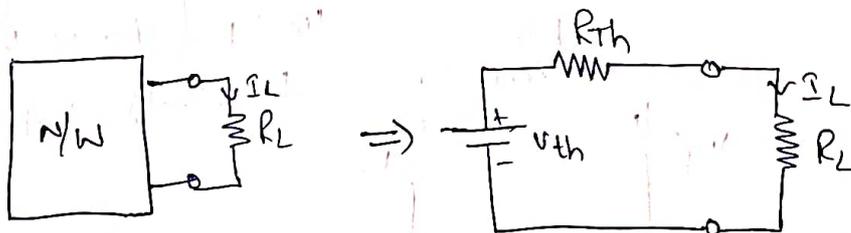
### Steps for solving Reciprocity Theorem:-

1. Identify the branches between which reciprocity is to be established.

2. Find the current in the branch when excitation and response are not interchanged.
3. Find the current in the branch when excitation and response are interchanged.
4. It may be observed that the currents obtained in step 2 & 3 are identical to each other.

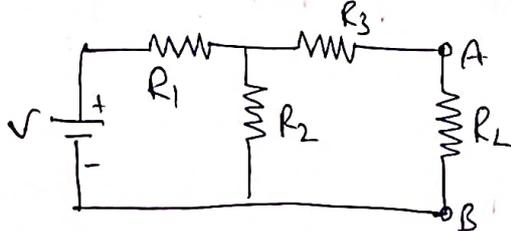
Thevenin's Theorem:-

Statement:- It states that "Any two terminal linear bilateral network containing a number of voltage sources and resistances can be replaced by an equivalent circuit consisting of a voltage source ( $V_{th}$ ) in series with a resistance  $R_{th}$ " and load resistance  $R_L$ .



Explanation:-

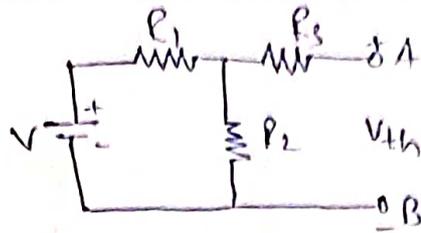
1. Consider a network as shown below



2. For finding load current through  $R_L$ , first remove the load resistor  $R_L$  from the network, and calculate open circuit voltage  $V_{th}$  across points A and B as shown below figure.

$$V_{th} = I_2 R_2$$

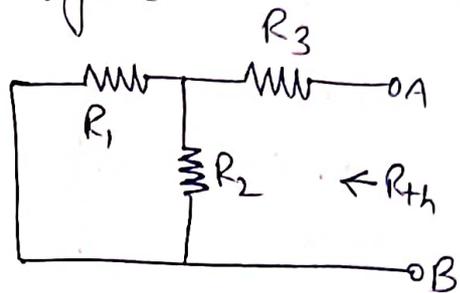
$$I_2 = \frac{V}{R} = \frac{V}{R_1 + R_2}$$



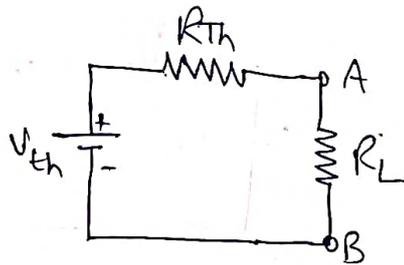
$$\therefore V_{th} = \left( \frac{V}{R_1 + R_2} \right) R_2$$

3. For finding series resistance  $R_{th}$ , replace the voltage source by a short circuit and calculate resistance between points A and B as shown below figure

$$R_{th} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$



4. Draw the thevenin's equivalent circuit and connect the load resistance back to its terminals at A and B.



5. Determine the current flowing through  $R_L$  by applying ohm's law

$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

6. The potential drop across the load will be

$$V_L = \left( \frac{V_{th}}{R_{th} + R_L} \right) R_L$$

7

### Steps for Solving Thevenin's Theorem:-

1. Remove the load Resistance  $R_L$ .
2. Find the open circuit voltage  $V_{Th}$  across points A and B.
3. Find the resistance  $R_{Th}$  as seen from points A and B.
4. Replace the network by a voltage source  $V_{Th}$  in series with Resistance  $R_{Th}$ .
5. Determine current through  $R_L$  and voltage across  $R_L$ .

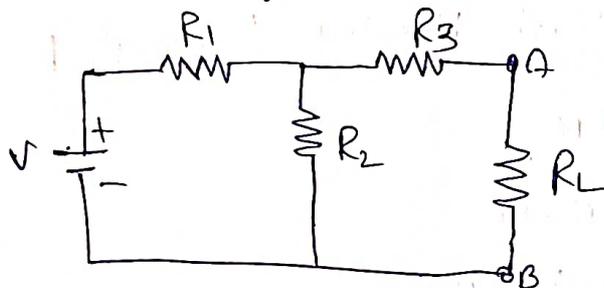
$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \quad \text{and} \quad V_L = \left( \frac{V_{Th}}{R_{Th} + R_L} \right) R_L$$

### Norton's Theorem:-

Statement:- It states that "Any two terminal linear bilateral network containing a number of voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source  $I_N$  in parallel with a resistance  $R_N$ ".

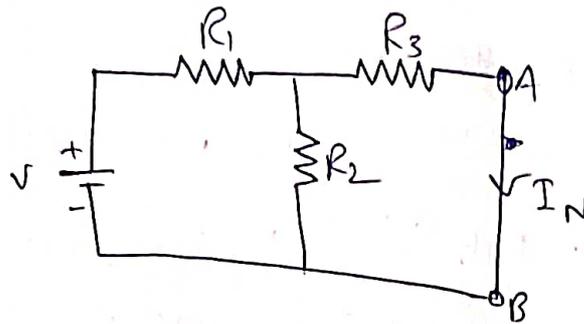
### Explanation:-

1. Consider a simple network as shown below.



2. For finding load current  $R_L$ , first remove the load resistance  $R_L$  from the network.

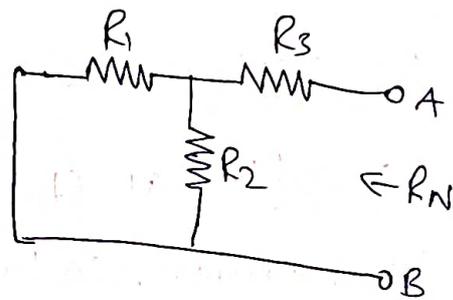
and calculate short-circuit current  $I_{sc}$  or  $I_N$



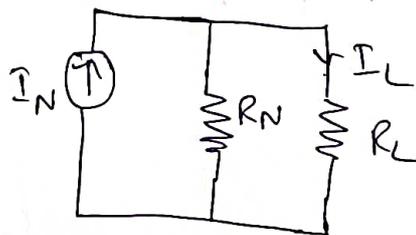
$$I_N = \frac{V}{R_{eq}}$$

3. For finding parallel resistance  $R_N$ , replace the voltage source by a short circuit and calculate resistance b/w points A and B.

$$R_N = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$



4. Draw the Norton's equivalent circuit and connect the load resistance  $R_L$



5. Determine the current flowing through  $R_L$

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

Steps for solving Norton's Theorem:

1. Remove the load resistance  $R_L$  and put a short ckt across the terminals.

2. Find the short-circuit current  $I_{SC}$  or  $I_N$ .
3. Find the resistance  $R_N$  as seen from point A and B.
4. Replace the network by a current source  $I_N$  in parallel with resistance  $R_N$ .
5. Determine current through  $R_L$  by current-division rule.

$$I_L = I_N \cdot \frac{R_N}{R_N + R_L}$$

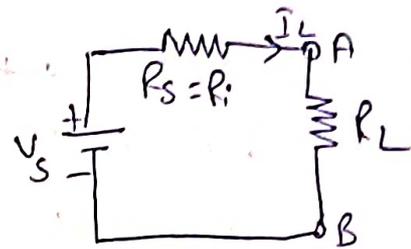
Maximum Power Transfer Theorem:-

Statement:- It states that - "Maximum power in a circuit can be transferred from a source to load when the load resistance is equal to the internal resistance of the source".

Let  $V_S =$  source voltage

$R_i = R_s =$  Internal or source resistance

$R_L =$  load resistance



Current supplied to the load,  $I_L = \frac{V_S}{R_s + R_L}$

Power drawn by the load,

$$P_L = I_L^2 R_L$$

$$P_L = \left( \frac{V_S}{R_s + R_L} \right)^2 R_L = \frac{V_S^2 R_L}{(R_s + R_L)^2}$$

power taken by the load will be maximum if  $\frac{d(P_L)}{dR_L} = 0$

$$\therefore \frac{d}{dR_L} \left[ \frac{V_S^2 R_L}{(R_s + R_L)^2} \right] = 0$$

$$(R_s + R_L)^2 V_s^2 \times 1 - V_s^2 R_L [2(R_s + R_L) \times 1] = 0$$

$$V_s^2 [(R_s + R_L)^2 - 2R_L(R_s + R_L)] = 0$$

$$(R_s + R_L)^2 - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + R_L^2 + 2R_L R_s - 2R_L(R_s + R_L) = 0$$

$$R_s^2 + R_L^2 + 2R_L R_s - 2R_L R_s - 2R_L^2 = 0$$

$$R_s^2 - R_L^2 = 0$$

$$R_s - R_L = 0$$

$$\therefore R_L = R_s \text{ (or)}$$

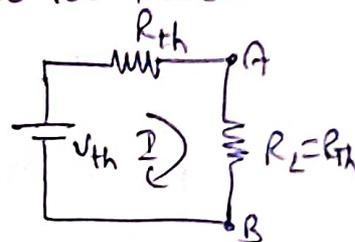
$$R_L = R_i$$

i.e. Load resistance = Internal resistance

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

Steps for Solving the Maximum Power Transfer Theorem:-

1. Remove the variable load resistor  $R_L$
2. Find the open circuit voltage  $V_{th}$  across A and B
3. Find the resistance  $R_{th}$  as seen from points A & B
4. Find the resistance  $R_L$  for maximum power transfer



$$R_L = R_{th}$$

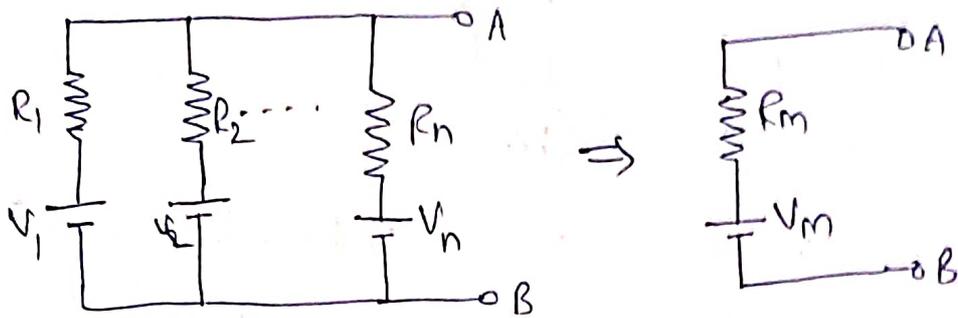
5. Find the maximum power

$$\left( \begin{array}{l} \because V_{th} = V_s \\ R_{th} = R_s \end{array} \right) \quad I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{V_{th}}{2R_L} \quad \left( \because R_{th} = R_s = R_L \right)$$

$$P_{max} = I_L^2 R_L = \frac{V_{th}^2}{(2R_L)^2} = \frac{V_{th}^2}{4R_L} = \frac{V_s^2}{4R_L}$$

## Milliman's Theorem:-

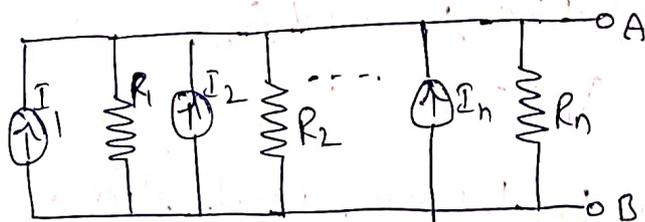
Statement:- It states that "If there are  $n$  voltage sources  $V_1, V_2, \dots, V_n$  with internal resistances  $R_1, R_2, \dots, R_n$  respectively connected in parallel then these voltage sources can be replaced by a single voltage source  $V_m$  and a single series resistance  $R_m$ ".



$$\text{where } V_m = \frac{V_1 G_1 + V_2 G_2 + \dots + V_n G_n}{G_1 + G_2 + \dots + G_n}$$

$$\text{and } R_m = \frac{1}{G_m} = \frac{1}{G_1 + G_2 + \dots + G_n}$$

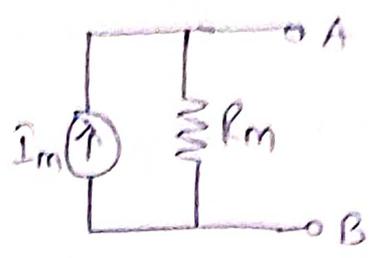
Explanation:- By source transformation, each voltage source in series with a resistance can be converted to a current source in parallel with a resistance as shown below.



Equivalent network

Let  $I_m$  be the resultant current of the parallel current source and  $R_m$  be the equivalent resistance as shown below

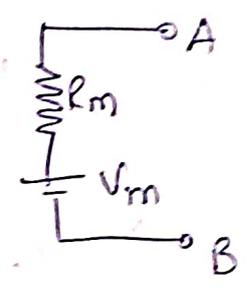
$$\begin{aligned} I_m &= I_1 + I_2 + \dots + I_n \\ &= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n} \\ &= V_1 G_1 + V_2 G_2 + \dots + V_n G_n \end{aligned}$$



$$\begin{aligned} \frac{1}{R_m} &= \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \\ G_m &= G_1 + G_2 + \dots + G_n \end{aligned}$$

By using source transformation, the parallel circuit can be converted into a series circuit as shown below

$$\begin{aligned} V_m &= I_m R_m \\ V_m &= \frac{I_m}{G_m} = \frac{V_1 G_1 + V_2 G_2 + \dots + V_n G_n}{G_1 + G_2 + \dots + G_n} \end{aligned}$$



Millman's equivalent v/w.

Steps for solving Millman's Theorem:-

1. Remove the load resistance  $R_L$
2. Find Millman's voltage across points A and B

$$V_m = \frac{V_1 G_1 + V_2 G_2 + \dots + V_n G_n}{G_1 + G_2 + \dots + G_n}$$

3. Find the resistance  $R_m$  between points A and B

$$R_m = \frac{1}{G_1 + G_2 + \dots + G_n}$$

4. Replace the network by a voltage source  $V_m$  in series with the resistance  $R_m$ .

5. Find the current through  $R_L$  using Ohm's Law.

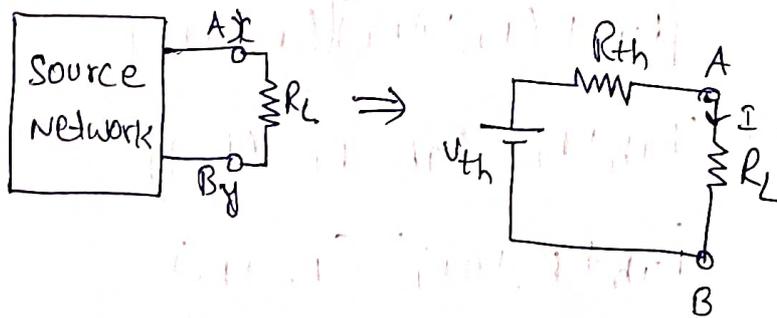
$$I_L = \frac{V_m}{R_m + R_L}$$

Compensation Theorem:-

Statement:- It states that "In any linear bilateral active network, if any branch carrying a current  $I$  has its resistance  $R$  changed by  $\Delta R$ , the resulting changes that occur in the other branches are the same as those which would have been produced by an opposing voltage source of value  $V_c (I \Delta R)$  introduced into the modified branch."

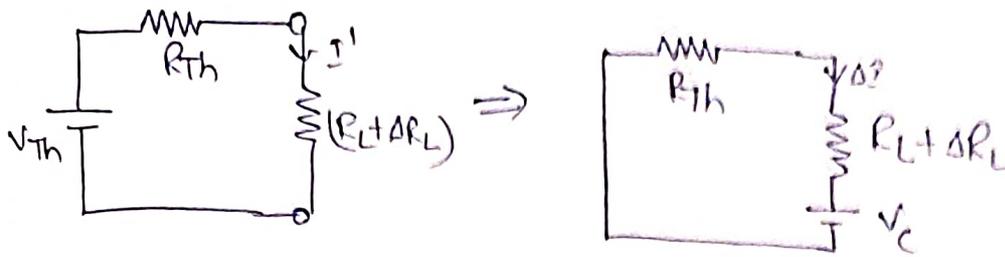
Explanation:-

Let us assume a load  $R_L$  be connected to a dc source network whose Thevenin's equivalent voltage and resistance are  $V_{th}$  and  $R_{th}$  as shown below.



$$\therefore I = \frac{V_{th}}{R_{th} + R_L}$$

Let the load resistance  $R_L$  be changed to  $(R_L + \Delta R_L)$ . Since the rest of the circuit remains unchanged, the Thevenin equivalent network remains the same as shown below.



$$\text{Hence } I' = \frac{V_{Th}}{R_{Th} + (R_L + \Delta R_L)}$$

The change of current - being termed as  $\Delta I$ ,

$$\Delta I = I' - I$$

$$= \frac{V_{Th}}{R_{Th} + (R_L + \Delta R_L)} - \frac{V_{Th}}{R_{Th} + R_L}$$

$$= \frac{V_{Th}(R_{Th} + R_L) - V_{Th}(R_{Th} + (R_L + \Delta R_L))}{(R_{Th} + (R_L + \Delta R_L))(R_{Th} + R_L)}$$

$$= \frac{V_{Th}[(R_{Th} + R_L) - (R_{Th} + (R_L + \Delta R_L))]}{(R_{Th} + (R_L + \Delta R_L))(R_{Th} + R_L)}$$

$$= \frac{V_{Th}[(R_{Th} + R_L) - R_{Th} - R_L - \Delta R_L]}{(R_{Th} + (R_L + \Delta R_L))(R_{Th} + R_L)}$$

$$= \frac{V_{Th}[-\Delta R_L]}{(R_{Th} + (R_L + \Delta R_L))(R_{Th} + R_L)}$$

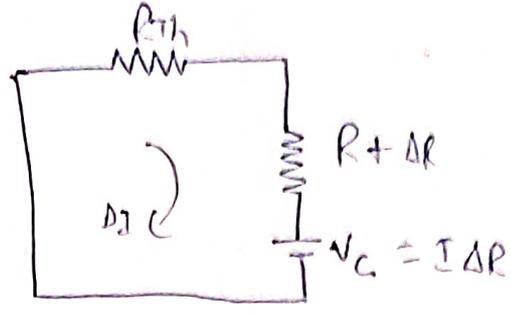
$$= - \frac{V_{Th} \Delta R_L}{(R_{Th} + R_L + \Delta R_L)(R_{Th} + R_L)}$$

$$= - \left[ \frac{V_{Th}}{R_{Th} + R_L} \right] \frac{\Delta R_L}{R_{Th} + R_L + \Delta R_L}$$

$$= - \frac{I \Delta R_L}{R_{Th} + R_L + \Delta R_L}$$

$$= - \frac{V_c}{R_{Th} + R_L + \Delta R_L}$$

where  $V_c = I \Delta R$  and is called the compensation voltage



Thus it has been proved that, with change of branch resistance, branch current is changed and the change is equivalent to an ideal compensating voltage source in series with the branch opposing the original current, all other sources in the network being replaced by their internal resistances.

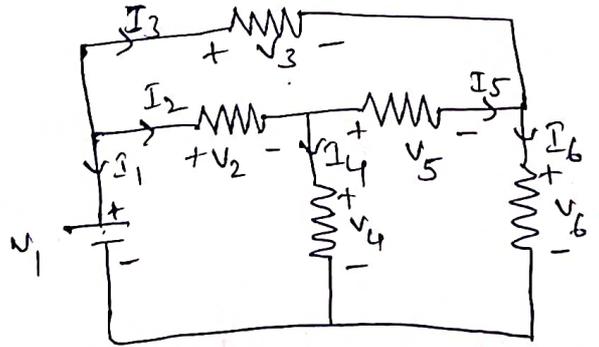
Tellegen's Theorem:-

statement:- It states that "the algebraic sum of the powers in all branches of the network at any instant is zero."

$$\sum_{k=1}^n V_k I_k = 0$$

This condition is valid for the network which obeys Kirchhoff's voltage and current laws.

Explanation:- Consider a network shown below.



Network illustrating Tellegen's Theorem

According to Tellegen's Theorem

$$\sum_{k=1}^n v_k i_k = 0$$

$$\therefore \sum_{k=1}^6 v_k i_k = v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 + v_5 i_5 + v_6 i_6$$

$$= v_1 i_1 + (v_1 - v_4) i_2 + (v_1 - v_6) i_3 + v_4 i_4 +$$

$$(v_4 - v_6) i_5 + v_6 i_6$$

$$\sum_{k=1}^6 v_k i_k = v_1 (i_1 + i_2 + i_3) + v_4 (-i_2 + i_4 + i_5) + v_6 (-i_3 - i_5 + i_6) \rightarrow \text{①}$$

Applying KCL at Node 1,

$$i_1 + i_2 + i_3 = 0$$

Applying KCL at Node 4,

$$i_2 = i_4 + i_5 \Rightarrow -i_2 + i_4 + i_5 = 0$$

Applying KCL at Node 6,

$$i_5 + i_3 = i_6 \Rightarrow -i_3 - i_5 + i_6 = 0$$

From Equation ①

$$\therefore \sum_{k=1}^6 v_k i_k = v_1 (0) + v_4 (0) + v_6 (0) = 0$$

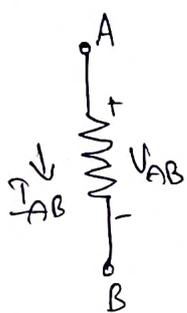
$$\therefore \sum_{k=1}^6 v_k i_k = 0$$

Hence, Tellegen's Theorem is verified.

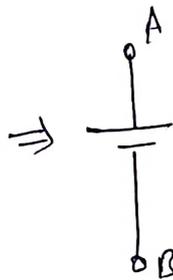
## Substitution Theorem:-

Statement:- The voltage across and current through any branch of a dc bilateral network being known, this branch can be replaced by any combination of elements that will make the same voltage across and current through the chosen branch.

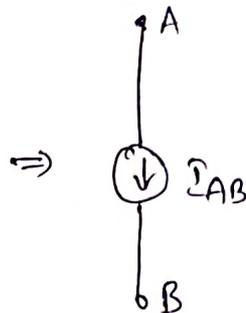
Explanation:- Any branch can be replaced by any combination of elements that will have the same voltage across it and same current through it as the original branch.



Branch of a n/w  
whose  $V_{AB}$  and  
 $I_{AB}$  are known



Branch replaced  
by a voltage  
source  $V_{AB}$



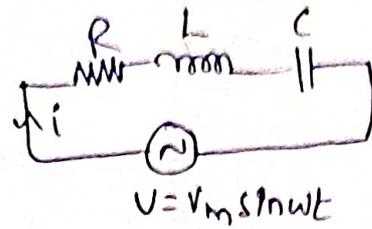
Branch replaced by a  
current source  $I_{AB}$

## Steps for solving substitution Theorem:-

1. Find the branch voltage and branch current in the n/w
2. Replace the branch by an independent voltage source and find the voltage and current in the n/w.
3. Replace the branch by an independent current source and find the voltage and current in the n/w.

## Resonance:-

Consider the series RLC circuit as shown figure. The impedance of the circuit is



$$Z = R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

At Resonance, the circuit is purely Resistive.

$$X_L - X_C = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where  $f_0$  is called the resonant frequency of the circuit.

1. power factor:

$$\text{Power factor} = \cos \phi = \frac{R}{Z}$$

$$\text{At Resonance } Z = R$$

$$\text{power factor} = \frac{R}{R} = 1$$

2. Current:-

since impedance is minimum, the current is maximum at Resonance. Thus, the circuit accepts more current and as

Such, an RLC circuit under Resonance is called an acceptor circuit.

$$I_0 = \frac{V}{Z} = \frac{V}{R}$$

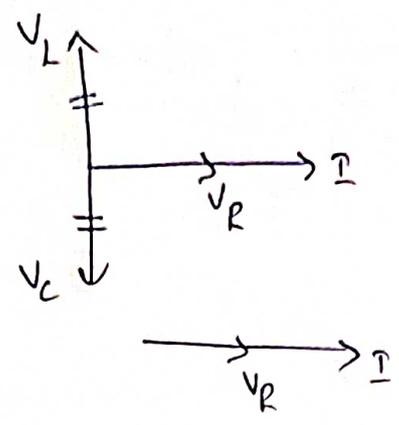
3. Voltage At Resonance

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 L I_0 = \frac{1}{\omega_0 C} I_0$$

$$V_{L0} = V_{C0}$$

4. Phasor diagram:-



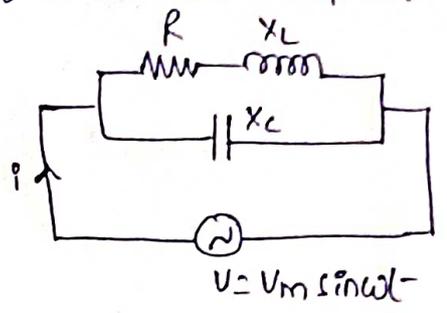
Parallel Resonance:-

Consider a parallel circuit consisting of a coil and capacitor as shown below figure.

The impedances of two branches are

$$z_1 = R + jX_L$$

$$z_2 = -jX_C$$



$$y_1 = \frac{1}{z_1} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$y_2 = \frac{1}{z_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$$

Admittance of the circuit

$$y = y_1 + y_2$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} - j \left( \frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} \right)$$

At resonance, the circuit is purely resistive.

$\therefore$  The condition for resonance is

$$\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$X_L X_C = R^2 + X_L^2$$

$$\omega_0 L \frac{1}{\omega_0 C} = R^2 + \omega_0^2 L^2$$

$$\frac{L}{C} = R^2 + \omega_0^2 L^2$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

where  $f_0$  is called the resonant-frequency of the circuit  
 if  $R$  is very small as compared to  $L$  then

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Dynamic Impedance of a parallel circuit:-

At resonance, the circuit is purely resistive.

The real part of admittance is  $\frac{R}{R^2 + X_L^2}$

Hence, the impedance at resonance is given by

$$Z_D = \frac{R^2 + X_L^2}{R}$$

At resonance,  $R^2 + X_L^2 = X_L X_C = \frac{L}{C}$

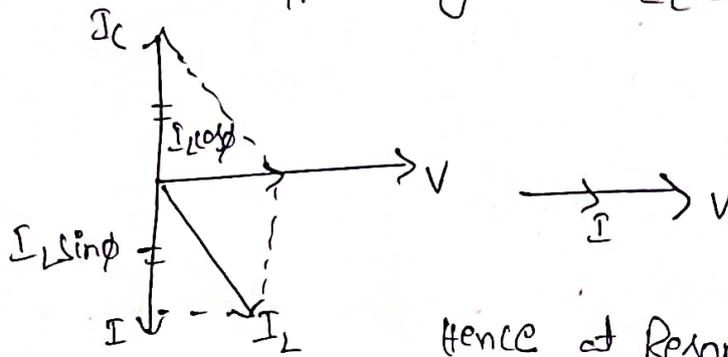
$$Z_D = \frac{L}{CR}$$

Current:- Since impedance is maximum at resonance, the current is minimum at resonance.

$$I_0 = \frac{V}{Z_0} = \frac{V}{\frac{L}{CR}} = \frac{VCR}{L}$$

Phasor Diagram:- At resonance, power factor of the circuit is unity and the total current drawn by the circuit is in phase with the voltage.

This will happen only when  $I_C = I_L \sin \phi$



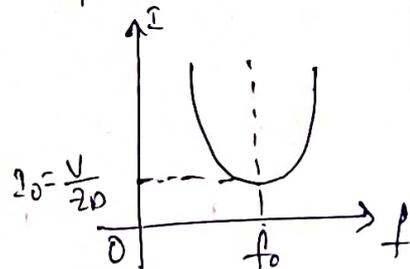
Hence, at Resonance

$$I_C = I_L \sin \phi$$

$$\text{and } I = I_L \cos \phi$$

Band Width:- The bandwidth of a parallel resonant circuit is defined in the same way as that for a series resonant circuit.

The figure shows the resonance curve of a parallel resonant circuit.



Quality factor:- It is a measure of current magnification in a parallel resonant circuit.

$$Q_0 = \frac{\text{current Through Inductor or capacitor}}{\text{current at Resonance}} = \frac{I_{C_0}}{I_0}$$

Substituting values of  $I_{c0}$  and  $I_0$ ,

$$Q_0 = \frac{\frac{V}{X_{c0}}}{\frac{VCR}{L}} = \frac{\frac{L}{X_{c0}}}{\frac{CR}{L}} = \frac{\omega_0 L}{\frac{CR}{L}} = \frac{\omega_0 L^2}{CR}$$

neglecting the resistance  $R$ , the resonant frequency  $\omega_0$  is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\therefore Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right)L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Expression for Bandwidth for series connection:-

Generally, at any frequency  $\omega$ ,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \rightarrow (1)$$

at half-power points

$$I = \frac{I_0}{\sqrt{2}}$$

$$\text{But } I_0 = \frac{V}{R}$$

$$\therefore I = \frac{V}{\sqrt{2}R} \rightarrow (2)$$

Equate Eqn (1) & Eqn (2)

$$\frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{V}{\sqrt{2}R}$$

$$\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{2}R$$

Squaring both the sides

$$R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2 = 2R^2$$

$$\left( \omega L - \frac{1}{\omega C} \right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} \pm R = 0$$

$$\omega^2 \pm \frac{R}{L} \omega - \frac{1}{LC} = 0$$

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

For low values of  $R$ , the term  $\left( \frac{R^2}{4L^2} \right)$  can be neglected in comparison with the term  $\frac{1}{LC}$ .

Then  $\omega$  is given by,  $\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{1}{LC}} = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$

Then  $\omega$  is given by,  $\omega = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}} = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$

The resonant frequency of this circuit is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega = \pm \frac{R}{2L} + \omega_0 \quad (\text{considering only +ve sign of } \omega_0)$$

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

$$\text{and } \omega_2 = \omega_0 + \frac{R}{2L}$$

$$f_1 = f_0 - \frac{R}{4\pi L}$$

$$\text{and } f_2 = f_0 + \frac{R}{4\pi L}$$

$$\text{Band width} = \omega_2 - \omega_1 = R/L$$

$$\text{or Band width} = f_2 - f_1 = \frac{R}{2\pi L}$$

## Quality factor for Series Resonance:-

It is measure of voltage magnification in the series resonant circuit. It is also a measure of selectivity or sharpness of the series resonant circuit.

$$Q_0 = \frac{\text{voltage across inductor or capacitor}}{\text{voltage at resonance}} = \frac{V_{L_0}}{V} = \frac{V_{C_0}}{V}$$

Substitute values of  $V_{L_0}$  and  $V$ ,

$$Q_0 = \frac{I_0 X_{L_0}}{I_0 R} = \frac{X_{L_0}}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

Substitute value of  $\omega_0$ ,

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right) L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

## Comparison of series and parallel Resonant circuit:-

Parameter	Series circuit	Parallel circuit
Current at resonance	$I = V/R$ and is maximum	$I = \frac{VCR}{L}$ and is minimum
Impedance at Resonance	$Z = R$ and is minimum	$Z = \frac{L}{CR}$ and is maximum
Power factor	unity	unity
Resonant frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Q-factor	$Q = \frac{2\pi f_0 L}{R}$	$Q = \frac{2\pi f_0 L}{R}$