UNIT – IV

NETWORK THEOREMS

SUPERPOSITION THEOREM

It states that '*in a linear network containing more than one independent source and dependent source, the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone, all the other independent sources being represented meanwhile by their respective internal resistances.*'

The independent voltage sources are represented by their internal resistances if given or simply with zero resistances, i.e., short circuits if internal resistances are not mentioned. The independent current sources are represented by infinite resistances, i.e., open circuits.

The dependent sources are not sources but dissipative components—hence they are active at all times. A dependent source has zero value only when its control voltage or current is zero.

A linear network is one whose parameters are constant, i.e., they do not change with voltage and current.

Explanation Consider the network shown in Fig. 3.1. Suppose we have to find current I_4 through resistor R_4.

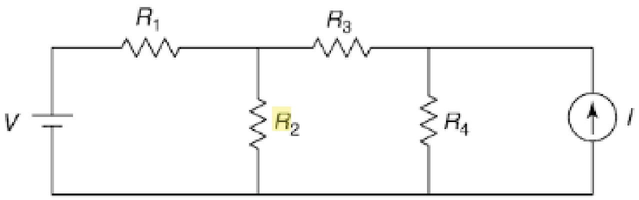


Fig. 3.1 *Network to illustrate superposition theorem*

The current flowing through resistor R_4 due to constant voltage source V is found to be say I_4' (with proper direction), representing constant current source with infinite resistance, i.e., open circuit.

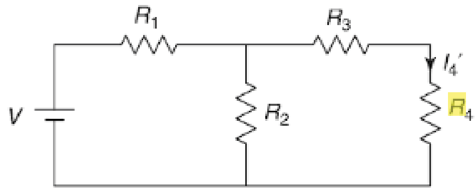


Fig. 3.2 *When voltage source V is acting alone*

The current flowing through resistor R_4 due to constant current source I is found to be say I_4'' (with proper direction), representing the constant voltage source with zero resistance or short circuit.

The resultant current I_4 through resistor R_4 is found by superposition theorem.

$$I_4 = I_4' + I_4''$$

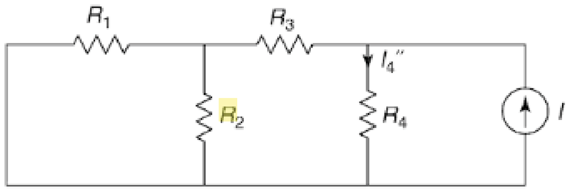


Fig. 3.3 *When current source I is acting alone*

Steps to be followed in Superposition Theorem

1. Find the current through the resistance when only one independent source is acting, replacing all other independent sources by respective internal resistances.
2. Find the current through the resistance for each of the independent sources.
3. Find the resultant current through the resistance by the superposition theorem considering magnitude and direction of each current.

Example 3.1 *Find the current through the 2 Ω resistor in Fig. 3.4.*

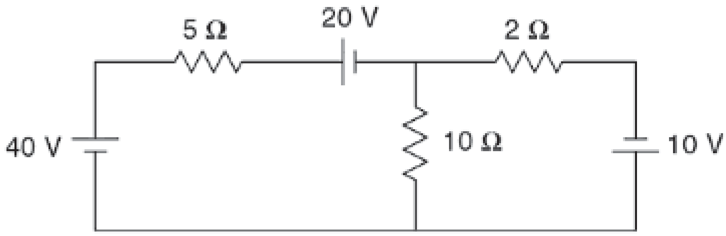


Fig. 3.4

Solution

Step I When the 40 V source is acting alone (Fig. 3.5)

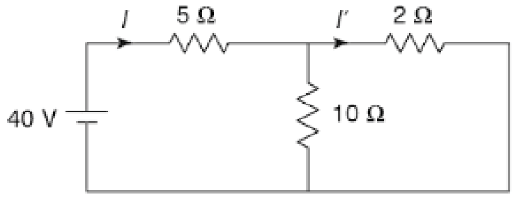


Fig. 3.5

By series parallel reduction technique (Fig. 3.6),

$$I = \frac{40}{5+1.67} = 6 \text{ A}$$

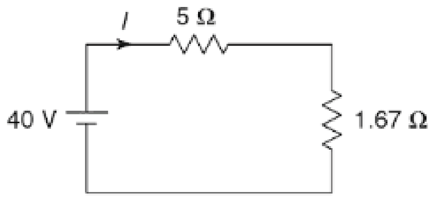


Fig. 3.6

From Fig. 3.5, by current-division rule,

$$I' = 6 \times \frac{10}{10+2} = 5 \text{ A } (\rightarrow)$$

Step II When the 20 V source is acting alone (Fig. 3.7)

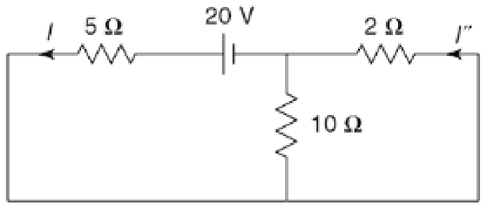


Fig. 3.7

By series–parallel reduction technique (Fig. 3.8),

$$I = \frac{20}{5+1.67} = 3 \text{ A}$$

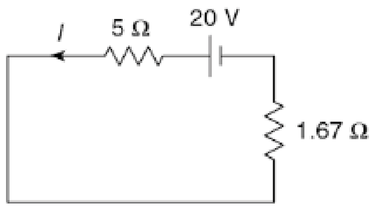


Fig. 3.8

From Fig. 3.7, by current-division rule,

$$I'' = 3 \times \frac{10}{10+2} = 2.5 \text{ A} (\leftarrow) = -2.5 \text{ A} \quad (\rightarrow)$$

Step III When the 10 V source is acting alone (Fig. 3.9)

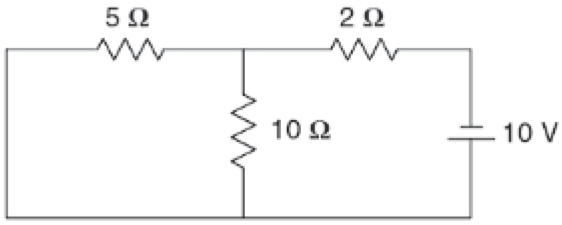


Fig. 3.9

By series–parallel reduction technique (Fig. 3.10),

$$I''' = \frac{10}{3.33+2} = 1.88\text{ A}(\rightarrow)$$

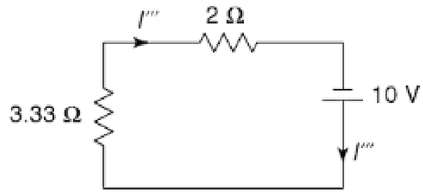


Fig. 3.10

Step IV By superposition theorem,

$$I = I' + I'' + I''' = 5 - 2.5 + 1.88 = 4.38\text{ A} \quad (\rightarrow)$$

THEVENIN'S THEOREM

It states that '*any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance, i.e., short circuit) and constant current source replaced by infinite resistance, i.e., open circuit.*'

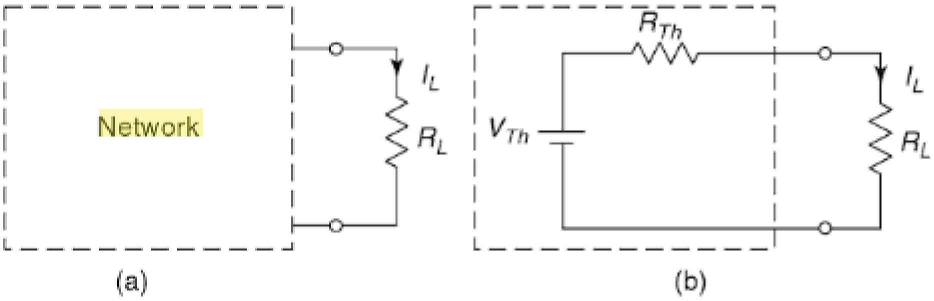


Fig. 3.109 *Network illustrating Thevenin's theorem*

Explanation Consider a simple network as shown in Fig. 3.110.

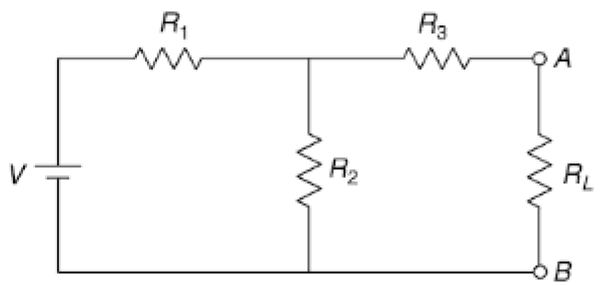


Fig. 3.110 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate open circuit voltage V_{Th} across points A and B as shown in Fig. 3.111.

$$V_{Th} = \frac{R_2}{R_1 + R_2} V$$

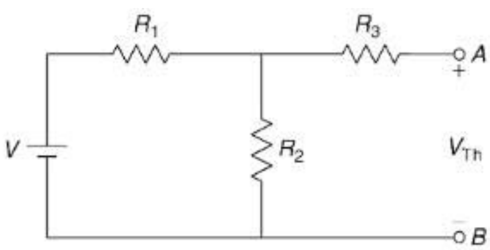


Fig. 3.111 *Calculation of V_{Th}*

For finding series resistance R_{Th}, replace the voltage source by a short circuit and calculate resistance between points A and B as shown in Fig. 3.112.

$$R_{Th} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

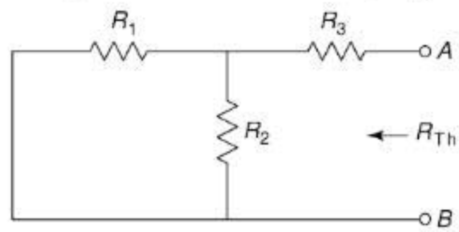


Fig. 3.112 *Calculation of R_{Th}*

Thevenin's equivalent network is shown in Fig. 3.113.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

If the network contains both independent and dependent sources, Thevenin's resistance R_{Th} is calculated as,

$$R_{Th} = \frac{V_{Th}}{I_N}$$

where I_N is the short-circuit current which would flow in a short circuit placed across the terminals A and B. Dependent sources are active at all times. They have zero values only when the control voltage or current is zero. R_{Th} may be negative in some cases which indicates negative resistance region of the device, i.e., as voltage increases, current decreases in the region and vice-versa.

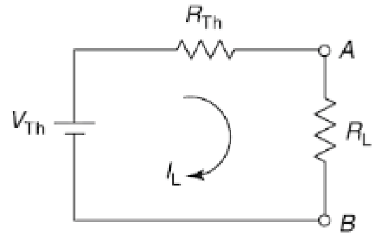


Fig 3.113 *Thevenin's equivalent network*

If the network contains only dependent sources then

$$V_{Th} = 0$$
$$I_N = 0$$

For finding R_{Th} in such a network, a known voltage V is applied across the terminals A and B and current is calculated through the path AB.

$$R_{Th} = \frac{V}{I}$$

or a known current source I is connected across the terminals A and B and voltage is calculated across the terminals A and B.

$$R_{Th} = \frac{V}{I}$$

Thevenin's equivalent network for such a network is shown in Fig. 3.114.

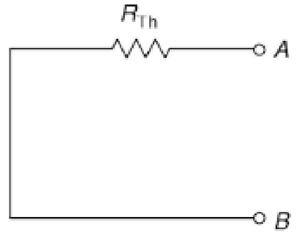


Fig. 3.114 *Thevenin's equivalent network*

Steps to be Followed in Thevenin's Theorem

1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Example 3.27 *Find the current through the 2 Ω resistor in Fig. 3.115.*

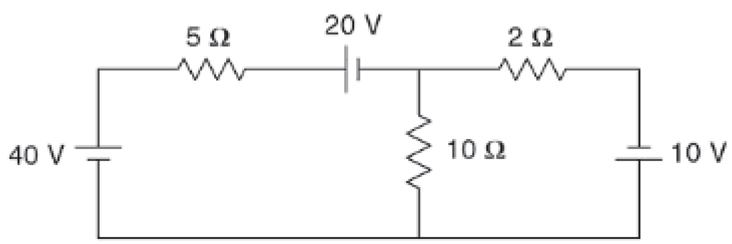


Fig. 3.115

Solution

Step I Calculation of V_{Th} (Fig. 3.116)

Applying KVL to the mesh,

$$40 - 5I - 20 - 10I = 0$$

$$15I = 20$$

$$I = 1.33 \text{ A}$$

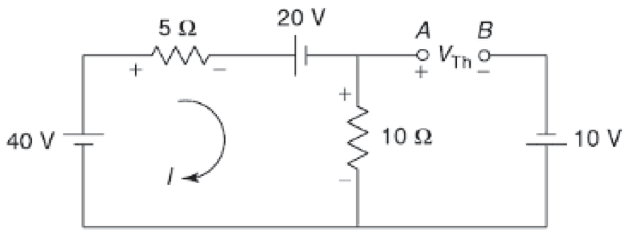


Fig. 3.116

Writing the V_{Th} equation,

$$10I - V_{Th} + 10 = 0$$

$$V_{Th} = 10I + 10 = 10(1.33) + 10 = 23.33 \text{ V}$$

Step II Calculation of R_{Th} (Fig. 3.117)

$$R_{Th} = 5 \,||\, 10 = 3.33\,\Omega$$

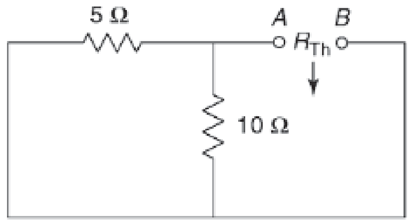


Fig. 3.117

Step III Calculation of I_L (Fig. 3.118)

$$I_L = \frac{23.33}{3.33 + 2} = 4.38 \text{ A}$$

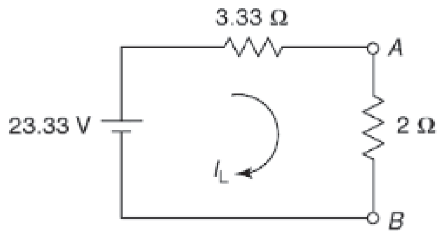


Fig. 3.118

NORTON'S THEOREM

It states that '*any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.*' The constant current is equal to the current which would flow in a short circuit placed across the terminals. The parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by internal resistances.

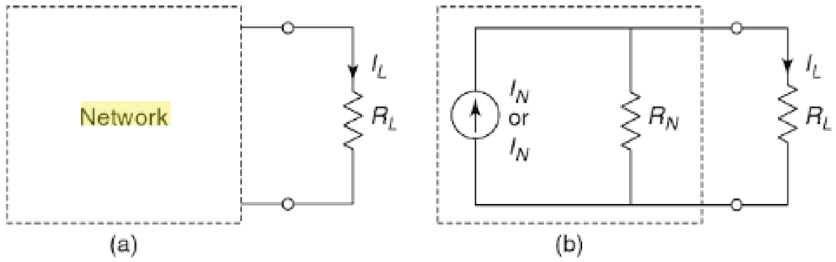


Fig. 3.251 *Network illustrating Norton's theorem*

Explanation Consider a simple network as shown in Fig.3.252

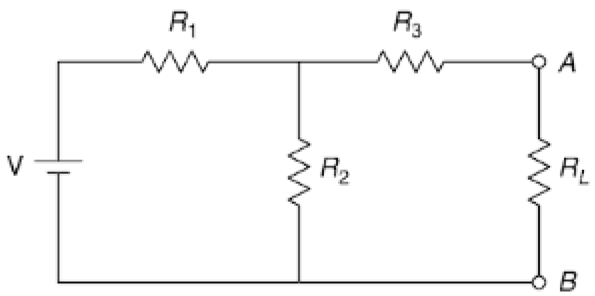


Fig. 3.252 *Network*

For finding load current through R_L, first remove the load resistor R_L from the network and calculate short circuit current I_{SC} or I_N which would flow in a short circuit placed across terminals A and B as shown in Fig. 3.253.

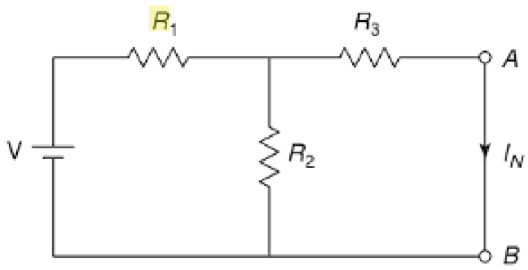


Fig. 3.253 *Calculation of I_N*

For finding parallel resistance R_N, replace the voltage source by a short circuit and calculate resistance between points A and B as shown in Fig. 3.254.

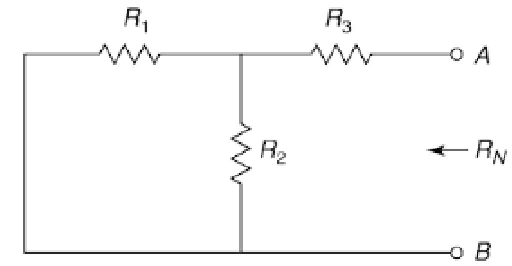


Fig. 3.254 *Calculation of R_N*

$$R_N = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Norton's equivalent network is shown in Fig.3.255.

$$I_L = I_N \frac{R_N}{R_N + R_L}$$

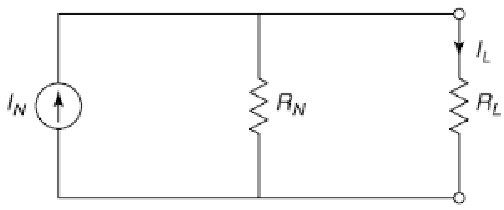


Fig. 3.255 *Norton's equivalent network*

If the network contains both independent and dependent sources, Norton's resistances R_N is calculated as

$$R_N = \frac{V_{Th}}{I_N}$$

where V_{Th} is the open-circuit voltage across terminals A and B. If the network contains only dependent sources, then

$$V_{Th} = 0$$

$$I_N = 0$$

To find R_{Th} in such network, a known voltage V or current I is applied across the terminals A and B, and the current I or the voltage V is calculated respectively.

$$R_N = \frac{V}{I}$$

Norton's equivalent network for such a network is shown in Fig. 3.256.

Fig. 3.256 *Norton's equivalent network*

Steps to be followed in Norton's Theorem

1. Remove the load resistance R_L and put a short circuit across the terminals.
2. Find the short-circuit current I_{SC} or I_N.
3. Find the resistance R_N as seen from points A and B.
4. Replace the network by a current source I_N in parallel with resistance R_N.
5. Find current through R_L by current–division rule.

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Example 3.59

Find the current through the 10 Ω resistor in Fig. 3.258.

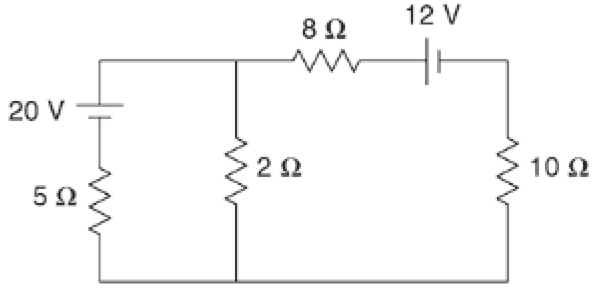


Fig. 3.261

Solution

Step I Calculation of I_N (Fig. 3.262)

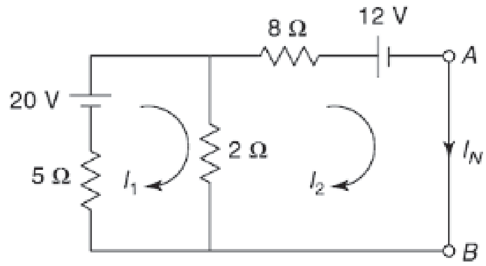


Fig. 3.262

Applying KVL to Mesh 1,

$$-5I_1 + 20 - 2(I_1 - I_2) = 0$$

$$7I_1 - 2I_2 = 20 \quad \text{...(i)}$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 8I_2 - 12 = 0$$

$$-2I_1 + 10I_2 = -12 \quad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = -0.67 \text{ A}$$

$$I_N = I_2 = -0.67 \text{ A}$$

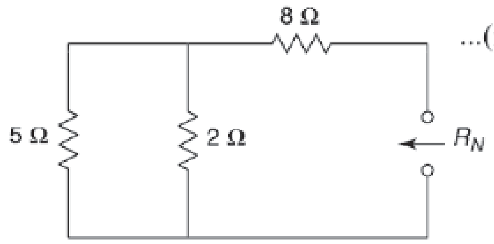


Fig. 3.263

Step II Calculation of R_N (Fig. 3.263)

$$R_N = (5 \,||\, 2) + 8 = 9.43\ \Omega$$

Step III Calculation of I_L (Fig. 3.264)

$$I_L = 0.67 \times \frac{9.43}{9.43 + 10} = 0.33 \text{ A} (\uparrow)$$

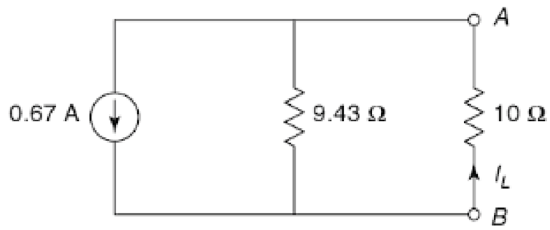


Fig. 3.264

MAXIMUM POWER TRANSFER THEOREM

It states that '*the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.*'

Proof From Fig. 3.363,

$$I = \frac{V}{R_s + R_L}$$

Power delivered to the load $R_L = P = I^2\, R_L = \dfrac{V^2 R_L}{(Rs + R_L)^2}$

To determine the value of R_L for maximum power to be transferred to the load,

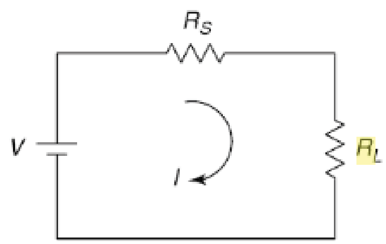


Fig. 3.363 *Network illustrating maximum power transfer theorem*

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \frac{V^2}{(R_s + R_L)^2} R_L$$

$$= \frac{V^2[(R_s + R_L)^2 - (2R_L)(R_s + R_L)]}{(R_s + R_L)^4}$$

$$(R_s + R_L)^2 - 2\,R_L(R_s + R_L) = 0$$

$$R_s^2 + R_L^2 + 2R_sR_L - 2R_LR_s - 2R_L^2 = 0$$

$$R_s = R_L$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

Steps to be followed in Maximum Power Transfer Theorem

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B.
4. Find the resistance R_L for maximum power transfer.

$$R_L = R_{Th}$$

5. Find the maximum power (Fig. 3.364).

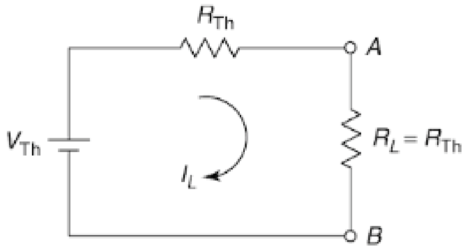


Fig. 3.364 *Thevenin's equivalent network*

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{V_{Th}}{2R_{Th}}$$

$$P_{max} = I_L^2\, R_L = \frac{V_{Th}^2}{4R_{Th}^2} \times R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$$

Example 3.82 *Find the value of resistance R_L in Fig. 3.365 for maximum power transfer and calculate maximum power.*

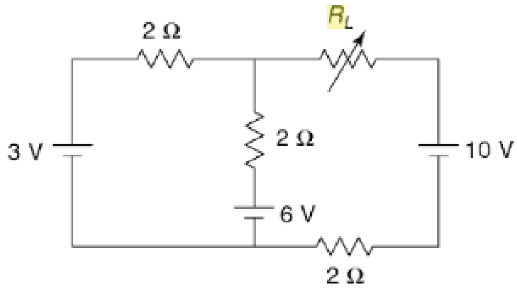


Fig. 3.365

Solution

Step I Calculation of V_{Th} (Fig. 3.366)

Applying KVL to the mesh,

$$3 - 2I - 2I - 6 = 0$$

$$I = -0.75\text{ A}$$

Writing the V_{Th} equation,

$$6 + 2I - V_{Th} - 10 = 0$$

$$V_{Th} = 6 + 2I - 10 = 6 + 2(-0.75) - 10 = -5.5\text{ V}$$

$$= 5.5\text{ V (terminal } B \text{ is positive w.r.t } A)$$

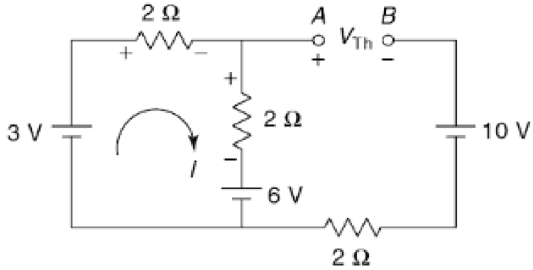


Fig. 3.366

Step II Calculation of R_{Th} (Fig. 3.367)

$$R_{Th} = (2 \,||\, 2) + 2 = 3\,\Omega$$

Step III Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 3\,\Omega$$

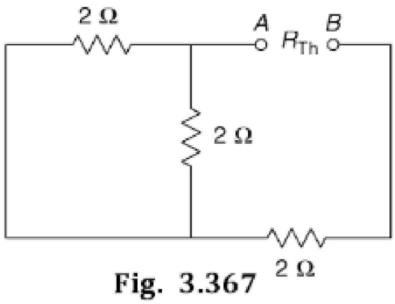


Fig. 3.367

Step IV Calculation of P_{max} (Fig. 3.368)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(5.5)^2}{4 \times 3} = 2.52\,W$$

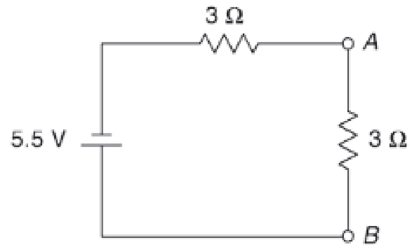


Fig. 3.368

Compensation Theorem

In circuit analysis, many times it is required to study the effect of change in impedance in one of its branches on the corresponding voltages and currents of the network. The compensation theorem provides a very simple way for studying such effects. The statement of compensation theorem is as follows.

Statement : *In any linear network consisting of linear and bilateral impedances and active sources, if the impedance Z of the branch carrying current I increases by δZ, then the increament of voltage or current in each branch of the network is that voltage or current that would be produced by an opposing voltage source of value $V_C(= I.\delta Z)$ introduced in the altered branch after replacing original sources by their internal impedances.*

7.6.1 Explanation of Compensation Theorem

Consider a network shown in the Fig. 7.21.

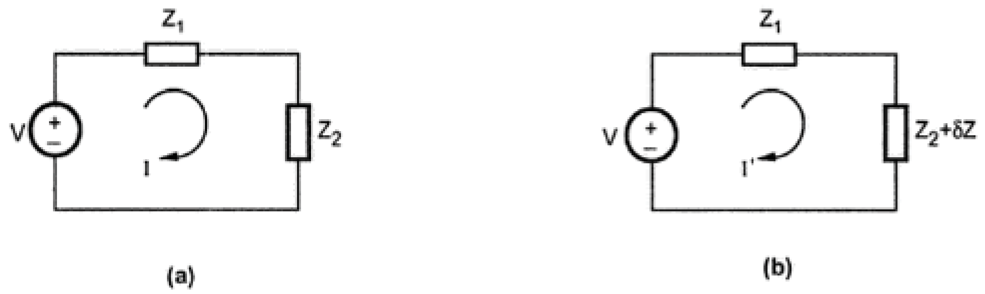


Fig. 7.21

V is the voltage applied to the network. I is the current flowing through Z_1 and Z_2. Consider that impedance Z_2 increases by δZ . Due to this, the current in the circuit changes to I′ as shown in the Fig. 7.21 (b).

Then the effect of change in impedance is the change in current which is given by,

$$\delta I = I - I'$$

Now this current can be directly calculated by using the compensation theorem. First modify the branch of which impedance is changed, by connecting a voltage source V_C of value $I \cdot \delta Z$. The new voltage source must be connected in the branch with proper polarity.

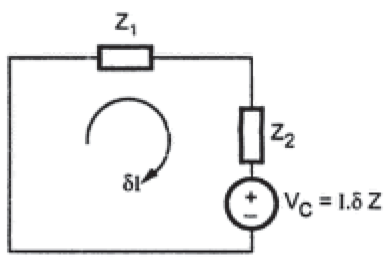


Fig. 7.22

Then replace original active source i.e. voltage source V by its internal impedance as shown in the Fig. 7.22.

The voltage source introduced in modified branch, V_C is called **compensation source** with value $I \cdot \delta Z$ where I is current through impedance before impedance of the branch is changed and δZ is the change in impedance.

Proof of Compensation Theorem

Consider a network shown in the Fig. 7.23.

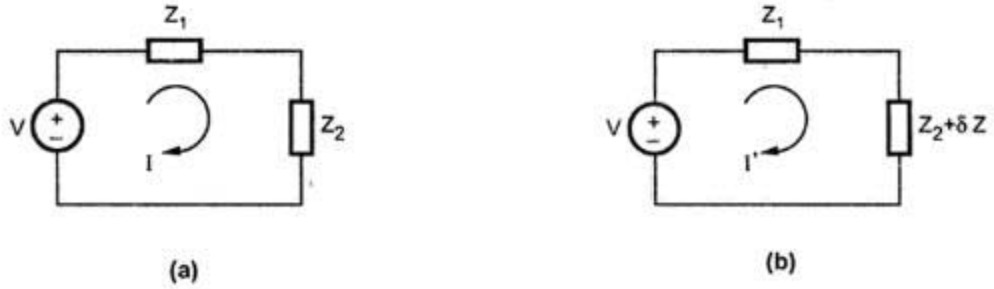


Fig. 7.23

The current flowing in the circuit is given by,

$$I = \frac{V}{Z_1 + Z_2} \quad \text{... (1)}$$

Consider that the impedance Z_2 changes by δZ, then the current changes to I' as shown in the Fig. 7.23 (b).

The current I' is given by,

$$I' = \frac{V}{Z_1 + (Z_2 + \delta Z)} \quad \text{... (2)}$$

The change in current due to change in impedance is given by,

$$\delta I = I - I'$$

$$= \frac{V}{Z_1 + Z_2} - \frac{V}{Z_1 + (Z_2 + \delta Z)} = \frac{V}{(Z_1 + Z_2)} - \frac{V}{(Z_1 + Z_2 + \delta Z)}$$

$$= \frac{V(Z_1 + Z_2 + \delta Z) - V(Z_1 + Z_2)}{(Z_1 + Z_2)(Z_1 + Z_2 + \delta Z)}$$

$$= \frac{V[Z_1 + Z_2 + \delta Z - Z_1 - Z_2]}{(Z_1 + Z_2)(Z_1 + Z_2 + \delta Z)}$$

$$= \frac{V}{(Z_1 + Z_2)} \cdot \frac{\delta Z}{(Z_1 + Z_2 + \delta Z)}$$

$$= \frac{I \cdot \delta Z}{(Z_1 + Z_2 + \delta Z)} \quad \text{... from equation (1)}$$

$$\therefore \quad \delta I = \frac{V_C}{(Z_1 + Z_2 + \delta Z)} \quad \text{... (3)}$$

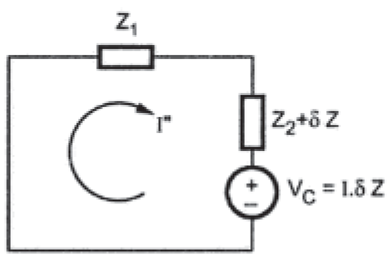


Fig. 7.24

Now consider that the branch is modified as shown in the Fig. 7.24 and also original voltage source is short circuited. Let the current in circuit be I".

Applying KVL to the loop,

$$-Z_1 \cdot I'' - (Z_2 + \delta Z) \cdot I'' + V_C = 0$$

$$\therefore \quad -(Z_1 + Z_2 + \delta Z)I'' = -I \cdot \delta Z$$

$$\therefore \quad I'' = \frac{I \cdot \delta Z}{(Z_1 + Z_2 + \delta Z)} = \frac{V_C}{(Z_1 + Z_2 + \delta Z)} \quad \text{... (4)}$$

From equations (3) and (4), $I'' = \delta I$

Thus, compensation theorem is proved.

Example 7.7 : *Calculate change in current in the network shown in the Fig. 7.25 by using compensation theorem when the reactance has changed to j35 Ω.*

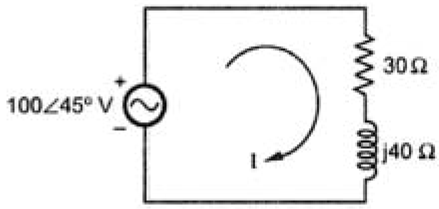


Fig. 7.25

Solution : Applying KVL, we get,

$$I = \frac{100\angle 45°}{30+j40} = \frac{100\angle 45°}{50\angle 53.13°} = 2\angle -8.13° \text{ A}$$

$$\therefore \quad I = (1.9798 - j0.2828) \text{ A} \quad \text{... (1)}$$

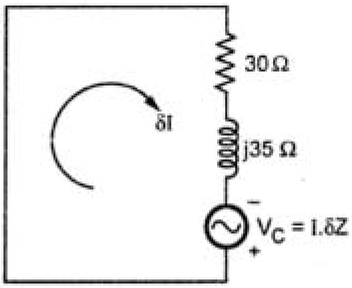


Fig. 7.25 (a)

Now the reactance has changed to j35. Hence the current in network will also change to I′. The change in the reactance is given by,

$$\delta Z = j40 - j35 = j5\ \Omega \quad \text{... (2)}$$

Now the reactance is decreased. Modifying the network by replacing voltage source by short circuit and introducing compensation source $V_C = I \cdot \delta Z$ in the branch altered as shown in the Fig. 7.25(a).

The compensation source is given by,

$$V_C = I \cdot \delta Z$$

$$= (2\angle -8.13°)(j5) = (2\angle -8.13°)(5\angle 90°)$$

$$\therefore \quad V_C = 10\angle 81.87° \text{ V} \quad \text{... (3)}$$

Thus, change in current is given by,

$$\delta I = \frac{V_C}{30+j35} = \frac{10\angle 81.87°}{46.0977\angle 49.4°} = \mathbf{0.2169\angle 32.47° \text{ A}}$$

Substitution Theorem

In network analysis many times it is needed to replace an impedance branch by another branch with different network elements without disturbing the voltages and currents in the network. The substitution theorem provides the convenient method to get the condition under which branch replacement is possible. The statement of substitution theorem is as below.

Statement : *In any network any branch of it may be replaced (substituted) by a branch with different network elements without disturbing the voltages and currents in the entire network, if the new branch has same set of terminal voltage and current as the original branch.*

Explanation of Substitution Theorem

Consider a network shown in the Fig. 7.26. Let the current through branch AB be I_{AB} and voltage across the branch A-B be V_{AB}.

The voltage across original branch AB is given by,

$$V_{AB} = Z_{AB} \cdot I_{AB} + E \quad \text{... (1)}$$

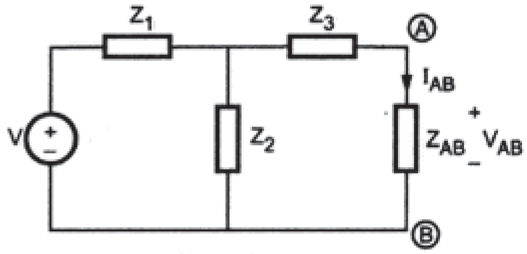


Fig. 7.26

This branch may be substituted by any other branch in many other ways where the branch voltage is given by

$$V_{AB} = Z'_{AB} I_{AB} + E' \quad \text{... (2)}$$

where Z'_{AB} and E' are chosen such that I_{AB} and V_{AB} are not changed.

Following are the important points in the accordance with the application of the substitution theorem :

1) The substitution theorem is applicable to both the types of the networks such as linear and nonlinear.
2) If the substitution theorem is applied in non-linear network then a modified network must have a unique solution. In linear networks it has number of solutions.
3) The substitution theorem is useful in proving other network theorems.
4) The substitution theorem is useful in analysis of a network having one non-linear element.

Example 7.8 : *For the network shown in the Fig. 7.27, substitute the branch A-B by a) a voltage source b) a current source.*

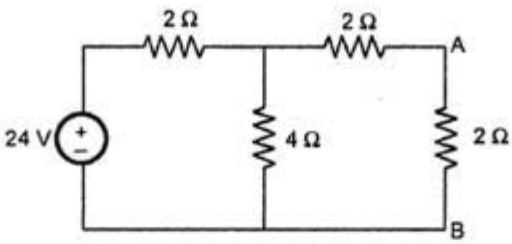


Fig. 7.27

Solution : Let current through branch A-B be I_{AB}.

Total resistance looking from source is given by

$$R_{eq} = 2+[4||(2+2)] = 2+[4||4] = 4\ \Omega$$

$$\therefore \text{ Total current } \quad I_T = \frac{24}{4} = 6\,A$$

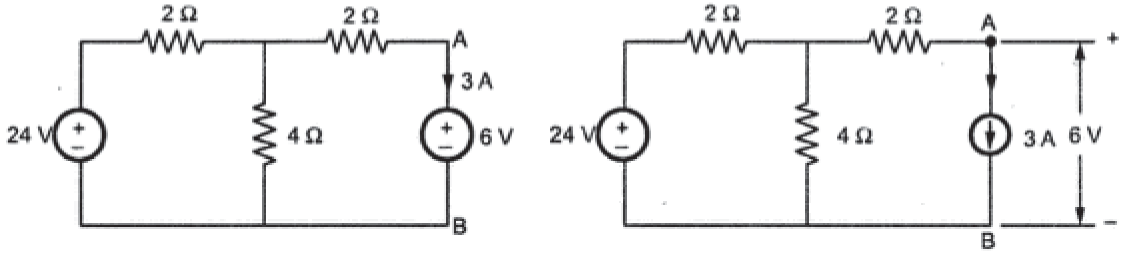
By current divider rule,

$$I_{AB} = 6\left[\frac{4}{4+4}\right] = 3\ A$$

∴ Voltage across branch A-B,

$$V_{AB} = I_{AB} \times 2 = 3 \times 2 = 6\ V$$

Therefore branch A-B may be substituted either by an independent voltage source of value 6 V with the branch current of 3 A or by an independent current source of value 3 A with the branch voltage of 6 V. The two substitutions are as shown in the Fig. 7.27 (a) and (b) and respectively.



(a) (b)

Fig. 7.27

Millman's Theorem

It is possible to combine number of voltage sources or current sources into a single equivalent voltage or current source, using Millman's theorem. The statement of the Millman's theorem is,

Statement : *If n voltage sources* $V_1, V_2, \ldots\ldots, V_n$ *having internal impedances (or series impedances)* $Z_1, Z_2, \ldots\ldots, Z_n$ *respectively, are in parallel, then these sources may be replaced by a single voltage source of voltage* V_M *having a series impedance* Z_M *where* V_M *and* Z_M *are given by,*

$$V_M = \frac{V_1 Y_1 + V_2 Y_2 + \ldots\ldots + V_n Y_n}{Y_1 + Y_2 + \ldots\ldots + Y_n} = \frac{\sum_{k=1}^{n} V_k Y_k}{\sum_{k=1}^{n} Y_k}$$

and

$$Z_M = \frac{1}{Y_1 + Y_2 + \ldots\ldots + Y_n} = \frac{1}{\sum_{k=1}^{n} Y_k}$$

where $Y_1, Y_2, \ldots\ldots, Y_n$ *are the admittances corresponding to the impedances* $Z_1, Z_2, \ldots\ldots, Z_n$.

Proof of Millman's Theorem

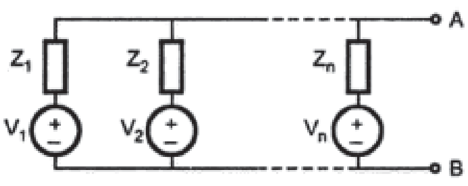


Fig. 7.33

Consider n voltage sources in parallel as shown in the Fig. 7.33.

Let us convert each voltage source into an equivalent current source. For source 1,

$$I_1 = \frac{V_1}{Z_1} = V_1 Y_1 \text{ as } Y_1 = \frac{1}{Z_1}$$

Similarly for the remaining sources we can write,

$$I_2 = V_2 Y_2, \quad I_3 = V_3 Y_3, \ldots\ldots I_n = V_n Y_n$$

where $Y_1, Y_2, \ldots\ldots Y_n$ are the admittances to be connected in parallel. Hence circuit reduces to,

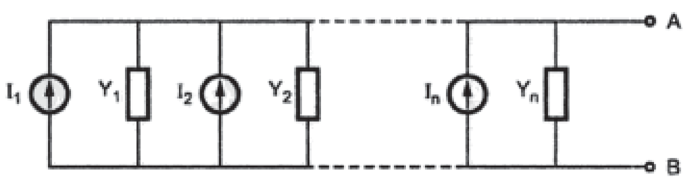


Fig. 7.33 (a)

Hence the effective current source across the terminals A-B is,

$$I_M = I_1 + I_2 + ...+ I_n \quad ... (1)$$

and $$Y_M = Y_1 + Y_2 + ...+ Y_n \quad ... (2)$$

This is because admittances in parallel get added to each other. Hence circuit reduces to, as shown in the Fig. 7.34.

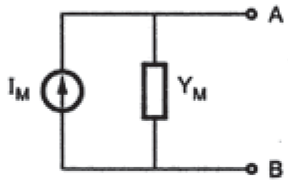


Fig. 7.34

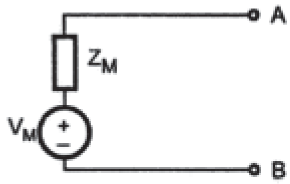


Fig. 7.34 (a)

Converting this equivalent current source into the voltage source we get,

$$V_M = \frac{I_M}{Y_M}$$

as $$Z_M = \frac{1}{Y_M}$$

$$V_M = I_M Z_M$$

Substituting I_M and Y_M from equations (1) and (2),

$$V_M = (I_1 + I_2 + ...+ I_n) \cdot \frac{1}{(Y_1 + Y_2 + ... + Y_n)}$$

but $$I_1 = \frac{V_1}{Z_1} = V_1 Y_1, \; I_2 = V_2 Y_2, ... I_n = V_n Y_n$$

$$\therefore \quad V_M = \frac{V_1 Y_1 + V_2 Y_2 + ... + V_n Y_n}{Y_1 + Y_2 + ... + Y_n}$$

$$\therefore \quad Z_M = \frac{1}{Y_M} = \frac{1}{Y_1 + Y_2 + ... + Y_n}$$

Thus Millman's theorem is proved.

Example 7.10 : *Use Millman's theorem to find the current through the 10 Ω resistance in the circuit of Fig. 7.35.*

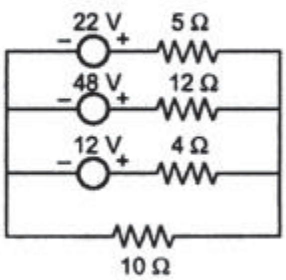


Fig. 7.35

Solution : From the given network we can write,

$$V_1 = 12 \text{ V}, Z_1 = 4 \, \Omega, V_2 = 48 \text{ V}, Z_2 = 12 \, \Omega, V_3 = 22 \text{ V}, Z_3 = 5 \, \Omega$$

$$\therefore \quad Y_1 = \frac{1}{4} \text{ mho}, Y_2 = \frac{1}{12} \text{ mho}, Y_3 = \frac{1}{5} \text{ mho}$$

According to Millman's theorem, the equivalent voltage source and impedance across 10 Ω is given by,

$$Z_M = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{\frac{1}{4} + \frac{1}{12} + \frac{1}{5}}$$

$$= 1.875 \, \Omega$$

$$V_M = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y_1 + Y_2 + Y_3} = \frac{12\times\frac{1}{4} + 48\times\frac{1}{12} + 22\times\frac{1}{5}}{05333}$$

$$= \mathbf{21.375\ V}$$

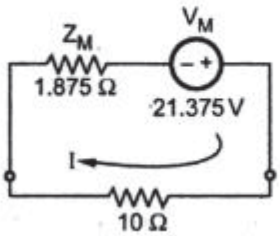


Fig. 7.35 (a)

The equivalent is shown in the Fig. 7.35 (a).

$$I_{10\Omega} = \frac{V_M}{Z_M + 10}$$

$$= \frac{21.375}{1.875 + 10} = \mathbf{1.8\ A}$$

Tellegan's Theorem

The Tellegan's theorem is valid for any lumped network which may be linear or nonlinear, active or passive, time varying or time invariant. The statement of the theorem is as below :

Statement : *In an arbitrary lumped network, the algebraic sum of the instantaneous powers in all the branches, at any instant is zero. All the branch currents and the voltages in that network must satisfy Kirchhoff's laws. In other words, it can be stated as, in a given network, the algebraic sum of the powers delivered by all the sources is equal to the algebraic sum of the powers absorbed by all the elements.*

Mathematically this theorem can be expressed as,

$$\sum_{k=1}^{b} v_k\, i_k = 0$$

where b is the number of branches in a network.

Explanation of Tellegan's Theorem

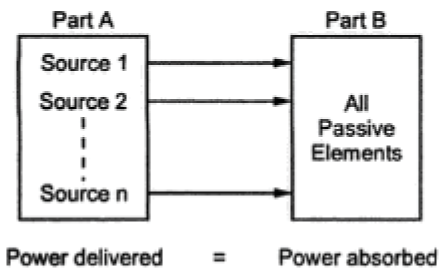


Fig. 7.36

Let the network is divided into two parts. The part A with 'n' active energy sources and second part B with all the passive elements. Then the power delivered by n sources of part A must be equal to the sum of the power absorbed (dissipated or stored) by the elements of part B. This is shown in the Fig. 7.36.

➠ **Example 7.11 :** *For the network shown in the Fig. 7.37 verify Tellegan's theorem.*

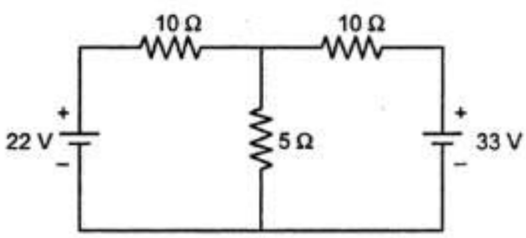


Fig. 7.37

Solution : Assuming loop currents as shown in the Fig. 7.37 (a).

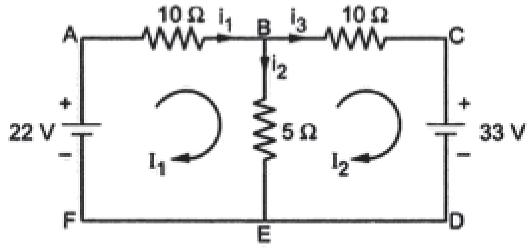


Fig. 7.37 (a)

Applying KVL to loop A-B-E-F-A,

$$-10I_1 - 5I_1 + 5I_2 + 22 = 0$$

$$\therefore \quad -15I_1 + 5I_2 = -22$$

$$\therefore \quad 15I_1 - 5I_2 = 22 \quad \text{... (1)}$$

Applying KVL to loop B-C-D-E-B,

$$-10I_2 - 33 - 5I_2 + 5I_1 = 0$$

$$\therefore \quad 5I_1 - 15I_2 = 33 \quad \text{... (2)}$$

Solving equations (1) and (2) simultaneously,

$$I_1 = 0.825 \text{ A}$$

$$I_2 = -1.925 \text{ A}$$

Current flowing through loop2 is negative which indicates that assumed direction of I_2 is exactly opposite to the actual direction. Hence I_2 flows in anticlockwise direction. $\therefore I_2$ = 1.925 A in anticlockwise direction.

Current through 10 Ω resistor between nodes A and B is given by,

$$i_1 = I_1 = 0.825 \text{ A....... from A to B}$$

Current flowing through 5 Ω resistor is given by

$$i_2 = I_1 + I_2 = (0.825) + (0.1925) = 2.75 \text{ A} \quad \text{....... From B to E}$$

Current flowing through 10 Ω resistor between nodes B and C is given by,

$$i_3 = I_2 = 1.925 \text{ A ...... from C to B.}$$

Total power delivered by sources,

$$P_{delivered} = (I_1)(22) + (I_2)(33)$$

$$= (0.825)(22) + (1.925)(33)$$

$$= \mathbf{81.675 \ W}$$

Total power absorbed by the elements

$$P_{absorbed} = (i_1^2 \times 10) + (i_2^2 \times 5) + (i_3^2 \times 10)$$

$$= [(0.825)^2 \times 10] + [(2.75)^2 \times 5] + [(1.925)^2 \times 10]$$

$$= \mathbf{81.675 \ W}$$

$$\therefore \quad P_{delivered} = P_{absorbed} \quad \text{..... Hence Tellegan's theorem is proved.}$$

Reciprocity Theorem

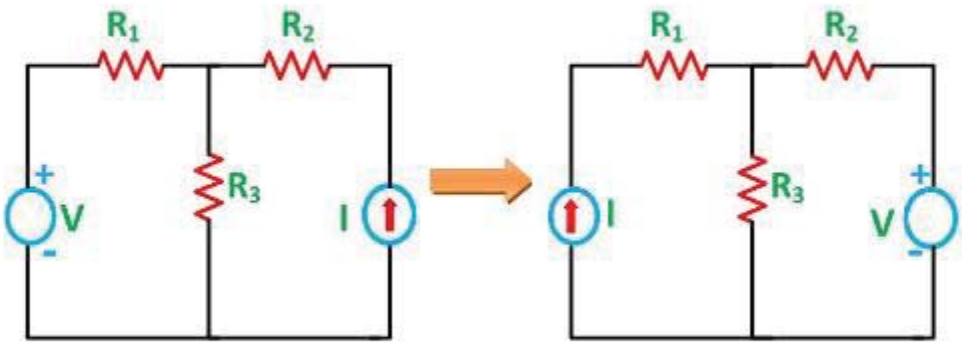
Reciprocity Theorem states that – In any branch of a network or circuit, the current due to a single source of voltage (V) in the network is equal to the current through that branch in which the source was originally placed when the source is again put in the branch in which the current was originally obtained.This theorem is used in the bilateral linear network which consists bilateral components.

In simple words, we can state the reciprocity theorem as when the places of voltage and current source in any network are interchanged the amount or magnitude of current and voltage flowing in the circuit remains the same. This theorem is used for solving many DC and AC network which have many applications in electromagnetism electronics.Their circuit does not have any time varying element.

Explanation of Reciprocity Theorem

The location of the voltage source and the current source may be interchanged without a change in current. However, the polarity of the voltage source should be identical with the direction of the branch current in each position.

The Reciprocity Theorem is explained with the help of the circuit diagram shown below



The various resistances R_1, R_2, R_3 is connected in the circuit diagram above with a voltage source (V) and a current source (I). It is clear from the figure above that the voltage source and current sources are interchanged for solving the network with the help of Reciprocity Theorem.

The limitation of this theorem is that it is applicable only to single source networks and not in the multi-source network. The network where reciprocity theorem is applied should be linear and consist of resistors, inductors, capacitors and coupled circuits. The circuit should not have any time-varying elements.

Steps for Solving a Network Utilizing Reciprocity Theorem

Step 1 – Firstly, select the branches between which reciprocity has to be established.

Step 2 – The current in the branch is obtained using any conventional network analysis method.

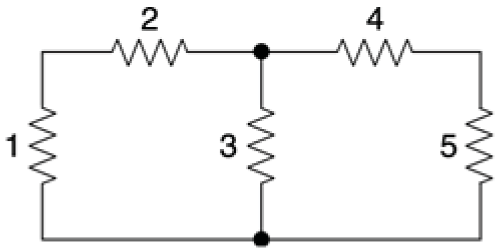
Step 3 – The voltage source is interchanged between the branch which is selected.

Step 4 – The current in the branch where the voltage source was existing earlier is calculated.

Step 5 – Now, it is seen that the current obtained in the previous connection, i.e., in step 2 and the current which is calculated when the source is interchanged, i.e., in step 4 are identical to each other.

Problems-14

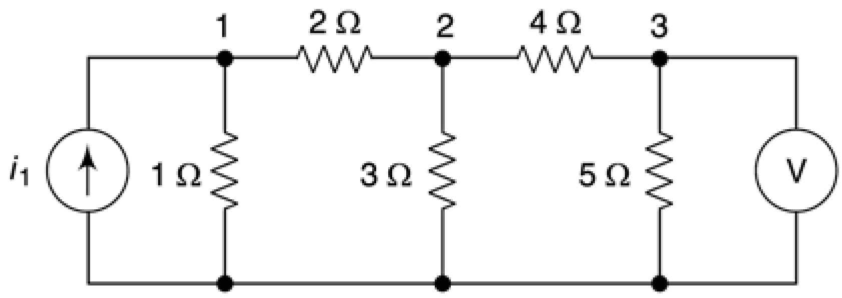
Verify the Reciprocity Theorem for the network shown in the figure using current source and a voltmeter. All the values are in ohm.



Solution

Using a current source and a voltmeter,

Let, e_1, e_2 be node voltages, v_1 be the voltmeter reading.



By KCL,

At node (1) $\Rightarrow 3e_1 - e_2 - 2i_1 = 0$ (i)

At node (2) $\Rightarrow -6e_1 + 13e_2 - 3v_1 = 0$ (ii)

At node (3) $9v_1 = 5e_2$ (iii)

From (ii) $\Rightarrow -6e_1 + 13 \times \frac{9}{5} v_1 - 3v_1 = 0$

$\Rightarrow -6e_1 + \left(\frac{117}{5} - 3\right) v_1 = 0$

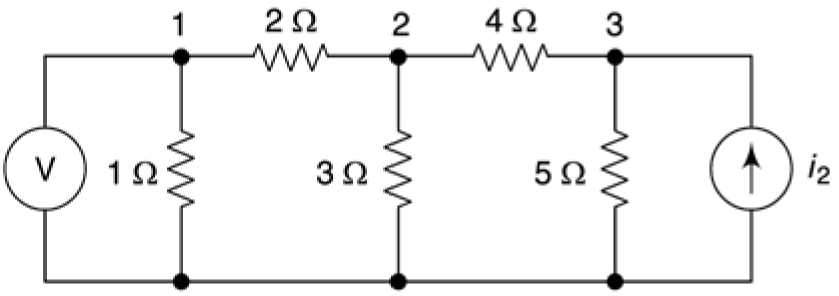
$\Rightarrow 6e_1 + \frac{102}{5} v_1 \Rightarrow e_1 = \frac{17}{5} v_1$

From (i) $\Rightarrow 3 \times \frac{17}{5} v_1 - \frac{9}{5} v_1 = 2i$

$\Rightarrow \left(\frac{i_1}{v_1}\right) = \left(\frac{21}{5}\right)$ (A)

Interchanging the positions of the current source and the voltmeter,

Now, let v_2 be the voltmeter reading



By KCL,

At node (1) $\Rightarrow 3v_2 = e_2$ (iv)

At node (2) $\Rightarrow -6v_2 + 13e_2 - 3e_3 = 0$

$\Rightarrow -6v_2 + 13 \times 3v_2 - 3e_3 = 0$

$\Rightarrow e_3 = 11v_2$ (v)

At node (3) $\Rightarrow 5e_3 - 5e_2 + 4e_3 - 20i_2 = 0$

$\Rightarrow 20i_2 = 9e_3 - 5e_2 = 9 \times 11v_2 - 5 \times 3v_2 = 84v_2$

$\Rightarrow \left(\frac{i_2}{v_2}\right) = \left(\frac{21}{5}\right)$ (B)

From equations (A) and (B), Reciprocity theorem is proved.