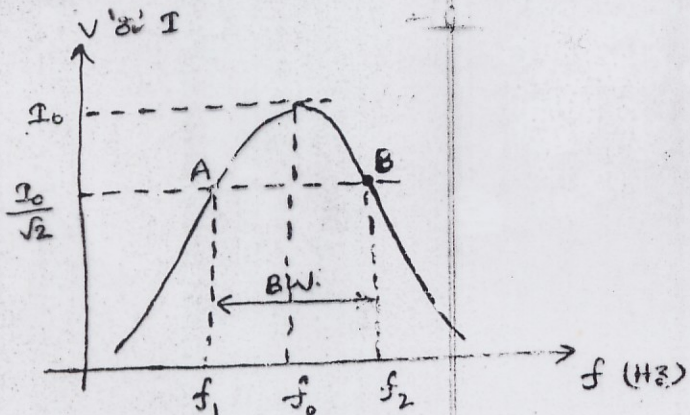
Bandwidth of a series Resonant ckt :-

The bandwidth of any system is the range of frequencies for which the current or output voltage is equal to 70.7% of its value at the resonant frequency.



Bandwidth (BW) is also defined as the frequency betw difference between 'f_2' and 'f_1'.

$$\therefore BW = f_2 - f_1$$

(OR)

$$\Delta f = f_2 - f_1$$

(OR)

$$\Delta \omega = \omega_2 - \omega_1$$

f_1 : Lower cut-off frequency.

f_2 : upper cut-off frequency.

power dissipated in the ckt at the frequency 'f_1' is,

$$P_A = I_1^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{I_0^2 R}{2}$$

power dissipated in the ckt at frequency 'f_2' is,

$$P_B = I_2^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{I_0^2 R}{2}$$

$$\therefore P_A = P_B = \frac{I_0^2 R}{2}$$

where $I_0^2 R = P_0$.

$$\therefore \boxed{P_A = P_B = \frac{P_0}{2}}$$

For this reason, the two points A & B on the frequency response curve are known as "half power points".

The Decibel power response :-

$$\text{power ratio} = 10 \log_{10} \left[\frac{\text{power at half power points}}{\text{power at resonance}} \right]$$

$$= 10 \log_{10} \left[\frac{P_0/2}{P_0} \right]$$

$$= 10 \log_{10} \left[\frac{1}{2} \right]$$

$$= -3\,dB.$$

Hence the half power points are also called '3dB' points. At these points, the power is '3dB' less than at resonance.

Quality factor (Q-factor) (or) Figure of Merit :-

Q-factor is defined as,

$$Q = 2\pi \left[\frac{\text{max. energy stored / cycle}}{\text{Energy dissipated / cycle}} \right]$$

For an inductor :-

$$\text{max energy stored} = \frac{1}{2} L I_m^2$$

$$\text{energy dissipated / cycle} = \left(\frac{I_m}{\sqrt{2}}\right)^2 RT$$

$$= \frac{I_m^2 RT}{2}$$

$$\therefore \quad Q = 2\pi \left[\frac{1}{2} L I_m^2 \Big/ \frac{I_m^2 R}{2}\left(\frac{1}{f}\right) \right]$$

$$Q = \frac{2\pi f L}{R}$$

$$\boxed{Q = \frac{\omega L}{R}}$$

For a capacitor :-

$$\text{Max. energy stored} = \frac{1}{2} C V_m^2$$

$$= \frac{1}{2} C \left[I_m X_c \right]^2$$

$$= \frac{1}{2} C \left[\frac{I_m^2}{\omega^2 C^2} \right]$$

$$= \frac{1}{2} \frac{I_m^2}{\omega^2 C}$$

$$\therefore \quad \text{Energy dissipated/cycle} = \left(\frac{I_m}{\sqrt{2}}\right)^2 R \times T$$

$$= \frac{I_m^2 R T}{2}$$

$$= \frac{I_m^2 R}{2}\left(\frac{1}{f}\right)$$

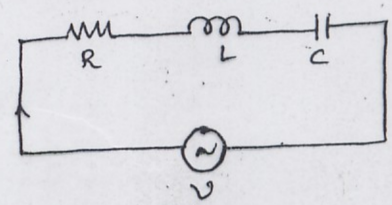
$$\therefore \quad Q = 2\pi \left[\frac{1}{2} \frac{I_m^2}{\omega^2 C} \Big/ \frac{I_m^2 R}{2}\left(\frac{1}{f}\right) \right]$$

$$\therefore \quad Q = \frac{2\pi f}{\omega^2 RC} = \frac{2\pi f}{(2\pi f)^2 RC} = \frac{1}{2\pi f RC}$$

$$\therefore \quad \boxed{Q = \frac{1}{\omega RC}}$$

$$\therefore \quad Q \text{ factor } 'Q' = \frac{\omega L}{R} = \frac{1}{\omega RC}.$$

Band width interms of ckt parameters :-



$$\text{Impedance} \quad Z = R + j\,(X_L - X_c)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\therefore \text{ current } \quad I = \frac{V}{|Z|}$$

$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \rightarrow ①$$

At half power points,

$$I_0 = \frac{V}{Z} \qquad \text{But } Z = R$$

$$\therefore \quad I_0 = V/R$$

$$\text{and} \quad \text{current } I = \frac{I_0}{\sqrt{2}}$$

$$\therefore \quad I = \frac{V/R}{\sqrt{2}} = \frac{V}{\sqrt{2}R} \quad \rightarrow ②$$

$\therefore$ Equate eq ① & eq ②;

$$\frac{V}{\sqrt{2}R} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\sqrt{2}R = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Squaring on both sides,

$$2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$2R^2 - R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\therefore \quad \omega L - \frac{1}{\omega C} = \pm R. \quad \rightarrow ③$$

We know that $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$

$\therefore Z = R \pm jR$

$\therefore \boxed{Z = R(1 \pm j1)} \rightarrow (4)$

The above equation gives the impedance at half power frequencies.

Let ω_1 & ω_2 be the lower half power and upper half power frequencies respectively.

At lower half-power frequency,

eq (3) becomes,

$\omega_1 L - \frac{1}{\omega_1 C} = -R$ ($\because$ "$-$" for capacitive nature)

$\frac{\omega_1^2 LC - 1}{\omega_1 C} = -R$

$\omega_1^2 LC + \omega_1 RC - 1 = 0.$

$\therefore \omega_1 = \frac{-(RC) \pm \sqrt{(RC)^2 - 4(LC)(-1)}}{2LC}$

Consider the +ve value for 'ω_1';

$$\boxed{\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}} \rightarrow (5)$$

Similarly, at higher half-power freq,

eq (3) becomes,

$\omega_2 L - \frac{1}{\omega_2 C} = +R$ ('+' for inductive nature)

$\omega_2^2 LC - \omega_2 RC - 1 = 0$

$\therefore \omega_2 = \frac{+(RC) \pm \sqrt{(-RC)^2 - 4(LC)(-1)}}{(2) LC}$

Consider the +ve value for 'ω_2';

$$\boxed{\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}} \rightarrow (6)$$

We know that,

BW, $\Delta f = f_2 - f_1$ (or)

$\Delta\omega = \omega_2 - \omega_1$

$\therefore$ BW = eq (6) $-$ eq (5)

$\Delta\omega = 2\left[\frac{R}{2L}\right]$

$\therefore \boxed{\Delta\omega = \frac{R}{L}}$

(OR)

$\boxed{\Delta f = \frac{R}{2\pi L}}$

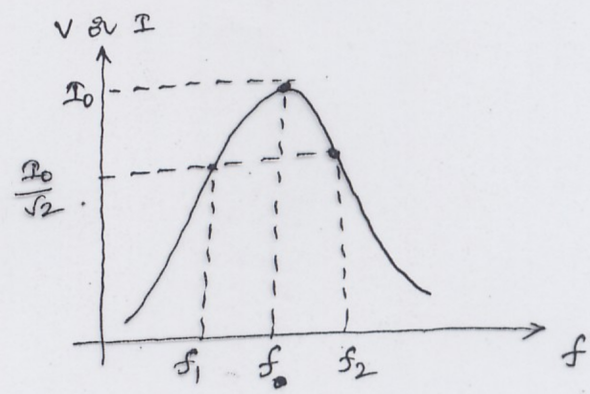
$\therefore$ The B.W. of a series RLC ckt depends upon the '$\frac{R}{L}$' ratio and independent of 'c'.

<u>Special case :</u>

We know that,

$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \rightarrow (1)$

$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \rightarrow (2)$



$\omega_1 \cdot \omega_2 = \left[\sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]^2 - \left[\frac{R}{2L}\right]^2$

$\therefore \omega_1\omega_2 = \left(\frac{1}{LC}\right) \rightarrow (3)$

We know that $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$\therefore \omega_0^2 = \frac{1}{LC}$

eq ③ can be written as,

$$\boxed{\omega_1 \cdot \omega_2 = \omega_0^2}$$

OR

$$\boxed{\omega_0 = \sqrt{\omega_1 \cdot \omega_2}}$$

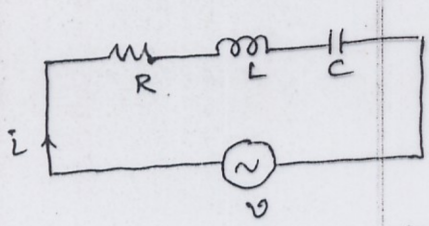
Resonant frequency in terms of half power frequency is given by,

$$2\pi f_0 = \sqrt{2\pi f_1 \cdot 2\pi f_2}$$

$$* \ \boxed{f_0 = \sqrt{f_1 f_2}} \ *$$

Hence, the "resonant frequency" is the geometric mean of the two half-power frequencies.

Relationship between Bandwidth, Resonant frequency & Quality factor :-



For a series R-L-C circuit,

$$Z = R + j(X_L - X_C)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= R\left[1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)\right]$$

$$= R\left[1 + j\left\{\frac{\omega L}{R} \times \frac{\omega_0}{\omega_0} - \frac{1}{\omega RC} \times \frac{\omega_0}{\omega_0}\right\}\right]$$

$$\therefore Z = R\left[1 + j\left(\frac{\omega_0 L}{R} \cdot \frac{\omega}{\omega_0} - \frac{1}{\omega_0 RC} \cdot \frac{\omega_0}{\omega}\right)\right]$$

We know that, at resonance,

$$\therefore Z = R\left[1 + j\left(Q_0 \frac{\omega}{\omega_0} - Q_0 \frac{\omega_0}{\omega}\right)\right]$$

$$= R\left[1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] \rightarrow ①$$

Let "δ" $= \frac{\omega - \omega_0}{\omega_0} = \frac{\omega}{\omega_0} - 1$

"δ" is called as "fractional freq variation" or "fractional detuning factor" or "per unit freq deviation"

$$\therefore \delta = \frac{\omega}{\omega_0} - 1 \Rightarrow (1+\delta) = \frac{\omega}{\omega_0}$$

from eq ①;

$$Z = R\left[1 + jQ_0\left[(1+\delta) - \frac{1}{(1+\delta)}\right]\right]$$

$$= R\left[1 + jQ_0\left[\frac{\delta(2+\delta)}{(1+\delta)}\right]\right]$$

Let $\delta \ll 1$; then

$$\boxed{Z \simeq R\left[1 + j2Q_0\delta\right]} \rightarrow ②$$

The above equation represents the relationship between 'R' and 'δ'.

At half power frequencies, the impedance is given by,

$$\boxed{Z = R(1 \pm j1)} \rightarrow ③$$

equate ② & ③;

$$2Q_0\delta = \pm 1$$

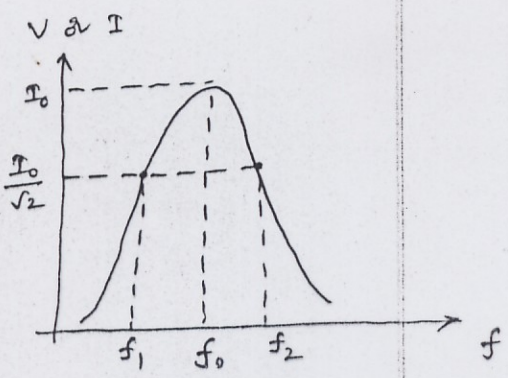
$$\boxed{\delta = \pm \frac{1}{2Q_0}} \rightarrow ④$$

At ω_1;

$$\delta = \frac{\omega - \omega_0}{\omega_0} = \frac{\omega_1 - \omega_0}{\omega_0}$$

At ω_2;

$$\delta = \frac{\omega - \omega_0}{\omega_0} = \frac{\omega_2 - \omega_0}{\omega_0}$$



$$\therefore \frac{\omega_1-\omega_0}{\omega_0} = \frac{f_1-f_0}{f_0} = -\frac{1}{2Q_0} \rightarrow (A)$$

$$\therefore \frac{\omega_2-\omega_0}{\omega_0} = \frac{f_2-f_0}{f_0} = +\frac{1}{2Q_0} \rightarrow (B)$$

(B) − (A) ;

$$\frac{1}{2Q_0}+\frac{1}{2Q_0} = \frac{f_2-f_0}{f_0} - \left(\frac{f_1-f_0}{f_0}\right)$$

$$\frac{1}{Q_0} = \frac{f_2-f_1}{f_0}$$

$$Q_0 = \frac{f_0}{f_2-f_1}$$

$$\therefore \boxed{Q_0 = \frac{f_0}{BW}}$$

$\therefore$ quality factor = $\frac{\text{Resonant frequency}}{\text{Bandwidth}}$.

$$Q_0 \propto \frac{1}{BW} \quad ; \quad Q_0 \propto f_0$$

Note :

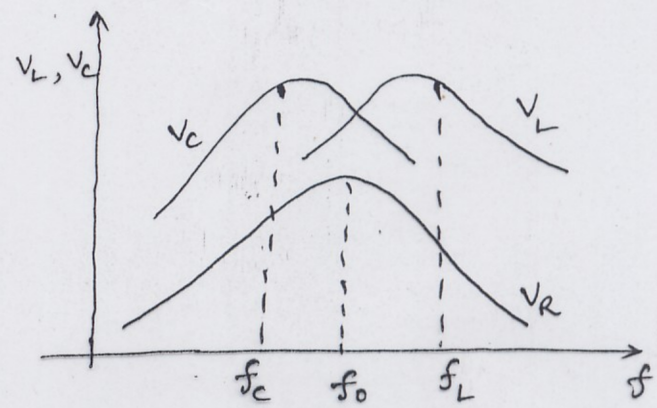
"Selectivity" of a resonant ckt is its ability to discriminate between signals of desired and undesired frequencies.

$$\text{Selectivity} = S = \frac{\omega_0 L}{R} = Q$$

Thus for a series RLC ckt, the selectivity is equal to the quality factor.

Variation of current & voltage with frequency :-

(Frequency at which V_C & V_L are max)



for a series R-L-C ckt,

$$Z = R + j(X_L - X_C)$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$I = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

(At resonance ; $Z = R$; then $I_0 = \frac{V}{R}$.

Hence current is max, at resonance).

Voltage across capacitor is

$$V_C = I \cdot X_C = I\left(\frac{1}{\omega C}\right)$$

$$\therefore V_C = \left[\frac{V}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}\right]\left(\frac{1}{\omega C}\right)$$

$$V_C^2 = \frac{V^2}{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2} \cdot \frac{1}{\omega^2 C^2}$$

$$V_C^2 = \frac{V^2}{\omega^2C^2\left(R^2+\left(\frac{\omega^2LC-1}{\omega C}\right)^2\right)}$$

$$= \frac{V^2}{\omega^2C^2\left[R^2+\frac{(\omega^2LC-1)^2}{\omega^2C^2}\right]}$$

$$= \frac{V^2}{\ldots}$$

$$\therefore V_c^2 = \frac{V^2}{R^2\omega^2C^2 + (\omega^2LC-1)^2} \rightarrow ①$$

The frequency at which 'V_c' is max, can be obtained by equating $\frac{dV_c^2}{d\omega} = 0$.

$$\frac{d}{d\omega}[V_c^2] = \frac{0 - V^2\left[2(\omega)[R^2C^2] + 2[\omega^2LC-1].(2\omega)(LC)\right]}{\left(\omega^2R^2C^2 + [\omega^2LC-1]^2\right)^2} = 0.$$

$$\therefore V^2\left[2\omega R^2C^2 + 2(\omega^2LC-1)\,2\omega LC\right] = 0$$

$V \neq 0$; But

$$2\omega R^2C^2 + 2(\omega^2LC-1)\,2\omega LC = 0.$$

$$2\omega R^2C^2 + 4\omega^3L^2C^2 - 4\omega LC = 0$$

$$2\omega C\left[R^2C + 2\omega^2L^2C - 2L\right] = 0$$

$$2\omega^2L^2C + R^2C - 2L = 0$$

$$2\omega^2L^2C = 2L - R^2C$$

$$\omega^2 = \frac{2L}{2L^2C} - \frac{R^2C}{2L^2C}$$

put, $\omega = \omega_c$;

$$\omega_c^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\therefore \boxed{\omega_c = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}} \text{ rad/sec.}$$

(or)

$$\boxed{f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}} \rightarrow ②$$

Similarly;

voltage across the inductor, $V_L = I.X_L$

But $I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

$$\therefore V_L = \left[\frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}\right].(\omega L)$$

$$V_L^2 = \frac{V^2\omega^2L^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$= \frac{V^2\omega^2L^2}{R^2 + \left(\frac{\omega^2LC-1}{\omega C}\right)^2}$$

$$\therefore V_L^2 = \frac{V^2\omega^2L^2}{\left[R^2\omega^2C^2 + (\omega^2LC-1)^2\right]/\omega^2C^2}$$

$$\therefore V_L^2 = \frac{V^2\omega^4L^2C^2}{R^2\omega^2C^2 + (\omega^2LC-1)^2} \rightarrow ③$$

The frequency at which 'V_L' max, can be obtained by equating $\frac{dV_L^2}{d\omega} = 0$.

Then we will get

$$2\omega^2LC - \omega^2R^2C^2 - 2 = 0$$

$$\omega^2\left[2LC - C^2R^2\right] = 2$$

$$\omega^2 = \frac{2}{2LC - C^2R^2} = \frac{1}{LC - \frac{C^2R^2}{2}}$$

put $\omega = \omega_L$

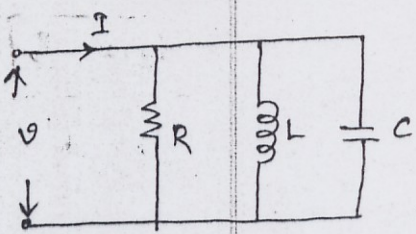
$$\therefore \boxed{\omega_L = \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}}} \text{ rad/sec.}$$

(or)

$$\boxed{f_L = \frac{1}{2\pi}\left(\frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}}\right)} \rightarrow ④$$

Eq ② & eq ④ represents the expressions of the frequencies at which 'V_c' & 'V_L' are maximum.

Parallel Resonance for ideal ckt :-



At resonance, in parallel ckt, the net susceptance must be zero. For analyzing parallel ckt, admittance-concept is more convenient.

The admittances of each branch are:

$$Y_1 = 1/R = G$$

$$Y_2 = \frac{1}{jX_L} = -j\left(\frac{1}{X_L}\right) = -jB_L$$

$$Y_3 = \frac{1}{-jX_C} = +j\left(\frac{1}{X_C}\right) = +jB_C$$

Total admittance $Y = Y_1 + Y_2 + Y_3$.

$$\therefore \boxed{Y = G + j(B_C - B_L)}$$

Condition for resonance is net susceptance should be zero.

$$\therefore B_C - B_L = 0$$

$$B_C = B_L$$

$$\frac{1}{X_C} = \frac{1}{X_L}$$

$$\omega_0 C = \frac{1}{\omega_0 L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \text{ rad/sec.}$$

(or)

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \text{ Hz.}$$

Bandwidth & Q-factor for a parallel R-L-C circuit :-

For a parallel R-L-C ckt,

$$Y = G + j(B_C - B_L)$$

we know that,

$$\omega L - \frac{1}{\omega C} = \pm R \quad (\text{for series ckt})$$

Similarly for parallel ckt,

$$\omega C - \frac{1}{\omega L} = \pm \frac{1}{R}$$

At $\omega = \omega_1$;

$$\omega_1 C - \frac{1}{\omega_1 L} = -\frac{1}{R}$$

$$\frac{\omega_1^2 LC - 1}{\omega_1 L} = \frac{-1}{R}$$

$$\omega_1^2 RLC - R = -\omega_1 L$$

$$\omega_1^2 RLC + \omega_1 L - R = 0.$$

$$\omega_1^2 + \frac{\omega_1}{RC} - \frac{1}{LC} = 0$$

$$\omega_1 = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4(1)\left(-\frac{1}{LC}\right)}}{(2)(1)}$$

$$\therefore \boxed{\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}} \rightarrow ①$$

Similarly,

$$\boxed{\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)}} \rightarrow ②$$

$$B.W = \Delta\omega = \omega_2 - \omega_1$$

$$\therefore \boxed{\Delta\omega = BW = \frac{1}{RC}}$$

(OR)

$$\boxed{\Delta f = \frac{1}{2\pi RC}}$$

$$\therefore \boxed{Q = \frac{\omega_0}{\Delta\omega} = \omega_0 RC}$$

where $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\therefore Q = \frac{\omega_0^2 RC}{\omega_0} = \left[\left(\frac{1}{LC}\right) RC\right] / \omega_0 .$$

$$\therefore \boxed{Q = \frac{R}{\omega_0 L}}$$

Resonant frequency for a practical parallel R-L-C circuit :-

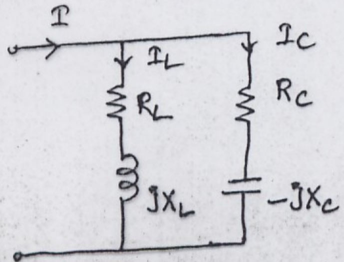


fig: (a)

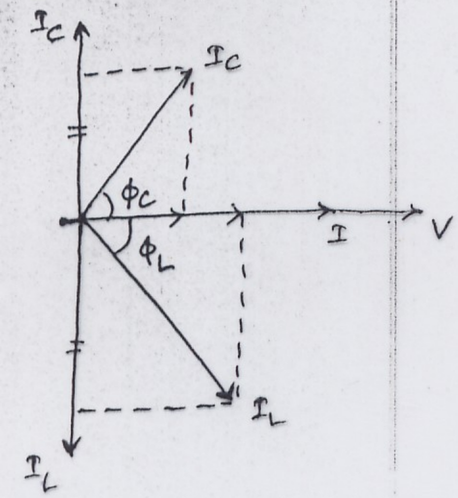


fig: (b)

The condition for resonance in the parallel ckt is $I_C \sin\phi_C = I_L \sin\phi_L$

From fig: (a)

$$Y = Y_L + Y_C$$

$$= \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$\frac{}{R_L^2 + X_L^2} + \frac{}{R_C^2 + X_C^2}$$

$$= \left[\frac{R_L}{R_L^2 + X_C^2} + \frac{R_C}{R_C^2 + X_C^2}\right] + j\left[\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right]$$

At resonance, the reactive part of $Y = 0$

i.e. $\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} = 0$

$$\therefore X_C\left[R_L^2 + X_L^2\right] - X_L\left[R_C^2 + X_C^2\right] = 0$$

$$\frac{1}{\omega_0 C}\left[R_L^2 + \omega_0^2 L^2\right] = \omega_0 L\left[R_C^2 + \frac{1}{\omega_0^2 C^2}\right]$$

$$R_L^2 + \omega_0^2 L^2 = \omega_0^2 LC\left[R_C^2 + \frac{1}{\omega_0^2 C^2}\right]$$

$$R_L^2 + \omega_0^2 L^2 = \omega_0^2 LC R_C^2 + \frac{L}{C}$$

$$\omega_0^2 L^2 - \omega_0^2 LC R_C^2 = \frac{L}{C} - R_L^2$$

$$\omega_0^2\left[L^2 - R_C^2 LC\right] = \frac{L}{C} - R_L^2$$

$$\omega_0^2 LC\left[\frac{L}{C} - R_C^2\right] = \frac{L}{C} - R_L^2$$

$$\omega_0^2 LC\left[R_C^2 - \frac{L}{C}\right] = R_L^2 - \frac{L}{C}$$

$$\omega_0^2 = \frac{1}{LC}\left[\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}\right]$$

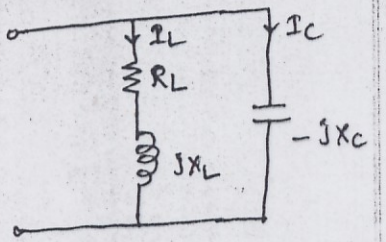
$$\boxed{\omega_0 = \sqrt{\frac{1}{LC}\left[\frac{R_L^2 - L/C}{R_C^2 - L/C}\right]}} \text{ rad/sec}$$

(or)

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}\left(\frac{R_L^2 - L/C}{R_C^2 - L/C}\right)}} \text{ Hz.}$$

<u>Special case :-</u>

In a more practical ckt, the resistance in the capacitance branch is negligible. " $R_c = 0$ "; then



$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}\left(\frac{R_L^2 - L/C}{R_C^2 - L/C}\right)}$$

where $R_C = 0$;

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}\left(\frac{R_L^2 - L/C}{(-L/C)}\right)}$$

$$\therefore \boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}\left(1 - \frac{R_L^2 C}{L}\right)}}$$

(OR)

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R_L}{L}\right)^2}}$$

<u>Impedance at Resonance :-</u>

$$Y_0 = \frac{1}{Z_0}$$

$$Y_0 = \frac{R_L}{R_L^2 + \omega_0^2 L^2} \rightarrow ①$$

we know that,

$$\omega_0 = \sqrt{\frac{1}{LC}\left(1 - \frac{R_L^2 C}{L}\right)}$$

$$\omega_0^2 = \frac{1}{LC}\left(1 - \frac{R_L^2 C}{L}\right)$$

$$\omega_0^2 LC = L - R_L^2 C$$

$$\omega_0^2 L^2 C + R_L^2 C = L$$

$$\therefore \frac{L}{C} = \omega_0^2 L^2 + R_L^2 \rightarrow ②$$

Substitute eq ② in eq ①;

$$\therefore Y_0 = \frac{R_L}{(L/C)}$$

$$\therefore Y_0 = \frac{R_L C}{L}$$

$$\therefore \boxed{Z_0 = \frac{L}{R_L C}}$$

The impedance at resonance, i.e. " $\frac{L}{R_L C}$ " is called "<u>dynamic impedance</u>" of the ckt.

<u>Definitions</u>

1. Voltage magnification (at Resonance) $= \frac{V_L}{V} = \frac{V_C}{V}$
2. Current magnification (at Resonance) $= \frac{I_L}{I} = \frac{I_C}{I}$

* Series Resonant ckt is also called as "SELECTOR CKT"

* Parallel Resonant ckt is also called as "REJECTOR CKT"
