UNIT-4

LOCUS DIAGRAMS, RESONANCE AND MAGNETIC CIRCUITS

- **Locus Diagrams with variation of various parameters**
- **Series RC and RL circuits**
- **Parallel RLC circuits**
- **Resonance**
- **Series and Parallel circuits**
- **Concept of Bandwidth and Quality factor**
- **Magnetic circuits**
- **Faradays Laws of Electromagnetic Induction**
- **Concepts of Self and Mutual Inductance**
- **Dot convention**
- **Coefficient of coupling**
- **Composite Magnetic circuits**
- **Analysis of Series and Parallel Magnetic circuits**
    - **Summary of important concepts and Formulae**
    - **Illustrative Examples**

Locus Diagrams with variation of various parameters:

Introduction: In AC electrical circuits the magnitude and phase of the current vector depends upon the values of R,L&C when the applied voltage and frequency are kept constant. The path traced by the terminus (tip) of the current vector when the parameters R,L&C are varied is called the current ***Locus diagram*** . Locus diagrams are useful in studying and understanding the behavior of the RLC circuits when one of these parameters is varied keeping voltage and frequency constant.

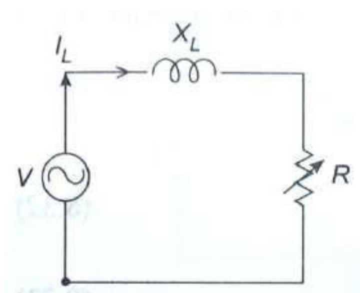
In this unit,Locus diagrams are developed and explained for series RC,RL circuits and Parallel LC circuits along with their internal resistances when the parameters R,L and C are varied.

The term circle diagram identifies locus plots that are either circular or semicircular. The defining equations of such circle diagrams are also derived in this unit for series RC and RL diagrams.

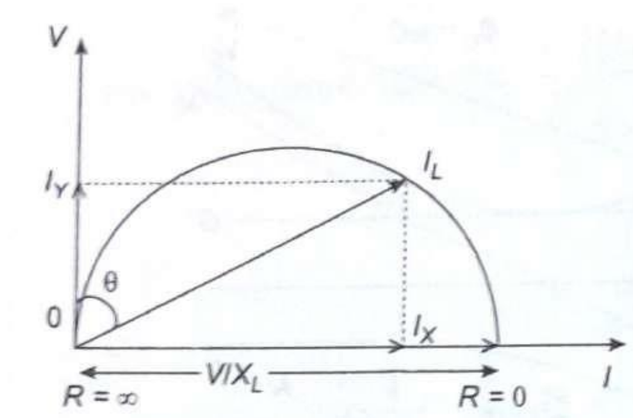
In both series RC,RL circuits and parallel LC circuits resistances are taken to be in series with L and C to highlight the fact that all practical L and C components will have at least a small value of internal resistance.

Series RL circuit with varying Resistance R:

Refer to the series RL circuit shown in the figure (a) below with constant X_L and varying R. The current I_L lags behind the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_L/R)$ for a given value of R as shown in the figure (b) below. When R=0 we can see that the current is maximum equal to V/X_L and lies along the I axis with phase angle equal to 90^0. When R is increased from zero to infinity the current gradually reduces from V/X_L to 0 and phase angle also reduces from 90^0 to 0^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve I axis.



Fig(a): Series RL circuit with Varying Resistance R



Fig(b): Locus of current vector I_L with variation of R

The related equations are:
$I_L = V/Z$ $\sin \Theta = X_L/Z$ or $Z = X_L/ \sin \Theta$ and $\cos \Theta = R / Z$
Therefore $I_L = (V/X_L) \sin \Theta$
For constant V and X_L the above expression for I_L is the polar equation of a circle with diameter (V/X_L) as shown in the figure above.

Circle equation for the RL circuit: (with fixed reactance and variable Resistance):

The X and Y coordinates of the current I_L are
$I_X = I_L \sin \Theta$ $I_Y = I_L \cos \Theta$
From the relations given above and earlier we get

$$I_X = (V/Z)(X_L/Z) = V X_L/Z^2 \text{ -------------------- } (1)$$

and

$$I_Y = (V/Z)(R/Z) = V R/Z^2 \text{ ------------------------ } (2)$$

Squaring and adding the above two equations we get

$$I_X^2 + I_Y^2 = V^2(X_L^2+R^2) / Z^4 = (V^2Z^2)/ Z^4 = V^2/Z^2 \text{ ------------------ } (3)$$

From equation (1) above we have $Z^2 = V X_L / I_X$ and substituting this in the above equation (3) we get :

$$I_X^2 + I_Y^2 = V^2/ (V X_L / I_X) = (V/X_L) I_X \quad \text{or}$$

$$I_X^2 + I_Y^2 - (V/X_L) I_X = 0$$

Adding $(V/2X_L)^2$ to both sides ,the above equation can be written as

$$[I_X - V/2X_L]^2 + I_Y^2 = (V/2X_L)^2 \text{ ---------------------------- } (4)$$

Equation (4) above represents a circle with a radius of $(V/2X_L)$ and with it's coordinates of the centre as $(V/2X_L, 0)$

Series RC circuit with varying Resistance R:

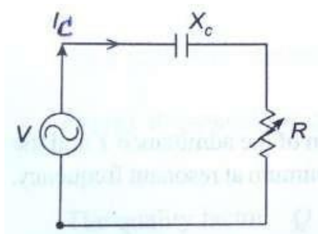
Refer to the series RC circuit shown in the figure (a) below with constant X_C and varying R. The current I_C leads the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below. When R=0 we can see that the current is maximum equal to $-V/X_C$ and lies along the negative I axis with phase angle equal to -90^0. When R is increased from zero to infinity the current gradually reduces from $-V/X_C$ to 0 and phase angle also reduces from -90^0 to 0^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle but now with its diameter along the negative I axis.

Circle equation for the RC circuit: (with fixed reactance and variable Resistance):

In the same way as we got for the Series RL circuit with varying resistance we can get the circle equation for an RC circuit with varying resistance as :

$$[I_X + V/2X_C]^2 + I_Y^2 = (V/2X_C)^2$$

whose coordinates of the centre are $(-V/2X_C, 0)$ and radius equal to $V/2X_C$



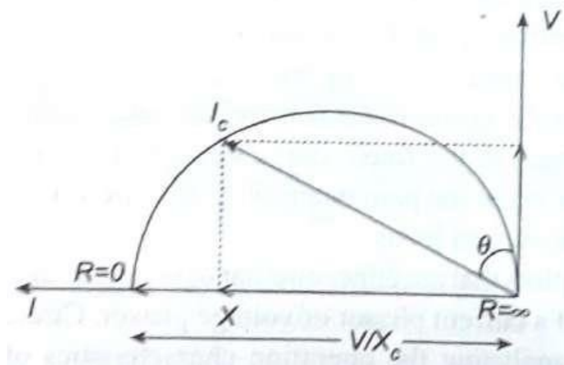
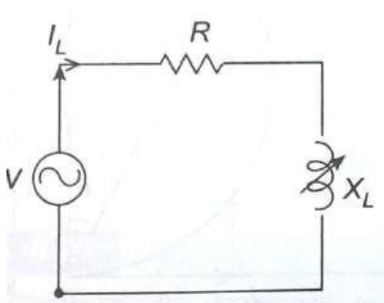


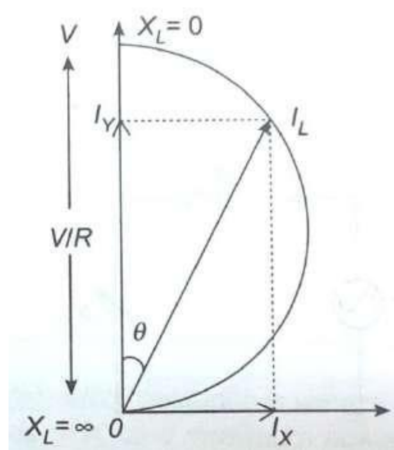
Fig: Series RC circuit with Varying Resistance R

Fig: Locus of current vector I_C with variation of R

Series RL circuit with varying Reactance X_L:

Refer to the series RL circuit shown in the figure (a) below with constant R and varying X_L. The current I_L lags behind the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_L/R)$ for a given value of R as shown in the figure (b) below. When $X_L = 0$ we can see that the current is maximum equal to V/R and lies along the +ve V axis with phase angle equal to 0^0. When X_L is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to 90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis and on to its right side.

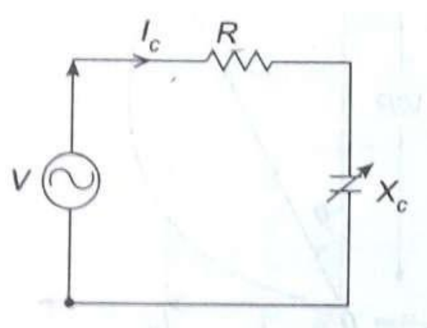


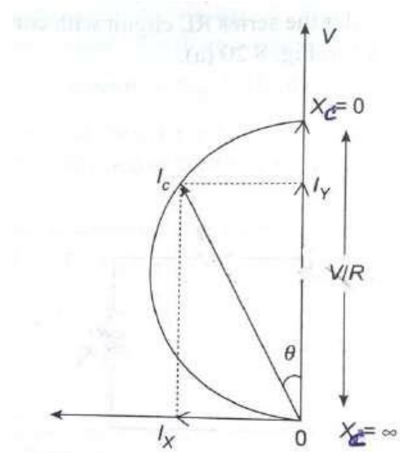


Fig(a): Series RL circuit with varying X_L **Fig(b) : Locus of current vector I_L with variation of X_L**

Series RC circuit with varying Reactance X_C:

Refer to the series RC circuit shown in the figure (a) below with constant R and varying X_C. The current I_C leads the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below. When $X_C = 0$ we can see that the current is maximum equal to V/R and lies along the V axis with phase angle equal to 0^0. When X_C is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to -90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis but now on to its left side.



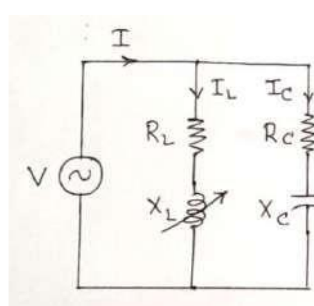


Fig(a): Series RC circuit with varying X_C Fig(b): Locus of current vector I_C with variation of X_C

Parallel LC circuits:

Parallel LC circuit along with its internal resistances as shown in the figures below is considered here for drawing the locus diagrams. As can be seen, there are two branch currents I_C and I_L along with the total current I. Locus diagrams of the current I_L or I_C (depending on which arm is varied)and the total current I are drawn by varying R_L, R_C , X_L and X_C one by one.

Varying X_L:



Fig(a): parallel LC circuit with Internal Resistances R_L and R_C in series with L (Variable) and C (fixed) respectively.

The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\Theta_C = \tan^{-1} (X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L . It's amplitude is maximum and equal to V/R_L when X_L is zero and it is in phase with the applied voltage V. When X_L is increased from zero to infinity it's amplitude decreases to zero and phase will be lagging the voltage by 90^0. In between, the phase angle will be lagging the voltage V by an angle $\Theta_L = \tan^{-1} (X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with X_L varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L)at the tip of the other vector (we will take constant amplitude vector I_C)and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semicircle I_CIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

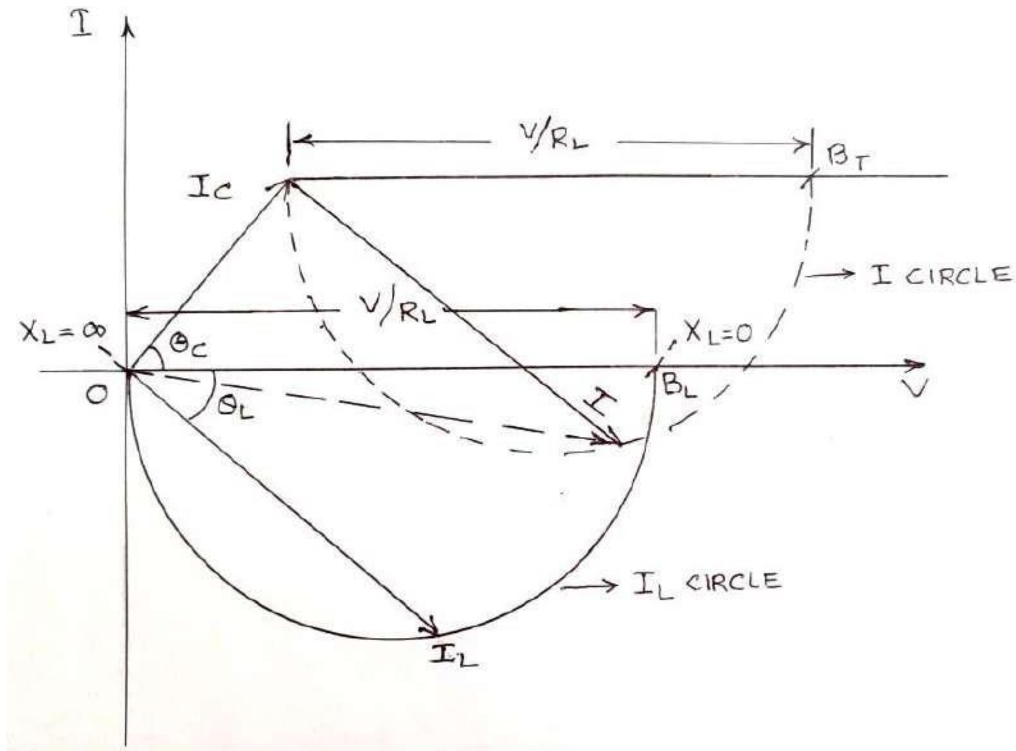


Fig (b): Locus of current vector I in Parallel LC circuit when X_L is varied from 0 to ∞

Varying X_C:

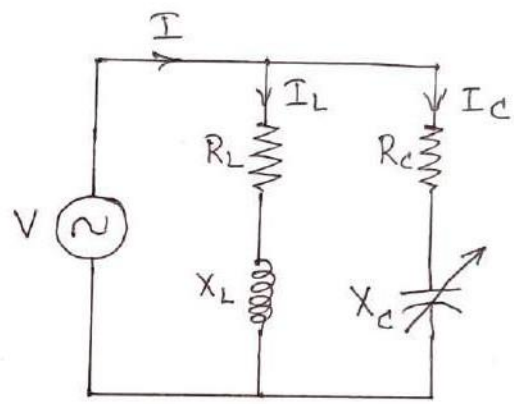
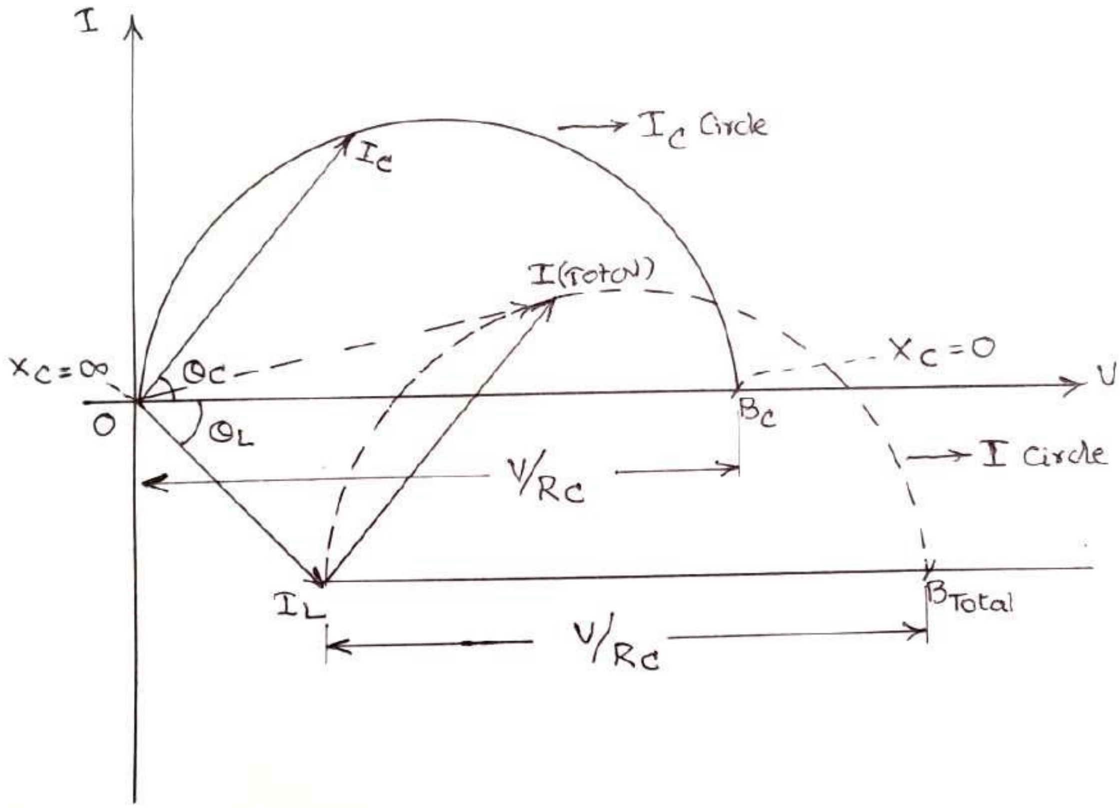


Fig.(a) parallel LC circuit with Internal Resistances R_L and R_C in series with L (fixed) and C (Variable) respectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OV by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C . It's amplitude is maximum and equal to V/R_C when X_C is zero and it is in phase with the applied voltage V. When X_C is increased from zero to infinity it's amplitude decreases to zero and phase will be leading the voltage by 90^0. In between, the phase angle will be leading the voltage V by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/R_C as shown in the figure below. Note that this is the same locus what we got earlier for the series RC circuit with X_C varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle I_LIB_T shown in the figure in dotted lines is the locus of the total current vector OI.



Fig(b) : Locus of current vector I in Parallel LC circuit when X_C is varied from 0 to ∞

***Varying R_L*:**

The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\Theta_C = \tan^{-1} (X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L . It's amplitude is maximum and equal to V/X_L when R_L is zero. Its phase will be lagging the voltage by 90^0. When R_L is increased from zero to infinity it's amplitude decreases to zero and it is in phase with the applied voltage V. In between, the phase angle will be lagging the voltage V by an angle $\Theta_L = \tan^{-1} (X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with R varying except that here V is shown horizontally.

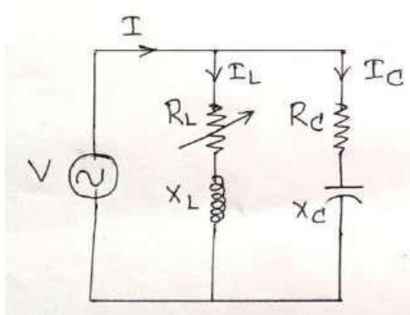
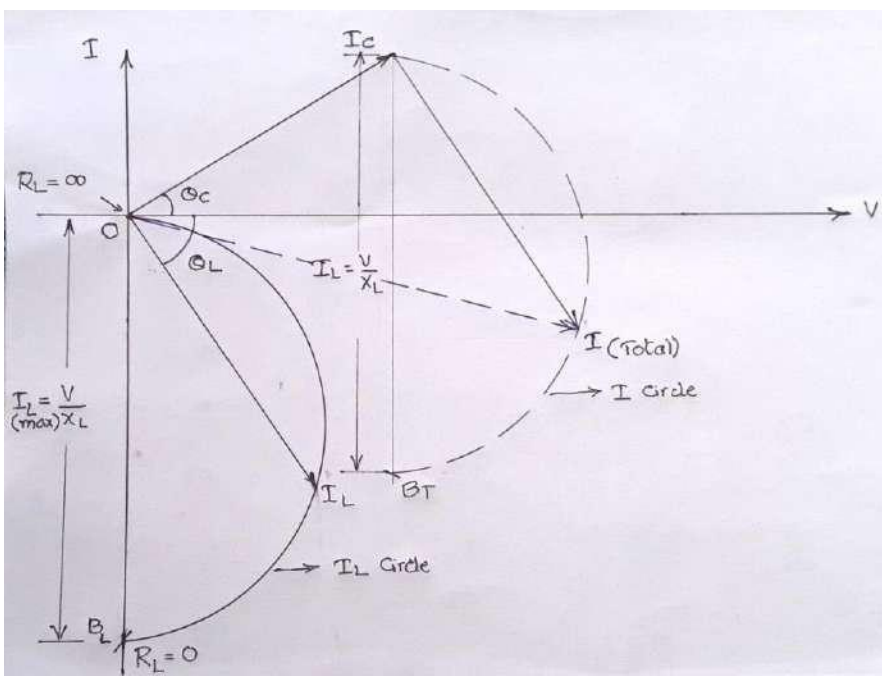


Fig.(a) parallel LC circuit with Internal Resistances R_L (Variable) and R_C (fixed) in series with L and C respectively.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L)at the tip of the other vector (we will take constant amplitude vector I_C)and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semicircle I_CIB_T shown in the figure in dotted lines is the locus of the total current vector OI.



Fig(b) : Locus of current vector I in Parallel LC circuit when R_L is varied from 0 to ∞

***Varying R_C*:**

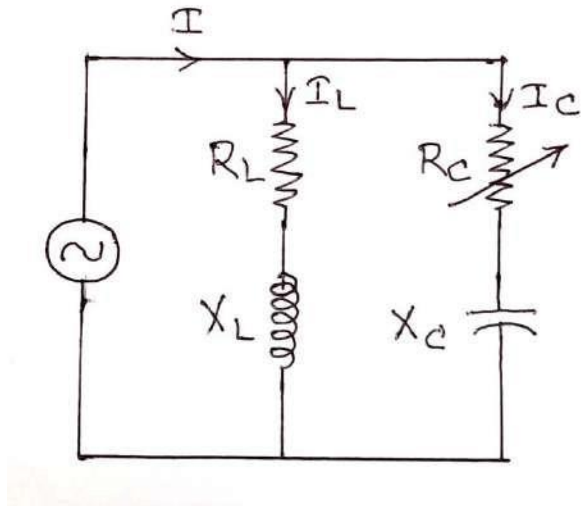
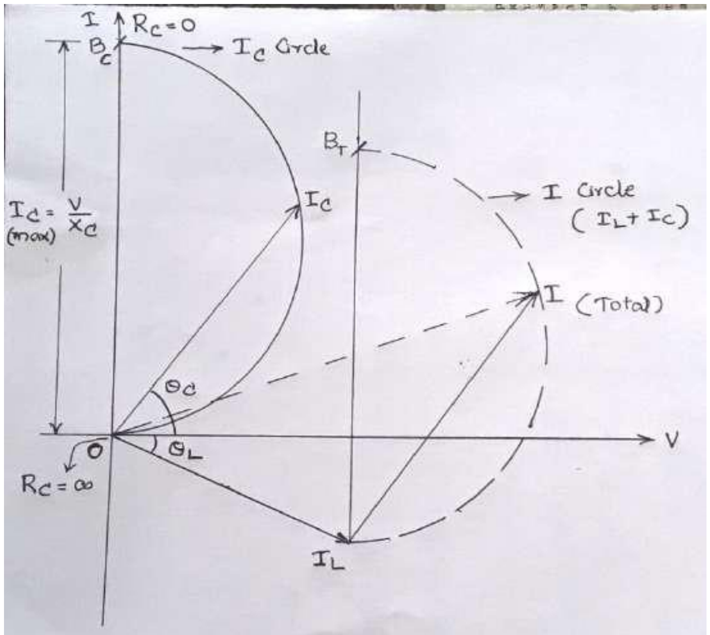


Fig.(a) parallel LC circuit with Internal Resistances R_L (fixed) and R_C (Variable) in series with L and C respectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OV by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C . It's amplitude is maximum and equal to V/X_C when R_C is zero and its phase will be leading the voltage by 90^0 . When R_C is increased from zero to infinity it's amplitude decreases to zero and it will be in phase with the applied voltage V. In between, the phase angle will be leading the voltage V by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/X_C as shown in the figure below. Note that this is the same locus what we got earlier for the series RC circuit with R varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle I_LIB_T shown in the figure in dotted lines is the locus of the total current vector OI.



Fig(b) : Locus of current vector I in Parallel LC circuit when R_C is varied from 0 to ∞

Resonance :

Series RLC circuit:

The impedance of the series RLC circuit shown in the figure below and the current I through the circuit are given by :

$$Z = R + j\omega L + 1/j\omega C = R + j(\omega L - 1/\omega C)$$

$$I = Vs/Z$$

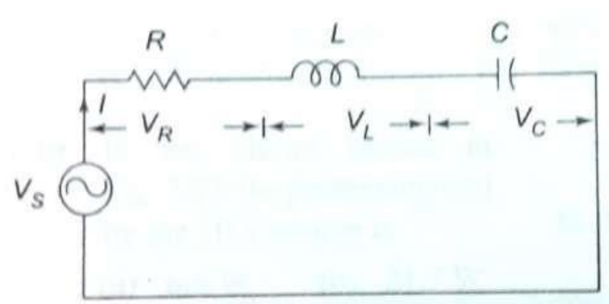


Fig: Series RLC circuit

The circuit is said to be in resonance when the Inductive reactance is equal to the Capacitive reactance. i.e. $X_L = X_C$ **or** $\omega L = 1/\omega C$. (i.e. Imaginary of the impedance is zero) The frequency at which the resonance occurs is called resonant frequency. In the resonant condition when X_L = X_C they cancel with each other since they are in phase opposition(180^0 out of phase) and net impedance of the circuit is purely resistive.In this condition the magnitudes of voltages across the Capacitance and the Inductance are also equal to each other but again since they are of opposite polarity they cancel with each other and the entire applied voltage appears across the Resistance alone**.**

Solving for the resonant frequency from the above condition of Resonance **: $\omega L = 1/\omega C$**

$$2\pi f_r L = 1/2\pi f_r C$$

$$f_r^2 = 1/4\pi^2 LC \quad \text{and} \quad f_r = 1/2\pi\sqrt{LC}$$

In a series RLC circuit, resonance may be produced by varying L or C at a fixed frequency or by varying frequency at fixed L and C.

Reactances, Impedance and Resistance of a Series RLC circuit as a function of frequency:

From the expressions for the Inductive and capacitive reactances we can see that when the frequency is zero, capacitance acts as an open circuit and Inductance as a short circuit. Similarly when the frequency is infinity inductance acts as an open circuit and the capacitance acts as a short circuit. The variation of Inductive and capacitive reactances along with Resistance R and the Total Impedance are shown plotted in the figure below.

As can be seen, when the frequency increases from zero to ∞ Inductive reactance X_L (directly proportional to ω) increases from zero to ∞ and Capacitive reactance X_C (inversely proportional to ω) decreases from $-\infty$ to zero. Whereas, the Impedance decreases from ∞ to Pure Resistance R as the frequency increases from zero to f_r (as capacitive reactance reduces from $-\infty$ and becomes equal to Inductive reactance) and then increases from R to ∞ as the frequency increases from f_r to ∞ (as inductive reactance increases from its value at resonant frequency to ∞)

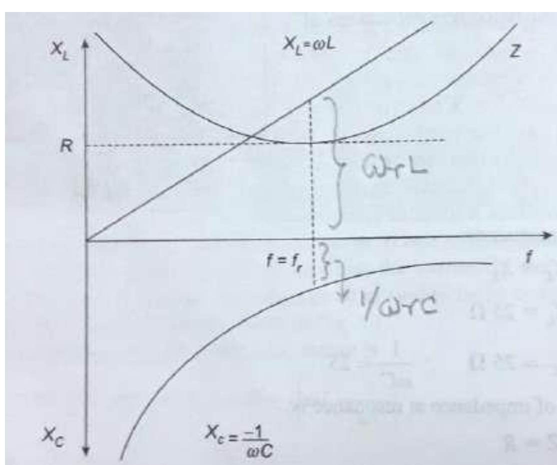


Fig : Reactance and Impedance plots of a Series RLC circuit

Phase angle of a Series RLC circuit as a function of frequency:

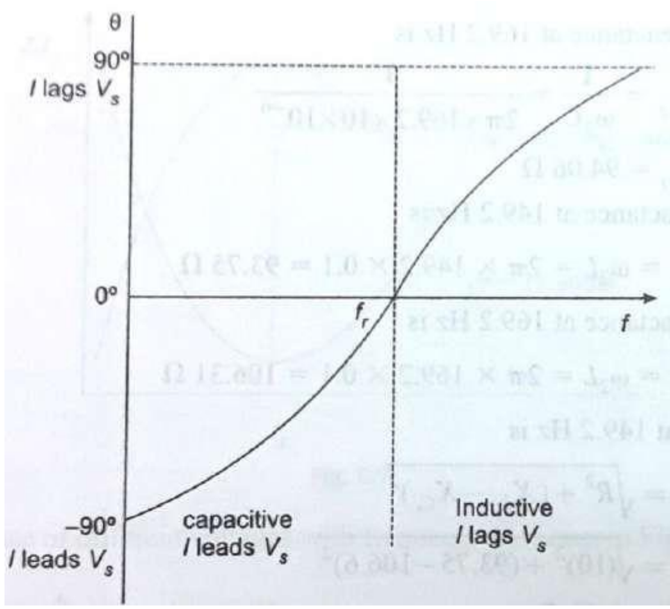


Fig : Phase plot of a Series RLC circuit

The following points can be seen from the Phase angle plot shown in the figure above:

- At frequencies below the resonant frequency capacitive reactance is higher than the inductive reactance and hence the phase angle of the current leads the voltage.
- As frequency increases from zero to $\mathbf{f_r}$ the phase angle changes from -90^0 to zero.
- At frequencies above the resonant frequency inductive reactance is higher than the capacitive reactance and hence the phase angle of the current lags the voltage.
- As frequency increases from $\mathbf{f_r}$ and approaches ∞, the phase angle increases from zero and approaches 90^0

Band width of a Series RLC circuit:

The band width of a circuit is defined as the Range of frequencies between which the output power is half of or 3 db less than the output power at the resonant frequency.These frequencies are called the cutoff frequencies, 3db points or half power points. But when we consider the output voltage or current, the range of frequencies between which the output voltage or current falls to 0.707 times of the value at the resonant frequency is called the Bandwidth BW. This is because voltage/current are related to power by a factor of √ 2 and when we are consider √ 2 times less it becomes 0.707. But still these frequencies are called as cutoff frequencies, 3db points or half power points. The lower end frequency is called ***lower cutoff frequency*** and the higher end frequency is called upper ***cutoff frequency.***

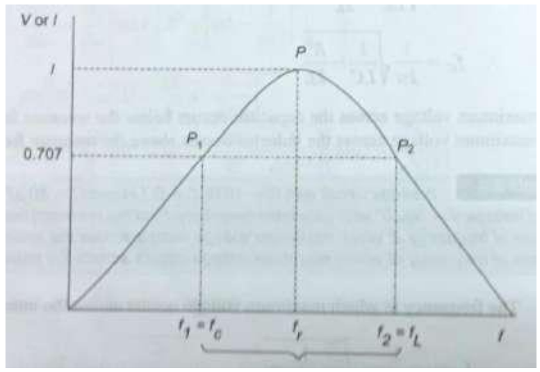


Fig: Plot showing the cutoff frequencies and Bandwidth of a series RLC circuit

Derivation of an expression for the BW of a series RLC circuit:

We know that BW = $f_2 - f_1$ Hz

If the current at points P_1 and P_2 are 0.707 ($1/\sqrt{2}$) times that of I_{max} (current at the resonant frequency) then the Impedance of the circuit at points P_1 and P_2 is $\sqrt{2}R$ (i.e. $\sqrt{2}$ times the impedance at f_r)

But Impedance at point P_1 is given by: $Z = \sqrt{R^2 + (1/\omega_1 C - \omega_1 L)^2}$ and equating this to $\sqrt{2}R$ we get : $(1/\omega_1 C) - \omega_1 L = R$ ------ (1)

Similarly Impedance at point P_2 is given by: $Z = \sqrt{R^2 + (\omega_2 L - 1/\omega_2 C)^2}$ and equating this to $\sqrt{2}R$ we get: $\omega_2 L - (1/\omega_2 C) = R$ ------ (2)

Equating the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L = \omega_2 L - 1/\omega_2 C$$

Rearranging we get $L(\omega_1 + \omega_2) = 1/C\,[(\omega_1 + \omega_2)/\omega_1\omega_2]$ i.e $\omega_1\omega_2 = 1/LC$

But we already know that for a series RLC circuit the resonant frequency is given by $\omega_r^2 = 1/LC$

Therefore: $\omega_1\omega_2 = \omega_r^2$ ---- (3) and $1/C = \omega_r^2 L$ ------ (4)

Next adding the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L + \omega_2 L - 1/\omega_2 C = 2R$$

$$(\omega_2 - \omega_1)L + (1/\omega_1 C - 1/\omega_2 C) = 2R$$

$$(\omega_2 - \omega_1)L + 1/C[(\omega_2 - \omega_1)/\omega_1\omega_2) = 2R \quad \text{------} \quad (5)$$

Using the values of $\omega_1\omega_2$ and $1/C$ from equations (3) and (4) above into equation (5) above we get: $(\omega_2 - \omega_1)L + \omega_r^2 L\,[(\omega_2 - \omega_1)/\omega_r^2) = 2R$

i.e. $2L(\omega_2 - \omega_1) = 2R$ i.e. $(\omega_2 - \omega_1) = R/L$ and $\mathbf{(f_2 - f_1) = R/2\pi L}$------------ (6)

Or finally Band width $\mathbf{BW = R/2\pi L}$--- (7)

Since f_r lies in the centre of the lower and upper cutoff frequencies f_1 and f_2 using the above equation (6) we can get:

$$f_1 = f_r - R/4\pi L \quad \text{------} \quad (8)$$

$$f_2 = f_r + R/4\pi L \quad \text{------} \quad (9)$$

Further by dividing the equation (6) above by f_r on both sides we get another **important** relation : $(f_2 - f_1) / f_r = R/2\pi f_r L$ **or** $BW / f_r = R/2\pi f_r L$ ------------------(10)

Here an important property of a coil i.e. **Q factor** or **figure of merit** is defined as the ratio of the reactance to the resistance of a coil.

$$Q = 2\pi f_r L / R \text{ ---------------------------------- } (11)$$

Now using the relation (11) we can rewrite the relation (10) as

$$Q = f_r / BW \text{ ------------------------------------- } (12)$$

Quality factor of a series RLC circuit:

The quality factor of a series RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance / Average power at Resonance

Reactive power in Inductor at resonance = I^2X_L

Reactive power in Capacitor at resonance = I^2X_C

Average power at Resonance = I^2R

Here the power is expressed in the form I^2X (not as V^2/X) since I is common through R.L and C in the series RLC circuit and it gets cancelled during the simplification.

Therefore $Q = I^2X_L / I^2R = I^2X_C / I^2R$

i.e. $Q = X_L / R = \omega_r L / R$ ---(1)

Or $Q = X_C / R = 1/\omega_r RC$ ---------------------------------------(2)

From these two relations we can also define Q factor as :

Q = Inductive (or Capacitive) reactance at resonance / Resistance

Substituting the value of $\omega_r = 1/\sqrt{LC}$ in the expressions (1) or (2) for **Q** above we can get the value of **Q** in terms of **R, L,C** as below.

$$Q = (1/\sqrt{LC})\, L/ R = (1/R)\,(\sqrt{L/C})$$

Selectivity:

Selectivity of a series **RLC** circuit indicates how well the given circuit responds to a given resonant frequency and how well it rejects all other frequencies. i.e. the selectivity is directly proportional to **Q** factor. A circuit with a good selectivity (or a high **Q** factor) will have maximum gain at the resonant frequency and will have minimum gain at other frequencies .i.e. it will have very low band width. This is illustrated in the figure below.

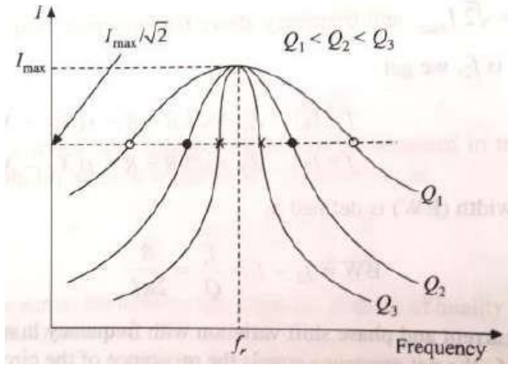


Fig: Effect of quality factor on bandwidthVoltage Magnification at resonance:

At resonance the voltages across the Inductance and capacitance are much larger than the applied voltage in a series RLC circuit and this is called voltage magnification at Resonance. The voltage magnification is equal to the **Q** factor of the circuit. This is proven below.
If we take the voltage applied to the circuit as **V** and the current through the circuit at resonance as **I** then
The voltage across the inductance **L** is: $V_L = IX_L = (V/R)\ \omega_r L$ and
The voltage across the capacitance **C** is: $V_C = IX_C = V/R\ \omega_r C$
But we know that the **Q** of a series RLC circuit = $\omega_r L/R = 1/R\ \omega_r C$
Using these relations in the expressions for V_L and V_C given above we get
$V_L = VQ$ and $V_C = VQ$
The ratio of voltage across the Inductor or capacitor at resonance to the applied voltage in a series RLC circuit is called Voltage magnification and is given by

$$\text{Magnification} = Q = V_L/V \quad \text{or } V_C/V$$

Important points In Series RLC circuit at resonant frequency :

- The impedance of the circuit becomes purely resistive and minimum i.e **Z = R**
- The current in the circuit becomes maximum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is **Q** times higher than the voltage across the resistor

Bandwidth and Q factor of a Parallel RLC circuit:

Parallel RLC circuit is shown in the figure below. For finding out the **BW** and **Q** factor of a parallel RLC circuit, since it is easier we will work with Admittance , Conductance and Susceptance instead of Impedance ,Resistance and Reactance like in series RLC circuit.

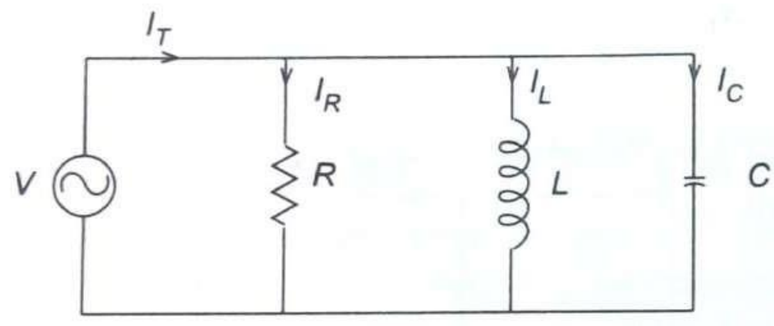


Fig: Parallel RLC circuit

Then we have the relation: $\mathbf{Y = 1/Z = 1/R + 1/j\omega L + j\omega C = 1/R + j (\omega C - 1/\omega L)}$

For the parallel RLC circuit also, at resonance, the imaginary part of the Admittance is zero and hence the frequency at which resonance occurs is given by: $\mathbf{\omega_r C - 1/\omega_r L = 0}$ **.** From this we get : $\mathbf{\omega_r C = 1/\omega_r L}$ **and** $\mathbf{\omega_r = 1/\sqrt{LC}}$
which is the same value for $\boldsymbol{\omega_r}$ as what we got for the series RLC circuit.

At resonance when the imaginary part of the admittance is zero the ***admittance*** becomes ***minimum***.(i.e ***Impedance*** becomes ***maximum*** as against Impedance becoming minimum in series RLC circuit) i.e. Current becomes minimum in the parallel RLC circuit at resonance (as against current becoming maximum in series RLC circuit) and increases on either side of the resonant frequency as shown in the figure below.

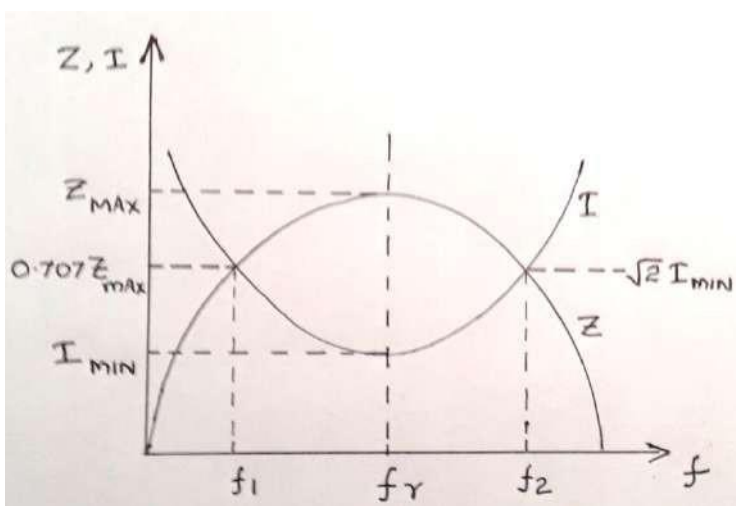


Fig: Variation of Impedance and Current with frequency in a Parallel RLC circuit

Here also the BW of the circuit is given by BW = f_2-f_1 where f_2 and f_1 are still called the upper and lower cut off frequencies but they are 3db higher cutoff frequencies since we notice that at these cutoff frequencies the amplitude of the current is √2 times higher than that of the amplitude of current at the resonant frequency.

The BW is computed here also on the same lines as we did for the series RLC circuit:

If the current at points P_1 and P_2 is √ 2 (3db) times higher than that of I_{min}(current at the resonant frequency) then the admittance of the circuit at points P_1 and P_2 is also √ 2 times higher than the admittance at f_r)

But amplitude of admittance at point P_1 is given by: $Y = \sqrt{1/R^2 + (1/\omega_1 L - \omega_1 C)^2}$ and equating this to √ 2 /R we get

$1/\omega_1 L - \omega_1 C = 1/R$ ----------------- (1)

Similarly amplitude of admittance at point P_2 is given by: $Y = \sqrt{1/R^2 + (\omega_2 C - 1/\omega_2 L)^2}$ and equating this to √ 2 /R we get

$\omega_2 C - 1/\omega_2 L = 1/R$ ----------------- (2)

Equating LHS of (1) and (2) and further simplifying we get

$$1/\omega_1 L - \omega_1 C = \omega_2 C - 1/\omega_2 L$$
$$1/\omega_1 L + 1/\omega_2 L = \omega_1 C + \omega_2 C$$
$$1/L\,[(\omega_1 + \omega_2)/\,\omega_1\omega_2] = (\omega_1 + \omega_2)C$$
$$1/L\,C = \omega_1\omega_2$$

Next adding the equations (1) and (2) above and further simplifying we get

$$1/\omega_1 L - \omega_1 C + \omega_2 C - 1/\omega_2 L = 2/R$$
$$(\omega_2 C - \omega_1 C) + (1/\omega_1 L - 1/\omega_2 L) = 2/R$$
$$(\omega_2 - \omega_1)C + 1/L\,[(\omega_2 - \omega_1)/\,\omega_1\omega_2] = 2/R$$

Substituting the value of $\omega_1\omega_2 = 1/LC$

$$(\omega_2 - \omega_1)C + LC/L\,[(\omega_2 - \omega_1)] = 2/R$$
$$(\omega_2 - \omega_1)C + C\,[(\omega_2 - \omega_1)] = 2/R$$
$$2\,C\,[(\omega_2 - \omega_1)] = 2/R$$
$$\text{Or } [(\omega_2 - \omega_1)] = 1/RC$$

From which we get the band width **BW = f_2-f_1 = 1/2π RC**

Dividing both sides by f_r we get : **(f_2-f_1)/ f_r = 1/2π f_r RC---------- (1)**

Quality factor of a Parallel RLC circuit:

The quality factor of a Parallel RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance / Average power at Resonance

Reactive power in Inductor at resonance = V^2/X_L

Reactive power in Capacitor at resonance = V^2/X_C

Average power at Resonance = V^2/R

Here the power is expressed in the form V^2/X (not as I^2X as in series circuit) since V is common across R,L and C in the parallel RLC circuit and it gets cancelled during the simplification.

Therefore **Q = $(V^2/X_L) / (V^2/R)$ = $(V^2/X_C) / (V^2/R)$**

i.e. **$Q = R/X_L = R/\omega_r L$** -- (1)
Or **$Q = R/X_C = \omega_r RC$** -- (2)
From these two relations we can also define **Q** factor as :

Q = Resistance /Inductive (or Capacitive) reactance at resonance

Substituting the value of **$\omega_r = 1/\sqrt{LC}$** in the expressions (1) or (2) for **Q** above we can get the value of **Q** in terms of R, L,C as below.

$$Q = (1/\sqrt{LC})\, RC = R\,(\sqrt{C/L})$$

Further using the relation **$Q = \omega_r RC$** (equation 2 above) in the earlier equation (1) we got in BW viz. **$(f_2-f_1)/f_r = 1/2\pi f_r RC$** we get : **$(f_2-f_1)/f_r = 1/Q$ or $Q = f_r/(f_2-f_1) = f_r/BW$**
i.e. In Parallel RLC circuit also the Q factor is inversely proportional to the BW.

Admittance, Conductance and Susceptance curves for a Parallel RLC circuit as a function of frequency :

- The effect of varying the frequency on the Admittance, Conductance and Susceptance of a parallel circuit is shown in the figure below.
- Inductive susceptance **B_L** is given by **$B_L = -1/\omega L$.** It is inversely proportional to the frequency ω and is shown in the in the fourth quadrant since it is negative.
- Capacitive susceptance **B_C** is given by **$B_C = \omega C$.** It is directly proportional to the frequency ω and is shown in the in the first quadrant as OP .It is positive and linear.
- Net susceptance **$B = B_C - B_L$** and is represented by the curve **JK**. As can be seen it is zero at the resonant frequency **f_r**
- The conductance **$G = 1/R$** and is constant
- The total admittance **Y** and the total current **I** are minimum at the resonant frequency as shown by the curve **VW**

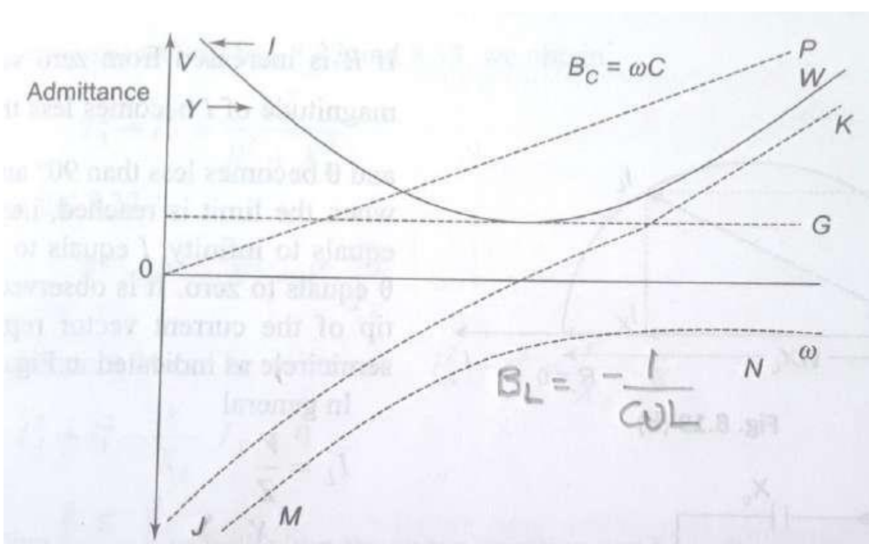


Fig: Conductance,Susceptance and Admittance plots of a Parallel RLC circuit

Current magnification in a Parallel RLC circuit:
Just as voltage magnification takes place across the capacitance and Inductance at the resonant frequency in a series RLC circuit , current magnification takes place in the currents through the capacitance and Inductance at the resonant frequency in a Parallel RLC circuit. This is shown below.
Voltage across the Resistance **= V = IR**
Current through the Inductance at resonance $I_L = V/ \omega_r L = IR / \omega_r L = I . R/ \omega_r L = I Q$
Similarly
Current through the Capacitance at resonance $I_C = V/ (1/\omega_r C) = IR / (1/\omega_r C) = I(R \omega_r C) = I Q$
From which we notice that the quality factor $Q = I_L / I$ **or** I_C / I and that the current through the inductance and the capacitance increases by **Q** times that of the current through the resistor at resonance. .

Important points In Parallel RLC circuit at resonant frequency :

- The impedance of the circuit becomes resistive and maximum i.e **Z = R**
- The current in the circuit becomes minimum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The current through the Capacitor becomes equal and opposite to the current through the Inductor at resonance and is Q times higher than the current through the resistor

Magnetic Circuits:

Introduction to the Magnetic Field:

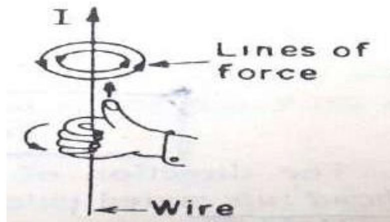
Magnetic fields are the fundamental medium through which energy is converted from one form to another in motors, generators and transformers. Four basic principles describe how magnetic fields are used in these devices.

1. A current-carrying conductor produces a magnetic field in the area around it.
   Explained in Detail by Fleming's Right hand rule and Amperes Law.
2. A time varying magnetic flux induces a voltage in a coil of wire if it passes through that coil.
   (basis of Transformer action)
   Explained in detail by the Faradays laws of Electromagnetic Induction.
3. A current carrying conductor in the presence of a magnetic field has a force induced in it (Basis of Motor action)
4. A moving wire in the presence of a magnetic field has a voltage induced in it (Basis of Generator action)

We will be studying in this unit the first two principles in detail and the other two principles in the next unit on DC machines.

Two basic laws governing the production of a magnetic field by a current carrying conductor :
The direction of the magnetic field produced by a current carrying conductor is given by the ***Flemings Right hand rule*** and its' amplitude is given by the ***Ampere's Law***.

Flemings right hand rule: *Hold the conductor carrying the current in your right hand such that the Thumb points along the wire in the direction of the flow of current, then the fingers will encircle the wire along the lines of the Magnetic force.*



Ampere's Law : *The line integral of the magnetic field intensity H around a closed magnetic path is equal to the total current enclosed by the path*.

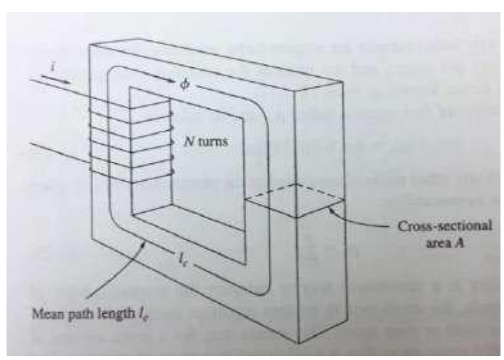
This is the basic law which gives the relationship between the Magnetic field Intensity H and the current I and is mathematically expressed as

$$\oint \boldsymbol{H}.\boldsymbol{dl} = I_{net}$$

where **H** is the magnetic field intensity produced by the current $\mathbf{I_{net}}$ and ***dl*** is a differential element of length along the path of integration. **H** is measured in *Ampere-turns per meter*.

Important parameters and their relation in magnetic circuits :

- Consider a current carrying conductor wrapped around a ferromagnetic core as shown in the figure below .



- Applying Ampere's law, the total amount of magnetic field induced will be proportional to the amount of current flowing through the conductor wound with N turns around the ferromagnetic material as shown. Since the core is made of ferromagnetic material, it is assumed that a majority of the magnetic field will be confined to the core.

- The path of integration in this case as per the Ampere's law is the mean path length of the core, l_C. The current passing within the path of integration I_{net} is then **Ni**, since the coil of wire cuts the path of integration **N** times while carrying the current **i**. Hence Ampere's Law becomes : $Hl_c = Ni$
  Therefore $H = Ni/l_c$

- In this sense**, H** (Ampere turns per meter) is known as the effort required to induce a magnetic field. The strength of the magnetic field flux produced in the core also depends on the material of the core. Thus: $B = \mu H$ where
  B = magnetic flux density [webers per square meter, or Tesla (T)]
  μ= magnetic permeability of material (Henrys per meter)
  H = magnetic field intensity (ampere-turns per meter)
- The constant **μ** may be further expanded to include ***relative permeability*** which can be defined as below:
  $\mu_r = \mu/\mu_o$
  where μ_o = permeability of free space (equal to that of air)
- Hence the permeability value is a combination of the relative permeability and the permeability of free space. The value of relative permeability is dependent upon the type of material used. The higher the amount permeability, the higher the amount of flux induced in the core. Relative permeability is a convenient way to compare the magnetizability of materials.
- Also, because the permeability of iron is so much higher than that of air, the majority of the flux in an iron core remains inside the core instead of travelling through the surrounding air, which has lower permeability. The small leakage flux that does leave the iron core is important in determining the flux linkages between coils and the self-inductances of coils in transformers and motors.
- In a core such as shown in the figure above

$$B = \mu H = \mu Ni/l_c$$

Now, to measure the total flux flowing in the ferromagnetic core, consideration has to be made in terms of its cross sectional area (CSA). Therefore:

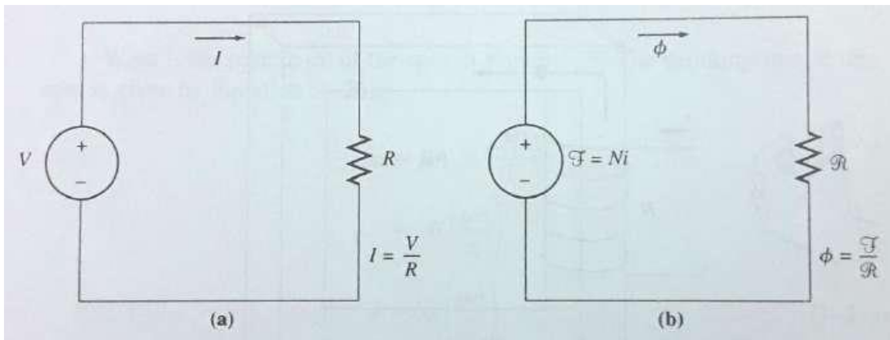
$\Phi = \int \boldsymbol{B}.\boldsymbol{dA}$ where: **A** = cross sectional area throughout the core.

Assuming that the flux density in the ferromagnetic core is constant throughout hence the equation simplifies to: $\Phi = B.A$

Taking the previous expression for **B** we get $\Phi = \mu NiA/l_c$

Electrical analogy of magnetic circuits:
The flow of magnetic flux induced in the ferromagnetic core is analogous to the flow of electric current in an electrical circuit hence the name magnetic circuit.

The analogy is as follows:



(a) Electric Circuit **(b) Electrical Analogy of Magnetic Circuit**

- Referring to the magnetic circuit analogy, **F** is denoted as **magnetomotive force** (mmf) which is similar to Electromotive force in an electrical circuit (emf). Therefore, we can say that **F** is the force which pushes magnetic flux around a ferromagnetic core with a value of **Ni** (refer to ampere's law). Hence **F** is measured in ampere turns. Hence the magnetic circuit equivalent equation is as shown:
  F = Ø.R (similar to V=IR)
  We already have the relation **Φ = µ NiA/l** and using this we get **R = F / Φ = Ni/ Φ**
  R = Ni /(µ NiA/l) = l/ µ A
- The polarity of the mmf will determine the direction of flux. To easily determine the direction of flux, the ***'right hand curl'*** rule is applied:
  When the direction of the curled fingers indicates the direction of current flow the resulting thumb direction will show the magnetic flux flow.
- The element of **R** in the magnetic circuit analogy is similar in concept to the electrical resistance. It is basically the measure of material resistance to the flow of magnetic flux. **Reluctance** in this analogy obeys the rule of electrical resistance (Series and Parallel Rules). Reluctance is measured in ***Ampere-turns per weber***.
- The inverse of electrical resistance is conductance which is a measure of conductivity of a material. Similarly the inverse of reluctance is known as **permeance P** which represents the degree to which the material permits the flow of magnetic flux.

- By using the magnetic circuit approach, calculations related to the magnetic field in a ferromagnetic material are simplified but with a little inaccuracy.

Equivalent Reluctance of a series Magnetic circuit : $R_{eqseries} = R_1 + R_2 + R_3 + ...$

Equivalent Reluctance of a Parallel Magnetic circuit: $1/R_{eqparallel} = 1/R_1 + 1/R_2 + 1/R_3 + ...$

Electromagnetic Induction and Faraday's law – Induced Voltage from a Time-Changing Magnetic Field:

Faraday's Law:

Whenever a varying magnetic flux passes through a turn of a coil of wire, voltage will be induced in the turn of the wire that is directly proportional to the rate of change of the flux linkage with the turn of the coil of wire.

$$e_{ind} \text{ a } -dØ/dt$$

$$e_{ind} = -k.\,dØ/dt$$

The negative sign in the equation above is in accordance to **Lenz' Law** which states:

The direction of the induced voltage in the turn of the coil is such that if the coil is short circuited, it would produce a current that would cause a flux which opposes the original change of flux.

And **k** is the constant of proportionality whose value depends on the system of units chosen. In the SI system of units **k=1** and the above equation becomes:

$$e_{ind} = -\,dØ/dt$$

Normally a coil is used with several turns and if there are N number of turns in the coil with the same amount of flux flowing through it then: $e_{ind} = -N\,dØ/dt$

*Change in the flux linkage **NØ** of a coil can be obtained in two ways:*

1. Coil remains stationary and flux changes with time (Due to AC current like in Transformers and this is called Statically induced e.m.f)
2. Magnetic flux remains constant and stationary in space, but the coil moves relative to the magnetic field so as to create a change in the flux linkage of the coil (Like in Rotating machines and this is a called Dynamically induced e.m.f.

Self inductance:

From the Faradays laws of Electromagnetic Induction we have seen that an e.m.f will be induced in a conductor when a time varying flux is linked with a conductor and the amplitude of the induced e.m.f is proportional to the rate of change of the varying flux.

If the time varying flux is produced by a coil of **N** turns then the coil itself links with the time varying flux produced by itself and an emf will be induced in the same coil. This is called self inductance .

The flux **Ø** produced by a coil of N turns links with its own N turns of the coil and hence the total flux linkage is equal to $NØ = (\mu N^2 A / l)\, I$ *[using the expression $\Phi = \mu NiA/l$ we already* *develop*ed] Thus we see that the total magnetic flux produced by a coil of N turns and linked with itself is proportional to the current flowing through the coil i.e.

$$N\emptyset \propto I \text{ or } N\emptyset = L I$$

From the Faradays law of electromagnetic Induction, the self induced e.m.f for this coil of N turns is given by:

$$e_{ind} = -N\, d\emptyset/dt = -L\, dI/dt$$

The constant of proportionality **L** is called the self Inductance of the coil or simply Inductance and its value is given by $L = (\mu N^2 A / l)$. If the radius of the coil is **r** then:

$$L = (\mu N^2 \pi r^2 / l)\ i$$

From the above two equations we can see that Self Inductance of a coil can be defined as the flux produced per unit current i.e *Weber/Ampere* (equation1) or the induced emf per unit rate of change of current i.e *Volt-sec/Ampere* (equation 2)

The unit of Inductance is named after Joseph Henry as ***Henry*** and is given to these two combinations as :

$$1H = 1WbA^{-1} = 1VsA^{-1}$$

Self Inductance of a coil is defined as one Henry if an induced emf of one volt is generated when the current in the coil changes at the rate of one Ampere per second.

Henry is relatively a very big unit of Inductance and we normally use Inductors of the size of mH (10^{-3} H) or µH (10^{-3}H)

Mutual inductance and Coefficient of coupling:

In the case of Self Inductance an emf is induced in the same coil which produces the varying magnetic field. The same phenomenon of Induction will be extended to a separate second coil if it is located in the vicinity of the varying magnetic field produced by the first coil. Faradays law of electromagnetic Induction is equally applicable to the second coil also. A current flowing in one coil establishes a magnetic flux about that coil and also about a second coil nearby but of course with a lesser intensity. The time-varying flux produced by the first coil and surrounding the second coil produces a voltage across the terminals of the second coil. This voltage is proportional to the time rate of change of the current flowing through the first coil.

Figure (*a)* shows a simple model of two coils L_1 and L_2, sufficiently close together that the flux produced by a current $i_1(t)$ flowing through L_1 establishes an open-circuit voltage $v_2(t)$ across the terminals of L_2. ***Mutual inductance,M*21**, is defined such that

$$v_2(t) = M_{21} di_1(t)/dt \text{ ------------------- [1]}$$

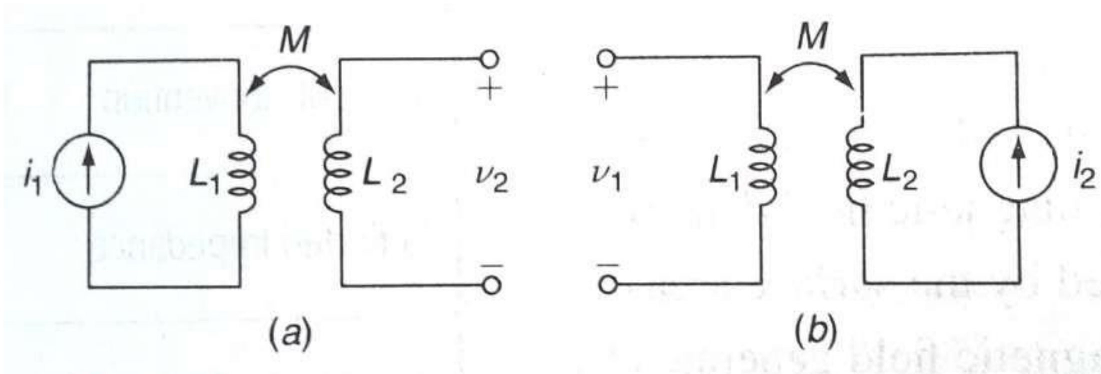


Figure (*a*) A current *i*1 through *L*1 produces an open-circuit voltage *v*2across *L*2. (*b*) A current *i*2 through *L*2 produces an open-circuit voltage *v*1 across *L*1.

The order of the subscripts on M_{21} indicates that a voltage response is produced at L_2 by a current source at L_1. If the system is reversed, as indicated in fig.(*b)* then we have

$$v_1(t) = M_{12} di_2(t)/dt \text{ --------------------[2]}$$

It can be proved that the two mutual inductances M_{12} and M_{21} are equal and thus, $M_{12} = M_{21} = M$. The existence of mutual coupling between two coils is indicated by a double-headed arrow, as shown in Fig. (*a*)and (*b*)

Mutual inductance is measured in Henrys and, like resistance, inductance, and capacitance, is a positive quantity. The voltage *M di/dt*, however, may appear as either a positive or a negative quantity depending on whether the current is increasing or decreasing at a particular instant of time.

Coefficient of coupling k **:** Is given by the relation $\mathbf{M = k\sqrt{L_1 L_2}}$ and its value lies between 0 and 1. It can assume the maximum value of 1 when the two coils are wound on the same core such that flux produced by one coil completely links with the other coil. This is possible in well designed cores with high permeability. Transformers are designed to achieve a coefficient of coupling of 1.

Dot Convention:

The polarity of the voltage induced in a coil depends on the sense of winding of the coil. In the case of Mutual inductance it is indicated by use of a method called "***dot convention".*** The dot convention makes use of a large dot placed at one end of each of the two coils which are mutually coupled. Sign of the mutual voltage is determined as follows:

A current entering the dotted terminal of one coil produces an open circuit voltage with a positive voltage reference at the dotted terminal of the second coil.

Thus in Fig(*a*) i_1 enters the dotted terminal of L_1, v_2 is sensed positively at the dotted terminal of L_2, and $v_2 = M\, di_1/dt$.

It may not be always possible to select voltages or currents throughout a circuit so that the passive sign convention is everywhere satisfied; the same situation arises with mutual coupling. For example, it may be more convenient to represent v_2 by a positive voltage reference at the undotted terminal, as shown in Fig (*b*). Then $v_2 = -M\, di_1/dt$. Currents also may not always enter the dotted terminal as indicated by Fig (*c*) and (*d*). Then we note that:

A current entering the undotted terminal of one coil provides a voltage that is positively sensed at the undotted terminal of the second coil.

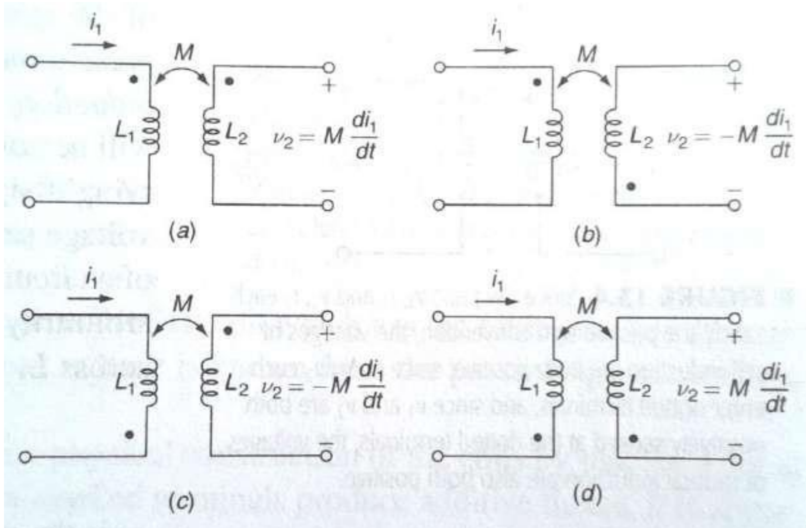


Figure : (a) and (b) Current entering the dotted terminal of one coil produces a voltage that is sensed positively at the dotted terminal of the second coil. **(c) and (d)** Current entering the undotted terminal of one coil produces a voltage that is sensed positively at the undotted terminal of the second coil.

Important Concepts and formulae:

Resonance and Series RLC circuit:

$\omega_r^2 = \omega_1\omega_2 = 1/LC \quad \therefore \omega_r = \sqrt{\omega_1\omega_2} = 1/\sqrt{LC}$

$BW = R/2\pi L$

$Q = \omega_r L / R = 1/ \omega_r RC$ **and in terms of R,L and C** $= (1/R)(\sqrt{L/C})$

$Q = f_r / BW$ **i.e.** inversely proportional to the BW
Voltage magnification Magnification $= Q = V_L/V$ **or** V_C / V

Important points In Series RLC circuit at resonant frequency :

- The impedance of the circuit becomes purely resistive and minimum i.e **Z = R**
- The current in the circuit becomes maximum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is **Q** times higher than the voltage across the resistor

Resonance and Parallel RLC circuit:

$\omega_r^2 = \omega_1\omega_2 = 1/LC \quad \therefore \omega_r = \sqrt{\omega_1\omega_2} = 1/\sqrt{LC}$ same as in series RLC circuit

$BW = 1/2\pi RC$

$Q = R / \omega_r L = \omega_r RC$ **and in terms of R, L and C** $= R(\sqrt{C/L})$ [Inverse of what we got in Series RLC circuit]
$Q = f_r / BW$ In Parallel RLC also inversely proportional to the BW

Current Magnification $= Q = I_L/I$ **or** I_C / I

Important points In Parallel RLC circuit at resonant frequency :

- The impedance of the circuit becomes resistive and maximum i.e **Z = R**
- The current in the circuit becomes minimum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The current through the Capacitor becomes equal and opposite to the current through the Inductor at resonance and is Q times higher than the current through the resistor

Magnetic circuits :

Ampere's Law: $\oint \boldsymbol{H.dl} = \boldsymbol{I}_{net}$ and in the case of a simple closed magnetic path of a ferromagnetic material it simplifies to ***Hl = Ni*** *or* ***H = Ni/l***

Magnetic flux density: **B = μH**
Magnetic field intensity: **H = Ni/l**
Total magnetic flux intensity**:** **Ø = BA = μHA = μ Ni A / l**
Reluctance of the magnetic circuit**:** **R = mmf/Flux = Ni/ Ø = l/μA**

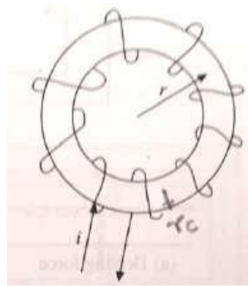
Faradays law of electromagnetic Induction:

Self induced e.m.f of a coil of N turns is given by**:** $\mathbf{e_{ind}} = -N\, d\text{Ø}/dt = -L\, dI/dt$ where L is the inductance of the coil of N turns with radius r and given by $\mathbf{L = (\mu N^2 \pi r^2 / l)\, i}$
Equivalent Reluctance of a series Magnetic circuit: $\mathbf{R_{eqseries} = R_1 + R_2 + R_3 + ...}$
Equivalent Reluctance of a Parallel Magnetic circuit: $\mathbf{1/R_{eqparallel} = 1/R_1 + 1/R_2 + 1/R_3 + ..}$
Coefficient of coupling *k* Is given by the relation**:** $\mathbf{M = k\sqrt{L_1\, L_2}}$

Illustrative examples:

Example 1: A toroidal core of radius 6 cms is having 1000 turns on it. The radius of cross section of the core 1cm.Find the current required to establish a total magnetic flux of 0.4mWb.When

(a) The core is nonmagnetic
(b) The core is made of iron having a relative permeability of 4000



Solution:

This problem can be solved by the direct application of the following formulae we know in magnetic circuits: $\mathbf{B = \Phi/A = \mu H}$ **and** $\mathbf{H = Ni/l}$

Where

B = magnetic flux density (Wb/mtr^2)

Φ = Total magnetic flux (Wb))

A = Cross sectional area of the core(mtr^2)

$\mu = \mu_r\mu_0$ = Permeability (Henrys/mtr)

μ_r = Relative permeability of the material (Dimensionless)

μ_0 = free space permeability = $4\pi \times 10^{-7}$ Henrys/mtr

H = Magenetic field intensity AT/mtr

N = Number of turns of the coil

i = Current in the coil (Amps)

l = Length of the coil (mtrs)

from the above relations we can get **i** as

$$\mathbf{i = H l/N = (1/\mu)(\Phi/A)\, l / N = (1/\mu)(\Phi/N)\, l / A = (1/\mu)(\Phi/N)\, [\, 2\pi r_T / \pi r_C^2] = [\, 2 r_T \Phi / \mu N r_C^2]}$$

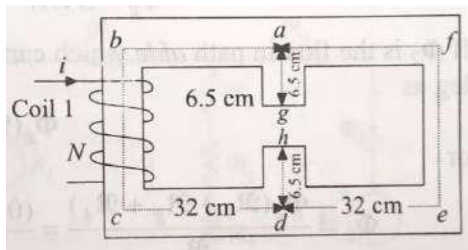
Where r_T is the radius of the toroid and r_C is the radius of cross section of the coil

Now we can calculate the currents in the two cases by substituting the respective values.

(a) Here $\mu = \mu_0$. Therefore $i = (2 \times 6 \times 10^{-2} \times 4 \times 10^{-4})/(4\pi \times 10^{-7} \times 1000 \times 10^{-4}) = 380$ Amps
(b) Here $\mu = \mu_r\mu_0$. Therefore $i = (2 \times 6 \times 10^{-2} \times 4 \times 10^{-4})/(4000 \times 4\pi \times 10^{-7} \times 1000 \times 10^{-4}) = 0.095$ Amps

Ex.2: (a) Draw the electrical equivalent circuit of the magnetic circuit shown in the figure below. The area of the core is 2x2 cm^2 .The length of the air gap is 1cm and lengths of the other limbs are shown in the figure. The relative permeability of the core is 4000.

(b) Find the value of the current **'i'** in the above example which produces a flux density of 1.2 Tesla in the air gap . The number f turns of the coil are 5000.



Solution: (a)

To draw the equivalent circuit we have to find the Reluctances of the various flux paths independently.

The reluctance of the path *abcd* is given by: R_1 = length of the path *abcd* /$\mu_r\mu_0$A

= $(32\times10^{-2}) / (4\pi \times 10^{-7} \times 4000 \times 4 \times 10^{-4})$ $=1.59 \times 10^5$ AT/Wb

The reluctance of the path *afed* is equal to the reluctance of the path *abcd* since it has the same length, same permeability and same cross sectional area. Thus $R_1 = R_2$

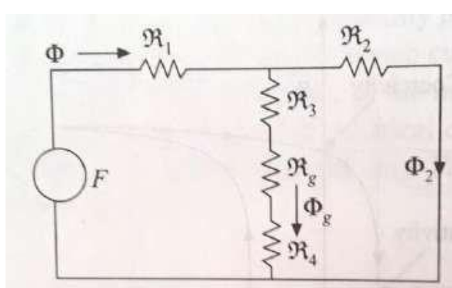
Similarly the reluctance of the path *ag* (R_3) is equal to that of the path *hd* (R_4) and can be calculated as:
$R_3 = R_4 = (6.5 \times 10^{-2}) / (4\pi \times 10^{-7} \times 4000 \times 4 \times 10^{-4})$ $= 0.32 \times 10^5$ AT/Wb

The reluctance of the air gap path *gh* R_G can be calculated as : R_G = length of the air gap path *gh*/μ_0A

(Here it is to be noted that μ is to be taken as μ_0 only and μ_r should not be included)

$R_G = (1 \times 10^{-2}) / (4\pi \times 10^{-7} \times 4 \times 10^{-4}) = 198.94 \times 10^5$ AT/Wb

The equivalent electrical circuit is shown in the figure below with the values of the reluctances as given below the circuit diagram.



$R_1 = R_2 = 1.59x10^5$ AT/Wb $R_3 = R_4 = 0.32x10^5$ AT/Wb $R_G = 198.94x10^5$ AT/Wb

Solution: (b) This problem is solved in the following steps:

1. **First the flux through the air gap Φ_G is found out.** The flux in the air gap Φ_G is given by the product of the Flux density in the air gap **B** and the cross sectional area of the core in that region **A** . Hence Φ_G = **B.A =** 1.2 x 4 x 10^{-4} = **0.00048 Wb**
   It is to be noted here that the same flux would be passing through the reluctances R_3,R_G & R_4
2. **Next,the Flux in the path *afed* Φ_2 is to be found out .** This can be found out by noticing that the mmf across the reluctance R_2 is same as the mmf across the sum of the reluctances **R_3,R_G, and R_4** coming in parallel with **R_4**. Hence by equating them we get

   Φ_G ($R_3 + R_G + R_4$) = $\Phi_2 R_2$ from which we get $\Phi_2 = \Phi_G (R_3 + R_G + R_4) / R_2$

   Hence Φ_2 = [0.00048 x (0.32 + 198.94 + 0.32)x10^5] / 1.59x10^5 = **0.06025 Wb**

3. **Next , the total flux Φ flowing through the reluctance of the path abcd R_1 produced by the winding is to be found out**. This is the sum of the air gap flux Φ_G and the flux in the outer limb of the core Φ_2 : i.e **Φ = $\Phi_G + \Phi_2$ =** (0.00048 + 0.06025) = **0.0607 Wb**
4. **Next , The total mmf F given by F = Ni is to be found out** . This is also equal to the sum of the mmfs across the reluctances **R_1** and **R_2 [or ($R_3 + R_G + R_4$)]** = **$\Phi R_1 + \Phi_2 R_2$** from which we can get **'i'** as : **'i'** = (**$\Phi R_1 + \Phi_2 R_2$) / N = [**0.0607 x 1.59x10^5 + 0.06025 x 1.59x10^5]/5000 = 3.847 Amps

is = 3.847 Amps