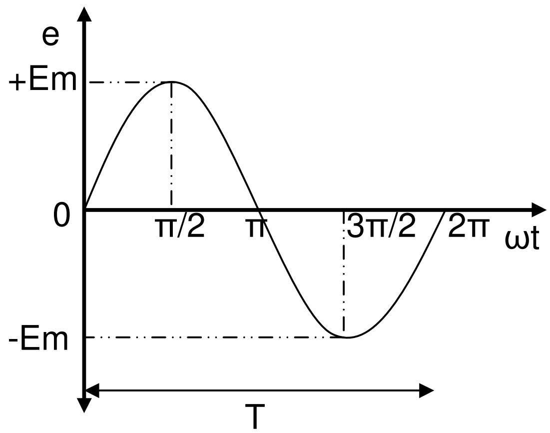
SINGLE PHASE AC CIRCUITS

Definition of Alternating Quantity



An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time. Important terms associated with an alternating quantity are defined below.

1. Amplitude

   It is the maximum value attained by an alternating quantity. Also called as maximum or peak value

2. Time Period (T)

   It is the Time Taken in seconds to complete one cycle of an alternating quantity

3. Instantaneous Value

   It is the value of the quantity at any instant

4. Frequency (f)

   It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec. The relationship between frequency and time period can be derived as follows.

   Time taken to complete f cycles = 1 second

   Time taken to complete 1 cycle = 1/f second

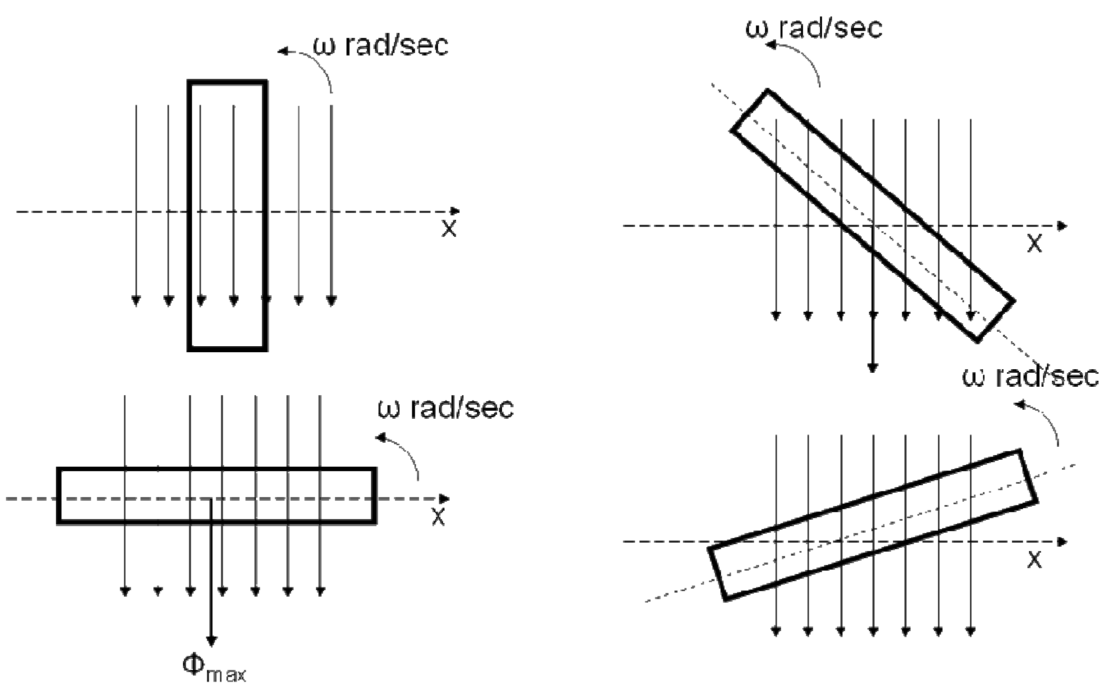
$$T = 1/f$$

Advantages of AC system over DC system

1. AC voltages can be efficiently stepped up/down using transformer
2. AC motors are cheaper and simpler in construction than DC motors
3. Switchgear for AC system is simpler than DC system

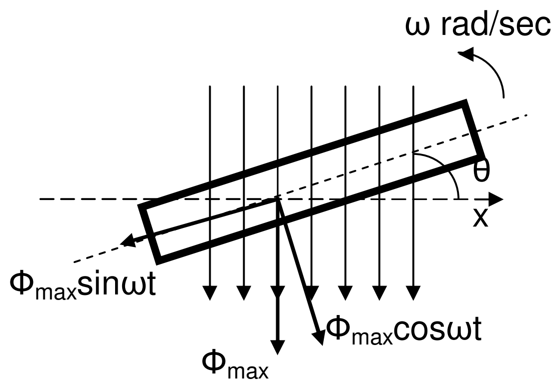
Generation of sinusoidal AC voltage

Consider a rectangular coil of N turns placed in a uniform magnetic field as shown in the figure. The coil is rotating in the anticlockwise direction at an uniform angular velocity of ω rad/sec.



When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero. When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil. When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces. Next when the coil comes to the vertical position, the emf induced becomes zero. After that the same cycle repeats and the emf is induced in the opposite direction. When the coil completes one complete revolution, one cycle of AC voltage is generated.

The generation of sinusoidal AC voltage can also be explained using mathematical equations. Consider a rectangular coil of N turns placed in a uniform magnetic field in the position shown in the figure. The maximum flux linking the coil is in the downward direction as shown in the figure. This flux can be divided into two components, one component acting along the plane of the coil $\Phi_{max}\sin\omega t$ and another component acting perpendicular to the plane of the coil $\Phi_{max}\cos\omega t$.



The component of flux acting along the plane of the coil does not induce any flux in the coil. Only the component acting perpendicular to the plane of the coil ie $\Phi_{max}\cos\omega t$ induces an emf in the coil.

$$\Phi = \Phi_{\max} \cos \omega t$$

$$e = -N \frac{d\Phi}{dt}$$

$$e = -N \frac{d}{dt} \Phi_{\max} \cos \omega t$$

$$e = N\Phi_{\max} \omega \sin \omega t$$

$$e = E_m \sin \omega t$$

Hence the emf induced in the coil is a sinusoidal emf. This will induce a sinusoidal current in the circuit given by

$$i = I_m \sin \omega t$$

Angular Frequency (ω)

Angular frequency is defined as the number of radians covered in one second(ie the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Problem 1

An alternating current i is given by

i = 141.4 sin 314t

Find i) The maximum value

ii) Frequency

iii) Time Period

iv) The instantaneous value when t=3ms

i = 141.4 sin 314t

$$i = I_m \sin \omega t$$

i) Maximum value Im=141.4 V

ii) ω = 314 rad/sec

f = ω/2π = 50 Hz

iii) T=1/f = 0.02 sec

iv) i=141.4 sin(314x0.003) = 114.35A

Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value

$$\text{Average value} = \frac{\text{Area under one cycle}}{\text{Base}}$$

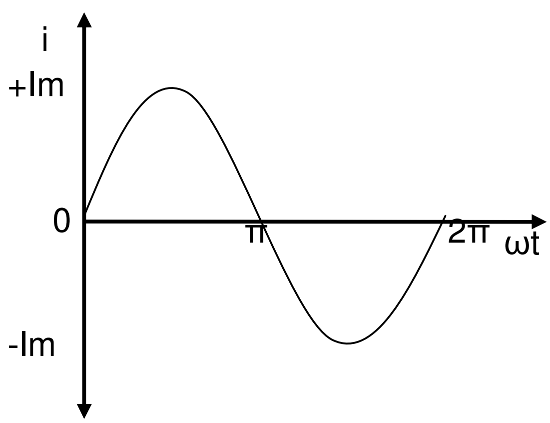
$$V_{av} = \frac{1}{2\pi} \int_0^{2\pi} v\,d(\omega t)$$

For Symmetrical waveforms, the average value calculated over one cycle becomes equal to zero because the positive area cancels the negative area. Hence for symmetrical waveforms, the average value is calculated for half cycle.

$$\text{Average value} = \frac{\text{Area under one half cycle}}{\text{Base}}$$

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v\,d(\omega t)$$

Average value of a sinusoidal current



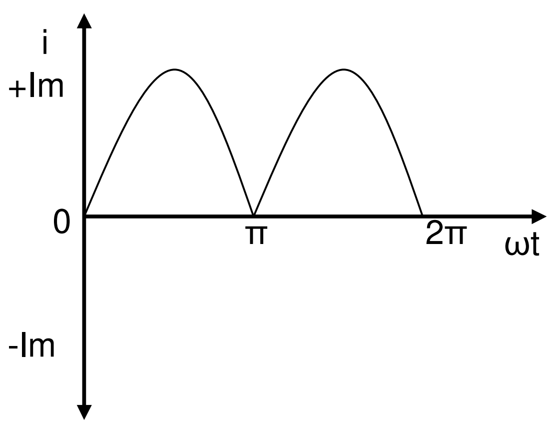
$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

Average value of a full wave rectifier output



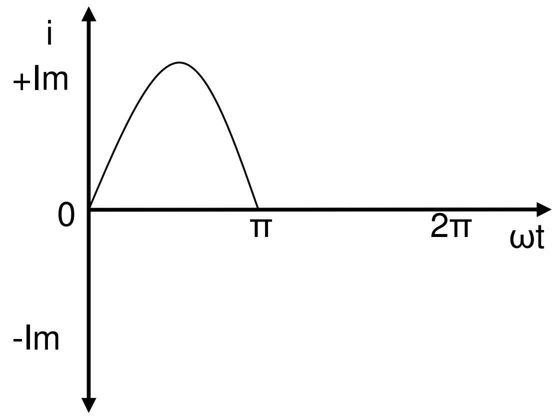
$$i = I_m \sin \omega t$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

Average value of a half wave rectifier output



$$i = I_m \sin \omega t$$

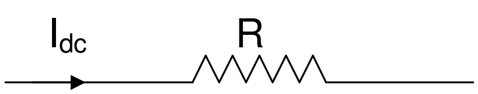
$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i\,d(\omega t)$$

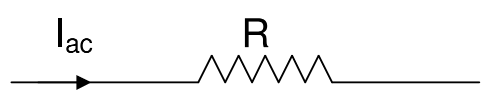
$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t\,d(\omega t)$$

$$I_{av} = \frac{I_m}{\pi} = 0.318 I_m$$

RMS or Effective Value

The effective or RMS value of an alternating quantity is that steady current (dc) which when flowing through a given resistance for a given time produces the same amount of heat produced by the alternating current flowing through the same resistance for the same time.

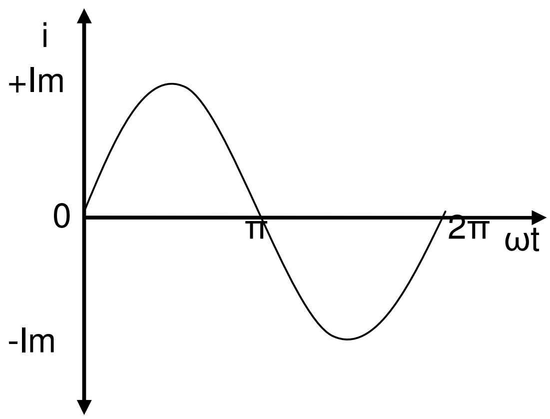




$$RMS = \sqrt{\frac{\text{Area under squared curve}}{base}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2 d(\omega t)}$$

RMS value of a sinusoidal current



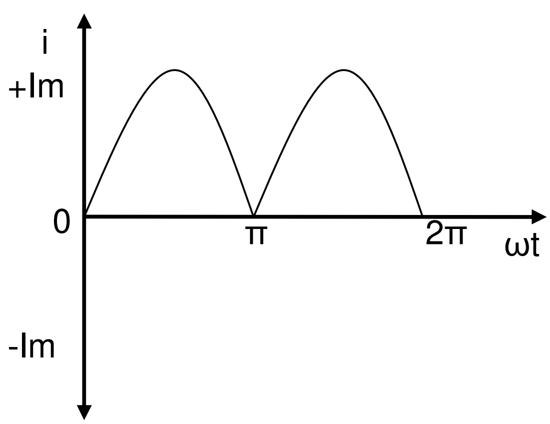
$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

RMS value of a full wave rectifier output



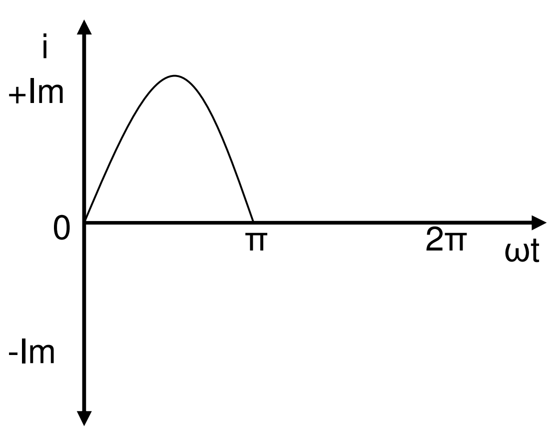
$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

RMS value of a half wave rectifier output



$$i = I_m \sin \omega t$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t d(\omega t)}$$

$$I_{rms} = \frac{I_m}{2} = 0.5 I_m$$

Form Factor

The ratio of RMS value to the average value of an alternating quantity is known as Form Factor

$$FF = \frac{RMSValue}{AverageValue}$$

Peak Factor or Crest Factor

The ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor

$$PF = \frac{MaximumValue}{RMSValue}$$

For a sinusoidal waveform

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$FF = \frac{I_{rms}}{I_{av}} = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

$$PF = \frac{I_m}{I_{rms}} = \frac{I_m}{0.707 I_m} = 1.414$$

For a Full Wave Rectifier Output

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$$FF = \frac{I_{rms}}{I_{av}} = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

$$PF = \frac{I_m}{I_{rms}} = \frac{I_m}{0.707 I_m} = 1.414$$

For a Half Wave Rectifier Output

$$I_{av} = \frac{I_m}{\pi} = 0.318 I_m$$

$$I_{rms} = \frac{I_m}{2} = 0.5 I_m$$

$$FF = \frac{I_{rms}}{I_{av}} = \frac{0.5 I_m}{0.318 I_m} = 1.57$$

$$PF = \frac{I_m}{I_{rms}} = \frac{I_m}{0.5 I_m} = 2$$

Phasor Representation

An alternating quantity can be represented using

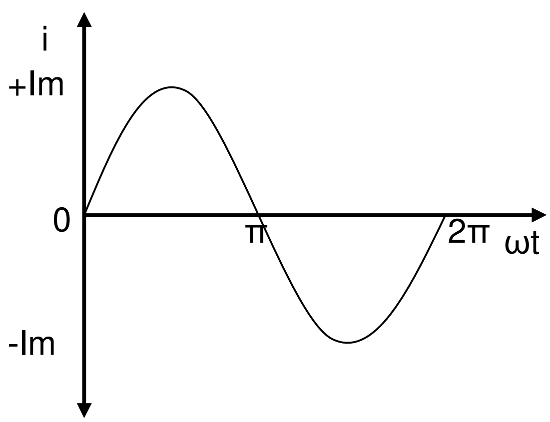
(i) Waveform

(ii) Equations

(iii) Phasor

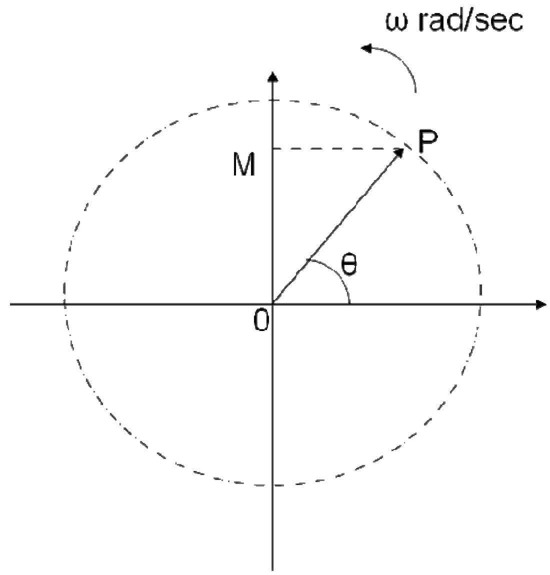
A sinusoidal alternating quantity can be represented by a rotating line called a **Phasor.** A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity

The waveform and equation representation of an alternating current is as shown. This sinusoidal quantity can also be represented using phasors.

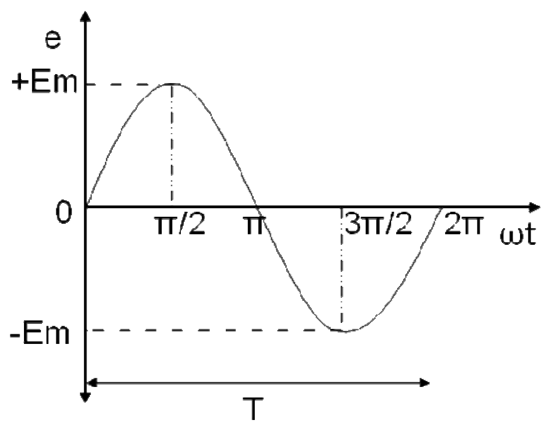


$$i = I_m \sin \omega t$$

Draw a line OP of length equal to I_m. This line OP rotates in the anticlockwise direction with a uniform angular velocity ω rad/sec and follows the circular trajectory shown in figure. At any instant, the projection of OP on the y-axis is given by $OM = OP\sin\theta = I_m \sin\omega t$. Hence the line OP is the phasor representation of the sinusoidal current



Phase

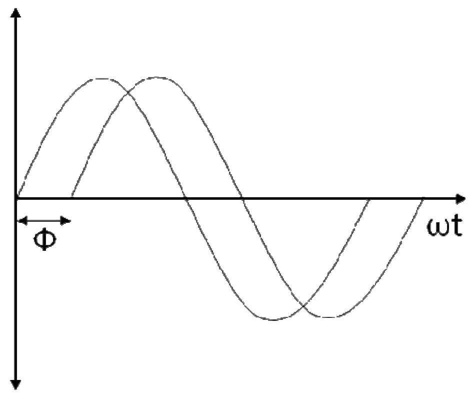


Phase is defined as the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference

Phase of $+E_m$ is $\pi/2$ rad or T/4 sec

Phase of $-E_m$ is $3\pi/2$ rad or 3T/4 sec

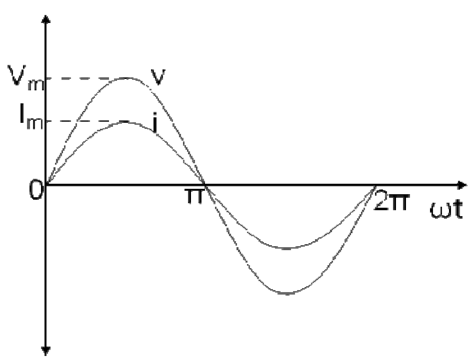
Phase Difference

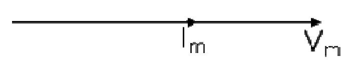


When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference.

In Phase

Two waveforms are said to be in phase, when the phase difference between them is zero. That is the zero points of both the waveforms are same. The waveform, phasor and equation representation of two sinusoidal quantities which are in phase is as shown. The figure shows that the voltage and current are in phase.



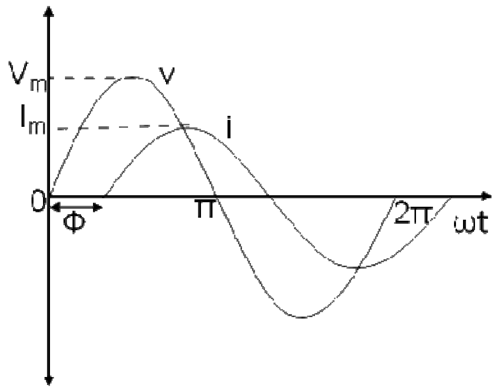


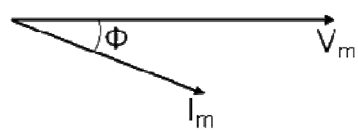
$$v = V_m \sin \omega t$$

$$i = I_m \sin \omega t$$

Lagging

In the figure shown, the zero point of the current waveform is after the zero point of the voltage waveform. Hence the current is lagging behind the voltage. The waveform, phasor and equation representation is as shown.



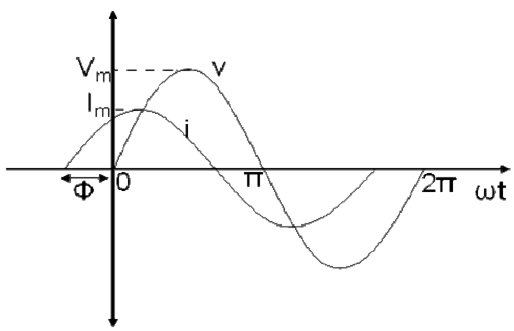


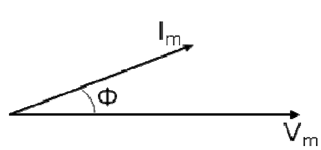
$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t - \Phi)$$

Leading

In the figure shown, the zero point of the current waveform is before the zero point of the voltage waveform. Hence the current is leading the voltage. The waveform, phasor and equation representation is as shown.

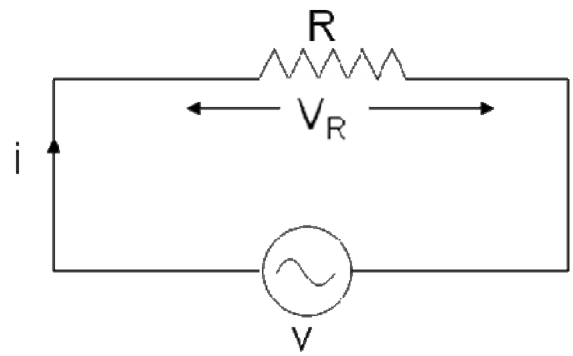




$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + \Phi)$$

AC circuit with a pure resistance



Consider an AC circuit with a pure resistance R as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \qquad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the resistor is given as V_R which is the same as v.

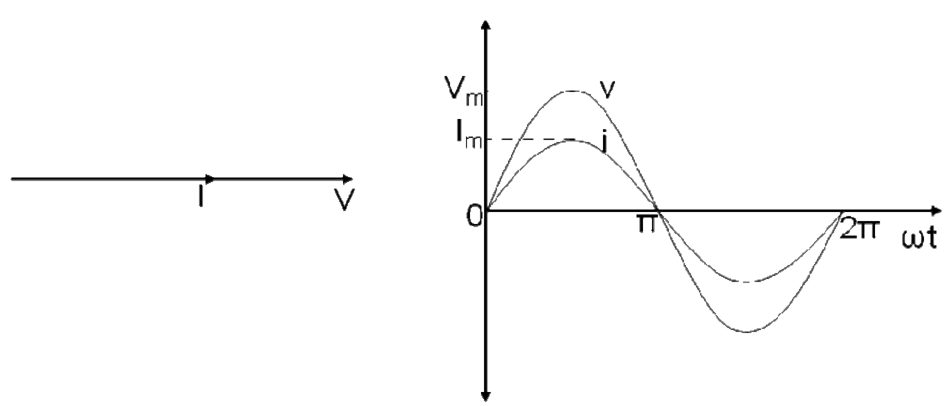
Using ohms law, we can write the following relations

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

$$i = I_m \sin \omega t \qquad \text{---------------(2)}$$

Where $I_m = \dfrac{V_m}{R}$

From equation (1) and (2) we conclude that in a pure resistive circuit, the voltage and current are in phase. Hence the voltage and current waveforms and phasors can be drawn as below.



Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$
$$p = (V_m \sin \omega t)(I_m \sin \omega t)$$
$$p = V_m I_m \sin^2 \omega t$$
$$p = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$
$$p = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

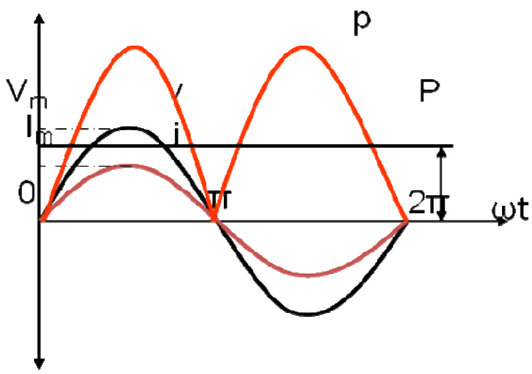
Average power

From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right] d\omega t$$
$$P = \frac{V_m I_m}{2} - \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos 2\omega t\right] d\omega t$$
$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}$$
$$P = V.I$$

As seen above the average power is the product of the rms voltage and the rms current.

The voltage, current and power waveforms of a purely resistive circuit is as shown in the figure.



As seen from the waveform, the instantaneous power is always positive meaning that the power always flows from the source to the load.

Phasor Algebra for a pure resistive circuit

$$\overline{V} = V\angle 0^\circ = V + j0$$

$$\overline{I} = \frac{\overline{V}}{R} = \frac{V + j0}{R} = I + j0 = I\angle 0^\circ$$

Problem 2

An ac circuit consists of a pure resistance of 10Ω and is connected to an ac supply of 230 V, 50 Hz. Calculate the (i) current (ii) power consumed and (iii) equations for voltage and current.

$$(i) I = \frac{V}{R} = \frac{230}{10} = 23A$$

$$(ii) P = VI = 230 \times 23 = 5260W$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

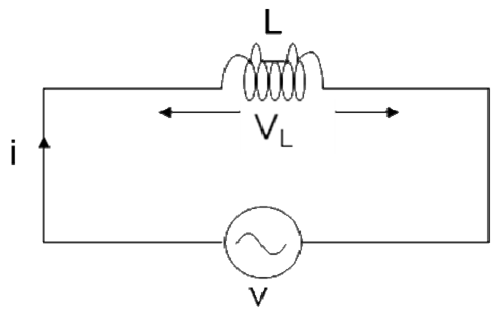
$$I_m = \sqrt{2}I = 32.52A$$

$$\omega = 2\pi f = 314 rad/\sec$$

$$v = 325.25 \sin 314t$$

$$i = 32.52 \sin 314t$$

AC circuit with a pure inductance



Consider an AC circuit with a pure inductance L as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \qquad \text{---------- (1)}$$

The current flowing in the circuit is i. The voltage across the inductor is given as V_L which is the same as v.

We can find the current through the inductor as follows

$$v = L\frac{di}{dt}$$

$$V_m \sin \omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\sin \omega t dt$$

$$i = \frac{V_m}{L}\int \sin \omega t dt$$

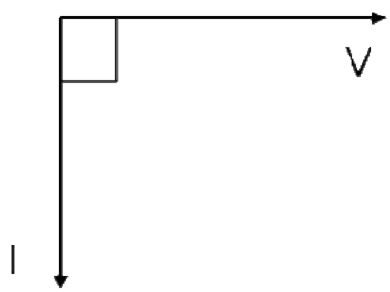
$$i = \frac{V_m}{\omega L}(-\cos \omega t)$$

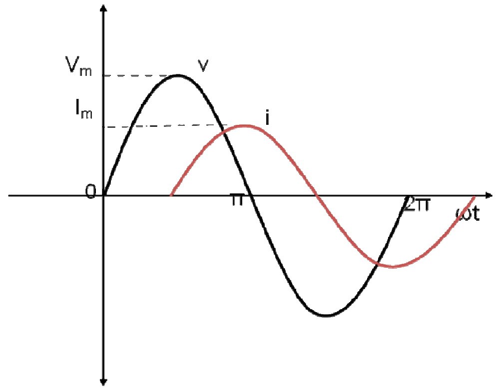
$$i = \frac{V_m}{\omega L}\sin(\omega t - \pi / 2)$$

$$i = I_m \sin(\omega t - \pi / 2) \qquad \text{-------------------(2)}$$

Where $I_m = \dfrac{V_m}{\omega L}$

From equation (1) and (2) we observe that in a pure inductive circuit, the current lags behind the voltage by 90°. Hence the voltage and current waveforms and phasors can be drawn as below.





Inductive reactance

The inductive reactance X_L is given as

$$X_L = \omega L = 2\pi f L$$

$$I_m = \frac{V_m}{X_L}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω)

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t - \pi / 2))$$

$$p = -V_m I_m \sin \omega t \cos \omega t$$

$$p = -\frac{V_m I_m}{2} \sin 2\omega t$$

As seen from the above equation, the instantaneous power is fluctuating in nature.

Average power

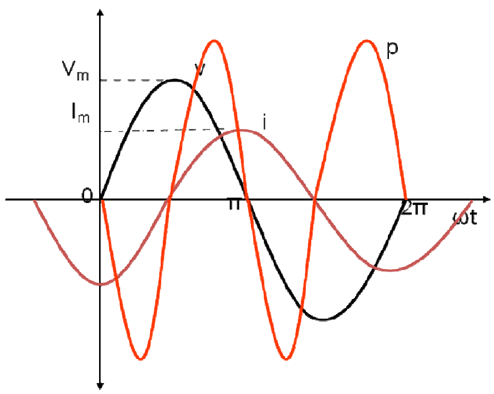
From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2}\sin 2\omega t \, d\omega t$$

$$P = 0$$

The average power in a pure inductive circuit is zero. Or in other words, the power consumed by a pure inductance is zero.

The voltage, current and power waveforms of a purely inductive circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the inductor and when the power in negative, the power flows from the inductor to the source. The positive power is equal to the negative power and hence the average power in the circuit is equal to zero. The power just flows between the source and the inductor, but the inductor does not consume any power.

Phasor algebra for a pure inductive circuit

$$\overline{V} = V\angle 0^\circ = V + j0$$

$$\overline{I} = I\angle -90^\circ = 0 - jI$$

$$\frac{\overline{V}}{\overline{I}} = \frac{V\angle 0^\circ}{I\angle -90} = X_L\angle 90^\circ$$

$$\overline{V} = \overline{I}(jX_L)$$

Problem 3

A pure inductive coil allows a current of 10A to flow from a 230V, 50 Hz supply. Find (i) inductance of the coil (ii) power absorbed and (iii) equations for voltage and current.

$$(i) X_L = \frac{V}{I} = \frac{230}{10} = 23\Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = 0.073H$$

$$(ii) P = 0$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

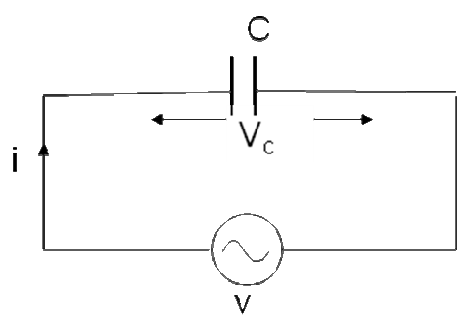
$$I_m = \sqrt{2}I = 14.14A$$

$$\omega = 2\pi f = 314 rad/\sec$$

$$v = 325.25 \sin 314t$$

$$i = 14.14 \sin(314t - \pi/2)$$

AC circuit with a pure capacitance



Consider an AC circuit with a pure capacitance C as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t \qquad \text{---------- (1)}$$

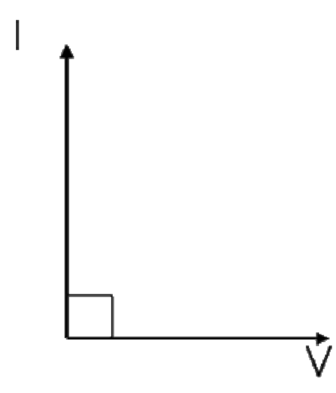
The current flowing in the circuit is i. The voltage across the capacitor is given as V_C which is the same as v.

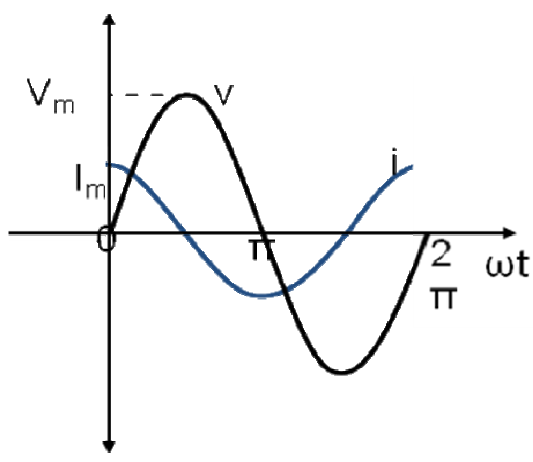
We can find the current through the capacitor as follows

$$q = Cv$$
$$q = CV_m \sin \omega t$$
$$i = \frac{dq}{dt}$$
$$i = CV_m \omega \cos \omega t$$
$$i = \omega CV_m \sin(\omega t + \pi / 2)$$
$$i = I_m \sin(\omega t + \pi / 2) \qquad \text{-------------------(2)}$$

Where $I_m = \omega CV_m$

From equation (1) and (2) we observe that in a pure capacitive circuit, the current leads the voltage by 90°. Hence the voltage and current waveforms and phasors can be drawn as below.





Capacitive reactance

The capacitive reactance X_C is given as

$$X_L = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$I_m = \frac{V_m}{X_C}$$

It is equivalent to resistance in a resistive circuit. The unit is ohms (Ω)

Instantaneous power

The instantaneous power in the above circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t + \pi / 2))$$

$$p = V_m I_m \sin \omega t \cos \omega t$$

$$p = \frac{V_m I_m}{2} \sin 2\omega t$$

As seen from the above equation, the instantaneous power is fluctuating in nature.

Average power

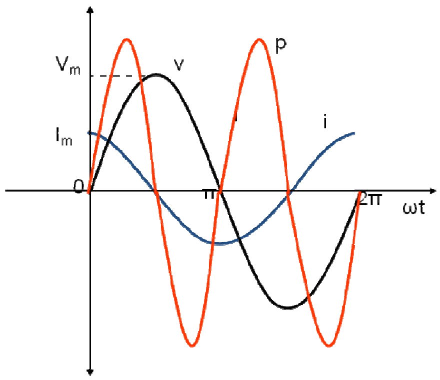
From the instantaneous power we can find the average power over one cycle as follows

$$P = \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t d\omega t$$

$$P = 0$$

The average power in a pure capacitive circuit is zero. Or in other words, the power consumed by a pure capacitance is zero.

The voltage, current and power waveforms of a purely capacitive circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the capacitor and when the power in negative, the power flows from the capacitor to the source. The positive power is equal to the negative power and hence the average power in the circuit is equal to zero. The power just flows between the source and the capacitor, but the capacitor does not consume any power.

Phasor algebra in a pure capacitive circuit

$$\bar{V} = V\angle 0^\circ = V + j0$$

$$\bar{I} = I\angle 90^\circ = 0 + jI$$

$$\frac{\bar{V}}{\bar{I}} = \frac{V\angle 0^\circ}{I\angle 90} = X_C\angle -90^\circ$$

$$\bar{V} = \bar{I}(-jX_C)$$

Problem 4

A 318μF capacitor is connected across a 230V, 50 Hz system. Find (i) the capacitive reactance (ii) rms value of current and (iii) equations for voltage and current.

$$(i) X_C = \frac{1}{2\pi fC} = 10\Omega$$

$$(ii) I = \frac{V}{X_C} = 23A$$

$$(iii) V_m = \sqrt{2}V = 325.27V$$

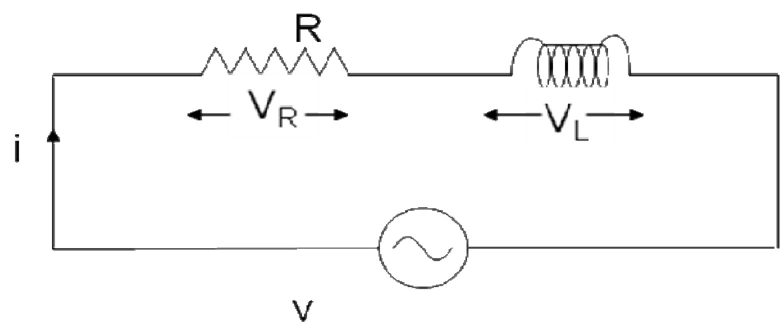
$$I_m = \sqrt{2}I = 32.53A$$

$$\omega = 2\pi f = 314 rad / \sec$$

$$v = 325.25 \sin 314t$$

$$i = 32.53 \sin(314t + \pi / 2)$$

R-L Series circuit



Consider an AC circuit with a resistance R and an inductance L connected in series as shown in the figure. The alternating voltage v is given by

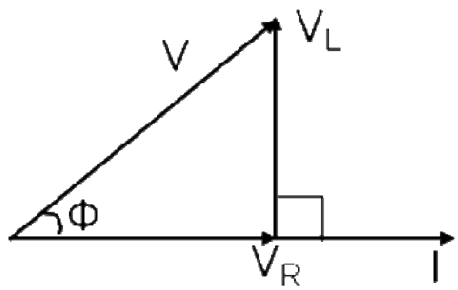
$$v = V_m \sin \omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the inductor is V_L.

V_R=IR is in phase with I

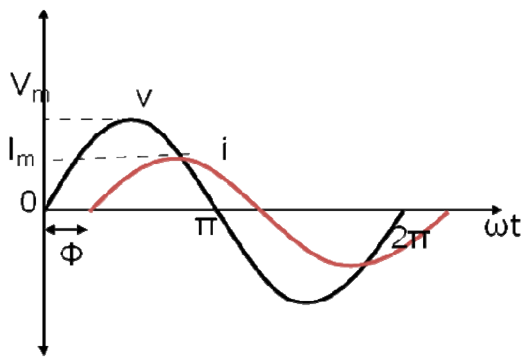
$V_L=IX_L$ leads current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_L leads the current by 90^o. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ.

The waveform and equations for an RL series circuit can be drawn as below.



$$V = V_m \sin \omega t$$
$$I = I_m \sin(\omega t - \Phi)$$

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_L^2}$$
$$V_R = IR$$
$$V_L = IX_L$$
$$V = \sqrt{(IR)^2 + (IX_L)^2}$$
$$V = I\sqrt{R^2 + X_L^2}$$
$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + X_L^2}$

The impedance in an AC circuit is similar to a resistance in a DC circuit. The unit for impedance is ohms (Ω).

Phase angle

$$\Phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{IX_L}{IR}\right)$$

$$\Phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Instantaneous power

The instantaneous power in an RL series circuit can be derived as follows

$$p = vi$$

$$p = (V_m \sin \omega t)(I_m \sin(\omega t - \Phi))$$

$$p = \frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)$$

The instantaneous power consists of two terms. The first term is called as the constant power term and the second term is called as the fluctuating power term.

Average power

From the instantaneous power we can find the average power over one cycle as follows

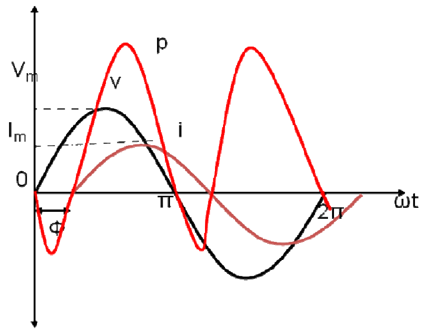
$$P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos\Phi - \frac{V_m I_m}{2}\cos(2\omega t - \Phi)\right]d\omega t$$

$$P = \frac{V_m I_m}{2}\cos\Phi$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\Phi$$

$$P = VI\cos\Phi$$

The voltage, current and power waveforms of a RL series circuit is as shown in the figure.



As seen from the power waveform, the instantaneous power is alternately positive and negative. When the power is positive, the power flows from the source to the load and when the power in negative, the power flows from the load to the source. The positive power is not equal to the negative power and hence the average power in the circuit is not equal to zero.

From the phasor diagram,

$$\cos\Phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

$$P = VI\cos\Phi$$

$$P = (IZ)\times I \times \frac{R}{Z}$$

$$P = I^2 R$$

Hence the power in an RL series circuit is consumed only in the resistance. The inductance does not consume any power.

Power Factor

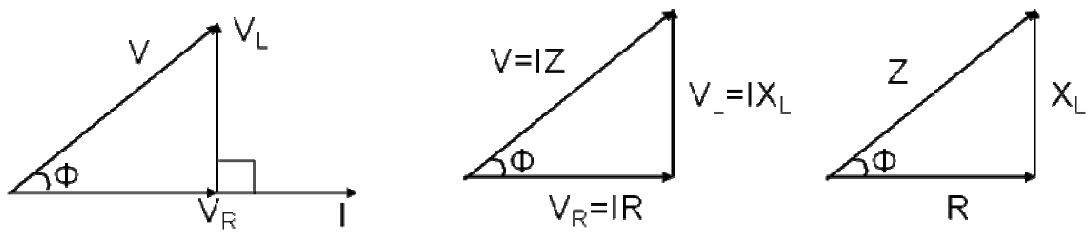
The power factor in an AC circuit is defined as the cosine of the angle between voltage and current ie cosΦ

$$P = VI\cos\Phi$$

The power in an AC circuit is equal to the product of voltage, current and power factor.

Impedance Triangle

We can derive a triangle called the impedance triangle from the phasor diagram of an RL series circuit as shown



The impedance triangle is right angled triangle with R and X_L as two sides and impedance as the hypotenuse. The angle between the base and hypotenuse is Φ. The impedance triangle enables us to calculate the following things.

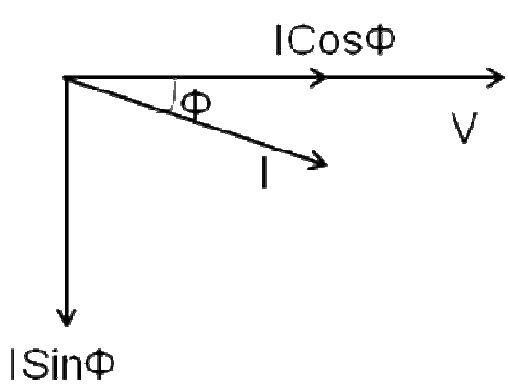
1. Impedance $Z = \sqrt{R^2 + X_L^2}$
2. Power Factor $\cos\Phi = \frac{R}{Z}$
3. Phase angle $\Phi = \tan^{-1}\left(\frac{X_L}{R}\right)$
4. Whether current leads or lags behind the voltage

Power

In an AC circuit, the various powers can be classified as

1. Real or Active power
2. Reactive power
3. Apparent power

Real or active power in an AC circuit is the power that does useful work in the cicuit. Reactive power flows in an AC circuit but does not do any useful work. Apparent power is the total power in an AC circuit.



From the phasor diagram of an RL series circuit, the current can be divided into two components. One component along the voltage IcosΦ, that is called as the active component of current and another component perpendicular to the voltage IsinΦ that is called as the reactive component of current.

Real Power

The power due to the active component of current is called as the active power or real power. It is denoted by P.

$P = V \times I\cos\Phi = I^2R$

Real power is the power that does useful power. It is the power that is consumed by the resistance.

The unit for real power in Watt(W).

Reactive Power

The power due to the reactive component of current is called as the reactive power. It is denoted by Q.

$Q = V \times I\sin\Phi = I^2X_L$

Reactive power does not do any useful work. It is the circulating power in th L and C components.

The unit for reactive power is Volt Amperes Reactive (VAR).

Apparent Power

The apparent power is the total power in the circuit. It is denoted by S.

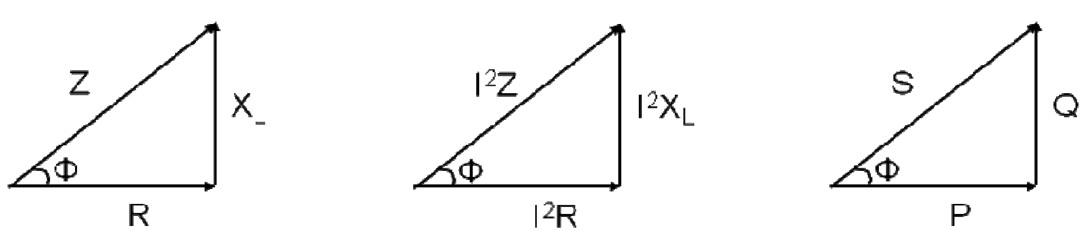
$S = V \times I = I^2Z$

$$S = \sqrt{P^2 + Q^2}$$

The unit for apparent power is Volt Amperes (VA).

Power Triangle

From the impedance triangle, another triangle called the power triangle can be derived as shown.



The power triangle is right angled triangle with P and Q as two sides and S as the hypotenuse. The angle between the base and hypotenuse is Φ. The power triangle enables us to calculate the following things.

1. Apparent power $S = \sqrt{P^2 + Q^2}$
2. Power Factor $Cos\Phi = \frac{P}{S} = \frac{\operatorname{Re}alPower}{ApparentPower}$

The power Factor in an AC circuit can be calculated by any one of the following methods

- Cosine of angle between V and I
- Resistance/Impedance R/Z
- Real Power/Apparent Power P/S

Phasor algebra in a RL series circuit

$$V = V + j0 = V\angle 0^\circ$$

$$\bar{Z} = R + jX_L = Z\angle\Phi$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle -\Phi$$

$$\bar{S} = VI^* = P + jQ$$

Problem 5

A coil having a resistance of 7Ω and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed

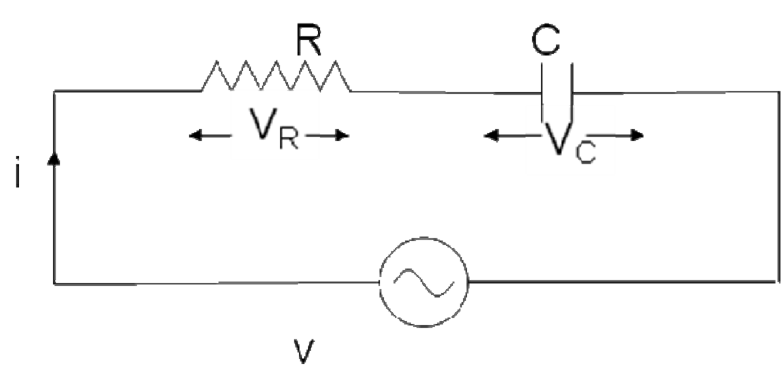
$$X_L = 2\pi f L = 2\times 3.14\times 50\times 31.8\times 10^{-3} = 10\Omega$$
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{7^2 + 10^2} = 12.2\Omega$$
$$(i) I = \frac{V}{Z} = \frac{230}{12.2} = 18.85A$$
$$(ii) \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{10}{7}\right) = 55^\circ\, lag$$
$$(iii) PF = \cos\Phi = \cos(55^\circ) = 0.573 lag$$
$$(iv) P = VI\cos\Phi = 230\times 18.85\times 0.573 = 2484.24W$$

Problem 6

A 200 V, 50 Hz, inductive circuit takes a current of 10A, lagging 30 degree. Find (i) the resistance (ii) reactance (iii) inductance of the coil

$$Z = \frac{V}{I} = \frac{200}{10} = 20\Omega$$
$$(i) R = Z\cos\phi = 20\times\cos 30^\circ = 17.32\Omega$$
$$(ii) X_L = Z\sin\phi = 20\times\sin 30^\circ = 10\Omega$$
$$(iii) L = \frac{X_L}{2\pi f} = \frac{10}{2\times 3.14\times 50} = 0.0318H$$

R-C Series circuit



Consider an AC circuit with a resistance R and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

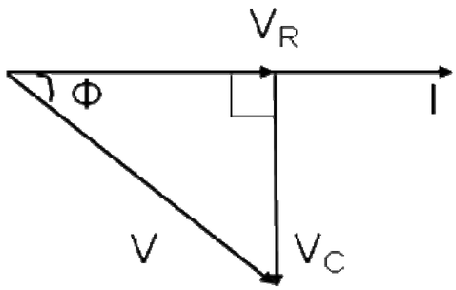
$$v = V_m \sin \omega t$$

The current flowing in the circuit is i. The voltage across the resistor is V_R and that across the capacitor is V_C.

V_R=IR is in phase with I

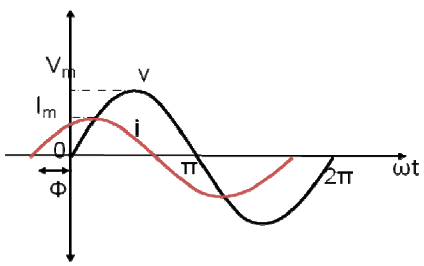
V_C=IX_C lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown.



The current I is taken as the reference phasor. The voltage V_R is in phase with I and the voltage V_C lags behind the current by 90^o. The resultant voltage V can be drawn as shown in the figure. From the phasor diagram we observe that the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

The waveform and equations for an RC series circuit can be drawn as below.



$$V = V_m \sin \omega t$$
$$I = I_m \sin(\omega t + \Phi)$$

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + V_C^2}$$
$$V_R = IR$$
$$V_C = IX_C$$
$$V = \sqrt{(IR)^2 + (IX_C)^2}$$
$$V = I\sqrt{R^2 + X_C^2}$$
$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + X_C^2}$

Phase angle

$$\Phi = \tan^{-1}\left(\frac{V_C}{V_R}\right)$$
$$\Phi = \tan^{-1}\left(\frac{IX_C}{IR}\right)$$
$$\Phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$
$$\Phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

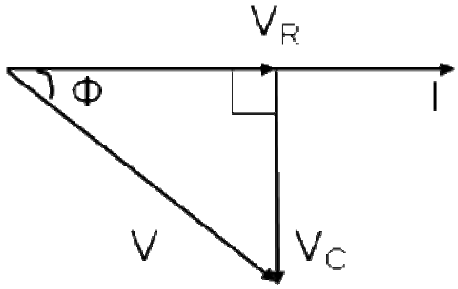
Average power

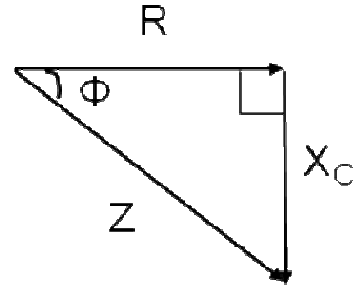
$$P = VI\cos\phi$$
$$P = (IZ) \times I \times \frac{R}{Z}$$
$$P = I^2 R$$

Hence the power in an RC series circuit is consumed only in the resistance. The capacitance does not consume any power.

Impedance Triangle

We can derive a triangle called the impedance triangle from the phasor diagram of an RC series circuit as shown





Phasor algebra for RC series circuit

$$V = V + j0 = V\angle 0^\circ$$

$$\bar{Z} = R - jX_C = Z\angle -\Phi$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z}\angle +\Phi$$

Problem 7

A Capacitor of capacitance 79.5µF is connected in series with a non inductive resistance of 30Ω across a 100V, 50Hz supply. Find (i) impedance (ii) current (iii) phase angle (iv) Equation for the instantaneous value of current

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 79.5\times 10^{-6}} = 40\Omega$$

$$(i) Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = 50\Omega$$

$$(ii) I = \frac{V}{Z} = \frac{100}{50} = 2A$$

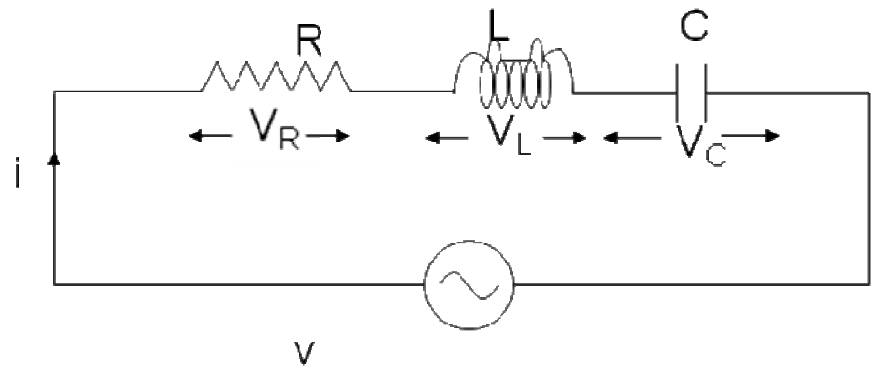
$$(iii) \Phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{40}{30}\right) = 53^\circ\, lead$$

$$(iv) I_m = \sqrt{2}I = \sqrt{2}\times 2 = 2.828A$$

$$\omega = 2\pi f = 2\times 3.14\times 50 = 314 rad/\sec$$

$$i = 2.828\sin(314t + 53^\circ)$$

R-L-C Series circuit



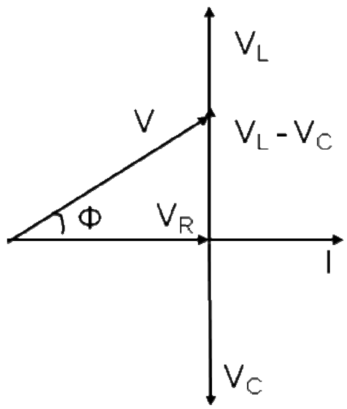
Consider an AC circuit with a resistance R, an inductance L and a capacitance C connected in series as shown in the figure. The alternating voltage v is given by

$$v = V_m \sin \omega t$$

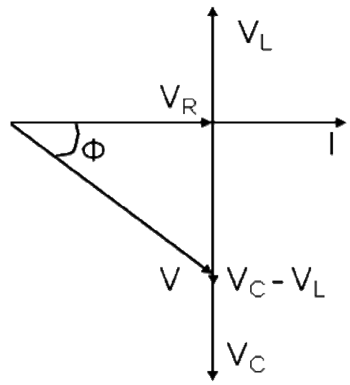
The current flowing in the circuit is i. The voltage across the resistor is V_R, the voltage across the inductor is V_L and that across the capacitor is V_C.

V_R=IR is in phase with I
V_L=IX_L leads the current by 90 degrees
V_C=IX_C lags behind the current by 90 degrees

With the above information, the phasor diagram can be drawn as shown. The current I is taken as the reference phasor. The voltage V_R is in phase with I, the voltage V_L leads the current by 90^o and the voltage V_C lags behind the current by 90^o. There are two cases that can occur $V_L>V_C$ and $V_L<V_C$ depending on the values of X_L and X_C. And hence there are two possible phasor diagrams. The phasor V_L-V_C or V_C-V_L is drawn and then the resultant voltage V is drawn.



$V_L > V_C$



$V_L < V_C$

From the phasor diagram we observe that when $V_L > V_C$, the voltage leads the current by an angle Φ or in other words the current lags behind the voltage by an angle Φ. When $V_L < V_C$,the voltage lags behind the current by an angle Φ or in other words the current leads the voltage by an angle Φ.

From the phasor diagram, the expressions for the resultant voltage V and the angle Φ can be derived as follows.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$V = IZ$$

Where impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phase angle

$$\Phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

$$\Phi = \tan^{-1}\left(\frac{IX_L - IX_C}{IR}\right)$$

$$\Phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

From the expression for phase angle, we can derive the following three cases

Case (i): When $X_L>X_C$

The phase angle Φ is positive and the circuit is inductive. The circuit behaves like a series RL circuit.

Case (ii): When XL<XC

The phase angle Φ is negative and the circuit is capacitive. The circuit behaves like a series RC circuit.

Case (iii): When XL=XC

The phase angle Φ = 0 and the circuit is purely resistive. The circuit behaves like a pure resistive circuit.

The voltage and the current can be represented by the following equations. The angle Φ is positive or negative depending on the circuit elements.

$$V = V_m \sin \omega t$$
$$I = I_m \sin(\omega t \pm \Phi)$$

Average power

$$P = VI\cos\phi$$
$$P = (IZ)\times I\times\frac{R}{Z}$$
$$P = I^2 R$$

Hence the power in an RLC series circuit is consumed only in the resistance. The inductance and the capacitance do not consume any power.

Phasor algebra for RLC series circuit

$$V = V + j0 = V\angle 0^\circ$$
$$\overline{Z} = R + j\left(X_L - X_C\right) = Z\angle\Phi$$
$$\bar{I} = \frac{\overline{V}}{\overline{Z}} = \frac{V}{Z}\angle -\Phi$$

Problem 8

A 230 V, 50 Hz ac supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6.8 μF capacitor. Calculate (i) Impedance (ii) Current (iii) Phase angle between current and voltage (iv) power factor (v) power consumed

$$X_L = 2\pi fL = 2\times 3.14\times 50\times 0.06 = 18.84\,\Omega$$
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 6.8\times 10^{-6}} = 468\,\Omega$$
$$(i) Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2.5^2 + (18.84 - 468)^2} = 449.2\Omega$$
$$(ii) I = \frac{V}{Z} = \frac{230}{449.2} = 0.512\,A$$
$$(iii)\Phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{18.84 - 468}{30}\right) = -89.7^\circ$$
$$(iv)\, pf = \cos\Phi = \cos 89.7 = 0.0056\, lead$$
$$(v) P = VI\cos\Phi = 230\times 0.512\times 0.0056 = 0.66W$$

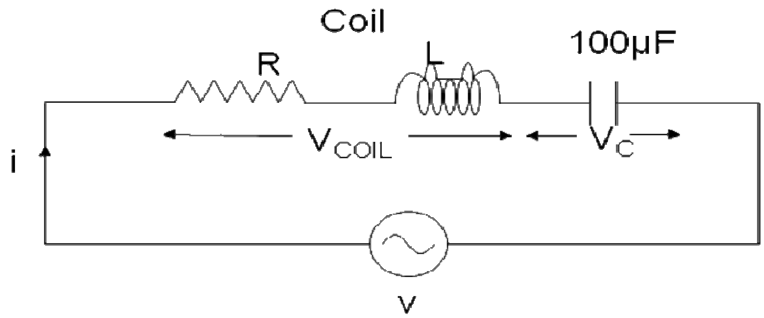
Problem 9

A resistance R, an inductance L=0.01 H and a capacitance C are connected in series. When an alternating voltage v=400sin(3000t-20º)is applied to the series combination, the current flowing is $10\sqrt{2}$ sin(3000t-65º). Find the values of R and C.

$$\Phi = 65^\circ - 20^\circ = 45^\circ lag$$
$$X_L = \omega L = 3000\times 0.01 = 30\Omega$$
$$\tan\Phi = \tan 45^\circ = 1$$
$$\tan\Phi = \frac{X_L - X_C}{R} = 1$$
$$R = X_L - X_C$$
$$Z = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = 28.3\Omega Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + R^2}$$
$$\sqrt{2}R = 28.3$$
$$R = 20\Omega$$
$$X_L - X_C = 20\Omega$$
$$X_C = 30 - 20 = 10\Omega$$
$$C = \frac{1}{\omega X_C} = \frac{1}{3000\times 10} = 33.3\mu F$$

Problem 10

A coil of pf 0.6 is in series with a 100µF capacitor. When connected to a 50Hz supply, the potential difference across the coil is equal to the potential difference across the capacitor. Find the resistance and inductance of the coil.



$\cos\Phi_{coil} = 0.6$
C=100µF
f = 50Hz
$V_{coil}=V_c$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 100\times 10^{-6}} = 31.83\Omega$$

$$V_{coil} = V_c$$

$$IZ_{coil} = IX_C$$

$$Z_{coil} = X_C = 31.83\Omega$$

$$R = Z_{coil}\cos\Phi_{coil} = 31.83\times 0.6 = 19.09\Omega$$

$$X_L = \sqrt{Z_{coil}^2 - R^2} = \sqrt{31.83^2 - 19.09^2} = 25.46\Omega$$

$$L = \frac{1}{2\pi fL} = \frac{1}{2\times 3.14\times 50\times 25.46} = 0.081H$$

Problem 11

A current of (120-j50)A flows through a circuit when the applied voltage is (8+j12)V. Determine (i) impedance (ii) power factor (iii) power consumed and reactive power

$$\overline{V} = 8 + j12$$
$$\overline{I} = 120 - j50$$
$$(i)\overline{Z} = \frac{\overline{V}}{\overline{I}} = \frac{8 + j12}{120 - j50} = 0.02 + j0.11 = 0.11\angle 79.7^{\circ}$$
$$Z = 0.11\Omega$$
$$\Phi = 79.7^{\circ}$$
$$(ii)\, pf = \cos\Phi = \cos 79.7^{\circ} = 0.179 lag$$
$$(iii) S = VI^{*} = (8 + j12)\times(120 + j50) = 360 + j1840$$
$$S = P + jQ$$
$$P = 360W$$
$$Q = 1840VAR$$

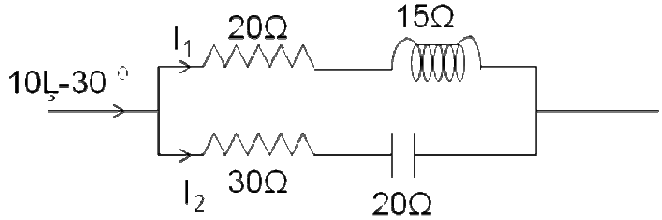
Problem 12

The complex Volt Amperes in a series circuit are (4330-j2500) and the current is (25+j43.3)A. Find the applied voltage.

$$\overline{S} = 4330 + j2500$$
$$\overline{I} = 25 + j43.3$$
$$\overline{V} = \frac{\overline{S}}{\overline{I^{*}}} = \frac{4330 + j2500}{25 - j43.3} = 86.6 + j50$$

Problem 13

A parallel circuit comprises of a resistor of 20Ω in series with an inductive reactance 15Ω in one branch and a resistor of 30Ω in series with a capacitive reactance of 20Ω in the other branch. Determine the current and power dissipated in each branch if the total current drawn by the parallel circuit is 10Ļ-30 ^{0}A



$$Z_1 = 20 + j15$$
$$Z_2 = 30 - j20$$
$$I = 10\angle -30^\circ = 8.66 - j5$$
$$I_1 = I\frac{Z_2}{Z_1 + Z_2} = (8.66 - j5)\times\frac{(30 - j20)}{(20 + j15) + (30 - j20)}$$
$$I_1 = 3.8 - j6.08 = 7.17\angle -60^\circ$$
$$I_2 = I - I_1 = (8.66 - j5) - (3.8 - j6.08)$$
$$I_2 = 4.86 + j1.08 = 4.98\angle -12.5^\circ$$
$$P_1 = I_1^2 R_1 = 7.17^2 \times 20 = 1028.2W$$
$$P_1 = I_2^2 R_2 = 4.98^2 \times 30 = 744W$$

Problem 14

A non inductive resistor of 10Ω is in series with a capacitor of 100μF across a 250V, 50Hz ac supply. Determine the current taken by the capacitor and power factor of the circuit

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 100\times 10^{-6}} = 31.83\Omega$$
$$Z = R - jX_C = 10 - j31.83$$
$$I = \frac{V}{Z} = \frac{250}{10 - j31.83} = 2.24 + j7.14 = 7.49\angle 72.5^\circ$$
$$\phi = 72.5^\circ$$
$$pf = \cos\phi = \cos 72.5^\circ = 0.3$$

Problem 15

An impedance coil in parallel with a 100μF capacitor is connected across a 200V, 50Hz supply. The coil takes a current of 4A and the power loss in the coil is 600W. Calculate (i) the resistance of the coil (ii) the inductance of the coil (iii) the power factor of the entire circuit.

$$Z_{coil} = \frac{V}{I} = \frac{200}{4} = 50\Omega$$

$$P = I^2 R = 600W$$

$$R = \frac{600}{I^2} = \frac{600}{4^2} = 37.5\Omega$$

$$X_L = \sqrt{Z_{coil}{}^2 - R^2} = \sqrt{50^2 - 37.5^2} = 33.07\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{33.07}{2\times 3.14\times 50} = 0.105H$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 100\times 10^{-6}} = 31.83\Omega$$

$$Z_1 = R + jX_L = 37.5 + j33.07$$

$$Z_2 = -jX_C = -j31.83$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(37.5 + j33.07)(-j31.83)}{(37.5 + j33.07) + (-j31.83)}$$

$$Z = 27 - j32.72 = 42.42\angle -50.5^\circ$$

$$\Phi = -50.5^\circ$$

$$pf = \cos\Phi = \cos(-50.5^\circ) = 0.6365$$

Problem 16

A series RLC circuit is connected across a 50Hz supply. R=100Ω, L=159.16mH and C=63.7μF. If the voltage across C is 150Ļ-90⁰V. Find the supply voltage

$$X_L = 2\pi fL = 2\times 3.14\times 50\times 159.16\times 10^{-3} = 50\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 63.7\times 10^{-6}} = 50\Omega$$

$$V_C = I(-jX_C) = 150\angle -90\circ = -j150$$

$$I = \frac{-j150}{-jX_C} = \frac{-j150}{-j50} = 3\angle 0^\circ A$$

$$Z = R + j(X_L - X_C) = 100 + j(50 - 50) = 100\Omega$$

$$V = IZ = 3\times 100 = 300V$$

Problem 17

A circuit having a resistance of 20Ω and inductance of 0.07H is connected in parallel with a series combination of 50Ω resistance and 60μF capacitance. Calculate the total current, when the parallel combination is connected across 230V, 50Hz supply.

$$X_L = 2\pi fL = 2\times 3.14\times 50\times 0.07 = 22\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times 3.14\times 50\times 60\times 10^{-6}} = 53\Omega$$

$$Z_1 = 20 + j22$$

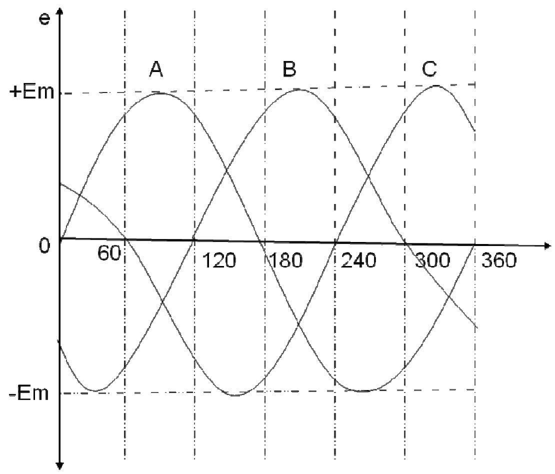
$$Z_2 = 50 - j53$$

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(20 + j22)(50 - j53)}{(20 + j22) + (50 - j53)} = 25.7 + \mathrm{j}11.9$$

$$I = \frac{V}{Z} = \frac{230}{Z} = 7.4 - \mathrm{j}3.4 = 8.13\angle -24.9^\circ$$

THREE PHASE AC CIRCUITS

A three phase supply is a set of three alternating quantities displaced from each other by an angle of 120°. A three phase voltage is shown in the figure. It consists of three phases- phase A, phase B and phase C. Phase A waveform starts at 0°. Phase B waveform stars at 120° and phase C waveform at 240°.



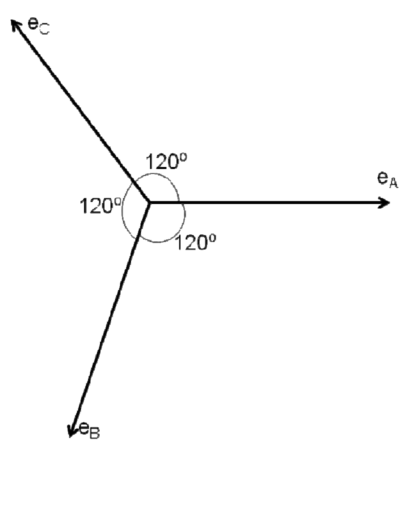
The three phase voltage can be represented by a set of three equations as shown below.

$$e_A = E_m \sin \omega t$$
$$e_B = E_m \sin(\omega t - 120^\circ)$$
$$e_C = E_m \sin(\omega t - 240^\circ) = E_m \sin(\omega t + 120^\circ)$$

The sum of the three phase voltages at any instant is equal to zero.

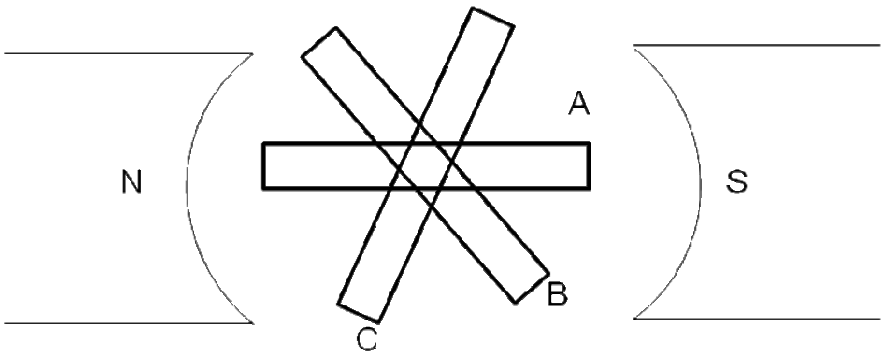
$$e_A + e_B + e_C = 0$$

The phasor representation of three phase voltages is as shown.



The phase A voltage is taken as the reference and is drawn along the x-axis. The phase B voltage lags behind the phase A voltage by 120°. The phase C voltage lags behind the phase A voltage by 240° and phase B voltage by 120°.

Generation of Three Phase Voltage



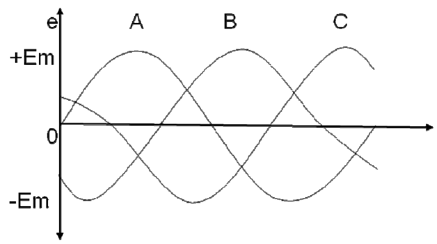
Three Phase voltage can be generated by placing three rectangular coils displaced in space by 120° in a uniform magnetic field. When these coils rotate with a uniform angular velocity of ω rad/sec, a sinusoidal emf displaced by 120° is induced in these coils.

Necessity and advantages of three phase systems

- 3Φ power has a constant magnitude whereas 1Φ power pulsates from zero to peak value at twice the supply frequency
- A 3Φ system can set up a rotating magnetic field in stationary windings. This is not possible with a 1Φ supply.
- For the same rating 3Φ machines are smaller, simpler in construction and have better operating characteristics than 1Φ machines
- To transmit the same amount of power over a fixed distance at a given voltage, the 3Φ system requires only 3/4th the weight of copper that is required by the 1Φ system
- The voltage regulation of a 3Φ transmission line is better than that of 1Φ line

Phase Sequence

The order in which the voltages in the three phases reach their maximum value

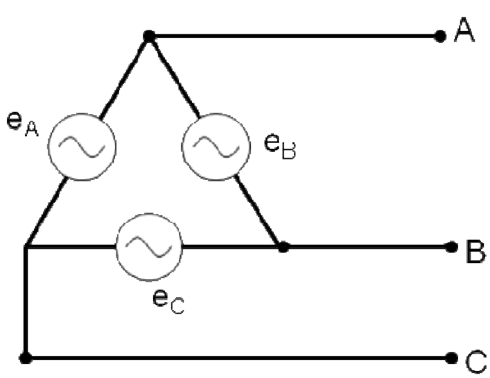


For the waveform shown in figure, phase A reaches the maximum value first, followed by phase B and then by phase C. hence the phase sequence is A-B-C.

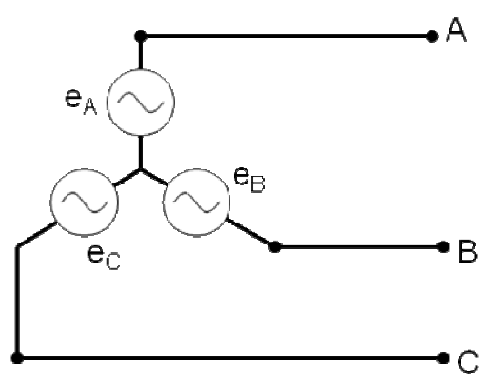
Balanced Supply

A supply is said to be balanced if all three voltages are equal in magnitude and displaced by 120^0

A three phase supply can be connected in two ways - Either in Delta connection or in Star connection as shown in the figure.



Delta Connection

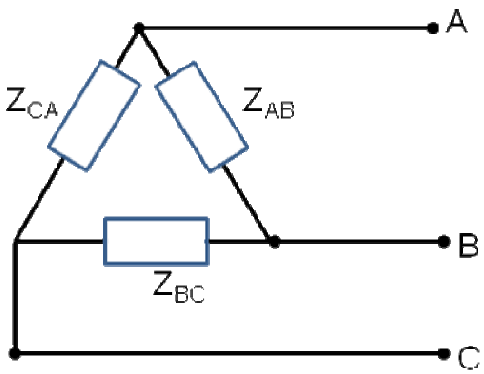


Star Connection

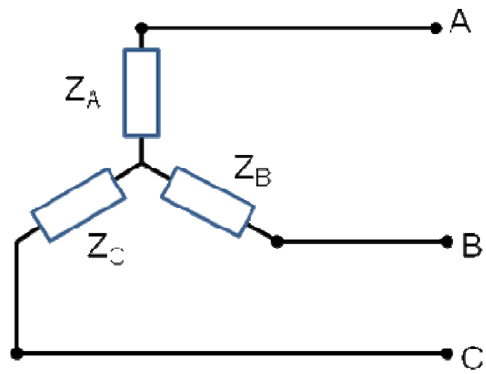
Balanced Load

A load is said to be balanced if the impedances in all three phases are equal in magnitude and phase

A three phase load can be connected in two ways - Either in Delta connection or in Star connection as shown in the figure.



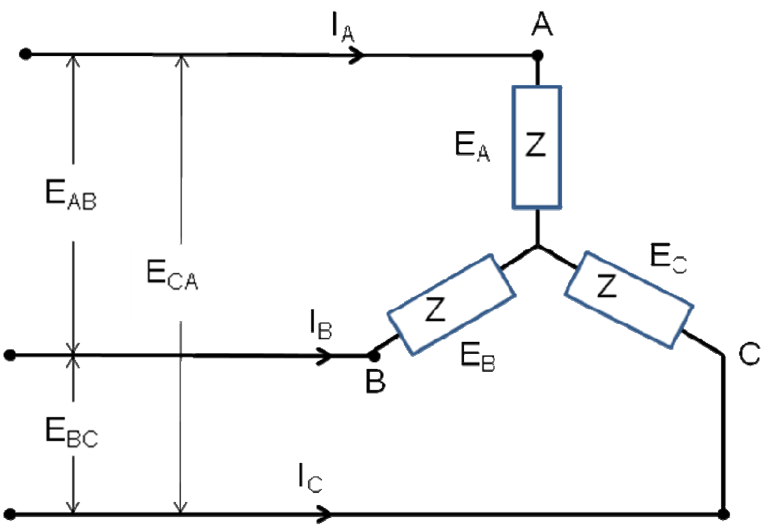
Delta Connection



Star Connection

Balanced Star Connected Load

A balanced star connected load is shown in the figure. A phase voltage is defined as voltage across any phase of the three phase load. The phase voltages shown in figure are E_A, E_B and E_C. A line voltage is defined as the voltage between any two lines. The line voltages shown in the figure are E_{AB}, E_{BC} and E_{CA}. The line currents are I_A, I_B and I_C. For a star connected load, the phase currents are same as the line currents.



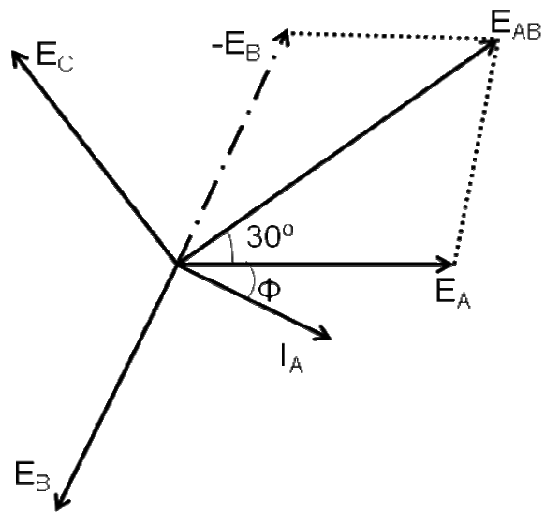
Using Kirchoff's voltage law, the line voltages can be written in terms of the phase voltages as shown below.

$$E_{AB} = E_A - E_B$$

$$E_{BC} = E_B - E_C$$

$$E_{CA} = E_C - E_A$$

The phasor diagram shows the three phase voltages and the line voltage E_{AB} drawn from E_A and $-E_B$ phasors. The phasor for current I_A is also shown. It is assumed that the load is inductive.



From the phasor diagram we see that the line voltage E_{AB} leads the phase voltage E_A by 30^o. The magnitude of the two voltages can be related as follows.

$$E_{AB} = 2E_A \cos 30^\circ = \sqrt{3}E_A$$

Hence for a balanced star connected load we can make the following conclusions.

$$E_l = \sqrt{3}E_{ph}$$

$$I_l = I_{ph}$$

Line voltage leads phase voltage by 30^0

Three phase Power

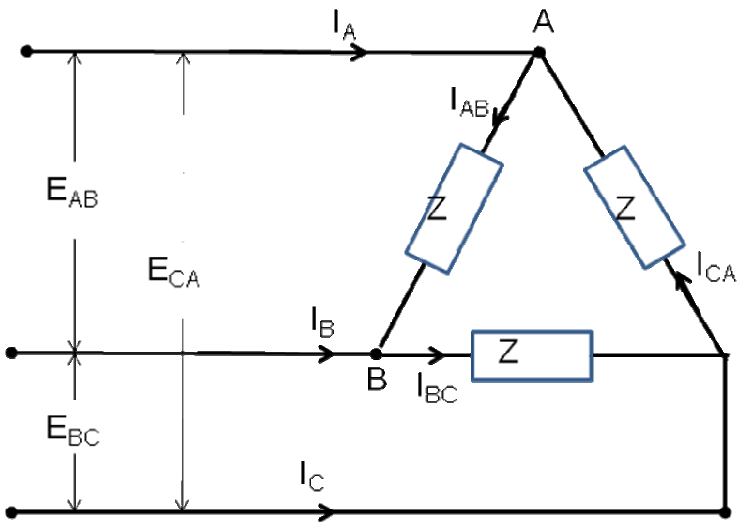
In a single phase circuit, the power is given by VIcosΦ. It can also be written as $V_{ph}I_{ph}\cos\Phi$. The power in a three circuit will be three times the power in a single phase circuit.

$$P = 3E_{ph}I_{ph}\cos\Phi$$

$$P = \sqrt{3}E_l I_l \cos\Phi$$

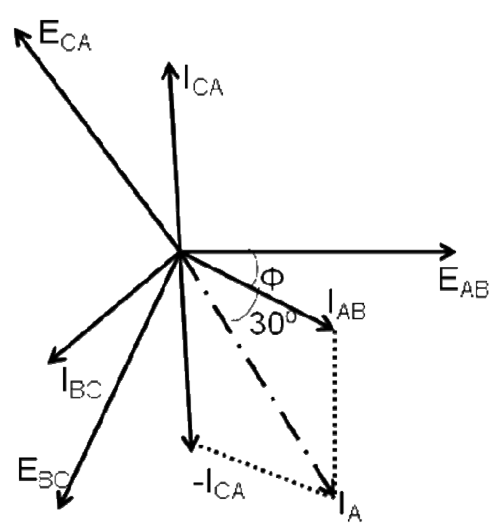
Balanced Delta Connected Load

A balanced delta connected load is shown in the figure. The phase currents are I_{AB}, I_{BC} and I_{CA}. The line currents are I_A, I_B and I_C. For a delta connected load, the phase voltages are same as the line voltages given by E_{AB}, E_{BC} and E_{CA}.



Using Kirchoff's current law, the line currents can be written in terms of the phase currents as shown below.

$$I_A = I_{AB} - I_{CA}$$
$$I_B = I_{BC} - I_{AB}$$
$$I_C = I_{CA} - I_{BC}$$



The phasor diagram shows the three voltages E_{AB}, E_{BC} and E_{CA} and the three phase currents I_{AB}, I_{BC} and I_{CA} lagging behind the respective phase voltages by an angle Φ. This is drawn by assuming that the load is inductive. From the phase currents I_{AB} and $-I_{CA}$, the line current I_A is drawn as shown in the figure.

From the phasor diagram we see that the line current I_A lags behind the phase phase current I_{AB} by 30^o. The magnitude of the two currents can be related as follows.

$$I_A = 2I_{AB}\cos 30^\circ = \sqrt{3}I_{AB}$$

Hence for a balanced delta connected load we can make the following conclusions.

$$I_l = \sqrt{3}I_{ph}$$

$$E_l = E_{ph}$$

Line current lags behind phase current by 30^0

Three phase Power

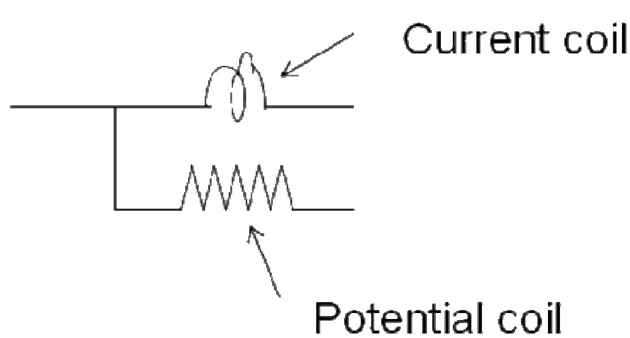
The three phase power for a delta connected load can be derived in the same way as that for a star connected load.

$$P = 3E_{ph}I_{ph}\cos\Phi$$

$$P = \sqrt{3}E_l I_l \cos\Phi$$

Measurement of power and power factor by two wattmeter method

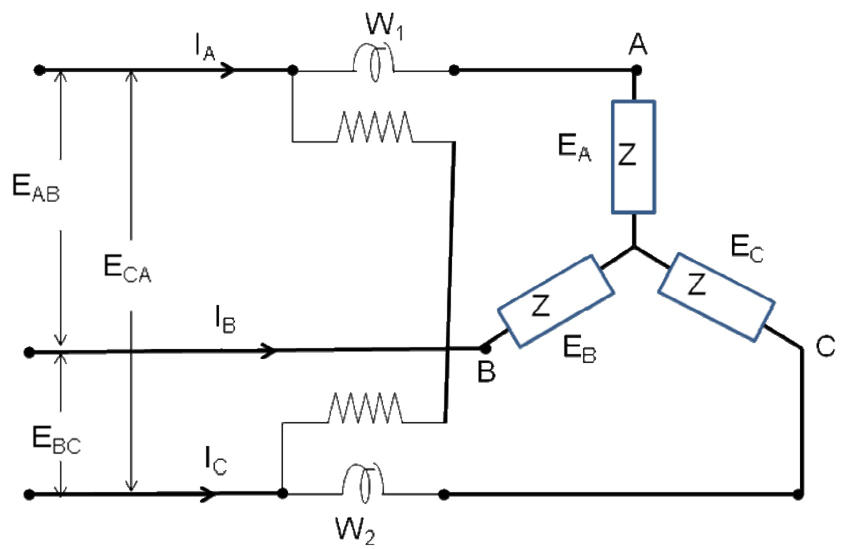
The power in a three phase circuit can be measured by connecting two wattmeters in any of the two phases of the three phase circuit. A wattmeter consists of a current coil and a potential coil as shown in the figure.



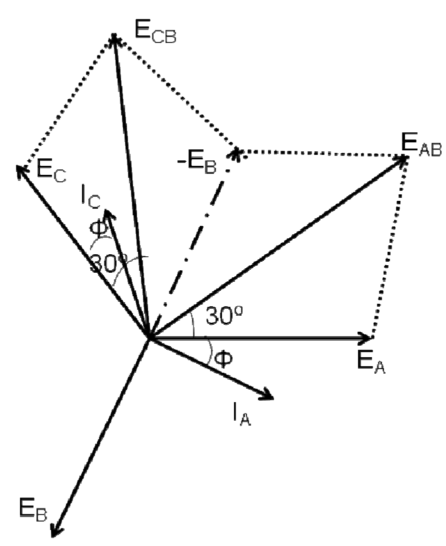
The wattmeter is connected in the circuit in such a way that the current coil is in series and carries the load current and the potential coil is connected in parallel across the load voltage. The wattmeter reading will then be equal to the product of the current carried by the current coil, the voltage across the potential coil and the cosine of the angle between the voltage and current.

The measurement of power is first given for a balanced star connected load and then for a balanced delta connected load.

(i) Balanced star connected load



The circuit shows a balanced star connected load for which the power is to be measured. Two wattmeter W_1 and W_2 are connected in phase A and phase C as shown in the figure.



The current coil of wattmeter W_1 carries the current I_A and its potential coil is connected across the voltage E_{AB}. A phasor diagram is drawn to determine the angle between I_A and E_{AB} as shown.
From the phasor diagram we determine that the angle between the phasors I_A and E_{AB} is (30+Φ). Hence the wattmeter reading W_1 is given by

$W_1=E_{AB}I_A\cos(30+\Phi)$

The current coil of wattmeter W_2 carries the current I_C and its potential coil is connected across the voltage E_{CB}. From the phasor diagram we determine that the angle between the phasors I_C and E_{CB} is (30-Φ). Hence the wattmeter reading W_2 is given by

$W_2=E_{CB}I_C\cos(30-\Phi)$

Line voltages $E_{AB}=E_{CB}=E_L$
And line currents $I_A=I_C=I_L$

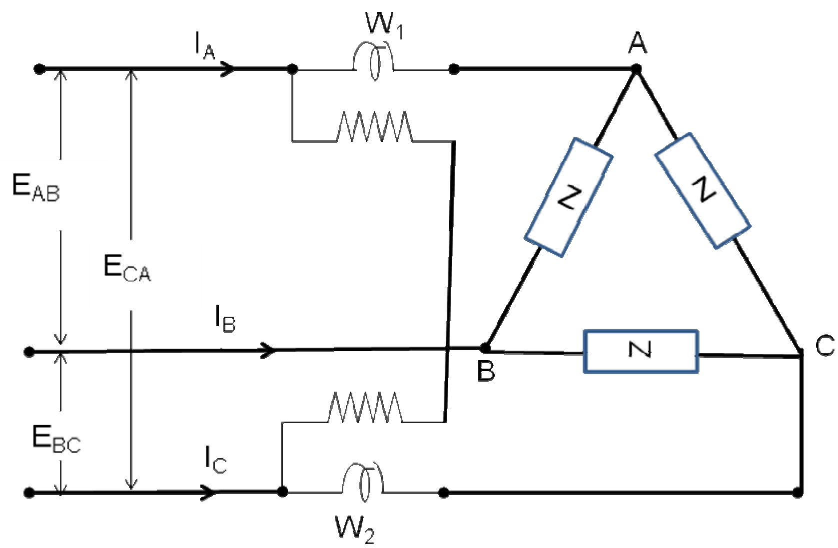
Hence

$$
\begin{aligned}
W_1 &= E_L I_L \cos(30+\Phi) \\
W_2 &= E_L I_L \cos(30-\Phi) \\
W_1+W_2 &= E_L I_L \cos(30+\Phi) + E_L I_L \cos(30-\Phi) \\
W_1+W_2 &= E_L I_L (2\cos 30^\circ \cos\Phi) \\
W_1+W_2 &= \sqrt{3} E_L I_L \cos\Phi
\end{aligned}
$$

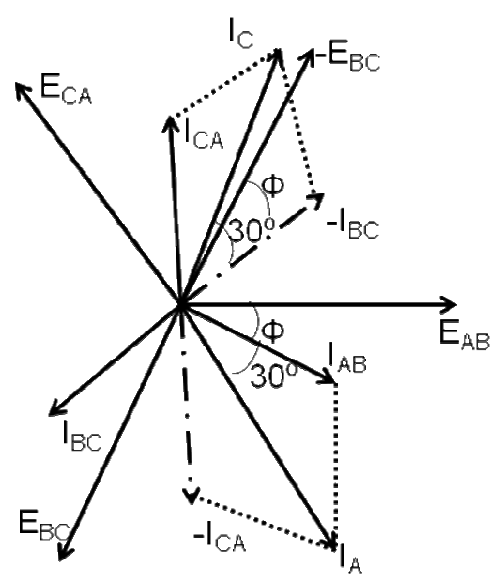
From the above equations we observe that the sum of the two wattmeter reading gives the three phase power.

(ii) Balanced delta connected load

The circuit shows a balanced delta connected load for which the power is to be measured. Two wattmeter W_1 and W_2 are connected in phase A and phase C as shown in the figure.



The current coil of wattmeter W_1 carries the current I_A and its potential coil is connected across the voltage E_{AB}. A phasor diagram is drawn to determine the angle between I_A and E_{AB} as shown.



From the phasor diagram we determine that the angle between the phasors I_A and E_{AB} is (30+Φ). Hence the wattmeter reading W_1 is given by

$W_1 = E_{AB} I_A \cos(30+\Phi)$

The current coil of wattmeter W_2 carries the current I_C and its potential coil is connected across the voltage E_{CB}. From the phasor diagram we determine that the angle between the phasors I_C and E_{CB} is (30-Φ). Hence the wattmeter reading W_2 is given by

$W_2=E_{CB}I_C\cos(30-\Phi)$

Line voltages $E_{AB}=E_{CB}=E_L$

And line currents $I_A=I_C=I_L$

Hence

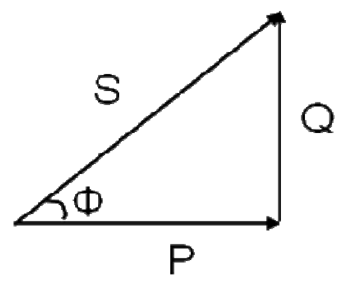
$$W_1 = E_L I_L \cos(30+\Phi)$$
$$W_2 = E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = E_L I_L \cos(30+\Phi) + E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = E_L I_L (2\cos 30^\circ \cos\Phi)$$
$$W_1 + W_2 = \sqrt{3} E_L I_L \cos\Phi$$

From the above equations we observe that the sum of the two wattmeter reading gives the three phase power.

Determination of Real power, Reactive power and Power factor

$$W_1 = E_L I_L \cos(30+\Phi)$$
$$W_2 = E_L I_L \cos(30-\Phi)$$
$$W_1 + W_2 = \sqrt{3} E_L I_L \cos\Phi$$
$$W_2 - W_1 = E_L I_L \sin\Phi$$
$$\tan\Phi = \sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)$$
$$\Phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\right]$$
$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_2 - W_1)$$
$$pf = \cos\Phi = \cos\left[\tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\right]\right]$$

The power factor can also be determined from the power triangle



From the power triangle,

$$P = W_1 + W_2$$
$$Q = \sqrt{3}(W_2 - W_1)$$
$$S = \sqrt{(W_1 + W_2)^2 + 3(W_2 - W_1)^2}$$
$$pf = \cos\Phi = \frac{P}{S} = \frac{W_1 + W_2}{\sqrt{(W_1 + W_2)^2 + 3(W_2 - W_1)^2}}$$

Wattmeter readings at different Power Factors

$$(i) upf$$
$$\Phi = 0^\circ$$
$$W_1 = E_L I_L \cos(30 + \Phi) = E_L I_L \cos(30) = \frac{\sqrt{3}}{2} E_L I_L$$
$$W_2 = E_L I_L \cos(30 - \Phi) = E_L I_L \cos(30) = \frac{\sqrt{3}}{2} E_L I_L$$
$$W_1 = W_2$$

$$(ii) pf = 0.866$$
$$\Phi = 30^\circ$$
$$W_1 = E_L I_L \cos(30 + \Phi) = E_L I_L \cos(30 + 30) = \frac{E_L I_L}{2}$$
$$W_2 = E_L I_L \cos(30 - \Phi) = E_L I_L \cos(30 - 30) = E_L I_L$$
$$W_2 = 2W_1$$

$$(iii)\ pf = 0.5$$
$$\Phi = 60^\circ$$
$$W_1 = E_L I_L \cos(30+\Phi) = E_L I_L \cos(30+60) = 0$$
$$W_2 = E_L I_L \cos(30-\Phi) = E_L I_L \cos(30-60) = \frac{\sqrt{3}}{2} E_L I_L$$

$$(iv)\ pf < 0.5$$
$$\Phi > 60^\circ$$
$$W_1 = E_L I_L \cos(30+\Phi) < 0$$
$$W_2 = E_L I_L \cos(30-\Phi) > 0$$

$$(v)\ pf = 0$$
$$\Phi = 90^\circ$$
$$W_1 = E_L I_L \cos(30+\Phi) = E_L I_L \cos(30+90) = \frac{E_L I_L}{2}$$
$$W_2 = E_L I_L \cos(30-\Phi) = E_L I_L \cos(30-90) = -\frac{E_L I_L}{2}$$
$$W_1 = -W_2$$

Problem 1

A balanced 3Φ delta connected load has per phase impedance of (25+j40)Ω. If 400V, 3Φ supply is connected to this load, find (i) phase current (ii) line current (iii) power supplied to the load.

$$Z_{ph} = \sqrt{25^2 + 40^2} = 47.17\Omega$$
$$\Phi = \tan^{-1}\left(\frac{40}{25}\right) = 60^\circ$$
$$Z_{ph} = 47.17\angle 60^\circ\,\Omega$$
$$E_L = 400V = E_{ph}$$
$$(i) I_{ph} = \frac{E_{ph}}{Z_{ph}} = \frac{400}{47.17\angle 60^\circ} = 8.48\angle -60^\circ A$$
$$(ii) I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 8.48 = 14.7\angle -90^\circ A$$
$$(iii) P = \sqrt{3} E_L I_L \cos\Phi = \sqrt{3} \times 400 \times 14.7 \times \cos 60^\circ$$
$$P = 5397.76W$$

Problem 2

Two wattmeter method is used to measure the power absorbed by a 3Φ induction motor. The wattmeter readings are 12.5kW and -4.8kW. Find (i) the power absorbed by the machine (ii) load power factor (iii) reactive power taken by the load.

$$W_1 = 12.5kW$$
$$W_2 = -4.8kW$$
$$(i) P = W_1 + W_2 = 12.5 - 4.8 = 7.7kW$$
$$(ii) \tan\Phi = \sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right) = \sqrt{3}\left(\frac{-4.8 - 12.5}{12.5 - 4.8}\right) = -3.89$$
$$\Phi = \tan^{-1}[-3.89] = -75.6^\circ$$
$$pf = \cos\Phi = \cos(-75.6^\circ) = 0.2487$$
$$(iii) Q = \sqrt{3}(W_2 - W_1) = \sqrt{3}(-4.8 - 12.5) = 29.96kVAR$$

Problem 3

Calculate the active and reactive components of each phase of a star connected 10kV, 3Φ alternator supplying 5MW at 0.8 pf.

$$E_L = 10kV$$
$$P = 5MW$$
$$pf = \cos\Phi = 0.8$$
$$\Phi = 36.87^\circ$$
$$P = \sqrt{3}E_L I_L \cos\Phi$$
$$I_L = \frac{P}{\sqrt{3}E_L \cos\Phi} = \frac{5\times10^6}{\sqrt{3}\times10\times10^3\times0.8} = 360.84A$$
$$P_{ph} = \frac{5\times10^6}{3} = 166.7MW$$
$$Q_{ph} = E_{ph}I_{ph}\sin\Phi = \frac{10\times10}{\sqrt{3}}\times360.8\times\sin 36.87^\circ = 1.25MVAR$$

Problem 4

A 3Φ load of three equal impedances connected in delta across a balanced 400V supply takes a line current of 10A at a power factor of 0.7 lagging. calculate (i) the phase current (ii) the total power (iii) the total reactive kVAR

$$E_L = 400V = E_{ph}$$
$$I_L = 10A$$
$$pf = \cos\Phi = 0.7 lag$$
$$\Phi = 45.57^\circ$$
$$(i) I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.8A$$
$$(ii) P = \sqrt{3} E_L I_L \cos\Phi = \sqrt{3} \times 400 \times 10 \times 0.7 = 4.84kW$$
$$(iii) Q = \sqrt{3} E_L I_L \sin\Phi = \sqrt{3} \times 400 \times 10 \times \sin 45.57^\circ = 4.94kVAR$$

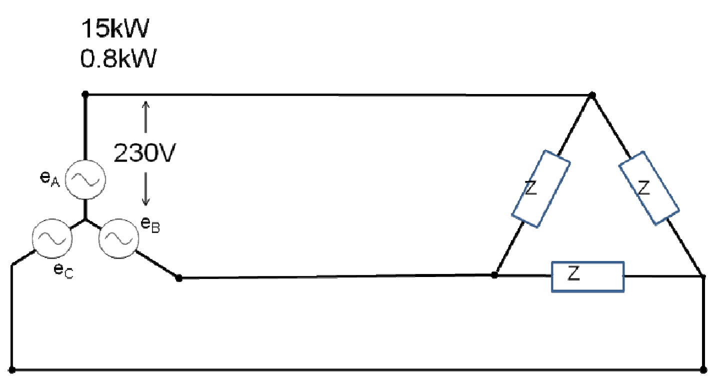
Problem 5

The power flowing in a 3Φ, 3 wire balanced load system is measured by two wattmeter method. The reading in wattmeter A is 750W and wattmeter B is 1500W. What is the power factor of the system?

$$W_1 = 750W$$
$$W_2 = 1500W$$
$$\Phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\right] = \tan^{-1}\left[\sqrt{3}\left(\frac{1500 - 750}{750 + 1500}\right)\right]$$
$$\Phi = 30^\circ$$
$$pf = \cos\Phi = \cos 30^\circ = 0.866$$

Problem 6

A 3Φ star connected supply with a phase voltage of 230V is supplying a balanced delta connected load. The load draws 15kW at 0.8pf lagging. Find the line currents and the current in each phase of the load. What is the load impedance per phase.



Alternator

$$E_{ph} = 230V$$

$$E_L = \sqrt{3} \times 230V = 398.37V$$

$$P = 15kW$$

$$pf = \cos\Phi = 0.8 lagging$$

$$I_L = \frac{P}{\sqrt{3} E_L \cos\Phi} = 27.17A$$

Load

$$E_{ph} = E_L = 398.37V$$

$$I_L = 27.17A$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = 15.68A$$

$$Z_{ph} = \frac{E_{ph}}{I_{ph}} = 25.4\Omega$$

SINGLE PHASE AC CIRCUITS

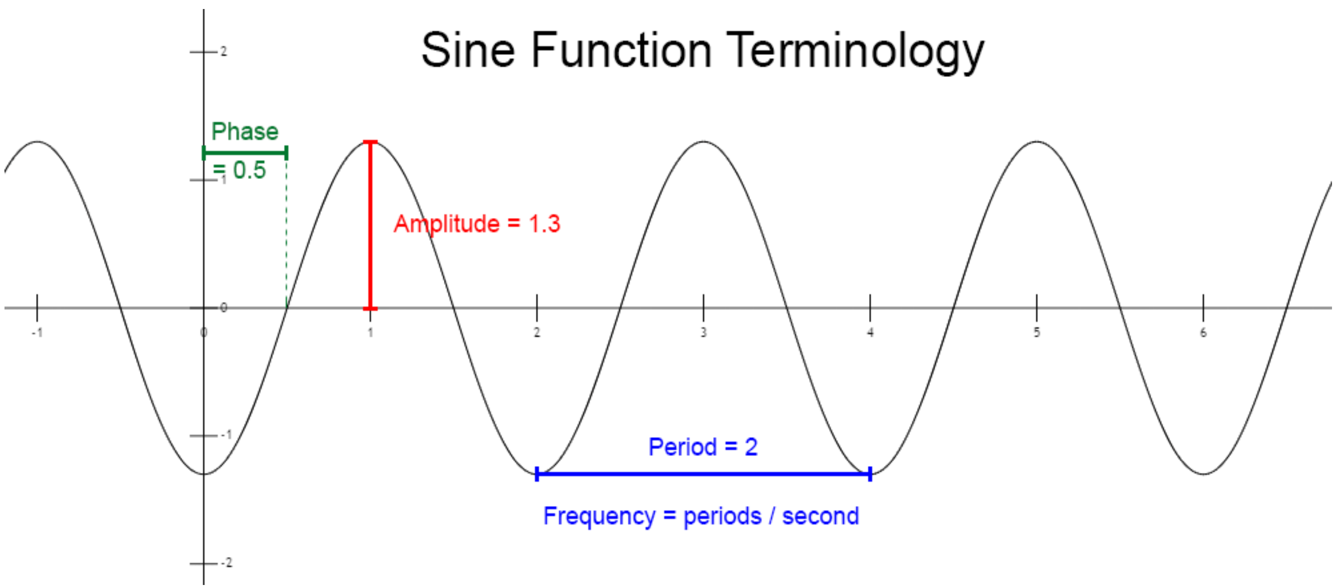
UNIT -3

Syllabus ……

- RMS and Average Values
- Form factor for different periodic wave forms
- Steady state analysis of R,L,C with sinusoidal excitation
- Concept of self and Mutual Inductances
- Dot convention
- Co-efficient of coupling
- Series Circuit Analysis with mutual inductance
- Resonance in series and parallel circuits
- Concept of Band width and Q Factor

SINGLE PHASE AC CIRCUITS

Till now, we have discussed about DC supply and DC Circuits. But, 90% of Electrical energy used now a days is AC in nature. Electrical supply used for Commercial purposes is alternative.



An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time.

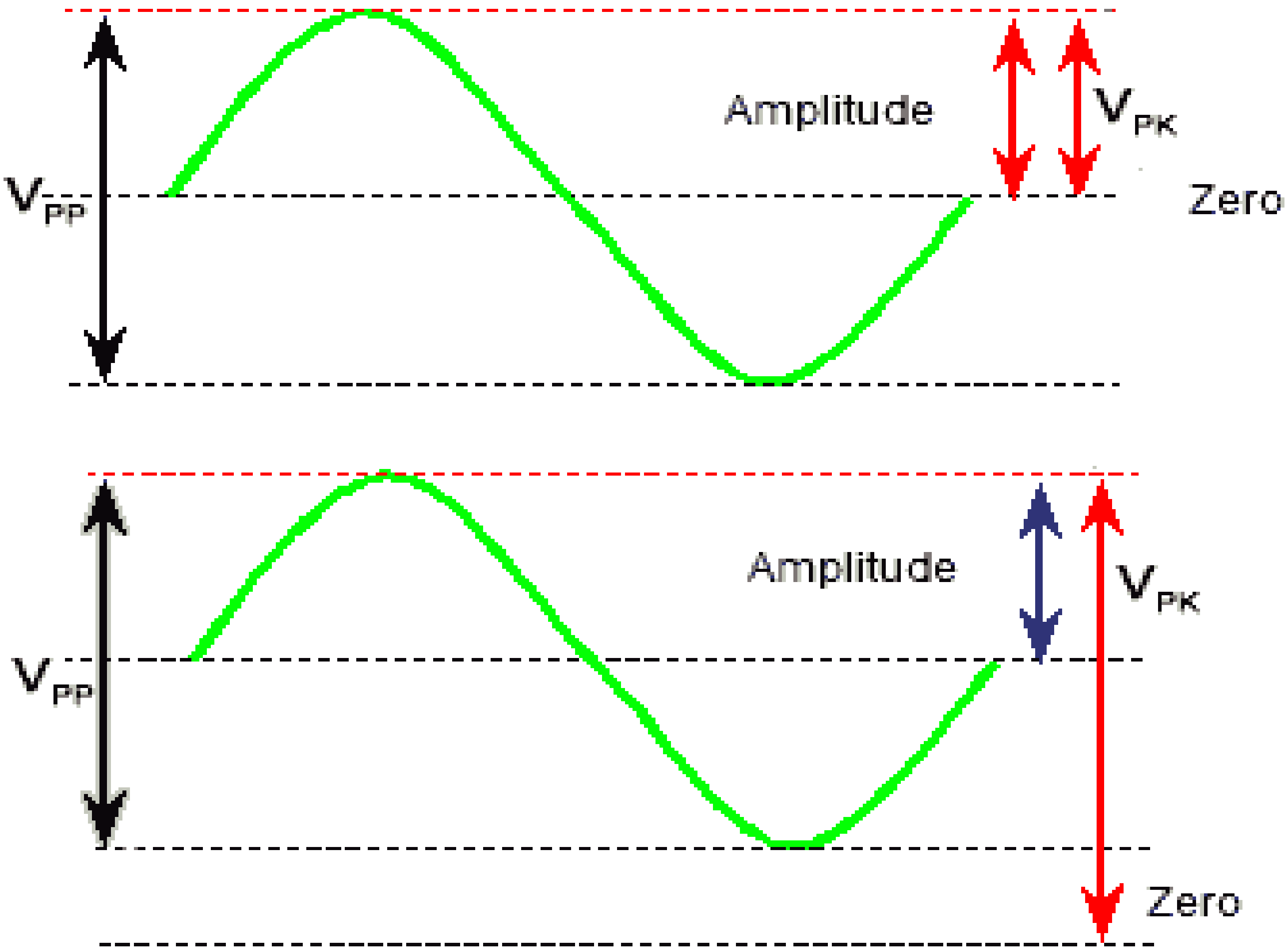
BASIC TERMINOLOGY

- WAVE FORM
- CYCLE & TIME PERIOD
- FREQUENCY & ANGULAR FREQENCY
- AMPLITUDE
- PHASE & PHASE DIFFERENCE
- PEAK & PEAK TO PEAK
- INSTANTANEOUS VALUE
- AVERAGE VALUE
- ROOT MEAN SQUARE(RMS)
- FORM & PEAK FACTORS

TERMINOLOGY	DEFINITION
Wave Form	A wave form is a graph in which the instantaneous value of any quantity is plotted against time
Cycle	One complete set of positive and negative values of an alternating quantity in termed as cycle
Frequency (f)	The number of cycles per second of an alternating quantity is known as frequency
Angular frequency ($\omega = 2\Pi f$)	Angular frequency is defined as the number of radians covered in one second(i.e the angle covered by the rotating coil). The unit of angular frequency is rad/sec.
Time period ($T = 1/f$)	The time taken by an alternating quantity to complete one complete cycle as called as Time period
Amplitude (A)	The AMPLITUDE of a sine wave is the maximum vertical distance reached, in either direction from the center line of the wave.
Phase	The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference ..
Phase difference	When two alternating quantities of the same frequency have different zero points, they are said to have a phase difference. The angle between the zero points is the angle of phase difference

TERMINOLOGY	DEFINITION
Peak value (pk)	Peak is the maximum value, either positive (pk+) or negative (pk-), that a waveform attains. Peak values can be expressed for V,I & P .
Peak to peak (pk-pk)	Peak-to-peak is the difference between the maximum positive and the maximum negative amplitudes of a waveform, as shown below. If there is no direct current (DC) component in an alternating current (AC) wave, then the pk-pk amplitude is twice the peak amplitude.
Instantaneous Value	This is the value (voltage or current) of a wave at any particular instant. often chosen to coincide with some other event. E.g. The instantaneous value of a sine wave one quarter of the way through the cycle will be equal to the peak value.
Average	The average of an alternating quantity is defined as the athematic mean of all the values over one complete cycle.
RMS	The RMS value of a set of values (or a continuous-time waveform) is the square root of the arithmetic mean of the squares of the values, or the square of the function that defines the continuous waveform.
Form Factor	The ratio of RMS value to Average value is called Form factor.
Peak Factor (Crest factor)	It is defined as the ratio of Maximum value to RMS value of given alternating quantity

**** Difference between peak & Amplitude



PEAK & AMPLITUDE

Example 1 : An alternating current i is given by $i = 141.4 \sin 314t$

Find i) The maximum value

ii) Frequency

iii) Time Period

iv) The instantaneous value when t=3ms

Solution

i) The maximum value $i = 141.4 \sin 314t$

Maximum value I_m=141.4 V

ii) Frequency $\omega = 2\prod f3 = 14$ rad/sec

$f = \omega / 2\prod = 50$ Hz

iii) Time Period $T=1/f = 0.02$ sec

iv) The instantaneous value when t=3ms

$i = 141.4 \sin(314 \times 0.003) = 114.35$A

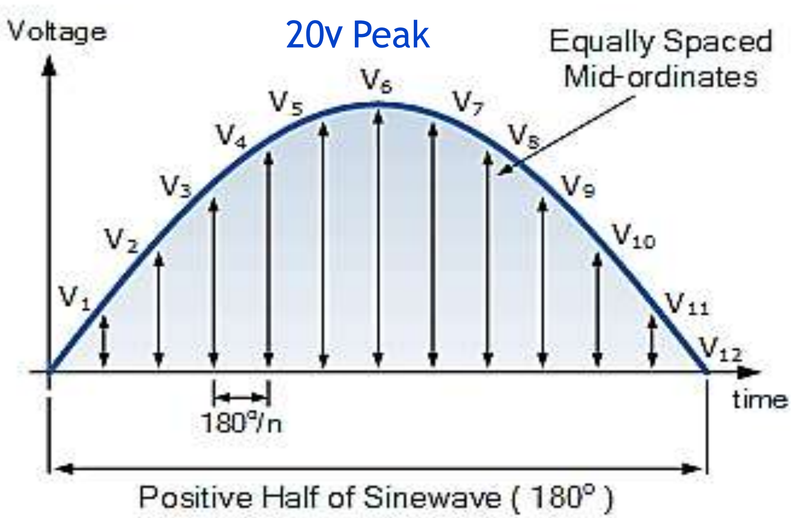
AVERAGE VALUE

Definition : The average of an alternating quantity is defined as the athematic mean of all the values over one complete cycle.

Methods of finding Average :

1. Graphical Method
2. Analytical Method

The Graphical Method of finding Average value of sine wave



$$V_{AV} = \frac{\text{sum of all the mid-ordinates}}{\text{number of mid-ordinates}}$$

$$V_{AV} = \frac{6.2+11.8+16.2+19+20+19+16.2+11.8+6.2+0}{10}$$

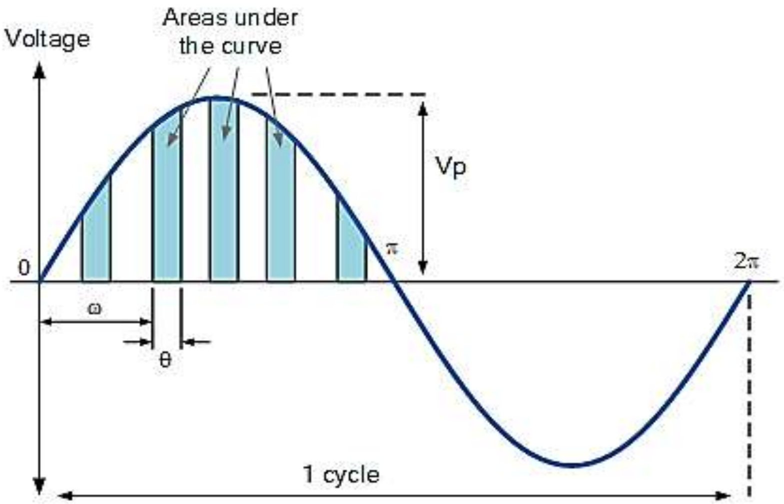
$$V_{AV} = \frac{126.4}{10} = 12.64\,\text{Volts}$$

Voltage	6.2V	11.8V	16.2V	19.0V	20.0V	19.0V	16.2V	11.8V	6.2V	0V
Angle	18°	36°	54°	72°	90°	108°	126°	144°	162°	180°

CONCEPT OF AVERAGE VALUE

Analytical Method of Finding Average Value of sine wave

The average value can be taken mathematically by taking the approximation of the area under the curve at various intervals to the distance or length of the base and this can be done using triangles or rectangles as shown.



$$\text{Area} = \int_0^{\pi} V_P \sin(\omega t)\, dt$$

$$V_{AVE} = \frac{1}{\pi}\int_0^{\pi} V_P \sin\theta \, d\theta$$

$$V_{AVE} = \frac{V_P}{\pi}\left(-\cos\theta\right)_0^{\pi}$$

$$= \frac{2V_P}{\pi} = \frac{2}{\pi}V_P = 0.637\, V_P$$

$$V_{AVE} = \frac{2V_P}{\pi} = 0.637V_P$$

For peak voltage of 20v
Average value =>

$V_{AV} = V_{pk} \times 0.637 = 20 \times 0.637 = 12.74$ volts

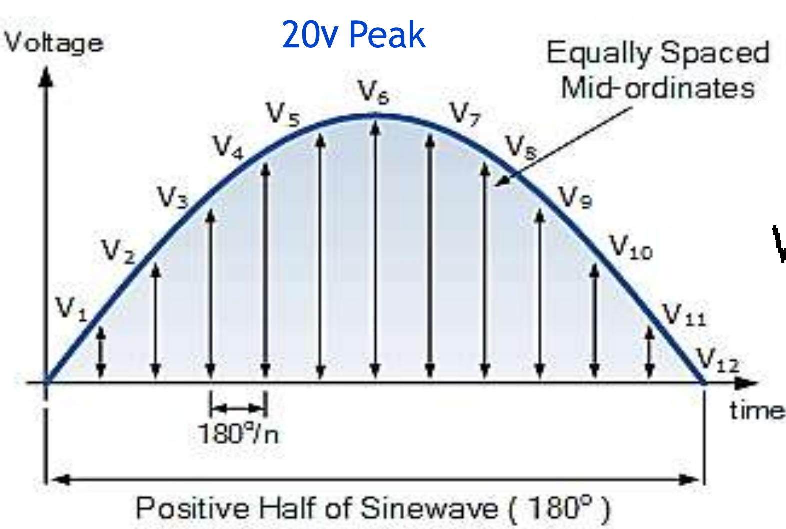
RMS/MEAN VALUE

Definition : The term "RMS" stands for "Root-Mean-Squared". Most books define this as the "amount of AC power that produces the same heating effect as an equivalent DC power", or something similar along these lines, but an RMS value is more than just that. The RMS value is the square root of the mean (average) value of the squared function of the instantaneous values. The symbols used for defining an RMS value are V_{RMS} or I_{RMS}.

Methods of finding Average :

1. **Graphical Method**
2. **Analytical Method**

The Graphical Method of finding RMS of sine wave



$$V_{RMS} = \sqrt{\frac{\text{sum of mid-ordinate (voltages)}^2}{\text{number of mid-ordinates}}}$$

$$V_{RMS} = \sqrt{\frac{6.2^2+11.8^2+16.2^2+19^2+20^2+19^2+16.2^2+11.8^2+6.2^2+0^2}{10}}$$

$$V_{RMS} = \sqrt{\frac{2000}{10}} = \sqrt{200} = 14.14\,\text{Volts}$$

Analytical Method of Finding RMS Value of sine wave

A periodic sinusoidal voltage is constant and can be defined as $V_{(t)} = V_m.\cos(\omega t)$ with a period of T. Then we can calculate the **root-mean-square** (rms) value of a sinusoidal voltage ($V_{(t)}$) as

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t)\, dt}$$

Integrating through with limits taken from 0 to 360º or "T", the period gives:

$$V_{RMS} = \sqrt{\frac{V_m^2}{2T}\left[t + \frac{1}{2\omega}\sin(2\omega t)\right]_0^T}$$

$$\boxed{V_{RMS} = V_m \frac{1}{\sqrt{2}} = V_m \times 0.7071}$$

For peak voltage (V_{pk}) of the waveform as 20 Volts using the analytical method just defined we can calculate the RMS voltage as being:

$$\boxed{V_{RMS} = V_{pk} \text{ X } 0.7071 = 20 \text{ X } 0.7071 = 14.14V}$$

COMPARISION BETWEEN AVERGAE & RMS VALUE

Mode of Classification	Average Value	RMS Value
Definition	The average of an alternating quantity is defined as the athematic mean of all the values over one complete cycle.	The RMS value is the square root of the mean (average) value of the squared function of instantaneous values.
Representation	V_{AVG} or I_{AVG}	V_{RMS} OR I_{RMS}
Formulae	$V_{AVG} = \frac{1}{T}\int_{O}^{T} V(t)dt$ $I_{AVG} = \frac{1}{T}t\int_{O}^{T} I(t)\,dt$	$V_{RMS} = \sqrt{\frac{1}{T}\int_{O}^{T}[V(t)]^2\,dt}$ $I_{RMS} = \sqrt{\frac{1}{T}\int_{O}^{T}[I(t)]^2\,dt}$
Properties	Average value of a periodic wave form over complete cycle is zero. Hence for symmetric periodic wave forms average value is calculated for half cycle only.	RMS value can be calculated conveniently even for a periodic wave form

Example 1 : A periodic current has values for equal time intervals changing suddenly from one value to next as 0,2,4,6,8,10, 8,6,4,2,0,-2, -4,-6,-8, -10,-8 . Calculate

1. Average value
2. RMS
3. Form Factor
4. Peak Factor

Solution

1. Average value = $\frac{(0+2+4+6+8+10+8+6+4+2)}{10}$ = **5A**

2. RMS = $\sqrt{\frac{0^2+2^2+4^2+6^2+8^2+10^2+ 8^2+6^2+4^2+2^2+0^2+(-2)^2+(-4)^2+(-6)^2+(-8)^2+(-10)^2+(-8)^2 +(-6)^2+(-4)^2+(-2)^2+(0)^2}{20}}$

= 5.8309A

3. Form Factor = $\frac{RMS}{AVERAGE}$ = $\frac{5.8309}{5}$ **= 1.1661**

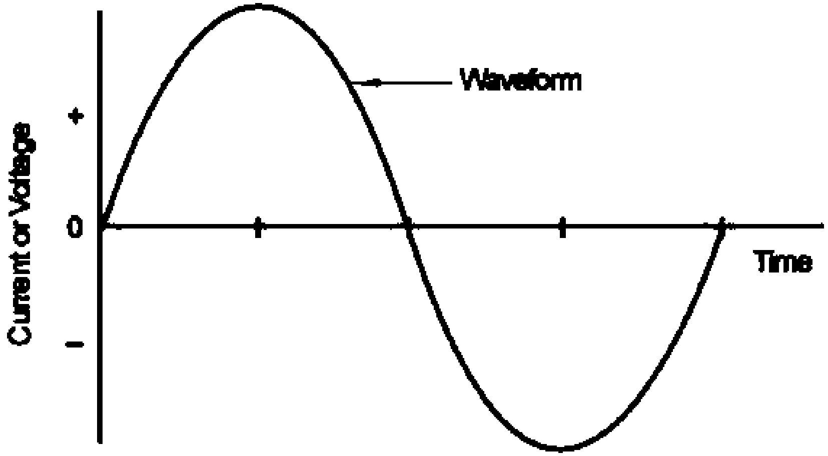
4. Peak Factor = $\frac{MAXIMUM\ VALUE}{RMS}$ = $\frac{10}{5.8309}$ = **1.715**

Example 1 : A For a Sine wave of peak vm, calculate

1. Average value
2. RMS
3. Form Factor
4. Peak Factor

Answer : Average Value = 0.637Vm
RMS = 0.707Vm
Form Factor = 1.11
Peak Factor =1.414

Solution



1. Average value

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta = \frac{V_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta = \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1] = \frac{2V_m}{\pi} = 0.637\ V_m$$

PROBLEMS ON AVERAGE & RMS VALUE ...

2. RMS

$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta} = \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

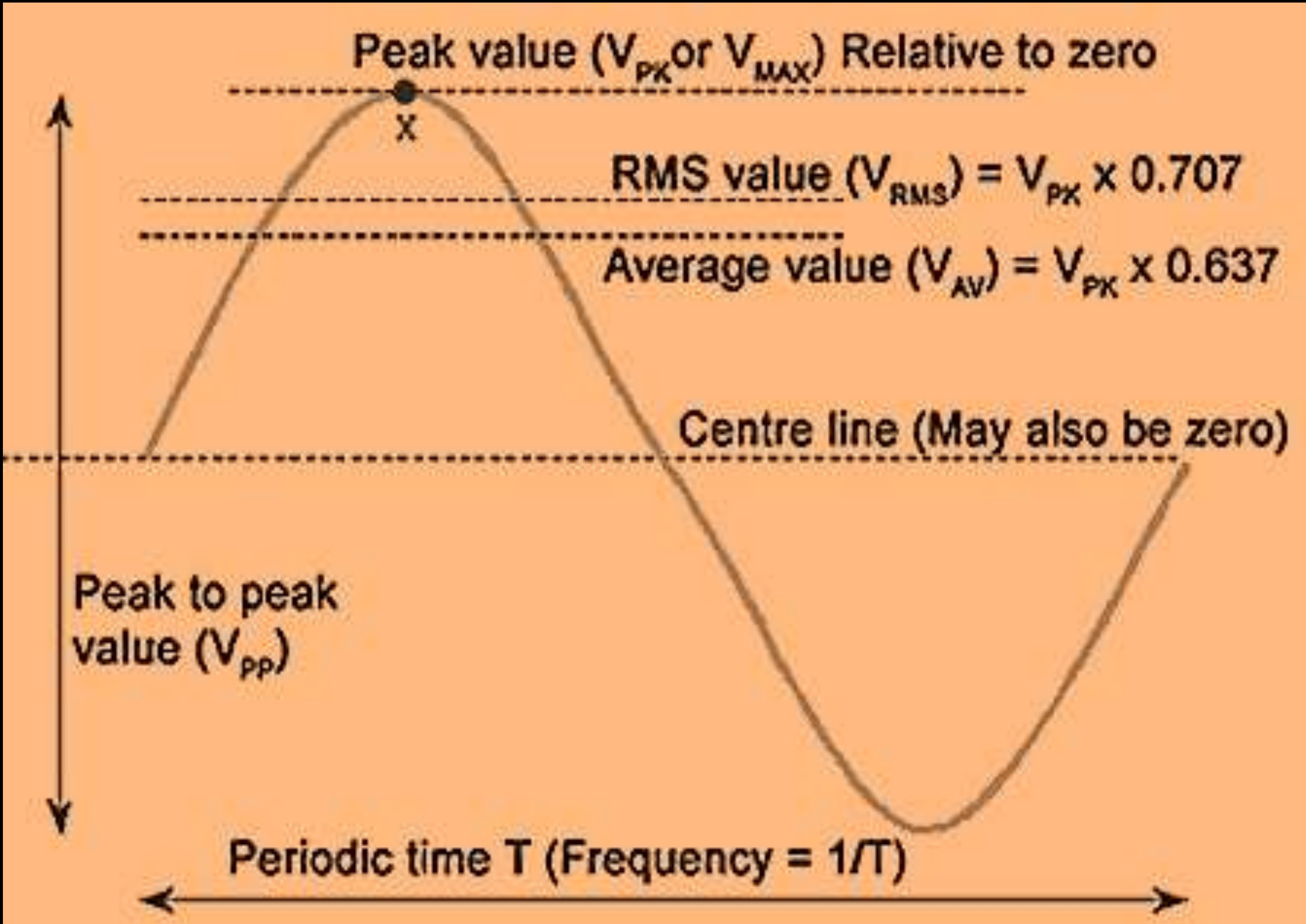
$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{2\pi}{2} - 0 - 0 + 0\right]} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

3.Form Factor

$$\text{Form Factor} = \frac{RMS}{AVERAGE} = \frac{0.707Vm}{0.637Vm} = \mathbf{1.11}$$

4.Peak Factor

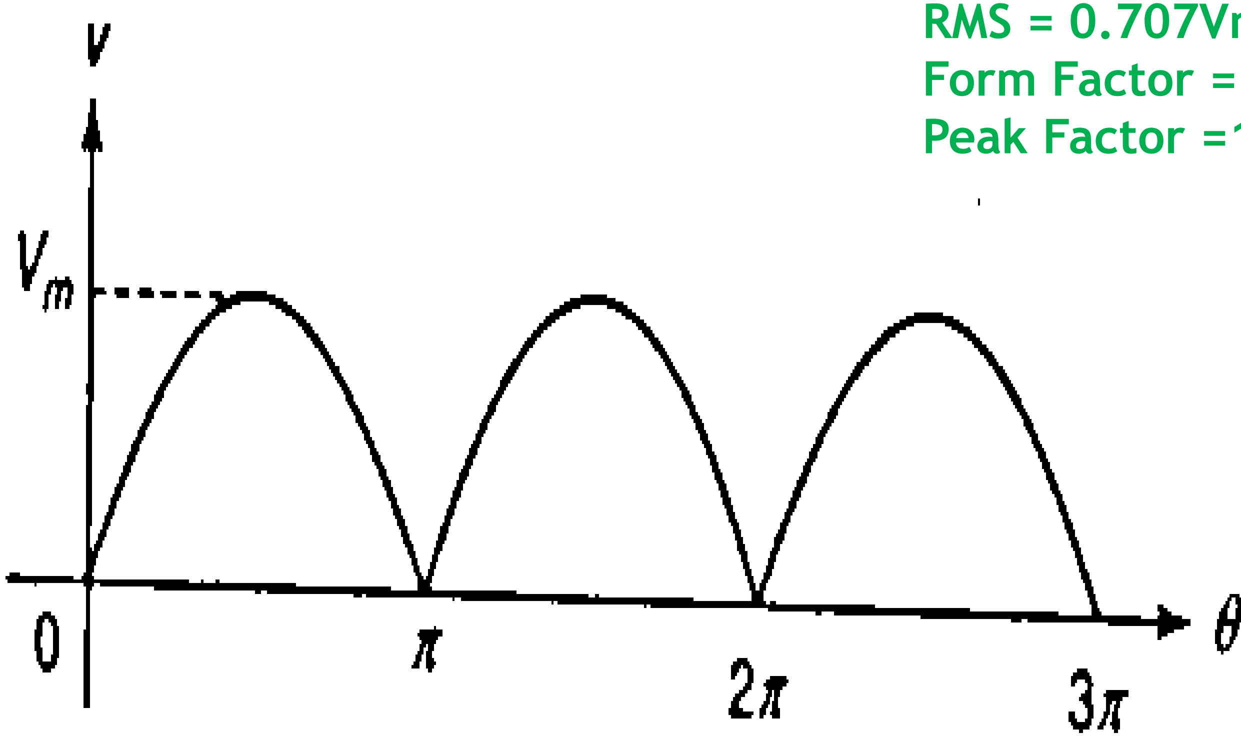
$$\text{Peak Factor} = \frac{MAXIMUM\ VALUE}{RMS} = \frac{Vm}{0.707Vm} = \mathbf{1.414}$$


Peak value (V_{PK} or V_{MAX}) Relative to zero
X
RMS value (V_{RMS}) = V_{PK} x 0.707
Average value (V_{AV}) = V_{PK} x 0.637
Centre line (May also be zero)
Peak to peak value (V_{PP})
Periodic time T (Frequency = 1/T)

Example 2 : For the full wave rectified wave form shown, calculate

1. Average value
2. RMS
3. Form Factor
4. Peak Factor

Answer : Average Value = 0.637Vm
RMS = 0.707Vm
Form Factor = 1.11
Peak Factor =1.414



$v = V_m \sin\theta \qquad 0 < \theta < \pi$

Solution

1. Average value

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta = \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1] = 0.637\, V_m$$

2. RMS

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\, d\theta} = \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta} = \sqrt{\frac{V_m^2}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}} = \sqrt{\frac{V_m^2}{\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]} = 0.707\, V_m$$

3.Form Factor

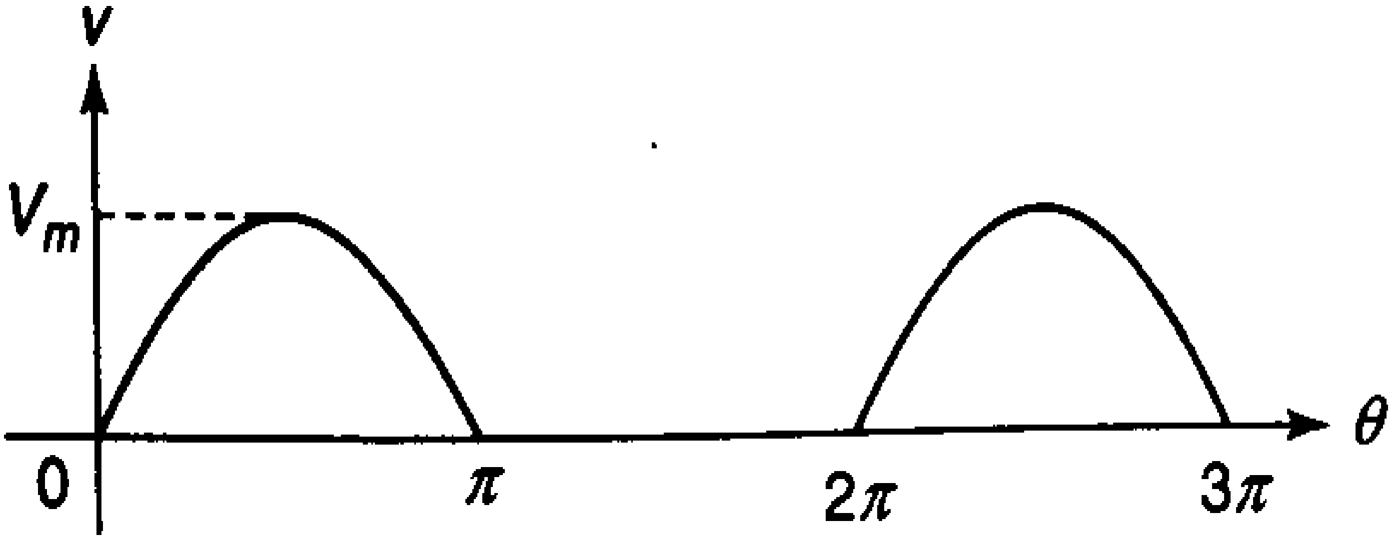
$$\text{Form Factor} = \frac{RMS}{AVERAGE} = \frac{0.707Vm}{0.637Vm} = 1.11$$

4.Peak Factor

$$\text{Peak Factor} = \frac{MAXIMUM\ VALUE}{RMS} = \frac{Vm}{0.707Vm} = 1.414$$

Example 3: For the wave shown, calculate

1. Average value
2. RMS
3. Form Factor
4. Peak Factor



$v = V_m \sin\theta \qquad 0 < \theta < \pi$

$= 0 \qquad \pi < \theta < 2\pi$

Answer : Average Value = 0.318Vm
RMS = 0.5Vm
Form Factor = 1.11
Peak Factor =1.414

Solution

1. Average value

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta\, d\theta = \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{V_m}{2\pi}[1+1] = 0.318\, V_m$$

2. RMS

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}} = \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]} = 0.5\, V_m$$

3.Form Factor

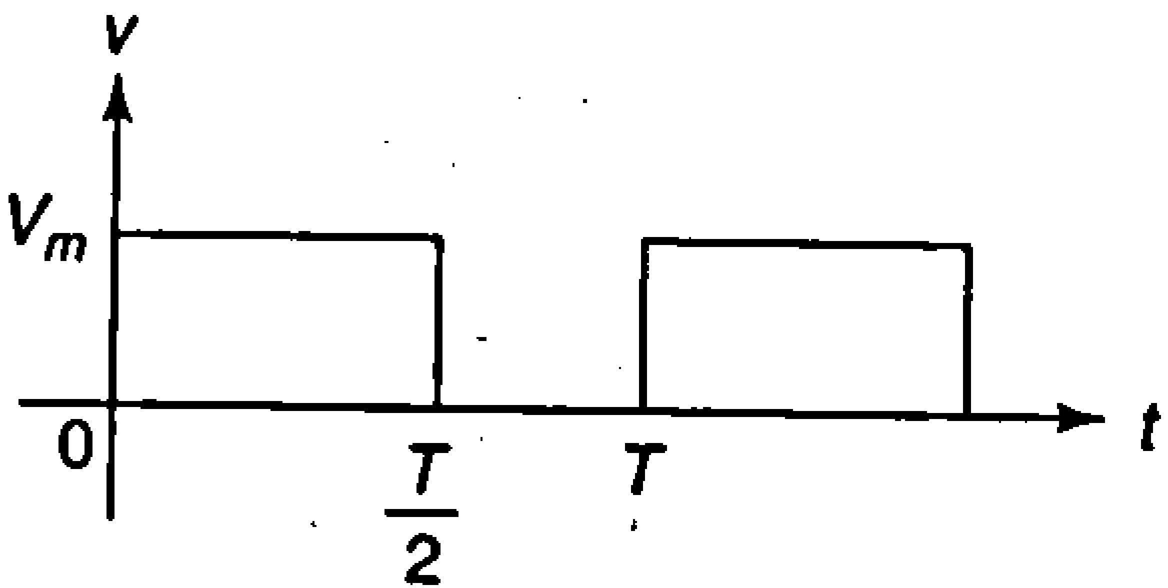
$$\text{Form Factor} = \frac{RMS}{AVERAGE} = \frac{0.5Vm}{0.318Vm} = 1.571$$

4.Peak Factor

$$\text{Peak Factor} = \frac{MAXIMUM\ VALUE}{RMS} = \frac{Vm}{0.2Vm} = 2$$

Example 4: For the wave shown, calculate

1. Average value
2. RMS
3. Form Factor
4. Peak Factor



Answer : Average Value = 0.5Vm

RMS = 0.707Vm

Form Factor = 1.414

Peak Factor =1.414

$$v = V_m \quad 0 < t < \frac{T}{2}$$

$$= 0 \quad \frac{T}{2} < t < T$$

Solution

1. Average value

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{T}\left[\int_0^{\frac{T}{2}} V_m\,dt + \int_{\frac{T}{2}}^{T} 0\,dt\right] = \frac{1}{T}\int_0^{\frac{T}{2}} V_m\,dt$$

$$= \frac{V_m}{T}[t]_0^{\frac{T}{2}} = \frac{V_m}{T}\cdot\frac{T}{2} = 0.5\,V_m$$

2. RMS

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} V_m^2\,dt} = \sqrt{\frac{V_m^2}{T}[t]_0^{\frac{T}{2}}}$$

$$= \sqrt{\frac{V_m^2}{T}\cdot\frac{T}{2}} = 0.707\,V_m$$

3.Form Factor

$$\text{Form Factor} = \frac{RMS}{AVERAGE} = \frac{0.707Vm}{0.5Vm} = 1.414$$

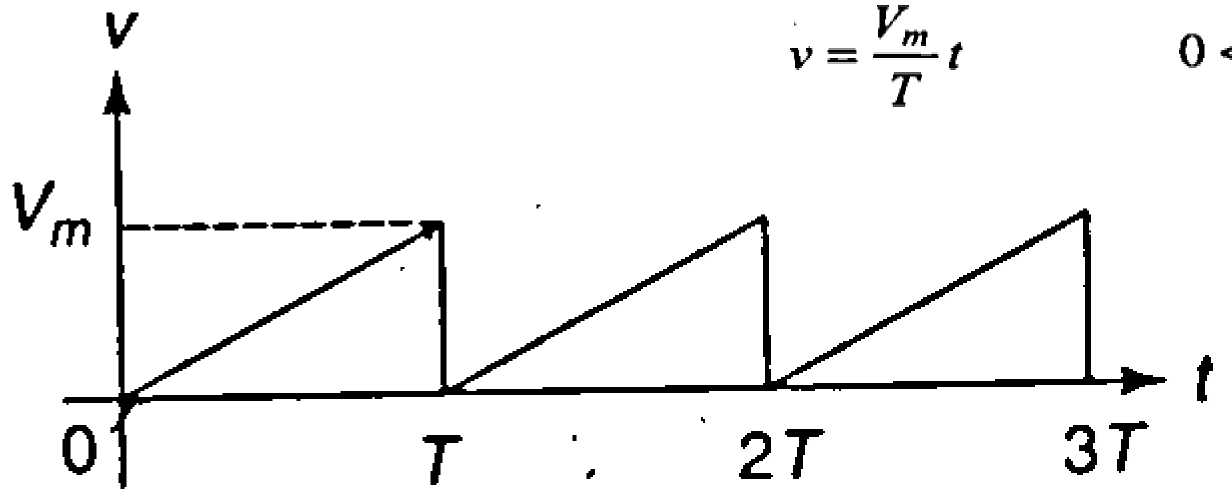
4.Peak Factor

$$\text{Peak Factor} = \frac{MAXIMUM\ VALUE}{RMS} = \frac{Vm}{0.707Vm} = 1.414$$

Example 5: For the wave shown, calculate

1. Average value
2. RMS
3. Form Factor
4. Peak Factor

$$v = \frac{V_m}{T} t \qquad 0 < t < T$$



Answer : Average Value = 0.5Vm
RMS = 0.577Vm
Form Factor = 1.154
Peak Factor =1.733

Solution

1. Average value

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{T}\int_0^T \frac{V_m}{T} t\,dt = \frac{V_m}{T^2}\left[\frac{t^2}{2}\right]_0^T = \frac{V_m}{T^2}\cdot\frac{T^2}{2} = 0.5\,V_m$$

2. RMS

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \sqrt{\frac{1}{T}\int_0^T \frac{V_m^2}{T^2}\cdot t^2\,dt}$$

$$= \sqrt{\frac{V_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T} = \sqrt{\frac{V_m^2}{T^3}\left[\frac{T^3}{3}\right]} = 0.577\,V_m$$

3.Form Factor

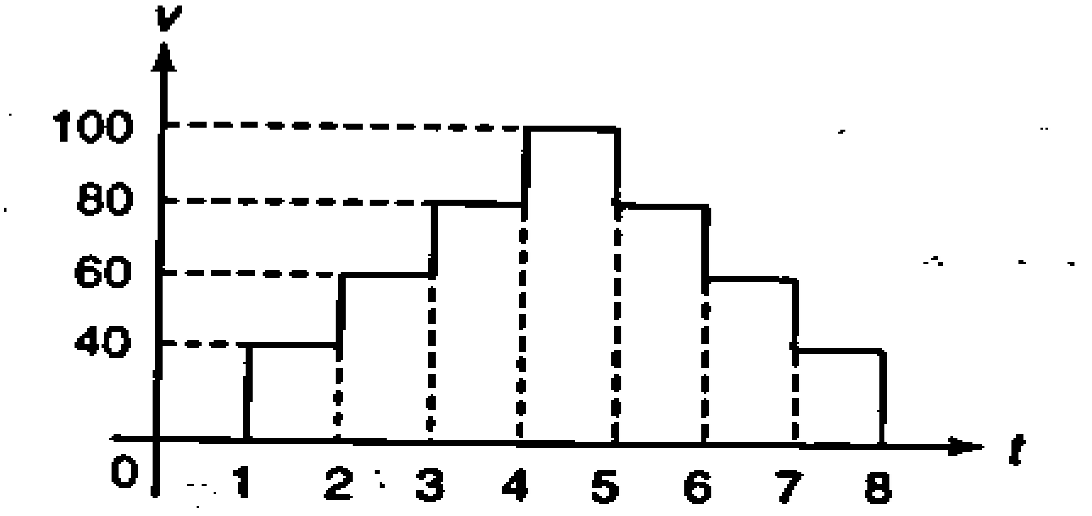
$$\text{Form Factor} = \frac{RMS}{AVERAGE} = \frac{0.577Vm}{0.5Vm} = 1.154$$

4.Peak Factor

$$\text{Peak Factor} = \frac{MAXIMUM\ VALUE}{RMS} = \frac{Vm}{0.577Vm} = 1.733$$

Example 6: For the wave shown, calculate

1. Average value
2. RMS
3. Form Factor
4. Peak Factor



Answer : Average Value = 57.5 V

RMS = 64.42V

Form Factor = 1.12

Peak Factor =1.552

PROBLEMS ON AVERAGE & RMS VALUE …

Solution

1. Average value

$$V_{AVG} = \frac{0+40+60+80+100+80+60+40}{8} = 57.5\ V$$

2. RMS

$$V_{RMS} = \sqrt{\frac{0^2+(40)^2+(60)^2+(80)^2+(100)^2+(80)^2+(60)^2+(40)^2}{8}} = 64.42\ V$$

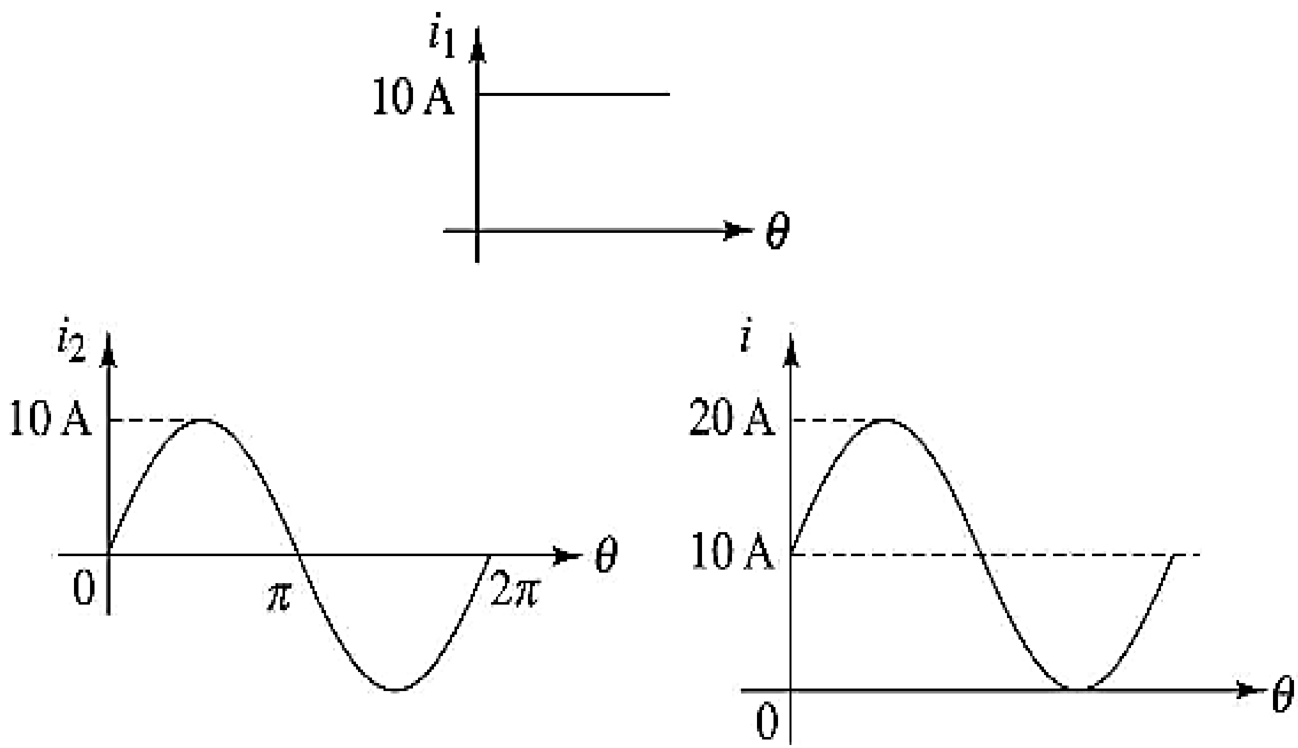
3.Form Factor

$$\text{Form Factor} = \frac{RMS}{AVERAGE} = \frac{64.42}{57.5} = 1.12$$

4.Peak Factor

$$\text{Peak Factor} = \frac{MAXIMUM\ VALUE}{RMS} = \frac{100}{64.42} = 1.552$$

Example 7: Find the effective value of the resultant current which carries simultaneously a direct current of 10A and a sinusoidally alternating current with a peak value of 10A.



Ans : 12.25A

Solution

$$i = 10 + 10 \sin \theta$$

$$I_{eff} = I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (10+10\sin\theta)^2\,d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (100+200\sin\theta+100\sin^2\theta)\,d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} (1+2\sin\theta+\sin^2\theta)\,d\theta}$$

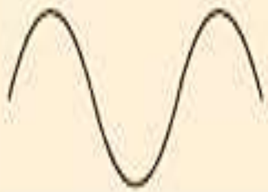
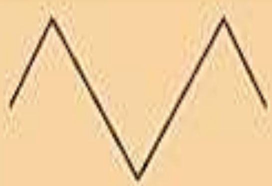
$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} \left[1+2\sin\theta+\left(\frac{1-\cos 2\theta}{2}\right)\right]d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\left[\theta-2\cos\theta+\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{100}{2\pi}\left[2\pi-2\cos 2\pi+\frac{2\pi}{2}-\frac{\sin 4\pi}{4}-0+2\cos 0-0+\frac{\sin 0}{4}\right]}$$

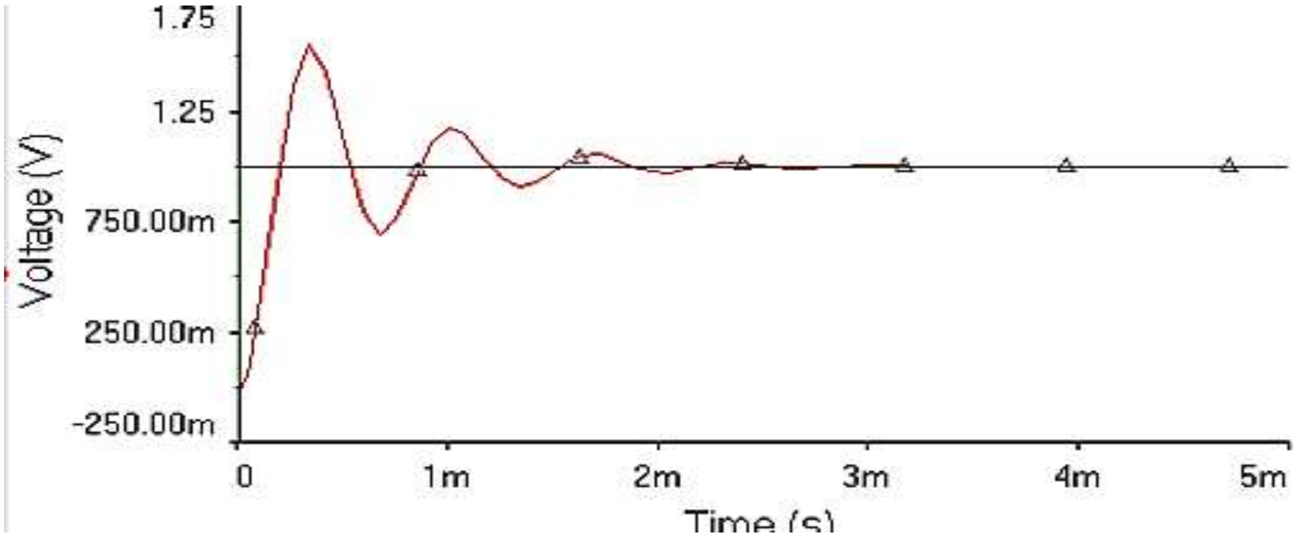
$$= \sqrt{\frac{100}{2\pi}\left[2\pi-2+\frac{2\pi}{2}+2\right]} = \sqrt{\frac{100}{2\pi}\times 3\pi}$$

$$= \sqrt{150} = 12.25\text{ A}$$

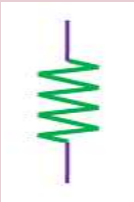
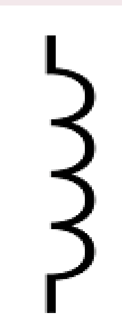
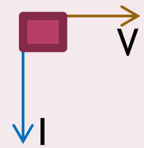
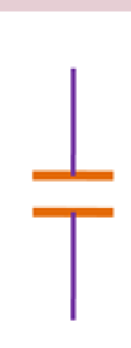
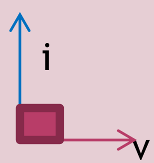
Wave type	Wave form	Mean value (rectified)	Waveform (factor)	RMS value	Crest factor
Sine wave		$\frac{2}{\pi} \approx 0.637$	$\frac{\pi}{2\sqrt{2}} \approx 1.11$	$\frac{1}{\sqrt{2}} \approx 0.707$	$\sqrt{2} \approx 1.414$
Half-wave rectified sine		$\frac{1}{\pi} \approx 0.318$	$\frac{\pi}{2} \approx 1.571$	$\frac{1}{2} = 0.5$	2
Full-wave rectified sine		$\frac{2}{\pi} \approx 0.637$	$\frac{\pi}{2\sqrt{2}} \approx 1.11$	$\frac{1}{\sqrt{2}} \approx 0.707$	$\sqrt{2} \approx 1.414$
Triangle wave		$\frac{1}{2} = 0.5$	$\frac{2}{\sqrt{3}} \approx 1.155$	$\frac{1}{\sqrt{3}} \approx 0.577$	$\sqrt{3} \approx 1.732$
Square wave		1	1	1	1

STEADY STATE ANALYSIS OF R,L,C WITH SINUSOIDAL EXCITATION

Definition : The steady state Analysis refers to analysis of circuit at a state or condition after the transient or natural response died out. Usually steady state is achieved after five time constants of the switching action and analysis at this state is carried out using phasors.

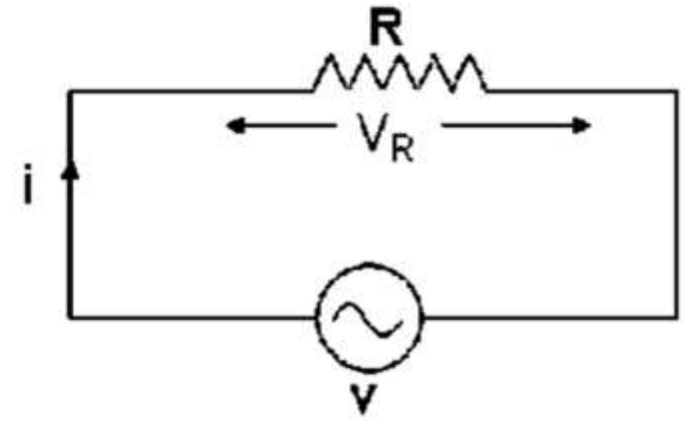


The term steady state response is used synonymously with forced response and the circuits we are about to analyze are commonly said to be in sinusoidal steady state

Element	Unit	Voltage	Current	Impedance	Power	Energy	= Characteristics
RESISTOR 	Ohm (C)	V= IR	$I = \frac{V}{R}$	$R = \frac{V}{I}$	$P = V * I$ $= I^2R$ $= \frac{V^2}{R}$	E =∫ P dt = V * I	R = 0 => I = ∞ , V = 0 , Short circuit condition (S/C) R = ∞ => I = 0 , V = ∞ , Open circuit condition (O/C)
INDUCTOR 	Henry (H)	$V \propto \frac{d\Phi}{dt} \propto \frac{di}{dt}$ $v(t) = L\frac{di}{dt}$	Integrating voltage of inductor $\int v(t)\,dt = \int L\frac{di}{dt}$ $i(t) = \frac{1}{L}\int v(t)\,dt$	$i(t) = I_m \sin \omega t$ $V(t) = L\frac{di}{dt}$ $= \omega L I_m \cos\omega t$ $X_L = \frac{v(t)}{i(t)}$ $= \frac{\omega L I_m \cos\omega t}{I_m \sin \omega t}$ $= \frac{\omega L \sin(wt+90)}{I_m \sin \omega t}$ $X_L = j\omega L$	$P = V * I$  $= L\, i \frac{di}{dt}$	E =∫ P dt $= \frac{1}{2} L I^2$	➢ $\frac{di}{dt}$=0 =>V =0 => S/C for DC ➢ $\frac{di}{dt}$ = + =>P = + L absorbs power, charging ➢ $\frac{di}{dt}$ = - =>P = - L supplies power, discharging ➢ $\frac{di}{dt}$ = 0 =>P=0 => L is idle ➢ $\frac{di}{dt}$ = ∞ => v=∞,i= ∞ => O/C L does not allow sudden change in current ➢ Energy across L is finite even if v=0 (since E=$\frac{1}{2}$ L I^2)
CAPACITOR 	Farad (F)	Integrating current of capacitor $\int i(t)dt = \int C\frac{dV}{dt}$ $v(t) = \frac{1}{C}\int i(t)\,dt$	$C = \frac{q}{V}$ => q=cv $\frac{dq}{dt} = C\frac{dv}{dt}$ $i(t) = C\frac{dV}{dt}$	$v(t) = V_m \sin \omega t$ $i(t) = C\frac{dV}{dt}$ $= \omega C V_m \cos\omega t$ $X_C = \frac{v(t)}{i(t)}$ $= \frac{V_m \sin\omega t}{\omega C V_m \cos \omega t}$ $X_C = \frac{1}{j\omega C}$	$P = V * I$  $= C\, v \frac{dv}{dt}$	E =∫ P dt $= \frac{1}{2} C V^2$	➢ $\frac{dV}{dt}$=0 =>i =0 => O/C for DC ➢ $\frac{dV}{dt}$ = + =>P = + C absorbs power, charging ➢ $\frac{dV}{dt}$ = - =>P = - C supplies power, discharging ➢ $\frac{dV}{dt}$ = 0 =>P=0 => C is idle ➢ $\frac{dV}{dt}$ = ∞ => I = ∞ => S/C C does not allow sudden change in voltage ➢ Energy across C is finite even if I =0 (since E=$\frac{1}{2}$ C V^2)

BEHAVIOUR OF PURE RESISTOR IN AC CIRCUIT

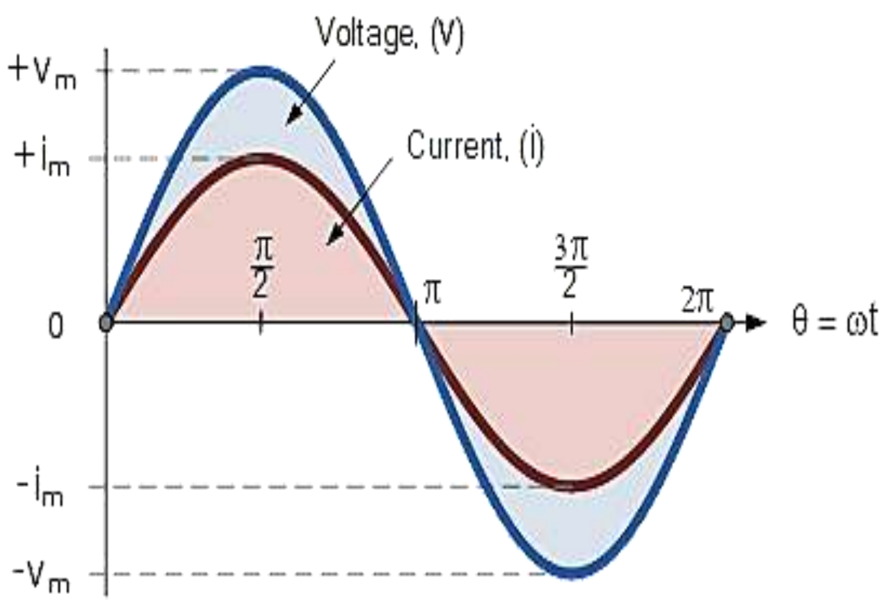
Let $V = V_m \sin\omega t \Rightarrow V = V_m\angle 0$

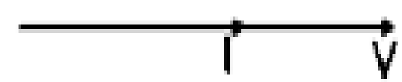


1.Current

$$I = \frac{V}{R} = \frac{V_m \sin\omega t}{R} = I_m \sin\omega t$$
$$= I_m \angle 0$$

2.Wave Forms & phasor representation





V in phase with I

ANALYSIS OF PURE RESISTOR...

3.Impedance

$$Z = \frac{V}{I} = \frac{V_m\angle 0}{I_m\ \angle 0} = \frac{V_m\angle 0}{\frac{Vm}{R}\angle 0} = \mathbf{R}$$

4.Phase Difference and power Factor

Phase difference = Φ = Angle $\angle V,I$ = 0

Power factor = $\cos\Phi = \cos 0 = 1$ (UPF)

5.Power

Instantaneous power P = V I

$$= (V_m \sin\omega t)(I_m \sin\omega t)$$

$$= V_m I_m \sin^2\omega t$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

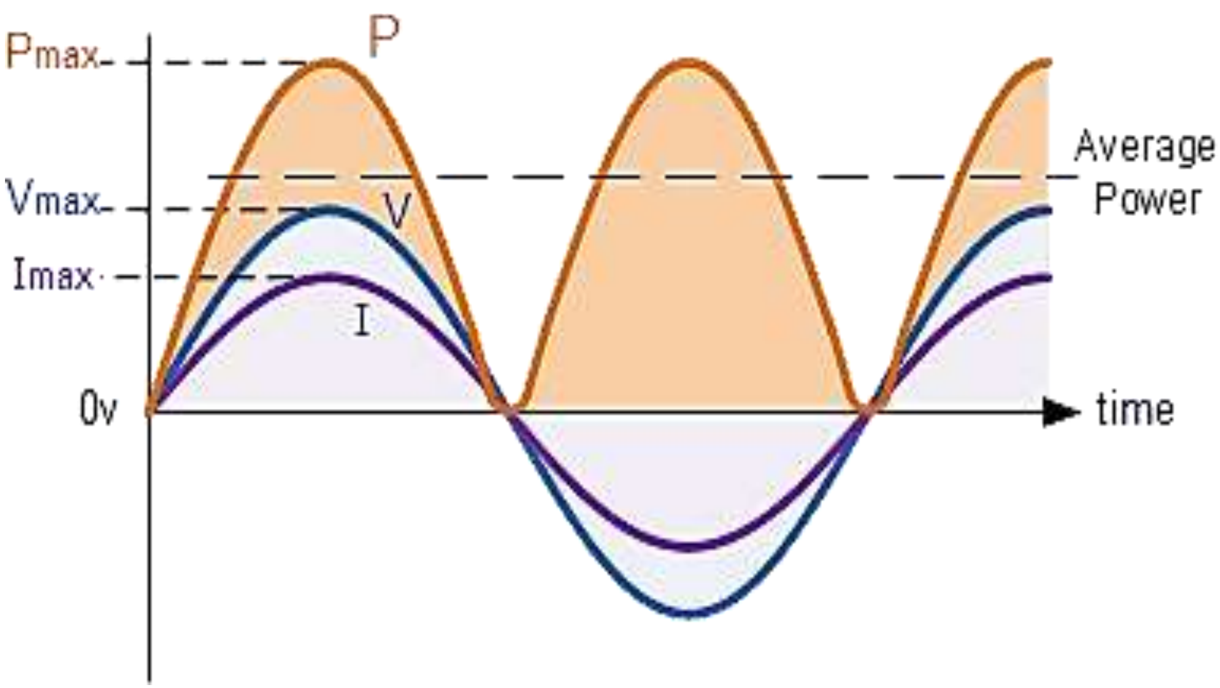
$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

ANALYSIS OF PURE RESISTOR..

$$P_{inst} = \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

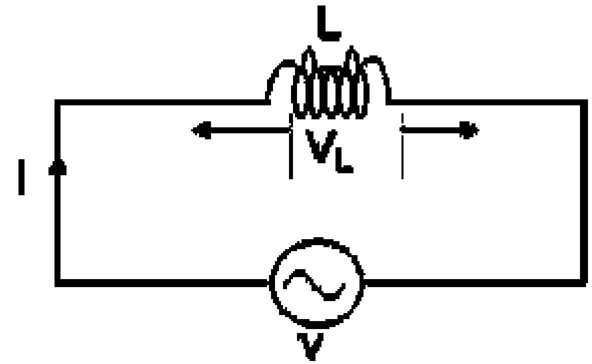
The power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}\cos 2\omega t$. The frequency of fluctuating part is twice the applied voltage frequency and its average value over one complete cycle is zero

$$\text{Average Power} = P_{Avg} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{RMS}\, I_{RMS}$$



BEHAVIOUR OF PURE INDUCTOR IN AC CIRCUIT

Let V =Vm sinωt => V=Vm∠0

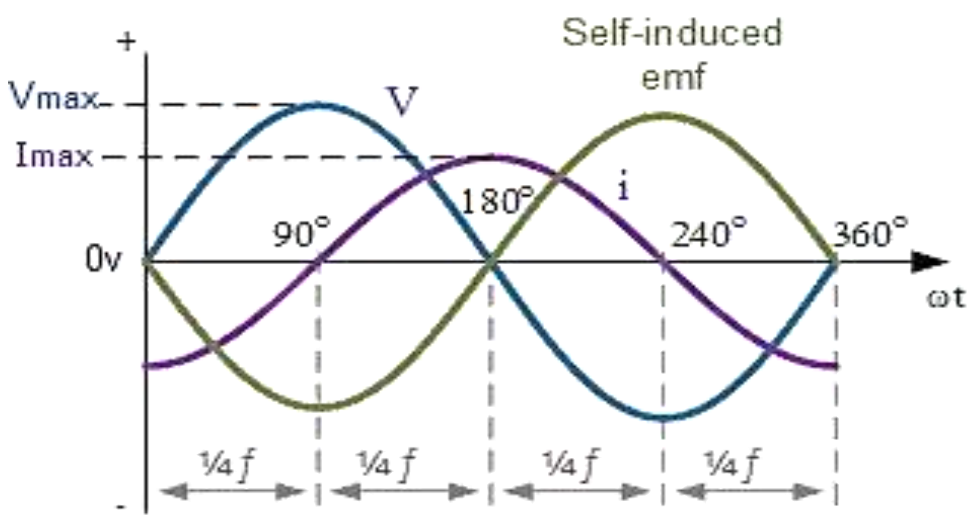


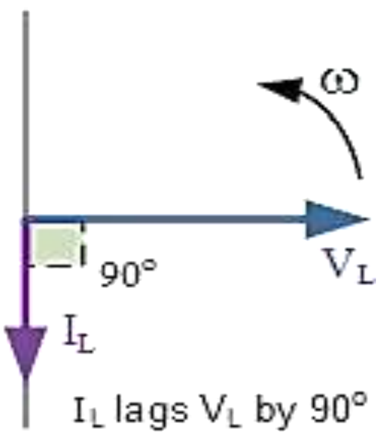
1.Current

$$i(t)= \frac{1}{L}\int v(t)\, dt = \frac{1}{L}\int V_m \sin\omega t\, dt$$
$$= \frac{V_m}{\omega L}(- \cos\omega t) = \frac{V_m}{\omega L}\sin(\omega t - \frac{\pi}{2})$$

$$i(t) = I_m \sin(\omega t - \frac{\pi}{2}) = I_m\angle{-90}$$ ...where $I_m = \frac{Vm}{\omega L}$

2.Wave Forms & phasor representation





3.Impedance

$$Z = \frac{V}{I} = \frac{V_m\angle 0}{I_m\ \angle -90} = \frac{V_m\angle 0}{\frac{Vm}{\omega_L}\angle -90} = \omega L\ \angle 90 = j\ \omega L = j\ X_L$$

The quantity ωL is called inductive reactance, is denoted by X_L and is measures in ohms

****** Significance of operator *j* :** The operator *j* is used in rectangular form. It is used to indicate anti clock wise rotation of phasor through 90 degrees . Mathematically *j* = ∠-1

4.Phase Difference and power Factor

Phase difference = Φ $= \text{Angle } \angle V,I$ = 90^0

Power factor = $\cos\Phi = \cos 90$ $= 0$

5.Power

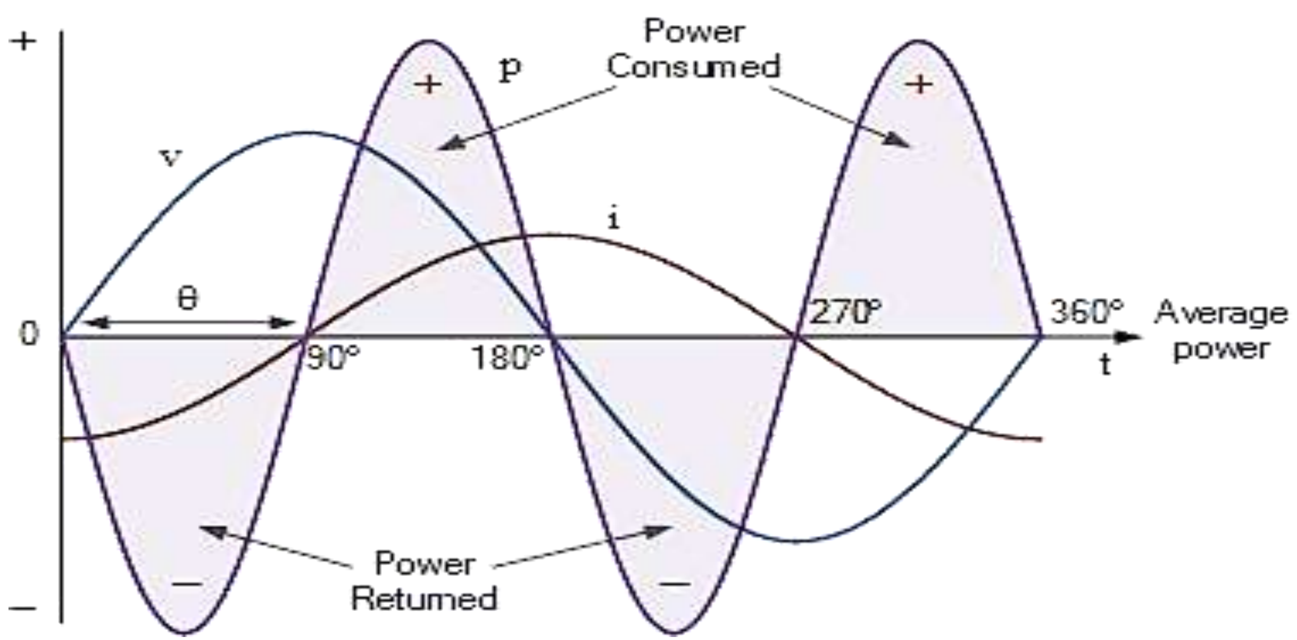
Instantaneous power P_{inst} = V I

$$= V_m \sin\omega t \quad I_m \sin(\omega t - \frac{\pi}{2})$$

$$= -V_m I_m \sin\omega t \cos\omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

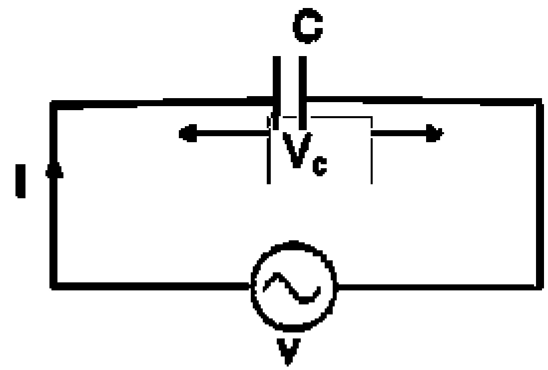
Average Power = P_{Avg} = $\int P_{inst}$ = 0



ANALYSIS OF PURE INDUCTOR...

BEHAVIOUR OF PURE CAPACITOR IN AC CIRCUIT

Let V =Vm sinωt => V=Vm∠0

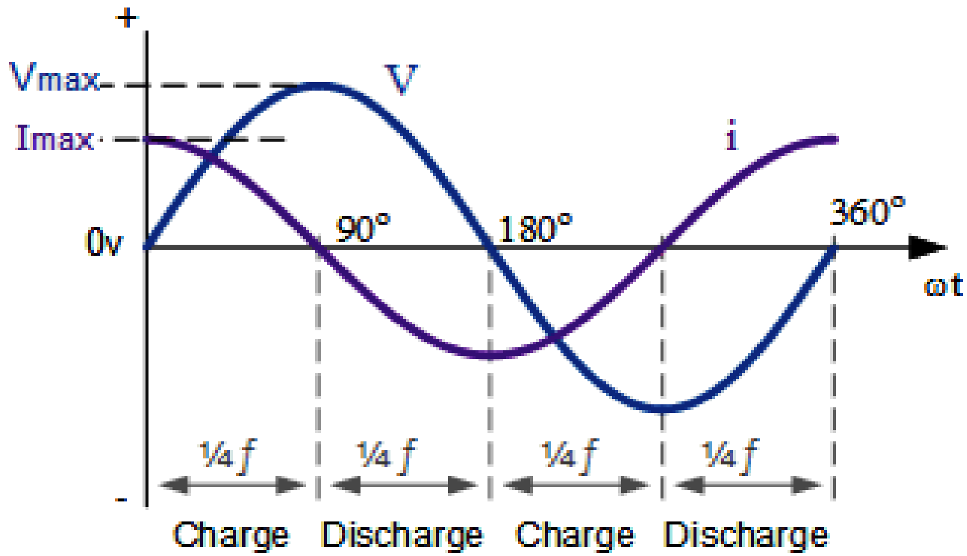


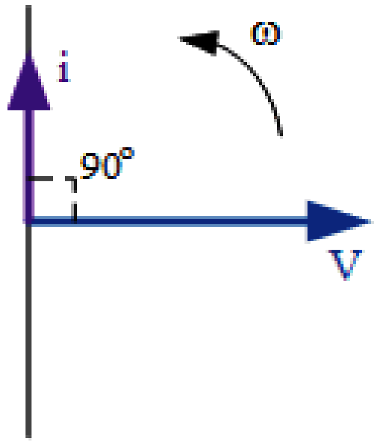
1.Current

$$i(t)= c\frac{dv}{dt} = c\frac{d}{dt}(V_m \sin\omega t) = \omega c\, V_m \cos \omega t$$

$$i(t) = I_m \sin(\omega t + \frac{\pi}{2}) = I_m\angle 90$$ ...where $I_m = V_m\,\omega c$

2.Wave Forms & phasor representation





ANALYSIS OF PURE CAPACITOR..

3.Impedance

$$Z = \frac{V}{I} = \frac{V_m \angle 0}{I_m \angle 90} = \frac{V_m \angle 0}{V_m \omega C \angle 90} = \frac{1}{\omega C} \angle -90 = \frac{-j}{\omega C} = -j\, X_c$$

The quantity $\frac{1}{\omega C}$ is called capacitive reactance, is denoted by X_c and is measures in ohms

****** Significance of operator- *j* :** The operator- *j* is used in rectangular form. It is used to indicate clock wise rotation of phasor through 90 degrees . Mathematically -*j* = -∠-1

4.Phase Difference and power Factor

$$\text{Phase difference} = \Phi = \text{Angle } \angle V,I = 90^0$$

$$\text{Power factor} = \cos\Phi = \cos 90 = 0$$

5.Power

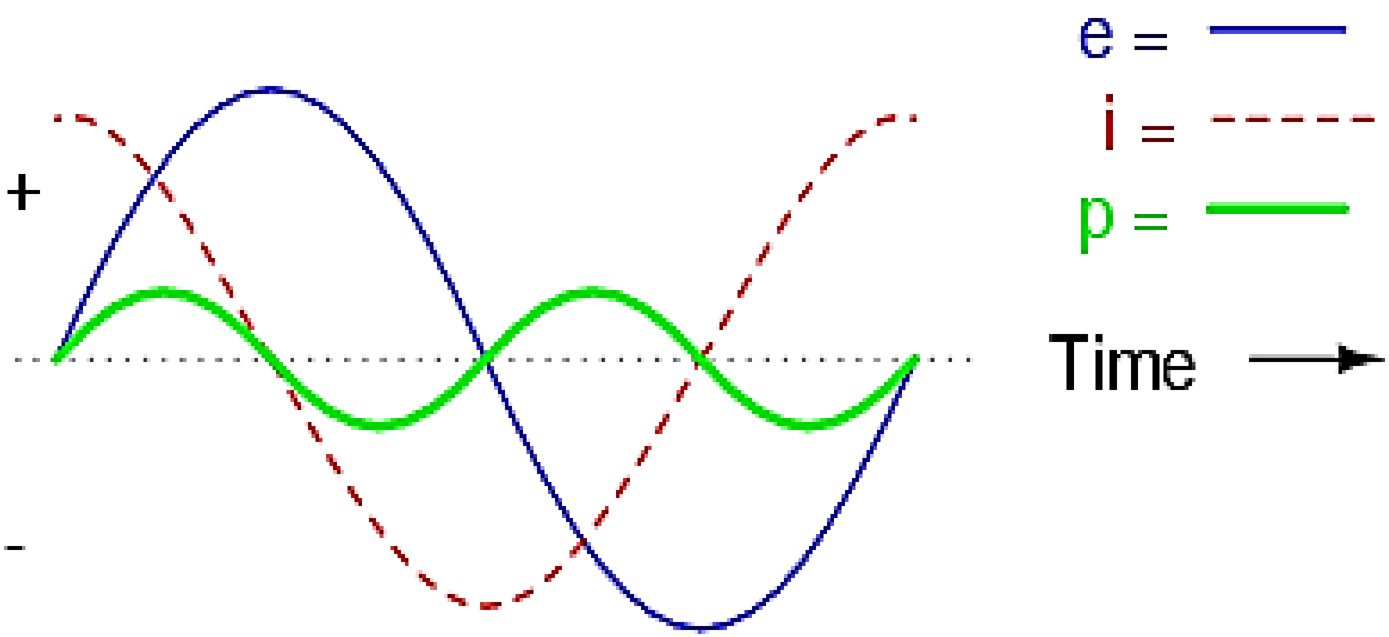
Instantaneous power P_{inst} = V I

$$= V_m \sin\omega t \quad I_m \sin(\omega t + \frac{\pi}{2})$$

$$= V_m I_m \sin\omega t \cos\omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

Average Power = P_{Avg} = $\int P_{inst}$ = 0



Example 1 : An AC circuit consists of a pure resistor of 10ohm and is connected across an AC supply of 230v,50Hz. Calculate

1. Current
2. Power consumed
3. Power factor
4. Write equations for voltage and current

Solution

$$R = 10\,\Omega, \quad V = 230\text{ V}, \quad f = 50\text{ Hz}$$

1. Current $I = \frac{V}{R} = \frac{230}{10} = 23\text{ A}$

2.Power consumed = $P = VI = 230 \times 23 = 5290\text{ W}$

3. Power Factor = $\text{pf} = \cos\phi = \cos(0°) = 1$

4. V & I

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 230 = 325.27\text{ V}$$

$$I_m = \sqrt{2}I = \sqrt{2} \times 23 = 32.53\text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16\text{ rad/s}$$

$$v = V_m \sin\omega t = 325.27 \sin 314.16\,t$$

$$i = I_m \sin\omega t = 32.53 \sin 314.16\,t$$

PROBLEMS ON SINGLE R,L,C CIRCUITS…

Example 2 : An AC Inductive coil consists of a negligible resistor and 0.1h inductance connected across an AC supply of 200v,50Hz. Calculate

1. Inductive Reactance
2. RMS value of current
3. Power
4. Power Factor
5. V & I Equations

Solution :

1.Inductive Reactance = $X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$

2.RMS of Current = $I = \frac{V}{X_L} = \frac{200}{31.42} = 6.37\ \text{A}$

3. Power = $P = VI\cos\phi = 200 \times 6.37 \times \cos(90°) = 0$

4. Power Factor = $\text{pf} = \cos\phi = \cos(90°) = 0$

5. V & I Equns =

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 200 = 282.84\ \text{V}$$

$$I_m = \sqrt{2}\,I = \sqrt{2} \times 6.37 = 9\ \text{A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16\ \text{rad/s}$$

$$v = V_m \sin\omega t = 282.84 \sin 314.16\,t$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) = 9 \sin\left(314.16\,t - \frac{\pi}{2}\right)$$

Example 3 : The voltage and current through circuit elements are

$$v = 100 \sin(314t + 45°) \text{ volts}$$
$$i = 10 \sin(314t + 315°) \text{ amperes}$$

1. Identify the circuit elements
2. Find the value of elements
3. Obtain expression for power

Solution

$$v = 100 \sin(314t + 45°)$$
$$i = 10 \sin(314t + 315°)$$
$$= 10 \sin(314t + 315° - 360°)$$
$$= 10 \sin(314t - 45°)$$

1. Circuit Elements : From the V and I equations ,it is clear that the current lags behind the voltage by 90 degree. Hence, the circuit Element is inductor
2. Value of elements :

$$X_L = \frac{V}{I} = \frac{V_m}{I_m} = \frac{100}{10} = 10\,\Omega \qquad X_L = \omega L$$
$$10 = 314L$$
$$L = 31.8 \text{ mH}$$

3. Expression for power :

$$p = -\frac{V_m I_m}{2} \sin 2\omega t = -\frac{100 \times 10}{2} \sin(2 \times 314t) = -500 \sin 628t$$

PROBLEMS ON PURE R,L,C CIRCUITS…

Example 4 : A Capacitor has a capacitance of 30 microfarads which is connected across a AC supply of 230v,50Hz. Calculate

1. Capacitive Reactance
2. RMS value of current
3. Power
4. Power Factor
5. V & I Equations

Solution $C = 30\ \mu F,\quad V = 230\ V,\quad f = 50\ Hz$

1. Capacitive Reactance $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 30 \times 10^{-6}} = 106.1\ \Omega$

2. RMS current $I = \frac{V}{X_C} = \frac{230}{106.1} = 2.17\ A$

3. Power $P = VI \cos\phi = 230 \times 2.17 \times \cos(90^\circ) = 0$

4. Power Factor $pf = \cos\phi = \cos(90^\circ) = 0$

5. V & I Equations

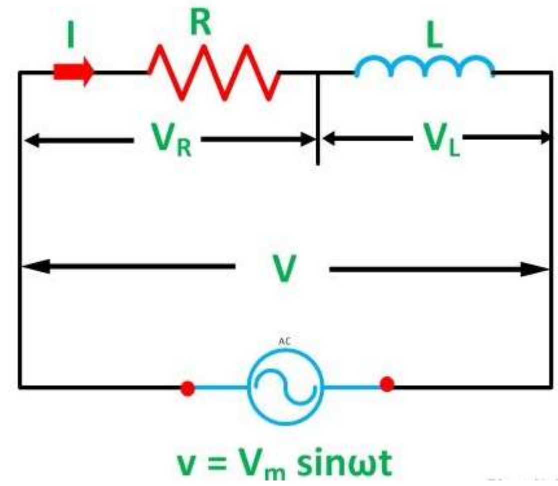
$$V_m = \sqrt{2}\ V = \sqrt{2} \times 230 = 325.27\ V$$
$$I_m = \sqrt{2}\ I = \sqrt{2} \times 2.17 = 3.07\ A$$
$$\omega = 2\pi f = 2\pi \times 50 = 314.16\ rad/s$$
$$v = V_m \sin \omega t = 325.27 \sin 314.16\, t$$
$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) = 3.07 \sin\left(314.16\, t + \frac{\pi}{2}\right)$$

PROBLEMS ON PURE R,L,C CIRCUITS...

BEHAVIOUR OF SERIES RL CIRCUIT

Let V(t) =Vm sinωt => V=Vm∠0

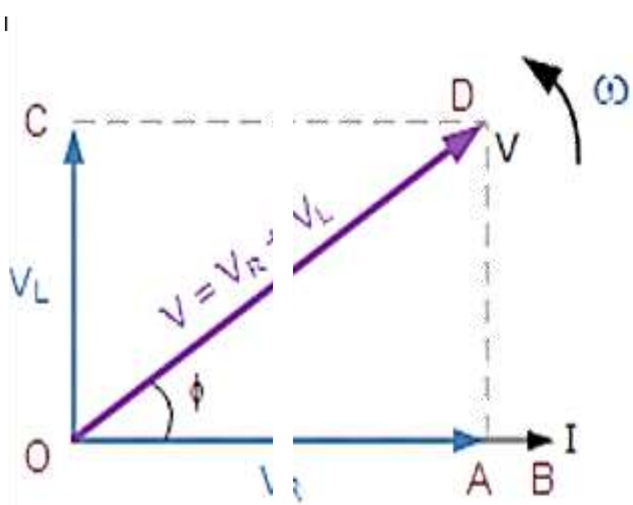
V & I are RMS Values of source voltage & current



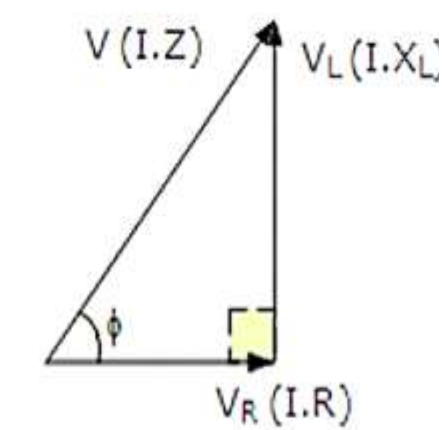
V_R = RI (V_R & I are in phase)

V_L = IX_L (I lags V_L by 90 degress)

1.Wave Form & phasor representation

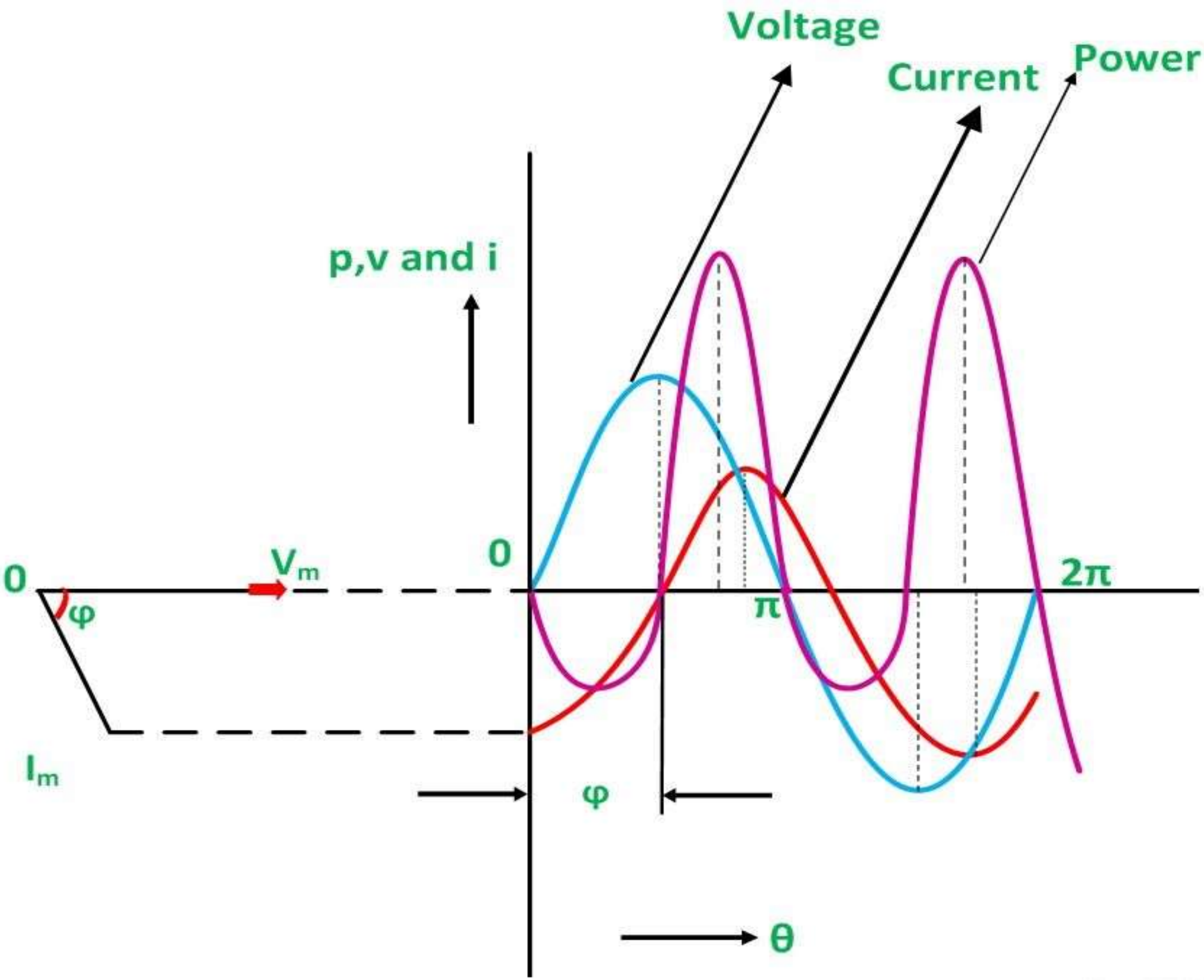


Vector Diagram



Voltage Triangle

ANALYSIS OF SERIES R L CIRCUITS…



2.Impedance (Z)

$$V = V_R + V_L = I\,(R + j\omega L) \;\Rightarrow\; Z = \frac{V}{I} = R + j\omega L = Z\angle\Phi$$

$$\text{Where } |Z| = \sqrt{R^2 + {X_L}^2} = \sqrt{R^2 + \omega L^2}$$

$$\Phi = tan^{-1}\left(\frac{X_L}{R}\right) = tan^{-1}\left(\frac{IX_L}{IR}\right) = tan^{-1}\left(\frac{V_L}{V_R}\right)$$

3.Impedance Triangle

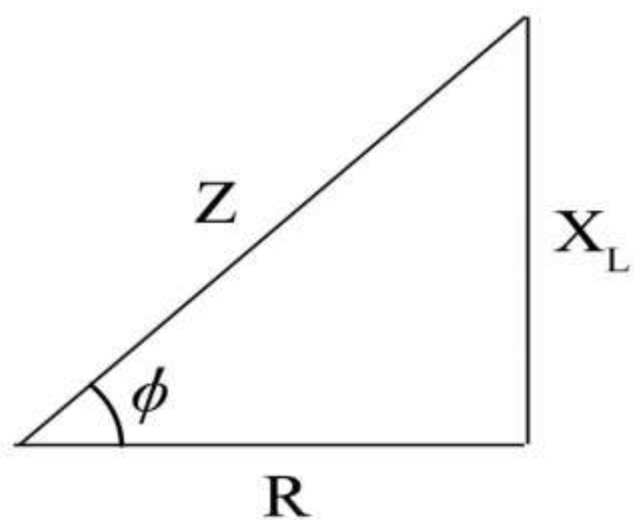
The right-angled triangle formed by the vectors representing the resistance drop, the reactance drop, and the impedance drop of a circuit carrying an alternating current is called Impedance Triangle

$$|Z| = \sqrt{R^2 + {X_L}^2}$$

$$\sin\Phi = \frac{X_L}{Z}$$

$$\cos\Phi = \frac{R}{Z}$$

$$\tan\Phi = \frac{X_L}{R}$$



4. Current ($I = I_R = I_L$)

i(t) = I_m sin(ωt-Φ) ... where $I_m = \frac{V_m}{Z}$ & $\Phi = tan^{-1}(\frac{X_L}{R})$

-

$I_{(RMS)} = \frac{V_R}{R} = \frac{V_L}{X_L} = \frac{V}{Z}$ *where z= R+ jX_L = R+ jωL*

5. Power

Instantaneous power P (t) = V I

$= (V_m \sin\omega t)(I_m \sin(\omega t - \Phi))$

$= V_m I_m \sin\omega t \sin(\omega t - \Phi)$

$= \frac{V_m I_m}{2} [\cos\Phi - \cos(2\omega t - \Phi)]$

$= \frac{V_m I_m}{2} \cos\Phi - \frac{V_m I_m}{2} \cos(2\omega t - \Phi)$

Average Power = $P_{Avg} = \int P_{inst} = \frac{V_m I_m}{2} \cos\Phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\Phi = V_{RMS} I_{RMS} \cos\Phi$

Average Power = P_{Avg} = $\int P_{inst}$ = $\frac{V_m I_m}{2}\cos\Phi$ = $\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\Phi$ = $V_{RMS}\ I_{RMS}\cos\Phi$

Active power : From the expression ,Average power is dependent on the in phase component of current. The average power is the use full power that is used to do work on the load. Active Power is also called as Average/Real/True power and is measured in watts.

$$\text{Active Power} = \text{P} = \text{V I Cos}\Phi$$

Reactive Power: We know that pure inductor and capacitor consume no power because all the power received from the source in a half cycle is returned to source in the next half cycle. This circulating power is called reactive power .It is the product of voltage and reactive part of current i.e I sinΦ and is measured in KVAR(volt-ampere-reactive)

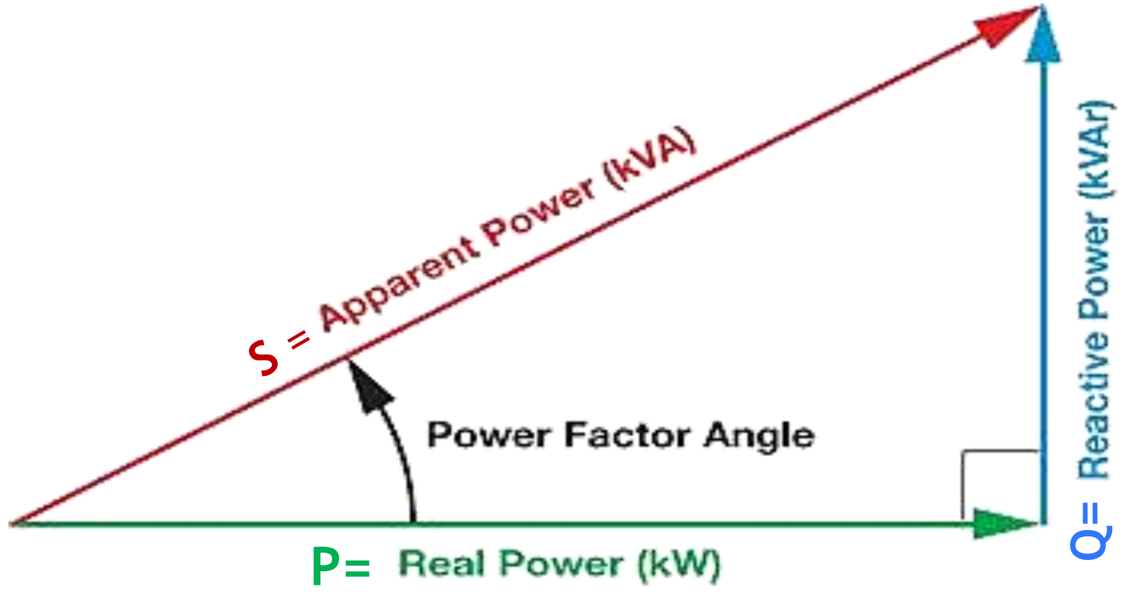
$$\text{Reactive Power} = \text{Q} = \text{V I Sin}\Phi$$

Apparent Power : The product of voltage and current is known as apparent power (s) and is measured in KVA (volt-ampere)

$$\text{Apparent power} = \text{S} = \text{V I} = \sqrt{P^2 + Q^2}$$

6.Power Triangle :

Power Triangle is the representation of a right angle triangle showing the relation between active power, reactive power and apparent power. When each component of the current that is the active component (Icosφ) or the reactive component (I sinφ) is multiplied by the voltage V, a power triangle is obtained shown in the figure below



P = Active Power = V I CosΦ

Q = Reactive Power =V I SinΦ

S = Apparent power = V I = $\sqrt{P^2 + Q^2}$

7.Power Factor : It is defined as the cosine of the angle between voltage and current

Power factor = cosΦ

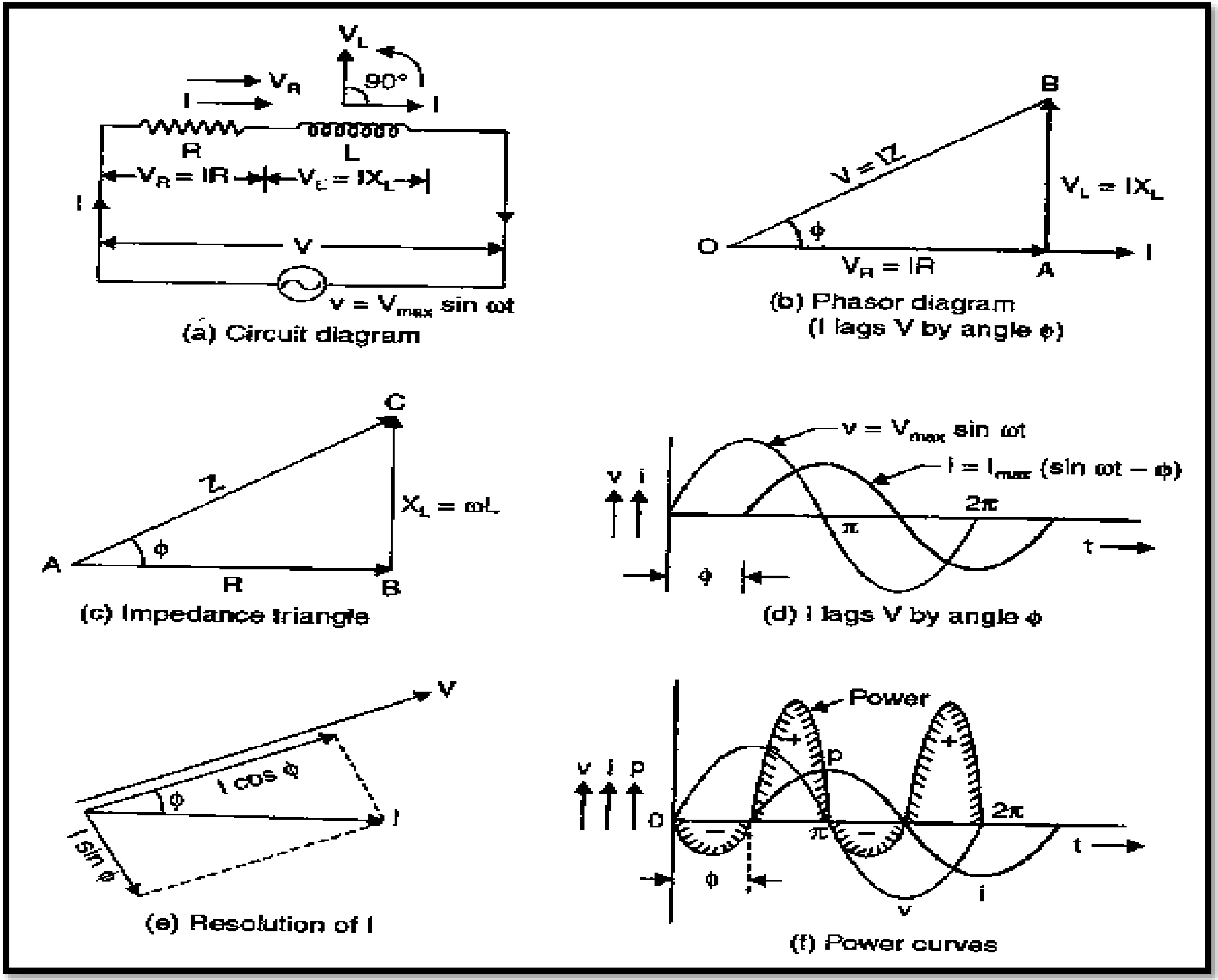
PF = $\frac{V_R}{V}$ … *From Voltage Triangle*

PF = $\frac{R}{Z}$ … *From Impedance Triangle*

PF = $\frac{P}{S}$ … *From Power Triangle*

In case of series RL circuit, the power Factor is *lagging* in nature

Synopsis Of RL Circuit



ANALYSIS OF SERIES R L CIRCUITS…

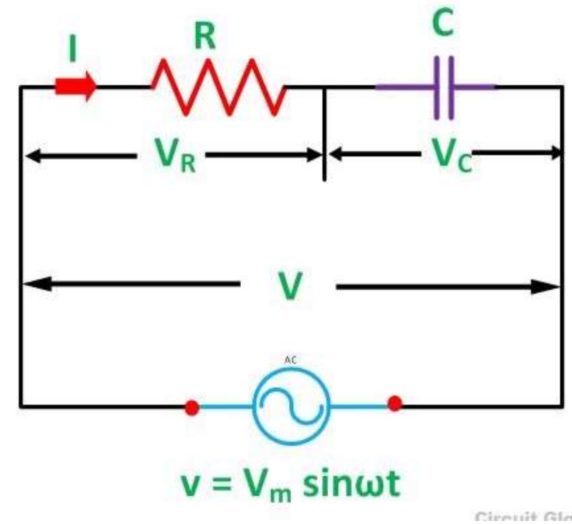
BEHAVIOUR OF SERIES RC CIRCUIT

Let V(t) =Vm sinωt => V=Vm∠0

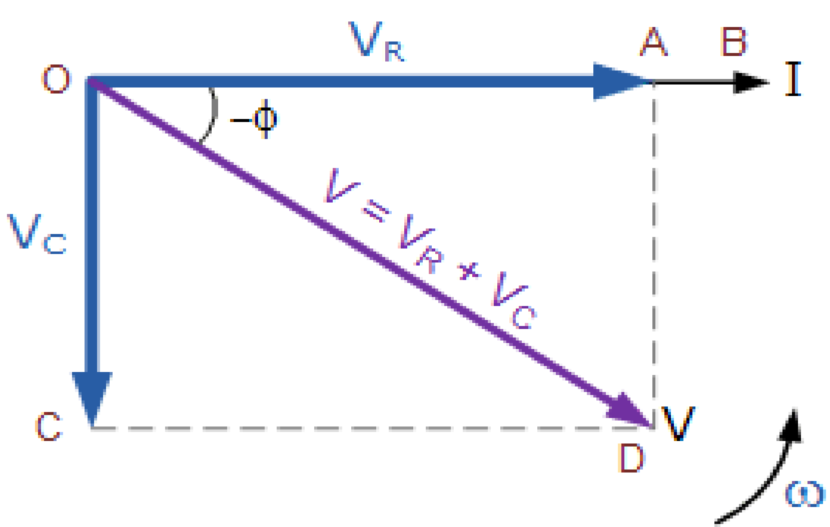
V & I are RMS Values of source voltage & current

V_R = RI (V_R & I are in phase)

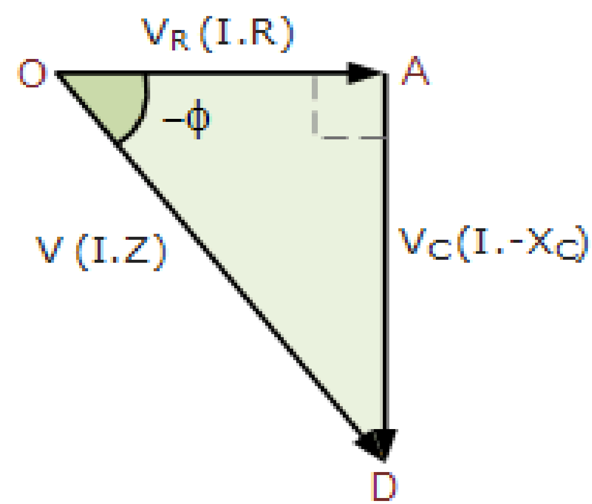
V_C = IX_C (I leads V_L by 90 degress)



1.Wave Form & phasor representation



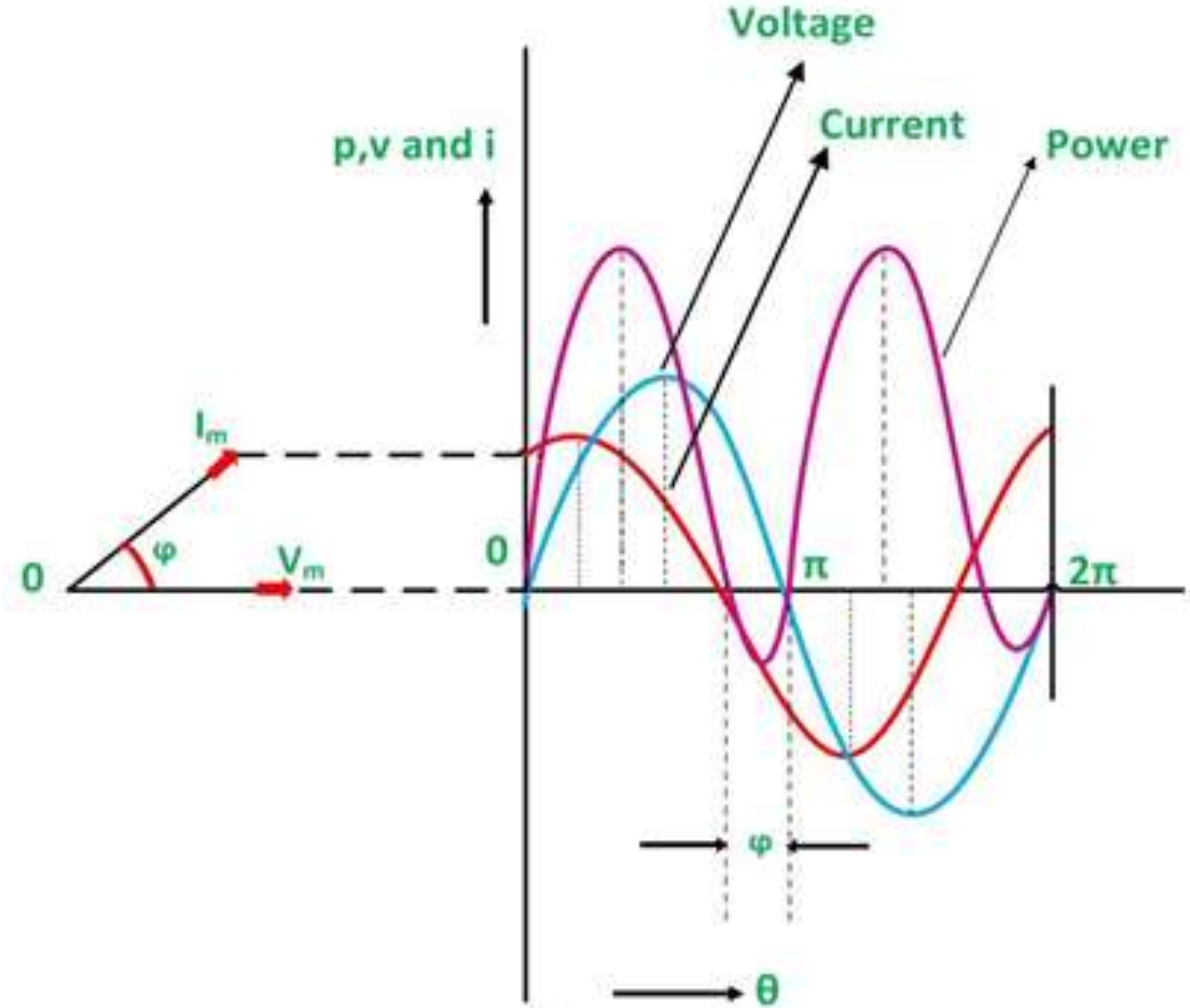
Vector Diagram



Voltage Triangle

ANALYSIS OF SERIES R C CIRCUITS…

ANALYSIS OF SERIES R C CIRCUITS…



2.Impedance (Z)

$$V = V_R + V_C = I\,(R - jX_C) \Rightarrow Z = \frac{V}{I} = R - \frac{j}{\omega C} = Z \angle \Phi$$

Where $|Z| = \sqrt{R^2 + {X_C}^2} = \sqrt{R^2 + (\frac{-1}{\omega C})^2}$

$$\Phi = tan^{-1}(\frac{X_C}{R}) = tan^{-1}(\frac{IX_C}{IR}) = tan^{-1}(\frac{V_C}{V_R})$$

3.Impedance Triangle

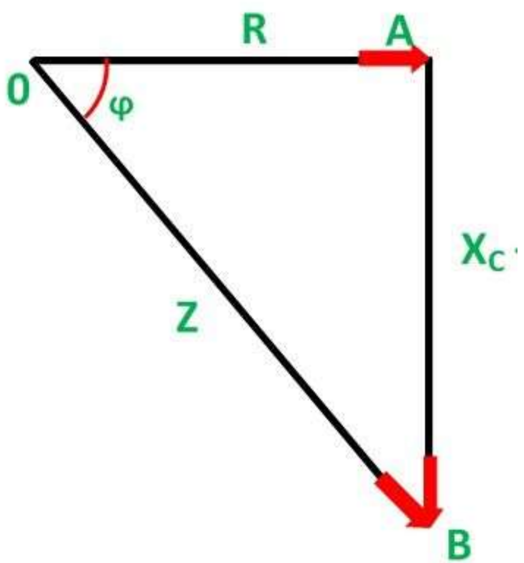
The right-angled triangle formed by the vectors representing the resistance drop, the reactance drop, and the impedance drop of a circuit carrying an alternating current is called Impedance Triangle

$|Z| = \sqrt{R^2 + {X_C}^2}$

$\sin\Phi = \frac{X_L}{Z}$

$\cos\Phi = \frac{R}{Z}$

$\tan\Phi = \frac{X_L}{R}$



4.Current ($I = I_R = I_L$)

$i(t) = I_m \sin(\omega t+\Phi)$ … where $I_m = \frac{V_m}{Z}$ & $\Phi = tan^{-1}(\frac{X_C}{R})$

-

$I_{(RMS)} = \frac{V_R}{R} = \frac{V_C}{X_C} = \frac{V}{Z}$ *where z= R-jX_C = R - $\frac{j}{\omega C}$*

5.Power

Instantaneous power P (t) = V I

$= (V_m \sin\omega t)(I_m \sin(\omega t+\Phi))$

$= V_m I_m \sin\omega t \sin(\omega t+\Phi)$

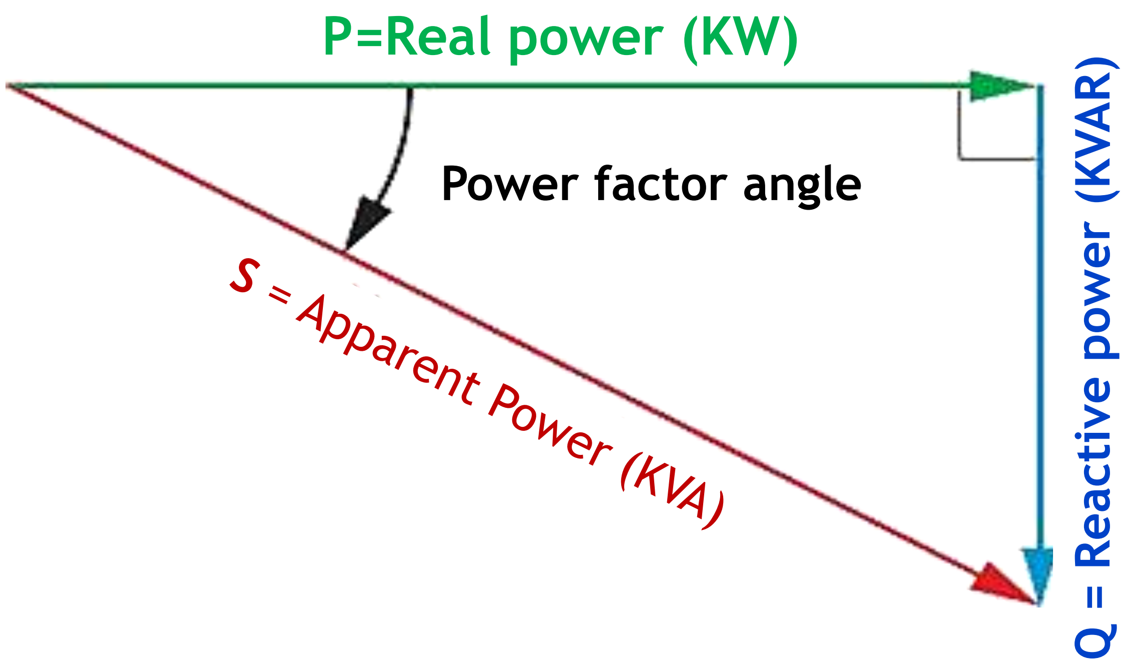
$= \frac{V_m I_m}{2} [\cos\Phi - \cos(2\omega t+\Phi)]$

$= \frac{V_m I_m}{2} \cos\Phi - \frac{V_m I_m}{2} \cos(2\omega t+\Phi)$

Average Power = $P_{Avg} = \int P_{inst} = \frac{V_m I_m}{2} \cos\Phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\Phi = V_{RMS} I_{RMS} \cos\Phi$

6.Power Triangle :

Power Triangle is the representation of a right angle triangle showing the relation between active power, reactive power and apparent power. When each component of the current that is the active component (Icosφ) or the reactive component (I sinφ) is multiplied by the voltage V, a power triangle is obtained shown in the figure below

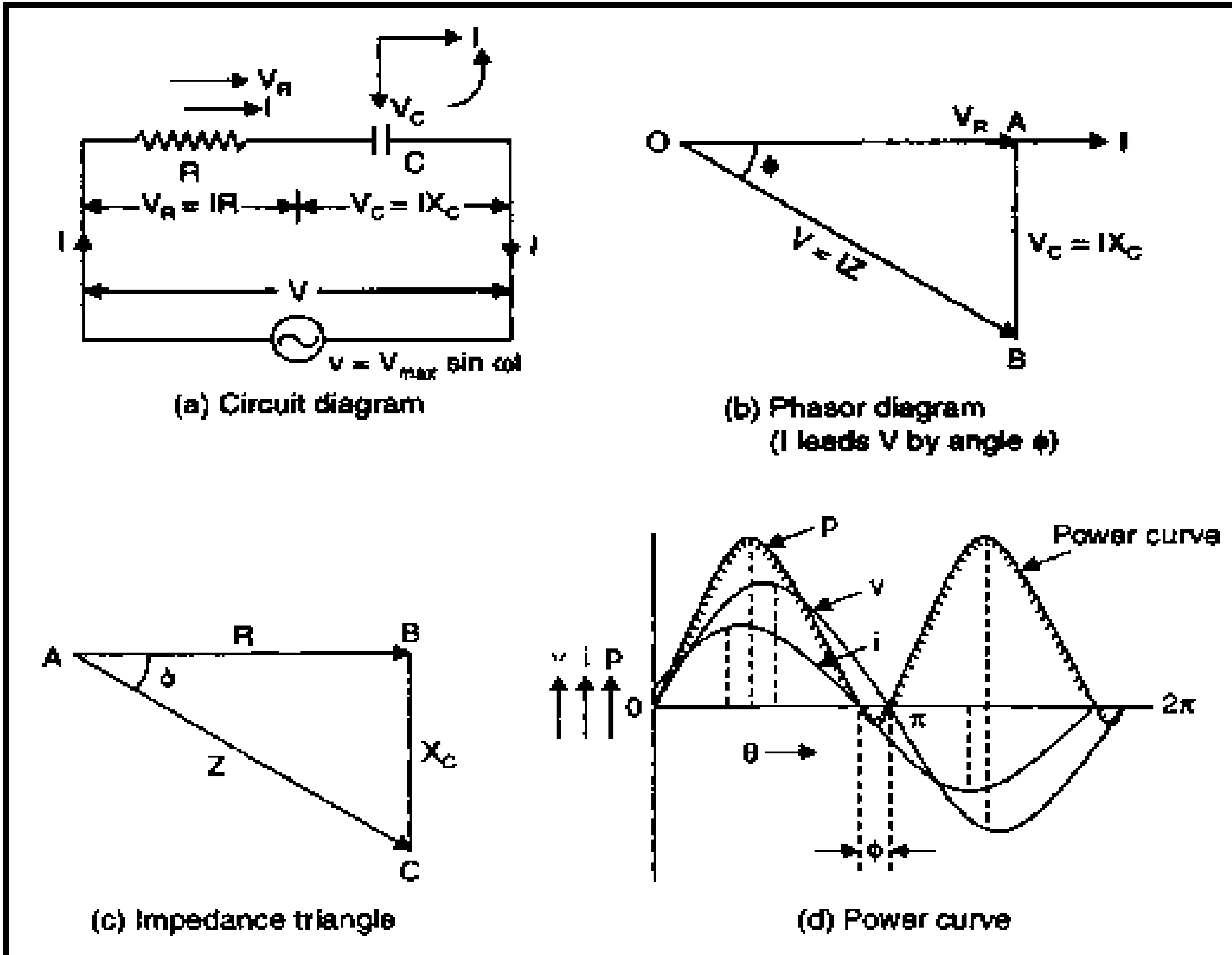


P = Active Power = V I CosΦ

Q = Reactive Power = V I SinΦ

S = Apparent power = V I = $\sqrt{P^2 + Q^2}$

Synopsis Of RC Circuit



(a) Circuit diagram

(b) Phasor diagram (I leads V by angle φ)

(c) Impedance triangle

(d) Power curve

ANALYSIS OF SERIES R C CIRCUITS…

Example 1 : An alternating voltage of 80+60j v is applied to a circuit and the current flowing is 4-2j.Find the

1. Impedance
2. Phase angle
3. Power factor
4. Power consumed

Answers:

Impedance = Z= 22.37 ohms

Phase angle = Φ = 63.43 deg

Power factor = 0.447 (lagging)

Power consumed = 199.81

Solution: V = 80+60j v , I = 4-2j A

= 100 ∠ 36.87 = 4.47 ∠-26.56

1.) Impedance = Z = $\frac{V}{I} = \frac{80+60j}{4-2j}$ = 22.37 ∠63.43

Z = 22.37

2.) Phase Angle = $\Phi = 63.43$

$3.)\text{Power Factor} = \cos\Phi = 0.447(\text{lagging})$

$4.)\text{Power consumed} = \text{P} = \text{V I} \cos\Phi = 100\text{x}4.47\text{x}0.447 = 199.81\text{W}$

Example 2 : An RMS voltage of 100 ∠0v is applied to a series combination of Z_1 and Z_2 where Z_1 =20 ∠ 30 ohms. The effective voltage drop across Z_1 is known to be 40 ∠-30v . Find the Reactive Component of Z_2

Answers : X_2 = 33.3ohms

Solution:

V = 100∠0v, Z_1 =20∠30 ohms , V_1 = 40∠-30v

$I = \frac{V_1}{Z_1} = \frac{40\angle -30}{20\angle 30} = 2 \angle -60$ A

$Z = \frac{V}{I} = \frac{100\angle 0}{2\angle -60} = 50 \angle 60$ ohms

$Z = Z_1 + Z_2 \Rightarrow Z_2 = Z - Z_1 = 50 \angle 60 - 20\angle 30$
$= 7.68 + 33.3j$

Reactive component of Z_2 =33.3ohms

Example 3 : When a sinusoidal voltage of 120v is applied to a series RL circuit , it is found that there occurs a power dissipation of 1200W and a current flow given by i(t)= 28.3sin(314t-Φ). Find the circuit Resistance and Inductance

Ans : R = 3 ohms
L = 0.0165 H

Solution :

$$
\begin{aligned}
V &= 120 \text{ V} \\
i(t) &= 28.3 \sin(314t - \phi) \\
P &= 1200 \text{ W} \\
I &= \frac{28.3}{\sqrt{2}} = 20.01 \text{ A} \\
P &= VI \cos\phi \\
1200 &= 120 \times 20.01 \times \cos\phi \\
\cos\phi &= 0.499 \\
\phi &= 60.02^\circ \\
Z &= \frac{V}{I} = \frac{120}{20.01} = 6\ \Omega \\
\overline{Z} &= Z\angle\phi = 6\angle 60.02^\circ = 3 + j5.2\ \Omega \\
\text{Resistance } R &= 3\ \Omega \\
\text{Reactance } X_L &= 5.2\ \Omega \\
X_L &= \omega L \\
5.195 &= 314 \times L \\
\text{Inductance } L &= 0.0165 \text{ H}
\end{aligned}
$$

Example 4 : When an inductive coil is connected to a dc supply at 240v, the current in it is 16A. When the same coil is connected to an AC supply at 240v, 50Hz, the current is 12.27A. Calculate

1. Resistance
2. Impedance
3. Reactance
4. Inductance of coil.

Answers:

Resistance = R= 15 ohms

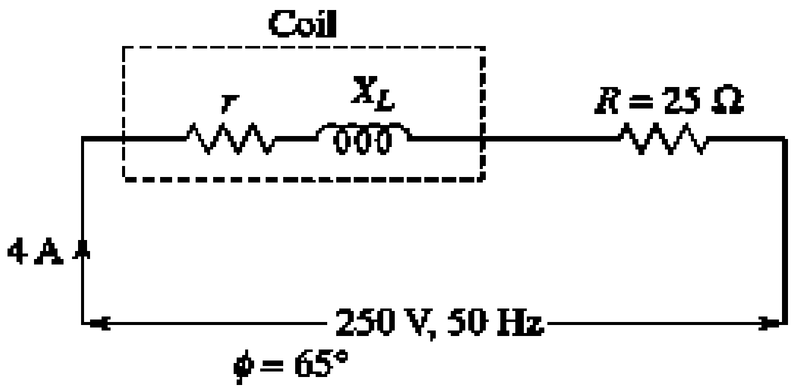
Impedance = Z = 19.56 ohms

Reactance = X_L = 12.55 ohms

Inductance = L = 0.04 H

Example 5 : A resistance of 25 ohm is connected in series with a choke coil. The series combination when connected across a 250v,50hz supply draws a current of 4A which lags behind the voltage by 65 deg. Calculate

1. Total power
2. Power consumed by resistance
3. Power consumed by choke coil
4. circuit Parameters of choke coil



Answers:

Total Power = 422.56W

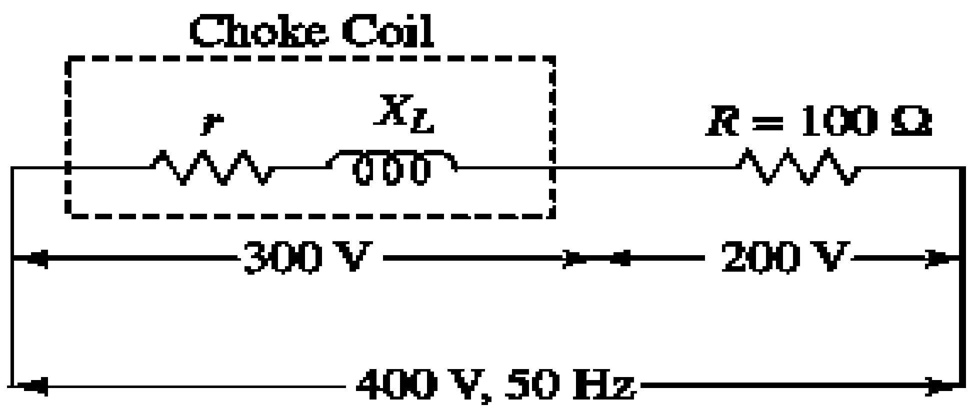
Power consumed by resistor = 400W

Power consumed by choke coil = 22.56W

R=1.41 Ohms & L= 0.18H

Example 6 : A 100 ohm resistor is connected in series with a choke coil. When a 400V,50Hz supply is applied to this combination, the voltage across the resistor and the choke coil are 200v and 300v respectively. Find the

1. power consumed by the choke coil
2. Power factor of the choke coil
3. Power factor of circuit.



Answers:

Power consumed by choke coil = 150W

Power Factor of choke coil = 0.25(lagging)

Power Factor of circuit = 0.6875 (lagging)

solution

$$\text{Current } I = \frac{200}{100} = 2 \text{ A}$$

Impedance of choke coil $Z_{coil} = \dfrac{300}{2} = 150\ \Omega$

$$\sqrt{r^2 + X_L^2} = 150$$

$$r^2 + X_L^2 = 22500$$

Total impedance $Z = \dfrac{400}{2} = 200\ \Omega$

$$\bar{Z} = (R + r) + jX_L$$

$$Z = \sqrt{(R + r)^2 + X_L^2} = 200$$

$$(100 + r)^2 + X_L^2 = 40000$$

Subtracting the Eq. (i) from (ii),

$$(100 + r)^2 - r^2 = 17500$$

$$10000 + 200r + r^2 - r^2 = 17500$$

$$200r = 7500$$

$$r = 37.5\ \Omega$$

Substituting the value of r in the Eq. (i),

$$(37.5)^2 + X_L^2 = 22500$$

$$X_L^2 = 21093.75$$

$$X_L = 145.24\ \Omega$$

Power consumed by choke coil $= I^2 r$

$$= (2)^2 \times 37.5 = 150 \text{ W}$$

Power factor of choke coil $= \dfrac{r}{Z_{coil}} = \dfrac{37.5}{150} = 0.25$ (lagging)

Power factor of circuit $= \dfrac{R + r}{Z} = \dfrac{100 + 37.5}{200} = 0.6875$ (lagging)

Example 7 : A series circuit consumes 2KW at 0.5leading power factor when connected to 230v,50 Hz AC supply. Calculate

1. KVA
2. KVAR
3. Current drawn.

Answers:

KVA = Apparent Power = S = 4KVA

KVAR = Reactive Power =Q = 3.464KVAR

Current Drawn = 17.39A

Solution

$$V = 230 \text{ V}$$

$$P = VI\cos\phi$$

$$2000 = 230 \times I \times 0.5$$

$$I = 17.39 \text{ A}$$

$$S = VI = \frac{P}{\cos\phi} = \frac{2000}{0.5} = 4000 \text{ VA} = 4 \text{ kVA}$$

$$\phi = \cos^{-1}(0.5) = 60°$$

$$Q = VI\sin\phi = 230 \times 17.39 \times \sin(60°) = 3.464 \text{ kVAR}$$

Example 8 : A resistor R in series with a capacitance C is connected to 240v,50Hz supply.

1. Find the value of C so that R absorbs 300W at 100V
2. Maximum Charge
3. Maximum stores Energy in C

Answers:

C = 43.77 µF

Maximum Charge = Q_{Max} = 0.0135C

Maximum Stored Energy = E_{Max} = 2.08 J

Solution

$$V = 240 \text{ V} \qquad V_R = 100 \text{ V}$$
$$P = 300 \text{ W} \qquad f = 50 \text{ Hz}$$

$$P = \frac{V_R^2}{R}$$

$$300 = \frac{(100)^2}{R}$$

$$R = 33.33\ \Omega$$

$$P = I^2 R$$

$$300 = I^2 \times 33.33$$

$$I = 3 \text{ A}$$

$$Z = \frac{V}{I} = \frac{240}{3} = 80\ \Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(80)^2 - (33.33)^2} = 72.72\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$72.72 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 43.77\ \mu\text{F}$$

Voltage across capacitor $V_C = \sqrt{V^2 - V_R^2} = \sqrt{(240)^2 - (100)^2} = 218.17 \text{ V}$

Maximum value of $V_C = 218.17 \times \sqrt{2} = 308.54 \text{ V}$

Maximum charge $Q_{max} = CV_{Cmax} = 43.77 \times 10^{-6} \times 308.54 = 0.0135 \text{ C}$

Maximum stored energy $E_{max} = \frac{1}{2} C (V_{Cmax})^2$

$$= \frac{1}{2} \times 43.77 \times 10^{-6} \times (308.54)^2 = 2.08 \text{ J}$$

Example 9 : A capacitor of 35μF is connected in series with a variable resistor .The circuit is connected across 50Hz main. Find the value of resistor for a condition when the voltage across capacitor is half the supply voltage.

Answer : R = 157.5 Ohms

Solution :

$$C = 35\ \mu F \qquad f = 50\ Hz$$

$$V_C = \frac{1}{2} V$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 35 \times 10^{-6}} = 90.946\ \Omega$$

$$V_C = \frac{1}{2} V$$

$$X_C . I = \frac{1}{2} Z . I$$

$$X_C = \frac{1}{2} Z$$

$$Z = 2X_C$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$(2X_C)^2 = R^2 + X_C^2$$

$$3X_C^2 = R^2$$

$$R^2 = 3 \times (90.946)^2 = 24813.35$$

$$R = 157.5\ \Omega$$

Example 10 : A resistor(R) and a capacitor(c) are connected across a 250V supply. When the supply frequency is 50Hz, the current drawn is 5A . When the frequency is increased to 60Hz. It draws 5.8A. Find

1. Value of R
2. Value of C
3. Power drawn in second case

Answers:

R = 19.96 Ohms
C= 69.4µF
P = 671.45W

Solution :

Data $V = 250\text{ V}$

$f_1 = 50\text{ Hz},\quad I_1 = 5\text{ A}$

$f_2 = 60\text{ Hz},\quad I_2 = 5.8\text{ A}$

For $f_1 = 50\text{ Hz},$

$$Z_1 = \frac{250}{5} = 50 = \sqrt{R^2 + \left(\frac{1}{2\pi f_1 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{100\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{100\pi C}\right)^2 = 2500$$

For $f_2 = 60\text{ Hz},$

$$Z_2 = \frac{250}{5.8} = 43.1\ \Omega$$

$$Z_2 = \sqrt{R^2 + \left(\frac{1}{2\pi f_2 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{120\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{120\pi C}\right)^2 = 1857.9\ \Omega$$

Solving Eqs (i) and (ii),

$R = 19.96\ \Omega$

$C = 69.4\ \mu\text{F}$

Power drawn in the second case= $I^2R = (5.8)^2 \times 19.96 = 671.45\text{ W}$

PROBLEMS ON SERIES RL & R C CIRCUITS…

BEHAVIOUR OF SERIES RLC CIRCUIT

Let V(t) =Vm sinωt => V=Vm∠0

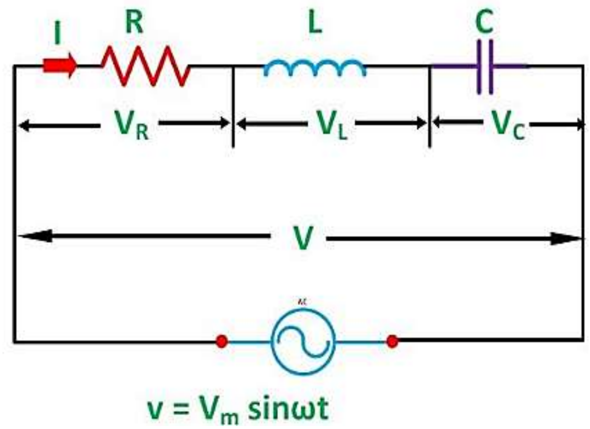
V & I are RMS Values of source voltage & current

V_R = RI (V_R & I are in phase)

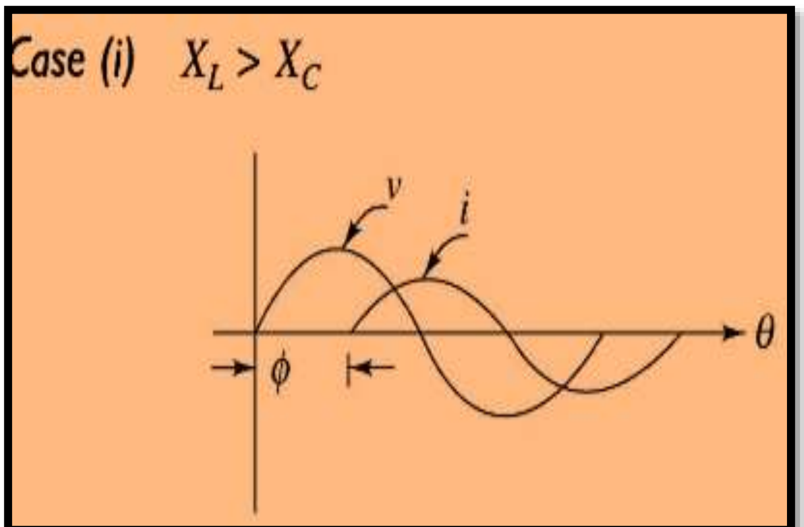
V_L = IX_L (I lags V_L by 90 degree)

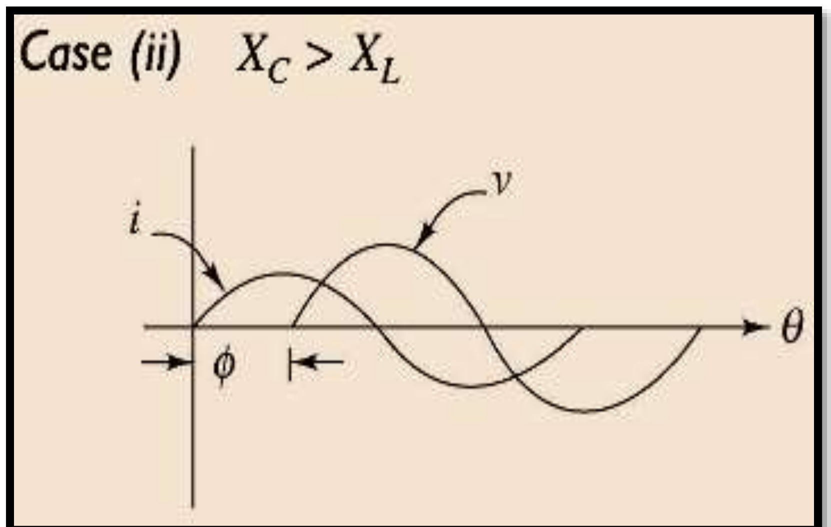
V_C = IX_C (I leads V_L by 90 degree)

Applied voltage $\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_C}$



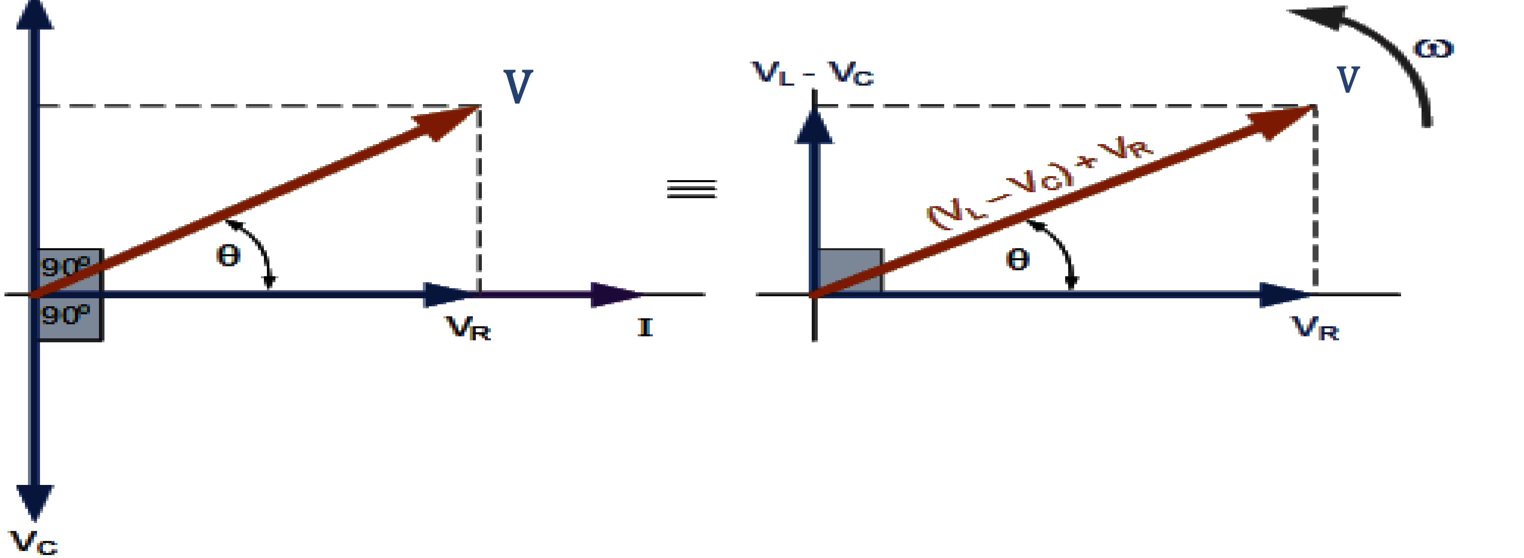
1.Wave Form & phasor representation



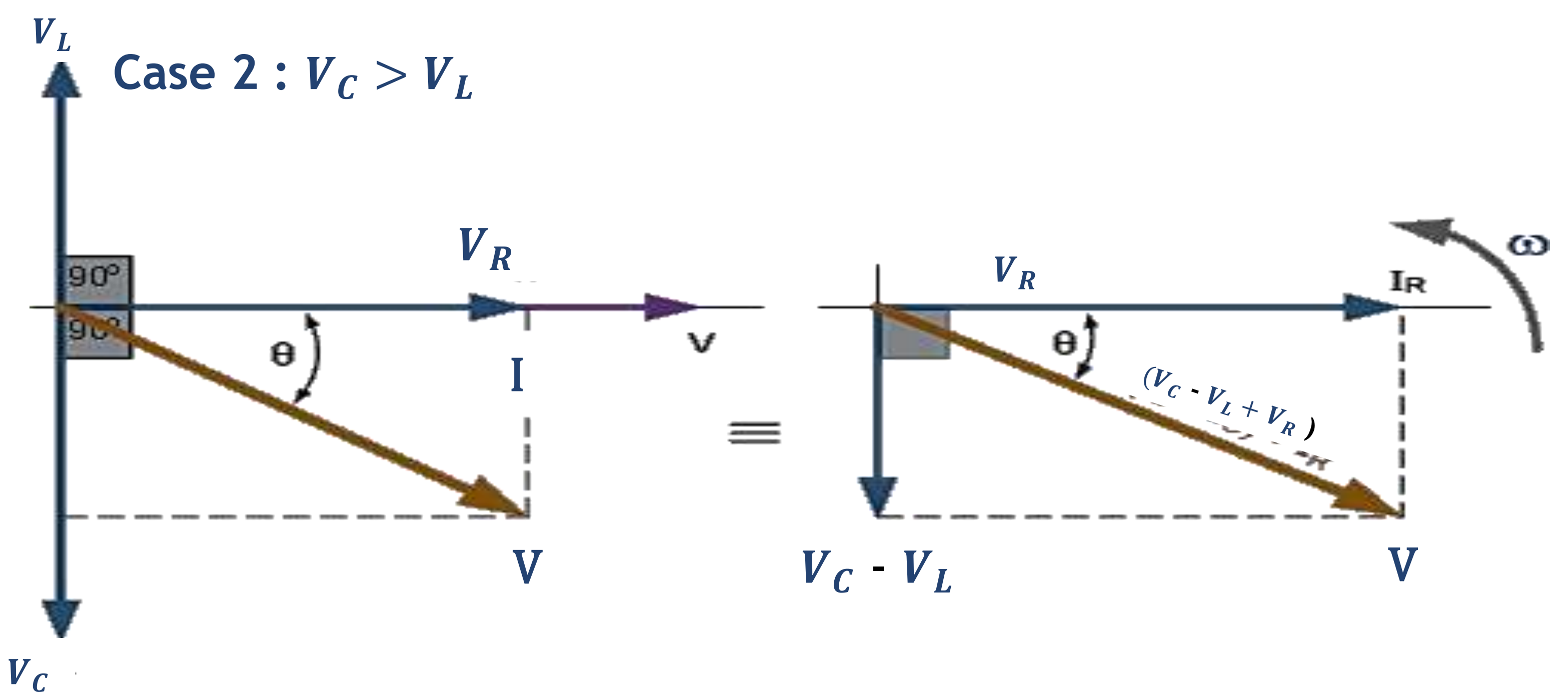


Vector Diagram

Case 1 : $V_L > V_C$



Case 2 : $V_C > V_L$



ANALYSIS OF SERIES R L C CIRCUIT…

2.Impedance (Z)

$$\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_C} = R\,\overline{I} + jX_L I - jX_C\,\overline{I} = [R + j\,(X_L - X_C)]\,\overline{I}$$

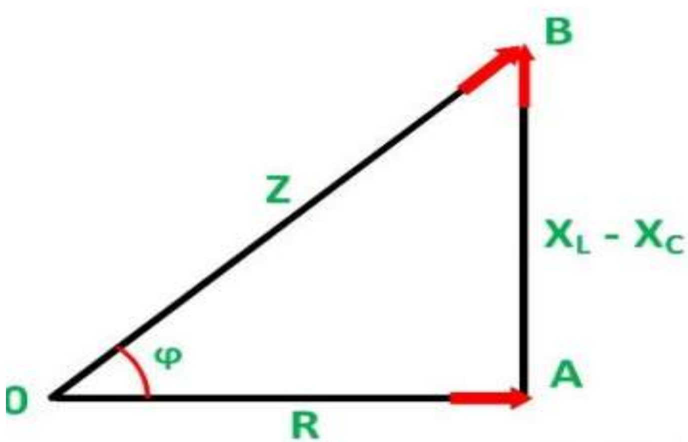
$$\frac{\overline{V}}{\overline{I}} = R + j\,(X_L - X_C) = \overline{Z}$$

$$\overline{Z} = Z \angle \phi$$

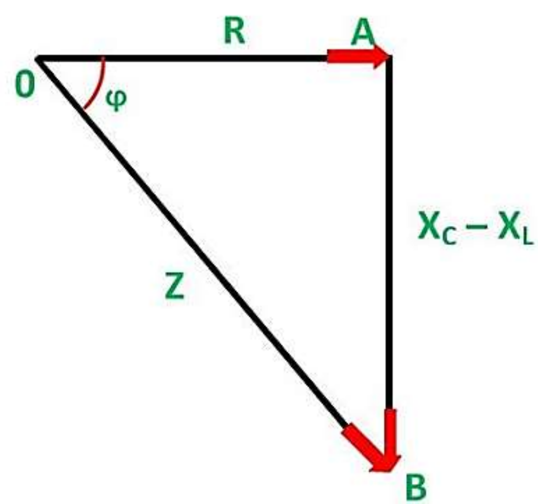
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

3.Impedance Triangle



$X_L > X_C$



$X_C > X_L$

4.Current ($I = I_R = I_L$)

$$i(t) = I_m \sin(\omega t \pm \Phi) \qquad \ldots \text{ where } I_m = \frac{V_m}{Z}$$

'–' sign is used when $X_L > X_C$

'+' sign is used when $X_C > X_L$

-

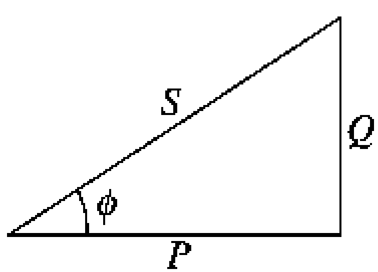
$$I_{(RMS)} = \frac{V_R}{R} = \frac{V_C}{X_C} = \frac{V_L}{X_L} = \frac{V}{Z} \qquad \text{where } z = R + jX_L - jX_C = R + j\omega L - \frac{j}{\omega C}$$

5.Power

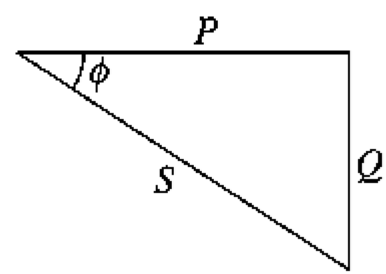
Average power	$P = VI\cos\phi = I^2R$
Reactive power	$Q = VI\sin\phi = I^2X$
Apparent power	$S = VI = I^2Z$

6.Power Triangle

Case (i) $X_L > X_C$



Case (ii) $X_C > X_L$



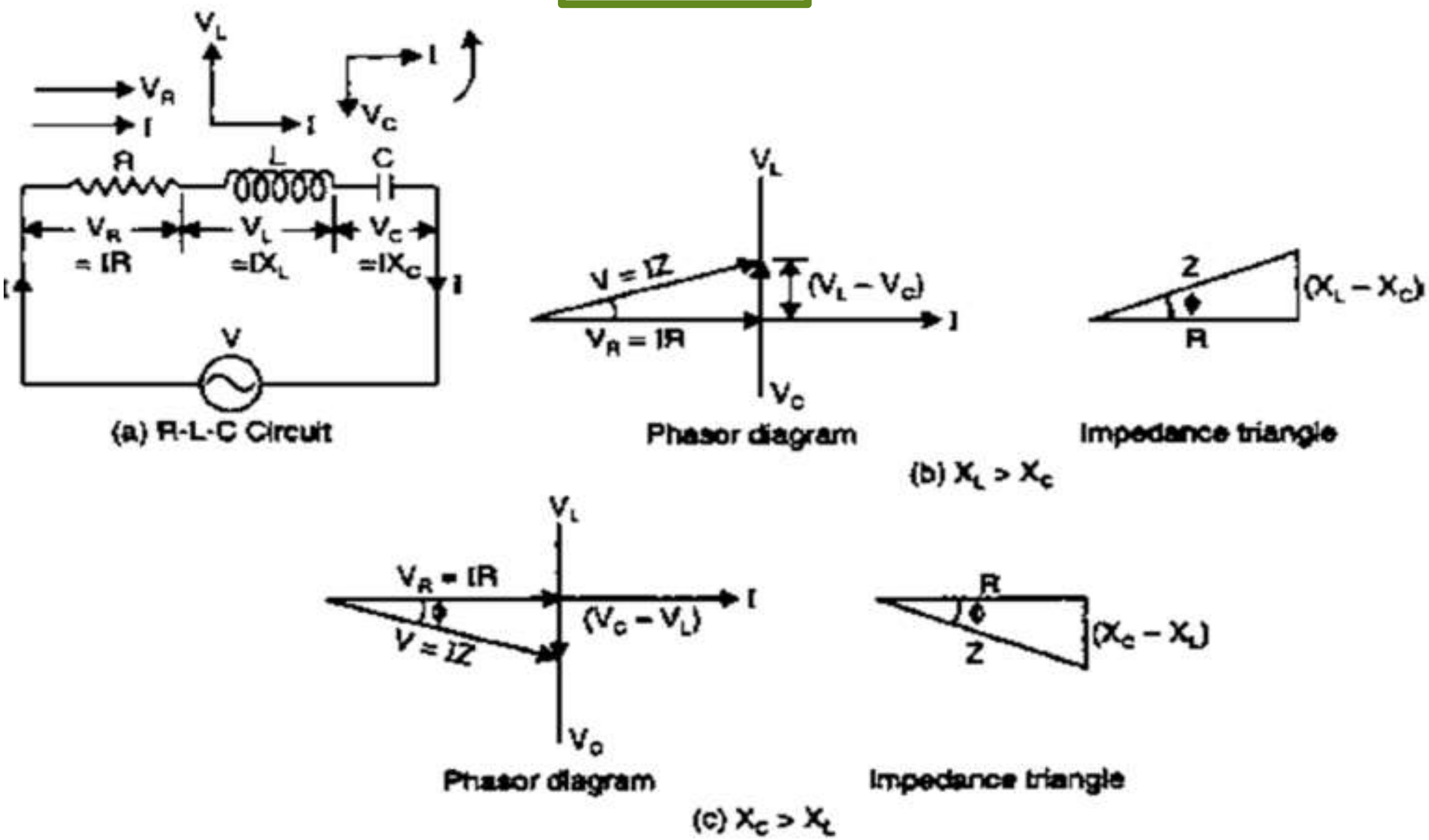
7.Power Factor

It is defined as the cosine of the angle between voltage and current phasor.

$$pf = \cos\phi$$

$$pf = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

Synopsis



(a) R-L-C Circuit

Phasor diagram — Impedance triangle

(b) $X_L > X_C$

Phasor diagram — Impedance triangle

(c) $X_C > X_L$

Example 1 : Two impedances Z_1 and Z_2 having same numerical values are connected in series .If Z_1 is having power factor 0.866 lagging of and Z_2 having power factor of 0.8 leading, calculate the power factor of series combination.

Answer:

PF = 0.9982 (leading)

Solution :

Data

$$pf_1 = 0.866 \text{ (lagging)}$$
$$pf_2 = 0.8 \text{ (leading)}$$
$$Z_1 = Z_2 = Z$$
$$\phi_1 = \cos^{-1}(0.866) = 30^\circ$$
$$\phi_2 = \cos^{-1}(0.8) = 36.87^\circ$$

$$\bar{Z}_1 = Z\angle\phi_1 = Z\angle 30^\circ = 0.866\,Z + j0.5\,Z$$

$$\bar{Z}_2 = Z\angle -\phi_2 = Z\angle -36.87^\circ = 0.8\,Z - j0.6\,Z$$

For a series combination,

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = 0.866\,Z + j0.5\,Z + 0.8\,Z - j0.6\,Z$$
$$= 1.6666\,Z - j0.1\,Z = Z\,(1.666 - j0.1) = 1.668\,Z\angle -3.43^\circ$$
$$pf = \cos(3.43^\circ) = 0.9982 \text{ (leading)}$$

Example 2 : An RLC series circuit has a current which lags the supply voltage by 45 degree. The voltage across the inductance has the maximum value equal to twice the maximum value across capacitor . Voltage across inductance is 300sin(1000t) and R= 20 ohms. Find the value of L and C.

Answers:
L = 0.04 H
C = 50 μF

Solution :

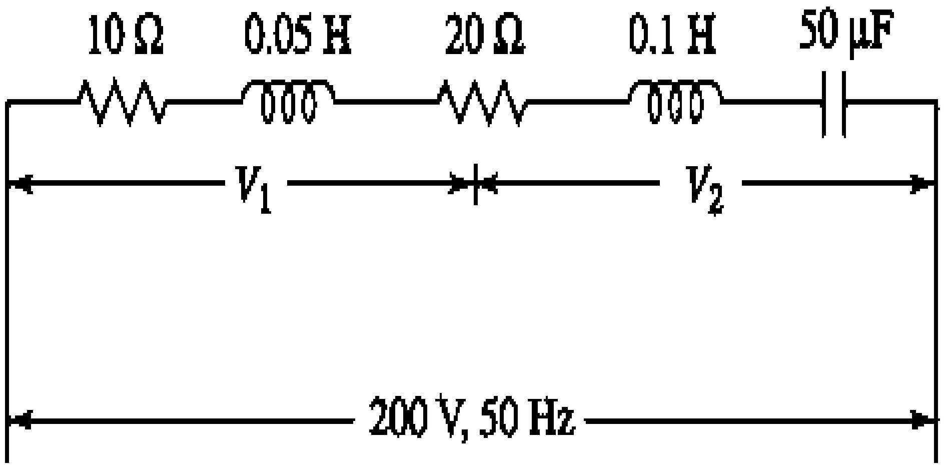
$$v_L = 300 \sin(1000t)$$
$$R = 20\ \Omega$$
$$\phi = 45°$$
$$V_{L(max)} = 2V_{C(max)}$$
$$\sqrt{2}\, V_L = 2\sqrt{2}\, V_C$$
$$I \times X_L = 2I \times X_C$$
$$X_L = 2X_C$$
$$\cos\phi = \frac{R}{Z}$$
$$\cos(45°) = \frac{20}{Z}$$
$$Z = 28.28\ \Omega$$

For a series *R-L-C* circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$(28.28)^2 = (20)^2 + (2X_C - X_C)^2$$
$$799.76 = 400 + X_C^2$$
$$X_C = 20\ \Omega$$
$$X_L = 2X_C = 40\ \Omega$$
$$X_L = \omega L$$
$$40 = 1000 \times L$$
$$L = \frac{40}{1000} = 0.04\ \text{H}$$
$$X_C = \frac{1}{\omega C}$$
$$20 = \frac{1}{1000 \times C}$$
$$C = 50\ \mu\text{F}$$

Example 3 : A Draw a vector diagram for the circuit shown in figure indicating terminal voltages V_1 and V_2 and the current. Find

1. Value of current
2. V_1 and V_2
3. Power Factor
4. Draw Vector diagram



Answers:

I = 5.48A

$V_1 = 108.74 \angle 57.52° \text{ V}$

$V_2 = 221.57 \angle -58.19° \text{ V}$

PF = 0.875 (lead)

PROBLEMS ON SERIES R L C CIRCUIT…

Solution :

$$X_{L_1} = 2\pi f L = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_{L_2} = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

Total impedance

$$\bar{Z} = 10 + j15.71 + 20 + j31.42 - j63.66$$
$$= 30 - j16.53 = 34.25\ \angle -28.85^\circ\ \Omega$$

$$I = \frac{V}{Z} = \frac{200}{34.25} = 5.84\ \text{A}$$

Let

$$\bar{I} = 5.84\ \angle 0^\circ\ \text{A}$$

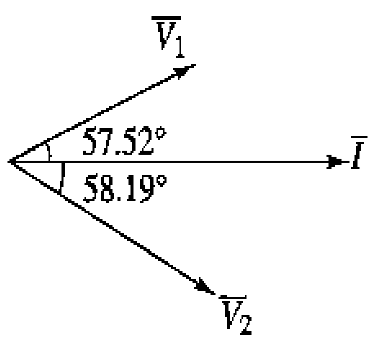
$$\bar{Z}_1 = 10 + j15.71 = 18.62\ \angle 57.52\ \Omega$$

$$\bar{V}_1 = \bar{Z}_1 \cdot \bar{I} = (18.62\ \angle 57.52^\circ)\ (5.84\ \angle 0^\circ) = 108.74\ \angle 57.52^\circ\ \text{V}$$

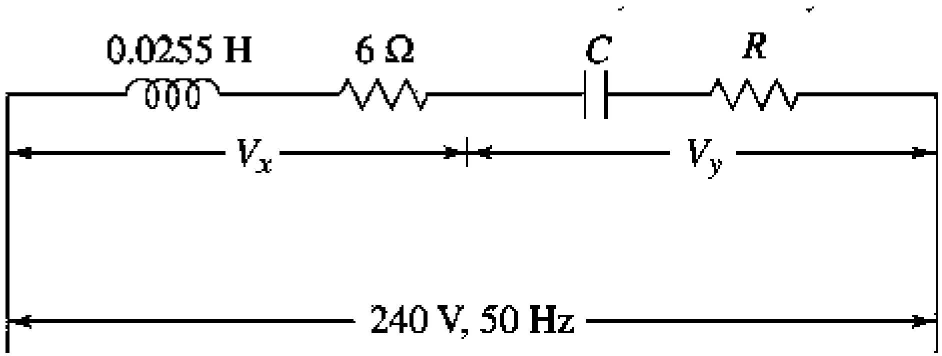
$$\bar{Z}_2 = 20 + j31.42 - j63.66$$
$$= 20 - j32.24 = 37.94\ \angle -58.19^\circ\ \Omega$$

$$\bar{V}_2 = \bar{Z}_2 \cdot \bar{I} = (37.94\ \angle -58.19^\circ)\ (5.84\ \angle 0^\circ) = 221.57\ \angle -58.19^\circ\ \text{V}$$

$$\text{pf} = \cos\phi = \cos(28.85^\circ) = 0.875\ \text{(leading)}$$



Example 4 : *Find the values of R and C so that $V_x = 3V_y$, V_x and V_y are in quadrature.*



Answers:
R = 2.66 Ω
C = 1.59mF

Solution :

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.0255 = 8\ \Omega$$

$$\bar{Z}_x = 6 + j8 = 10\ \angle 53.13°\ \Omega$$

$$V_x = 3V_y$$

$$I \times Z_x = 3 \times I \times Z_y$$

$$Z_x = 3Z_y$$

V_x and V_y are in quadrature, i.e., phase angle between V_x and V_y is 90°. Hence, the angle between Z_x and Z_y will be 90°. The impedance Z_y is capacitive in nature.

$$\bar{Z}_y = Z_y \angle -\phi$$

$$\bar{Z}_y = \frac{10}{3} \angle (53.13 - 90)° = 3.33\ \angle -36.87° = 2.66 - j2\ \Omega$$

$$R = 2.66\ \Omega$$

$$X_C = 2\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$C = \frac{1}{2\pi \times 50 \times 2} = 1.59\text{ mF}$$

Example 5 : A 250v,50Hz voltage is applied to a coil having resistance of 5 ohm and an inductance of 9.55H in series with a capacitor C. If the voltage across the coil is 300v, Find value of C.

Answers:
C = 6.37μF : $X_L > X_C$
C = 0.58μ F : $X_C > X_L$

Solution :

$$V = 250\text{ V}$$
$$R = 5\ \Omega$$
$$L = 9.55\text{ H}$$
$$V_{\text{coil}} = 300\text{ V}$$
$$X_L = 2\pi f L = 2\pi \times 50 \times 9.55 = 3000\ \Omega$$
$$Z_{\text{coil}} = \sqrt{R^2 + X_L^2} = \sqrt{(5)^2 + (3000)^2} = 3000\ \Omega$$
$$I = \frac{V_{\text{coil}}}{Z_{\text{coil}}} = \frac{300}{3000} = 0.1\text{ A}$$

Total impedance $Z = \frac{V}{I} = \frac{250}{0.1} = 2500\ \Omega$

When $X_L > X_C$,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$(2500)^2 = (5)^2 + (3000 - X_C)^2$$
$$(3000 - X_C) = 2500$$
$$X_C = 500$$
$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 500} = 6.37\ \mu\text{F}$$

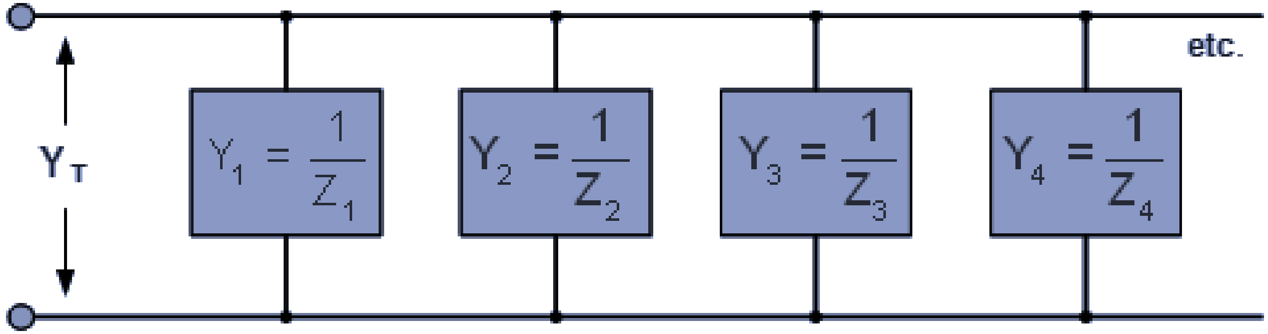
When $X_C > X_L$,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$
$$(2500)^2 = (5)^2 + (X_C - 3000)^2$$
$$2500 = X_C - 3000$$
$$X_C = 5500$$
$$C = \frac{1}{2\pi \times 50 \times 5500} = 0.58\ \mu\text{F.}$$

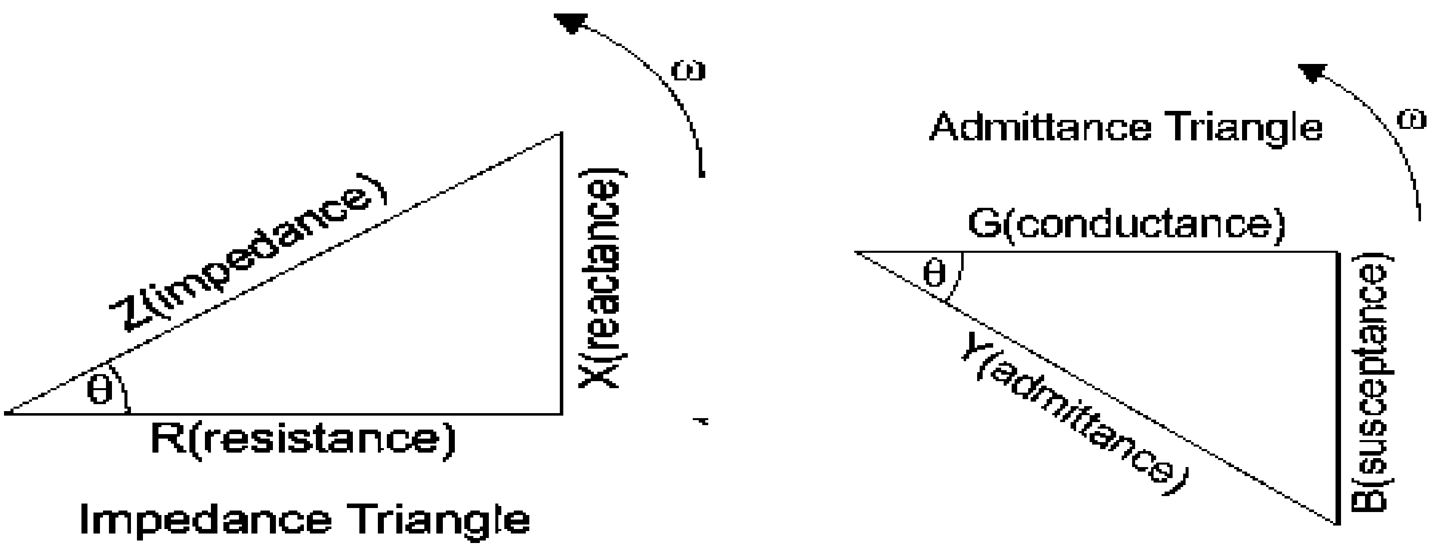
ADMITTANCE AND ITS COMPONENTS

	IMPEDANCE (Z)	ADMITTANCE(Y)
Definition	Electrical impedance is the measure of the opposition that a circuit presents to a current when a voltage is applied.	Admittance of a circuit is defined as the reciprocal of impedance
Unit	Ohm (Ω)	Mho(℧) or siemens (s)
Mathematical expression	$Z = \frac{V}{I}$ = R+jX	$Y = \frac{I}{V}$ = G+jB
Its components	Real {Z} = R = Resistance Imag {Z }=X = Impedance	Real {Y} = G = Conductance Imag {Y}= B = Susceptance
Properties	Impedances in series add up to get net impedance	Admittance in parallel add up to get net admittance

$$\frac{1}{Z_T} = Y_T = Y_1 + Y_2 + Y_3 + Y_4 + \ldots\ldots \text{etc}$$



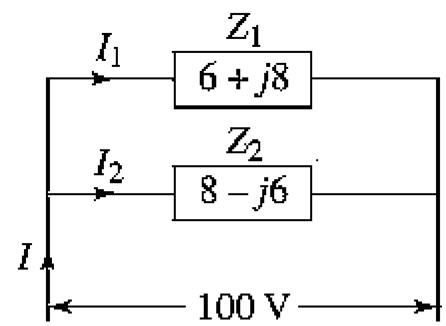
IMPEDANCE AND ADMITTANCE TRIANGLE



Example 1 : Two circuits the impedances are given by Z_1= (6+8j)Ω and Z_2 = (8-6j) are connected in parallel. If the applied voltage to the combination is 100v, find

1. Current and PF of each branch
2. Over all current and power factor of combination
3. Power consumed by each impedance

Solution :



$$\bar{Z}_1 = 6 + j8\ \Omega \qquad \bar{Z}_2 = 8 - j6\ \Omega$$

$$V = 100\ V$$

$$\bar{V} = 100\ \angle 0°\ \text{V}$$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{100\angle 0°}{6 + j8} = 10\ \angle{-53.13°}\ \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_1} = \frac{100\angle 0°}{8 - j6} = 10\ \angle 36.9°\ \text{A}$$

$$\cos\phi_1 = \cos(53.13°) = 0.6\ \text{(lagging)}$$

$$\cos\phi_2 = \cos(36.9°) = 0.8\ \text{(leading)}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 10\ \angle{-53.13°} + 10\ \angle 36.9°$$

$$= 14.14\ \angle{-8.13°}\ \text{A}$$

$$\text{pf} = \cos\phi = \cos(8.13°) = 0.989\ \text{(lagging)}$$

$$P_1 = I_1^2 R_1 = (10)^2 \times (6) = 600\ \text{W}$$

$$P_2 = I^2 R_2 = (10)^2 \times (8) = 800\ \text{W}$$

Answers:

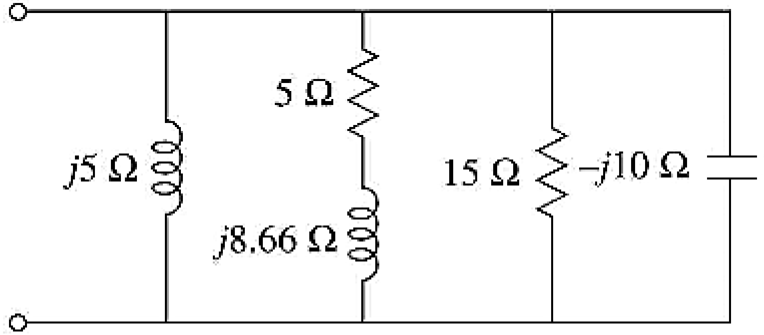
I_1= 10∠-53.13 & I_2= 10∠36.9

PF1 = 0.6 (Lag) & PF2 = 0.8(Lead)

I = 14.14∠-8.13 , PF = 0.989 (Lag)

P_1= 600w & P_1 = 800w

Example 2 : *Compute Z_{eq} and Y_{eq} for the given circuit.*



Answers:

Z_{eq}= 4.54∠57.99

Y_{eq}= 0.22∠-57.99

Solution :

$$\bar{Z}_1 = j5\,\Omega$$

$$\bar{Z}_2 = 5 + j8.66\ \Omega$$

$$\bar{Z}_3 = 15\ \Omega$$

$$\bar{Z}_4 = -j10\ \Omega$$

$$\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_2 + \bar{Y}_4$$

$$= \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3} + \frac{1}{\bar{Z}_4}$$

$$= \frac{1}{j5} + \frac{1}{5 + j8.66} + \frac{1}{15} + \frac{1}{-j10}$$

$$= 0.22 \angle -57.99^\circ\ \mho$$

$$\bar{Z}_{eq} = \bar{Y}_{eq} = \frac{1}{0.22\angle -57.99^\circ} = 4.54 \angle 57.99^\circ\ \Omega$$

Example 3 : A voltage of 200∠ 25v is applied to a circuit composed of two parallel branches. If the branch currents are 10 ∠ 40 and 20∠-30, determine

1. KW,KVA & KVAR of each branch
2. PF of combined load

Answers:

P_1 = 1.94 KW
Q_1 = 0.52 KVAR
S_1 = 2 KVA

P_1 = 2.28 KW
Q_1 = 3.28 KVAR
S_1 = 4 KVA

PF = 0.84(Lag)

Solution :

Data $\overline{I_1} = 10\angle 40^\circ$ A $\overline{I_2} = 20\angle -30^\circ$ A

$$\overline{V} = 200\angle 25^\circ \text{ V}$$

Phase difference between V and I_1, $\phi_1 = 40^\circ - 25^\circ = 15^\circ$

Phase difference between V and I_2, $\phi_2 = 25^\circ - (-30^\circ) = 55^\circ$

$$\cos\phi_1 = \cos(15^\circ) = 0.97 \text{ (leading)}$$

$$\cos\phi_2 = \cos(55^\circ) = 0.57 \text{ (lagging)}$$

For the branch current of $10\angle 40^\circ$ A,

$$P_1 = VI_1\cos\phi_1 = 200\times 10\times 0.97 = 1.94 \text{ kW}$$

$$Q_1 = VI_1\sin\phi_1 = 200\times 10\times \sin(15^\circ) = 0.52 \text{ kVAR}$$

$$S_1 = VI_1 = 200\times 10 = 2 \text{ kVA}$$

For the branch current of $20\angle -30^\circ$ A,

$$P_2 = VI_2\cos\phi_2 = 200\times 20\times 0.57 = 2.28 \text{ kW}$$

$$Q_2 = VI_2\sin\phi_2 = 200\times 20\times \sin(55^\circ) = 3.28 \text{ kVAR}$$

$$S_2 = VI_2 = 200\times 20 = 4 \text{ kVA}$$

For the combined load,

$$\overline{I_1} = 10\angle 40^\circ \text{ A}$$

$$\overline{I_2} = 20\angle -30^\circ \text{ A}$$

$$\overline{I} = \overline{I_1} + \overline{I_2}$$

$$= 10\angle 40^\circ + 20\angle -30^\circ = 25.24\angle -8.14^\circ \text{ A}$$

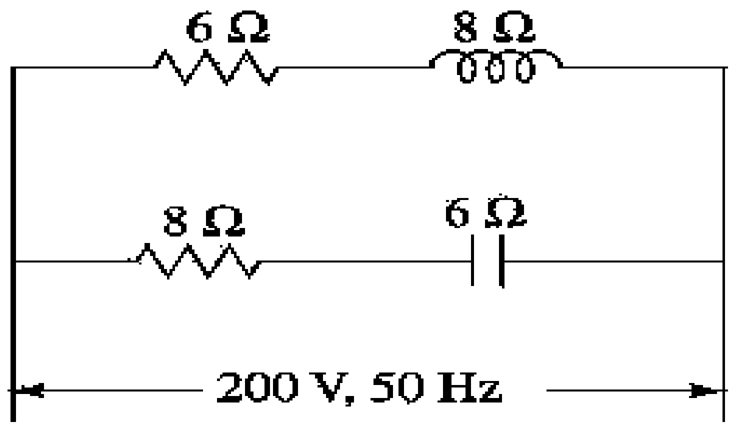
$$\text{Phase difference} = 25^\circ - (-8.14^\circ)$$

$$= 33.14^\circ$$

$$\text{pf} = \cos(33.13^\circ) = 0.84 \text{ (lagging)}$$

Example 4 : For the circuit shown, calculate

1. Total admittance, conductance and succeptance
2. Total current and total power factor
3. Value of pure capacitance to be connected in parallel with the above combination to make total power factor unity



Answers:

Y = 0.14 ℧ , G=0.14 ℧ , B=0.02 ℧

I = 28 ∠ -8.13 A, PF =0.989(lagging)

C = 63.667μF

Solution : (i)

$$\bar{Z}_1 = 6 + j8 = 10 \angle 53.13° \ \Omega$$

$$\bar{Z}_2 = 8 - j6 = 10 \angle -36.87° \ \Omega$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{10\angle 53.13°} = 0.1 \angle -53.13° \ ℧$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10\angle -36.87°} = 0.1 \angle 36.87° \ ℧$$

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2$$

$$= 0.1 \angle -53.13° + 0.1 \angle 36.87°$$

$$= 0.14 - j0.02 \ ℧ = 0.14 \angle -8.13° \ ℧$$

Total admittance $Y = 0.14 \ ℧$

Total conductance $G = 0.14 \ ℧$

Total susceptance $B = 0.02 \ ℧$

Let $\bar{V} = 200 \angle 0° \text{ V}$

(ii) $I = \bar{V} \cdot \bar{Y}$

$$= (200 \angle 0°)(0.14 \angle -8.13°) = 28 \angle -8.13° \text{ A}$$

Total pf = cos (8.13°) = 0.989 (lagging)

(iii) Since the current lags behind voltage, the circuit is inductive in nature. In order to make the total pf unity, a pure capacitor is connected in parallel so that pf becomes unity and imaginary part of $\bar{Y}_{req}$ becomes zero.

$$\bar{Y}_{req} = 0.14 - j0.02 + j0.02 = 0.14$$

$$\frac{1}{X_C} = 0.02$$

$$X_C = 50 \ \Omega$$

$$C = \frac{1}{2\pi \times 50 \times 50} = 63.66 \ \mu\text{F}$$

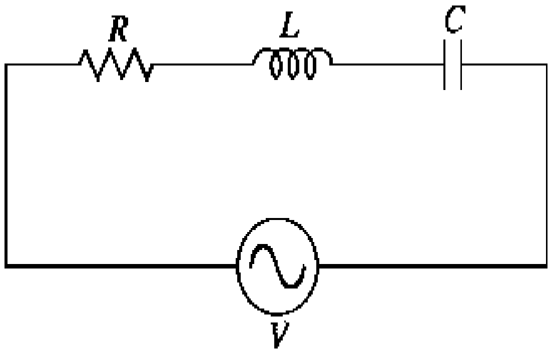
RESONANCE

1. Definition
2. Resonance frequency
3. Power Factor
4. Current
5. Voltage
6. Phasor Diagram
7. Behavior of RLC with change in frequency
8. Band width
9. Quality Factor

1 Definition

A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with the current in the circuit. At resonance, the circuit thus behaves as a pure resistor and net reactance is zero.

Consider an R-L-C series circuit shown in fig. The impedance is given by



$$\bar{Z} = R + jX_L - jX_C$$

$$= R + j\omega L - \frac{j}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

At resonance, Z must be resistive. Therefore the condition for resonance is

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

2 Resonance frequency

The frequency at which a circuit containing reactance acts as purely resistive is called resonant frequency (f_0).

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

3 Power Factor

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

But at resonance $\omega L = \frac{1}{\omega C}$

$$\text{Power factor} = \frac{R}{R} = 1$$

4 Current

Since impedance is minimum , the current is maximum at resonance. Thus the circuit accepts more current and as such, an RLC under resonance is called an *acceptor circuit* .

$$I_0 = \frac{V}{Z} = \frac{V}{R}$$

5 Voltage

At resonance,

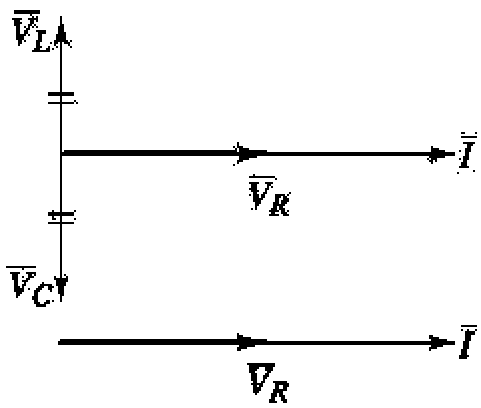
$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 L\, I_0 = \frac{1}{\omega_0 C}\, I_0$$

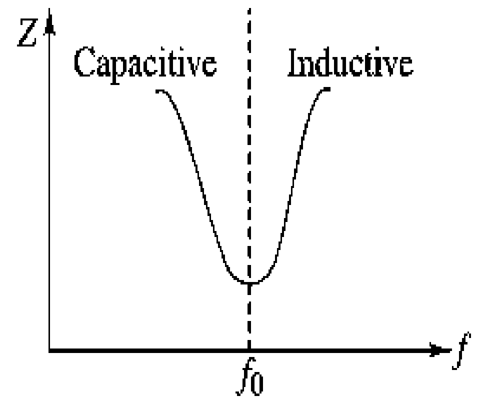
$$V_{L_0} = V_{C_0}$$

At resonance potential across inductance and capacitor are equal and opposite, hence cancel each other. Also as I_0 is maximum, voltage across capacitor and inductor are also maximum. Thus R-L-C series circuit at resonance is also called as *voltage magnification circuit* .

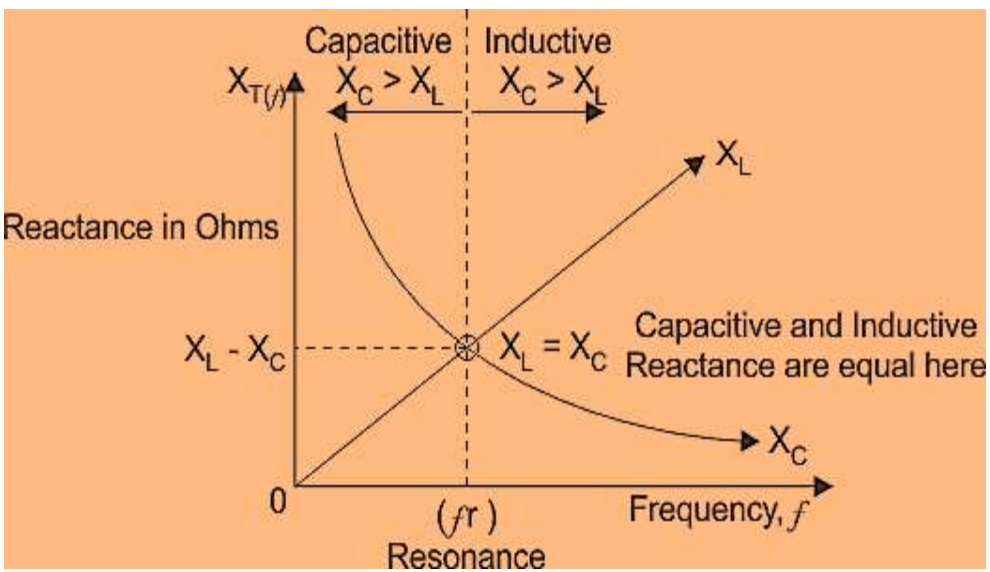
6 Phasor Diagram



$$\overline{Z} = R + j\,(X_L - X_C)$$



7 Behavior of R-L-C with change in frequency



At $f<f_0$

Impedance is capacitive and decreases up to f_0. PF is leading

At $f=f_0$

Impedance is resistive and PF is unity

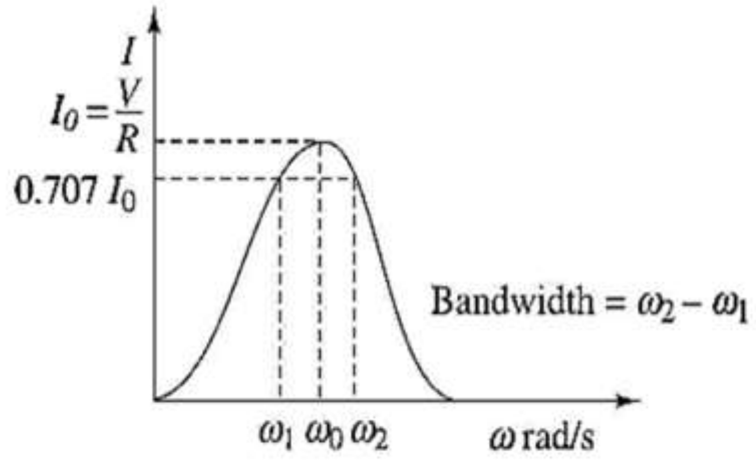
At $f>f_0$

Impedance is inductive and goes on increasing beyond f_0.PF is lagging

8 Band width & Half power frequencies

Half power frequencies

The frequencies for which the power delivered to a resistor is half or more than half the power at resonance are called half power frequencies



Band width

The range of frequencies for which the power delivered to a resistor is greater than or equal to half the power delivered at resonance is called band width

Hence $$I_1^2 R = \frac{1}{2} I_0^2 R = I_2^2 R$$

where the subscript 1 denotes the lower half point and the subscript 2, the higher half point. It follows then that

$$I_1 = I_2 = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

Alternately , the range of frequencies for which the magnitude of current is greater than or equal to 0.707 times the current at resonance is called band width

Expression for the bandwidth Generally, at any frequency ω,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{... (4.1)}$$

At half-power points,

$$I = \frac{I_0}{\sqrt{2}}$$

But

$$I_0 = \frac{V}{R}$$

$$I = \frac{V}{\sqrt{2}R} \quad \text{... (4.2)}$$

From Eqs (4.1) and (4.2), we get

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}R}$$

$$\frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}R}$$

Squaring both sides we get,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} \pm R = 0$$

$$\omega^2 \pm \frac{1}{2}\omega - \frac{1}{LC} = 0$$

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

For low values of R, the term $\left(\frac{R^2}{4L^2}\right)$ can be neglected in comparison with the term $\frac{1}{LC}$.

Then ω is given by, $\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{1}{LC}} = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$

The resonant frequency for this circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega = \pm \frac{R}{2L} + \omega_0$$

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

and $\omega_2 = \omega_0 + \frac{R}{2L}$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/s}$$

or $\text{Bandwidth} = f_2 - f_1 = \text{c/s}$

9 Quality Factor or Voltage magnification factor

The ratio of resonant frequency to the bandwidth is called Quality Factor. It is the measure of selectivity or sharpness of tuning of series R-L-C circuit.

$$Q_0 = \frac{\omega_0}{\text{Bandwidth}}$$

$$Q_0 = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R} = \frac{X_{L0}}{R} = \frac{1}{\omega_0 RC}$$

$$Q_0 = \frac{1/\sqrt{LC}}{R/L} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q_0 = \frac{V_{L_0}}{V} = \frac{V_{C_0}}{V}$$

where V_{L_0} and V_{C_0} are both measured at resonance. Hence, Q_0 is also called *voltage magnification factor.*

Example 1 : An RLC Series circuit with a resistance of 10ohms, inductance of 0.2H and a capacitance of 40 μF is supplied with a 100V , variable frequency source. Find the following with respect to series resonance circuit

1. Frequency at which resonance takes place
2. current at resonance
3. Power at resonance
4. Power Factor
5. Voltage across R,L,C at resonance
6. Quality factor
7. Half power points
8. Resonance and phasor diagram

Answers:

1. f_0 = 56.3 Hz
2. I_0 =10A
3. P_0 =1000W
4. $\text{PF} = 1$
5. V_R=100V, V_L=707.5V, V_C=707.5v
6. Q_0 = 7.07
7. $f_1 = 52.32Hz$ $f_2 = 60.3Hz$

Solution

Data

$R = 10\ \Omega$
$L = 0.2\ \text{H}$
$C = 40\ \mu\text{F}$
$V = 100\ \text{V}$

(i) Resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.2 \times 40 \times 10^{-6}}} = 56.3\ \text{Hz}$

(ii) Current $I_0 = \dfrac{V}{R} = \dfrac{100}{10} = 10\ \text{A}$

(iii) Power $P_0 = I_0^2 R = (10)^2 \times 10 = 1000\ \text{W}$

(iv) Power factor pf = 1

(v) Voltage across $R = R \cdot I = 10 \times 10 = 100\ \text{V}$

Voltage across $L = X_L \cdot I = 2\pi \times 56.3 \times 0.2 \times 10 = 707.5\ \text{V}$

Voltage across $C = X_C \cdot I = \dfrac{1}{2\pi \times 56.3 \times 40 \times 10^{-6}} \times 10 = 707.5\ \text{V}$

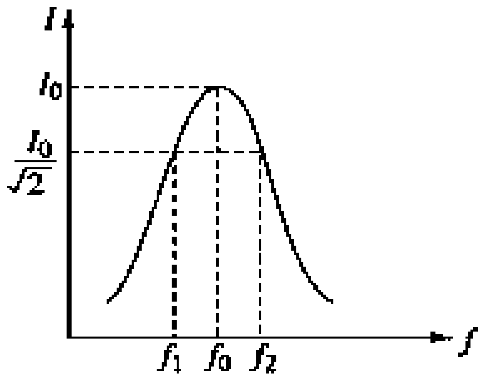
(vi) Quality factor $Q = \dfrac{1}{R} \times \sqrt{\dfrac{L}{C}} = \dfrac{1}{10}\sqrt{\dfrac{0.2}{40 \times 10^{-6}}} = 7.07$

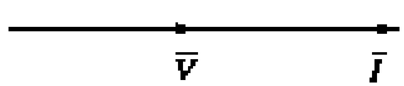
(vii) Half-power points

$$f_1 = f_0 - \frac{R}{4\pi L} = 56.3 - \frac{10}{4\pi(0.2)} = 52.32\ \text{Hz}$$

$$f_2 = f_0 + \frac{R}{4\pi L} = 56.3 + \frac{10}{4\pi(0.2)} = 60.3\ \text{Hz}$$

(viii) *Resonance and phasor diagram*





Example 2 : A RLC Series has the following parameter values : R=10ohm, L= 0.01H ,C=100μF. Compute

1. Resonance frequency
2. Band width
3. Lower and upper frequency of band width.

Answers:

1. f_0 = 159.15 Hz
2. BW=159.15 Hz
3. $f_1 = 79.58\ Hz$ $f_2 = 238.73\ Hz$

Solution

Data $R = 10\ \Omega$

$L = 0.01\ \text{H}$

$C = 100\ \mu\text{F}$

Resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.01 \times 100 \times 10^{-6}}} = 159.15\ \text{Hz}$

Bandwidth $BW = \dfrac{R}{2\pi L} = \dfrac{10}{2\pi \times 0.01} = 159.15\ \text{Hz}$

Lower frequency of bandwidth $f_1 = f_0 - \dfrac{BW}{2}$

$$= 159.15 - \frac{159.15}{2} = 79.58\ \text{Hz}$$

Higher frequency of bandwidth $f_2 = f_0 + \dfrac{BW}{2} = 159.15 + \dfrac{159.15}{2} = 238.73\ \text{Hz}$

Example 3 : A Resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a variable 230V, 50hz supply, the maximum current obtained by varying inductance is 2A. Calculate R,L,C of circuit.

Answers:

1. R = 115 Ohm
2. L = 0.795H
3. C =12.73 μF

Solution

Data

$$V = 230\ \text{V}$$
$$f_0 = 50\ \text{Hz}$$
$$I_0 = 2\ \text{A}$$
$$V_{C_0} = 500\ \text{V}$$

Resistance

$$R = \frac{V}{I_0} = \frac{230}{2} = 115\ \Omega$$

$$X_{C_0} = \frac{V_{C0}}{I_0} = \frac{500}{2} = 250\ \Omega$$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$250 = \frac{1}{2\pi \times 50 \times C}$$

Capacitance

$$C = 12.73\ \mu\text{F}$$

At resonance

$$X_{C_0} = X_{L_0}$$

$$X_{L_0} = 250\ \Omega$$

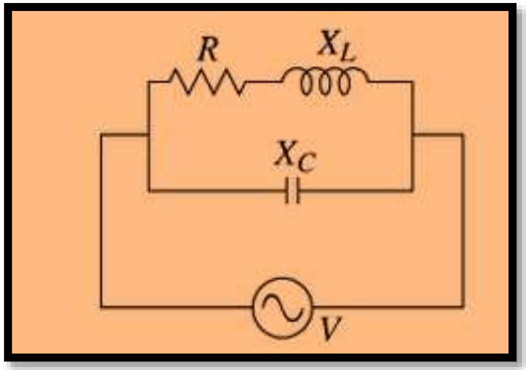
$$X_{L_0} = 2\pi f_0 L$$

$$250 = 2\pi \times 50 \times L$$

Inductance

$$L = 0.795\ \text{H}$$

PARALLEL RESONANCE



$$\bar{Z}_1 = R + jX_L$$

$$\bar{Z}_2 = -jX_C$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{R + jX_L}$$

$$= \frac{R - jX_L}{R^2 + X_L^2}$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$$

Admittance of the circuit $\bar{Y} = \bar{Y}_1 + \bar{Y}_2$

$$= \frac{R - jX_L}{R^2 + X_L^2} + j\frac{1}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} - j\left(\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive. Therefore the condition for resonance is

$$\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$X_L . X_C = R^2 + X_L^2$$

$$\omega_0 L \cdot \frac{1}{\omega_0 C} = R^2 + \omega_0^2 L^2$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

where f_0 is called the resonant frequency of the circuit.

If R is very small as compared to L, then

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Dynamic Impedance of Parallel Circuit At resonance, the circuit is purely resistive. The real part of admittance is $\frac{R}{R^2 \times X_L^2}$. Hence, the dynamic impedance at resonance is given by

$$Z = \frac{R^2 + X_L^2}{R}$$

At resonance,

$$R^2 + X_L^2 = X_L \cdot X_C = \frac{L}{C}$$

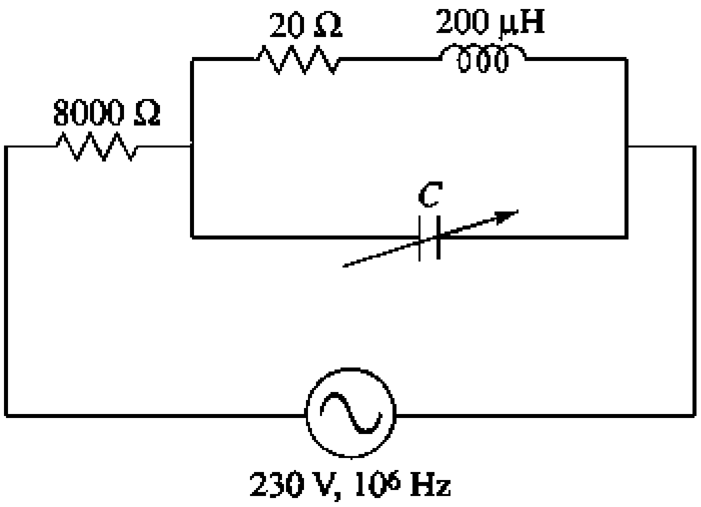
$$Z = \frac{L}{CR}$$

COMPARISION OF SERIES AND PARALLEL RESONANCE

Parameter	*Series Circuit*	*Parallel Circuit*
Current at resonance	$I = \frac{V}{R}$ and is maximum	$I = \frac{V}{(L/CR)}$ and is minimum
Impedance at resonance	$Z = R$ and is minimum	$Z = \frac{L}{CR}$ and is maximum
Power factor at resonance	Unity	Unity
Resonant frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Q-factor	$Q = \frac{2\pi fL}{R}$	$Q = \frac{2\pi fL}{R}$
It magnifies	Voltage across L and C	Current through L and C

Example 1 : A coil having resistance of 20ohm and inductance of 200μH is connected in parallel with a variable capacitor. This parallel combination is connected in series with a resistance of 8000 ohm. A voltage of 230V at a frequency of 10^6 Hz is applied across the circuit. Calculate

1. Value of capacitance at resonance
2. Q factor of circuit
3. Dynamic impedance of the circuit
4. Total circuit current



Answers:

1. C = 126.65pF
2. Q = 62.83
3. Z =78958 Ohm
4. I = 2.65 mA

Solution

Data

$$R = 20\ \Omega \qquad L = 200\ \mu\text{H}$$

$$f = 10^6\ \text{Hz} \qquad V = 230\ \text{V}$$

$$R_S = 8000\ \Omega$$

230 V

Fig

$$X_L = 2\pi f L = 2 \times \pi \times 10^6 \times 200 \times 10^{-6} = 1256.6\ \Omega$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{200\times10^{-6}\times C} - \frac{(20)^2}{(200\times10^{-6})^2}}$$

$$C = 126.65 \times 10^{-12}\ \text{F} = 126.65\ \text{pF}$$

Quality Factor

$$Q_0 = \frac{2\pi fL}{R}$$

$$= \frac{2\pi\times10^6\times200\times10^{-6}}{20} = 62.83$$

Dynamic Impedance

$$Z = \frac{L}{CR}$$

$$= \frac{200\times10^{-6}}{126.65\times10^{-12}\times20} = 78958\ \Omega$$

Total equivalent impedance of the circuit at resonance

$$= 78958 + 8000 = 86958\ \Omega$$

$$\text{Total circuit current} = \frac{230}{86958}$$

$$= 2.645 \times 10^{-3}\ \text{A}$$

$$= 2.65\ \text{mA}$$

Example 2 *A coil of 20-Ω resistance has an inductance of 0.2 H and is connected in parallel with a condenser of 100 μF capacitance. Calculate the frequency at which this circuit will behave as a non-inductive resistance. Find also the value of dynamic resistance.*

ANS : R= 100 ohms

Solution

Data $R = 20\ \Omega$ $L = 0.2$ H

$C = 100\ \mu F$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 100 \times 10^{-6}} - \left(\frac{20}{0.2}\right)^2} = 31.83 \text{ Hz}$$

Dynamic resistance $= \dfrac{L}{CR}$

$$= \frac{0.2}{100 \times 10^{-6} \times 20} = 100\ \Omega$$

MAGNETIC CIRCUITS

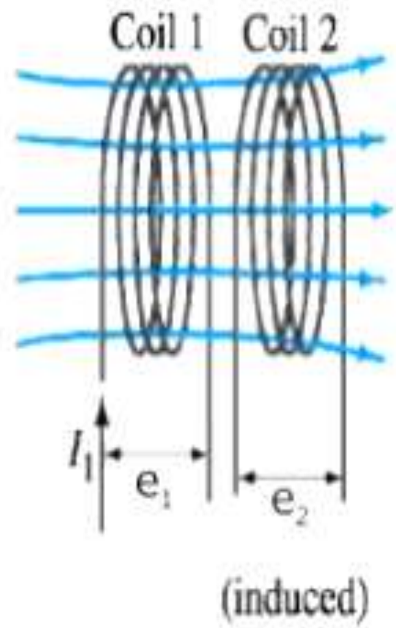
- The induced emf, e, in a coil is proportional to the rate of the change of the magnetic flux passing through it due to its own current. This emf is termed as **Self Induced EMF**
- The induced emf e is proportional to the rate of change of current through coil and this proportionality constant is called the **self inductance**, L.

$$e_1 = -L \frac{di}{dt}$$

- The negative sign is used to indicate that EMF is opposing the cause producing it

- If two coils of wire are placed near each other, a change of current in one coil will induce emfs e_1 in the first coil and e_2 in the second coil.

- The induced emf, e_2, in coil 2 is proportional to the rate of the change of the magnetic flux passing through it and hence proportional to rate of change of current in first coil and is termed as **Mutually induced EMF**.



The induced emf e_2 is proportional to the rate of change of current through coil 1 and this proportionality constant is called the mutual inductance, M

The mutually induced emf is expressed as

$$e_2 = M \frac{di_1}{dt}$$

This induced emf can also be expressed as

$$e_2 = N_2 \frac{d\varphi_{12}}{dt}$$

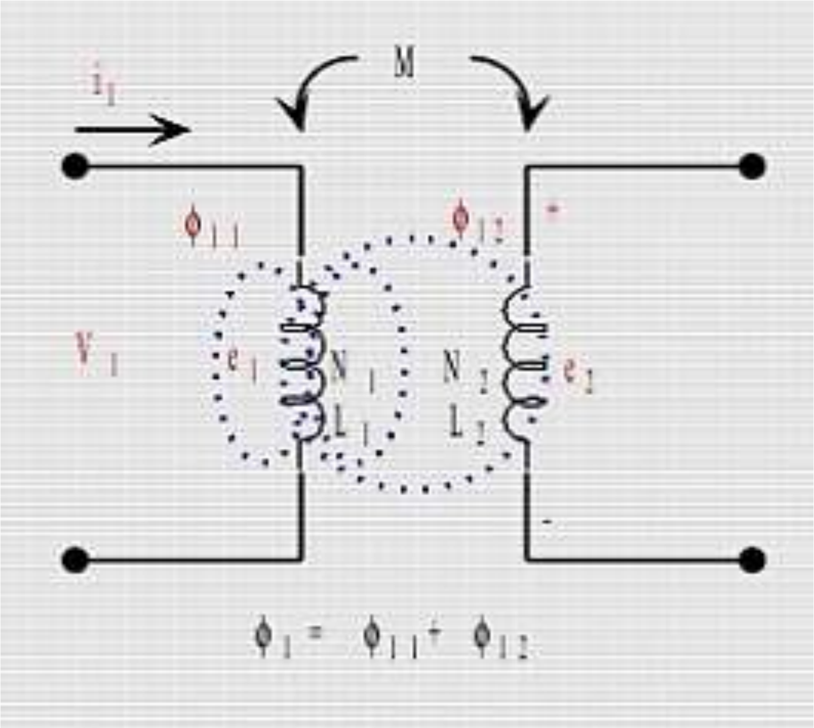
- Therefore

$$\mathrm{M} = \mathrm{N}_2 \frac{d\phi_{12}}{di_1}$$

If μ_r is constant, $\frac{d\phi_{12}}{di_1}$ is constant and

$$\mathrm{M} = \mathrm{N}_2 \frac{\Phi_{12}}{\mathrm{I}_1}$$

Unit: Henry (H)



Self Inductances L_1 and L_2 are

$$L_1 = \frac{N_1 \Phi_1}{I_1} \quad \text{and} \quad L_2 = \frac{N_2 \Phi_2}{I_2}$$

Mutual Inductance M

$$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_1 \Phi_{21}}{I_2}$$

where $\Phi_{12} = k\,\Phi_1$; $\Phi_{21} = k\,\Phi_2$ and

k is the coupling coeficient

$$L_1 L_2 = \frac{M^2}{k^2} \quad \text{or} \quad k = \frac{M}{\sqrt{L_1 L_2}}$$

Coil 1 of a pair of coupled coils has a continuous current of 5A, and the corresponding fluxes ϕ_1 and ϕ_{12} are 0.6mWb and 0.4 mWb respectively. If the turns are N_1=500 and N_2=1500, find L_1, L_2, M and k.

Ans:

- $k = \Phi_{12}/\Phi_1 = 0.667$
- $M = N_2\Phi_{12}/I_1 = 0.12H$
- $L_1 = N_1\Phi_1/I_1 = 0.06\ H$
- $L_2 = 0.539H$