UNIT_2

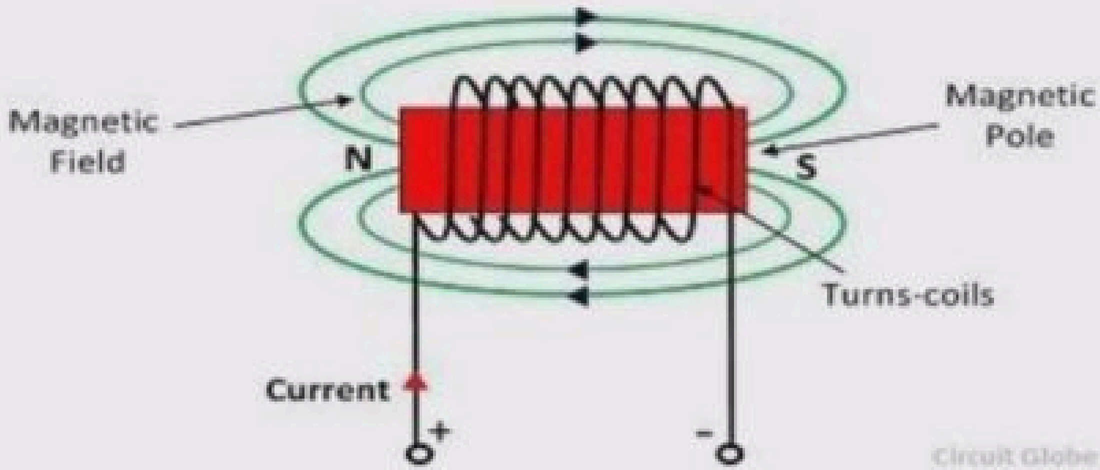
MAGNETIC CIRCUITS

TOPICS

Basic definition of MMF, flux and reluctance, analogy between electrical and magnetic circuits, Faraday's laws of electromagnetic induction – concept of self and mutual inductance, Dot convention – coefficient of coupling and composite magnetic circuit, analysis of series and parallel magnetic circuits.

Magneto motive force (MMF)

Definition: The current flowing in an electric circuit is due to the existence of electromotive force similarly **magneto motive force** (MMF) is required to drive the magnetic flux in the magnetic circuit. The magnetic pressure, which sets up the magnetic flux in a magnetic circuit is called Magneto motive Force. The SI unit of MMF is Ampere-turn (AT), and their CGS unit is G (gilbert). The MMF for the inductive coil shown in the figure below is expressed as



Where, N – numbers of turns of the inductive coil I – current

- The strength of the MMF is equivalent to the **product of the current around the turns and the number of turns of the coil**. As per work law, the MMF is defined as the work done in moving the unit magnetic pole (1weber) once around the magnetic circuit.
- The MMF is also known as the magnetic potential. It is the property of a material to give rise to the magnetic field. The magneto motive force is the product of the magnetic flux and the magnetic reluctance.

The MMF regarding reluctance and magnetic flux is given as

$$F = \phi R$$

The reluctance is the opposition offers by the magnetic field to set up the magnetic flux on it.

The magneto motive force can measure regarding magnetic field intensity and the length of the substance. The magnetic field strength is the force act on the unit pole placed on the magnetic field. MMF regarding field intensity is expressed as

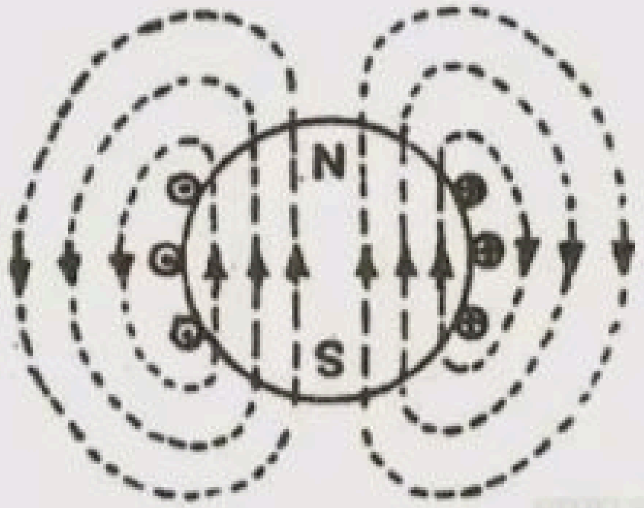
$$F = Hl$$

Where H is the magnetic field strength, and l is the length of the substance.

Flux is the presence of a force field in a specified physical medium, or the flow of energy through a surface

Magnetic Flux

Definition: The number of magnetic lines of forces set up in a magnetic circuit is called **Magnetic Flux**. It is analogous to electric current, I in an electric circuit. Its SI unit is Weber (Wb) and its CGS unit is Maxwell. It is denoted by φ_m. The magnetic flux measures through flux meter. The flux meter has to measure coil which measures the variation of voltage to measure the flux.



Net number of lines passing through the surface are called **magnetic lines of forces**.

If the magnetic field is constant then the magnetic flux passing through a surface (S) is

$$\varphi_B = B.S\ \cos\theta$$

where

B – the magnitude of the magnetic field

S – area of surface

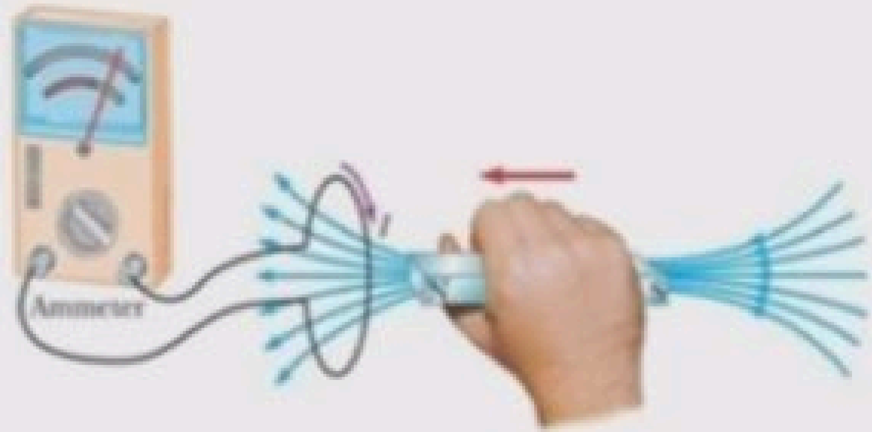
θ – angle between the magnetic field lines and perpendicular distance normal to the surface area Magnetic flux for a closed surface

Table 10.1 Analogy between magnetic and electric circuit

Electric circuit	*Magnetic circuit*
Exciting force = emf in volts	mmf in AT
Response = current in amps	flux in webers
Voltage drop = VI volts	mmf drop = $\Re\phi$ AT
Electric field density = $\frac{V}{l}$ volt/m	Magnetic field Intensity = $\frac{\Im}{1}$ AT/m
Current(I) = $\frac{E}{R}$ A	Flux (ϕ) = $\frac{\Im}{R}$ Web
Current density(J) = $\frac{I}{a}$ Amp/m^2	Flux density (B) = $\frac{\phi}{A}$ Web/m^2
Resistance (R) = $\frac{\rho_l}{a}$ ohm	Reluctance ($\Re$) = $\frac{1}{\mu}\cdot\frac{l}{a}$ AT/Web
Conductance (G) = $\frac{1}{R}$ Mho	Permeance = $\frac{1}{\Re} = \frac{\mu a}{\mu}\cdot\frac{l}{a}$ Web/AT

EMF Produced by a Changing Magnetic Field, 1

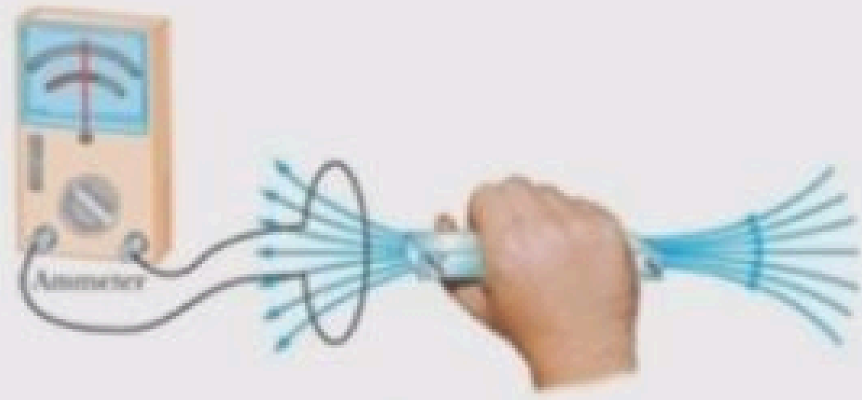
- A loop of wire is connected to a sensitive ammeter
- When a magnet is moved toward the loop, the ammeter deflects



(a)

EMF Produced by a Changing Magnetic Field, 2

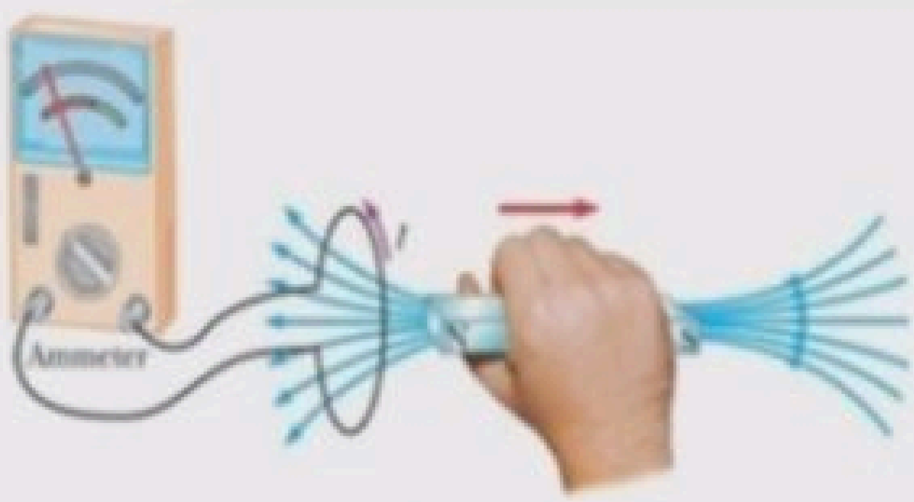
- When the magnet is held stationary, there is no deflection of the ammeter
- Therefore, there is no induced current
  - Even though the magnet is in the loop



(b)

EMF Produced by a Changing Magnetic Field, 3

- The magnet is moved away from the loop
- The ammeter deflects in the opposite direction



(c)

Faraday's Law – Statements

- Faraday's law of induction states that "the emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit"
- Mathematically,

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Faraday's Law – Statements, cont

- Remember Φ_B is the magnetic flux through the circuit and is found by

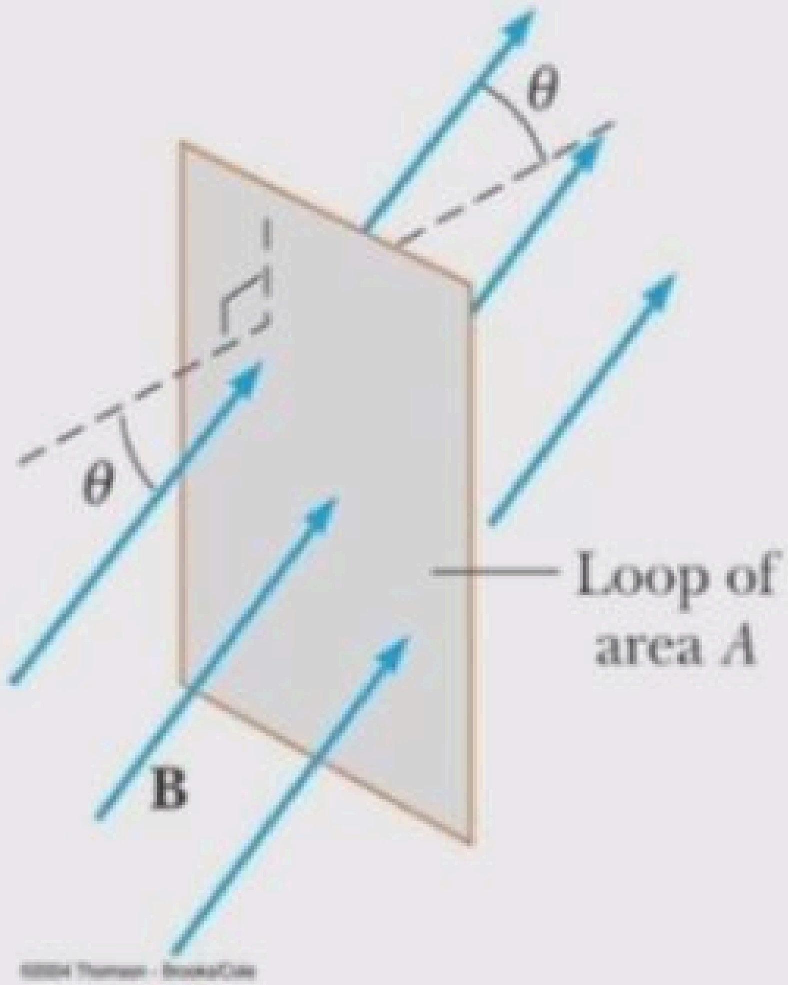
$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

- If the circuit consists of N loops, all of the same area, and if Φ_B is the flux through one loop, an emf is induced in every loop and Faraday's law becomes

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Faraday's Law – Example

- Assume a loop enclosing an area *A* lies in a uniform magnetic field **B**
- The magnetic flux through the loop is $\Phi_B = BA\cos\theta$
- The induced emf is
  $\varepsilon = -\,d/dt\,(BA\cos\theta)$



Ways of Inducing an emf

- The magnitude of **B** can change with time
- The area enclosed by the loop can change with time
- The angle θ between **B** and the normal to the loop can change with time
- Any combination of the above can occur

Self Induced emf & Self Inductance

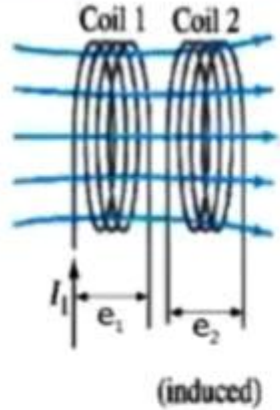
- The induced emf, e, in a coil is proportional to the rate of the change of the magnetic flux passing through it due to its own current. This emf is termed as **Self Induced EMF**
- The induced emf e is proportional to the rate of change of current through coil and this proportionality constant is called the **self inductance**, L.

$$e_1 = -L \frac{di}{dt}$$

- The negative sign is used to indicate that EMF is opposing the cause producing it

Mutually Induced emf & Mutual Inductance

- If two coils of wire are placed near each other, a change of current in one coil will induce emfs e_1 in the first coil and e_2 in the second coil.



- The induced emf, e_2, in coil 2 is proportional to the rate of the change of the magnetic flux passing through it and hence proportional to rate of change of current in first coil and is termed as **Mutually induced EMF**.

Self & Mutual Inductance…

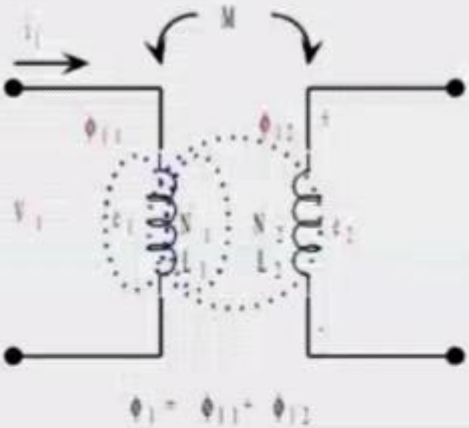
- The induced emf e_2 is proportional to the rate of change of current through coil 1 and this proportionality constant is called the mutual inductance, M

  The mutually induced emf is expressed as

$$e_2 = M\frac{di_1}{dt}$$

  This induced emf can also be expressed as

$$e_2 = N_2\frac{d\varphi_{12}}{dt}$$



$e2 = M\, di1/dt \Rightarrow 1$

$e2 = N2\, d\varnothing 12/dt \Rightarrow 2$

equation 1&2

$M\,(di1/dt) = N2\,(d\varnothing 12/dt)$

both sides "dt" get cancel

$M\, di1 = N2.\, d\varnothing 12$

$M = N2.\,(d\varnothing 12/di1)$

Mutual Inductance...

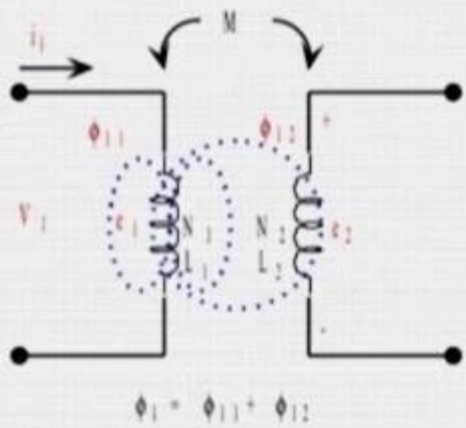
- Therefore

$$M = N_2 \frac{d\phi_{12}}{di_1}$$

If μ_r is constant, $\frac{d\phi_{12}}{di_1}$ is constant and

$$M = N_2 \frac{\Phi_{12}}{I_1}$$

Unit: Henry (H)



Coupling Coefficient

Self Inductances L_1 and L_2 are

$$L_1 = \frac{N_1 \Phi_1}{I_1} \quad \text{and} \quad L_2 = \frac{N_2 \Phi_2}{I_2}$$

Mutual Inductance M

$$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_1 \Phi_{21}}{I_2}$$

where $\Phi_{12} = k\,\Phi_1;\ \Phi_{21} = k\,\Phi_2$ and

k is the coupling coeficient

$$L_1 L_2 = \frac{M^2}{k^2} \quad \text{or} \quad k = \frac{M}{\sqrt{L_1 L_2}}$$

$$L_1 L_2 = \frac{N_1 \Phi_1}{I_1} \cdot \frac{N_2 \Phi_2}{I_2}$$

$$= \frac{N_1 \cdot \Phi_{12}}{I_1 \cdot K} \cdot \frac{N_2 \cdot \Phi_{21}}{I_2 K}$$

$$= \frac{1}{K^2} \cdot \frac{N_1 \Phi_{12}}{I_1} \cdot \frac{N_2 \Phi_{21}}{I_2}$$

$$= \frac{1}{K^2} \cdot M \cdot M$$

$$L_1 L_2 = \frac{M^2}{K^2}$$

Example

Coil 1 of a pair of coupled coils has a continuous current of 5A, and the corresponding fluxes ϕ_1 and ϕ_{12} are 0.6mWb and 0.4 mWb respectively. If the turns are N_1=500 and N_2=1500, find L_1, L_2, M and k.

Ans:

- $k = \Phi_{12}/\Phi_1 = 0.667$
- $M = N_2\Phi_{12}/I_1 = 0.12H$
- $L_1 = N_1\Phi_1/I_1 = 0.06\ H$
- $L_2 = 0.539H$

$$\Phi_{12} = K\Phi_1$$

$$K = \frac{\Phi_{12}}{\Phi_1}$$

$$= \frac{0.4m}{0.6m}$$

$$K = 0.667$$

given

$I = 5$

$\Phi_1 = 0.6m$

$\Phi_{12} = 0.4m$

$N_1 = 500$

$N_2 = 1500$

$$M = \frac{N_2 \Phi_{12}}{I_1}$$

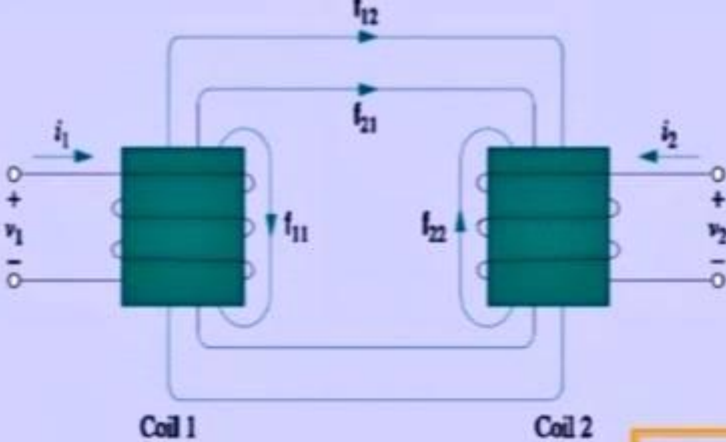
$$= \frac{1500 \times 0.4m}{5} = 0.21H$$

$$L_1 = \frac{N_1 \Phi_1}{I_1} = \frac{500 \times 0.6m}{5} = 0.06H$$

$$L_2 = \frac{N_2 \Phi_2}{I_2}, \quad L_1 L_2 = \frac{M^2}{K^2}$$

$$L_2 = \frac{1}{0.06} \times \frac{(0.2}{(0.}$$

$$L_2 = 0.539H$$

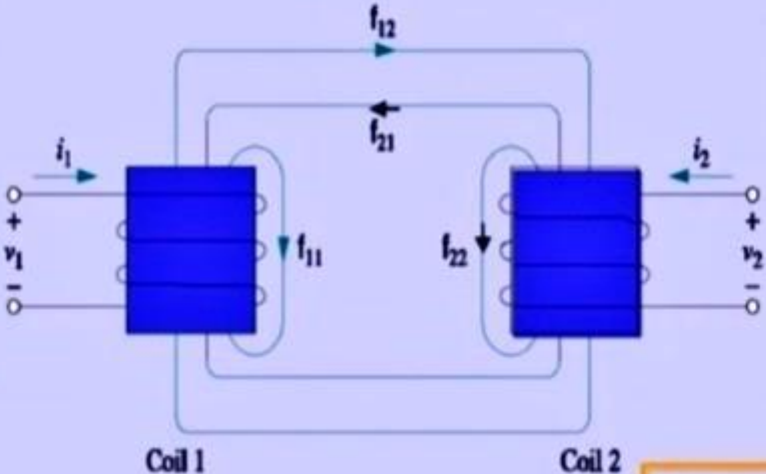


$$\phi_{coil1} = \phi_1 + \phi_{21} \quad and \; v_1 = N_1 \frac{d\phi_{coil1}}{dt}$$

$$\phi_{coil2} = \phi_2 + \phi_{12} \quad and \; v_2 = N_2 \frac{d\phi_{coil2}}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

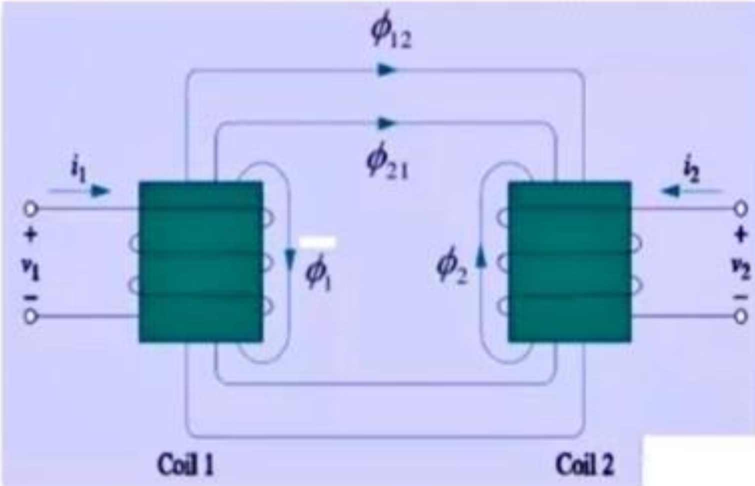


$$\phi_{coil1} = \phi_1 - \phi_{21} \quad and \; v_1 = N_1 \frac{d\phi_{coil1}}{dt}$$

$$\phi_{coil2} = \phi_2 - \phi_{12} \quad and \; v_2 = N_2 \frac{d\phi_{coil2}}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

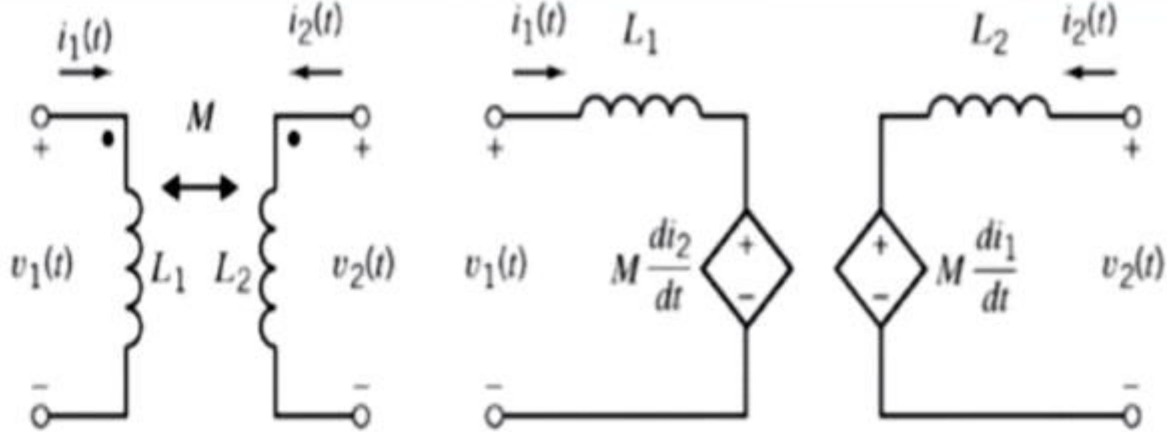


$$\phi_{coil1} = \phi_1 + \phi_{21} \quad and \; v_1 = N_1 \frac{d\phi_{coil1}}{dt}$$

$$\phi_{coil2} = \phi_2 + \phi_{12} \quad and \; v_2 = N_2 \frac{d\phi_{coil2}}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



Using KVL

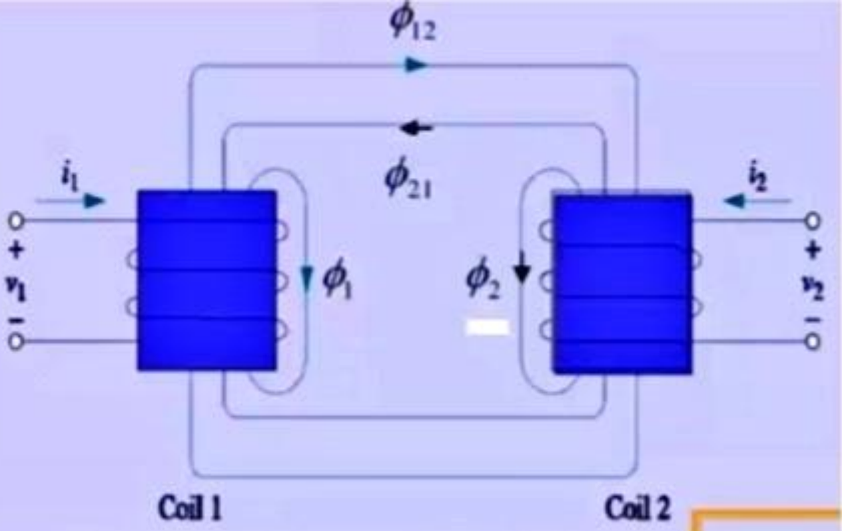
$$-v_1(t) + L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt} = 0$$

$$v_1(t) = L_1\frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

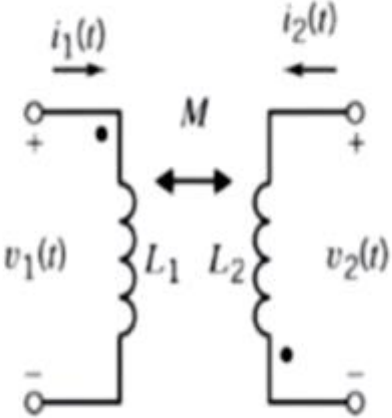
$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

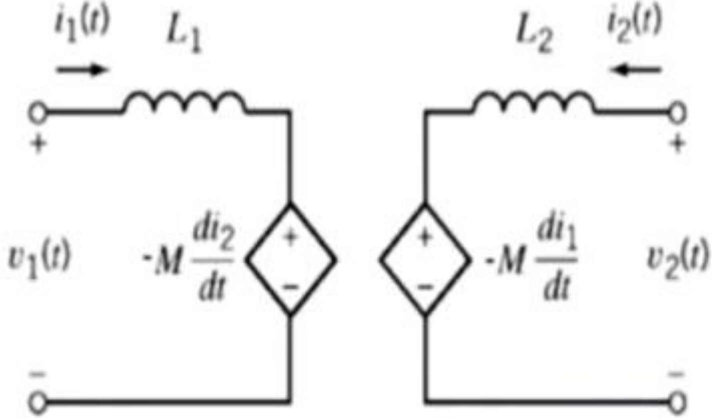
$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$



ϕ_{12}
ϕ_{21}
i_1
i_2
+
v_1
−
ϕ_1
ϕ_2
+
v_2
−
Coil 1
Coil 2





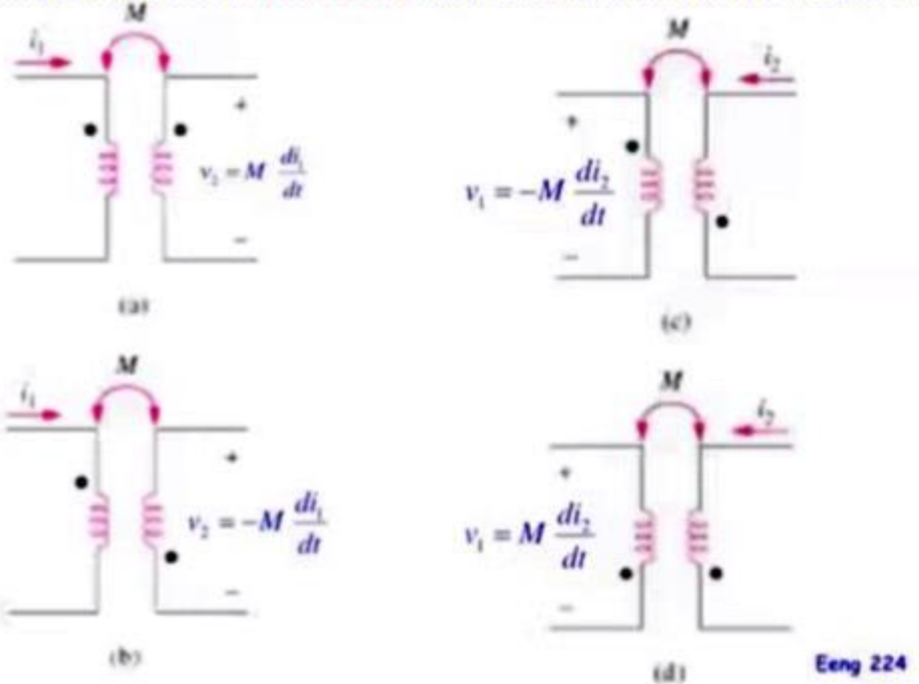
$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

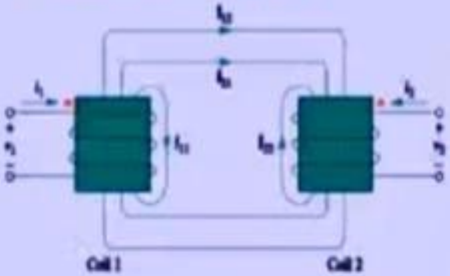
Dot Convention

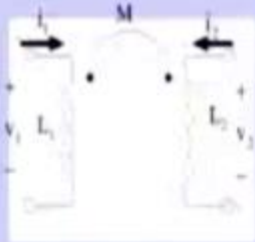
- **If the current ENTERS the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is POSITIVE at the dotted terminal of the second coil.**

  If the current LEAVES the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is NEGATIVE at the dotted terminal of the second coil.



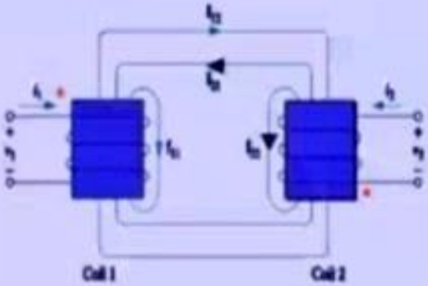
Dot convention

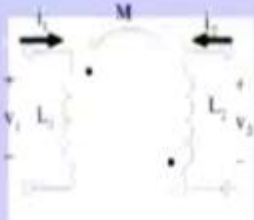




$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



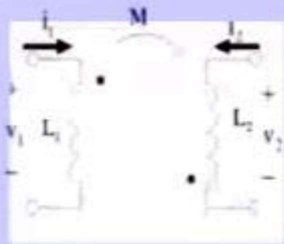


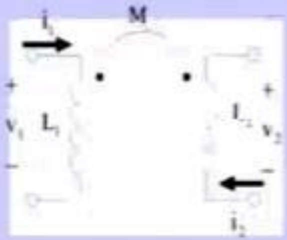
$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.

Examples

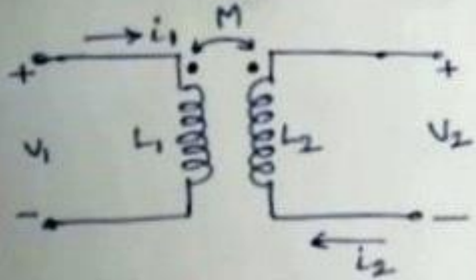


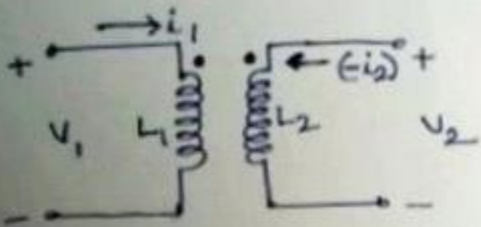


$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$
$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$
$$v_2 = +M \frac{di_1}{dt} - L_2 \frac{di_2}{dt}$$

How could we determine dot markings if we don't know?





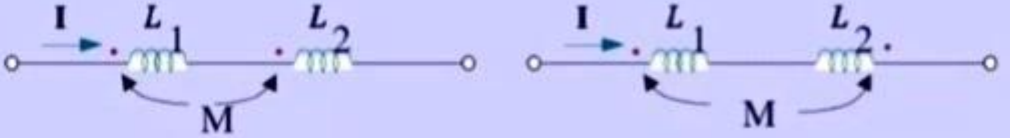
$$v_1 = L_1 \frac{di_1}{dt} + M \cdot \frac{d(-i_2)}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \quad \text{—— (1)}$$

$$v_2 = L_2 \frac{d}{dt}(-i_2) + M \cdot \frac{di_1}{dt}$$

$$v_2 = -L_2 \frac{di_2}{dt} + M \cdot \frac{di_1}{dt} \quad \text{—— (2)}$$

Series connection



(a)mutually coupled coils in series-aiding connection

(b)mutually coupled coils in series–opposing connection

Total inductance

$$L_T=L_1+L_2+2M$$

$$L_T=L_1+L_2-2M$$

Series Aiding

$$v = L_1 \frac{di}{dt} + L_2 \cdot \frac{di}{dt} + M \cdot \frac{di}{dt} + M \cdot \frac{di}{dt}$$

$$L_T \cdot \frac{di}{dt} = (L_1 + L_2 + M + M) \frac{di}{dt}$$

$$L_T = L_1 + L_2 + 2M$$

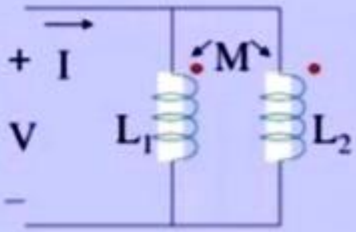
Series - opposing

$$v = L_1 \frac{di}{dt} - M \cdot \frac{di}{dt} + L_2 \cdot \frac{di}{dt} - M \cdot \frac{di}{dt}$$

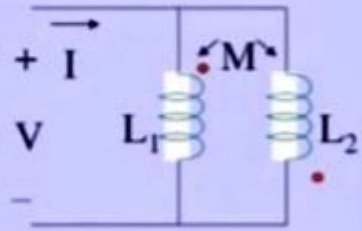
$$L_T \cdot \frac{di}{dt} = (L_1 - M + L_2 - M) \frac{di}{dt}$$

$$L_T = L_1 + L_2 - 2M$$

Parallel connection



(a)mutually coupled coils in parallel-aiding connection

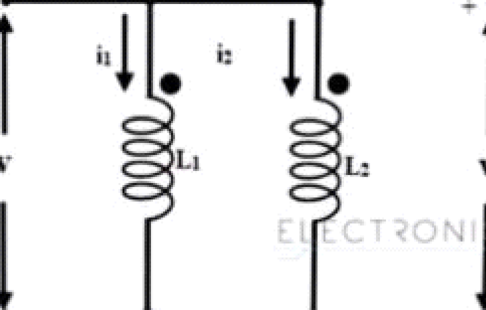


(b)mutually coupled coils in parallel–opposing connection

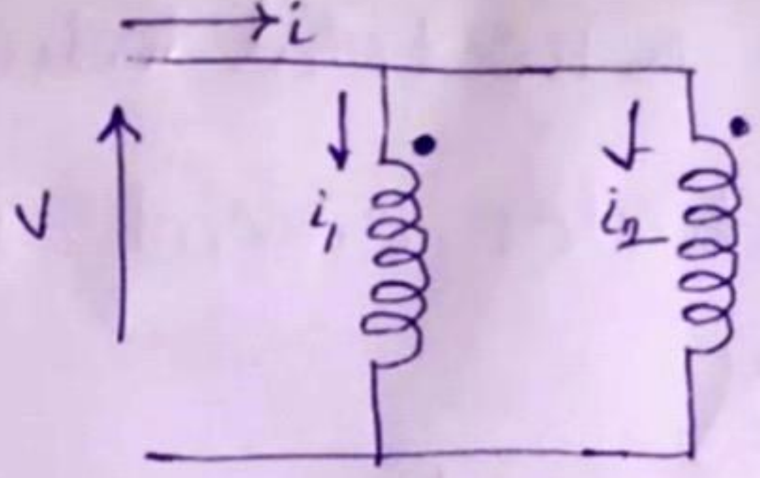
Equivalent inductance

$$L_e = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

$$L_e = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$



i1
i2
L1
L2
V
V
ELECTRONI



$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad \text{——} \ (1)$$

voltage across inductor

$$V = L_1 \frac{di_1}{dt} + M \cdot \frac{di_2}{dt}$$

(or)

$$= L_2 \frac{di_2}{dt} + M \cdot \frac{di_1}{dt}$$

$$L_1 \frac{di_1}{dt} + M \cdot \frac{di_2}{dt} = L_2 \cdot \frac{di_2}{dt} + M \cdot \frac{di_1}{dt}$$

$$(L_1 - M) \frac{di_1}{dt} = (L_2 - M) \frac{di_2}{dt}$$

$$\frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M} \right) \frac{di_2}{dt} \quad \text{——} \ (2)$$

By substituting Eq (2) in Eq (1)

$$\frac{di}{dt} = \left(\frac{L_2 - M}{L_1 - M} \right) \frac{di_2}{dt} + \frac{di_2}{dt}$$

$$\frac{di}{dt} = \left[\frac{L_2 - M}{L_1 - M} + 1 \right] \frac{di_2}{dt} \quad \text{——} \ (3)$$

If L_T is the total inductance of the parallel inductor circuit, then voltage is given by

$$V = L_T \cdot \frac{di}{dt}$$

$$L_T \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \cdot \frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L_T}\left[L_1 \frac{di_1}{dt} + M \cdot \frac{di_2}{dt}\right]$$

By substituting Eq ② in above Eq

$$\frac{di}{dt} = \frac{1}{L_T}\left[L_1\left[\frac{di_2}{dt}\left[\frac{L_2-M}{L_1-M}\right]\right] + M \frac{di_2}{dt}\right]$$

$$= \frac{1}{L_T}\left[L_1\left(\frac{L_2-M}{L_1-M}\right) + M\right]\frac{di_2}{dt} \quad — ④$$

By equating ③ & ④

$$\frac{di_2}{dt}\left\{\frac{L_2-M}{L_1-M} + 1\right\} = \frac{1}{L_T}\left[L_1\left(\frac{L_2-M}{L_1-M}\right) + M\right]\frac{di_2}{dt}$$

$$\frac{L_2-M}{L_1-M} + 1 = \frac{1}{L_T}\left[L_1\left(\frac{L_2-M}{L_1-M}\right) + M\right]$$

$$\frac{L_2-M+L_1-M}{L_1-M} = \frac{1}{L_T}\left[L_1\left(\frac{L_2-M}{L_1-M}\right) + M\right]$$

$$L_T = \frac{L_1 - M}{L_1 + L_2 - 2M}\left[\frac{L_1 L_2 - L_1 M + M L_1 - M^2}{L_1 - M}\right]$$

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

UNIT _2 (29/06/2021)

COEFFICIENT OF COUPLING

<u>Definition</u>:

The fraction of magnetic flux produced by the current in one coil that links with the other coil is called coefficient of coupling between the two coils.

It is denoted by “k”.

Coefficient of coupling

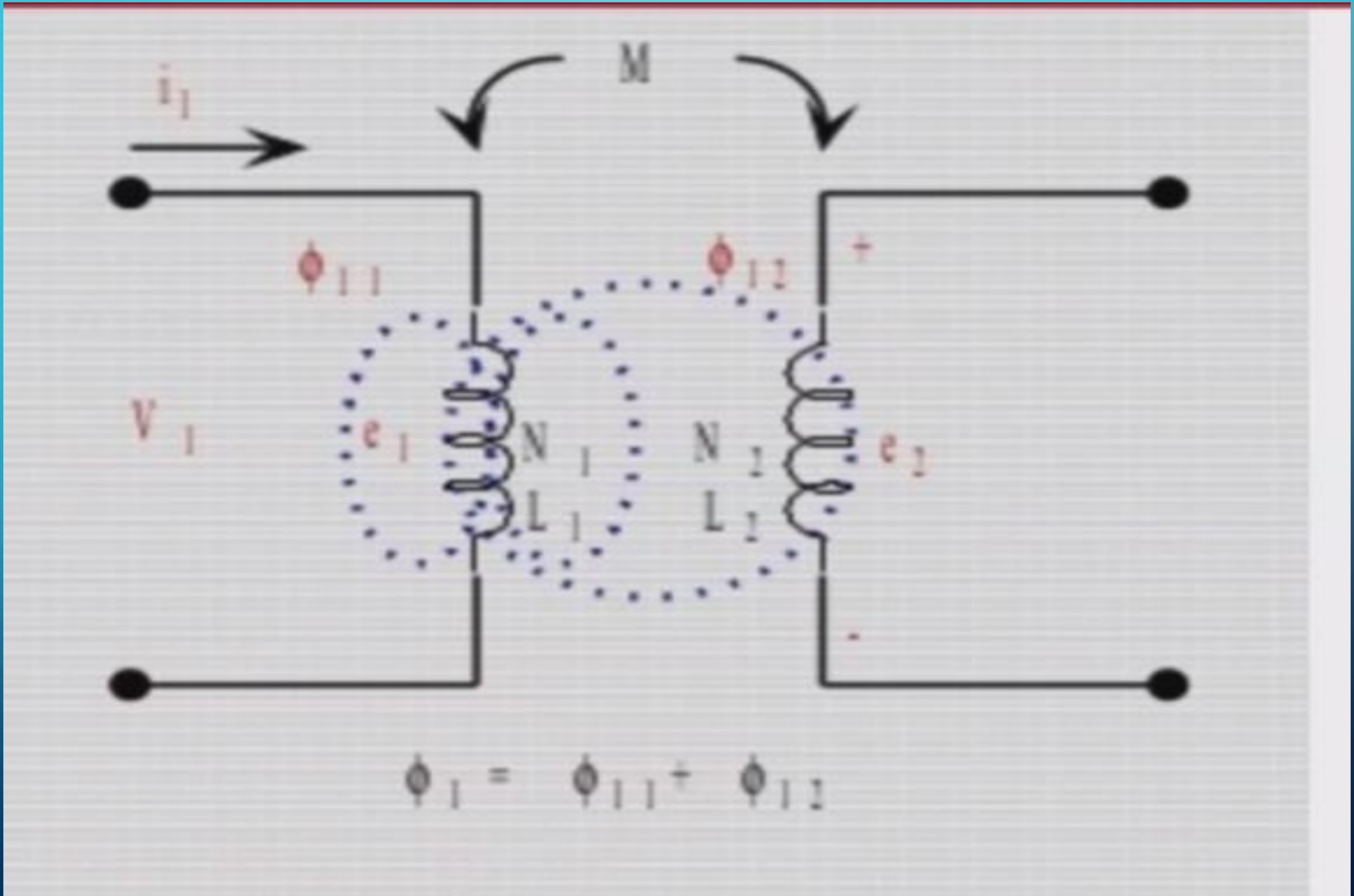
The coupling coefficient k is a measure of the magnetic coupling between two coils

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$0 \le k \le 1$$

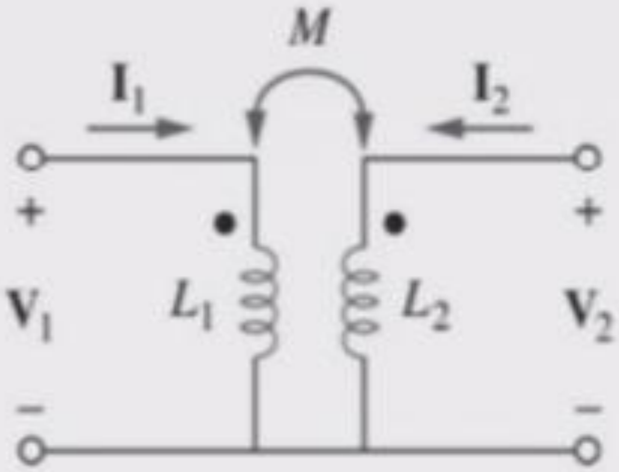
$k < 0.5$ loosely coupled;

$k > 0.5$ tightly coupled.


M
i1
ϕ11
ϕ12
+
V1
e1
N1
L1
N2
L2
e2
-
ϕ1 = ϕ11 + ϕ12

Equivalent T (or Wye) Circuit for Linear Transformer

- The coupled transformer can equivalently be represented by an equivalent T (or Wye) circuit using UNCOUPED INDUCTORS.



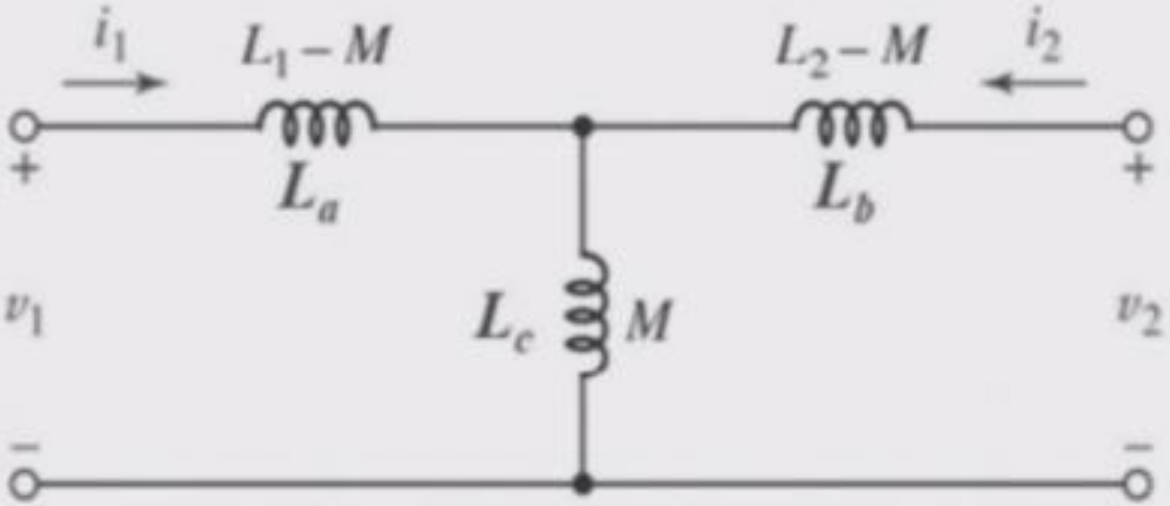
Transformer circuit.

Consider the mesh-current equations to show that these circuits are equivalent.

$$L_a = L_1 - M$$

$$L_b = L_2 - M$$

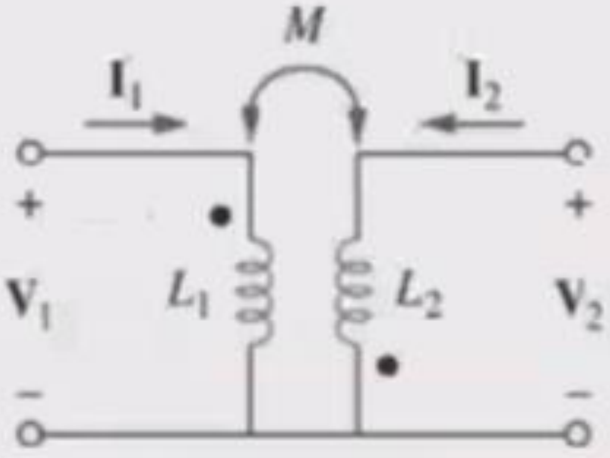
$$L_c = M$$



Equivalent T circuit of the transformer.

Equivalent T (or Wye) Circuit for Linear Transformer

- The coupled transformer can equivalently be represented by an equivalent T (or Wye) circuit using <u>UNCOUPED</u> INDUCTORS.



Transformer circuit

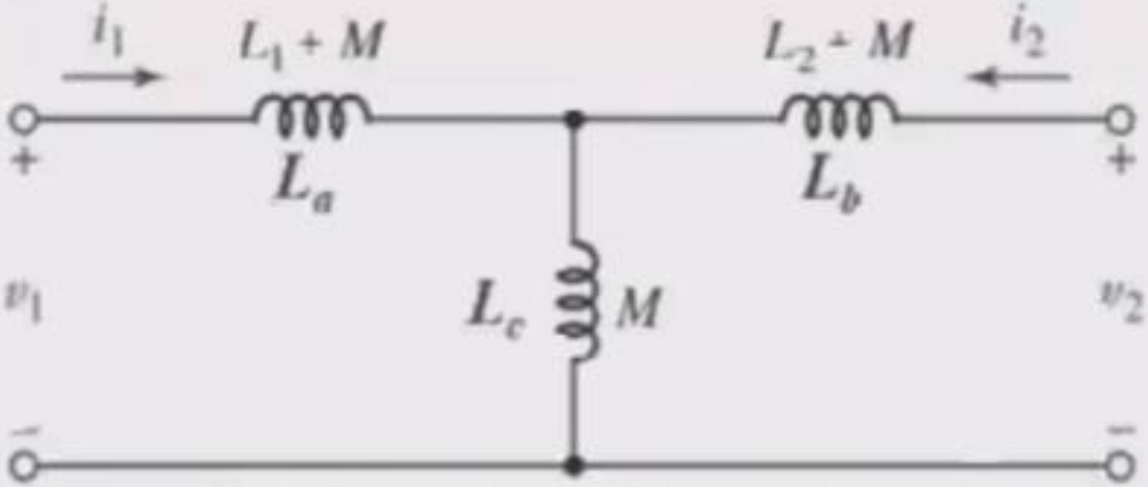
Note:

The current **direction** will **<u>not</u>** affect the equivalent circuit, but if the location of one dote is changed then M will become ($-M$). Its impossible to build this circuit practically, but one can deal with it mathematically.

$$L_a = L_1 + M$$

$$L_b = L_2 + M$$

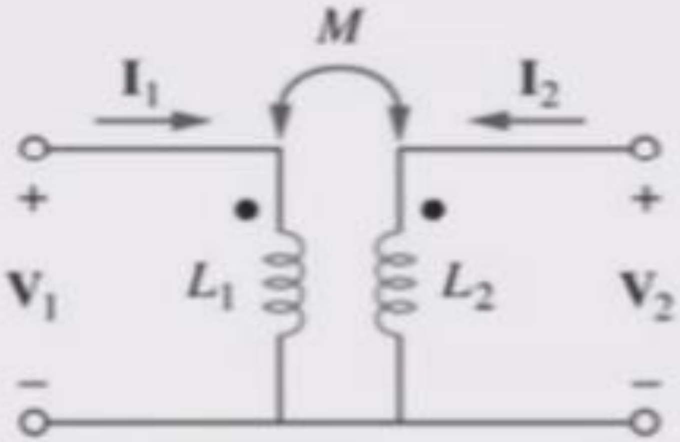
$$L_c = -M$$



Equivalent T circuit of the transformer.

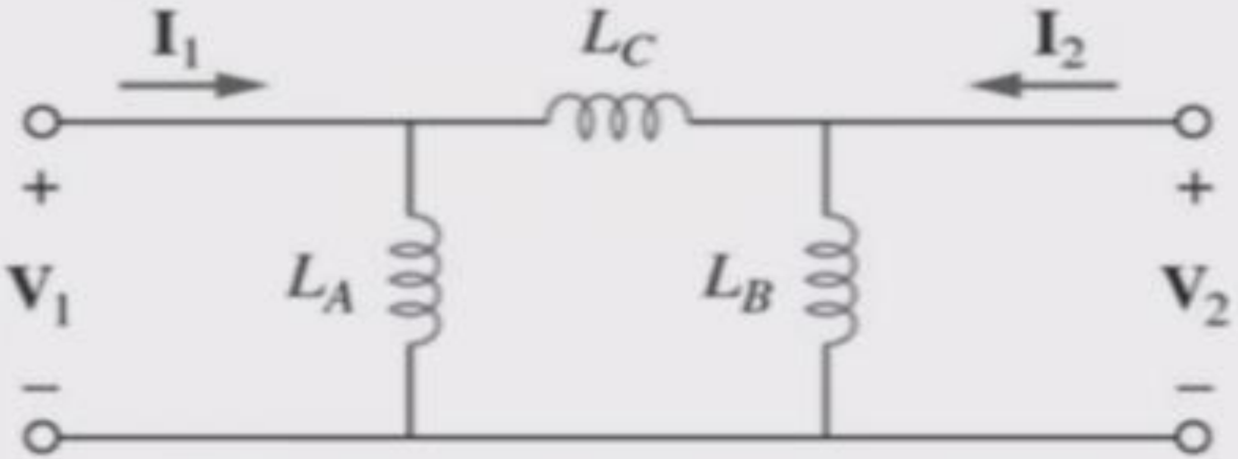
Equivalent Π (or Delta) Circuit for Linear Transformer

- The coupled transformer can equivalently be represented by an equivalent Π (π or Delta) circuit using <u>UNCOUPED</u> INDUCTORS.



Transformer circuit.

Consider the mesh-current equations to show that these circuits are equivalent.



Equivalent Π (π) circuit of the transformer.

$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M}, \quad L_B = \frac{L_1 L_2 - M^2}{L_1 - M}$$

$$L_C = \frac{L_1 L_2 - M^2}{M}$$

Problem 13.2 *[13.1]* For the three coupled coils in Figure 13.1, calculate the total inductance.

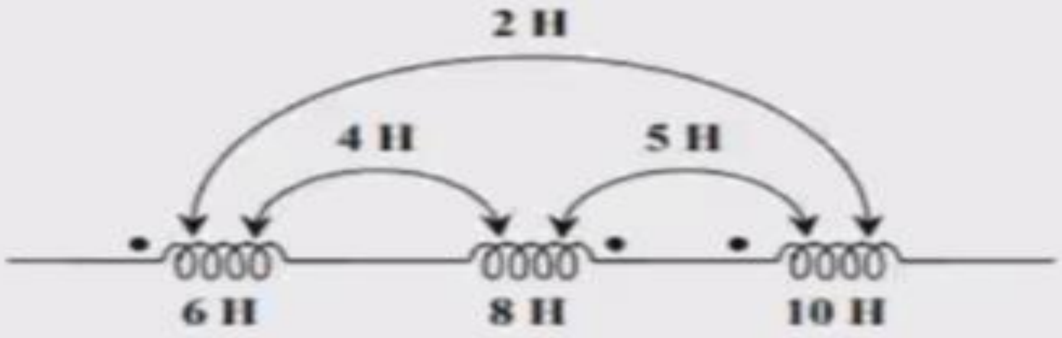


Figure 13.1

For coil 1, $L_1 - M_{12} + M_{13} = 6 - 4 + 2 = 4$

For coil 2, $L_2 - M_{21} - M_{23} = 8 - 4 - 5 = -1$

For coil 3, $L_3 + M_{31} - M_{32} = 10 + 2 - 5 = 7$

$$L_T = 4 - 1 + 7 = \underline{\mathbf{10\ H}}$$

or

$$L_T = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{12}$$
$$L_T = 6 + 8 + 10 - (2)(4) - (2)(5) + (2)(2)$$
$$L_T = 6 + 8 + 10 - 8 - 10 + 4 = \underline{\mathbf{10\ H}}$$

Two inductors whose self-inductances are of 75mH and 55mH respectively are connected together in parallel aiding. Their mutual inductance is given as 22.5mH. Calculate the total inductance of the parallel combination.

$$L_T = \frac{L_1 \times L_2 - M^2}{L_1 + L_2 - 2M}$$

$$L_T = \frac{75mH \times 55mH - 22.5mH^2}{75mH + 55mH - 2 \times 22.5mH}$$

$$L_T = 42.6mH$$