→ 5. Magnetic fl...

The

* MAGNETIC CIRCUITS *

Definitions :-

→ 1. Magneto Motive Force (MMF) :-

In an electric circuit, the current is due to the existence of an electromotive force (emf) or voltage.

Similarly, in the magnetic circuit, the magnetic flux is due to the existence of magneto motive force (mmf).

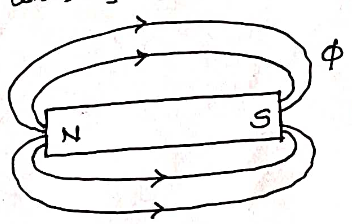
MMF is defined as the product of no. of turns and current passing through the coil.

$$MMF = NI$$

Units : Amp-turns.

→ 2. Magnetic Flux (ϕ) :-

Magnetic flux is the total no. of magnetic lines of force in a magnetic field.



$$\text{magnetic flux } \phi \propto MMF$$

Here constant of proportionality is called as "reluctance".

$$\therefore \phi = \frac{MMF}{Reluctance} = \frac{NI}{S}$$

(or)

$$MMF = \phi \cdot S$$

The above expression is called as ohm's law for magnetic circuit.

Units: weber

* 1 weber = 10^8 lines

→ 3. Reluctance (S) :-

It is a measure of opposition offered by a magnetic circuit to the flux.

Reluctance (S) is the ratio of magnetomotive force (mmf) to the flux (ϕ).

$$\therefore S = \frac{mmf}{\phi}$$

Units : Amp.turns/wb.

Reluctance of a magnetic circuit also depends on the physical dimensions of the magnetic path.

i) $S \propto l$

ii) $S \propto \frac{1}{A}$

$$\therefore S = \frac{l}{\mu A}$$

μ = Absolute permeability of material.

$\mu = \mu_0 \mu_r$

* μ_0 = Absolute permeability of free space 'or' vacuum.
$= 4\pi \times 10^{-7}$ H/m.

μ_r = Relative permeability of material.

* 'μ_r' for air, vaccum or non-magnetic material, $\mu_r = 1$ (no units)

→ 4. Magnetic field strength (H) :-

It is defined as the magneto-motive force per unit length of the magnetic circuit.

$$H = \frac{MMF}{\text{Length of magnetic ckt}} = \frac{NI}{l}$$

Units : AT/m

5. Magnetic flux density (B) :-

The magnetic flux density in any material is defined as the magnetic flux established per unit area of the cross section.

$$B = \frac{\text{Total flux}}{\text{C.S. area}} = \frac{\phi}{A}$$

Units : wb/m^2 or Tesla. (S.I system)

maxwell/cm^2 or guass (C.G.S.)

★ 1 Tesla = 10^4 guass

★ Relationship between "B" and "H" is given by $B = \mu H$

6. permeance :-

Reciprocal of reluctance is called as "permeance".

$$\therefore \text{permeance} = \frac{1}{S} = \frac{\phi}{mmf} = \frac{\mu A}{l}$$

Units : wb/AT

7. Relative permeability (μ_r) :-

It is defined as the ratio of flux density established in the magnetic material to the ratio of flux density established in air or vacum for the same magnetic field strength 'H'.

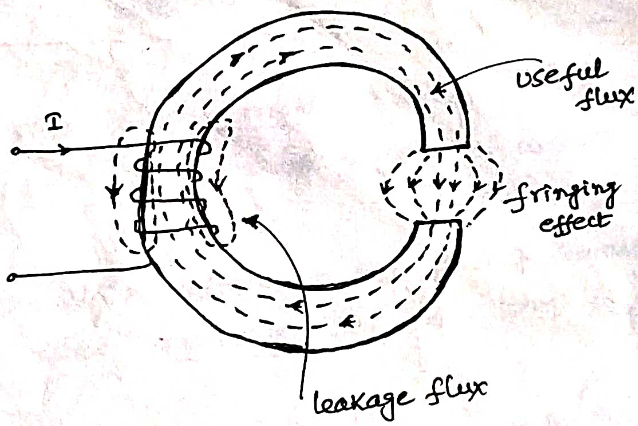
$$\therefore \mu_r = \frac{B_{material}}{B_{air\ or\ vacuum}} = \frac{\mu}{\mu_0}$$

★ Relative permeability changes with the material.

★ Units : No - units.

★ $\mu = \mu_0 \mu_r$

8. Magnetic leakage & Fringing :-



Consider an iron ring with a smaller airgap as shown in above figure. whenever the coil wound on the ring, carries a current and magnetic flux is established.

Let us consider that the total flux produced is "ϕ". But this total flux actually doesnot complete its path after crossing the airgap.

★ The flux wasted is called the "leakage flux" and the flux utilised is called "useful flux". The phenomenon of wastage of some amount of flux is called "magnetic leakage".

★ The ratio of total flux produced by the magnet to the useful flux is called "leakage coefficient" (λ).

$$\therefore \text{leakage coefficient } \lambda = \frac{\text{Total flux produced}}{\text{useful flux}}$$

★★ It is always greater than 1.

Magnetic Fringing :-

★ The useful flux in the airgap tends to spread out at the edges of the airgap. This effect is called "fringing"

★ Fringing increases the area of cross section and there by it reduces the flux density in the air gap.

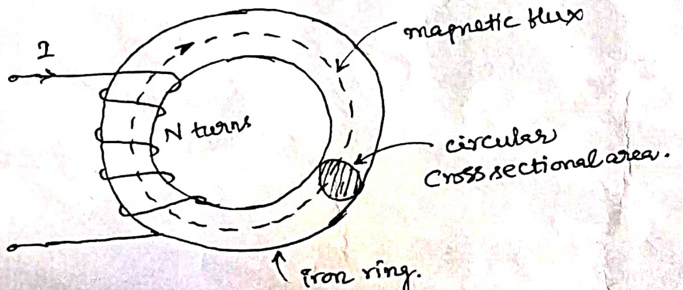
Similarities and dissimilarities in magnetic circuit and electric circuit :-

Magnetic circuit	Electric circuit
Similarities:	
1. Flow is of flux ϕ (weber)	1. Flow is of current (amp)
2. MMF is the cause of flow of flux (amp-turns)	2. EMF is the cause of flow of current. (volts)
3. Opposition offered to the flow of flux is called reluctance (S) (AT/wb)	3. Opposition offered to the flow of current is called resistance (R) (ohm)
4. $S = \frac{l}{\mu A}$; μ = permeability	4. $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$. σ = conductance.
5. Flux density $B = \frac{\phi}{A}$ wb/m^2	5. electric flux density $(D) = \frac{I}{A}$ amp/m^2
6. ohms law : $mmm = \phi . S$.	6. ohms law : $emf = I.R.$
7. magnetic field strength $(H) = \frac{NI}{l}$ (AT/m)	7. Electric field strength (E) volt/m.
Dissimilarities:	
1. flux doesnot actually flow in a magnetic circuit.	1. Current actually flows in an electric circuit.
2. Energy is initially needed to create the magnetic flux, but not to maintain it.	2. Energy is needed as long as current flows.
3. Permeability of a material varies with flux density and magnetic field strength.	3. The conductivity or resistivity of a material is constant.

Types of magnetic circuits :-

Definition: The complete closed path followed by any group of magnetic flux lines is referred to as a magnetic circuit.

One of the simplest forms of a magnetic circuit is the circular ring as shown in figure.



Types of magnetic circuits are :

a) Series magnetic circuit
b) parallel magnetic circuit
c) Composite magnetic circuit.

a) Series magnetic circuit :-

* A series magnetic circuit is analogous to a series electric circuit.
* Kirchoff's laws are applicable to magnetic circuits also.

* Flux law: $\Sigma \phi = 0$
* MMF law: $\Sigma mmf = \Sigma H.l.$

Examples for series magnetic circuits :-

Ex: 1

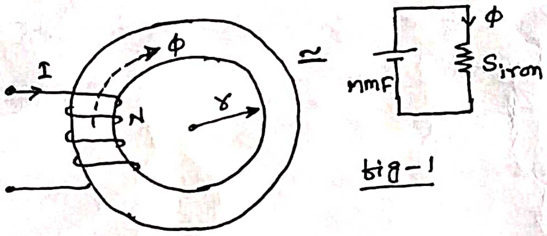


fig-1

for fig-1

$$H = \frac{mmf}{l} = \frac{NI}{l} \quad AT/m. \rightarrow (1)$$

where l = mean length of magnetic circuit

$= 2\pi r$. (Circumfrance of circle)

$$B = \mu_0 \mu_r H$$

$$= \mu_0 \cdot \mu_r \frac{NI}{l} \quad wb/m^2 \rightarrow (2)$$

$$\phi = \mu H A \quad wb \rightarrow (3)$$

(or)

$$= B \cdot A \quad wb.$$

$$S = \frac{l}{\mu A} \quad AT/wb \rightarrow (4)$$

Ex: 2

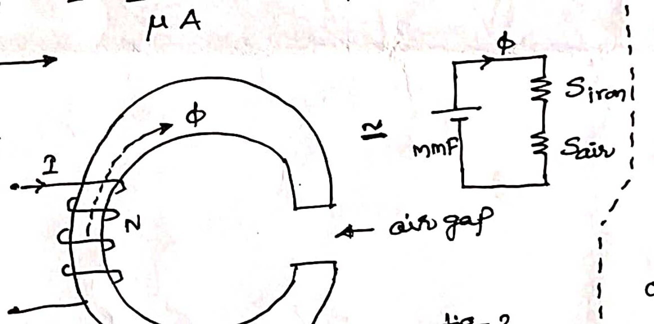


fig-2

for fig-2

$$\Sigma S = S_{iron} + S_{air}$$

$$= \frac{l_{iron}}{\mu_0 \mu_{r_{iron}} A} + \frac{l_{airgap}}{\mu_0 \mu_{r_{air}} A} \rightarrow (1)$$

$$MMF = (\phi)(\Sigma S) \rightarrow (2)$$

$$H = \frac{MMF}{l} \rightarrow (3)$$

Note: A series magnetic circuit carries the same flux.

b) Parallel magnetic circuits :-

* A parallel magnetic circuit is analogous to a parallel electric ckt.
* Kirchoff's laws are applicable.

Example : 1

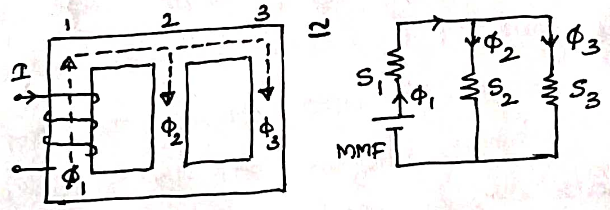


fig-3

for fig-3 :

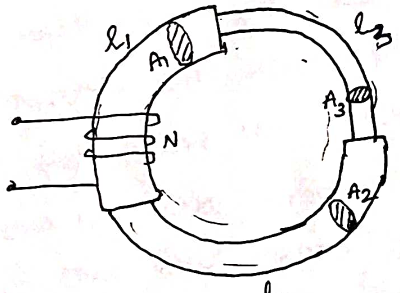
$$\phi_1 = \phi_2 + \phi_3 \quad (or) \quad \phi_1 - \phi_2 - \phi_3 = 0 \rightarrow (1)$$

$$\Sigma S = \left(\frac{S_2 \cdot S_3}{S_2 + S_3}\right) + S_1$$

$$Flux = \frac{mmF}{Total\ Reluctance} = \frac{NI}{S_1 + \left(\frac{S_2 \cdot S_3}{S_2 + S_3}\right)}$$

(c) Composite Series magnetic ckt :-

Consider a toroid, composed of three different magnetic materials of different permeabilities, areas and lengths excited by a coil of 'N' turns.



$$\Sigma S = S_1 + S_2 + S_3$$

$$= \frac{l_1}{\mu_0 \mu_{r_1} A_1} + \frac{l_2}{\mu_0 \mu_{r_2} A_2} + \frac{l_3}{\mu_0 \mu_{r_3} A_3}$$

$$\phi = \frac{mmF}{Total\ Reluctance} = \frac{NI}{\Sigma S}$$

Electro Magnetic Induction :-

The phenomenon of the production of induced emf in a circuit, due to a change in the number of magnetic lines of force in it, is called "electro-magnetic induction".

Laws of electro magnetic induction :-

* **Faraday's First law** states that "whenever the no. of magnetic lines of force (or magnetic flux) passing through a circuit changes, an induced emf is setup in the circuit".

* **Faraday's Second law** states that "the magnitude of induced emf is proportional to the rate of change of flux".

Suppose the flux of a coil having 'N' turns, changes from initial value of ϕ_1 wb. to the final value of ϕ_2 wb in 't' sec, then,

$$e \propto \frac{d\phi}{dt}$$

$$\boxed{e = N.\frac{d\phi}{dt}}$$

* **Lenz's law** states that, the direction of induced current or induced emf is such that it opposes the very cause producing this current or emf. i.e. it opposes the change in magnetic flux.

$$\therefore \boxed{e = -N.\frac{d\phi}{dt}}$$

Flemings Right hand Rule :-

According to fleming's right hand rule,

"hold the right hand in such a manner that the thumb, the fore finger and the middle finger are at right angles to each other. If the forefinger points towards the direction of the magnetic field, and the thumb points towards the direction of motion of the conductor, then the middle finger points towards the direction of induced emf (or induced current)".

This rule is also called as Generator rule.

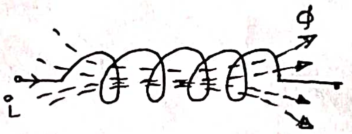
Flemings Left hand rule :-

According to fleming's left hand rule,

"hold the left hand in such a manner that the thumb, the fore finger and the middle finger are at right angles to each other. If the forefinger points towards the direction of the magnetic field, and the middle finger points towards the direction of the current, then the thumb indicates the direction of the mechanical force acting on the conductor.

This rule is also called as motor rule.

Self Inductance (L) :-



When a current changes in a circuit, the magnetic flux linking the same circuit changes and an emf is induced in the circuit. This induced emf is proportional to the rate of change of current.

$$e \propto \frac{di}{dt}$$

(or)

$$\boxed{e = L \cdot \frac{di}{dt}} \rightarrow (1)$$

where e = induced voltage

L = const. of proportionality, called as "Self-inductance"

units: Henry

Self inductance (L) is given by,

$$L = \frac{N\phi}{i} \; ; \quad i = \frac{N\phi}{L}$$

from eq (1);

$$e = L \cdot \frac{di}{dt} \Rightarrow L \cdot \frac{d}{dt}\left(\frac{N\phi}{L}\right)$$

$$\therefore \boxed{e = N \cdot \frac{d\phi}{dt}} \rightarrow (2)$$

By equating ① & ②;

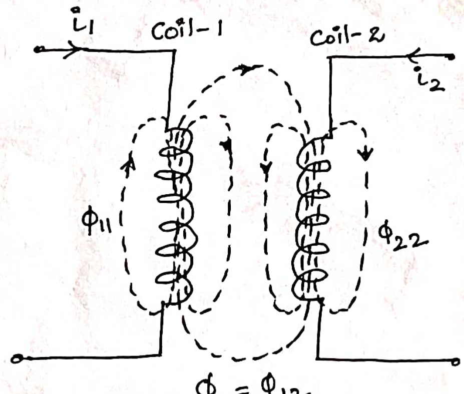
$$L \cdot \frac{di}{dt} = N \cdot \frac{d\phi}{dt}$$

$$\therefore \boxed{L = N \cdot \frac{d\phi}{di}} \rightarrow (3)$$

Equation (3) represents the magnitude of self inductance.

Units: Henry.

Mutual Inductance (M) :-



Let the two coils carrying currents of "i_1" and "i_2". Each coil will have leakage flux; (ϕ_{11} & ϕ_{22} for coil-1 and coil-2 respectively) as well as mutual flux (ϕ_{21} 'or' ϕ_{12}, where the flux of coil-2 links coil-1 'or' flux of coil-1 links coil-2).

The induced voltage of coil-2 is given by,

$$e_2 = N \cdot \frac{d\phi_{12}}{dt} \rightarrow (1)$$

Since "ϕ_{12}" is related to the current of coil-1 and the induced voltage is proportional to the rate of change of current "i_1";

$$e_2 \propto \frac{di_1}{dt}$$

(or)

$$e_2 = M \cdot \frac{di_1}{dt} \rightarrow (2)$$

where, M = Const of proportionality termed as mutual inductance between the two coils.

comparing ① & ②;

$$N \cdot \frac{d\phi_{12}}{dt} = M \cdot \frac{di_1}{dt}$$

$$\therefore \boxed{M = N_2 \cdot \frac{d\phi_{12}}{di_1}} \rightarrow (3)$$

Similarly;

$$M = N_1 \frac{d\phi_{21}}{di_2} \rightarrow (4)$$

When the coils are linked with air as medium, the flux and current are linearly related and the expressions of mutual inductance are modified as,

$$M = \frac{N_2 \phi_{12}}{i_1}$$

$$M = \frac{N_1 \phi_{21}}{i_2}$$

Henry.

Coefficient of Coupling (K) :-

It is defined as the fraction of total flux that links the coils.

i.e;

$$K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

where,

ϕ_{12} = flux of coil-1 that links coil-2

ϕ_{21} = flux of coil-2 that links coil-1

ϕ_1 = Total flux of coil-1

ϕ_2 = Total flux of coil-2

Since, $\phi_{12} < \phi_1$ and $\phi_{21} < \phi_2$,

hence the maximum value of 'K' is unity.

$$M = \frac{N_2 \phi_{12}}{i_1} \rightarrow (1)$$

$$M = \frac{N_1 \phi_{21}}{i_2} \rightarrow (2)$$

eq ① × eq ②;

$$M^2 = N_1 N_2 \frac{\phi_{12} \cdot \phi_{21}}{i_1 \cdot i_2}$$

$$\therefore M^2 = N_1 N_2 \cdot \frac{K \phi_1}{i_1} \cdot \frac{K \cdot \phi_2}{i_2}$$

$$= K^2 \cdot \frac{N_1 \phi_1}{i_1} \cdot \frac{N_2 \phi_2}{i_2}$$

$$M^2 = K^2 \cdot L_1 \cdot L_2$$

$$\star \quad M = K \cdot \sqrt{L_1 L_2} \quad \star$$

The above equation represents the relationship between mutual inductance, self inductances (L_1 & L_2) and coefficient of coupling, between the two coils.

★ The value of "K" ranges from 0 to 1.

★ For tightly coupled circuit (or 100% coupling) $k = 1$.

★ A transformer is a device, which is based on the principle of mutual induction.

Types of Induced emf :-

* Whenever there is a change of magnetic flux linking or cutting a conductor, then there will be an induced emf in the conductor.

* The induced emf in the conductor, is directly proportional to the rate of change of magnetic flux with the conductor.

** An induced emf due to a change of flux produced by relative motion between the conductor and flux, is called "dynamically induced emf."

The magnitude of dynamically induced emf in a conductor is directly proportional to

i) the strength of magnetic field (B)
ii) length of the conductor
iii) relative velocity between conductor and the flux.

∴ dynamically induced emf is

$$\boxed{e = Blv} \text{ volts.}$$

** An induced emf due to a change of flux produced by a time varying current i(t) flowing through a conductor is called "statically induced emf".

The magnitude of statically induced emf in a coil having 'N' turns and carrying a current i(t) is proportional to the rate of of change of flux linkages.

$$e \propto \frac{d\phi(t)}{dt}$$

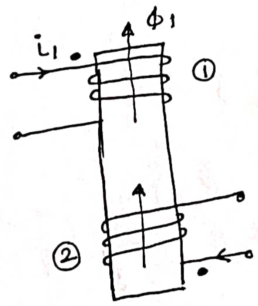
$$\boxed{e = -N \cdot \frac{d\phi(t)}{dt}}$$

The "−" ve sign indicates the nature of induced emf.

** Dot Convention :-

** Dot Convention is used to determine the correct polarity for the mutually induced voltages in the coupled coils.

** Circular dot marks and/or special symbols are placed at one end of each of two coils which are mutually coupled to simplify the diagrammatic representation of the windings around its core.



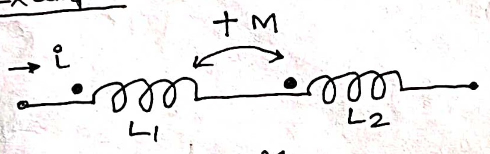
1. Place the dot at one end of coil (1) and assume that the current enters at that dotted end in coil-1

2. place another dot at one of the ends of coil (2) such that the current entering at that end in coil (2) establishes magnetic flux in the same direction.

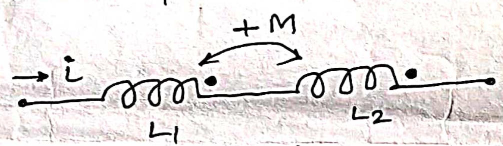
In order that the flux produced by I_2 flowing in coil (2) produce flux in the same upward direction, it should enter at lower end of coil (2). Hence place a dot at that end of coil (2).

** When the currents through each of the mutually coupled coils are going away from the dot 'or' towards the dot, the mutual inductance is <u>positive</u>, while for the case when the current through the coil is leaving the dot for one coil and entering the other, the mutual inductance is <u>negative</u>.

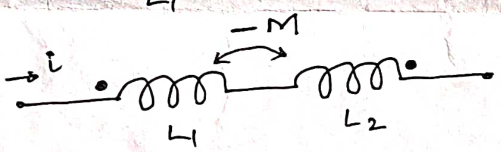
<u>Examples:</u>

a) 

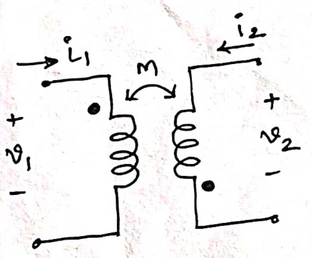
+M

b) 

+M

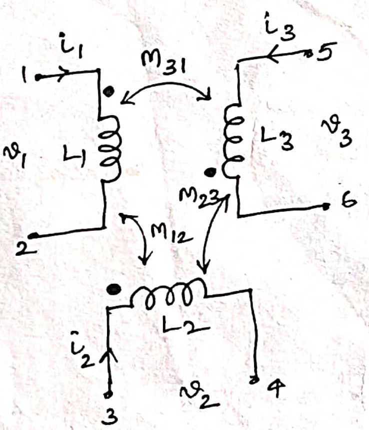
c) 

−M

d) 

$$v_1 = L_1 \frac{di_1}{dt} - M \cdot \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} - M \cdot \frac{di_1}{dt}$$

e) 

$$v_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} - M_{13} \frac{di_3}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M_{12} \frac{di_1}{dt} - M_{23} \frac{di_3}{dt}$$

$$v_3 = L_3 \frac{di_3}{dt} - M_{23} \frac{di_2}{dt} - M_{13} \frac{di_1}{dt}$$