Module
2
Measurement Systems

Lesson
5
Pressure and Force Measurement

Instructional Objectives

The reader, after going through the lesson would be able to

1. Name different methods for pressure measurement using elastic transducers.
2. Explain the construction and principle of operation of a Bourdon tube pressure gage.
3. Define gage factor of a strain gage
4. Name different strain gage materials and state their gage factors.
5. Will be able to draw the connection diagram of an unbalanced bridge with four strain gages so as to obtain maximum sensitivity and perfect temperature compensation.
6. Name different methods for force measurement with strain gages.

1. Introduction

In this lesson, we will discuss different methods for measurement of pressure and force. Elastic elements, namely diaphragms and Bourdon tubes are mainly used for pressure measurement. On the other hand, strain gages are commonly used for measurement of force. The constructions and principles of operation of different elastic elements for pressure measurement have been discussed in the next section. This is followed by principle of strain gage and measurement of force using strain gages.

2. Pressure Measurement

Measurement of pressure inside a pipeline or a container in an industrial environment is a challenging task, keeping in mind that pressure may be very high, or very low (vacuum); the medium may be liquid, or gaseous. We will not discuss the vacuum pressure measuring techniques; rather try to concentrate on measurement techniques of pressure higher than the atmospheric. They are mainly carried out by using elastic elements: diaphragms, bellows and Bourdon tubes. These elastic elements change their shape with applied pressure and the change of shape can be measured using suitable deflection transducers. Their basic constructions and principle of operation are explained below.

2.1 Diaphragms

Diaphragms may be of three types: Thin plate, Membrane and Corrugated diaphragm. This classification is based on the applied pressure and the corresponding displacements. Thin plate (fig. 1(a)) is made by machining a solid block and making a circular cross sectional area with smaller thickness in the middle. It is used for measurement of relatively higher pressure. In a membrane the sensing section is glued in between two solid blocks as shown in fig. 1(b). The thickness is smaller; as a result, when pressure is applied on one side, the displacement is larger. The sensitivity can be further enhanced in a corrugated diaphragm (fig. 1(c)), and a large deflection can be obtained for a small change in pressure; however at the cost of linearity. The materials used are Bronze, Brass, and Stainless steel. In recent times, Silicon has been extensively used the diaphragm material in MEMS (Micro Electro Mechanical Systems) pressure sensor. Further, the natural frequency of a diaphragm can be expressed as:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m_{eq}}} \qquad (1)$$

where m_{eq} = equivalent mass, and

k= elastic constant of the diaphragm.

The operating frequency of the pressure to be measured must be less than the natural frequency of the diaphragm.

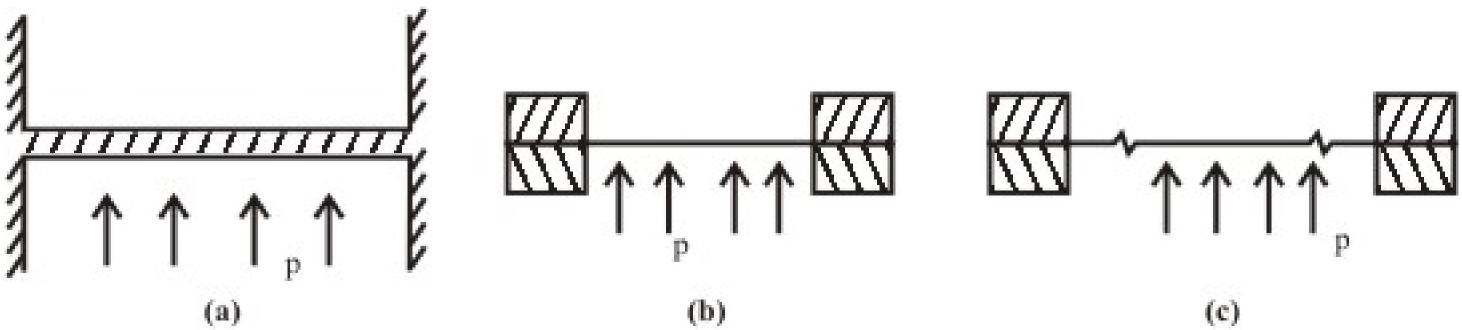


Fig. 1 (a) Thin plate, (b) Membrane and (c) Corrugated diaphragm.

When pressure is applied to a diaphragm, it deflates and the maximum deflection at the centre (y_0) can measured using a displacement transducer. For a Thin plate, the maximum deflection y_0 is small ($y_0 < 0.3t$) and referring fig. 2, a linear relationship between p and y_0 exist*s* as:

$$y_0 = \frac{3}{16} p \frac{(1-\nu^2)}{Et^3} R^4 \qquad (2)$$

where, E= Modulus of elasticity of the diaphragm material, and

ν = Poisson's ratio.

However, the allowable pressure should be less than:

$$p_{\max} = 1.5\left(\frac{t}{R}\right)^2 \sigma_{\max} \qquad (3)$$

where, $\sigma_{\max}$ is the safe allowable stress of the material.

For a membrane, the deflection is larger, and the relationship between p and y_0 is nonlinear and can be expressed as (for ν = 0.3):

$$p = 3.58 \frac{E\,t^3}{R^4} {y_0}^3 \qquad (4)$$

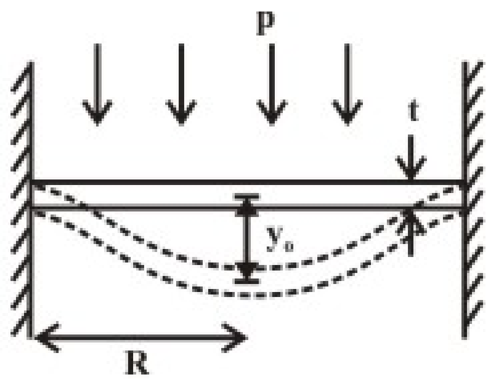


Fig. 2 Displacement of a diaphragm

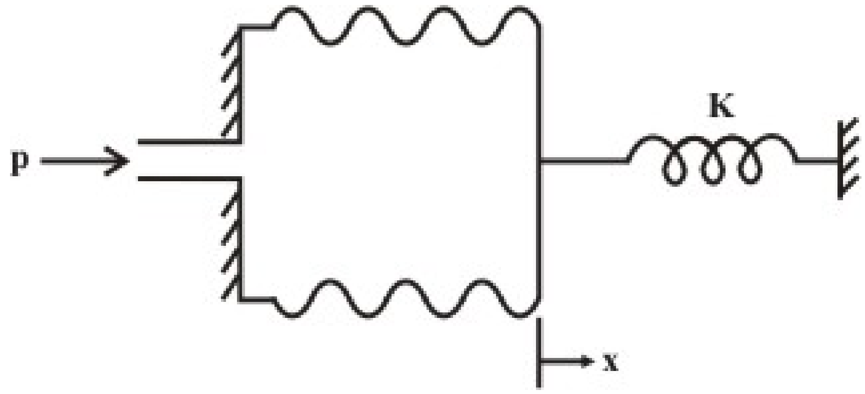


Fig. 3 Bellows

For a corrugated diaphragm, it is difficult to give any definite mathematical relationship between p and y_0; but the relationship is also highly nonlinear.

As the diaphragm deflates, strains of different magnitudes and signs are generated at different locations of the diaphragm. These strains can also be measured by effectively placing four strain gages on the diaphragm. The principle of strain gage will be discussed in the next section.

2.2 Bellows

Bellows (fig. 3) are made with a number of convolutions from a soft material and one end of it is fixed, wherein air can go through a port. The other end of the bellows is free to move. The displacement of the free end increases with the number of convolutions used. Number of convolutions varies between 5 to 20. Often an external spring is used opposing the movement of the bellows; as a result a linear relationship can be obtained from the equation:

$$p\,A = k\,x \tag{5}$$

where, A is the area of the bellows,
k is the spring constant and
x is the displacement of the bellows.

Phosphor Bronze, Brass, Beryllium Copper, Stainless Steel are normally used as the materials for bellows. Bellows are manufactured either by (i) turning from a solid block of metal, or (ii) soldering or welding stamped annular rings, or (iii) rolling (pressing) a tube.

2.3 Bourdon Tube

Bourdon tube pressure gages are extensively used for local indication. This type of pressure gages were first developed by E. Bourdon in 1849. Bourdon tube pressure gages can be used to measure over a wide range of pressure: form vacuum to pressure as high as few thousand psi. It is basically consisted of a C-shaped hollow tube, whose one end is fixed and connected to the pressure tapping, the other end free, as shown in fig. 4. The cross section of the tube is elliptical. When pressure is applied, the elliptical tube tries to acquire a circular cross section; as a result, stress is developed and the tube tries to straighten up. Thus the free end of the tube moves up, depending on magnitude of pressure. A deflecting and indicating mechanism is attached to the free end that rotates the pointer. The materials used are commonly Phosphor Bronze, Brass and Beryllium Copper. For a $2''$ overall diameter of the C-tube the useful travel of the free end is approximately $\frac{1}{8}''$. Though the C-type tubes are most common, other shapes of tubes, such as helical, twisted or spiral tubes are also in use.

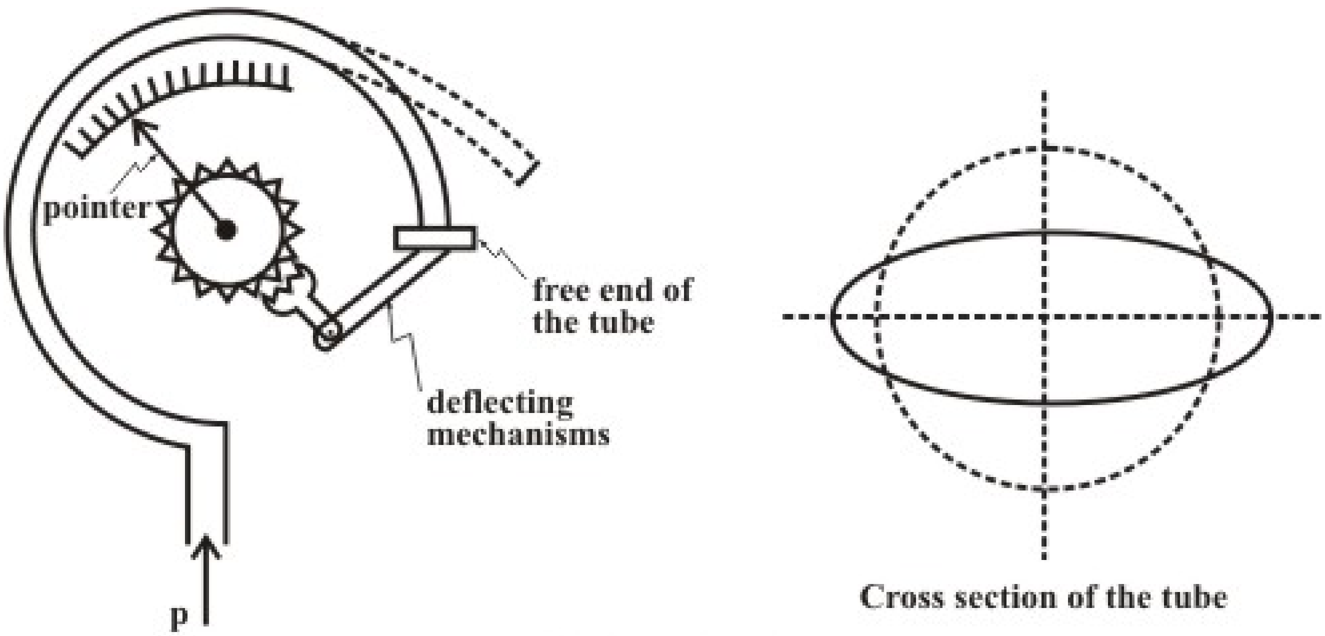


Fig. 4 Bourdon tube

3. Measurement of Force

The most popular method for measuring force is using strain gage. We measure the strain developed due to force using strain gages; and by multiplying the strain with the effective cross sectional area and Young's modulus of the material, we can obtain force. Load cells and Proving rings are two common methods for force measurement using strain gages. We will first discuss the principle of strain gage and then go for the force measuring techniques.

3.1 Strain Gage

Strain gage is one of the most popular types of transducer. It has got a wide range of applications. It can be used for measurement of force, torque, pressure, acceleration and many other parameters. The basic principle of operation of a strain gage is simple: when strain is applied to a thin metallic wire, its dimension changes, thus changing the resistance of the wire. Let us first investigate what are the factors, responsible for the change in resistance.

3.1.1 Gage Factor

Let us consider a long straight metallic wire of length l circular cross section with diameter d (fig. 5). When this wire is subjected to a force applied at the two ends, a strain will be generated and as a result, the dimension will change (l changing to $l+\Delta l$, d changing to $d+\Delta d$ and A changing to $A+\Delta A$). For the time being, we are considering that all the changes are in positive direction. Now the resistance of the wire:

$R = \dfrac{\rho l}{A}$, where ρ is the resistivity.

From the above expression, the change in resistance due to strain:

$$\Delta R = \left(\frac{\partial R}{\partial l}\right)\Delta l + \left(\frac{\partial R}{\partial A}\right)\Delta A + \left(\frac{\partial R}{\partial \rho}\right)\Delta \rho$$

$$= \frac{\rho}{A}\Delta l - \frac{\rho}{A^2}\Delta A + \frac{l}{A}\Delta \rho$$

$$= R\frac{\Delta l}{l} - R\frac{\Delta A}{A} + R\frac{\Delta \rho}{\rho}$$

or,

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho} \qquad (6)$$

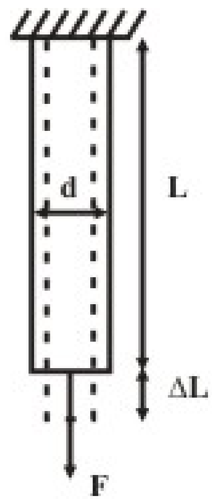


Fig. 5 Change of resistance with strain

Now, for a circular cross section, $A = \dfrac{\pi d^2}{4}$; from which, $\Delta A = \dfrac{\pi d}{2}\Delta d$. Alternatively,

$$\frac{\Delta A}{A} = 2\frac{\Delta d}{d}$$

Hence,

$$\frac{\Delta R}{R} = \frac{\Delta l}{l} - 2\frac{\Delta d}{d} + \frac{\Delta \rho}{\rho} \qquad (7)$$

Now, the *Poisson's Ratio* is defined as:

$$\upsilon = -\frac{lateral \quad strain}{longitudinal \quad strain} = -\frac{\Delta d/d}{\Delta l/l}$$

The Poisson's Ratio is the property of the material, and does not depend on the dimension. So, (6) can be rewritten as:

$$\frac{\Delta R}{R} = (1+2\upsilon)\frac{\Delta l}{l} + \frac{\Delta \rho}{\rho}$$

Hence,

$$\frac{\Delta R / R}{\Delta l / l} = 1 + 2\upsilon + \frac{\Delta\rho / \rho}{\Delta l / l}$$

The last term in the right hand side of the above expression, represents the change in resistivity of the material due to applied strain that occurs due to the *piezo-resistance property* of the material. In fact, all the elements in the right hand side of the above equation are independent of the geometry of the wire, subjected to strain, but rather depend on the material property of the wire. Due to this reason, a term *Gage Factor* is used to characterize the performance of a strain gage. The Gage Factor is defined as:

$$G := \frac{\Delta R / R}{\Delta l / l} = 1 + 2\upsilon + \frac{\Delta\rho / \rho}{\Delta l / l} \qquad (8)$$

For normal metals the Poisson's ratio υ varies in the range:

$0.3 \le \upsilon \le 0.6$,

while the piezo-resistance coefficient varies in the range:

$$0.2 \le \frac{\Delta\rho / \rho}{\Delta l / l} \le 0.6 .$$

Thus, the Gage Factor of metallic strain gages varies in the range 1.8 to 2.6. However, the semiconductor type strain gages have a very large Gage Factor, in the range of 100-150. This is attained due to dominant piezo-resistance property of semiconductors. The commercially available strain gages have certain fixed resistance values, such as, 120Ω, 350 Ω, 1000 Ω, etc. The manufacturer also specifies the Gage Factor and the maximum gage current to avoid self-heating (normally in the range 15 mA to 100 mA).

The choice of material for a metallic strain gage should depend on several factors. The material should have low temperature coefficient of resistance. It should also have low coefficient for thermal expansion. Judging from all these factors, only few alloys qualify for a commercial metallic strain gage. They are:

Advance (55% Cu, 45% Ni): Gage Factor between 2.0 to 2.2

Nichrome (80% Ni, 20% Co): Gage Factor between 2.2 to 2.5

Apart from these two, *Isoelastic* -another trademarked alloy with Gage Factor around 3.5 is also in use. Semiconductor type strain gages, though having large Gage Factor, find limited use, because of their high sensitivity and nonlinear characteristics.

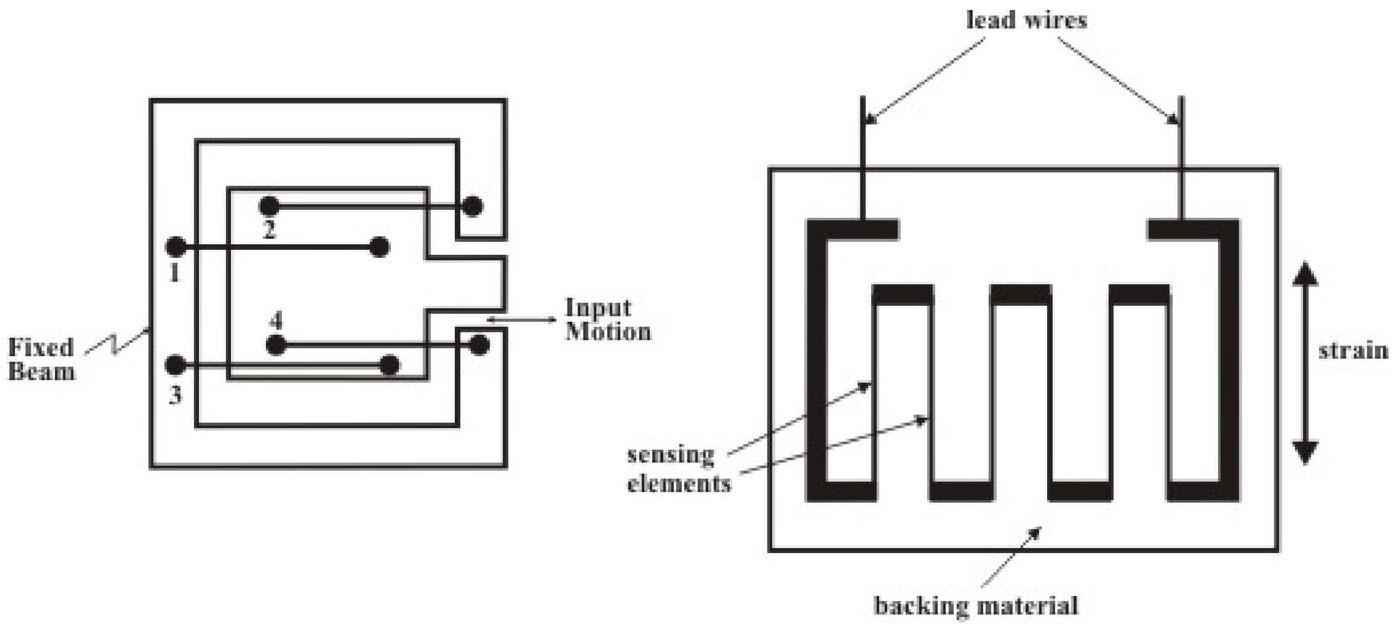


Fig. 6 (a) Unbonded metallic strain gage, (b) bonded metal foil type strain gage

3.1.2 Metallic Strain Gage

Most of the strain gages are metallic type. They can be of two types: *unbonded* and *bonded*. The unbonded strain gage is normally used for measuring strain (or displacement) between a fixed and a moving structure by fixing four metallic wires in such a way, so that two are in compression and two are in tension, as shown in fig. 6 (a). On the other hand, in the bonded strain gage, the element is fixed on a backing material, which is permanently fixed over a structure, whose strain has to be measured, with adhesive. Most commonly used bonded strain gages are *metal foil type*. The construction of such a strain gage is shown in fig. 6(b). The metal foil type strain gage is manufactured by photo-etching technique. Here the thin strips of the foil are the active elements of the strain gage, while the thick ones are for providing electrical connections. Because of large area of the thick portion, their resistance is small and they do not contribute to any change in resistance due to strain, but increase the heat dissipation area. Also it is easier to connect the lead wires with the strain gage. The strain gage in fig. 6(b) can measure strain in one direction only. But if we want to measure the strain in two or more directions at the same point, strain gage *rosette*, which is manufactured by stacking multiple strain gages in different directions, is used. Fig. 7 shows a three-element strain gage rosette stacked at 45^0.

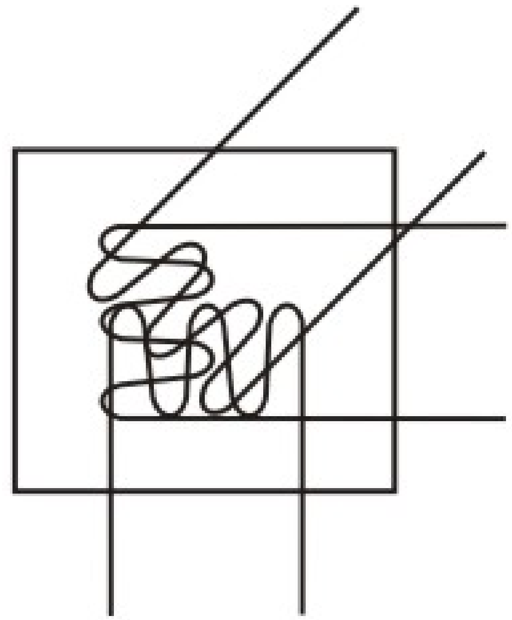


Fig. 7 Three-element strain gage rosette- 45° stacked.

The *backing material*, over which the strain gage is fabricated and which is fixed with the strain measuring structure has to satisfy several important properties. Firstly, it should have high mechanical strength; it should also have high dielectric strength. But the most important it should have is that it should be non-hygroscopic, otherwise, absorption of moisture will cause bulging and generate local strain. The backing materials normally used are impregnated paper, fibre glass, etc. The *bonding material* used for fixing the strain gage permanently to the structure should also be non-hygroscopic. Epoxy and Cellulose are the bonding materials normally used.

3.1.3 Semiconductor type Strain Gage

Semiconductor type strain gage is made of a thin wire of silicon, typically 0.005 inch to 0.0005 inch, and length 0.05 inch to 0.5 inch. They can be of two types: *p*-type and *n*-type. In the former the resistance increases with positive strain, while, in the later the resistance decreases with temperature. The construction and the typical characteristics of a semiconductor strain gage are shown in fig.8.

MEMS pressure sensors is now a days becoming increasingly popular for measurement of pressure. It is made of a small silicon diagram with four piezo-resistive strain gages mounted on it. It has an in-built signal conditioning circuits and delivers measurable output voltage corresponding to the pressure applied. Low weight and small size of the sensor make it suitable for measurement of pressure in specific applications.

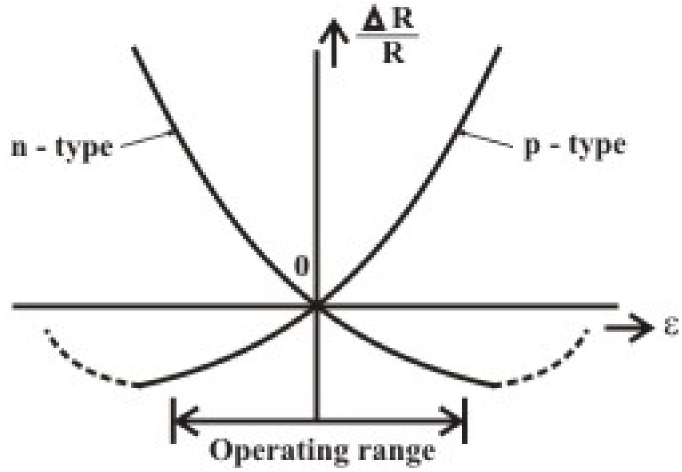


Fig. 8 (a) construction and (b) characteristics of a semiconductor strain gage

3.1.4 Strain Gage Bridge

Normal strain experienced by a strain gage is in the range of micro strain (typical value: 100 x 10^{-6}). As a result, the change in resistance associated with it is small ($\Delta R/R = G\varepsilon$). So if a single strain gage is connected to a wheatstone bridge, with three fixed resistances, the bridge output voltage is going to be linear (recall, that we say the bridge output voltage would be linearly varying with $\Delta R/R$, if $\Delta R/R$ does not exceed 0.1). But still then, a single strain gage is normally never used in a wheatstone bridge. This is not because of improving linearity, but for obtaining *perfect temperature compensation*. Suppose one strain gage is connected to a bridge with three fixed arms. Due to temperature rise, the strain gage resistance will change, thus making the bridge unbalance, thus giving an erroneous signal, even if no strain is applied. If two identical strain gages are fixed to the same structure, one measuring compressional strain and the other tensile strain, and connected in the adjacent arms of the bridge, temperature compensation can be achieved. If the temperature increases, both the strain gage resistances will be affected in the same way, thus maintaining the bridge balance under no strain condition. One more advantage of using the push-pull configuration is increasing the sensitivity. In fact, all the four arms of the bridge can be formed by four active gages; this will improve the sensitivity further, while retaining the temperature compensation property. A typical strain gage bridge is shown in fig. 9. It can be shown that if nominal resistances of the strain gages are same and also equal gage factor *G*, then the unbalanced voltage is given be:

$$e_0 = \frac{EG}{4}\left(\varepsilon_1 + \varepsilon_3 - \varepsilon_2 - \varepsilon_4\right) \qquad (9)$$

where ε_1, ε_2, ε_3, ε_4 are the strains developed with appropriate signs.

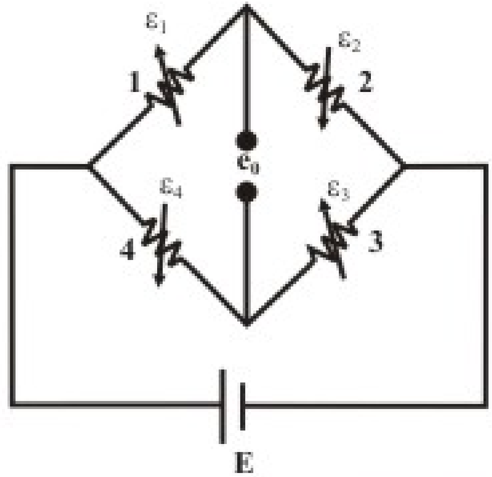


Fig. 9 Strain gage bridge with four active gages

3.2 Load Cell

Load cells are extensively used for measurement of force; weigh bridge is one of the most common applications of load cell. Here two strain gages are fixed so as to measure the longitudinal strain, while two other measuring the transverse strain, as shown in fig. 10. The strain gages, measuring the similar strain (say, tensile) are placed in the opposite arms, while the adjacent arms in the bridge should measure opposite strains (one tensile, the other compressional). If the strain gages are identical in characteristics, this will provide not only the perfect temperature coefficient, but also maximum obtainable sensitivity from the bridge. The longitudinal strain developed in the load cell would be compressional in nature, and is given by: $\varepsilon_1 = -\frac{F}{A\ E}$, where F is the force applied, A is the cross sectional area and Y is the Young's modulus of elasticity. The strain gages 1 and 3 will experience this strain, while for 2 and 4 the strain will be $\varepsilon_2 = \frac{\nu F}{A E}$, where ν is the Poisson's ratio.

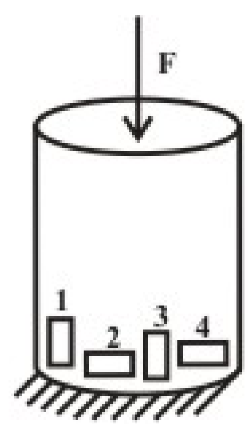


Fig. 10 Load cell with four strain gages

3.3 Proving Ring

Proving Rings can be used for measurement of both compressional and tensile forces. The advantage of a Proving Ring is that, because of its construction more strain can be developed compared to a load cell. The typical construction of a Proving Ring is shown in fig.11. It consists of a hollow cylindrical beam of radius *R*, thickness *t* and axial width *b*. The two ends of the ring are fixed with the structures between which force is measured. Four strain gages are mounted on the walls of the proving ring, two on the inner wall, and two on the outer wall. When force is applied as shown, gages 2 and 4 will experience strain $-\varepsilon$ (compression), while gages 1 and 3 will experience strain $+\varepsilon$ (tension). The magnitude of the strain is given by the expression:

$$\varepsilon = \frac{1.08FR}{Ebt^2} \qquad (10)$$

The four strain gages are connected in a bridge and the unbalanced voltage can easily be calibrated in terms of force to be measured.

3.4 Cantilever Beam

Cantilever beam can be used for measurement up to 10 kg of weight. One end of the cantilever is fixed, while the other end is free; load is applied at this end, as shown in fig. 12. The strain developed at the fixed end is given by the expression:

$$\varepsilon = \frac{6Fl}{Ebt^2} \qquad (11)$$

where,

l = length of the beam
t = thickness of the cantilever
b = width of the beam
E = Young's modulus of the material

The strain developed can be measured by fixing strain gages at the fixed end: two on the top side of the beam, measuring tensile strain $+\varepsilon$ and two on the bottom measuring compressional strain $-\varepsilon$ (as shown in fig. 12) and using eqn. (9).

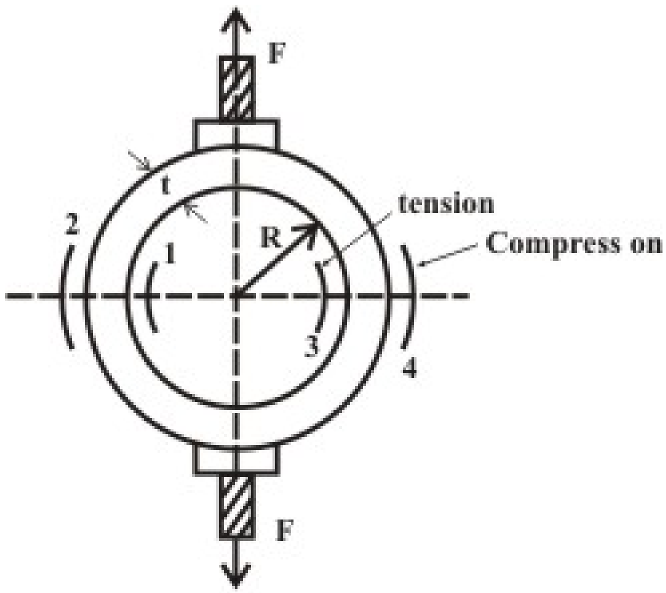


Fig. 11 Proving Ring

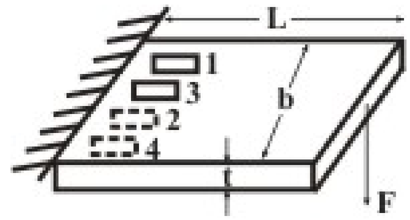


Fig. 12 Cantilever Beam

4. Conclusion

In this lesson, we have studied the commonly used sensing elements for measurement of pressure and force. Elastic elements are used for measurement of pressure, where the pressure signal is converted into displacement signal. Displacement sensors are further used to convert this to appropriate electrical signal. Strain gages are also sometimes used to measure strain developed on the diaphragm.

On the other hand, load cells, Proving Rings and Cantilever Beams are used for force measurement. Here strain gages mounted on the sensing elements measure strains, and the unbalanced voltage of a strain gage bridge can be effectively calibrated in terms of force. Another method of force measurement is using *magnetostrictive transducers*; but its principle of operation is beyond the scope of this lesson.

Review Exercise

1. Which one of the elastic transducers: Bellows, Thin Plate and Corrugated Diaphragm, can be used for measurement of high pressure?
2. Bellows are commonly used in conjunction with a spring. Why?
3. Explain the construction and principle of operation of a Bourdon tube pressure gage.
4. Define gage factor of strain gage. What are the strain gage materials normally used? Which one of them is having maximum gage factor?
5. What is a strain gage rosette?
6. A 120 Ω strain gage of Gage Factor 2.0 is subjected to a positive strain of 1×10^{-6}. Find the change in resistance.
7. How the effect of temperature variation can be compensated in a strain gage bridge?

8. How would you connect four strain gages on a cantilever beam so as to achieve maximum sensitivity and perfect temperature compensation? Show the arrangement of placing the strain gages and the bridge arrangement.

Answer

Q6. 0.24 mΩ (increase).

Strain gauge and rosettes

Introduction

A **strain gauge** is a device which is used to measure strain (deformation) on an object subjected to forces.

Strain can be measured using various types of devices classified depending upon their principle of operation. Some of them are as follows:

1. Mechanical type,

2. Optical type

3. Pneumatic type

4. Electrical type

Earlier, mechanical type of device such as extensometer or extension meter was used to measure strain by measuring change in length.

Photoelectric strain gauge was also introduced which uses a light beam to produce electric current corresponding to deformation.

The most commonly used strain gauge is an **electrical resistance strain gauge**.

This strain gauge works on the principle that when a metallic wire type gauge is strained (here due to forces on object in contact), the resistance of the wire will be changed due to changes in its length, diameter and resistivity.

$$\text{Resistance (R)} = \frac{\rho L}{A}$$

Where ρ is resistivity; L is length of wire; A is area of crossection of wire.

This change in resistance will be in proportion with the strain produced which can be easily measured using Wheatstone bridge.

Drawbacks of Strain gauge

1. A strain gauge is capable only of measuring strain in the direction in which gauge is oriented.

2. There is no direct way to measure the shear strain or to directly measure the principal strains as directions of principal planes are not generally known.

Strain rosettes

Since for strain analysis in biaxial state of stress we should know strain in three directions and due to drawbacks in a strain gauge, Strain rosettes came in to picture.

Strain rosette can be defined as the arrangement of strain gauges in three arbitrary directions.

These strain gauges are used to measure the **normal strain** in those three directions.

Depending on the arrangement of strain gauges, strain rosettes are classified in to:-

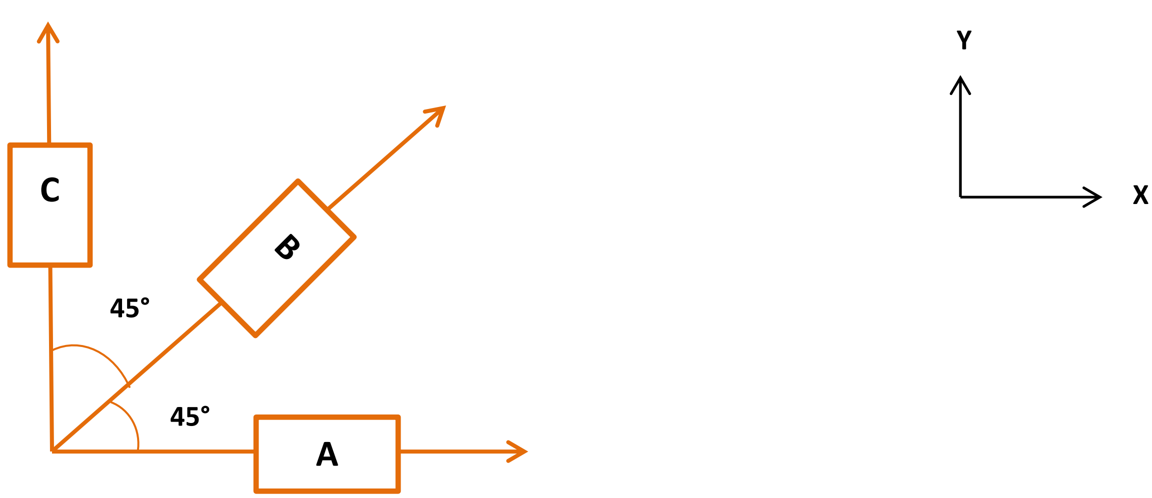
1. Rectangular strain gauge rosette

2. Delta strain gauge rosette

3. Star strain gauge rosette

Rectangular strain gauge rosette

A rectangular strain rosette consists of three strain gauges arranged as follows:-



We know normal strain in any direction (θ) is given by

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

where ε_x = normal strain at a point in x-direction

ε_y = normal strain at a point in y- direction

γ_{xy} = shear strain at a point on x face in y direction

So, normal strain at θ = 0°

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0° + \frac{\gamma_{xy}}{2}\sin 0°$$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)$$

$$\boldsymbol{\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \varepsilon_x} \quad \text{———————} \quad \textbf{Eqn (1)}$$

Normal strain at θ = 45° (with respect to strain guage A, anticlockwise)

$$\varepsilon_B = (\varepsilon_n)_{\theta=45°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 90° + \frac{\gamma_{xy}}{2}\sin 90°$$

$$\varepsilon_B = (\varepsilon_n)_{\theta=45°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{\gamma_{xy}}{2}$$

$$\boldsymbol{\varepsilon_x + \varepsilon_y + \gamma_{xy} = 2\,\varepsilon_B} \quad \text{———————} \quad \textbf{Eqn (2)}$$

Similarly, normal strain at θ= 90° (with respect to strain guage A, anticlockwise)

$$\varepsilon_C = (\varepsilon_n)_{\theta=90°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 180° + \frac{\gamma_{xy}}{2}\sin 180°$$

$$\varepsilon_c = (\varepsilon_n)_{\theta=90^\circ} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) - \frac{1}{2}(\varepsilon_x - \varepsilon_y)$$

$$\boldsymbol{\varepsilon_c = (\varepsilon_n)_{\theta=90^\circ} = \varepsilon_y} \quad \text{\_\_\_\_\_\_\_\_\_\_} \quad \textbf{Eqn (3)}$$

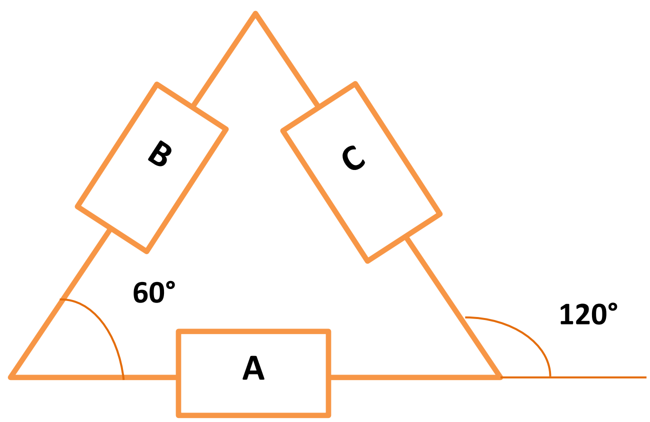
From Eqn 1 , 2 and 3

$$\boldsymbol{\gamma_{xy} = 2\,\varepsilon_B - \varepsilon_A - \varepsilon_c}$$

Note :- With the help of rectangular strain rosette we get strains in three arbitrary directions which in turn give us $\boldsymbol{\varepsilon_x, \varepsilon_y}$ **and** $\boldsymbol{\gamma_{xy}}$ and hence principal strains at a point on the surface of the object can be determined.

Delta strain gauge rosette

A delta strain gauge also consist of three strain gauges arranged as shown below



Delta strain gauge rosette

$$\boldsymbol{\varepsilon_A = (\varepsilon_n)_{\theta=0^\circ}}$$

$$\boldsymbol{\varepsilon_B = (\varepsilon_n)_{\theta=60^\circ}}$$

$$\boldsymbol{\varepsilon_c = (\varepsilon_n)_{\theta=120^\circ}}$$

We know normal strain in any direction (θ) is given by

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

So, normal strain at $\theta = 0°$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0° + \frac{\gamma_{xy}}{2}\sin 0°$$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)$$

$$\boldsymbol{\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \varepsilon_x} \quad \text{———— } \textbf{Eqn (4)}$$

Normal strain at $\theta = 60°$ (with respect to strain gauge A, anticlockwise)

$$\boldsymbol{\varepsilon_B = (\varepsilon_n)_{\theta=60°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 120° + \frac{\gamma_{xy}}{2}\sin 120°} \quad \text{————}$$

Eqn (5)

Similarly, normal strain at $\theta = 120°$ (with respect to strain gauge A , anticlockwise)

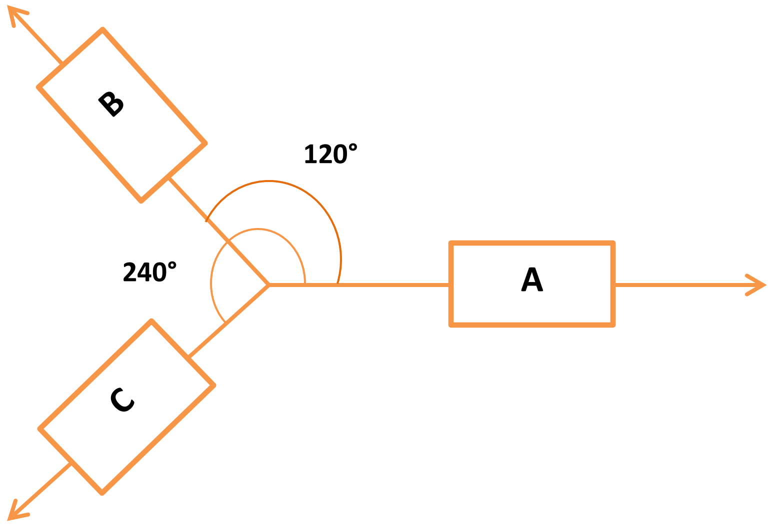
$$\boldsymbol{\varepsilon_C = (\varepsilon_n)_{\theta=120°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 240° + \frac{\gamma_{xy}}{2}\sin 240°} \quad \text{————}$$

Eqn (6)

Similarly, $\boldsymbol{\varepsilon_x}$ **,** $\boldsymbol{\varepsilon_y}$ **and** $\boldsymbol{\gamma_{xy}}$ can be determined from equations 4, 5 and 6 and hence principal strains at a point on the surface of the object.

Star strain gauge rosette

This rosette consist of three strain gauges in three directions as shown below



Star strain gauge rosette

$$\boldsymbol{\varepsilon_A = (\varepsilon_n)_{\theta=0°}}$$

$$\boldsymbol{\varepsilon_B = (\varepsilon_n)_{\theta=120°}}$$

$$\boldsymbol{\varepsilon_c = (\varepsilon_n)_{\theta=240°}}$$

We know normal strain in any direction (θ) is given by

$$\varepsilon_n = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\,\mathrm{Sin}\,2\theta$$

So, normal strain at $\theta = 0°$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0° + \frac{\gamma_{xy}}{2}\,\mathrm{Sin}\,0°$$

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)$$

$$\boldsymbol{\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \varepsilon_x} \quad \text{———} \quad \textbf{Eqn (7)}$$

Normal strain at $\theta = 120°$ (with respect to strain gauge A, anticlockwise)

$$\varepsilon_B = (\varepsilon_n)_{\theta=120°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 240° + \frac{\gamma_{xy}}{2}\sin 240° \text{ ——— }$$

Eqn (8)

Similarly, normal strain at θ= 240° (with respect to strain gauge A, anticlockwise)

$$\varepsilon_c = (\varepsilon_n)_{\theta=240°} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 480° + \frac{\gamma_{xy}}{2}\sin 480° \text{ ——— }$$

Eqn (9)

Similarly, by solving equations 7, 8 and 9 we can determine ε_x, ε_y **and** γ_{xy} and hence principal strains at a point on the surface of the object.

Sample Question for Concept brush-up

S-1. Determine maximum shear stress at a point by using the following strain gauge readings of rectangular strain rosette

$\varepsilon_{0°} = 400 * 10^{-6}$	**Young's Modulus = E= 200 GPa**
$\varepsilon_{45°} = 375 * 10^{-6}$	**Poisson ratio = 0.25**
$\varepsilon_{90°} = 200 * 10^{-6}$	

Solution : We know from Eqn 1 ,2 and 3

$$\varepsilon_A = (\varepsilon_n)_{\theta=0°} = \varepsilon_x = 400 * 10^{-6}$$

$$\varepsilon_c = (\varepsilon_n)_{\theta=90°} = \varepsilon_y = 200 * 10^{-6}$$

$$\varepsilon_B = (\varepsilon_n)_{\theta=45°} = 375 * 10^{-6}$$

and $\varepsilon_x + \varepsilon_y + \gamma_{xy} = 2\,\varepsilon_B$

$\gamma_{xy} = 150 * 10^{-6}$

Principal strain, $\varepsilon_{1,2} = \frac{1}{2}[(\varepsilon_x + \varepsilon_y) \pm \sqrt{(\varepsilon x - \varepsilon y)^2 + (\gamma xy)^{\wedge}2}]$

$\varepsilon_1 = 425 * 10^{-6}$

$\varepsilon_2 = 175 * 10^{-6}$

Principal stress, $\sigma_1 = \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\ \varepsilon_2)$

$\sigma_1 = 100$ Mpa

$\sigma_2 = \frac{E}{1-\mu^2}(\varepsilon_2 + \mu\ \varepsilon_1)$

$\sigma_2 = 60$ Mpa

Absolute $\tau_{max} = \frac{\sigma_1}{2}$ Or $\frac{\sigma_1 - \sigma_2}{2}$

As principal stresses like in nature therefore

Ans. **Absolute $\tau_{max} = \frac{\sigma_1}{2} = 50$ Mpa**

References

1. **Mechanics of Materials by B.C. Punmia**

2. **Strength of Materials by R.K. Bansal**