COURSE MATERIAL

II Year B. Tech I- Semester

MECHANICAL ENGINEERING

KINEMATICS OF MACHINERY

R18A0307

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

(Autonomous Institution-UGC, Govt. of India)
Secunderabad-500100, Telangana State, India.
www.mrcet.ac.in

CONTENTS

LIVE TO LEARN & LEARN TO SHARE

MRCET CAMPUS

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

VISION

- To establish a pedestal for the integral innovation, team spirit, originality and competence in the students, expose them to face the global challenges and become technology leaders of Indian vision of modern society.

MISSION

- To become a model institution in the fields of Engineering, Technology and Management.
- To impart holistic education to the students to render them as industry ready engineers.
- To ensure synchronization of MRCET ideologies with challenging demands of International Pioneering Organizations.

QUALITY POLICY

- To implement best practices in Teaching and Learning process for both UG and PG courses meticulously.
- To provide state of art infrastructure and expertise to impart quality education.
- To groom the students to become intellectually creative and professionally competitive.
- To channelize the activities and tune them in heights of commitment and sincerity, the requisites to claim the never - ending ladder of **SUCCESS** year after year.

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VISION

To become an innovative knowledge center in mechanical engineering through state-of-the-art teaching-learning and research practices, promoting creative thinking professionals.

MISSION

The Department of Mechanical Engineering is dedicated for transforming the students into highly competent Mechanical engineers to meet the needs of the industry, in a changing and challenging technical environment, by strongly focusing in the fundamentals of engineering sciences for achieving excellent results in their professional pursuits.

Quality Policy

- ✓ To pursuit global Standards of excellence in all our endeavors namely teaching, research and continuing education and to remain accountable in our core and support functions, through processes of self-evaluation and continuous improvement.
- ✓ To create a midst of excellence for imparting state of art education, industry-oriented training research in the field of technical education.

PROGRAM OUTCOMES

Engineering Graduates will be able to:

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. **Life-long learning**: Recognize the need for and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSOs)

PSO1 Ability to analyze, design and develop Mechanical systems to solve the Engineering problems by integrating thermal, design and manufacturing Domains.

PSO2 Ability to succeed in competitive examinations or to pursue higher studies or research.

PSO3 Ability to apply the learned Mechanical Engineering knowledge for the Development of society and self.

Program Educational Objectives (PEOs)

The Program Educational Objectives of the program offered by the department are broadly listed below:

PEO1: PREPARATION

To provide sound foundation in mathematical, scientific and engineering fundamentals necessary to analyze, formulate and solve engineering problems.

PEO2: CORE COMPETANCE

To provide thorough knowledge in Mechanical Engineering subjects including theoretical knowledge and practical training for preparing physical models pertaining to Thermodynamics, Hydraulics, Heat and Mass Transfer, Dynamics of Machinery, Jet Propulsion, Automobile Engineering, Element Analysis, Production Technology, Mechatronics etc.

PEO3: INVENTION, INNOVATION AND CREATIVITY

To make the students to design, experiment, analyze, interpret in the core field with the help of other inter disciplinary concepts wherever applicable.

PEO4: CAREER DEVELOPMENT

To inculcate the habit of lifelong learning for career development through successful completion of advanced degrees, professional development courses, industrial training etc.

PEO5: PROFESSIONALISM

To impart technical knowledge, ethical values for professional development of the student to solve complex problems and to work in multi-disciplinary ambience, whose solutions lead to significant societal benefits.

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

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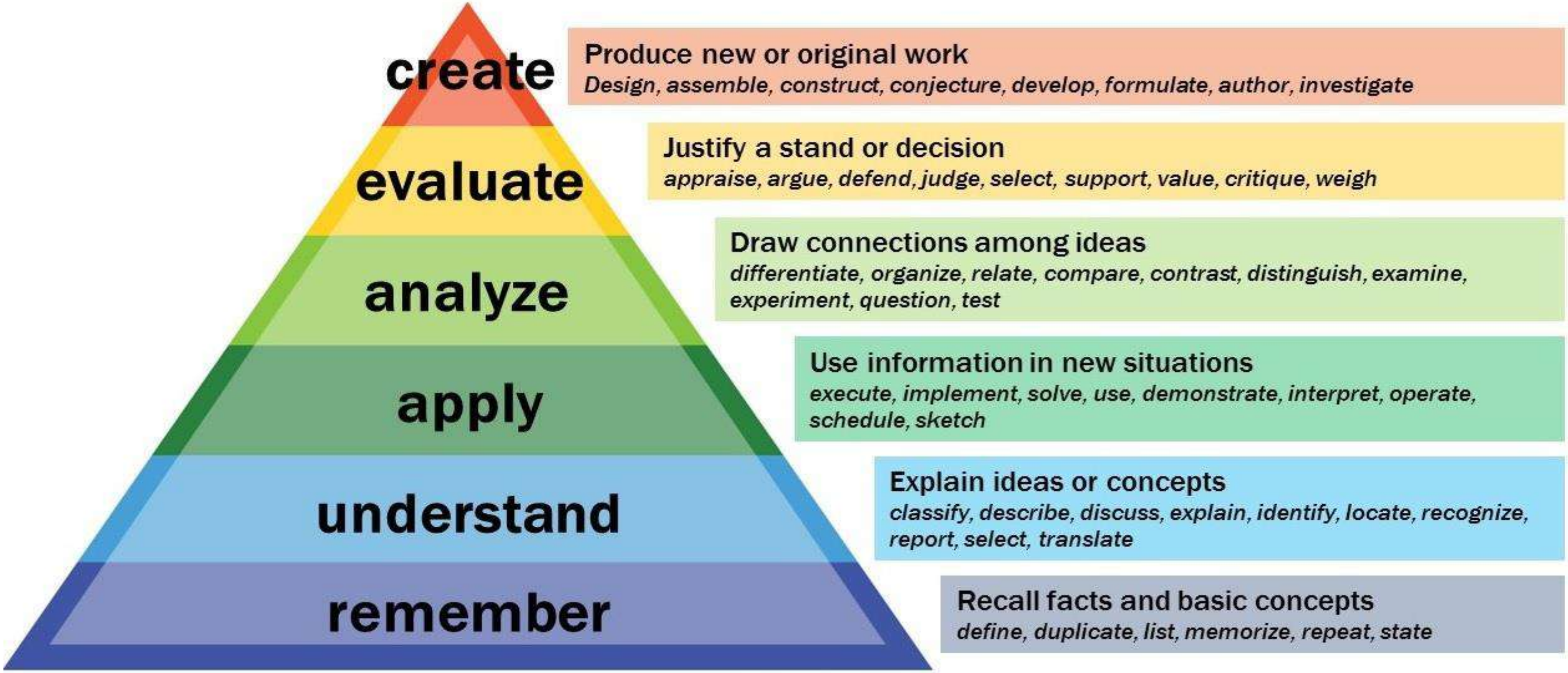
Department of Mechanical Engineering

Blooms Taxonomy

Bloom’s Taxonomy is a classification of the different objectives and skills that educators set for their students (learning objectives). The terminology has been updated to include the following six levels of learning. These 6 levels can be used to structure the learning objectives, lessons, and assessments of a course.

1. **Remembering**: Retrieving, recognizing, and recalling relevant knowledge from long- term memory.
2. **Understanding**: Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.
3. **Applying**: Carrying out or using a procedure for executing or implementing.
4. **Analyzing**: Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing.
5. **Evaluating**: Making judgments based on criteria and standard through checking and critiquing.
6. **Creating**: Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing.



Produce new or original work
Design, assemble, construct, conjecture, develop, formulate, author, investigate

Justify a stand or decision
appraise, argue, defend, judge, select, support, value, critique, weigh

Draw connections among ideas
differentiate, organize, relate, compare, contrast, distinguish, examine, experiment, question, test

Use information in new situations
execute, implement, solve, use, demonstrate, interpret, operate, schedule, sketch

Explain ideas or concepts
classify, describe, discuss, explain, identify, locate, recognize, report, select, translate

Recall facts and basic concepts
define, duplicate, list, memorize, repeat, state

II Year B. Tech, ME-I Sem

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(R18A0307) KINEMATICS OF MACHINERY

Course Objectives:

1. To impart knowledge on various types of Mechanisms and synthesis
2. To Synthesize and analyze 4 bar mechanisms
3. To impart skills to analyse the position, velocity and acceleration of mechanisms
4. To perform synthesis of mechanism by analytical and graphical method
5. To familiarize higher pairs like cams and principles of cams design
6. To study the relative motion analysis and design of gears, gear trains.

UNIT-I

Mechanisms : Elements or Links , Classification, Rigid Link, flexible and fluid link, Types of kinematic pairs , sliding, turning, rolling, screw and spherical pairs lower and higher pairs, closed and open pairs, constrained motion, completely, partially or successfully constrained and incompletely constrained .

Machines : Mechanism and machines, classification of machines, kinematic chain , inversion of mechanism, inversion of mechanism , inversions of quadric cycle, chain , single and double slider crank chains.

UNIT-II

Straight Line Motion Mechanisms: Exact and approximate copiers and generated types Peaucellier, Hart and Scott Russul Grasshopper Watt T. Chebicheff and Robert Mechanisms and straight line motion, Pantograph.

Steering Mechanisms: Conditions for correct steering Davis Steering gear, Ackermans steering gear velocity ratio.

Hooke's Joint: Single and double Hookes joint Universial coupling application problems.

UNIT-III

Kinematics: Velocity and acceleration - Motion of link in machine - Determination of Velocity and acceleration diagrams - Graphical method - Application of relative velocity method four bar chain.

Plane motion of body: Instantaneous center of rotation, centroids and axodes - relative motion between two bodies - Three centres in line theorem - Graphical determination of instantaneous centre, diagrams for simple mechanisms and determination of angular velocity of points and links.

UNIT-IV

Cams: Definitions of cam and followers their uses Types of followers and cams Terminology Types of follower motion - Uniform velocity Simple harmonic motion and uniform acceleration. Maximumm velocity and maximum acceleration during outward and return strokes in the above 3 cases.

Analysis of motion of followers: Roller follower circular cam with straight, concave and convex flanks.

UNIT-V

Gears: Higher pairs, friction wheels and toothed gears types law of gearing, condition for constant velocity ratio for transmission of motion, Form of teeth: cycloidal and involute profiles. Velocity of sliding phenomena of interferences. Methods of interference. Condition for minimum number of teeth to avoid interference, expressions for arc of contact and path of contact - Introduction to Helical, Bevel and worm gearing.

Gear Trains: Introduction - Train value - Types - Simple and reverted wheel train

Epicycle gear Train. Methods of finding train value or velocity ratio - Epicycle gear trains. Selection of gear box-Differential gear for an automobile.

TEXT BOOKS:

1. Kinematics of Machinery – Special Edition. MRCET, McGrahill Publishers.
2. Theory of Machines by Thomas Bevan/ CBS
3. Theory of machines/ PL. Balaney/khanna publishers.

REFERENCE BOOKS:

1. The theory of Machines /Shiegley/ Oxford.
2. Mechanism and Machine Theory / JS Rao and RV Dukkipati / New Age International Publishers
3. Theory of Machines / R.K Bansal/Fire Wall media Publisher

Course Outcomes:

1. Understand the principles of kinematic pairs, chains and their classification, DOF, inversions, equivalent chains and planar mechanisms.
2. Analyze the planar mechanisms for position, velocity and acceleration.
3. Synthesize planar four bar and slider crank mechanisms for specified kinematic conditions.
4. Design cams and followers for specified motion profiles
5. Evaluate gear tooth geometry and select appropriate gears for the required applications.

UNIT 1

MECHANISMS & MACHINES

COURSE OBJECTIVE

To impart knowledge on various types of Mechanisms and synthesis.

COURSE OUTCOME

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	**Mechanisms**	Definition of Mechanism Definition of Machine	• Understanding the mechanics of rigid, fixed, deformable bodies (B2)
2	Kinematic Link and Classification of Links	Definition of Link Definition of Pair Classification of Links Classification of Pairs	• State the basic concept of link and pair (B1) • Understanding the classification of links and pairs (B2)
3	Constrained Motion and Classification	Definition of Constrained Motion Classification of Constrained Motion	• Describe the constrained motion (B1) • Understanding the direction of motion (B2)
4	**Mechanism and Machines**	Definition of Machine. Determine the nature of chain. Definition of Grashof's law.	• Analyse machine and structure (B4) • Evaluate the nature of mechanism (B5)
5	Inversion of Mechanism	Definition of inversion. Classification of inversion of mechanism	• Understanding the inversion of mechanisms and its classifications (B2)
6	Inversions of Quadric Cycle	Working of 4-bar chain mechanisms	• Understanding the important inversions of 4-bar mechanism (B2) • Analyse the inversion of 4-bar mechanism (B4)
7	Inversion of Single Slider Crank Chains	Working of Single slider crank chain mechanisms	• Understanding the important inversions of single mechanism (B2) • Analyse the inversion of single mechanism (B4)
8	Inversion of Double Slider Crank Chains	Working of Double slider crank chain mechanisms	• Understanding the important inversions of single mechanism (B2)

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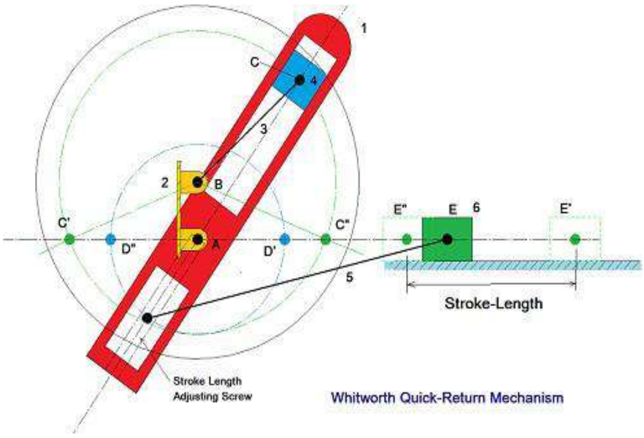
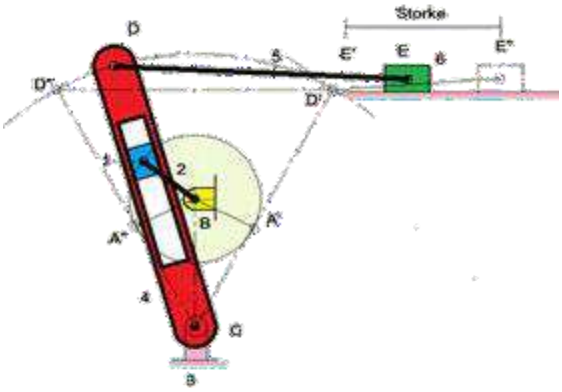
Machines and Mechanisms



Whitworth Quick-Return Mechanism



Crank-Rocker Quick-Return Mechanism for Shaping Machine

Course Contents

1.1 Machine and Mechanism:

- **Mechanism:**
  - If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism*.
- **Machine:**
  - A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.
- **Analysis:**
  - *Analysis* is the study of motions and forces concerning different parts of an existing mechanism.
- **Synthesis:**
  - *Synthesis* involves the design of its different parts.

1.2 Types of constrained motion:

1.2.1 Completely constrained motion:

- When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.
- For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (*i.e.* it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank.

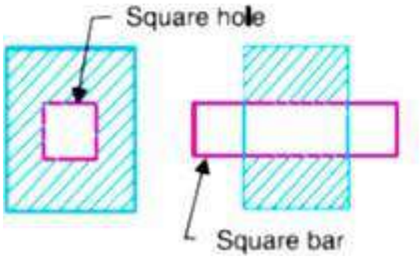


Fig. 1.1

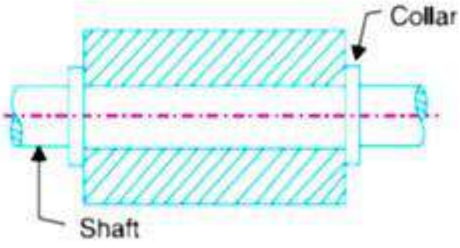


fig. 1.2

- The motion of a square bar in a square hole, as shown in Fig. 1.1, and the motion of a shaft with collars at each end in a circular hole, as shown in Fig. 1.2, are also examples of completely constrained motion.

1.2.2 Incompletely constrained motion:

- When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig. 1.3, is an example of an incompletely constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other.

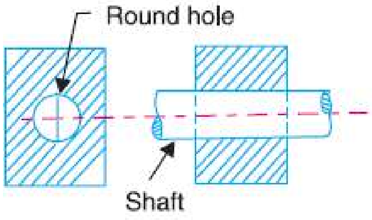


Fig. 1.3

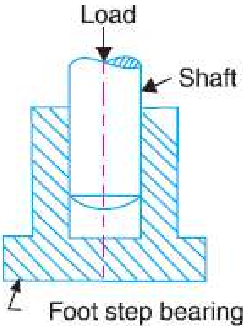


FIG. 1.4

1.2.3 Successfully constrained motion:

- When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig. 1.4.
- The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine

1.3 Types of Links:

- A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movements is known as a link.
- A link may also define as a member or a combination of members of a mechanism, connecting other members and having motion relative to them.
- Links may be classified into binary, ternary and quaternary.

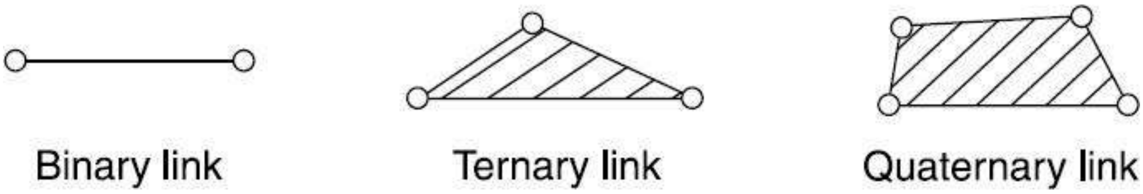


FIG. 1.4 Types of link

1.4 Kinematic Pair:

- When two kinematic links are connected in such a way that their motion is either completely or successfully constrained, these two links are said to form a kinematic pair.
- Kinematic pairs can be classified according to:

1.4.1 Kinematic pairs according to nature of contact:

a. Lower Pair:

- A pair of links having surfaced or area contact between the members is known as a lower pair. The contact surfaces of two links are similar.
- Examples: Nut turning on a screw, shaft rotating in a bearing.

b. Higher Pair:

- When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of two links are similar.
- Example: Wheel rolling on a surface, Cam and Follower pair etc.

1.4.2 Kinematic pairs according to nature of contact:

a. Closed Pair:

- When the elements of a pair are held together mechanically, it is known as a closed pair. The two elements are geometrically identical; one is solid and full and the other is hollow or open. The latter not only envelops the former but also encloses it. The contact between the two can be broken only by destruction of at least one of the members.

b. Unclosed Pair:

- When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically, e.g. cam and follower pair.

1.4.3 Kinematic pairs according to Nature of Relative Motion:

a. Sliding pair:

- When two links have a sliding motion relative to another; the kinematic pair is known as sliding pair.

b. Turning pair:

- When one link is revolve or turn with respect to the axis of first link, the kinematic pair formed by two links is known as turning pair.

c. Rolling pair:

- When the links of a pair have a rolling motion relative to each other, they form a rolling pair.

d. Screw pair:

- If two mating links have a turning as well as sliding motion between them, they form a screw pair.

e. Spherical pair:

- When one link in the form of sphere turns inside a fixed link, it is a spherical pair.

1.5 Types of Joint:

- The usual types of joints in a chain are:
    - ○ Binary Joint
    - ○ Ternary Joint
    - ○ Quaternary Joint

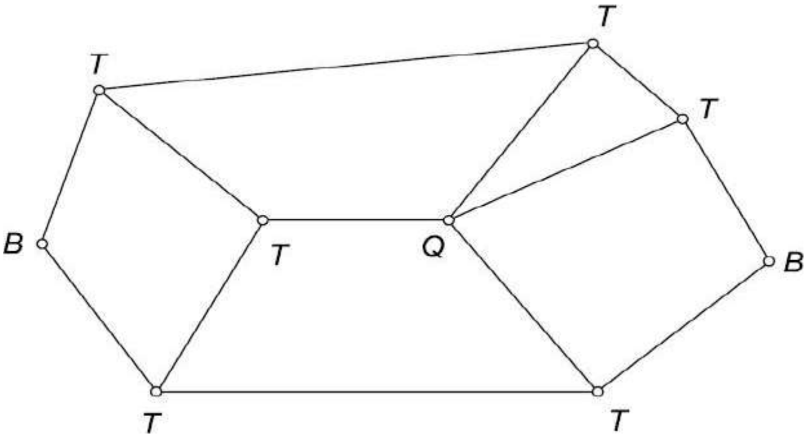


Fig1.5. Types of joint

a. **Binary Joint:**
   - If two links are joined at the same connection, it is called a binary joint. For example, in fig. at joint B

b. **Ternary Joint:**
   - If three links joined at a connection, it is known as a ternary link. For example point T in fig.

c. **Quaternary Joint:**
   - If four links joined at a connection, it is known as a quaternary link. For example point Q in fig.

1.6 Degrees of Freedom:

- An unconstrained rigid body moving in space can describe the following independent motion:
  a. Translational motion along any three mutually perpendicular axes x, y and z.
  b. Rotational motion about these axes

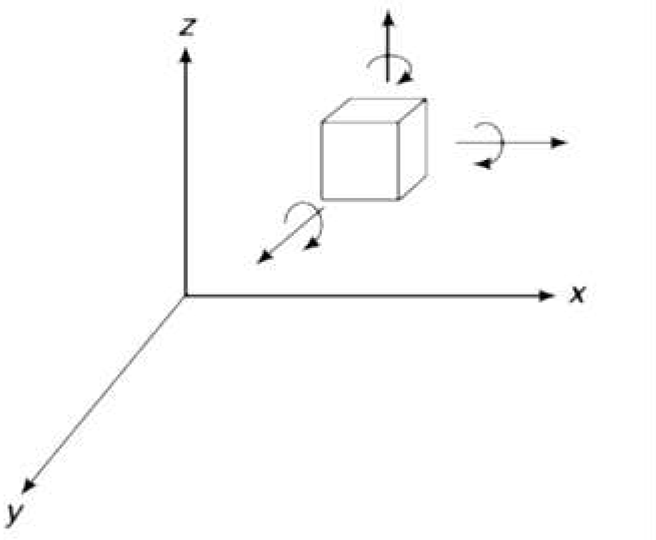


Fig.1.6 Degrees of freedom

- A rigid body possesses six degrees of freedom.
- Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.
- DOF = 6 – Number of Restraints

1.7 Kinematic chain

- Kinematic chain is defined as the combination of kinematic pairs in which each links forms a part of two kinematic pairs and the relative motion between the links is either completely constrained or successfully constrained.
- Examples: slider-crank mechanism
- For a kinematic chain

$$N = 2P - 4 = 2(j + 2) / 3$$

- Where N = no. of links, P = no. of Pairs and j = no. of joints
- When,

LHS > RHS, then the chain is locked
LHS = RHS, then the chain is constrained
LHS < RHS, then the chain is unconstrained

1.8 Kutzbach Criterion

- DOF of a mechanism in space can be determined as follows:
- In mechanism one link should be fixed. Therefore total no. of movable links are in mechanism is (N-1)
- Any pair having 1 DOF will impose 5 restraints on the mechanism, which reduces its total degree of freedom by 5P1.
- Any pair having 2 DOF will impose 4 restraints on the mechanism, which reduces its total degree of freedom by 4P2
- Similarly, the other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of mechanism. Thus,
- Thus,

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5 - 0P_6$$

- Hence,

$$F = 6(N-1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - 1P_5$$

- The above equation is the general form of **Kutzbach criterion.** This is applicable to any type of mechanism including a spatial mechanism.

1.9 Grubler's criterion

- If we apply the Kutzbach criterion to planer mechanism, then equation of Kutzbach criterion will be modified and that modified equation is known as Grubler's Criterion for planer mechanism.
- Therefore in planer mechanism if we consider the links having 1 to 3 DOF, the total number of degree of freedom of the mechanism considering all restraints will becomes,

$$F = 3 (N-1) - 2 P_1 - 1 P_2$$

- The above equation is known as **Grubler's criterion** for planer mechanism.
- Sometimes all the above empirical relations can give incorrect results, e.g. fig (a) has 5 links, 6 turning pairs and 2 loops. Thus, it is a structure with zero degree of freedom.

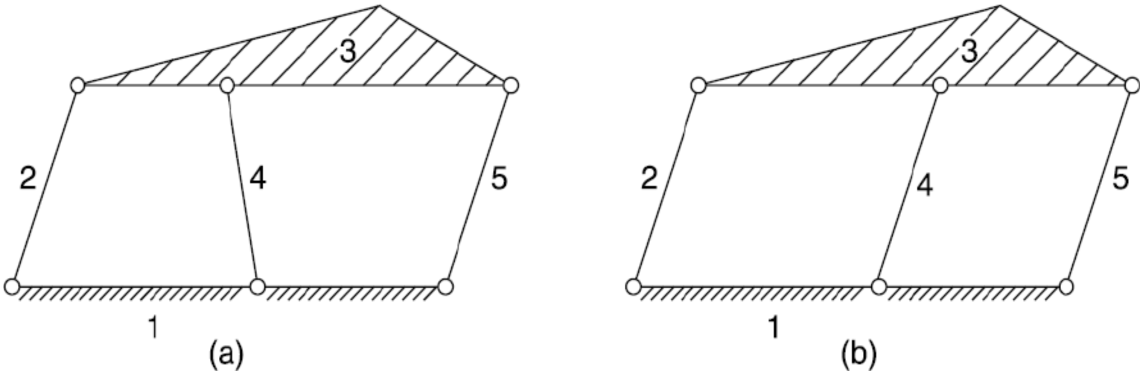


Fig. 1.7

- However, if the links are arranged in such a way as shown in fog. (b), a double parallelogram linkage with one degree of freedom is obtained. This is due to the reason that the lengths of links or other dimensional properties are not considered in these empirical relations.
- Sometimes a system may have one or more link which does not introduce any extra constraint. Such links are known as redundant links and should not be counted to find the degree of freedom. For example fig. (B) has 5 links, but the function of the mechanism is not affected even if any one of the links 2, 4 and 5 are removed. Thus, the effective number of links in this case is 4 with 4 turning pairs, and thus 1 degree of freedom.
- In case of a mechanism possessing some redundant degree of freedom, the effective degree of freedom is given by,

$$F = 3 (N - 1) - 2 P_1 - 1P_2 - F_r$$

- Where F = no. of redundant degrees of freedom

1.10 The Four-Bar chain

- A four bar chain is the most fundamental of the plane kinematic chains. It is a much proffered mechanical device for the mechanisation and control of motion due to its simplicity and versatility. Basically, it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints.
- When one of the link fixed, it is known as mechanism or linkage. A link that makes complete revolution is called the crank. The link opposite to the fixed link is called coupler, and the forth link is called a lever or rocker if it oscillates or another crank if it rotates.
- It is impossible to have a four-bar linkage if the length of one of the link is greater than the sum of other three. This has been shown in fig.

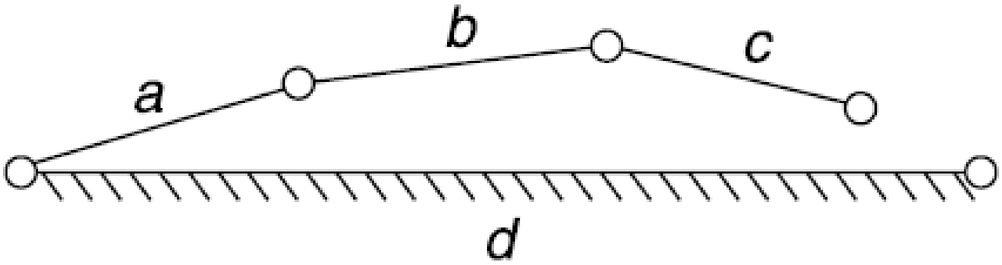


Fig. 1.7 Four bar chain

1.11 Grashof's law:

- We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig. 5.18. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths.
- According to Grashof's 's law for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

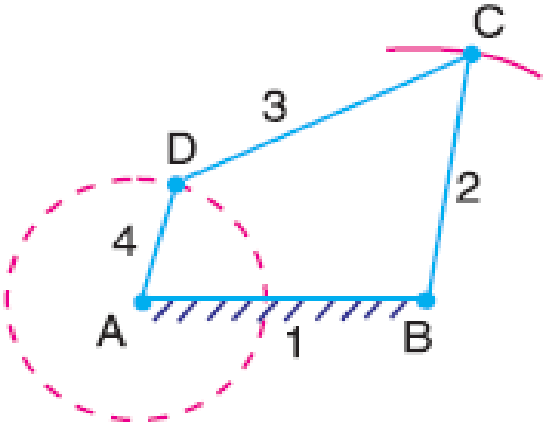


Fig. 1.8 Grashof's law

- A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof's law. Such a link is known as crank or driver. In Fig.5.18, AD (link 4) is a crank.
- The link BC (link 2) which makes a partial rotation or oscillates is known as lever or rocker or follower and the link CD (link 3) which connects the crank and lever is called connecting rod or coupler. The fixed link AB (link 1) is known as frame of the mechanism.

1.12 Inversion of Mechanism:

- When the number of links in kinematic chain is more than three, the chain is known as mechanism. When one link of the kinematic chain at a time is fixed, give the different mechanism of the kinematic chain. The method of generating different mechanism by fixing a link is called the inversion of mechanism.
- The number of inversion is equal to the numbers of links in the kinematic chain.
- The inversion of mechanism may be classified as:
  - **a.** Inversion of four-bar chain
  - **b.** Inversion of single slider crank chain
  - **c.** Inversion of double slider crank chain

1.13 Inversion of Four-Bar chain

1.13.1 First inversion: coupled wheel of locomotive

- The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links is shown in Fig.

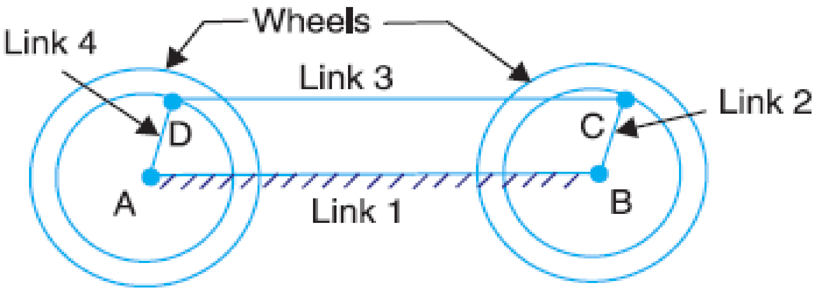


Fig. 1.9 coupled wheel of locomotive

- In this mechanism, the links *AD* and *BC* (having equal length) act as cranks and are connected to the respective wheels. The link *CD* acts as a coupling rod and the link *AB* is fixed in order to maintain a constant centre to Centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

1.13.2 Second inversion: Beam Engine

- A part of the mechanism of a beam engine (also known as cranks and lever mechanism) which consists of four links is shown in Fig. 1.10.
- In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

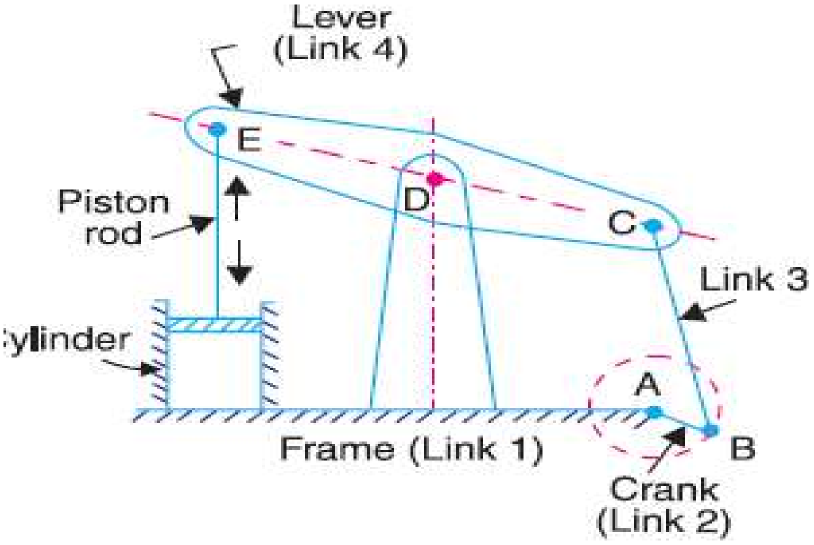


Fig. 1.10 beam engine

- In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

1.13.3 Third inversion: watts indicator mechanism

- A Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links is shown in Fig.
- The four links are: fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers.
- The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.

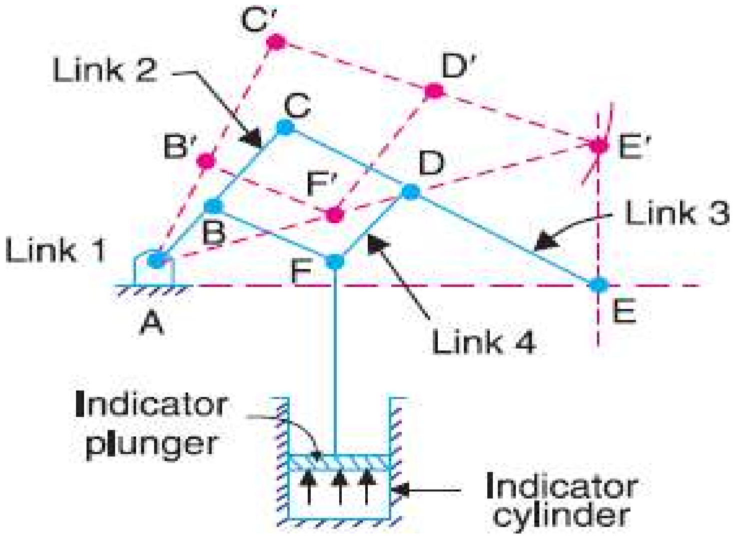


Fig. 1.11 watts indicator mechanism

1.14 The slider-crank chain

- When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a single slider-crank chain or simply a slider-crank chain.
- It is also possible to replace two sliding pairs of a four-bar chain to get a double slider-crank chain. In a slider-crank chain, the straight line path of the slider may be passing through the fixed pivot O or may be displaced.
- The distance e between the fixed pivot O and the straight line path of the slider is called the offset and the chain so formed an offset slider-crank chain.
- Different mechanisms obtained by fixing different links of a kinematic chain are known as its inversions.

1.14.1 First inversion

- This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and slider respectively. (fig.a)
- **Applications:**
  - **a** Reciprocating engine
  - **b** Reciprocating compressor

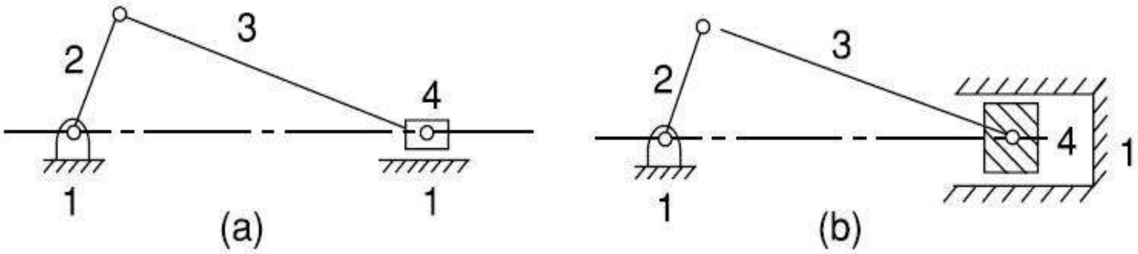


Fig. 1.12 first inversion

1.14.2 Second inversion

- Fixing of the link 2 of a slider-crank chain results in the second inversion.
- **Applications:**
  - **a** Whitworth quick-return mechanism
  - **b** Rotary engine

1.14.3 Third Inversion

- By Fixing of the link 3 of the slider-crank mechanism, the third inversion is obtained. Now the link 2 again acts as a crank and the link 4 oscillates.
- **Applications:**
  - **a** **Oscillating cylinder engine**
  - **b** Crank and slotted-lever mechanism

1.14.4 Fourth Inversion

- If the link 4 of the slider-crank mechanism is fixed, the fourth inversion is obtained. Link 3 can oscillates about the fixed pivot B on the link 4. This makes the end A of the link 2 to oscillate about B and the end O to reciprocate along the axis of the fixed link 4.

- **Application: Hand Pump**

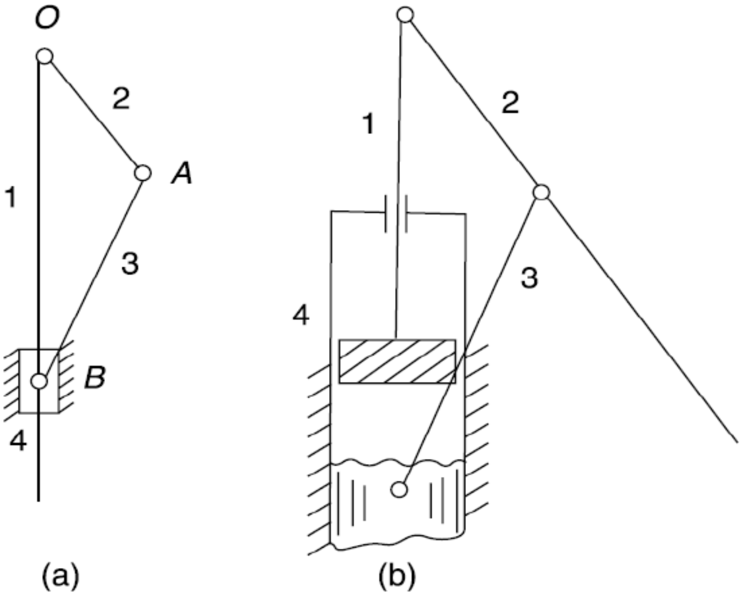


Fig. 1.13 hand pump

- Fig.1.13 shows a hand-pump. Link 4 is made in the form of a cylinder and a plunger fixed to the link 1 reciprocates in it.

1.15 Whitworth Quick-Return Mechanism:

- This mechanism used in shaping and slotting machines.
- In this mechanism the link CD (link 2) forming the turning pair is fixed; the driving crank CA (link 3) rotates at a uniform angular speed and the slider (link 4) attached to the crank pin at a slides along the slotted bar PA (link 1) which oscillates at D.
- The connecting rod PR carries the ram at R to which a cutting tool is fixed and the motion of the tool is constrained along the line RD produced.

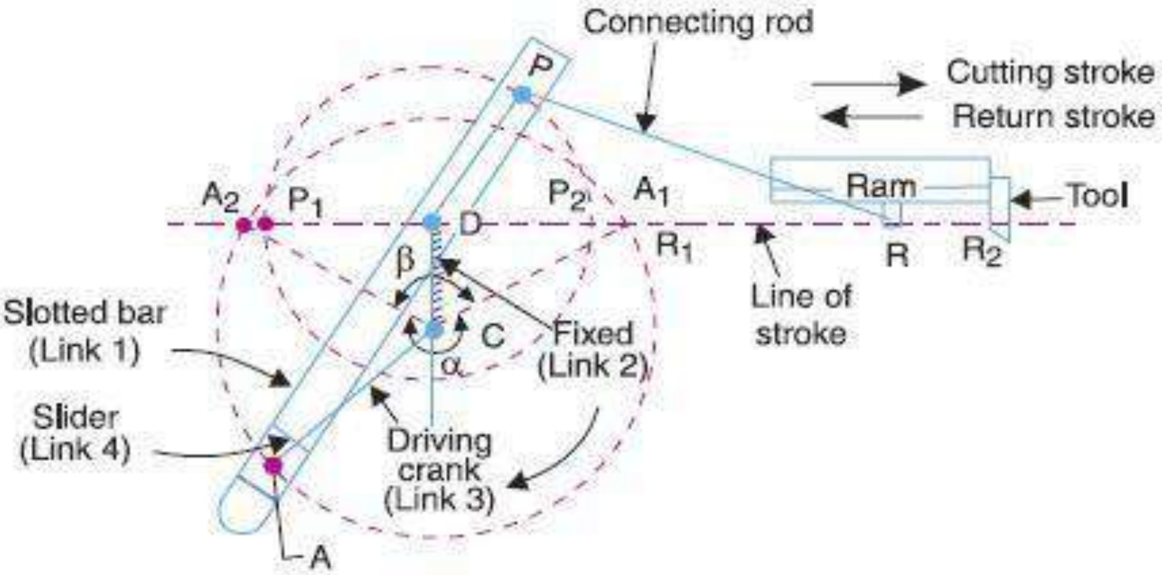


Fig. 1.14 Whitworth quick returns mechanism

- The length of effective stroke = 2 PD. And mark P1R1 = P2 R2 = PR.

$$\frac{time\ of\ cutting\ stroke}{time\ of\ return} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} = \frac{360° - \beta}{\beta}$$

1.16 Rotary engine

- Sometimes back, rotary internal combustion engines were used in aviation. But now- a-days gas turbines are used in its place.

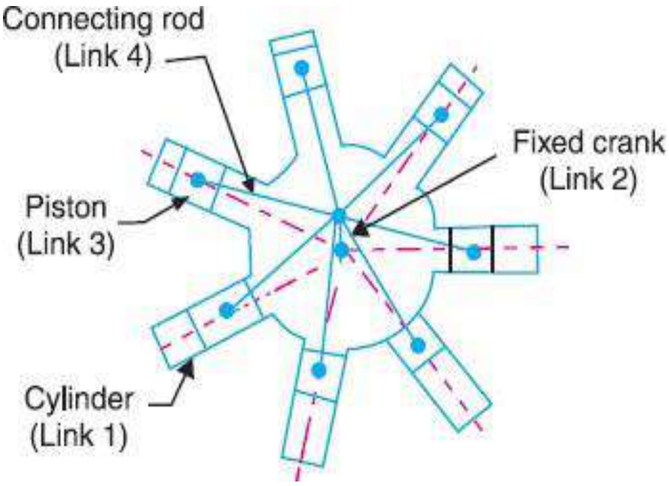


Fig. 1.15 rotary engine

- It consists of seven cylinders in one plane and all revolves about fixed center D, as shown in Fig. 5.25, while the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

1.17 Oscillating cylinder engine

- The arrangement of oscillating cylinder engine mechanism, as shown in Fig. Is used to convert reciprocating motion into rotary motion.

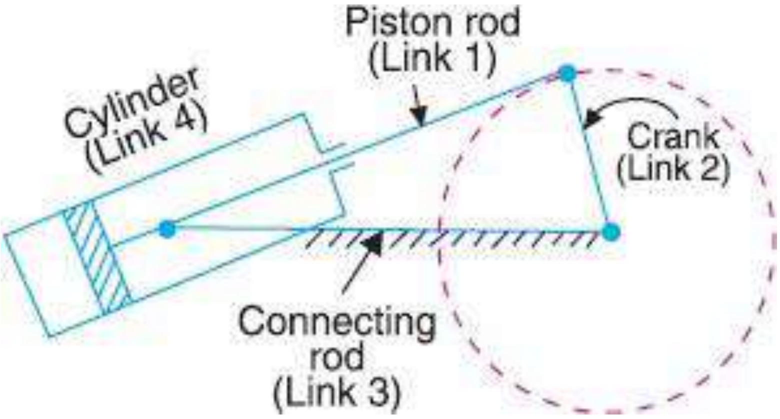


Fig. 1.16 oscillating cylinder engine

- In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at *A*.

1.18 Crank and slotted-lever Mechanism

- This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.
- In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed center C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A.
- A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced.

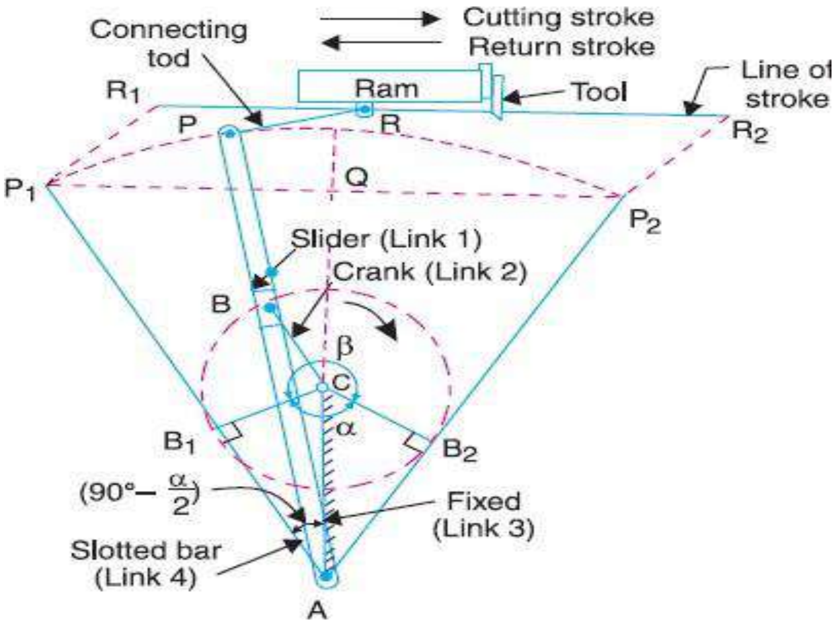


Fig.*1.17 Crank and slotted lever mechanism*

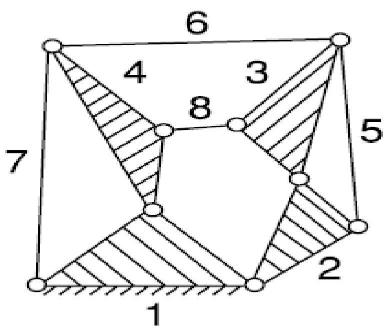
- In the extreme positions, *AP*1 and *AP*2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position *CB*1 to *CB*2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position *CB*2 to *CB*1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\textit{time of cutting stroke}}{\textit{time o} \quad \textit{return}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} = \frac{360° - \alpha}{\alpha}$$

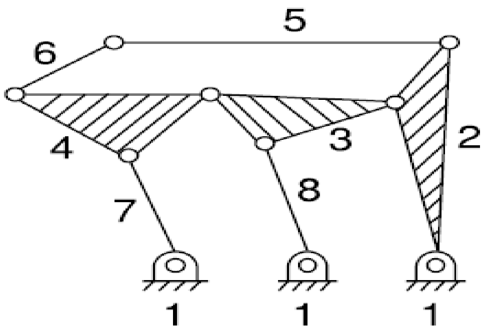
1.19 Example based on Degrees of Freedom:

1 For the kinematic linkages shown in following fig. calculate the following:

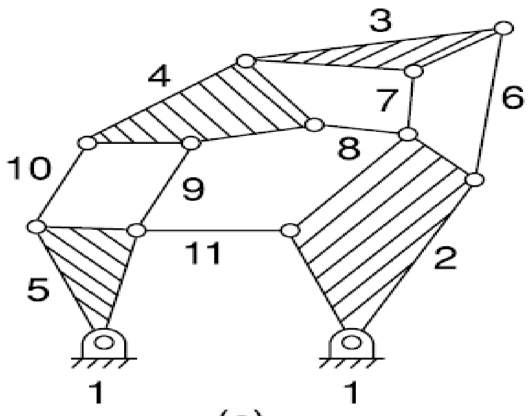
The numbers of binary links (N_b)
The numbers of ternary links (Nt)
The numbers of other (quaternary) links (No)
The numbers of total links (n)
The numbers of loops (L)
The numbers of joints or pairs (P1)
The numbers of degrees of freedom (F)



(a)



(b)



(c)

a $N_b = 4$; $N_t = 4$; $N_0 = 0$; $N = 8$; $L = 4$; $P_1 = 11$ (by counting) $P_1 = (N + L - 1) = 11$

$F = 3 (N - 1) - 2P_1$

$F = 3 (8 - 1) - 2 \times 11 = -1$ or,

v $F = N - (2 L + 1)$

$F = 8 - (2 \times 4 + 1) = -1$

b $N_b = 4$; $N_t = 4$; $N_0 = 0$; $N = 8$; $L = 3$; $P_1 = 10$ (by counting) $P_1 = (N + L - 1) = 10$

$F = 3 (N - 1) - 2P_1$
$F = 3 (8 - 1) - 2 \times 10 = 1$
or, $F = N - (2 L + 1)$
$F = 8 - (2 \times 3 + 1) = 1$

c $N_b = 7$; $N_t = 2$; $N_0 = 2$; $N = 11$; $L = 5$; $P_1 = 15$ (by counting) $F = N - (2 L + 1)$
$F = 11 - (2 \times 5 + 1) = 0$
Therefore the linkage is a structure.

LECTURE 1

Mechanisms

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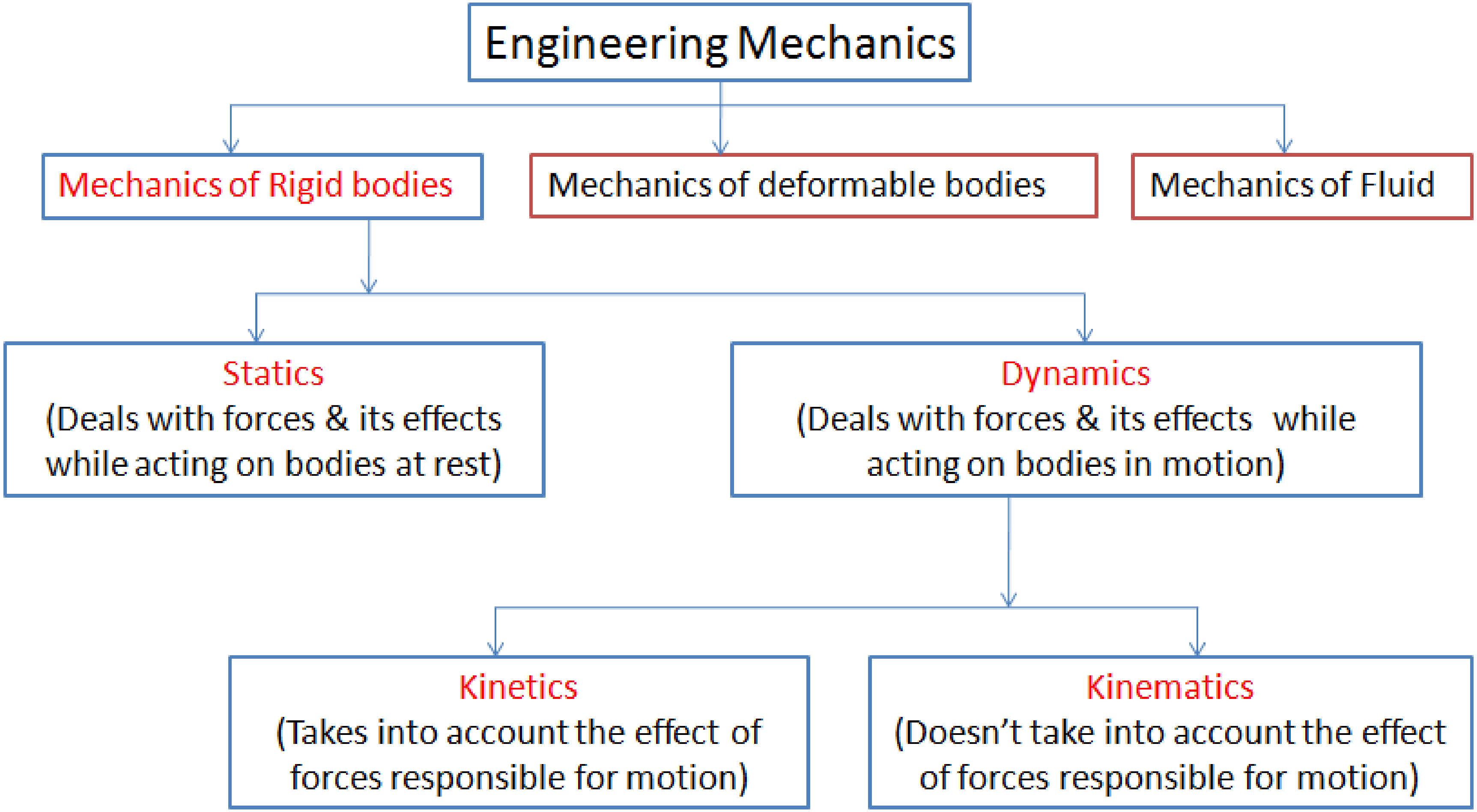


Engineering Mechanics

Mechanics of Rigid bodies

Mechanics of deformable bodies

Mechanics of Fluid

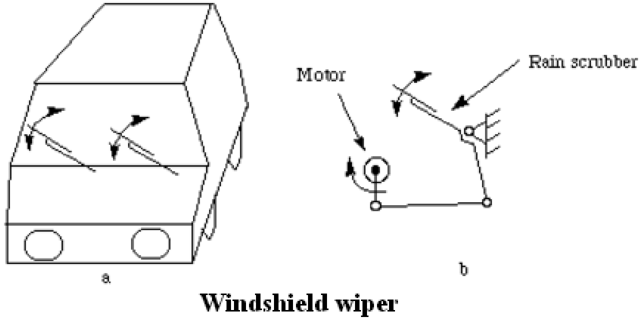
Statics
(Deals with forces & its effects while acting on bodies at rest)

Dynamics
(Deals with forces & its effects while acting on bodies in motion)

Kinetics
(Takes into account the effect of forces responsible for motion)

Kinematics
(Doesn't take into account the effect of forces responsible for motion)

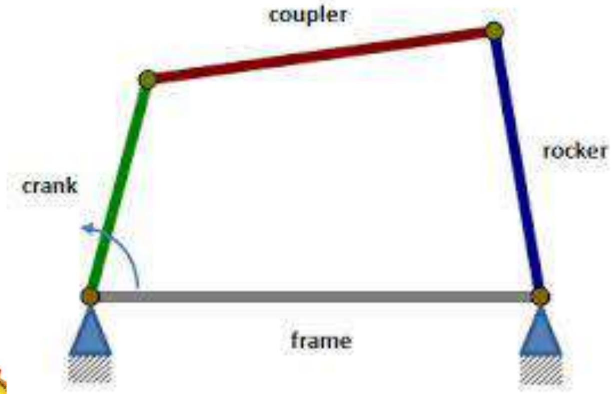
BASICS

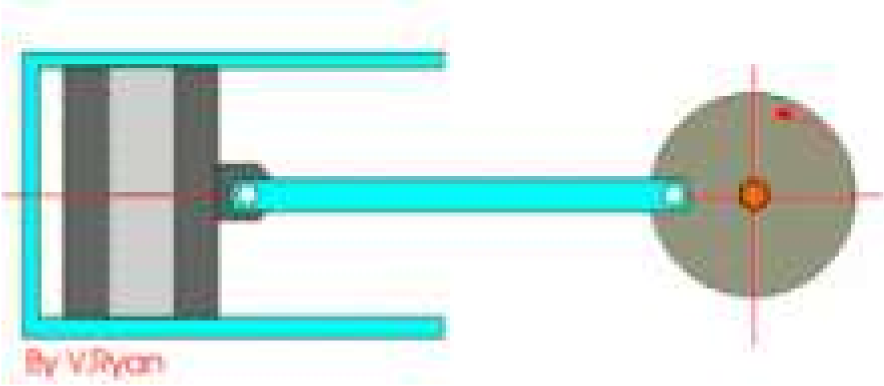


Windshield wiper

Mechanism:

- A number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others.
- A mechanism transmits and modifies a motion.
- Example: 4 bar mechanism, Slider crank mechanism





BASICS

Machine: (Combinations of Mechanisms)

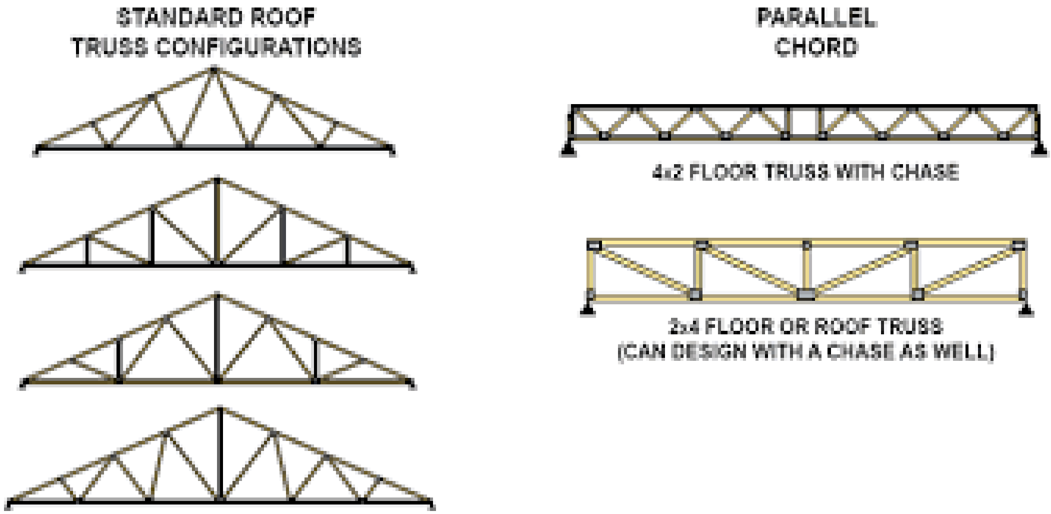
Transforms energy available in one form to another to do certain type of desired useful work.

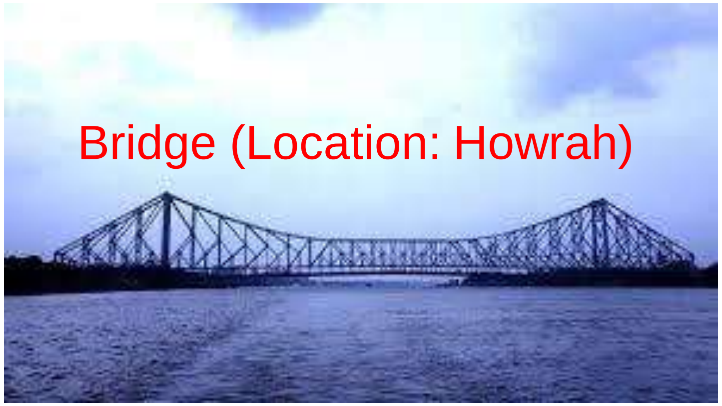
Lathe Machine

BASICS

Structure:

- ➢ Assembly of a number of resistant bodies meant to take up loads.
- ➢ No relative motion between the members




Bridge (Location: Howrah)

Truss

LECTURE 2

KINEMATIC LINK AND CLASSIFICATION OF LINKS

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BASICS

Kinematic Link (element): It is a Resistant body i.e. transmitting the required forces with negligible deformation.

Types of Links

1. **Rigid Link**

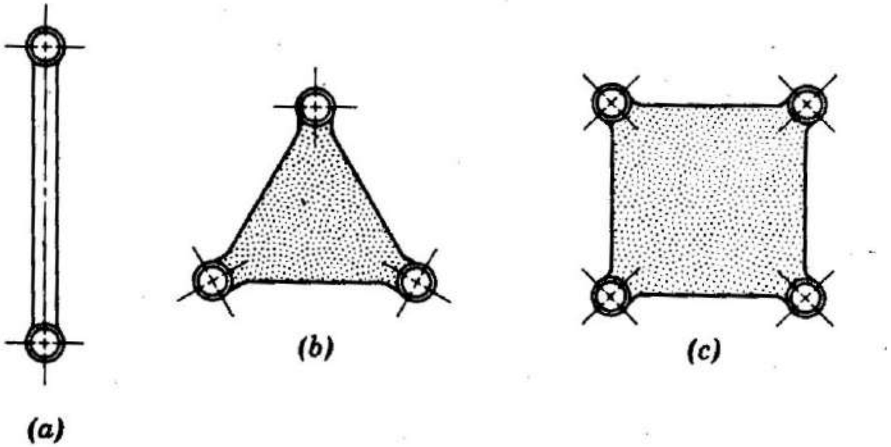
Doesn't undergo deformation. Example: Connecting rod, crank

2. **Flexible Link**

Partially deformed link. Example: belts, Ropes, chains

3. **Fluid Link**

Formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only. Example: Jacks, Brakes



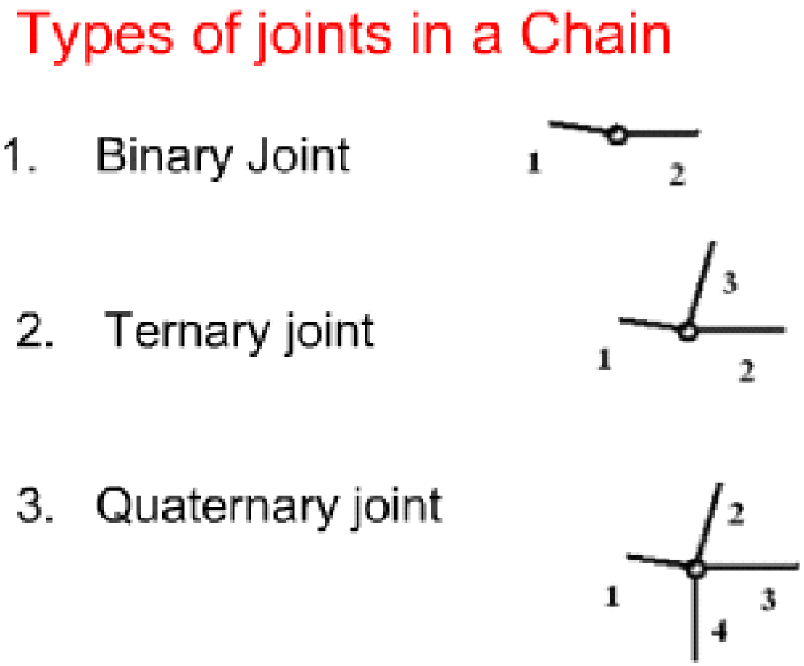
Binary link (2 vertices) **Ternary link (3 vertices)** **Quaternary link (4 vertices)**

BASICS

Kinematic Joint: Connection between two links by a pin

Types of Joints:

- Binary Joint (2 links are connected at the joint)
- Ternary Joint (3 links are connected)
- Quaternary Joint. (4 links are connected)



Types of joints in a Chain

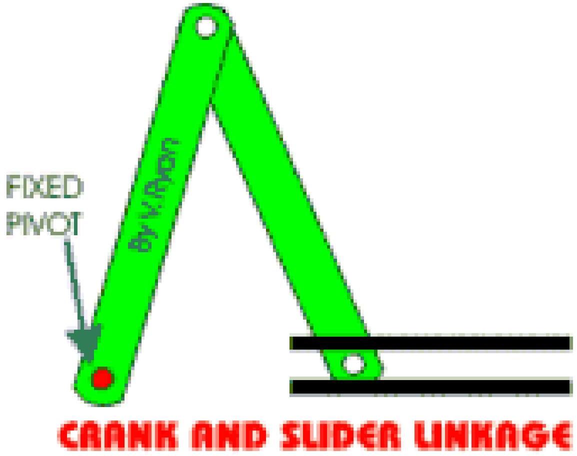
1. Binary Joint
2. Ternary joint
3. Quaternary joint

Note: if ‘ɩ’ number of links are connected at a joint, it is equivalent to (ɩ-1) binary joints.

BASICS

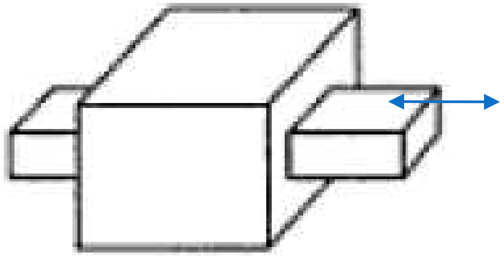
Kinematic Pair:

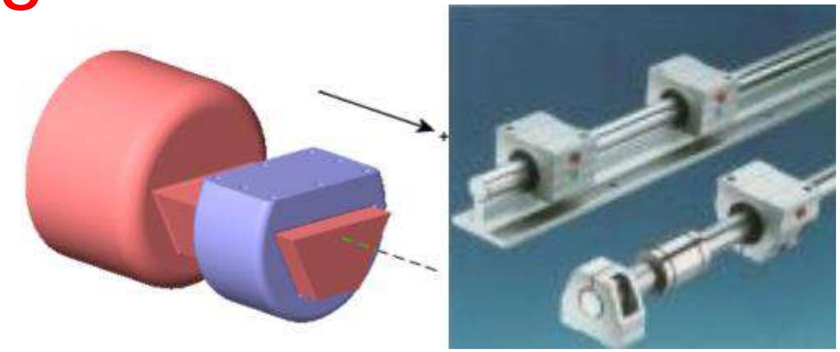
- The two links (or elements) of a machine, when in contact with each other, are said to form a pair.
- If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair



KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

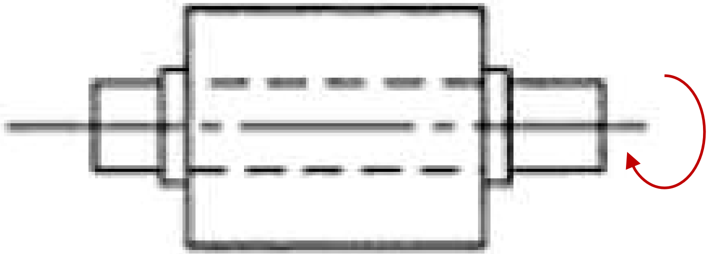
1. Sliding Pair





Rectangular bar in a rectangular hole

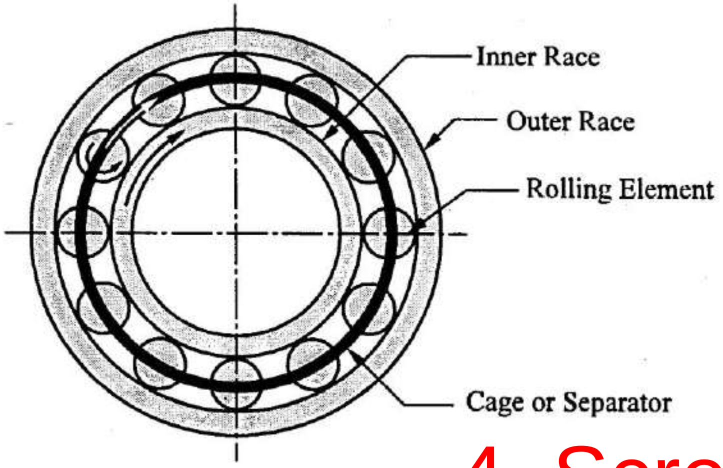
2. Turning or Revolving Pair



Collared shaft revolving in a circular hole

KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

3. Rolling Pair



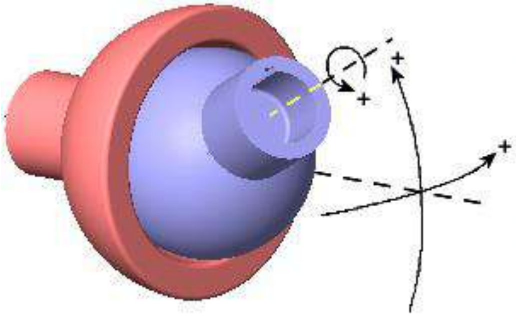
Links of pairs have a rolling motion relative to each other.

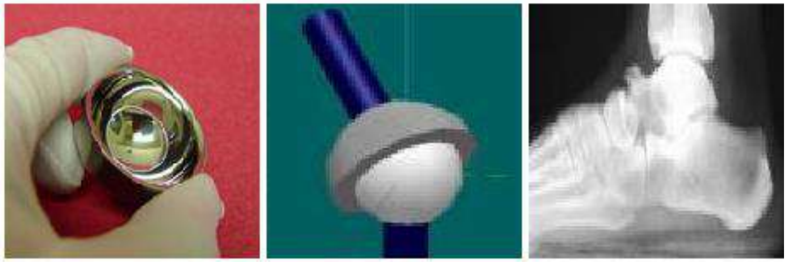
4. Screw or Helical Pair

if two mating links have a turning as well as sliding motion between them.

KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

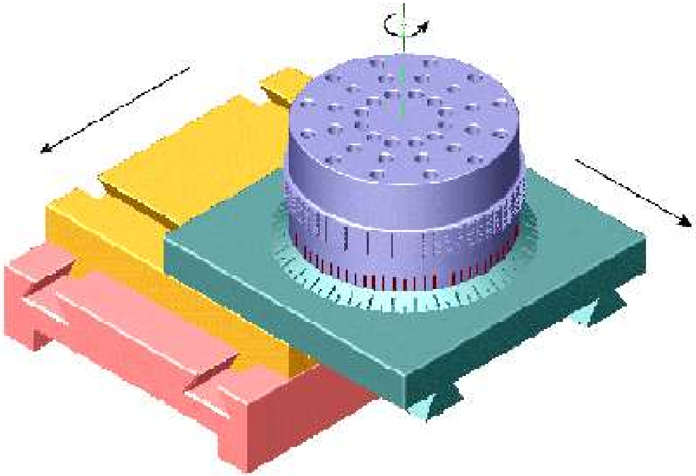
5. Spherical Pair





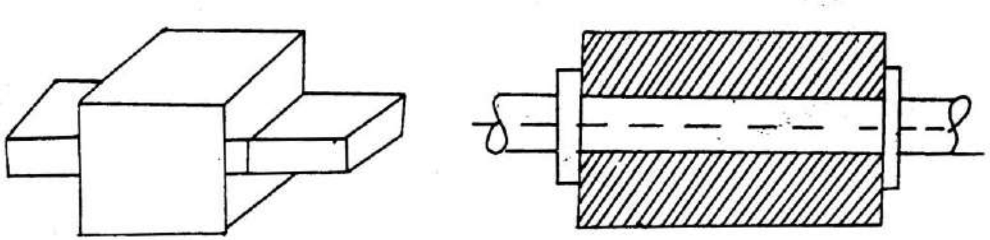
When one link in the form of a sphere turns inside a fixed link

6. Planar Pair



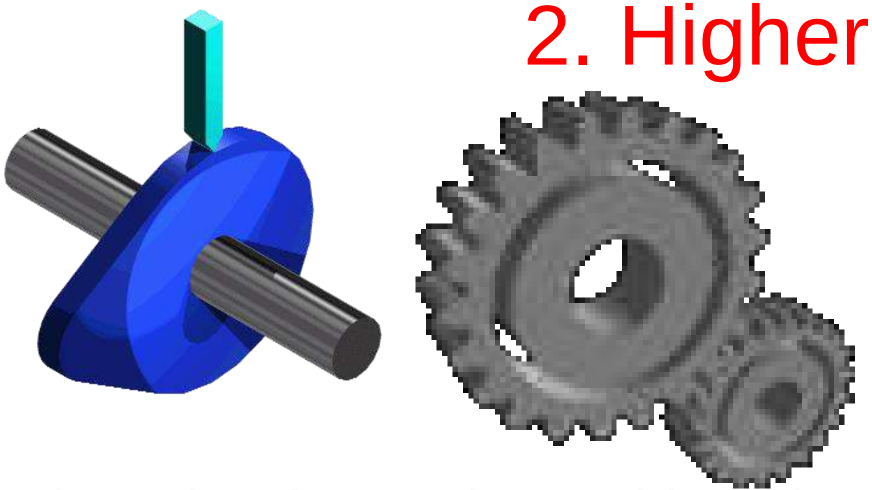
KINEMATIC PAIRS ACCORDING TO TYPE OF CONTACT

1. Lower Pair



The joint by which two members are connected has surface (Area) contact

2. Higher Pair



The contact between the pairing elements takes place at a point or along a line.

Toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs

LECTURE 3

CONSTRAINED MOTION AND CLASSIFICATION

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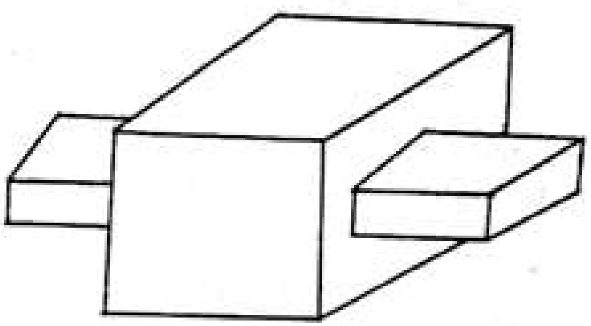
KINEMATIC PAIRS ACCORDING TO TYPE OF CONSTRAINT

1. Closed Pair



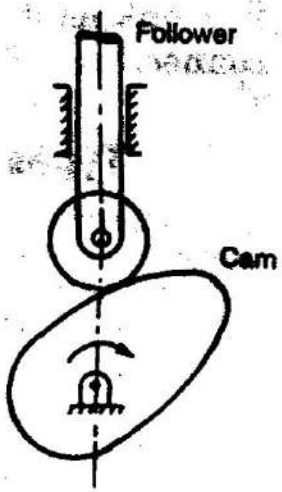
Two elements of pair are held together mechanically to get required relative motion.

Eg. All lower pairs

2. Unclosed Pair

- Elements are not held mechanically.
- Held in contact by the action of external forces.

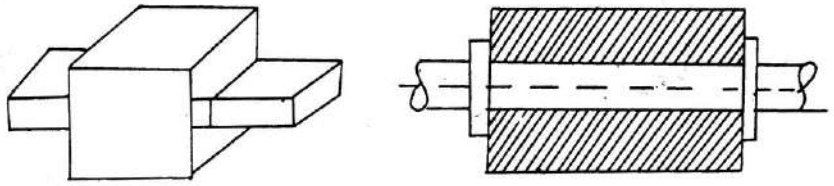
Eg. Cam and spring loaded follower pair



CONSTRAINED MOTION

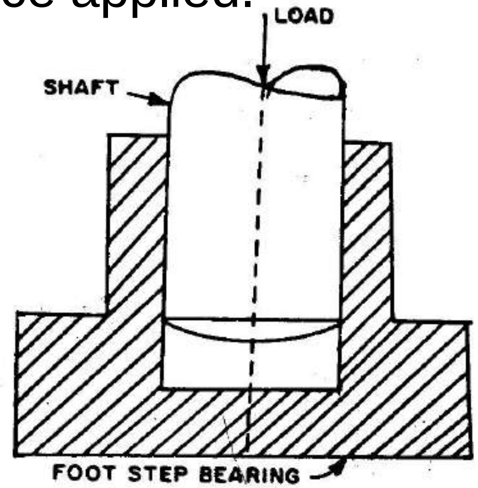
1.Completely constrained Motion:

Motion in definite direction irrespective of the direction of the force applied.



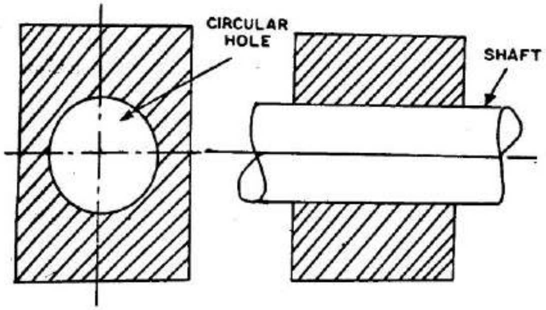
2. Successfully (partially) constrained Motion:

- Constrained motion is not completed by itself but by some other means.
- Constrained motion is successful when compressive load is applied on the shaft of the foot step bearing



3. Incompletely constrained motion:

Motion between a pair can take place in more than one direction.



Circular shaft in a circular hole may have rotary and reciprocating motion. Both are independent of each other.

KINEMATIC CHAIN

Group of links either joined together or arranged in a manner that permits them to move relative (i.e. completely or successfully constrained motion) to one another.

Example: 4 bar chain

The following relationship holds for kinematic chain

$$l = 2p - 4$$

$$j = \frac{3}{2}l - 2$$

Where

p = number of lower pairs

l = number of links

j = Number of binary joints

KINEMATIC CHAIN

$$l = 2p - 4$$

$$j = \frac{3}{2}l - 2$$

If LHS > RHS, Locked chain or redundant chain; no relative motion possible.

LHS = RHS, Constrained chain .i.e. motion is completely constrained

LHS < RHS, unconstrained chain. *i.e. the relative motion is not completely constrained.*

NUMERICAL EXAMPLE-1

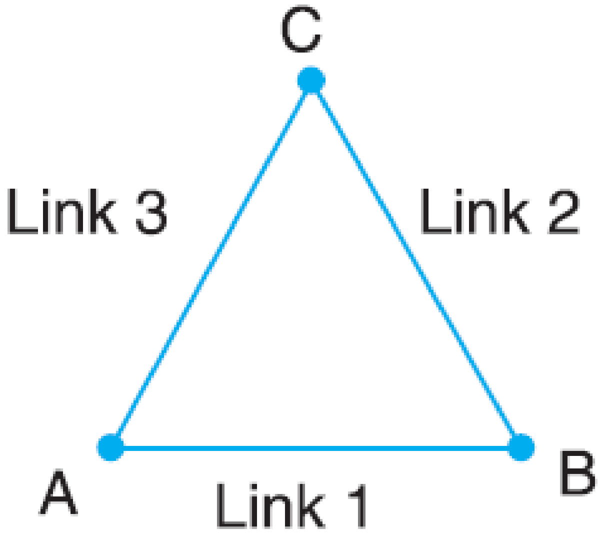
Determine the nature of the chain (K2:U)

$l = 3 \quad p = 3 \quad j = 3$

From equation

$$l = 2p - 4$$
$$= 2 \times 3 - 4 = 2$$

L.H.S. > R.H.S.



$$j = \frac{3}{2} l - 2$$
$$= \frac{3}{2} \times 3 - 2 = 2.5$$

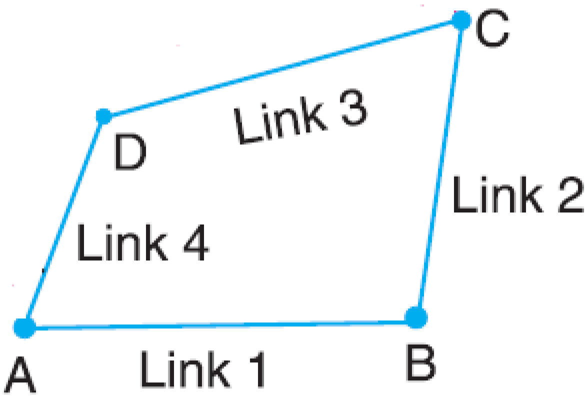
L.H.S. > R.H.S.

Therefore it is a locked Chain

EXERCISE

Determine the nature of the chains given below (K2:U)

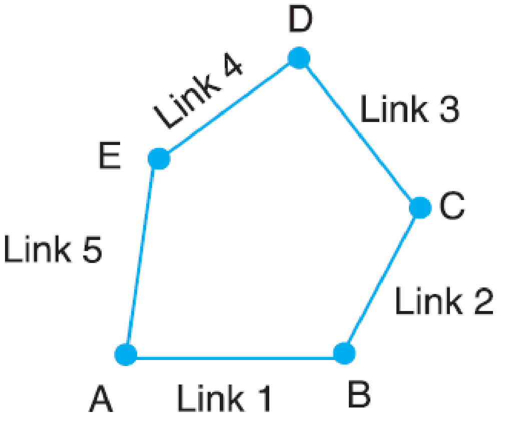
Hint: Check equations $l = 2p - 4$, $j = \frac{3}{2}l - 2$



$l = 4, p = 4, \text{ and } j = 4$

L.H.S. = R.H.S.

constrained kinematic chain



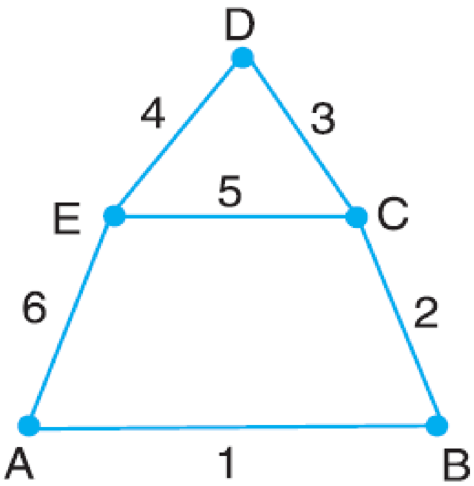
$l = 5, p = 5, \text{ and } j = 5$

L.H.S. < R.H.S.

unconstrained chain

NUMERICAL EXAMPLE-2

Determine the nature of the chain (K2:U)



- l = 6
- j = 3 Binary joints (A, B & D) + 2 ternary joints (E & C)
- We know that, 1 ternary joint = (3-1) = 2 Binary Joints
- Therefore, j = 3 + (2*2) = 7

$$j = \frac{3}{2} l - 2$$

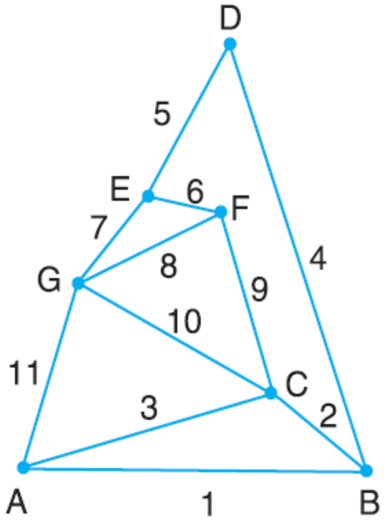
$$= \frac{3}{2} \times 6 - 2 = 7$$

L.H.S. = R.H.S.

Therefore, the given chain is a kinematic chain or constrained chain.

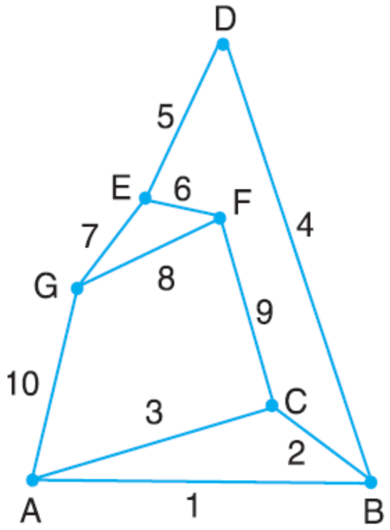
EXERCISE

Determine the number of joints (equivalent binary) in the given chains (K2:U)



Number of Binary Joints = 1 (D)
No. of ternary joints = 4 (A, B, E, F)
No. of quaternary joints = 2 (C & G)

Therefore, j = 1 + 4 (2) + 2 (3) = 15



No. of Binary Joints = 1 (D)
No. of ternary joints = 6 (A, B, C, E,F, G)

j = 1 + 6 (2) = 13

KINEMATIC CHAIN

- For a kinematic chain having higher pairs, each higher pair is taken equivalent to two lower pairs and an additional link.

- In this case to determine the nature of chain, the relation given by A.W. Klein, may be used

$$j + \frac{h}{2} = \frac{3}{2}l - 2$$

where j = Number of binary joints,

h = Number of higher pairs, and

l = Number of links.

LECTURE 4

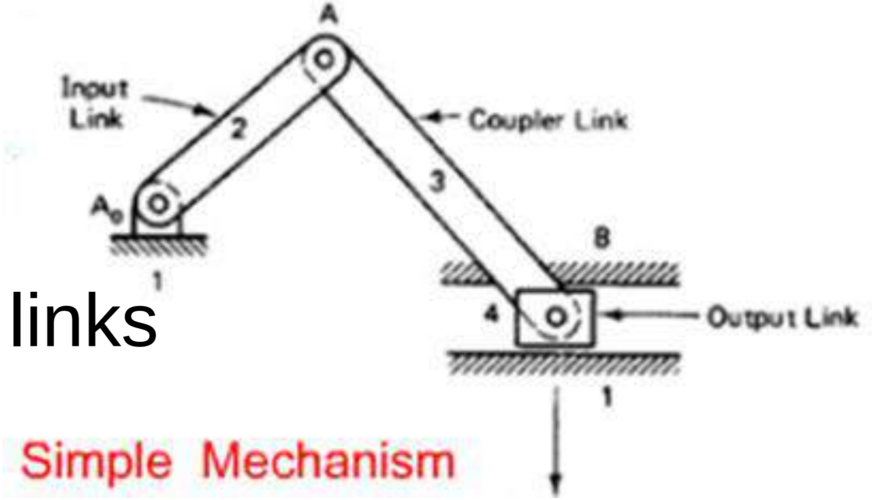
MECHANISM AND MACHINES

DEPARTMENT OF MECHANICAL ENGINEERING

CLASSIFICATION OF MECHANISMS

Mechanism:

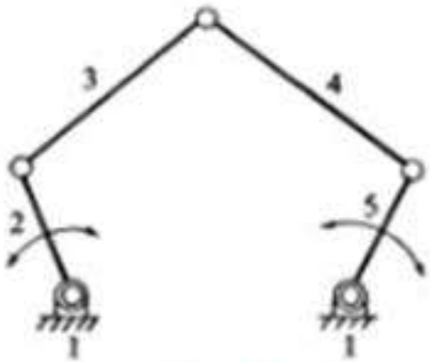
When one of the links of a kinematic chain is fixed, the chain is called Mechanism.



Simple Mechanism

Types:

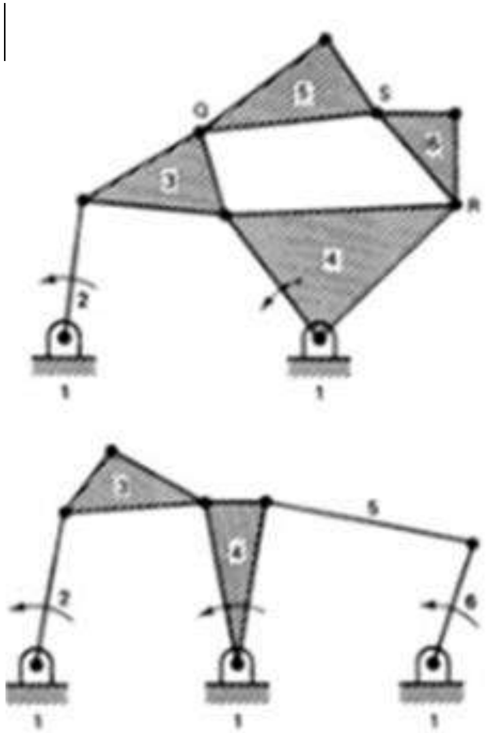
- ➢Simple - 4 Links
- ➢Compound - More than 4 links



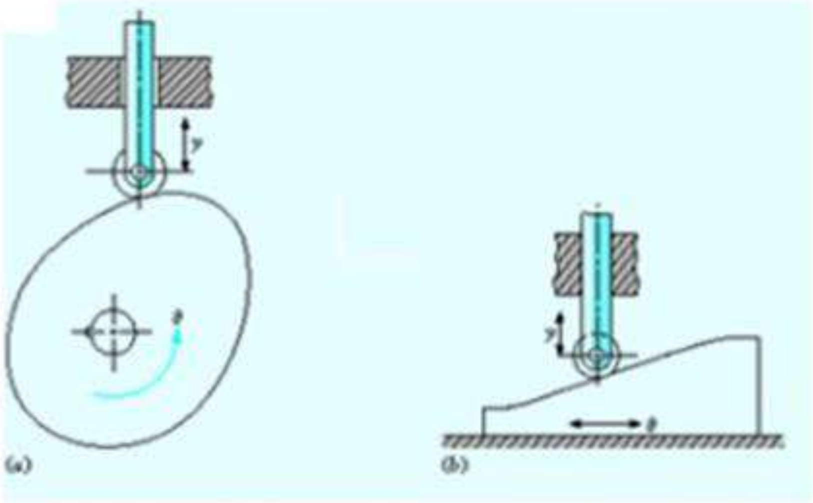
Compound Mechanism

Classification of mechanisms

- Complex - Ternary or Higher order fl Links
- Planar - All links lie in the same plane



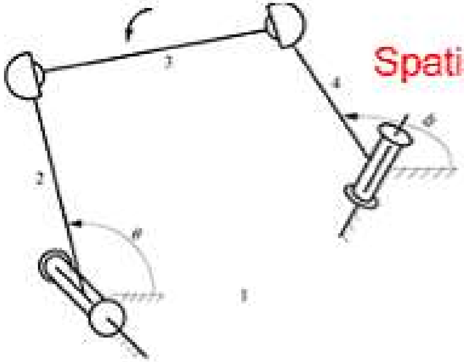
Complex Mechanism



Planar Mechanism

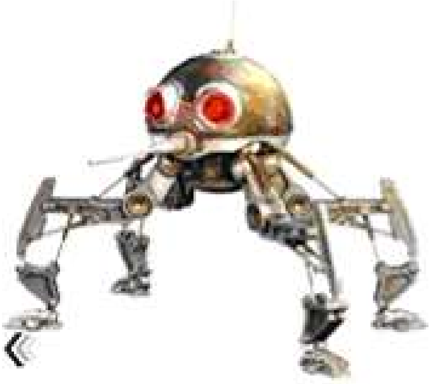
Classification of mechanisms

- ➢Spatial - Links of a mechanism lie in different planes



Spatial Mechanism

Parallel robot

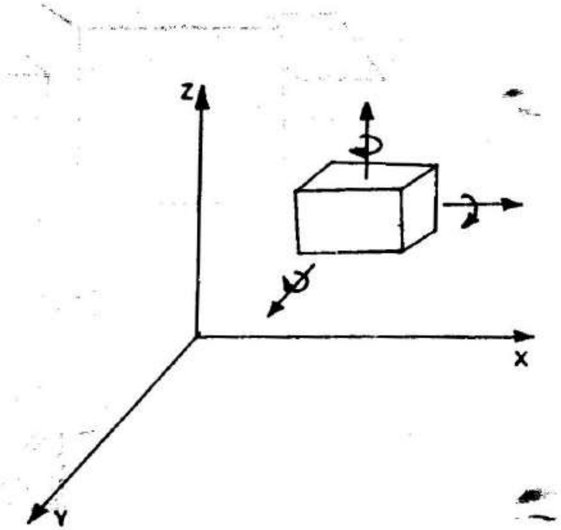


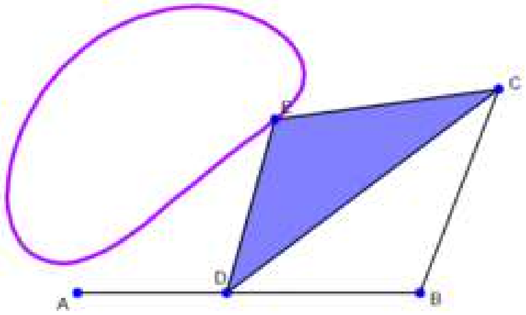
Machine

When a mechanism is required to transmit power or to do some particular type of work, it then becomes a **machine**. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely.

DEGREES OF FREEDOM (DOF) / MOBILITY

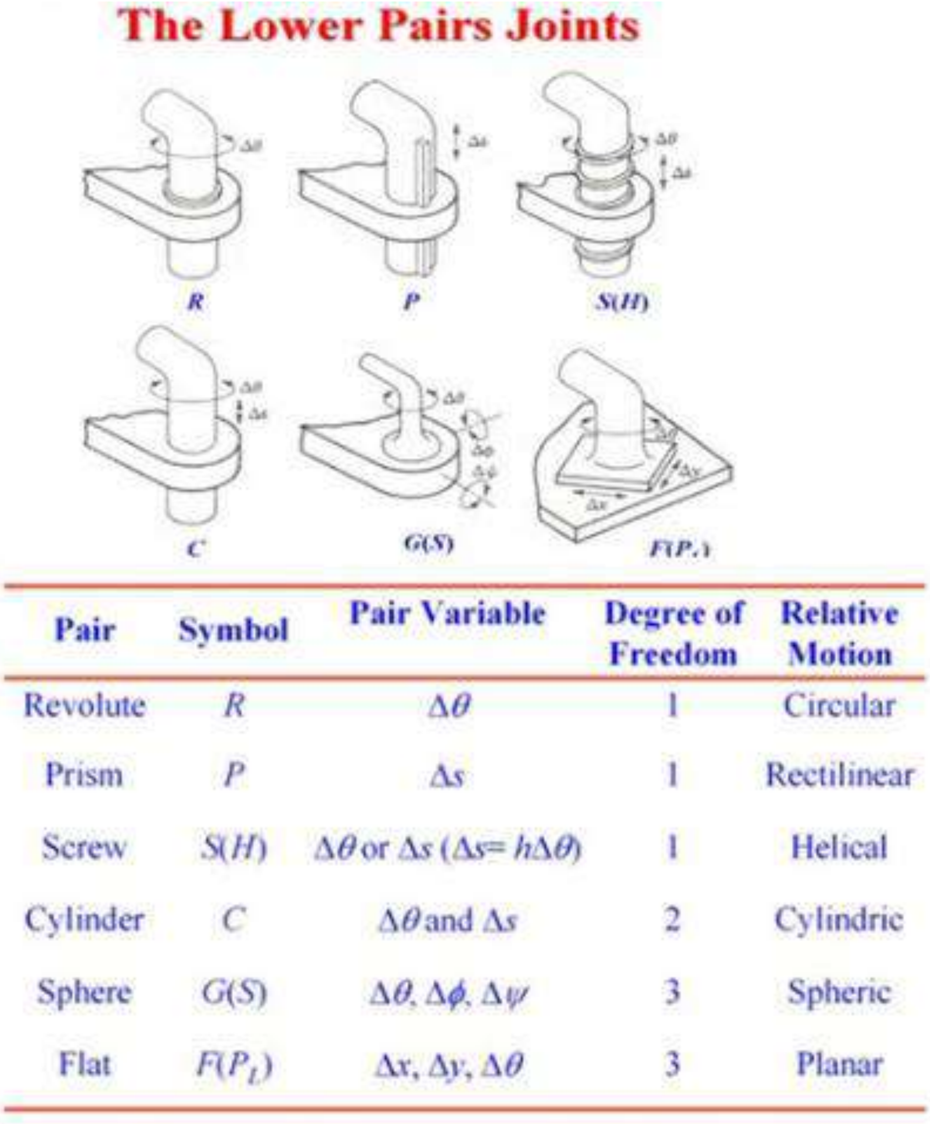
It is the number of **independent coordinates** required to describe the **position of a body**.





4 bar Mechanism has 1 DoF as the angle turned by the crank AD is fully describing the position of the every link of the mechanism

DOF



The Lower Pairs Joints

Pair	Symbol	Pair Variable	Degree of Freedom	Relative Motion
Revolute	R	$\Delta\theta$	1	Circular
Prism	P	Δs	1	Rectilinear
Screw	$S(H)$	$\Delta\theta$ or Δs ($\Delta s = h\Delta\theta$)	1	Helical
Cylinder	C	$\Delta\theta$ and Δs	2	Cylindric
Sphere	$G(S)$	$\Delta\theta, \Delta\phi, \Delta\psi$	3	Spheric
Flat	$F(P_L)$	$\Delta x, \Delta y, \Delta\theta$	3	Planar

DEGREES OF FREEDOM/MOBILITY OF A MECHANISM

It is the number of inputs (number of independent coordinates) required to describe the configuration or position of all the links of the mechanism, with respect to the fixed link at any given instant.

KUTZBACH CRITERION

For mechanism having plane motion

$$\text{DoF} = n = 3\,(l - 1) - 2\,j - h$$

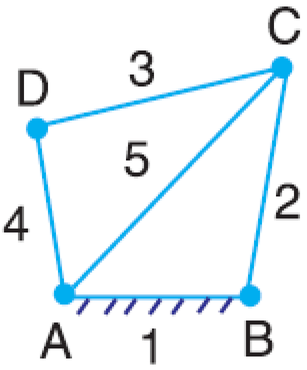
l = number of links

j = number of binary joints or lower pairs (1 DoF pairs)

h = number of higher pairs (i.e. *2 DoF pairs)*

NUMERICAL EXAMPLE -1 &2

Determine the DoF of the mechanism shown below:

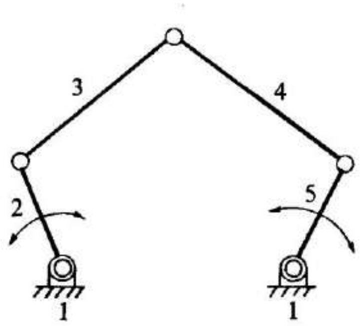


$$n = 3\,(l - 1) - 2\,j - h$$ **Kutzbach Criterion**

$l = 5$; $j = 2 + 2 * (3-1) = 6$; $h = 0$

$$n = 3\,(5 - 1) - 2 \times 6 = 0$$

DoF = 0, means that the mechanism forms a structure



$l = 5$; $j = 5$; $h = 0$

$n = 3\,(5-1) – 2*5 – 0 = 2$

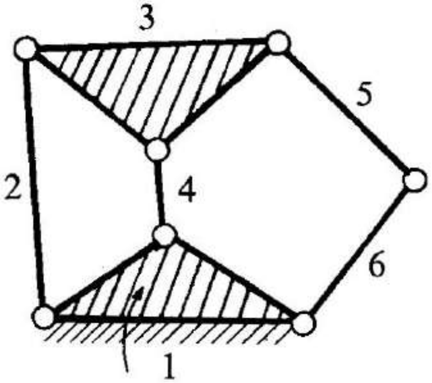
Two inputs to any two links are required to yield definite motions in all the links.

NUMERICAL EXAMPLE -3 &4

Determine the Dof for the links shown below:

$$n = 3\,(l - 1) - 2\,j - h$$

Kutzbach Criterion

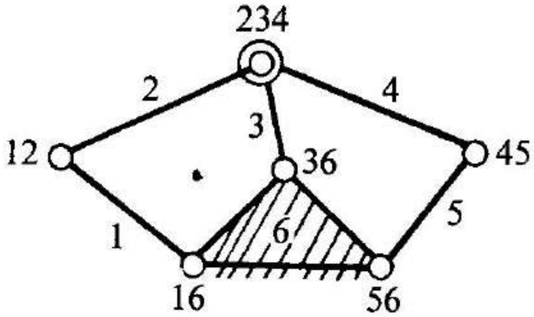


$l = 6$; j = 7 ; h = 0

n = 3 (6-1) – 2 (7) – 0 = 1

Dof = 1

i.e., one input to any one link will result in definite motion of all the links.



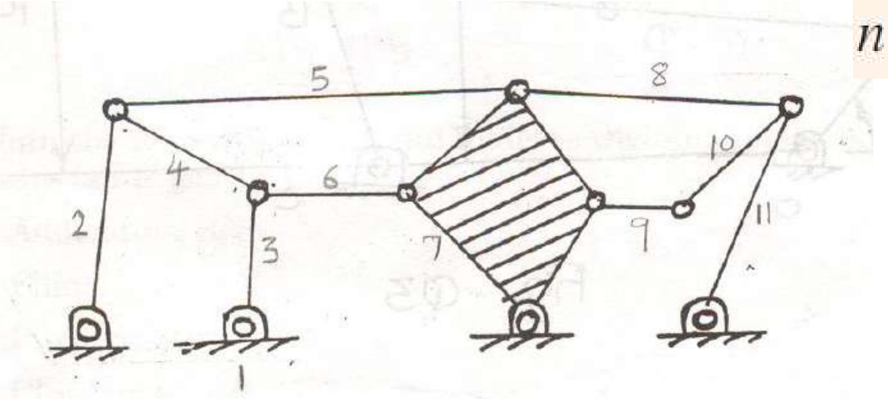
Note: at the intersection of 2, 3 and 4, two lower pairs are to be considered

$l = 6$; j = 5 + 1 (3-1) = 7 ; h = 0

n = 3 (6-1) – 2 (7) – 0 = 1

Dof = 1

NUMERICAL EXAMPLE - 5



$n = 3\,(l - 1) - 2\,j - h$ **Kutzbach Criterion**

l = 11 ; j = 7 + 4 (3-1) = 15 ; h = 0

n = 3 (11-1) – 2 (15) – 0 = 0

Dof = 0

Here, j= 15 (two lower pairs at the intersection of 3, 4, 6; 2, 4, 5; 5, 7, 8; 8, 10, 11) and h = 0.

Summary

Dof = 0, Structure

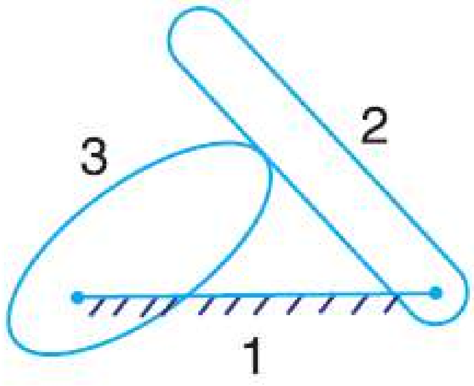
Dof = 1, mechanism can be driven by a single input motion

Dof = 2, two separate input motions are necessary to produce constrained motion for the mechanism

Dof = -1 or less, redundant constraints in the chain and it forms a statically indeterminate structure

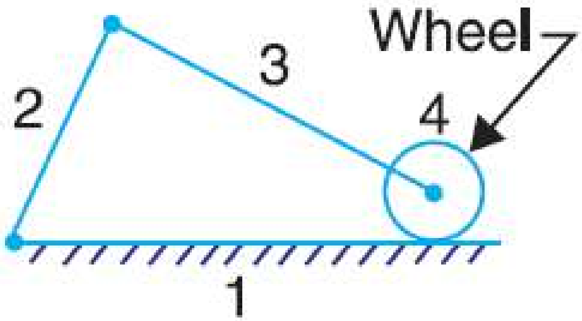
KUTZBACH CRITERION FOR HIGHER PAIRS

$$n = 3\,(l - 1) - 2\,j - h$$



$l = 3, j = 2$ and $h = 1$

$$n = 3\,(3 - 1) - 2 \times 2 - 1 = 1$$

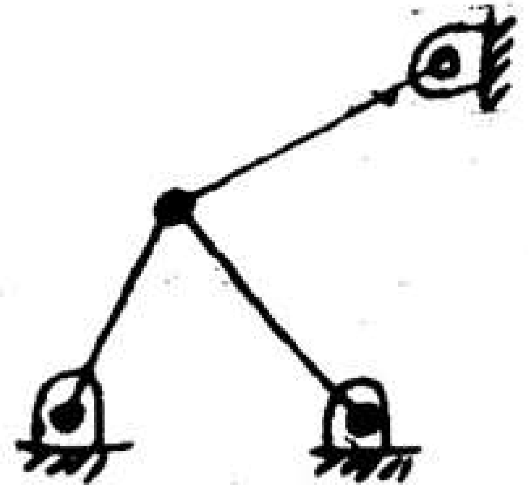


$l = 4,\ j = 3$ and $h = 1$

$$n = 3\,(4 - 1) - 2 \times 3 - 1 = 2$$

KUTZBACH CRITERION

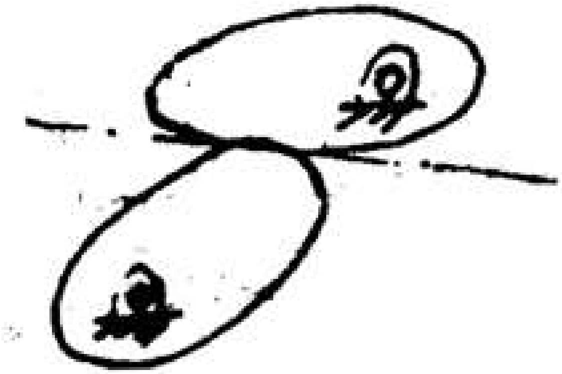
$$n = 3\,(l - 1) - 2\,j - h$$



l = 4, j = 5, h=0

n = 3 (4-1) – 2 (5) – 0 = -1

Indeterminate structure



l = 3, j = 2, h=1

n = 3 (3-1) – 2 (2) – 1 = 1

GRUBLER'S CRITERION FOR PLANE MECHANISMS

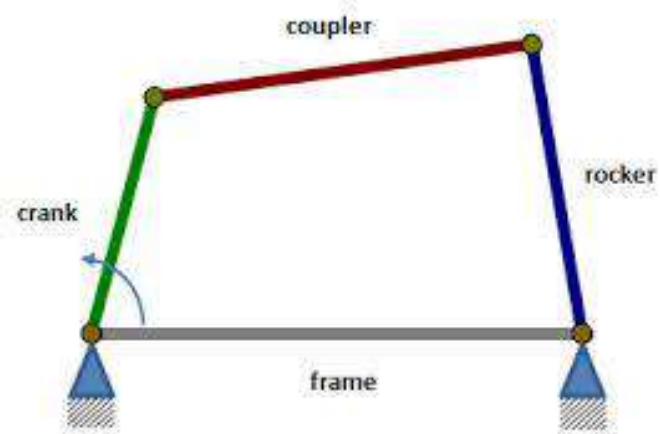
Kutzbach Criterion $n = 3\,(l - 1) - 2\,j - h$

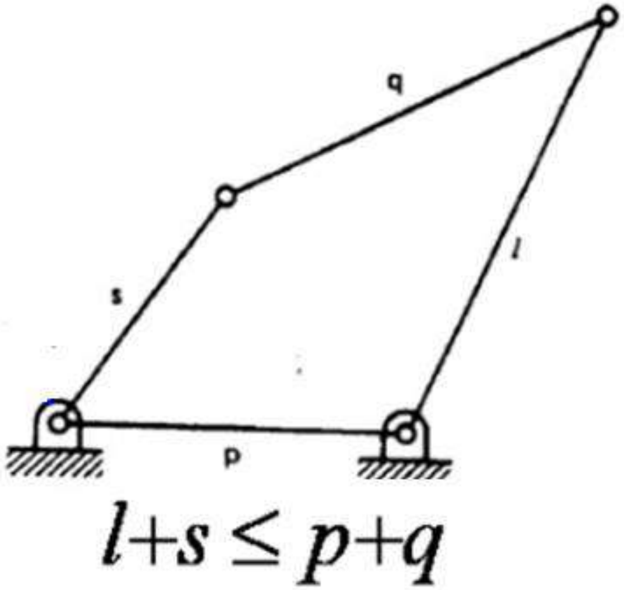
Grubler's criterion applies to mechanisms having 1 DoF.

Substituting n = 1 and h=0 in Kutzbatch equation, we can have Grubler's equation.

$$1 = 3\,(l - 1) - 2\,j \qquad \text{or} \qquad 3l - 2j - 4 = 0$$

GRASHOF’S LAW

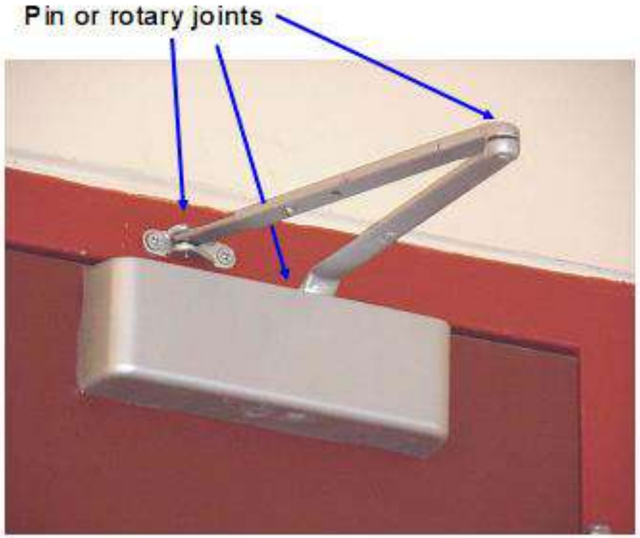


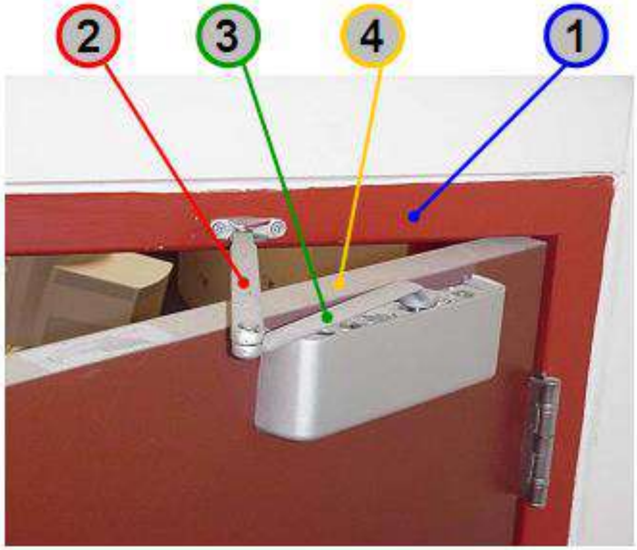


$$l+s \leq p+q$$

According to **Grashof ’s law for a four bar mechanism,** the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

Example: 4 bar door damper linkage





① = Wall or Link 1 This is the grounded (held still)

② = Bar 2 or Link 2

③ = Bar 3 or Link 3

④ = Door or Link 4

LECTURE 5

INVERSION OF MECHANISM

LIVE TO LEARN & LEARN TO SHARE

MRCET CAMPUS

UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

INVERSIONS OF MECHANISM

- A mechanism is one in which one of the links of a kinematic chain is fixed.

- Different mechanisms can be obtained by fixing different links of the same kinematic chain.

- It is known as inversions of the mechanism.

INVERSIONS OF FOUR BAR CHAIN

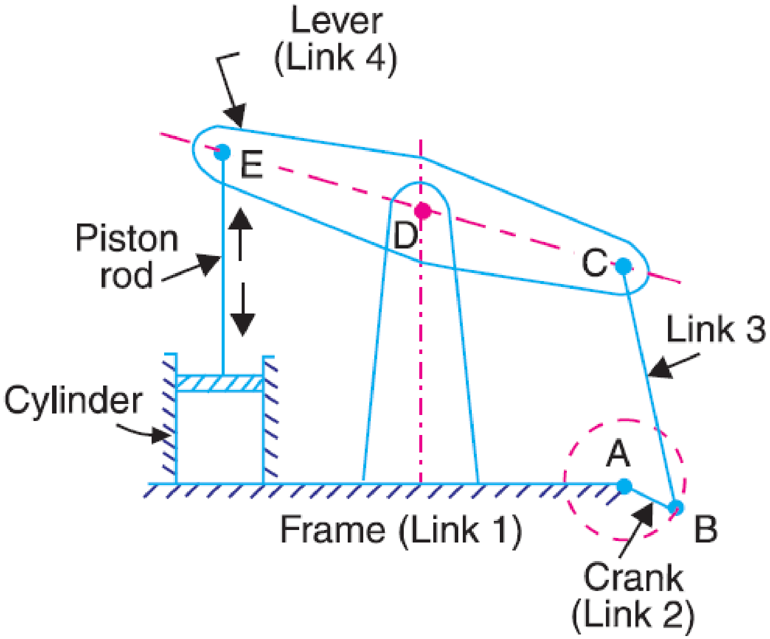
- Beam engine (crank and lever mechanism)

- Coupling rod of a locomotive (Double crank mechanism)

- Watt’s indicator mechanism (Double lever mechanism)

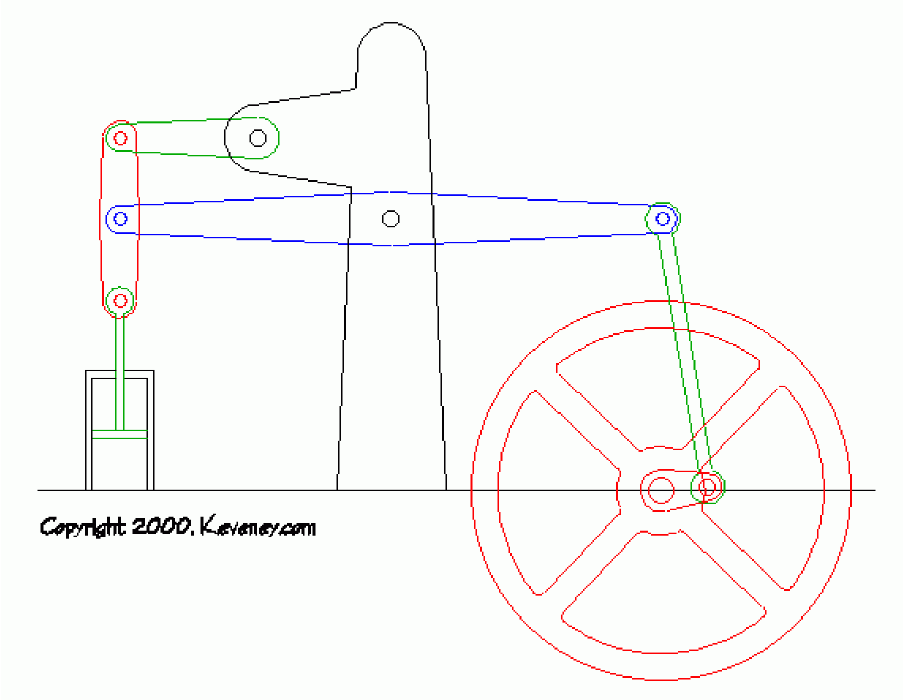
INVERSIONS OF FOUR BAR CHAIN

1. Beam engine (crank and lever mechanism)



Beam engine.

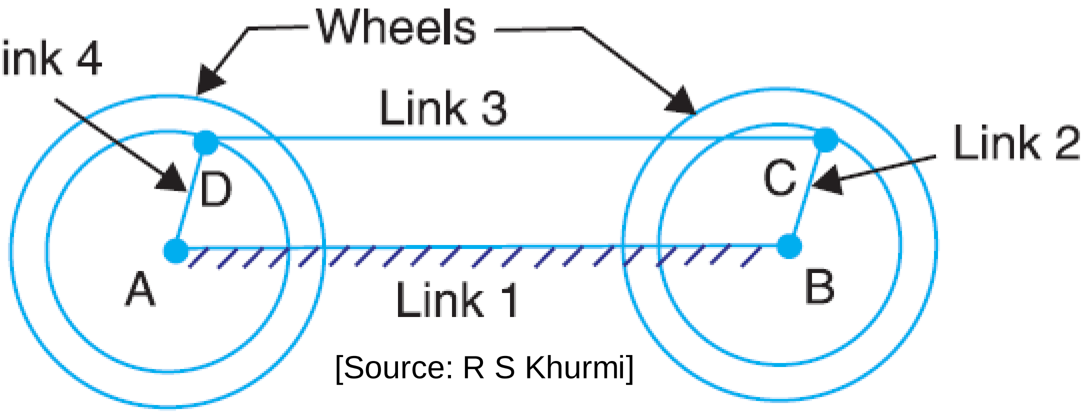
[Source: R S Khurmi]



The purpose of this mechanism is to convert rotary motion into reciprocating motion.

INVERSIONS OF FOUR BAR CHAIN

2. Coupling rod of a locomotive (Double crank mechanism).

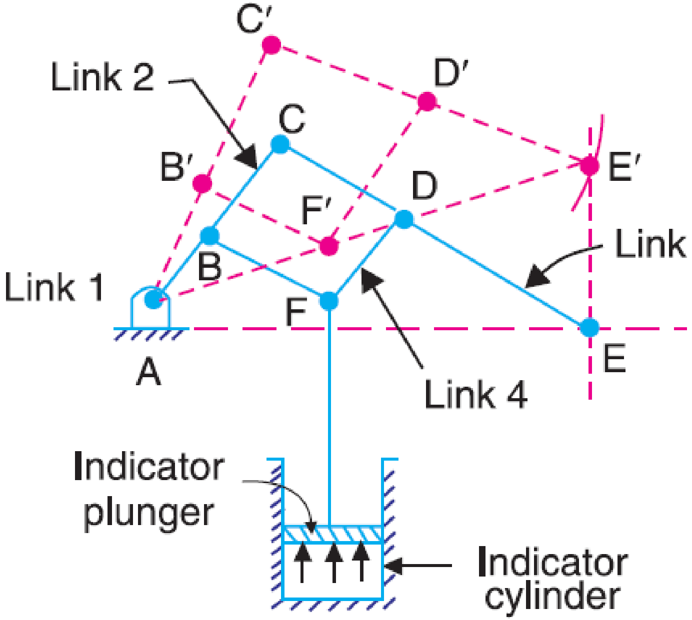


[Source: R S Khurmi]

- ➢links *AD and BC* (*having equal length*) *act as cranks and are connected* to the respective wheels.
- ➢The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant centre to centre distance between them.
- ➢This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

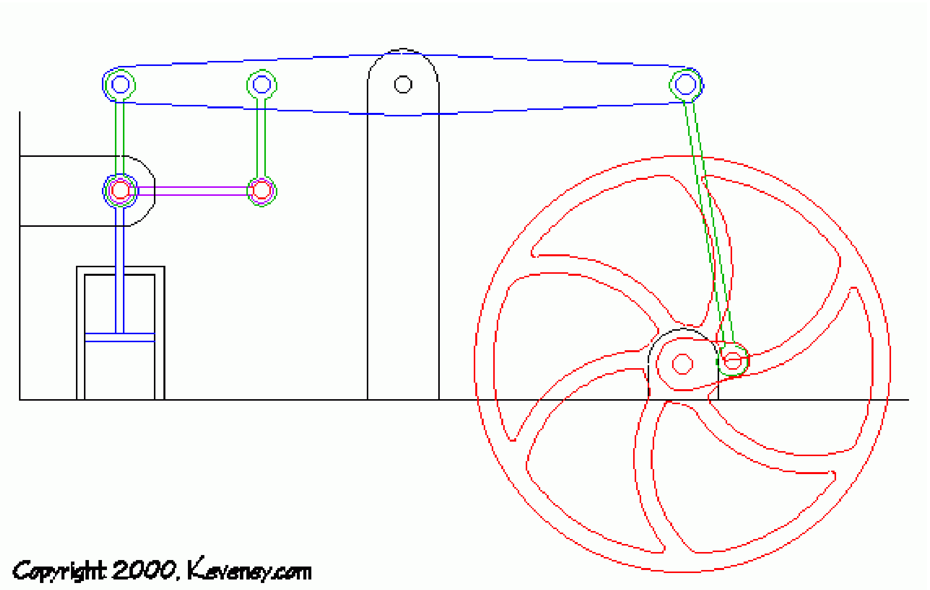
INVERSIONS OF FOUR BAR CHAIN

3.Watt's indicator mechanism (Double lever mechanism)



Watt's indicator mechanism.

[Source: R S Khurmi]



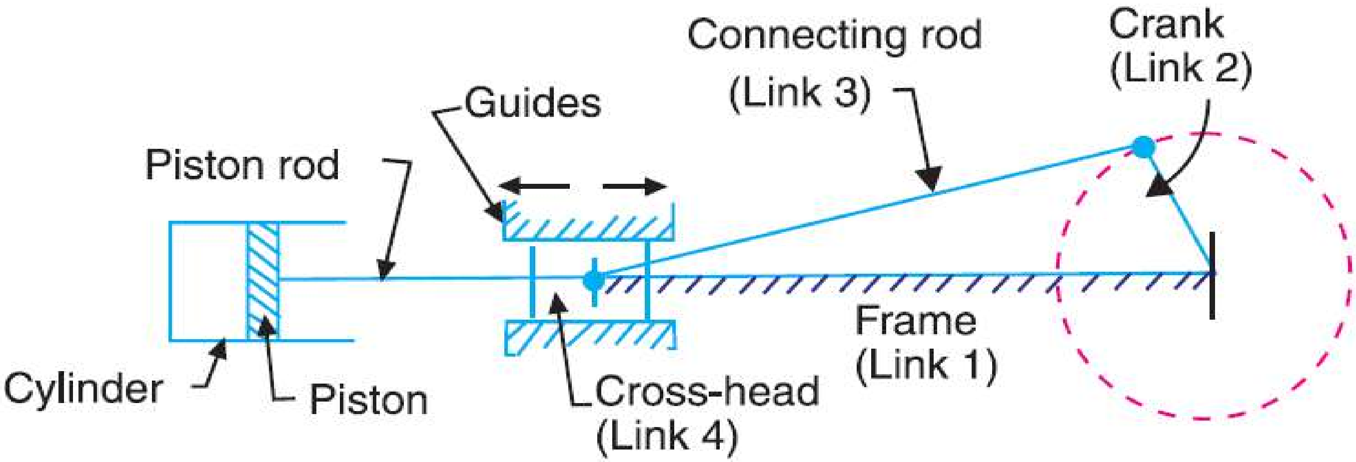
On any small displacement of the mechanism, the tracing point *E at the end of the link CE* traces out approximately a straight line

LECTURE 7

INVERSION OF SINGLE SLIDER CRANK CHAINS

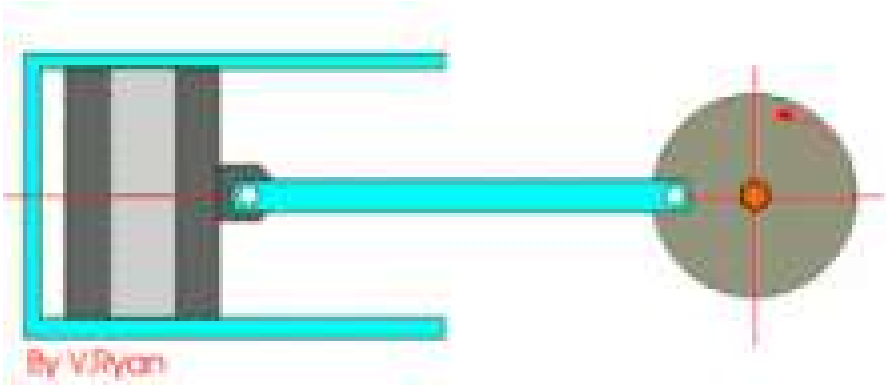
DEPARTMENT OF MECHANICAL ENGINEERING

SINGLE SLIDER CRANK CHAIN

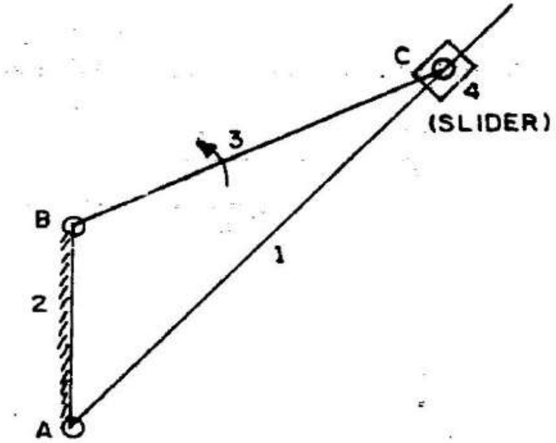


Single slider crank chain

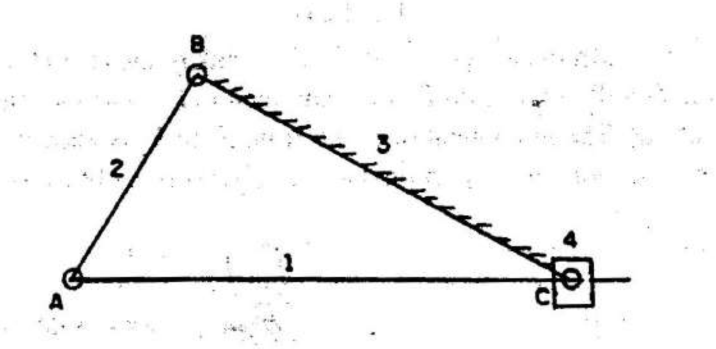
Links 1-2, 2-3, 3-4 = Turning pairs;   Link 4-1 = Sliding pair



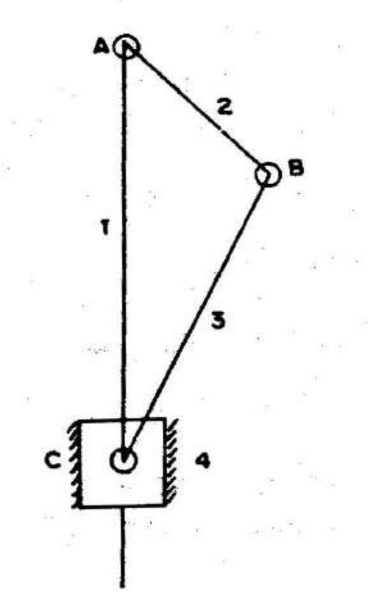
INVERSIONS OF SINGLE SLIDER CRANK CHAIN



crank fixed



connecting rod fixed

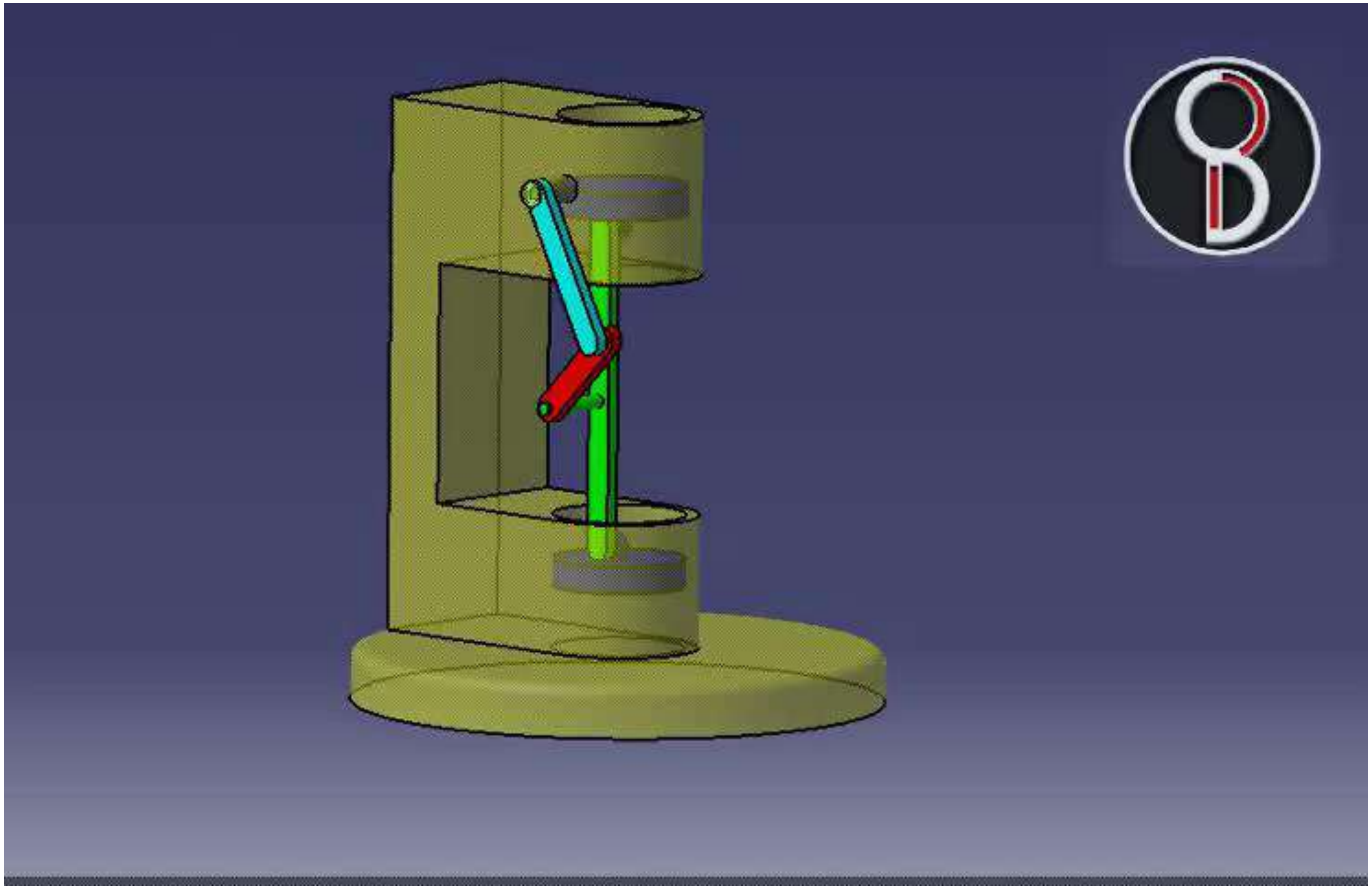


slider fixed

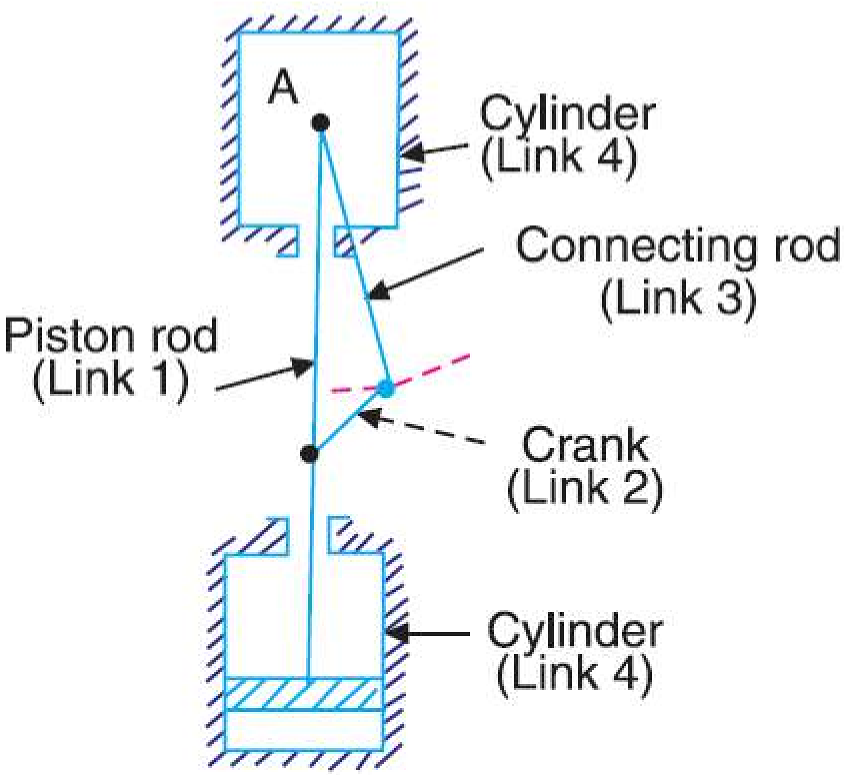
INVERSIONS OF SINGLE SLIDER CRANK CHAIN

- Pendulum pump or Bull engine
- Oscillating cylinder engine
- Rotary internal combustion engine (or) Gnome engine
- Crank and slotted lever quick return motion mechanism
- Whitworth quick return motion mechanism

PENDULUM PUMP OR BULL ENGINE



PENDULUM PUMP OR BULL ENGINE



[Source: R S Khurmi]

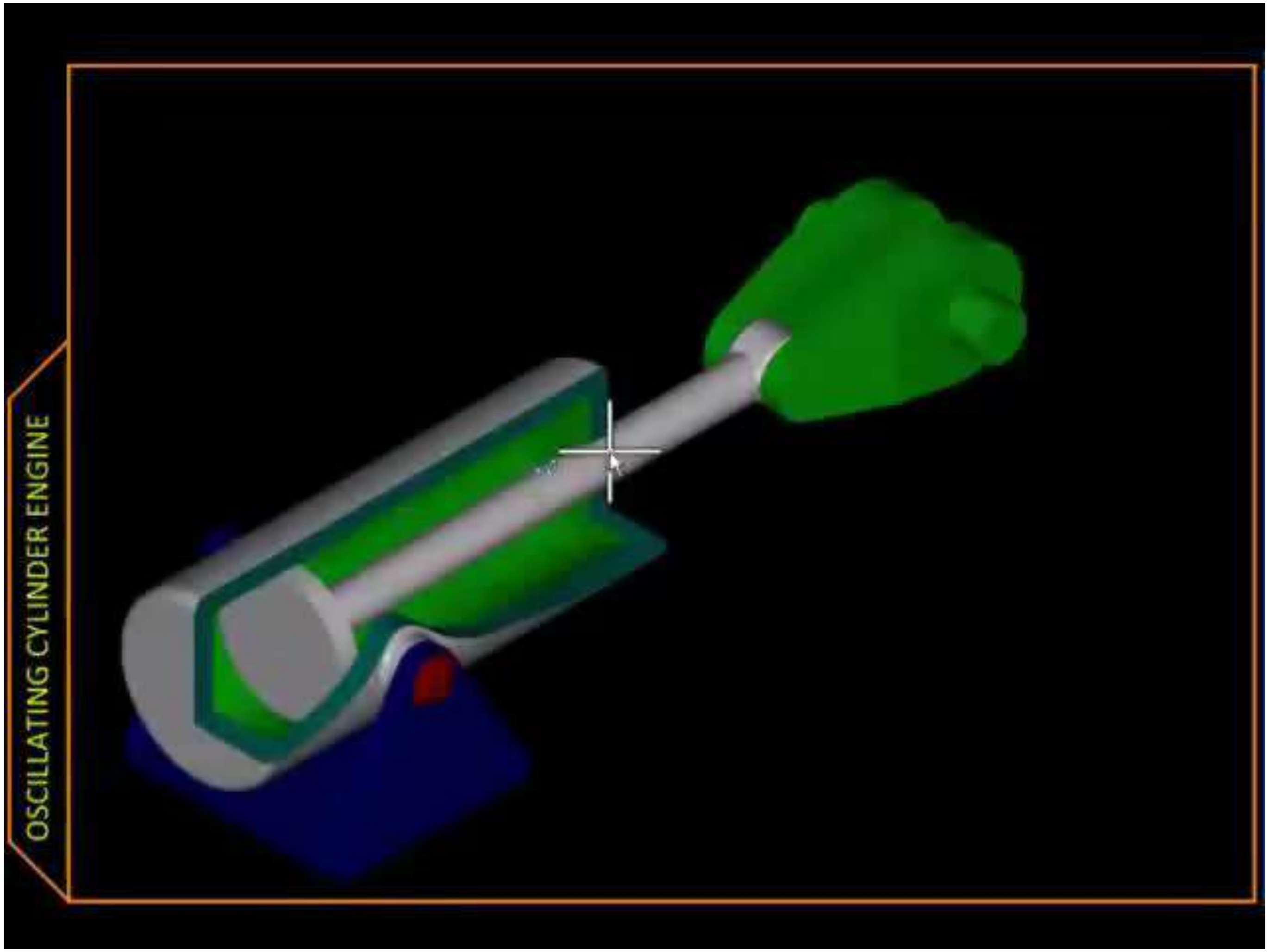
This inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair)

when the crank (link 2) rocks, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A.

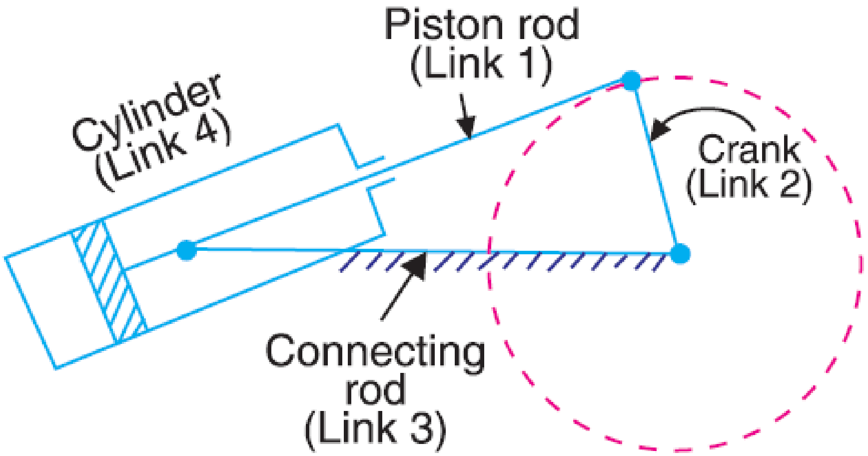
The piston attached to the piston rod (link 1) reciprocates.

It supplies water to a boiler.

OSCILLATING CYLINDER ENGINE



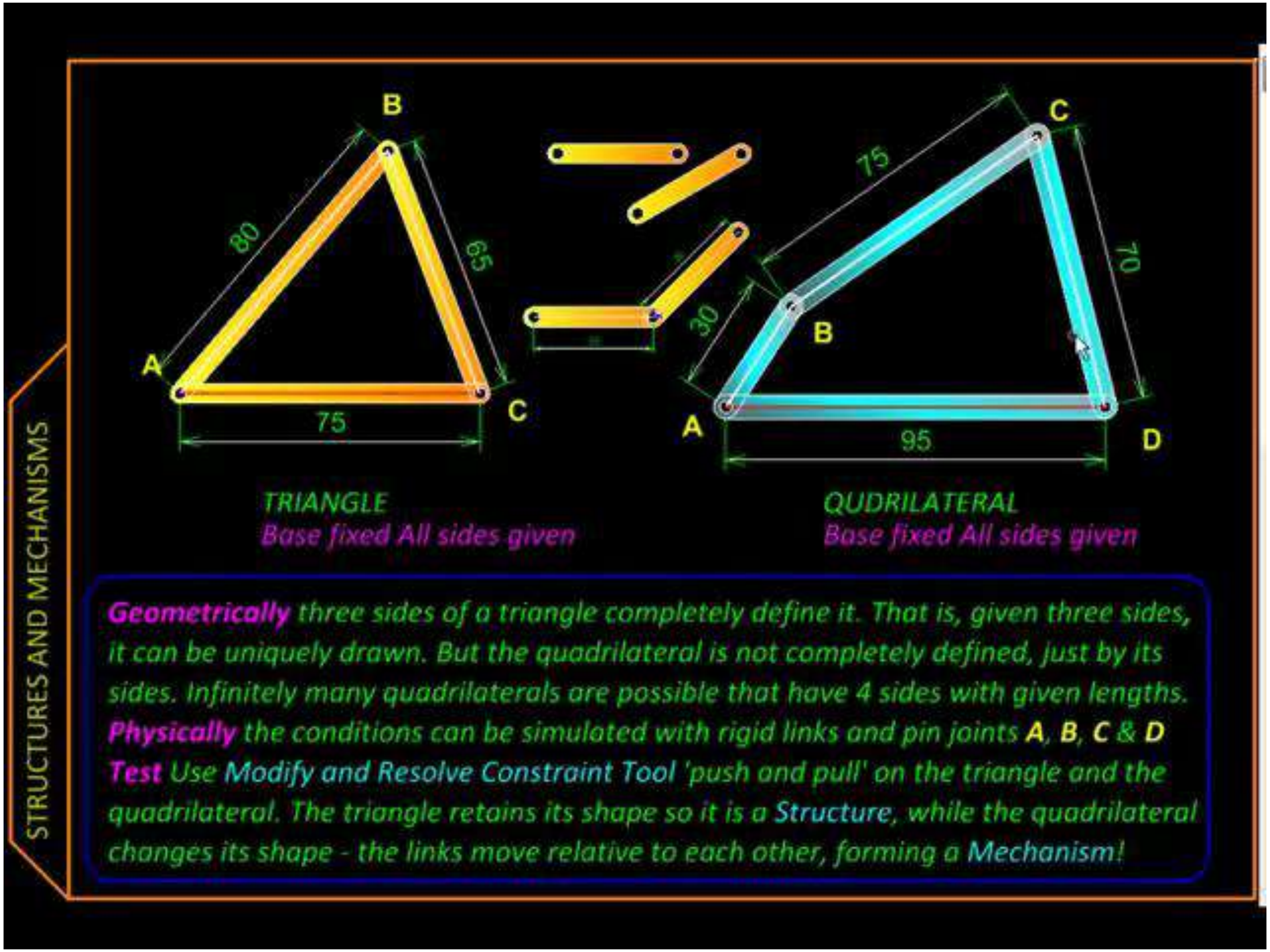
OSCILLATING CYLINDER ENGINE



[Source: R S Khurmi]

- ➢used to convert reciprocating motion into rotary motion
- ➢the link 3 (Connecting Rod) forming the turning pair is fixed.

MULTI-CYLINDER RADIAL IC ENGINE



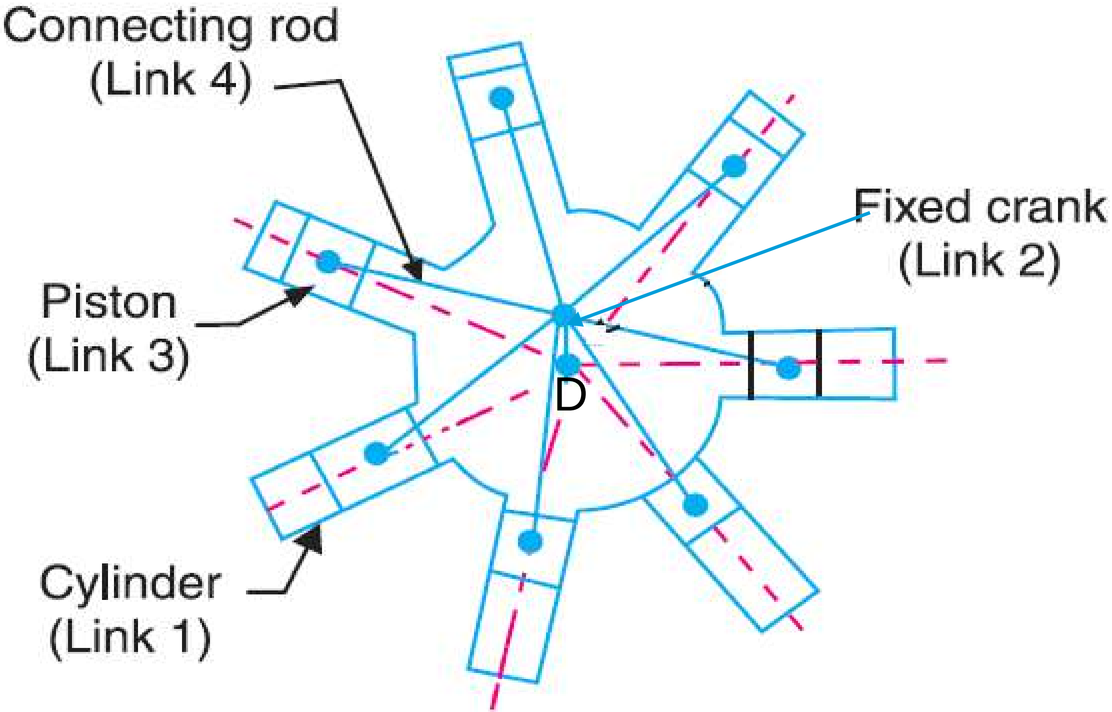
STRUCTURES AND MECHANISMS

TRIANGLE
Base fixed All sides given

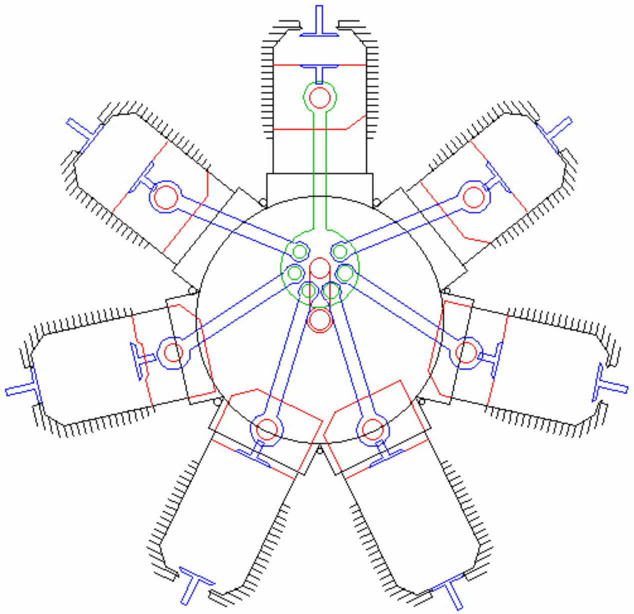
QUDRILATERAL
Base fixed All sides given

Geometrically three sides of a triangle completely define it. That is, given three sides, it can be uniquely drawn. But the quadrilateral is not completely defined, just by its sides. Infinitely many quadrilaterals are possible that have 4 sides with given lengths. **Physically** the conditions can be simulated with rigid links and pin joints **A**, **B**, **C** & **D** **Test** Use Modify and Resolve Constraint Tool 'push and pull' on the triangle and the quadrilateral. The triangle retains its shape so it is a Structure, while the quadrilateral changes its shape - the links move relative to each other, forming a Mechanism!

ROTARY INTERNAL COMBUSTION ENGINE (OR) GNOME ENGINE

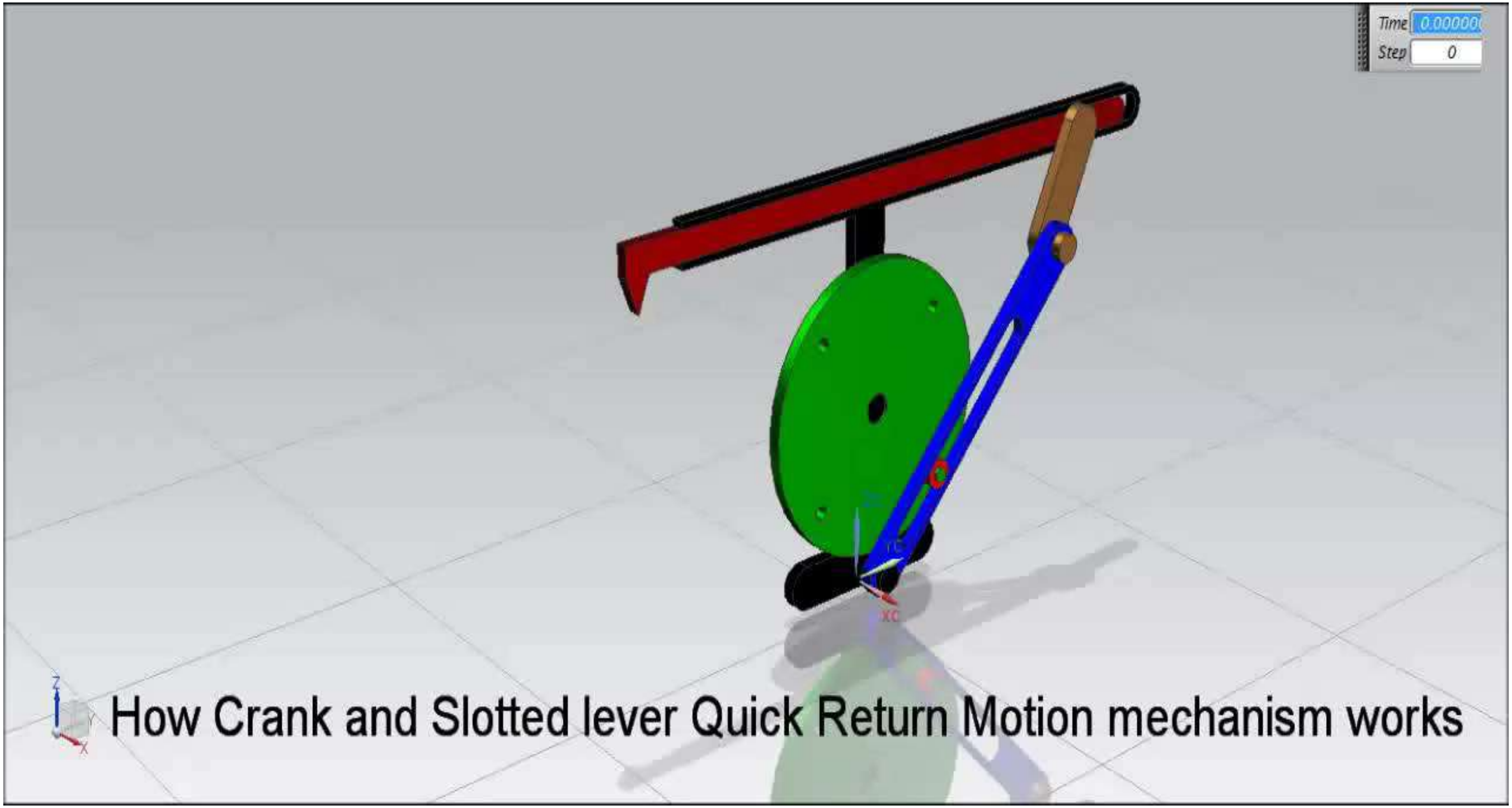


[Source: R S Khurmi]



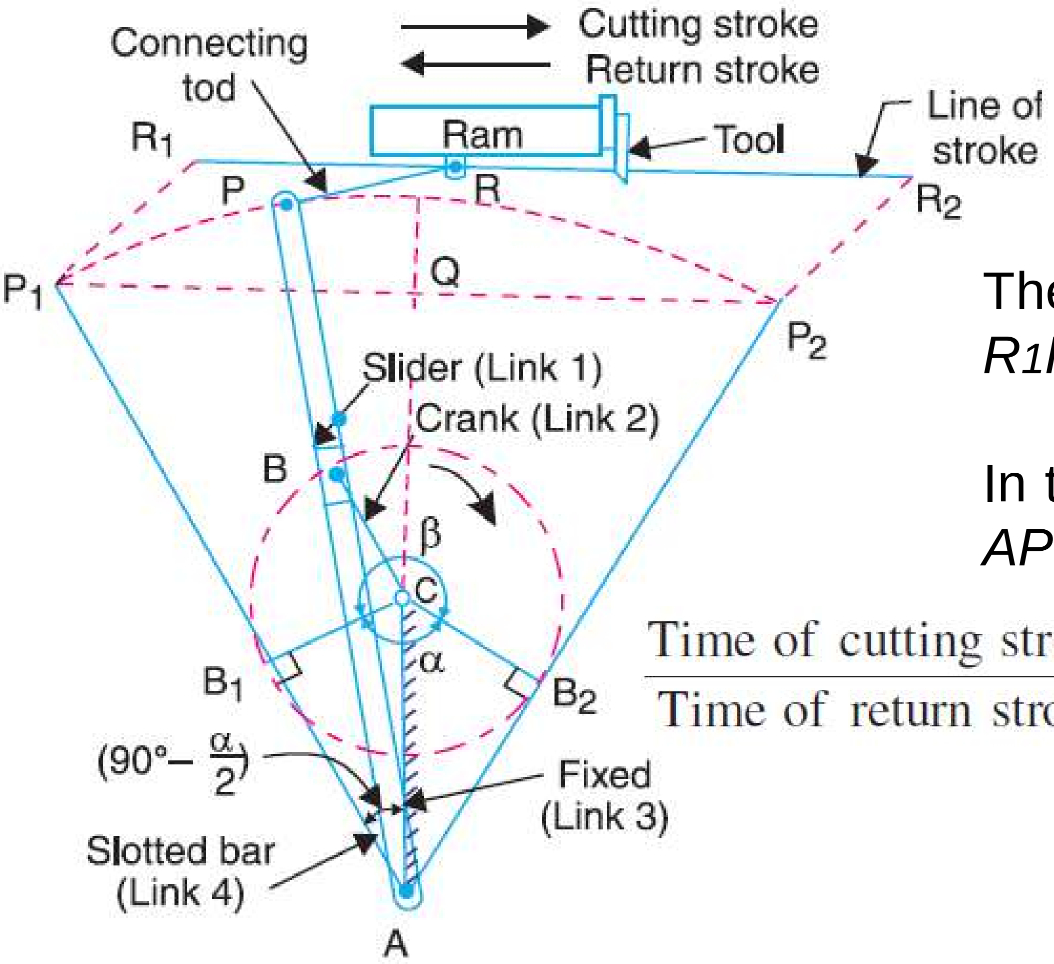
- ➢Crank is fixed at center D
- ➢Cylinder reciprocates
- ➢Engine rotates in the same plane

CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM



How Crank and Slotted lever Quick Return Motion mechanism works

CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM



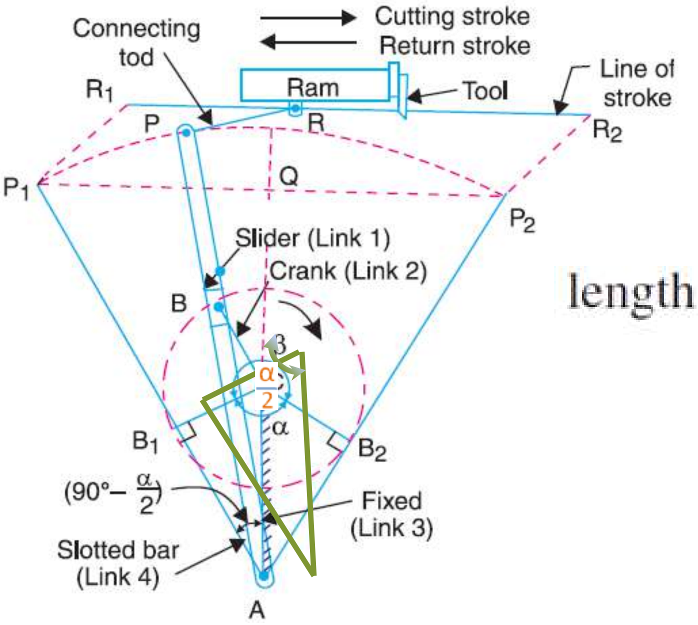
[Source: R S Khurmi]

The line of stroke of the ram (*i.e.* R_1R_2) is perpendicular to AC

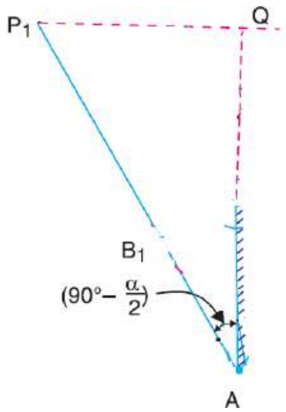
In the extreme positions, AP_1 and AP_2 are tangential to the circle

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} \quad \text{or} \quad \frac{360° - \alpha}{\alpha}$$

CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM



[Source: R S Khurmi]



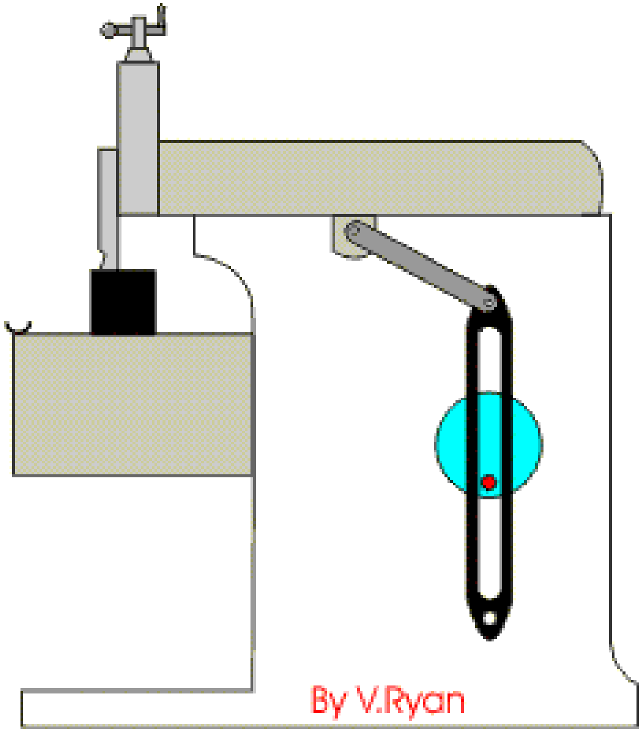
length of stroke $= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1AQ$

$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2}$ ..... $(\because AP_1 = AP)$

$= 2AP \times \frac{CB_1}{AC}$ ... $\left(\because \cos\frac{\alpha}{2} = \frac{CB_1}{AC}\right)$

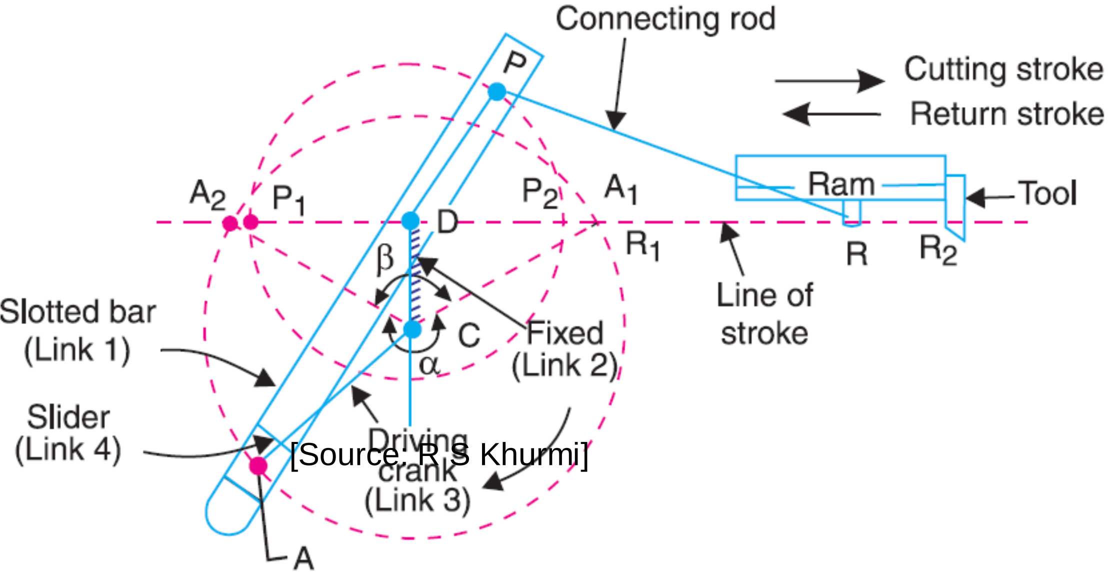
$= 2AP \times \frac{CB}{AC}$ ... $(\because CB_1 = CB)$

Crank and slotted lever quick return mechanism is mostly used in shaping machines & slotting machines

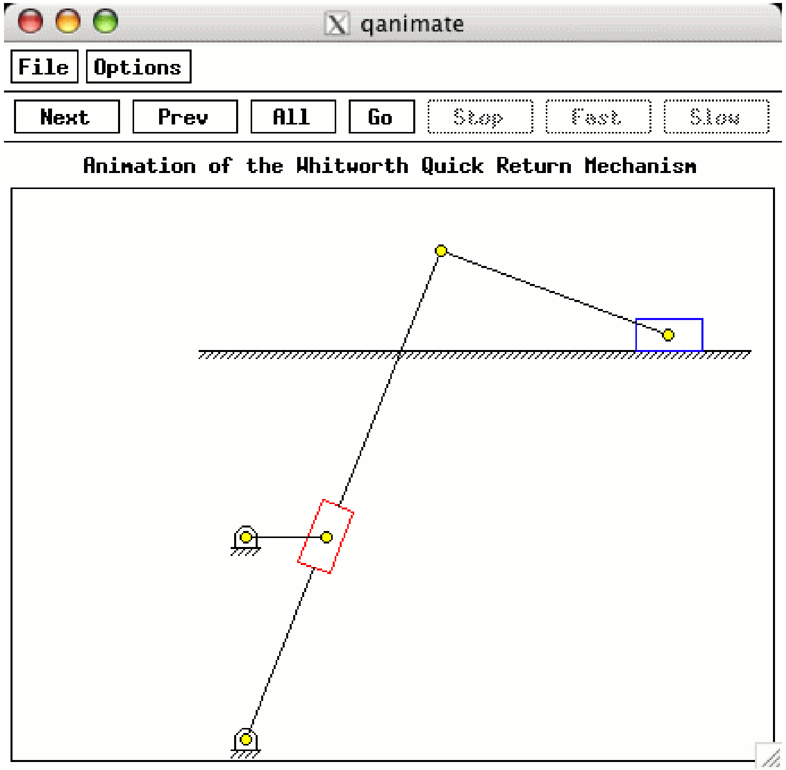


THE SHAPING MACHINE

WHITWORTH QUICK RETURN MOTION MECHANISM



[Source: R S Khurmi]



Animation of the Whitworth Quick Return Mechanism

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360° - \alpha} \quad \text{or} \quad \frac{360° - \beta}{\beta}$$

LECTURE 8

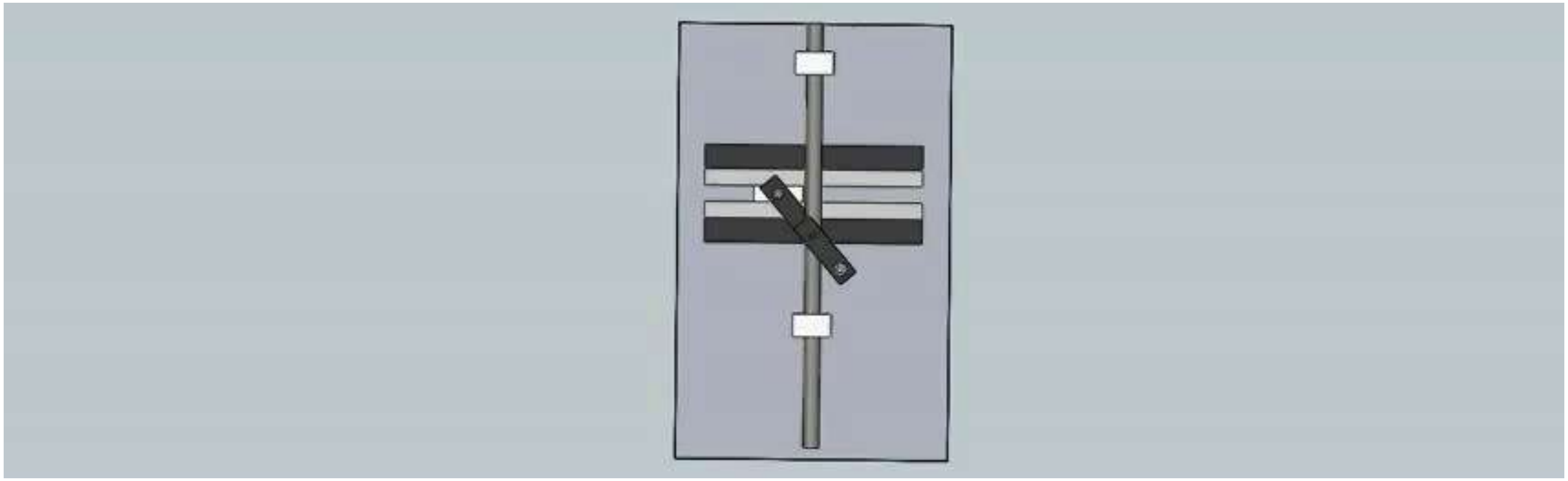
INVERSION OF DOUBLE SLIDER CRANK CHAINS

MRCET CAMPUS
UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

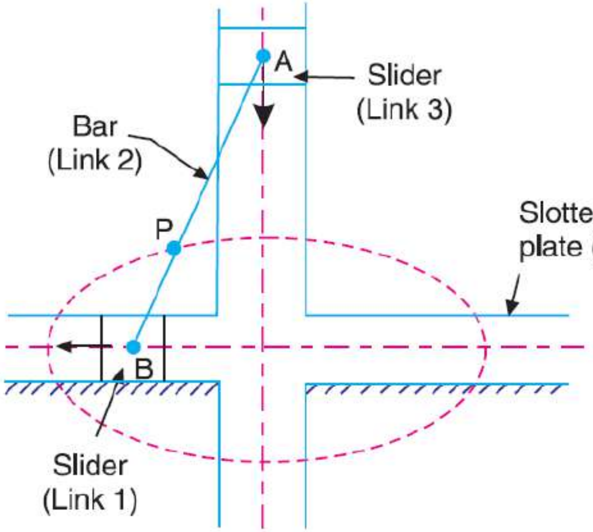
INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

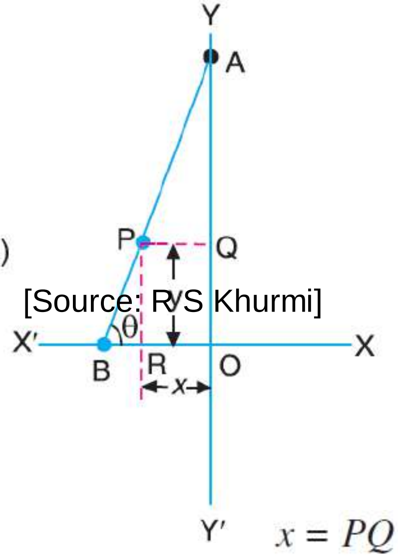
(1. ELLIPTICAL TRAMMELS)



INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(1. ELLIPTICAL TRAMMELS)





[Source: R/S Khurmi]

$$x = PQ = AP\cos\theta;\ \text{and}\ y = PR = BP\sin\theta$$

or

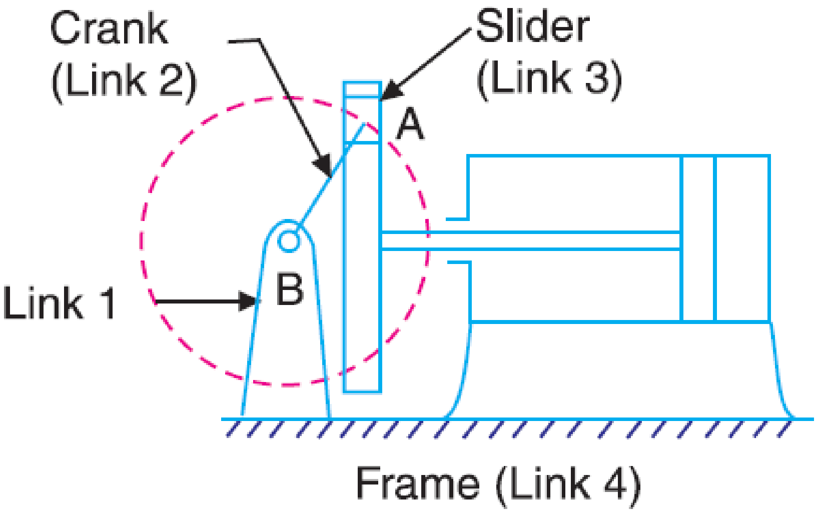
$$\frac{x}{AP} = \cos\theta;\ \text{and}\ \frac{y}{BP} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

- used for drawing ellipses
- any point on the link 2 such as P traces out an ellipse on the surface of link 4
- AP - semi-major axis;
- BP - semi-minor axis

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN
(2. SCOTCH YOKE MECHANISM)



Scotch yoke mechanism.

[Source: R S Khurmi]

- This mechanism is used for converting rotary motion into a reciprocating motion.

- Link 1 is fixed.

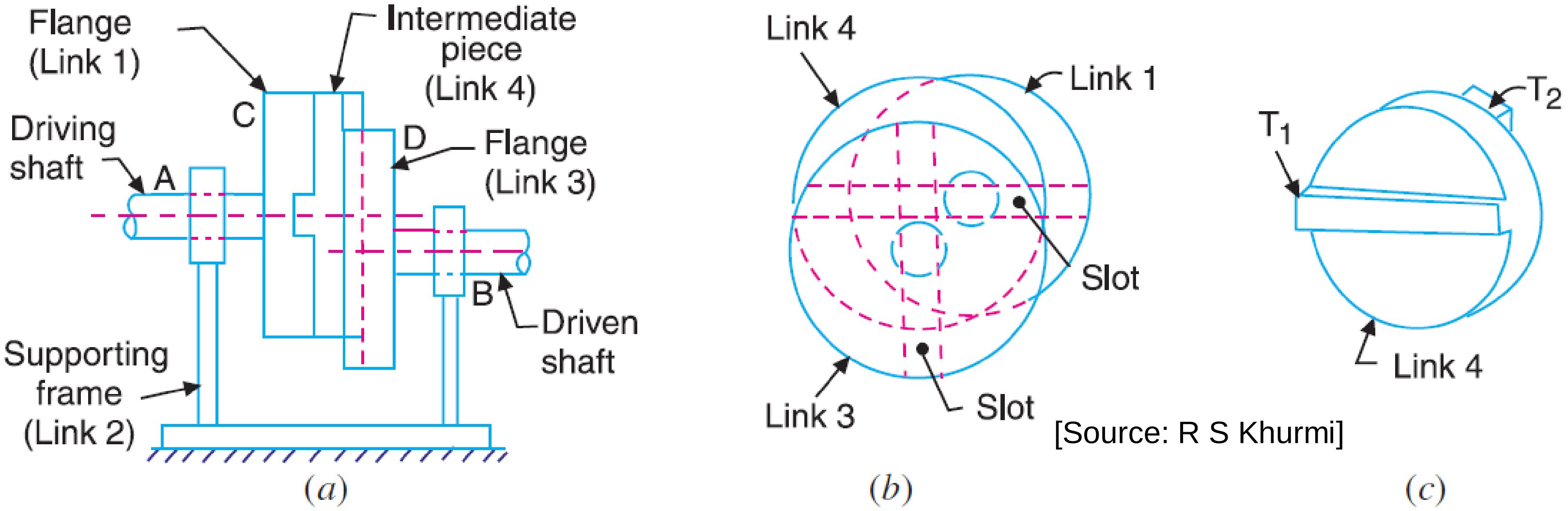
- when the link 2 (crank) rotates about B as centre, reciprocation motion taking place.

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(3. OLDHAM'S COUPLING)

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(3. OLDHAM’S COUPLING)



[Source: R S Khurmi]

Oldham’s coupling.

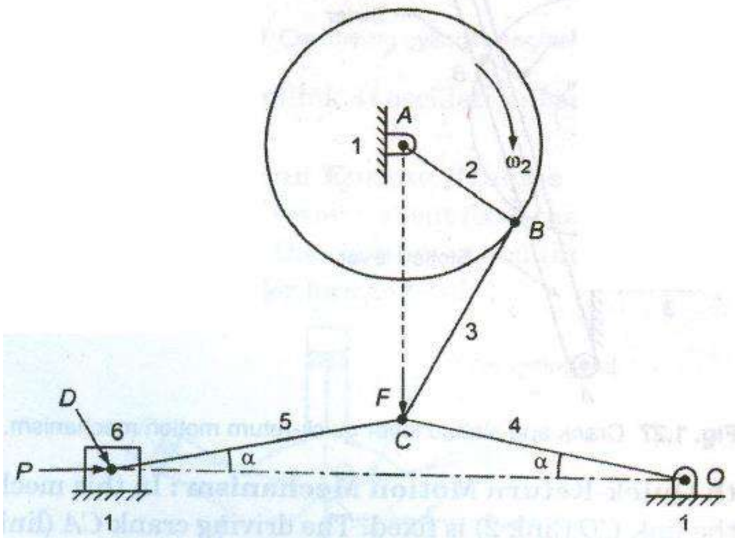
T_1 and T_2 two tongues (*i.e.* diametrical projections) on each face at right angles to each other

used for connecting two parallel shafts whose axes are at a small distance apart.

SOME COMMON MECHANISMS :TOGGLE MECHANISM

TOGGLE MECHANISM

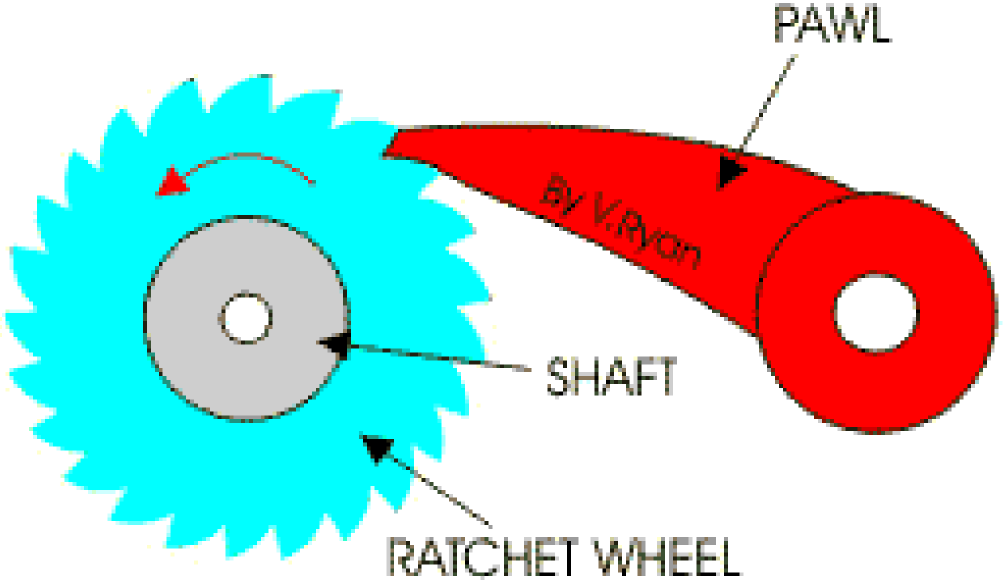
$$F = 2P \tan\alpha$$



- ➢If α approaches to zero, for a given F, P approaches infinity.
- ➢A <u>stone crusher</u> utilizes this mechanism to overcome a large resistance with a small force.
- ➢It is used in numerous <u>toggle clamping devices</u> for holding work pieces.
- ➢Other applications are: Clutches, Pneumatic riveters etc.,

INTERMEDIATE MOTION MECHANISM

RATCHET AND PAWL MECH



➢There are many different forms of ratchets and escapements which are used in:

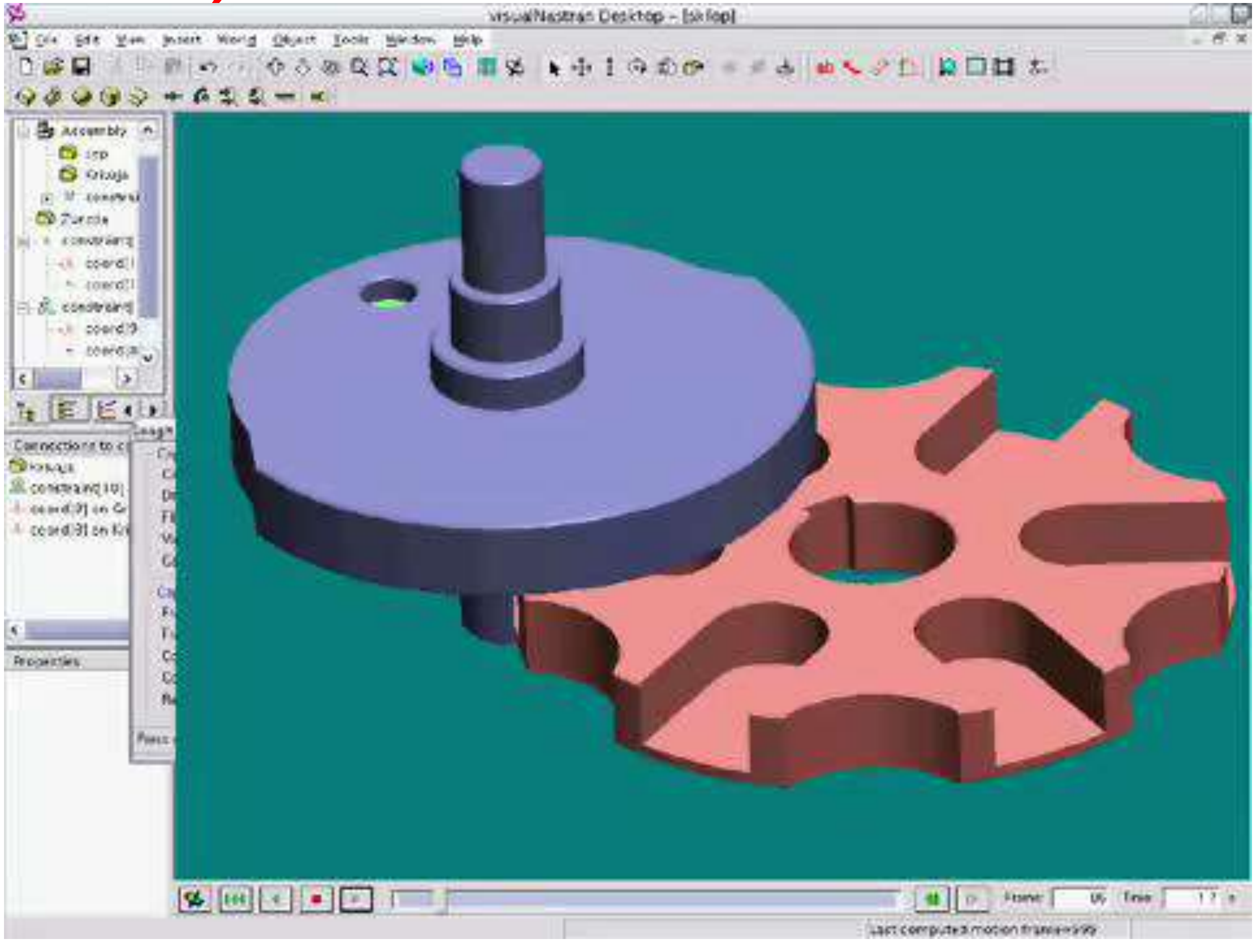
➢ locks, jacks, clockwork, and other applications requiring some form of intermittent motion.

APPLICATION OF RATCHET PAWL MECHANISM

Used in Hoisting Machines as safety measure

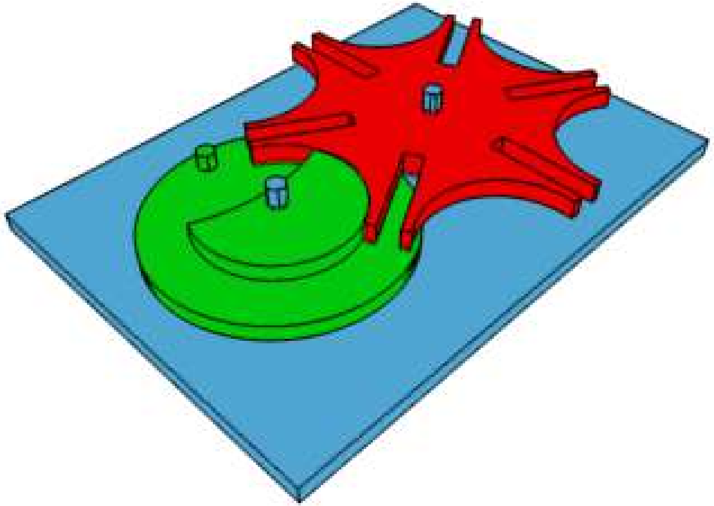
INTERMEDIATE MOTION MECHANISM

GENEVA MECHANISM (INDEXING MECHANISM)

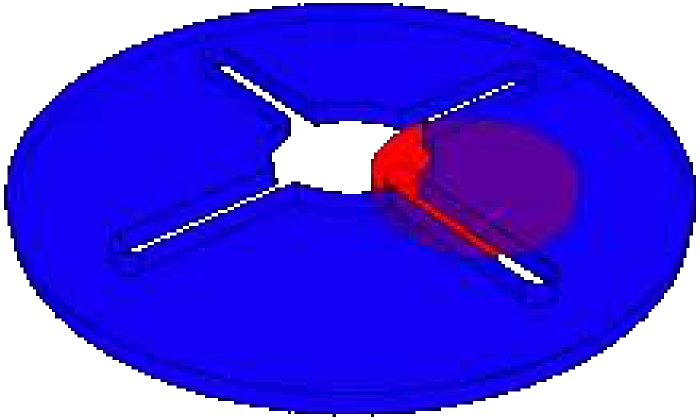


INTERMEDIATE MOTION MECHANISM

GENEVA MECHANISM



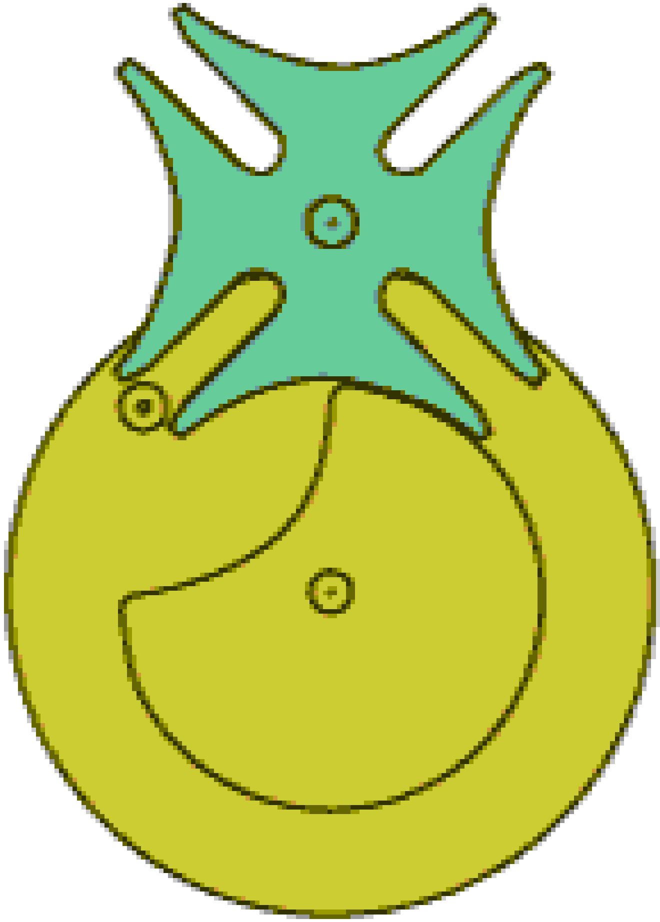
Animation showing a six-position external Geneva drive in operation

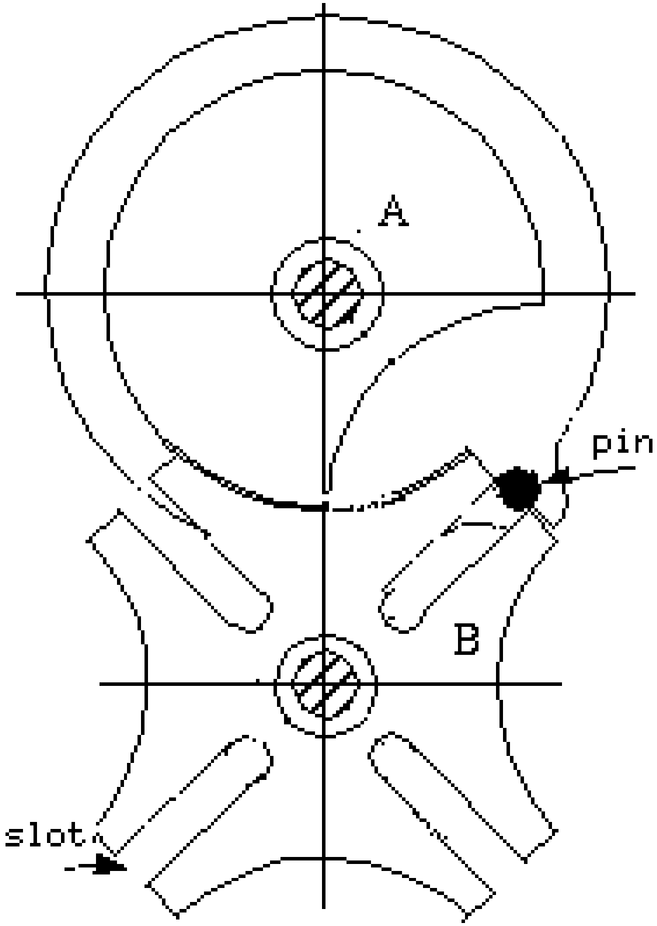


Animation showing an internal Geneva drive in operation.

INTERMITTENT MOTION MECHANISMS

GENEVA WHEEL MECHANISM





APPLICATIONS OF GENEVA MECHANISM

- Locating and locking mechanism
- Indexing system of a multi-spindle machine tool

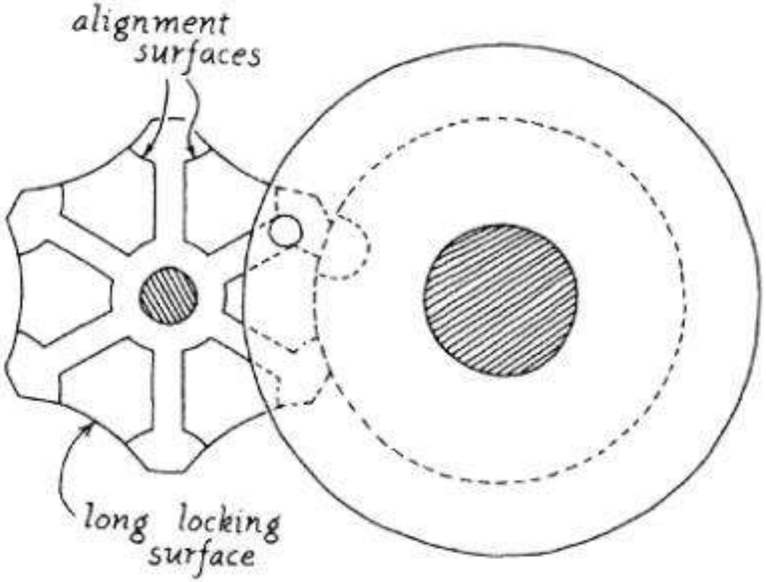
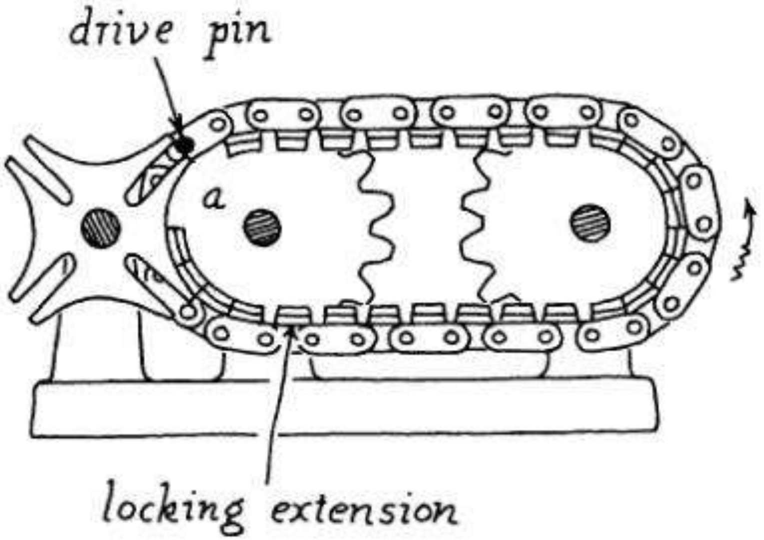


Fig. 9–15. Six-slot external Geneva used for light-duty instrument applications.



Drawing courtesy of PRODUCT ENGINEERING Magazine; June 8, 1964: pp. 67, 68

Fig. 9–18. Chain-mounted drive pins with blocks for locking during dwells.

Industry applications

1. Compliant mechanisms used in new age industries.
2. Mechanical Components form Specialized Motion-Control Systems
3. Mechanism for Planar Manipulation with Simplified Kinematics
4. Five linkages for straight-line motion
5. Seven linkages for transport mechanisms

Question Bank for Assignments

1. Explain inversions of a four bar chain in detail?
2. Explain the working of any two inversions of a single slider crank chain with neat sketches.
3. What is inversion of mechanism? Describe various inversions of double slider crank mechanism with sketches.
4. Explain with neat sketch the working of crank and slotted lever quick return motion mechanism. Deduce the expression for length of stroke in terms of link lengths.
5. State and explain Whitworth quick return mechanism. Also derive an equation for ratio of time taken for return strokes and forward strokes.
6. Define Kinematic pair and discuss various types of kinematic pairs with example.

Tutorial Questions

1. What is a machine? Giving example, differentiate between a machine and a structure.

2. Write notes on complete and incomplete constraints in lower and higher pairs, illustrating your answer with neat sketches.

3. Explain different kinds of kinematic pairs giving example for each one of them.

4. Explain the terms: 1. Lower pair, 2. Higher pair, 3. Kinematic chain, and 4. Inversion.

5. In what way a mechanism differ from a machine?

6. What is the significance of degrees of freedom of a kinematic chain when it functions as a mechanism? Give examples.

7. Explain Grubler's criterion for determining degree of freedom for mechanisms. Using Grubler's criterion for plane mechanism, prove that the minimum number of binary links in a constrained mechanism with simple hinges is four.

8. Sketch and explain the various inversions of a slider crank chain.

9. Sketch and describe the four bar chain mechanism. Why it is considered to be the basic chain?

10. Show that slider crank mechanism is a modification of the basic four bar mechanism.

11. Sketch slider crank chain and its various inversions, stating actual machines in which these are used in practice.

12. Sketch and describe the working of two different types of quick return mechanisms. Give examples of their applications. Derive an expression for the ratio of times taken in forward and return stroke for one of these mechanisms.

13. Sketch and explain any two inversions of a double slider crank chain.

UNIT 2

SPECIAL MECHANISMS

COURSE OBJECTIVE

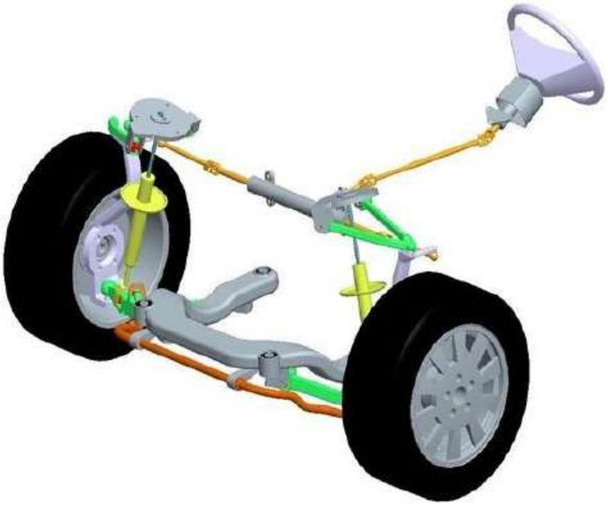
To Synthesize and analyze 4 bar mechanisms.

COURSE OUTCOME

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	**Straight Line Motion Mechanisms**	Definition of Straight line motion mechanism Classification of exact straight line motion mechanism	Understanding the inversion of mechanisms and its classifications (B2)
2	Approximate Straight Line Mechanism	Working of approximate straight-line mechanisms	Understanding the inversions of approximate straight line mechanism (B2) Analyse the approximate straight line mechanisms (B4)
3	Pantograph	Purpose of mechanisms Applications of Mechanism	Understanding the Pantograph mechanism (B2) List the applications of Pantograph (B4)
4	**Davi's Steering Gear Mechanism**	Definition of Davi's Steering Gear Mechanism Condition for correct steering condition	Understanding the working of Davi's Steering gear mechanism (B2) Apply the mechanism for correct steering condition (B3)
5	Ackerman's Steering Gear Mechanism	Definition of Ackerman's Steering Gear Mechanism Condition for correct steering condition	Understanding the working of Ackerman's Steering gear mechanism (B2) Apply the mechanism for correct steering condition (B3)
6	**Single and Double Hooke Joint**	Describe single and double Hooke's Joint List of applications using single and double Hooke's Joint	Remember the working single and double Hooke's Joint (B1) Apply single and double Hooke's joint for various applications (B3)
7	Ratio of Shaft Velocities	Derive the ratio of shaft velocities	Evaluate the velocity ratio for shafts (B5)

2

Special Mechanisms



Course Contents

2.1 Straight Line Mechanisms

- It permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called ***straight line mechanisms***.
  1. In which only turning pairs are used
  2. In which one sliding pair is used.
- These two types of mechanisms may produce exact straight line motion or approximate straight line motion.
- **Need of Straight Line:**
  1. Sewing Machine converts rotary motion to up/down motion.
  2. Want to constrain pistons to move only in a straight line.
  3. How do you create the first straight edge in the world? (Compass is easy)
  4. Windshield wipers, some flexible lamps made of solid pieces connected by flexible joints.

2.2 Exact Straight Line Motion Mechanisms Made Up Of Turning Pairs

- The principle adopted for a mathematically correct or exact straight line motion is described in Fig.2.1
- Let O be a point on the circumference of a circle of diameter OP. Let OA be any chord and B is a point on OA produced, such that

$$\mathrm{OA} \times \mathrm{OB} = \text{constant}$$

- The triangles OAP and OBQ are similar.

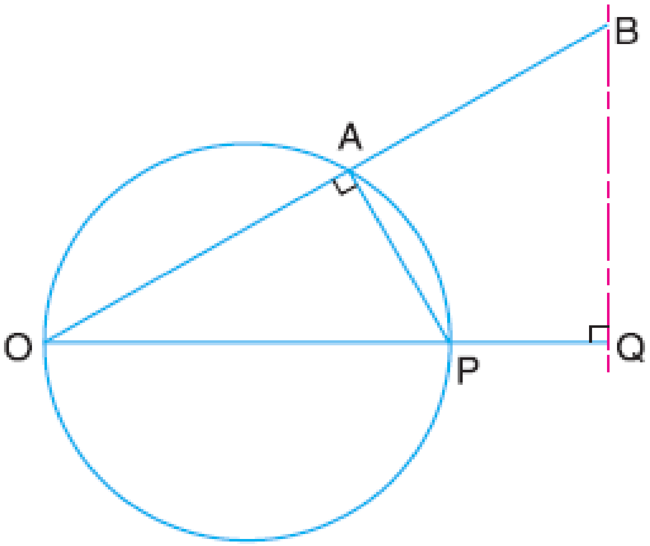


Fig. 2.1 Exact straight line motion mechanism

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

$$OP \times OQ = OA \times OB$$

$$OQ = \frac{OA \times OB}{OP}$$

- But *OP* is constant as it is the diameter of a circle; therefore, if *OA* × *OB* is constant, then *OQ* will be constant.
- Hence

$$OA \ \times OB \ = constant$$

- So point B moves along the straight line.

2.3 Peaucellier Mechanism (Exact Straight Line)

- It consists of a fixed link OO1 and the other straight links O1A, OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in Fig. 2.2
- The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O1A. In Fig. 2.2
- AC = CB = BD = DA
- OC = OD
- $OO_1 = O_1A$

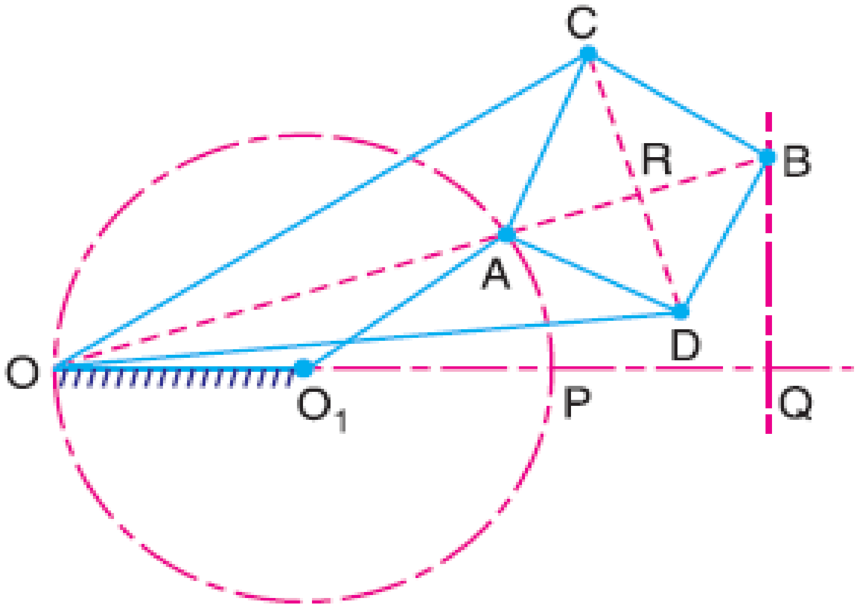


Fig. 2.2 Peaucellier Mechanism

- From right angled triangles *ORC* and *BRC*, we have

$$OC^2 = OR^2 + RC^2 \qquad \text{(I)}$$

$$BC^2 = RB^2 + RC^2 \qquad (ii)$$

- From (i) and (ii)

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR - RB)(OR + RB)$$

$$= OB \times OA$$

- Since *OC* and *BC* are of constant length, therefore the product *OB* × *OA* remains constant.

Hart's Mechanism

- This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.
- It consists of a fixed link OO1 and other straight links O1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig. 2.3.
- The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio. A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to FD and CE.

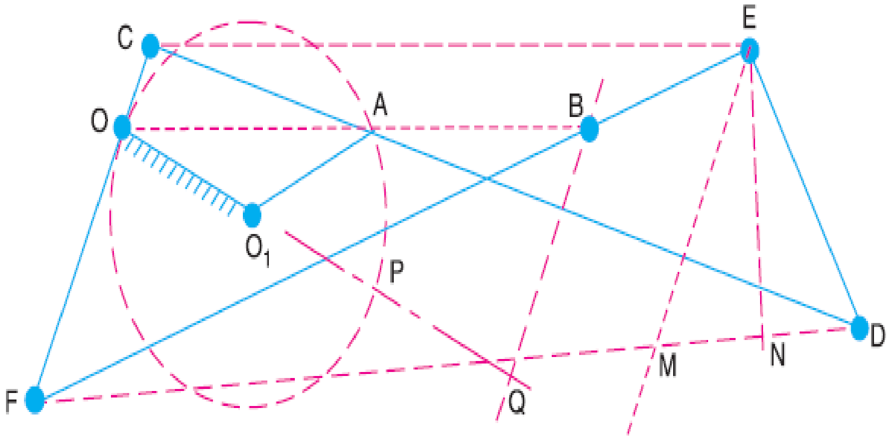


Fig. 2.3 Hart's Mechanism

- Here, FC = DE & CD = EF
- The point O, A and B divide the links FC, CD and EF in the same ratio.
- From similar triangles CFE and OFB,

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or CB} = \frac{CE \times OF}{FC} \ldots \ldots \text{(i)}$$

- From similar triangle FCD and OCA

$$\frac{FD}{FC} = \frac{OA}{OC} \quad or\ OA = \frac{FD \times OC}{FC} \quad \ldots .\ (ii)$$

- From above equations,

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC}$$

$$= FD \times CE \times \frac{OC \times OF}{FC^2}$$

- Since the lengths of OC, OF and FC are fixed, therefore

$$OA \times OB = FD \times CE \times cons. \ldots (iii)$$

- From point E, draw EM parallel to CF and EN perpendicular to FD.

$$FD \times CE = FD \times FM \quad (CE = FM)$$
$$= (FN + ND)(FN - MN)$$
$$= FN^2 - ND^2 \text{ (MN = ND)}$$
$$= (FE^2 - NE^2) - (ED^2 - NE^2)$$ (From right angle triangles FEN and EDN)
$$= E^2 - ED^2 = constant \qquad (iv)$$

- From equation (iii) and (iv),

$$OA \times OB = constant$$

Exact Straight Line Motion consisting of one sliding pair-Scott Russell's Mechanism

- A is the middle point of PQ and OA = AP = AQ. The instantaneous center for the link PAQ lies at I in OA produced and is such that IP is perpendicular to OP.

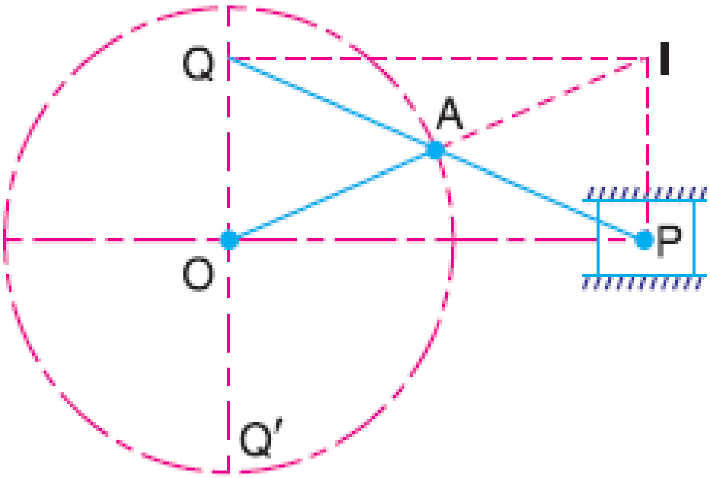


Fig. 2.4 Scott Russell's Mechanism

- Join IQ. Then Q moves along the perpendicular to IQ. Since OPIQ is a rectangle and IQ is perpendicular to OQ, therefore Q moves along the vertical line OQ for all positions of QP. Hence Q traces the straight line OQ′.
- If OA makes one complete revolution, then P will oscillate along the line OP through a distance 2 OA on each side of O and Q will oscillate along OQ′ through the same distance 2 OA above and below O. Thus, the locus of Q is a copy of the locus of P.

Approximate straight line motion mechanisms

Watt's Mechanism

- It has four links as shown in fig. OB, O1A, AB and OO1.

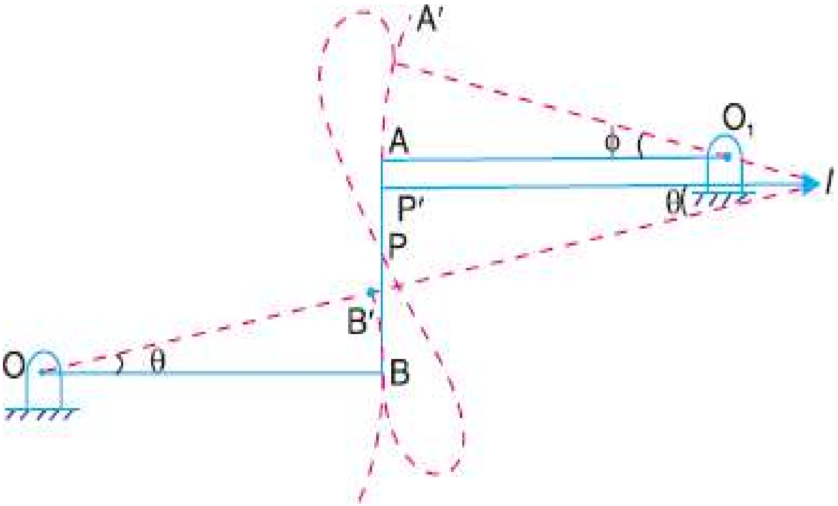


Fig. 2.5 watt's mechanism

- OB and O1A oscillates about centers O and O1 respectively. P is a point on AB such that,

$$\frac{O_1}{OB} = \frac{PB}{PA}$$

- As OB oscillates the point P will describe an approximate straight line.

Modified Scott-Russel Mechanism

- This is similar to Scott-Russel mechanism but in this case AP is not equal to AQ and the points P and Q are constrained to move in the horizontal and vertical directions.

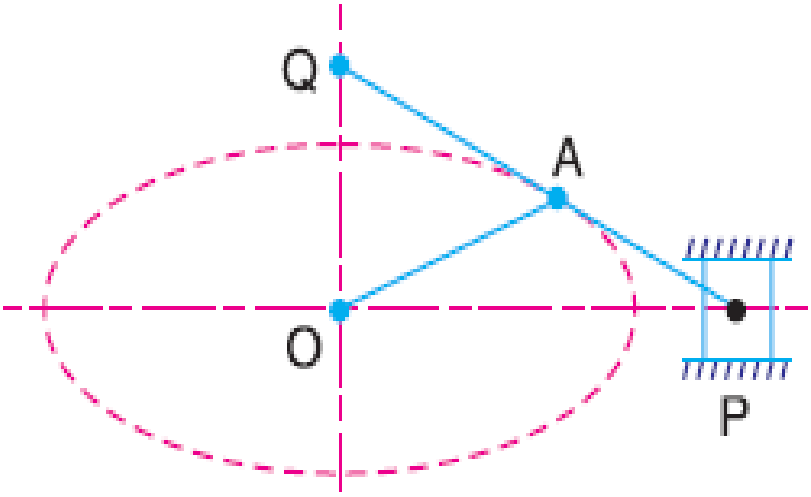


Fig. 2.6 Modified Scott-Russel Mechanisms

- A little consideration will show that it forms an elliptical trammel, so that any point *A* on *PQ* traces an ellipse with semi-major axis *AQ* and semi minor axis *AP*.
- If the point *A* moves in a circle, then for point *Q* to move along an approximate straight line, the length *OA* must be equal (AP)2 / AQ. This is limited to only small displacement of *P*.

Grasshopper Mechanism

- In this mechanism, the centers O and O1 are fixed. The link OA oscillates about O through an angle AOA1 which causes the pin P to move along a circular arc with O1 as center and O1P as radius.

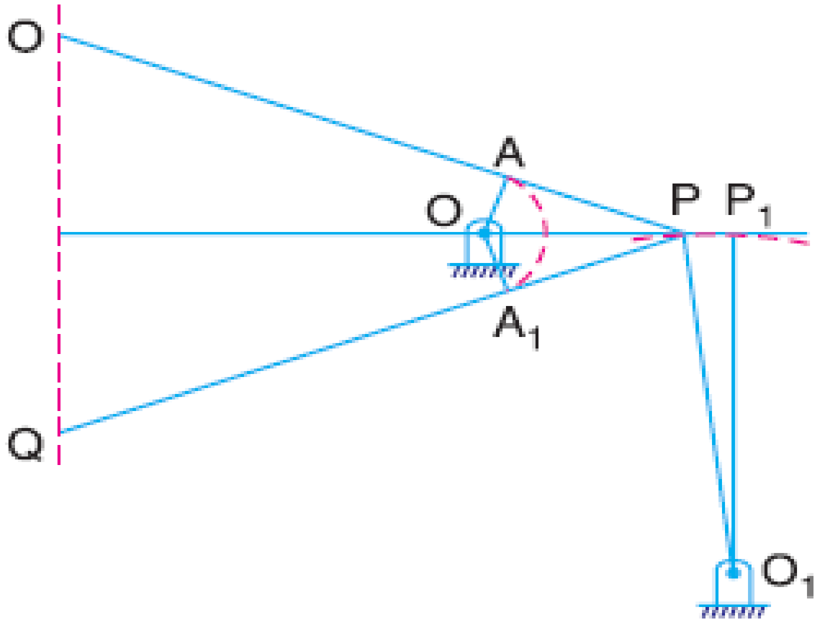


Fig. 2.7 Grasshopper Mechanism

- For small angular displacements of OP on each side of the horizontal, the point Q on the extension of the link PA traces out an approximately a straight path QQ′. if the lengths are such that

$$OA = \frac{AP^2}{AQ}$$

Tchebicheff’s Mechanism

- It is a four bar mechanism in which the crossed links OA and O1B are of equal length, as shown in Fig. 2.8.
- The point P, which is the mid-point of AB, traces out an approximately straight line parallel to OO1.

- The proportions of the links are, usually, such that point P is exactly above O or O1 in the extreme positions of the mechanism i.e. when BA lies along OA or when BA lies along BO_1.

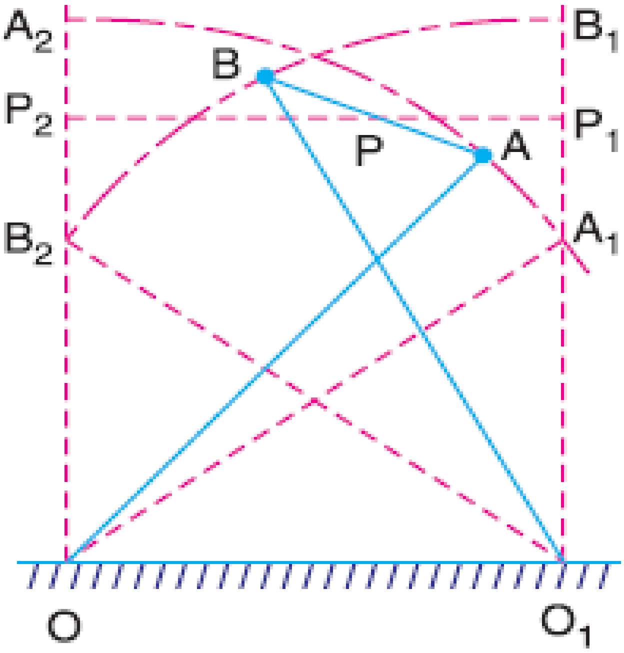


Fig. 2.8 Tchebicheff's mechanism

- It may be noted that the point P will lie on a straight line parallel to OO_1, in the two extreme positions and in the mid position, if the lengths of the links are in proportions

$$AB: OO_1 : OA = 1 : 2 : 4.5.$$

Roberts Mechanism

- It is also a four bar chain mechanism, which, in its mean position, has the form of a trapezium. The links OA and O1 B are of equal length and OO1 is fixed. A bar PQ is rigidly attached to the link AB at its middle point P.

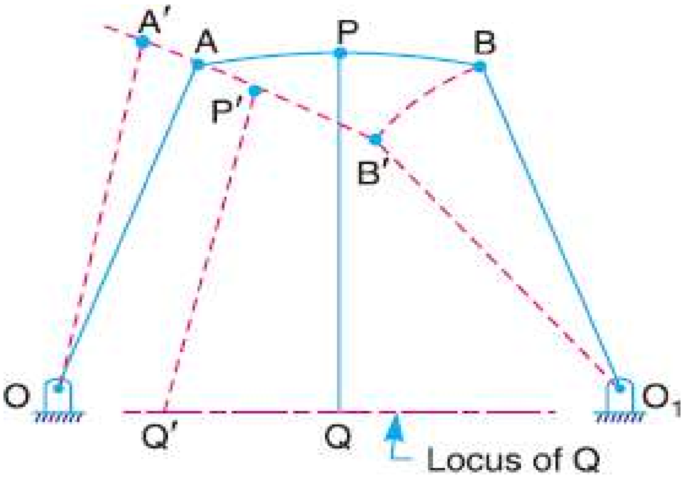


Fig. 2.9 Robert's Mechanism

- A little consideration will show that if the mechanism is displaced as shown by the dotted lines in Fig. the point *Q* will trace out an approximately straight line.

Steering gear mechanism

- The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path.
- Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.
- In automobiles, the front wheels are placed over the front axles, which are pivoted at the points A and B, as shown in Fig. 2.10.

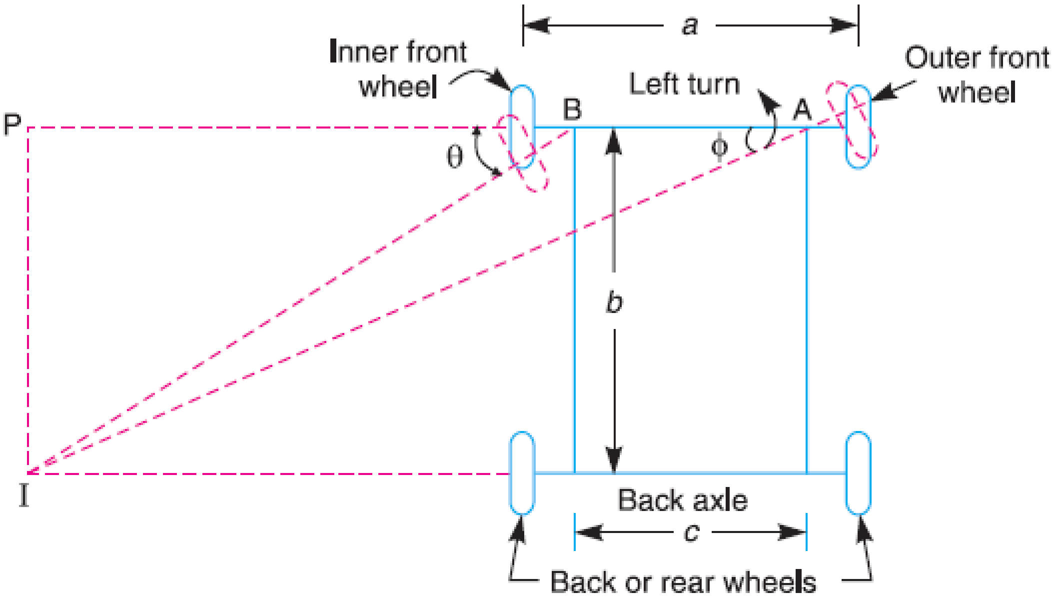


Fig. 2.10 steering gear mechanism

- These points are fixed to the chassis. The back wheels are placed over the back axle, at the two ends of the differential tube. When the vehicle takes a turn, the front wheels along with the respective axles turn about the respective pivoted points. The back wheels remain straight and do not turn. Therefore, the steering is done by means of front wheels only.
- In order to avoid skidding (i.e. slipping of the wheels sideways), the two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheels. If the instantaneous centre of the two front wheels do not coincide with the instantaneous Centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear and tear of the tyres

- Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre. The axis of the inner wheel makes a larger turning angle θ than the angle φ subtended by the axis of outer wheel.
- Let, a = wheel track

  b = wheel base

  c = Distance between the pivots A and B of the front axle.
- Now from triangle IBP,

$$\cot\theta = \frac{BP}{IP}$$

- And from triangle IAP,

$$\cot \emptyset = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{c}{b} + \cot\theta$$

$$\boldsymbol{\cot \emptyset - \cot \theta = \frac{c}{b}}$$

- This is the fundamental equation for correct steering.

Devis Steering Mechanism

- The Davis steering gear is shown in Fig. 2.11. It is an exact steering gear mechanism. The slotted links AM and BH are attached to the front wheel axle, which turn on pivots A and B respectively.
- The rod CD is constrained to move in the direction of its length, by the sliding members at P and Q. These constraints are connected to the slotted link AM and BH by a sliding and a turning pair at each end. The steering is affected by moving CD to the right or left of its normal position. C ′D′ shows the position of CD for turning to the left.

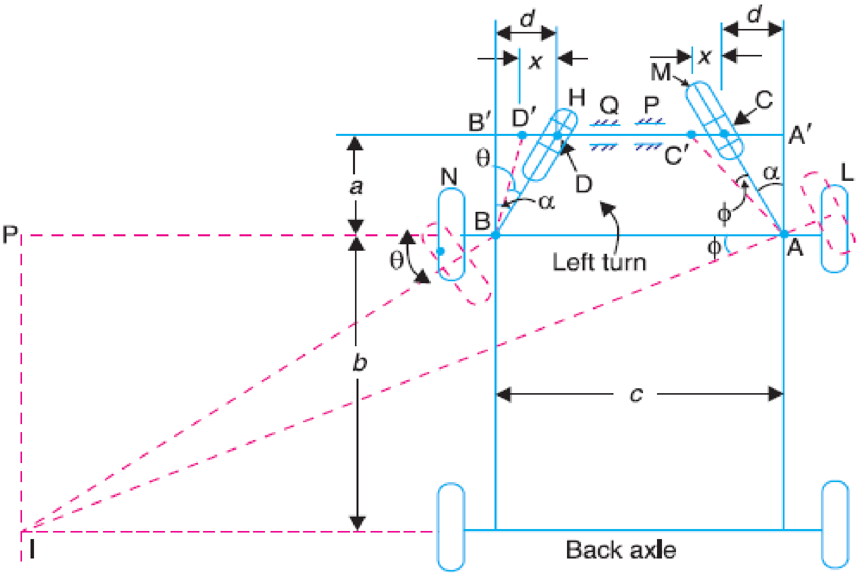


Fig. 2.11 Devis steering gear mechanism

- Let,

  a = Vertical distance between AB and CD,
  b = Wheel base,
  d = Horizontal distance between AC and BD,
  c = Distance between the pivots A and B of the front axle.
  x = Distance moved by AC to AC ′ = CC ′ = DD′, and
  α = Angle of inclination of the links AC and BD, to the vertical.

- From triangle AA’C ′

$$\tan(\alpha + \emptyset) = \frac{\mathrm{A'C'}}{\mathrm{A'A'}} = \frac{\mathrm{d + x}}{\mathrm{a}} \ldots\ldots \ldots \ldots \ldots \ldots \ldots (i)$$

- From triangle AA’C

$$\tan \alpha = \frac{A'C}{A'A'} = \frac{d}{a} \ldots \ldots \ldots\ldots \ldots \ldots \ldots .\,. \ldots \ldots .\,(ii)$$

- From triangle BB’D’

$$\tan(\alpha - \theta) = \frac{B'D'}{BB'} = \frac{d - x}{a} \ldots \ldots\ldots \ldots \ldots\ldots \ldots \ldots \ldots\ldots .\ldots (iii)$$

- We know that,

$$\tan(\alpha + \emptyset) = \frac{\tan \alpha + \tan \emptyset}{1 - \tan \alpha \times \tan \emptyset}$$

$$\frac{d + x}{a} = \frac{{}^{d}/_{a} + \tan \emptyset}{1 - \ /_{\ a} \times \tan \emptyset} = \frac{d + (\ \times \ \tan \emptyset)}{a - (\ \times \ \tan \emptyset)}$$

$$d \cdot x \times (a - d \times \tan \emptyset) = a \times (d + a \times \tan \emptyset)$$

$$a \cdot d - d^2 \times \tan \emptyset + a \cdot x - d \times x \times \tan \emptyset = a \cdot d + \ ^2 \times \ \tan \emptyset$$

$$\tan \emptyset \times (a^2 + d^2 + d \cdot x) = a \cdot \ x$$

$$\tan \emptyset \ = \frac{a \ \cdot x}{(a^2 + \ d^2 + d \cdot x)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

- Similarly from $\tan(\alpha - \theta) = \frac{d-x}{a}$, we get

$$\tan \theta = \frac{a \cdot x}{(a^2 + d^2 - d \cdot x)} \ldots \ldots\ldots \ldots \ldots \ldots \ldots .\,(v)$$

- We know that for correct steering,

$$\cot\emptyset - \cot\theta = \frac{c}{b}$$

$$\frac{(a^2 + d^2 + d \cdot x)}{a \ \cdot x} - \frac{(a^2 + \ ^2 - d \ \cdot x)}{a \ \cdot x} = \frac{c}{b}$$

$$\frac{2\ d}{a} = \frac{c}{b}$$

$$2\ \tan \alpha = \frac{c}{b}$$

$$\tan \alpha = \frac{c}{2\, b}$$

Ackerman steering Gear

- The Ackerman steering gear mechanism is much simpler than Davis gear. The difference between the Ackerman and Davis steering gears are :
  1. The whole mechanism of the Ackerman steering gear is on back of the front wheels; whereas in Davis steering gear, it is in front of the wheels.
  2. The Ackerman steering gear consists of turning pairs, whereas Davis steering gear consists of sliding members.

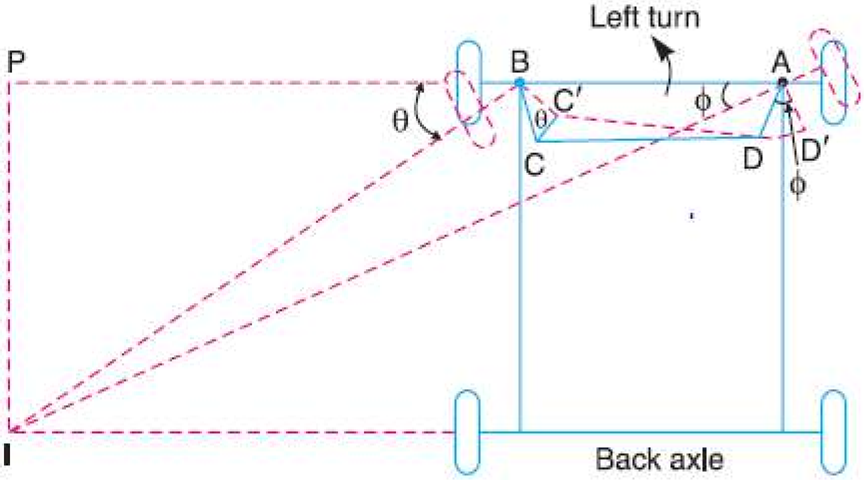


Fig. 2.12 Ackerman steering mechanism

- In Ackerman steering gear, the mechanism ABCD is a four bar crank chain, as shown in Fig. 2.12. The shorter links BC and A D are of equal length and are connected by hinge joints with front wheel axles. The longer links A B and CD are of unequal length.
- The following are the only three positions for correct steering.
  1. When the vehicle moves along a straight path, the longer links A B and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by firm lines in Fig. 2.12.
  2. When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. 2.12. In this position, the lines of the front wheel axle intersect on the back wheel axle at I, for correct steering.
  3. When the vehicle is steering to the right, the similar position may be obtained.

Universal or Hooke’s Joint

- A Hooke’s joint is used to connect two shafts, which are intersecting at a small angle, as shown in Fig.2.10. The end of each shaft is forked to U-type and each fork provides two bearings for the arms of a cross.
- The arms of the cross are perpendicular to each other. The motion is transmitted from the driving shaft to driven shaft through a cross.

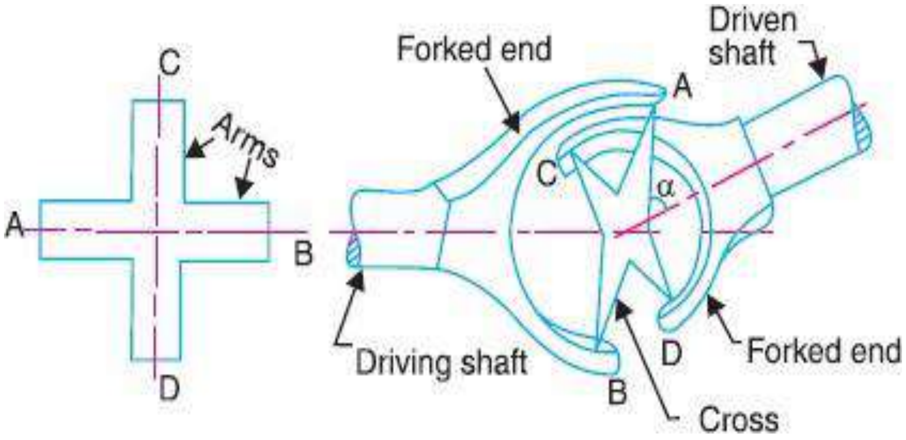


Fig. 2.13 Hooke's Joint

- The main application of the Universal or Hooke's joint is found in the transmission from the gear box to the differential or back axle of the automobiles. It is also used for transmission of power to different spindles of multiple drilling machines.

Ratio of shaft velocities

- The top and front views connecting the two shafts by a universal joint are shown in Fig. 2.11. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shafts.
- Let the driving shaft rotates through an angle θ, so that the arm AB moves in a circle to a new position A1 B1 as shown in front view.
- A little consideration will show that the arm CD will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse. Therefore the arm CD takes new position C1D1 on the ellipse, at an angle θ. But the true angle must be on the circular path.
- To find the true angle, project the point C1 horizontally to intersect the circle at C2. Therefore the angle COC2 (equal to φ) is the true angle turned by the driven shaft.

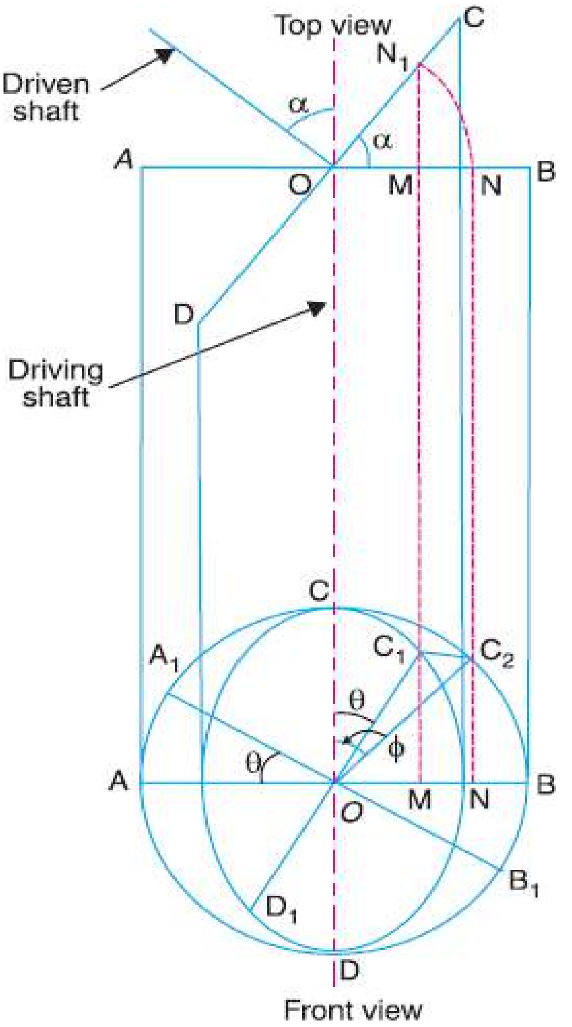


Fig. 2.14 ration of shaft velocities

- In triangle OC_1M, angle OC_1M = Θ

$$\tan\theta = \frac{OM}{MC_1} \dots\dots\dots\dots..(i)$$

- In triangle OC_2N, angle OC_2N= Ø

$$\tan\emptyset = \frac{ON}{NC_2} = \frac{ON}{MC_1}\dots\dots..(ii) \quad (NC_2 = MC_1)$$

- Dividing eq. (i) by (ii)

$$\tan\emptyset = \frac{ON}{NC_2} = \frac{ON}{MC_1}$$

- But

  $OM = N_1 \cos\alpha = ON\cos\alpha$ (α = angle of inclination of driving and driven shaft)

$$\frac{\tan\theta}{\tan\emptyset} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

$$\tan\theta = \tan\emptyset \times \cos\alpha \,................(iii)$$

- Let,

ω = angular velocity of driving shaft = $\frac{d\theta}{dt}$

ω_1 = angular velocity of driven shaft = $\frac{d\emptyset}{dt}$

- Differentiating both side of eq. (iii)

$$\sec^2\theta \times \frac{d\theta}{dt} = \cos\alpha \times \sec^2\emptyset \times \frac{\emptyset}{dt}$$

$$\sec^2\theta \times = \cos\alpha \times \sec^2\emptyset \times \omega_1$$

$$\frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha \times \sec^2\emptyset}$$

$$= \frac{1}{\cos^2\theta \times \cos\alpha \times \sec^2\emptyset} \dots\dots.(iv)$$

- We know that,

$$\sec^2\emptyset = 1 + \tan^2\emptyset = 1 + \frac{\tan^2\theta}{\cos^2\alpha}$$

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta \times \cos^2\alpha + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta \times (1 - \sin^2\alpha) + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{\cos^2\theta - \sin^2\alpha \times \cos^2\theta + \sin^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

$$= \frac{1 - \sin^2\alpha \times \cos^2\theta}{\cos^2\theta \times \cos^2\alpha}$$

- Substituting this value in eq. (iv)

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta \times \cos\alpha} \times \frac{\cos^2\theta \times \cos^2\alpha}{1 - \sin^2\alpha \times \cos^2\theta}$$

Maximum and Minimum speed of Driven Shaft

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \sin^2\alpha \times \cos^2\theta}$$

$$\omega_1 = \frac{\omega \times \cos\alpha}{1 - \sin^2\alpha \times \cos^2\theta} \dots\dots\dots\dots\dots\dots\dots\dots (i)$$

- The value of ω_1 will be minimum for a given value of α, if the denominator of eq. (I) is minimum.

$\cos^2\theta = 1$, i.e. Θ = 0°, 180°, 360° etc.

- Maximum speed of the driven shaft,

$$\omega_{1(\max)} = \frac{\omega\cos\alpha}{1 - \sin^2\alpha} = \frac{\omega \times \cos\alpha}{\cos^2\alpha} = \frac{\omega}{\cos\alpha}$$

$$N_{1(\max)} = \frac{N}{\cos}$$

- Similarly, the value of ω_1 is minimum , if the denominator of eq. (i) is maximum, this will happen, when $(\sin^2 \alpha \times \cos^2 \theta)$ is maximum, or

$$\cos^2 \theta = 0, \text{ i.e. } \Theta = 90^o, 270^o \text{ etc.}$$

Polar diagram – salient features of driven shaft speed

- For one complete revolution of the driven shaft, there are two points i.e. at 0° and 180° as shown by points 1 and 2 in Fig. Where the speed of the driven shaft is maximum and there are two points i.e. at 90° and 270° as shown by point 3 and 4 where the speed of the driven shaft is minimum.

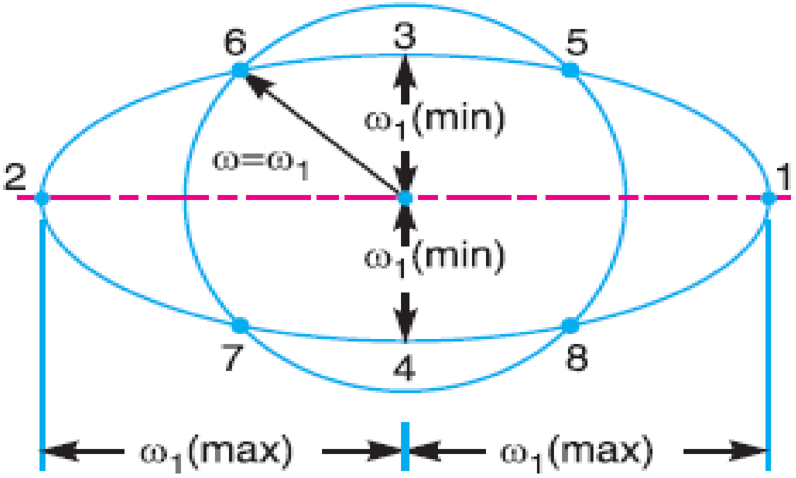


Fig. 2.15 polar diagram

- Since there are two maximum and two minimum speeds of the driven shaft, therefore there are four points when the speeds of the driven and driver shaft are same. This is shown by points, 5, 6, 7 and 8 in Fig.
- Since the angular velocity of the driving shaft is usually constant, therefore it is represented by a circle of radius ω. The driven shaft has a variation in angular velocity, the maximum value being ω/cos α and minimum value is ω cos α. Thus it is represented by an ellipse of semi-major axis ω/cos α and semi-minor axis ω cos α, as shown in Fig.2.15.

Double Hooke's Joint

- The velocity of the driven shaft is not constant, but varies from maximum to minimum values. In order to have a constant velocity ratio of the driving and driven shafts, an intermediate shaft with a Hooke's joint at each end as shown in Fig.2.16, is used. This type of joint is known as double Hooke's joint.

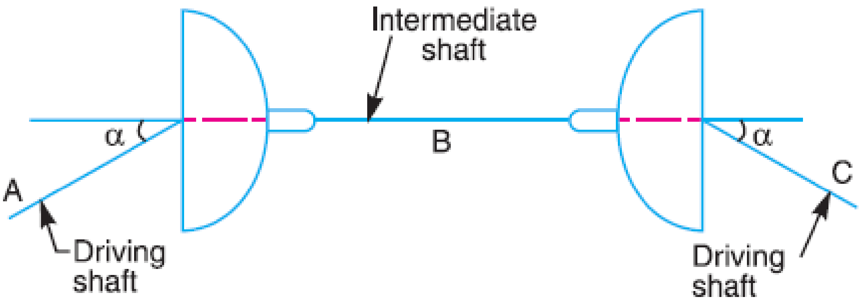


Fig. 2.16 double Hooke's joint

- For shaft A and B,

$$\tan\theta = \tan\emptyset \times \cos\alpha$$

- For shaft B and C,

$$\tan\gamma = \tan\emptyset \times \cos\alpha$$

- This shows that the speed of the driving and driven shaft is constant. In other words, this joint gives a velocity ratio equal to unity, if
  1. The axes of the driving and driven shafts are in the same plane, and
  2. The driving and driven shafts make equal angles with the intermediate shaft.

Examples:

1. In a Davis steering gear, the distance between the pivots of the front axle is 1.2 metres and the wheel base is 4.7 metres. Find the inclination of the track arm to the longitudinal axis of the car, when it is moving along a straight path .

- **Given:** c = 1.2 m ; b = 4.7 m
- Let, α = Inclination of the track arm to the longitudinal axis.
- We know that

$$\tan\alpha = \frac{C}{2\,b} = \frac{1.2}{2 \times 4.7} = 0.222$$

$$= 14.5°$$

2. Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg and 100 mm radius of gyration. Find the maximum angular acceleration of the driven shaft and the maximum torque required.

- **Given:** N = 1500 rpm ; m = 12 kg; k = 100 mm ; α = 20^{o}
- We know that angular speed of driving shaft,

$$\omega = 2\,\pi\,\frac{1500}{60} = 157\,\frac{rad}{s}$$

- The mass moment of inertia of the driven shaft,

$$I = m \times K^2 = 12 \times 0.1^2 = 0.12\ kg.m$$

Max. angular acceleration of driven shaft,

$$\cos 2\theta = \frac{\sin^2 \alpha \times 2}{2 - \sin^2 \alpha} = \frac{\sin^2 20 \times 2}{2 - \sin^2 20} = 0.124$$

$$= 41.45°$$

$$\frac{d\omega_1}{dt} = \frac{\omega^2 \times \cos \alpha \times \sin 2\theta \times \sin^2 \alpha}{(1 - \sin^2 \alpha \times \cos^2 \theta)^2}$$

$$= \frac{157^2 \times \cos 20 \times \sin 84.9 \times \sin^2 20}{(1 - \sin^2 20 \times \cos^2 44.45)^2} = 3090\ \frac{rad}{s^2}$$

- Max torque req.

$$= I \times \frac{d\omega_1}{} = 0.12 \times 3090 = 371\ N.m$$

LECTURE 1

STRAIGHT LINE MOTION MECHANISMS

DEPARTMENT OF MECHANICAL ENGINEERING

STRAIGHT LINE MOTION MECHANISMS

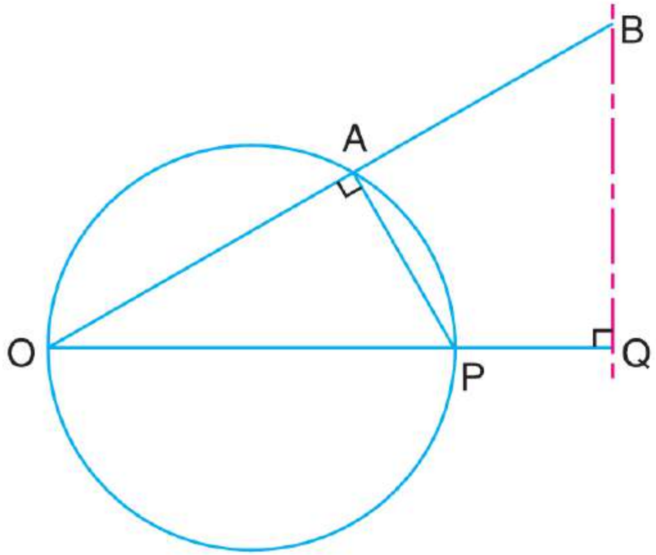
- One of the most common forms of the constraint mechanisms is that it permits only relative motion of an oscillatory nature along a straight line.
- The mechanisms used for this purpose are called straight line mechanisms. These mechanisms are of the following two types:
    - in which only turning pairs are used, an
    - in which one sliding pair is used.

These two types of mechanisms may produce **exact straight line motion or approximate straight line motion**, as discussed in the following articles.

EXACT STRAIGHT LINE MOTION MECHANISMS MADE UP OF TURNING PAIRS

- Let O be a point on the circumference of a circle of diameter OP.
- Let OA be any chord and B is a point on OA produced, such that,

$$OA \times OB = \text{constant}$$



Exact straight line motion mechanism

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

$$OP \times OQ = OA \times OB$$

$$OQ = \frac{OA \times OB}{OP}$$

But OP is constant as it is the diameter of a circle, therefore, if OA×OB is constant, then OQ will be constant.
Hence the point B moves along the straight path BQ which is perpendicular to OP.

PEAUCELLIER MECHANISM

- It consists of a fixed link OO1 and the other straight links O1A , OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in Fig.
- The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O1A .

$$AC = CB = BD = DA \; ; \; OC = OD \; ; \text{ and } OO_1 = O_1A$$

$$OC^2 = OR^2 + RC^2 \qquad \ldots(i)$$

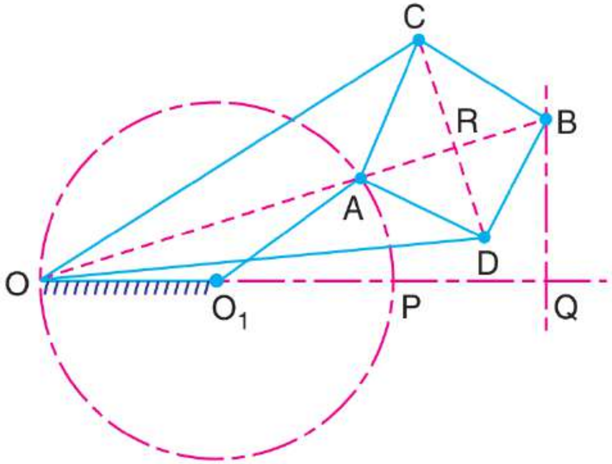
$$BC^2 = RB^2 + RC^2 \qquad \ldots(ii)$$

Subtracting equation (*ii*) from (*i*), we have

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR + RB)\,(OR - RB)$$

$$= OB \times OA$$



HART’S MECHANISM

- This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.
- It consists of a fixed link OO1 and other straight links O1A , FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig.
- The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio.
- A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to *FD and CE.
- Hence OAB is a straight line. It may be proved now that the product OA× OB is constant.

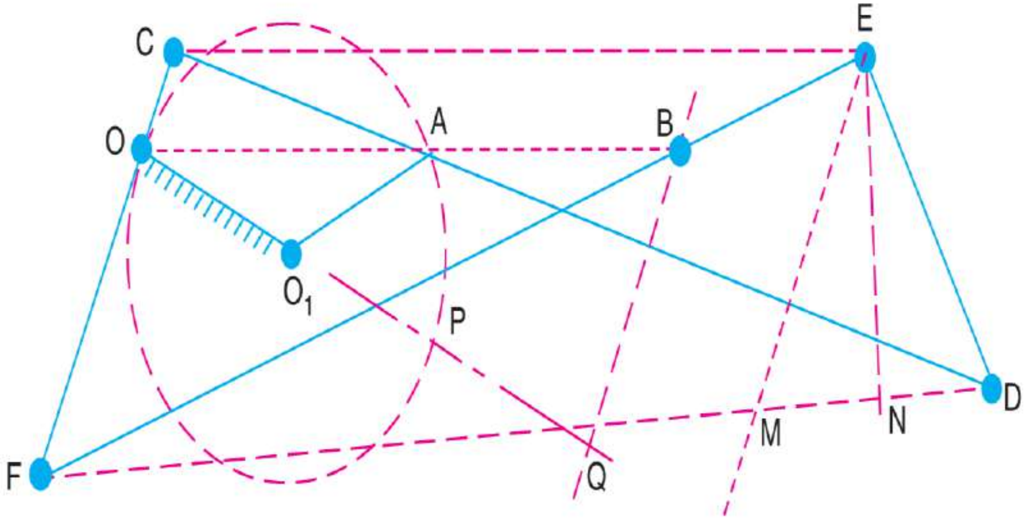
HART'S MECHANISM

From similar triangles CFE and OFB,

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or} \quad OB = \frac{CE \times OF}{FC}$$

and from similar triangles FCD and OCA

$$\frac{FD}{FC} = \frac{OA}{OC} \quad \text{or} \quad OA = \frac{FD \times OC}{FC}$$



It therefore follows that if the mechanism is pivoted about O as a fixed point and the point A is constrained to move on a circle with centre O1, then the point B will trace a straight line perpendicular to the diameter OP produced.

HART’S MECHANISM

Multiplying equations (*i*) and (*ii*), we have

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC} = FD \times CE \times \frac{OC \times OF}{FC^2}$$

Since the lengths of *OC*, *OF* and *FC* are fixed, therefore

$$OA \times OB = FD \times CE \times \text{constant} \qquad \ldots(iii)$$

$$\ldots\left(\text{substituting } \frac{OC \times OF}{FC^2} = \text{constant}\right)$$

Now from point *E*, draw *EM* parallel to *CF* and *EN* perpendicular to *FD*. Therefore

$$FD \times CE = FD \times FM \qquad \ldots(\because CE = FM)$$

$$= (FN + ND)(FN - MN) = FN^2 - ND^2 \qquad \ldots(\because MN = ND)$$

$$= (FE^2 - NE^2) - (ED^2 - NE^2)$$

...(From right angled triangles *FEN* and *EDN*)

$$= FE^2 - ED^2 = \text{constant} \qquad \ldots(iv)$$

...($\because$ Length *FE* and *ED* are fixed)

From equations (*iii*) and (*iv*),

$$OA \times OB = \text{constant}$$

LECTURE 2

APPROXIMATE STRAIGHT LINE MECHANISM

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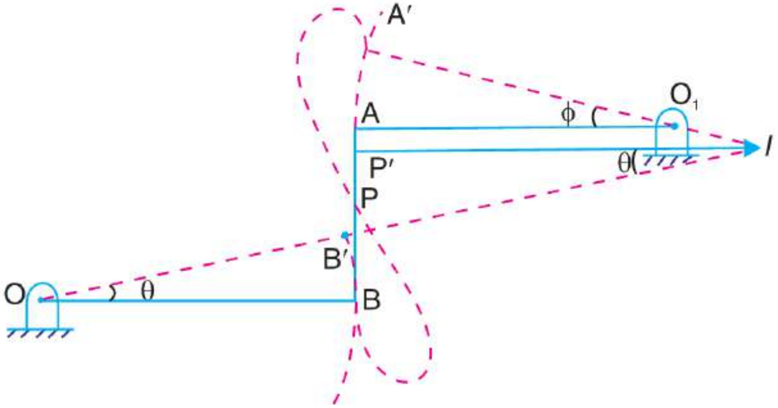
APPROXIMATE STRAIGHT LINE MOTION MECHANISMS

- The approximate straight line motion mechanisms are the modifications of the four-bar chain mechanisms. Following mechanisms to give approximate straight line motion, are important from the subject point of view:
- **Watt's mechanism:** It is a crossed four bar chain mechanism and was used by Watt for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion.



$$\text{arc } B\,B' = \text{arc } A\ A' \qquad \text{or} \qquad OB \times \theta = O_1A \times \phi$$

$$\therefore \qquad OB / O_1A = \phi / \theta$$

$$\text{Also} \qquad A'P' = IP' \times \phi, \text{ and } B'P' = IP' \times \theta$$

$$\therefore \qquad A'P' / B'P' = \phi / \theta$$

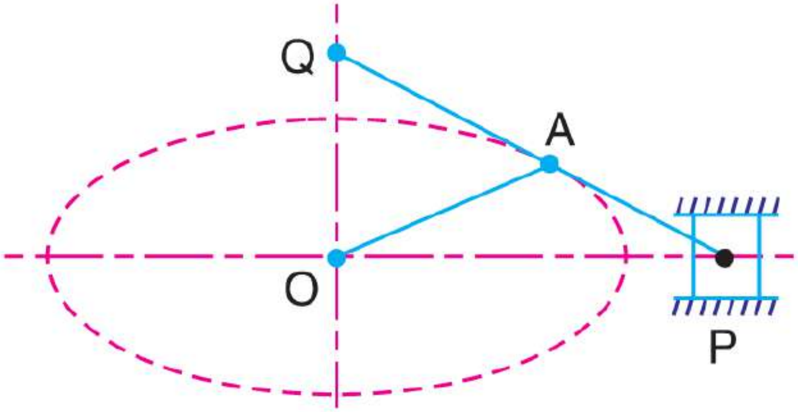
From equations (*i*) and (*ii*),

$$\frac{OB}{O_1A} = \frac{A'P'}{B'P'} = \frac{AP}{BP} \qquad \text{or} \qquad \frac{O_1A}{OB} = \frac{PB}{PA}$$

MODIFIED SCOTT-RUSSEL MECHANISM

- This mechanism is similar to Scott-Russel mechanism but in this case AP is not equal to AQ and the points P and Q are constrained to **move in the horizontal and vertical directions.**
- A little consideration will show that it forms an elliptical trammel, so that any point A on PQ traces an ellipse with semi-major axis AQ and semi-minor axis AP.

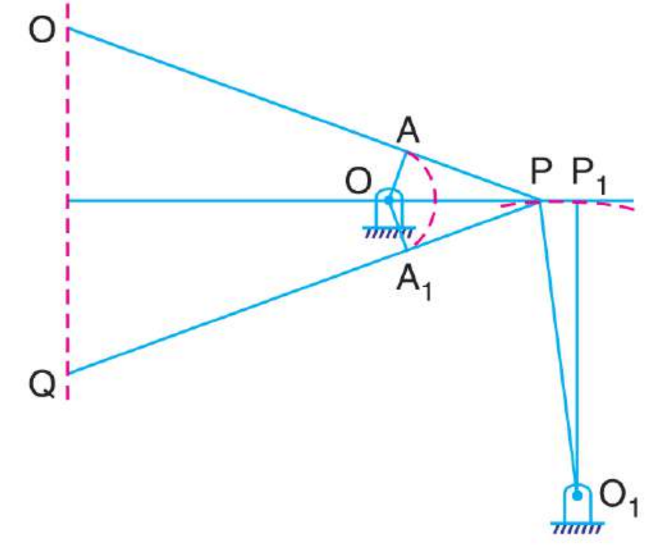
If the point A moves in a circle, then for point Q to move along an approximate straight line, the length OA must be equal (AP)2/AQ. This is limited to only small displacement of P.



Modified Scott-Russel mechanism

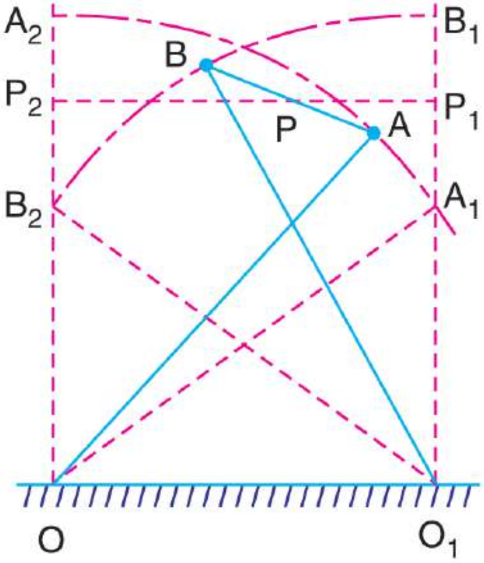
GRASSHOPPER MECHANISM

- This mechanism is a **modification of modified Scott-Russel’s mechanism** with the difference that the point P does not slide along a straight line, but moves in a circular arc with centre O.
- It is a four bar mechanism and **all the pairs are turning pairs** as shown in Fig. In this mechanism, the centres O and O1 are fixed. The link OA oscillates about O through an angle AOA1 which causes the pin P to move along a circular arc with O1 as centre and O1P as radius. OA= (AP)2/ AO.



TCHEBICHEFF’S MECHANISM

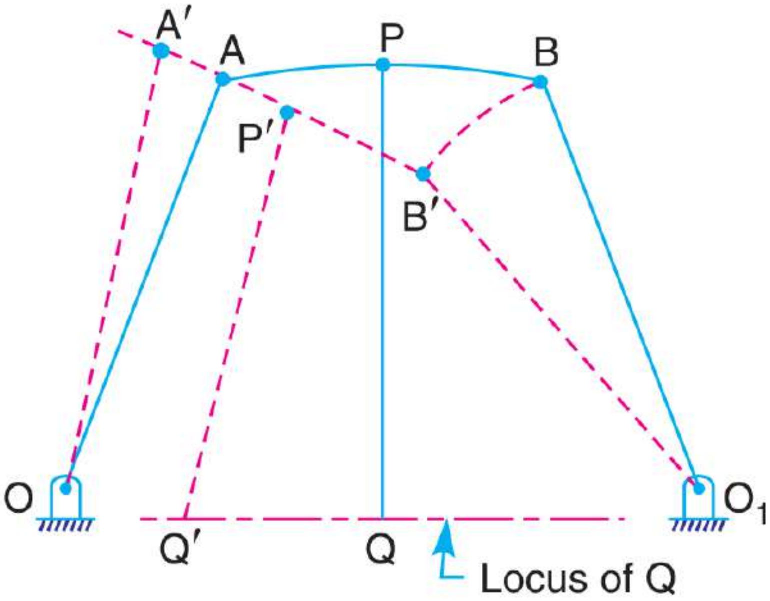
- It is a four bar mechanism in which the **crossed links OA and O1B are of equal length,** as shown in Fig. The point P, which is the mid-point of AB traces out an approximately straight line parallel to OO1.
- The proportions of the links are, usually, such that point P is exactly above O or O1 **in the extreme positions of the mechanism** i.e. when BA lies along OA or when BA lies along BO1.



It may be noted that the point P will lie on a straight line parallel to OO1, in the two extreme positions and in the mid position, if the **lengths of the links are in proportions AB: OO1: OA= 1 : 2 : 2.5.**

ROBERTS MECHANISM

- **It is also a four bar chain mechanism,** which, in its mean position, has the form of a trapezium.
- The links OA and O1 Bare of equal length and OO1 is fixed. A bar PQ is rigidly attached to the link AB at its middle point P.



LECTURE 3

PANTOGRAPH

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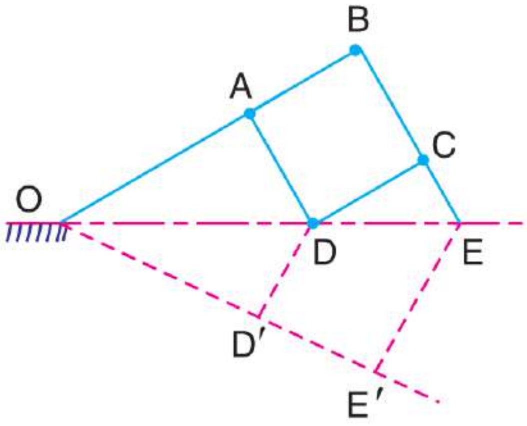
PANTOGRAPH

- A pantograph is an **instrument used to reproduce to an enlarged or a reduced scale** and as exactly as possible the path described by a given point.
- It consists of a jointed parallelogram ABCD as shown in Fig. It is made up of bars **connected by turning pairs.** The bars BA and BC are extended to O and E respectively, such that $OA/OB = AD/BE$
  - Thus, for all relative positions of the bars, the triangles OAD and OBE are similar and the points O, D and E are in one straight line.
  - It may be proved that point E traces out the same path as described by point D.

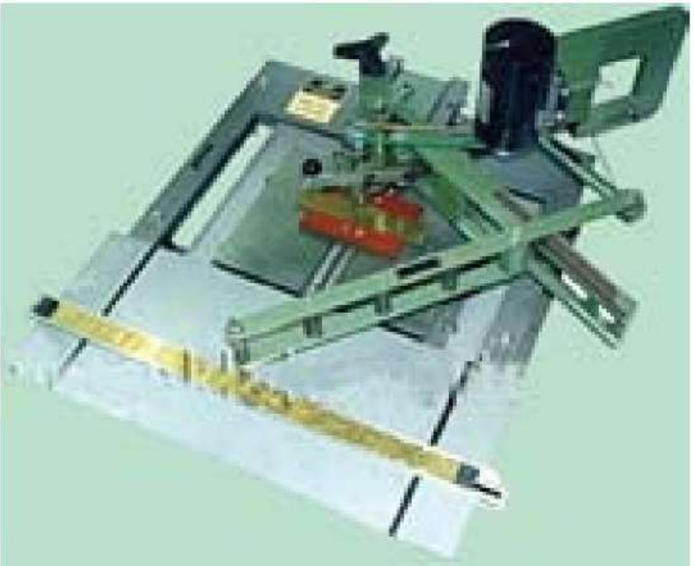


PANTOGRAPH

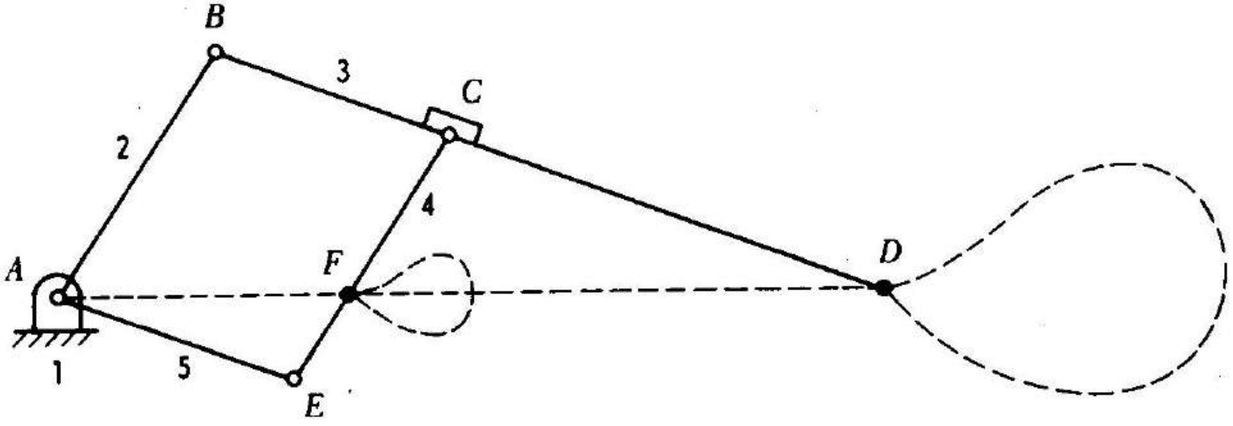
- From similar triangles OAD and OBE, we find that, $OD/OE = AD/BE$

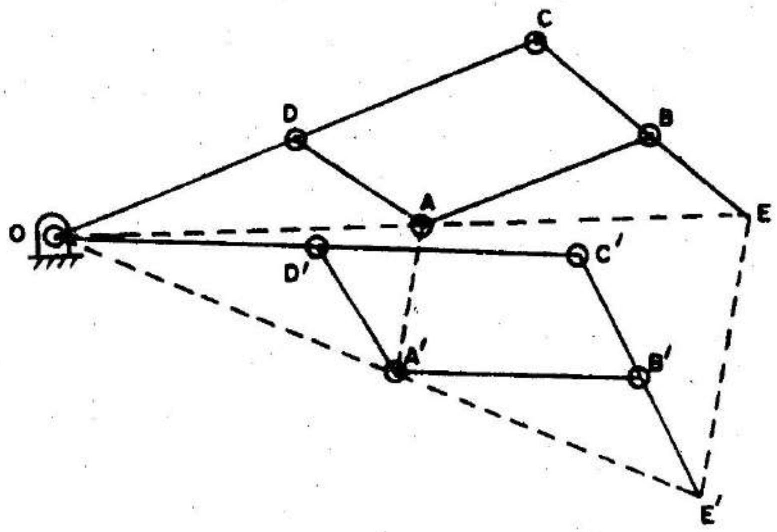
Let point O be fixed and the points D and E move to some new positions D′ and E′. Then $OD/OE = OD'/OE'$

- A pantograph is mostly used for the reproduction of **plane areas and figures such as maps, plans etc., on enlarged or reduced scales.**
- It is, sometimes, used as an indicator rig in order to reproduce to a **small scale the displacement of the crosshead and therefore of the piston of a reciprocating steam engine.** It is also used to guide cutting tools.
- A modified form of pantograph is used to collect power at the top of an electric locomotive.

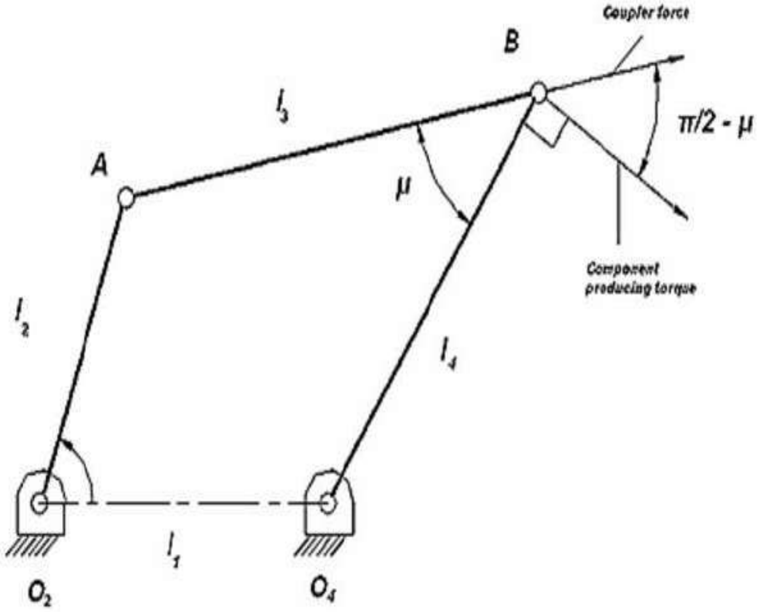


PANTOGRAPH





TRANSMISSION ANGLE



For a 4 R linkage, the transmission angle (μ) is defined as the acute angle between the coupler (AB) and the follower

For a given force in the coupler link, the torque transmitted to the output bar (about point O_4) is maximum when the angle μ between coupler bar AB and output bar BO_4 is π/2. Therefore, angle ABO_4 is called **transmission angle.**

LECTURE 4

DAVI'S STEERING GEAR MECHANISM

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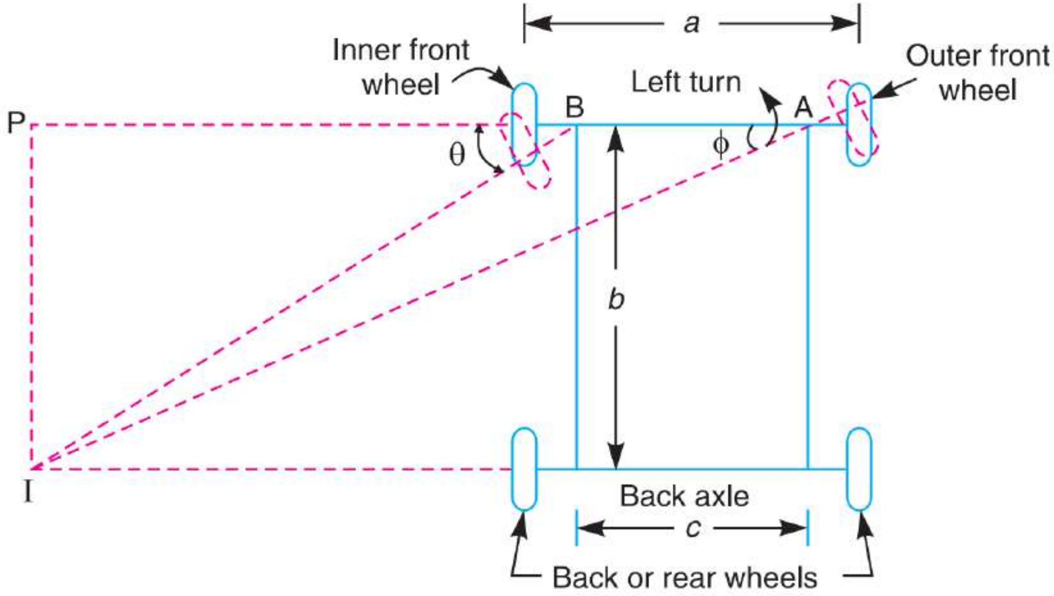
STEERING GEAR MECHANISM

- The steering gear mechanism is used for changing the direction of two or more of the **wheel axles with reference to the chassis, so as to move the automobile in any desired path.**
- Usually the **two back wheels have a common axis, which is fixed in direction with reference to the chassis** and the steering is done by means of the front wheels.



STEERING GEAR MECHANISM

Thus, the condition for correct steering is that all the four wheels **must turn about the same instantaneous centre.** The axis of the inner wheel makes a larger turning angle θ than the angle φ subtended by the axis of outer wheel.

Let a = Wheel track,

b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

Now from triangle IBP,

$$\cot\theta = \frac{BP}{IP}$$

and from triangle IAP,

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta \qquad \ldots(\because IP = b)$$

$$\therefore \cot\phi - \cot\theta = c / b$$

This is the **fundamental equation for correct steering.** If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

DAVIS STEERING GEAR

- It is an **exact steering gear mechanism**. The slotted links AM and BH are attached to the front wheel axle, which turn on pivots A and B respectively.
- The rod CD is constrained to move in the direction of its length, by the sliding members at P and Q. These **constraints are connected to the slotted link** AM and BH by a sliding and a turning pair at each end.

a = Vertical distance between AB and CD,

b = Wheel base,

d = Horizontal distance between AC and BD,

c = Distance between the pivots A and B of the front axle.

x = Distance moved by AC to $AC' = CC' = DD'$, and

α = Angle of inclination of the links AC and BD, to the vertical.

DAVIS STEERING GEAR

From triangle $A\,A'\,C'$,

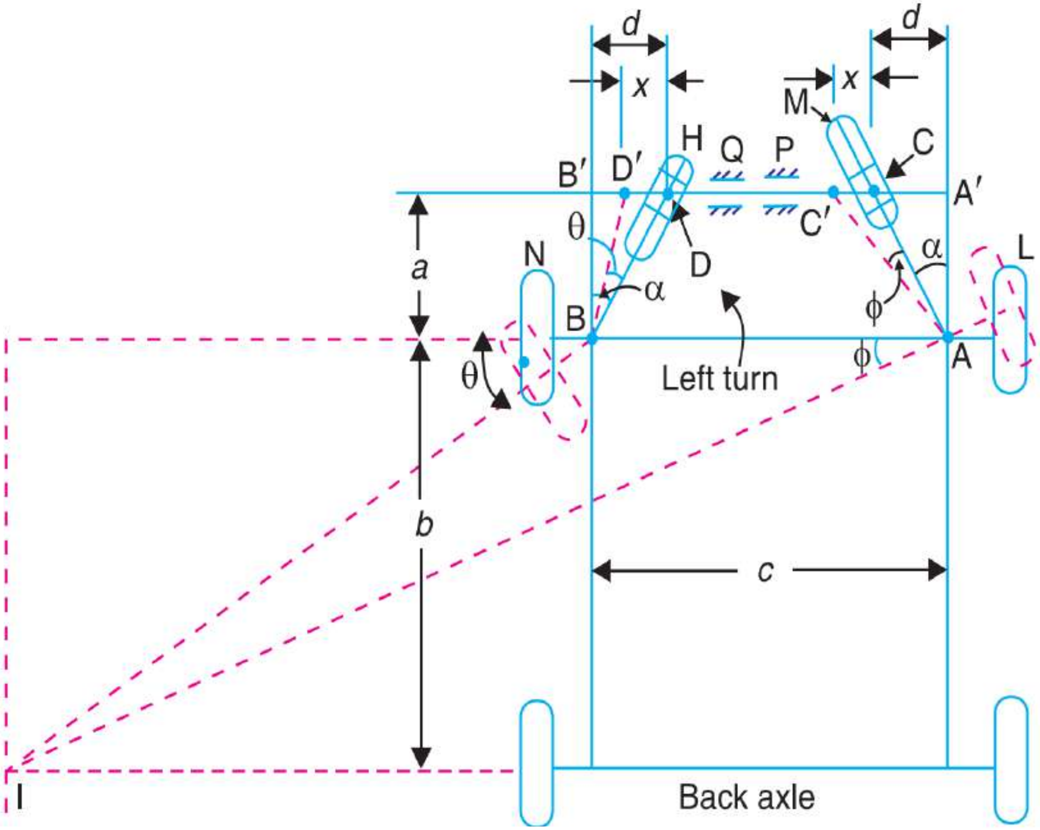
$$\tan(\alpha+\phi)=\frac{A'C'}{A\,A'}=\frac{d+x}{a}$$

From triangle $A\,A'C$,

$$\tan\alpha=\frac{A'C}{A\,A'}=\frac{d}{a}$$

From triangle $B\,B'D'$,

$$\tan(\alpha-\theta)=\frac{B'D'}{BB'}=\frac{d-x}{a}$$



We know that $\tan(\alpha+\phi)=\dfrac{\tan\alpha+\tan\phi}{1-\tan\alpha.\tan\phi}$

or

$$\frac{d+x}{a}=\frac{d/a+\tan\phi}{1-d/a\times\tan\phi}=\frac{d+a\tan\phi}{a-d\tan\phi}$$

...[From equations (*i*) and (*ii*)]

DAVIS STEERING GEAR

$$(d + x)\,(a - d \tan \phi) = a\,(d + a \tan \phi)$$

$$a.\,d - d^2 \tan \phi + a.\,x - d.x \tan \phi = a.d + a^2 \tan \phi$$

$$\tan \phi\,(a^2 + d^2 + d.x) = ax \quad \text{or} \quad \tan \phi = \frac{a.x}{a^2 + d^2 + d.x} \qquad \text{...}(iv)$$

Similarly, from $\tan\,(\alpha - \theta) = \dfrac{d - x}{a}$, we get

$$\tan \theta = \frac{ax}{a^2 + d^2 - d.x} \qquad \text{...}(v)$$

We know that for correct steering,

$$\cot \phi - \cot \theta = \frac{c}{b} \quad \text{or} \quad \frac{1}{\tan \phi} - \frac{1}{\tan \theta} = \frac{c}{b}$$

$$\frac{a^2 + d^2 + d.x}{a.x} - \frac{a^2 + d^2 - d.x}{a.x} = \frac{c}{b} \qquad \text{...[From equations } (iv) \text{ and } (v)\text{]}$$

$$\frac{2d.x}{a.x} = \frac{c}{b} \quad \text{or} \quad \frac{2d}{a} = \frac{c}{b}$$

$$2 \tan \alpha = \frac{c}{b} \quad \text{or} \quad \tan \alpha = \frac{c}{2b} \qquad \text{...}(\because\ d / a = \tan \alpha)$$

LECTURE 5

ACKERMAN'S STEERING GEAR MECHANISM

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ACKERMAN’S STEERING GEAR MECHANISM

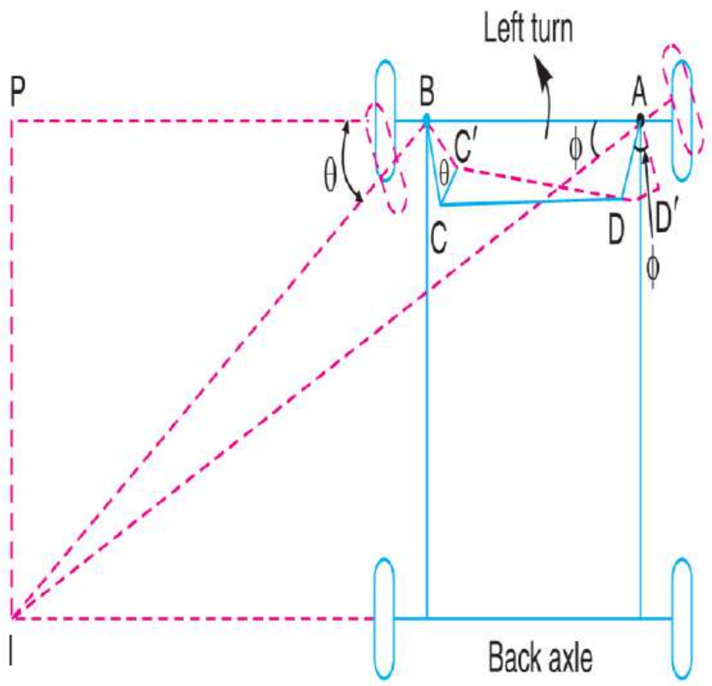
- The Ackerman steering gear **mechanism is much simpler than Davis gear.** The difference between the Ackerman and Davis steering gears are:
- The whole mechanism of the **Ackerman steering gear is on back of the front wheels; whereas in Davis steering gear, it is in front of the wheels.**
- The Ackerman **steering gear consists of turning pairs,** whereas Davis steering gear consists of sliding members.
- The **shorter links BC and AD are of equal length** and are connected by hinge joints with front wheel axles.
- The **longer links AB and CD are of unequal length.**

ACKERMAN'S STEERING GEAR MECHANISM

1. When the vehicle moves along a straight path, the **longer links AB and CD are parallel and the shorter links BC and AD are equally inclined** to the longitudinal axis of the vehicle, as shown by firm lines in Fig.

2. When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. In this position, the lines of the **front wheel axle intersect on the back wheel axle at I, for correct steering.**

3. When the vehicle is **steering to the right, the similar position may be obtained.**



LECTURE 6

SINGLE AND DOUBLE HOOKE JOINT

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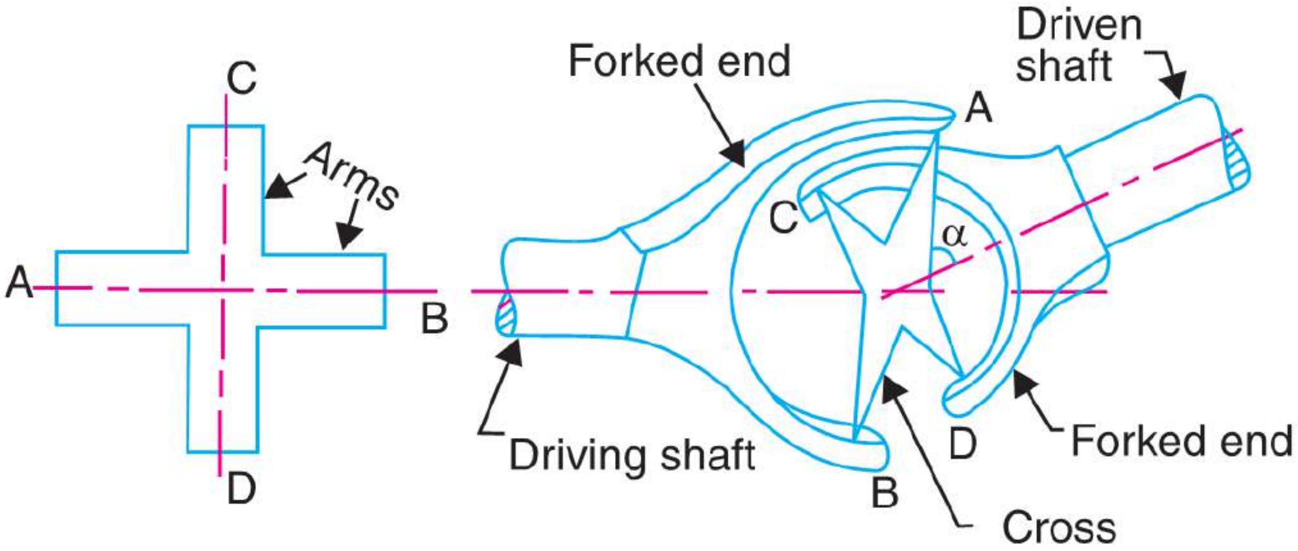
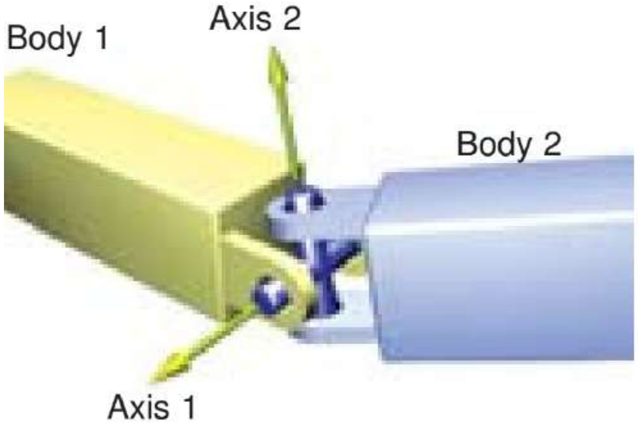
UNIVERSAL OR HOOKE’S JOINT

- A Hooke’s joint is **used to connect two shafts**, which are intersecting at a small angle, as shown in Fig.
- The end of each shaft is **forked to U-type and each fork provides two bearings for the arms of a cross**. The arms of the cross are perpendicular to each other.
- The motion is transmitted from the driving shaft to driven shaft through a cross. **The inclination of the two shafts may be constant**, but in actual practice it varies, when the motion is transmitted.
- The main application of the **Universal or Hooke’s joint is found in the transmission from the gear box to the differential or back axle** of the automobiles.
- It is also used for transmission of power to **different spindles of multiple drilling machine**. It is also used as a knee joint in milling machines.

UNIVERSAL OR HOOKE'S JOINT





UNIVERSAL OR HOOKE’S JOINT

- The arms of the cross are perpendicular to each other. The motion is **transmitted from the driving shaft to driven shaft** through a cross. The **inclination of the two shafts may be constant,** but in actual practice it varies, when the motion is transmitted.
- The main application of the Universal or Hooke’s joint is found in the **transmission from the gear box to the differential or back axle of the automobiles.**
- It is also used for **transmission of power to different spindles of multiple drilling machine.** It is also used as a knee joint in milling machines.

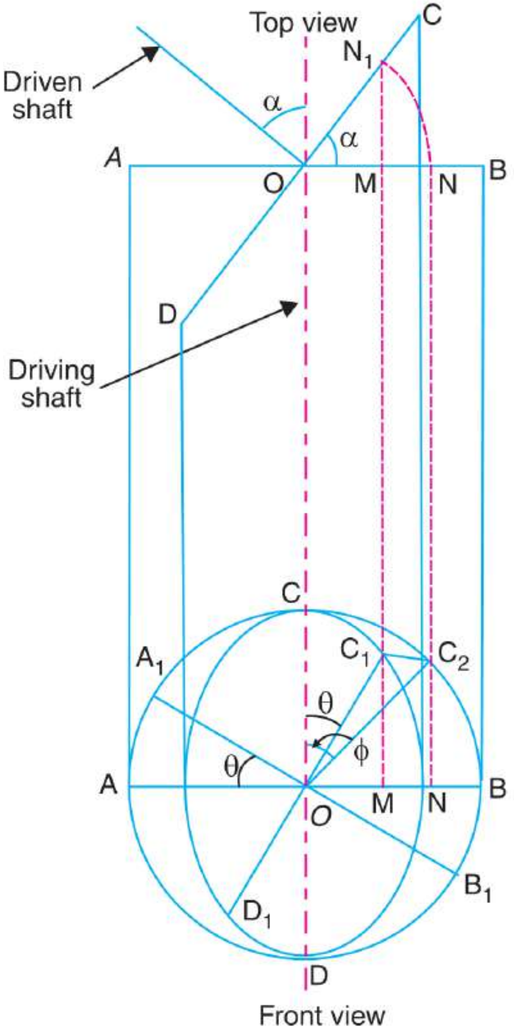
LECTURE 7

RATIO OF SHAFT VELOCITIES

DEPARTMENT OF MECHANICAL ENGINEERING

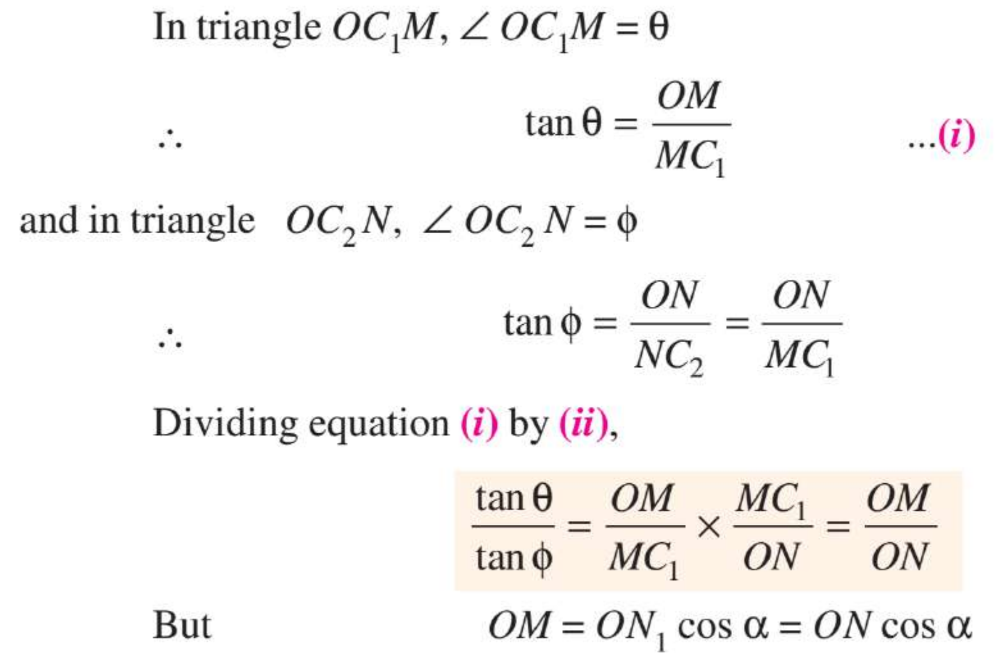
RATIO OF SHAFT VELOCITIES

- The top and front views connecting the two shafts by a universal joint are shown in Fig. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shafts.
- Let the driving shaft rotates through an angle θ, so that the arm AB moves in a circle to a new position A1B1 as shown in front view.
- A little consideration will show that the arm CD will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse.



RATIO OF SHAFT VELOCITIES

- Therefore the arm CD takes new position C1D1 on the ellipse, at an angle θ. But the true angle must be on the circular path.
- To find the true angle, project the point C1 horizontally to intersect the circle at C2.
- Thus when the driving shaft turns through an angle θ, the driven shaft turns through an angle φ.



In triangle OC_1M, $\angle OC_1M = \theta$

$$\therefore \qquad \tan\theta = \frac{OM}{MC_1} \qquad \ldots(i)$$

and in triangle OC_2N, $\angle OC_2N = \phi$

$$\therefore \qquad \tan\phi = \frac{ON}{NC_2} = \frac{ON}{MC_1}$$

Dividing equation (*i*) by (*ii*),

$$\boxed{\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \times \frac{MC_1}{ON} = \frac{OM}{ON}}$$

$$\text{But} \qquad OM = ON_1 \cos\alpha = ON \cos\alpha$$

RATIO OF SHAFT VELOCITIES

$$\therefore \quad \frac{\tan\theta}{\tan\phi} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

$$\tan\theta = \tan\phi \, . \cos\alpha$$

Let $\omega =$ Angular velocity of the driving shaft $= d\theta / dt$

$\omega_1 =$ Angular velocity of the driven shaft $= d\phi / dt$

Differentiating both sides of equation (*iii*),

$$\sec^2\theta \times d\theta / dt = \cos\alpha \, . \sec^2\phi \times d\phi / dt$$

$$\sec^2\theta \times \omega = \cos\alpha \, . \sec^2\phi \times \omega_1$$

$$\therefore \quad \frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha . \sec^2\phi} = \frac{1}{\cos^2\theta . \cos\alpha . \sec^2\phi}$$

RATIO OF SHAFT VELOCITIES

We know that $\sec^2 \phi = 1 + \tan^2 \phi = 1 + \dfrac{\tan^2 \theta}{\cos^2 \alpha}$ ...[From equation (*iii*)]

$$= 1 + \frac{\sin^2 \theta}{\cos^2 \theta.\cos^2 \alpha} = \frac{\cos^2 \theta.\cos^2 \alpha + \sin^2 \theta}{\cos^2 \theta.\cos^2 \alpha}$$

$$= \frac{\cos^2 \theta (1 - \sin^2 \alpha) + \sin^2 \theta}{\cos^2 \theta.\cos^2 \alpha} = \frac{\cos^2 \theta - \cos^2 \theta.\sin^2 \alpha + \sin^2 \theta}{\cos^2 \theta.\cos^2 \alpha}$$

$$= \frac{1 - \cos^2 \theta.\sin^2 \alpha}{\cos^2 \theta.\cos^2 \alpha} \qquad \ldots(\because \cos^2 \theta + \sin^2 \theta = 1)$$

Substituting this value of $\sec^2 \phi$ in equation (*iv*), we have veloity ratio,

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2 \theta.\cos \alpha} \times \frac{\cos^2 \theta.\cos^2 \alpha}{1 - \cos^2 \theta.\sin^2 \alpha} = \frac{\cos \alpha}{1 - \cos^2 \theta.\sin^2 \alpha} \qquad \ldots(v)$$

e: If N = Speed of the driving shaft in r.p.m., and

N_1 = Speed of the driven shaft in r.p.m.

Then the equation (*v*) may also be written as

$$\frac{N_1}{N} = \frac{\cos \alpha}{1 - \cos^2 \theta.\sin^2 \alpha}.$$

Industry applications

1. An Evans’ linkage has an oscillating drive arm that should have a maximum operating angle of about 40. For a relatively short guideway, the reciprocating output stroke is large. Output motion is on a true straight line in true harmonic motion. If an exact straight-line motion is not required, a link can replace the slide. The longer this link, the closer the output motion approaches that of a true straight line. If the link-length equals the output stroke, deviation from straight-line motion is only 0.03% of the output stroke.
2. A simplified Watt’s linkage generates an approximate straight-line motion. If the two arms are of equal length, the tracing point describes a symmetrical figure 8 with an almost straight line throughout the stroke length. The straightest and longest stroke occurs when the connecting-link length is about two thirds of the stroke, and arm length is 1.5 times the stroke length.
3. Four-bar linkage produces an approximately straight-line motion. This arrangement provides motion for the stylus on self-registering measuring instruments. A comparatively small drive displacement results in a long, almost-straight line.
4. A D-drive is the result when linkage arms are arranged as shown here. The output link point describes a path that resembles the letter D, so there is a straight part of its cycle. This motion is ideal for quick engagement and disengagement before and after a straight driving stroke.

Question Bank for Assignments

1. Draw a neat sketch and explain Peaucellier's exact straight line mechanism.
2. Explain Hart's straight line mechanism in detail.
3. Draw a neat sketch and explain any three approximate straight line generating mechanisms.
4. With a neat sketch, explain the Ackermann steering gear of an automobile.
5. With a neat sketch, explain the Davis steering gear mechanism in detail.
6. Two shafts are connected by universal Hooke's joint. The driving shaft rotates at uniform speed of 1200 rpm. Determine the greatest permissible angle between the shaft axis so that the total fluctuation of speed does not exceed 100 rpm also calculate the maximum and minimum speeds of driven shaft.
7. Derive an expression for the ratio of shafts velocities for Hooke's joint and draw the polar diagram.

Tutorial Questions

1. Sketch a pantograph, explain its working and show that it can be used to reproduce to an enlarged scale a given figure.

2. A circle has OR as its diameter and a point Q lies on its circumference. Another point P lies on the line OQ produced. If OQ turns about O as centre and the product OQ× OP remains constant, show that the point P moves along a straight line perpendicular to the diameter OR.

3. What are straight line mechanisms? Describe one type of exact straight line motion mechanism with the help of a sketch.

4. Describe the Watt’s parallel mechanism for straight line motion and derive the condition under which the straight line is traced.

5. Sketch an intermittent motion mechanism and explain its practical applications.

6. Give a neat sketch of the straight line motion ‘Hart mechanism’ Prove that it produces an exact straight line motion.

10. What is the condition for correct steering? Sketch and show the two main types of steering gears and discuss their relative advantages.

11. Explain why two Hooke’s joints are used to transmit motion from the engine to the differential of an automobile.

UNIT 3

VELOCITY AND ACCELERATION

COURSE OBJECTIVE

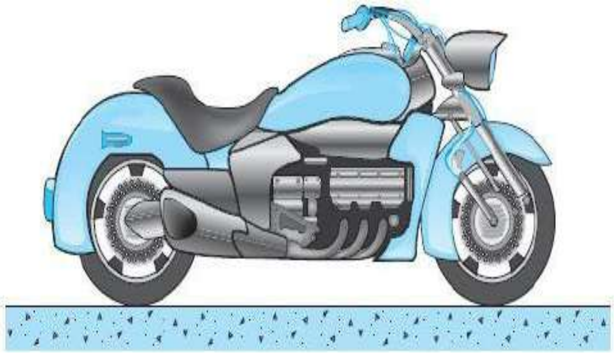
To impart skills to analyze the position, velocity and acceleration of mechanisms and synthesis of mechanism by analytical and graphical method.

COURSE OUTCOME

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	**Motion of link in machine**	Introduction to motion of link Methods to determine the motion of the link	Analyse the motion of the link (B4) Remember the types of motion (B1)
2	Velocity and acceleration diagrams	Relative velocity method Instantaneous Center Method	Remember the expressions for velocity and accelerations (B1) Calculate the Instantaneous centres (B4)
3	Graphical method	Velocity and acceleration on a Link by Relative Velocity Method Rubbing Velocity at a Pin Joint	Understanding different types of graphical method for velocity and acceleration calculation (B2) Apply graphical method for various methods (B3)
4	Relative velocity method four bar chain	Numerical examples to estimate the velocity and acceleration using relative velocity method	Apply relative velocity method to estimate the velocity and acceleration for four bar mechanisms (B3)
5	Instantaneous centre of rotation	Definition of instantaneous centre of rotation Types of instantaneous centre of rotation	Understanding the Instantaneous axis (B2) Compare the two components of acceleration (B1)
6	Three centers in line theorem	Aronhold Kennedy Theorem	Understanding the Three centers in line theorem (B2) Locate the instantaneous centres by Aronhold Kennedy’s theorem (B5)
7	Graphical determination of instantaneous center	Number of Instantaneous Centres in a Mechanism Numerical Examples using instantaneous centre of rotation	Evaluate instantaneous centers of the slider crank mechanism (B5) Apply graphical method for Instantaneous Centres (B3)

3

Velocity and Acceleration Analysis



Course Contents

3.1 Introduction

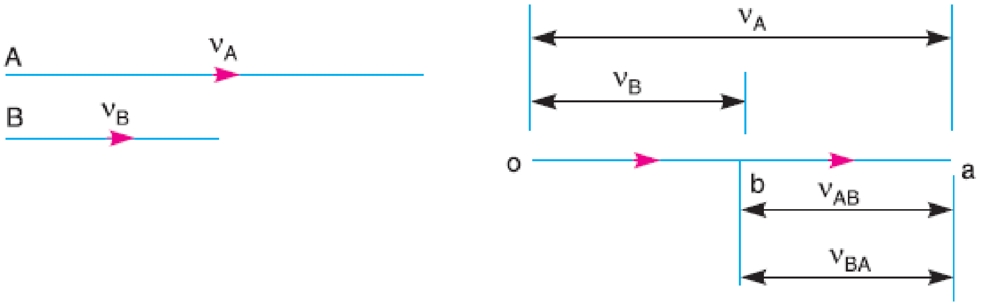
- There are many methods for determining the velocity of any point on a link in a mechanism whose direction of motion (i.e. path) and velocity of some other point on the same link is known in magnitude and direction, yet the following two methods :
  1. Instantaneous centre method
  2. Relative velocity method
- The instantaneous centre method is convenient and easy to apply in simple mechanisms, whereas the relative velocity method may be used to any configuration diagram.

3.2 Velocity of Two Bodies Moving In Straight Lines

- Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig. 3.1 (a) and 3.2 (a) respectively.
- Consider two bodies A and B moving along parallel lines in the same direction with absolute velocities v_A and v_B such that $v_A > v_B$, as shown in Fig. 3.1 (a). The relative velocity of A with respect to B,

$$\boldsymbol{v_{AB} = vector\ difference\ of\ v_A\ and\ v_B = \vec{v}_A - \vec{v}_B}$$

- From Fig. 3.1 (b), the relative velocity of A with respect to B (i.e. v_{AB}) may be written in the vector form as follows :



(a) (b)

Fig. 3.1 relative velocity of two bodies moving along parallel line

- Similarly, the relative velocity of B with respect to A,

$$\boldsymbol{v_{AB} = vector\ difference\ of\ v_A\ and\ v_B}$$

- Now consider the body B moving in an inclined direction as shown in Fig. 3.2 (a). The relative velocity of A with respect to B may be obtained by the law of parallelogram of velocities or triangle law of velocities. Take any fixed point o and draw vector oa to represent v_A in magnitude and direction to some suitable scale. Similarly, draw vector ob to represent v_B in magnitude and direction to the same scale. Then vector ba represents the relative velocity of A with respect to B as shown in Fig. 3.2 (b). In the similar way as discussed above, the relative velocity of A with respect to B,

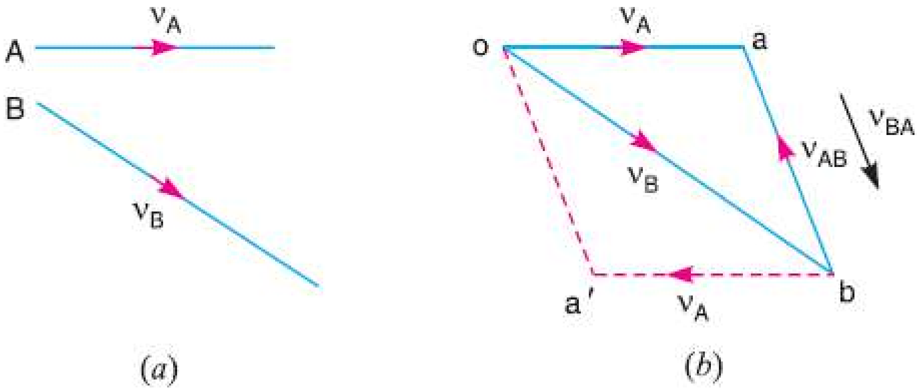


Fig. 3.2 relative velocity of two bodies moving along inclined line

$$v_{AB} = vector\ differece\ of\ v_A\ and\ v_B$$

- Similarly, the relative velocity of B with respect to A

$$v_{BA} = vector\ differece\ of\ v_B\ and\ v_A$$

- From above, we conclude that the relative velocity of a point A with respect to B (v_{AB}) and the relative velocity of point B with respect to A (v_{BA}) are equal in magnitude but opposite in direction

$$v_{AB} = -v_{BA}$$

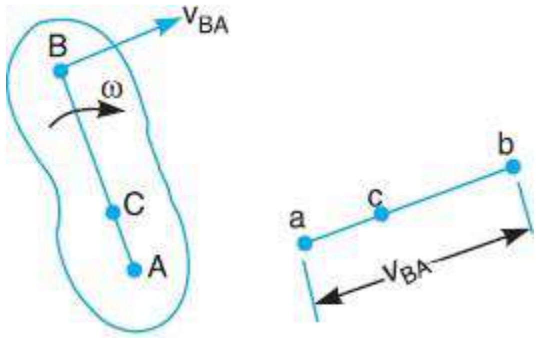
3.3 Motion Of A Link

- Consider two points A and B on a rigid link A B, as shown in Fig. 3.3 (a). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB.
- Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.
- The relative velocity of B with respect to A (i.e. v_{BA}) is represented by the vector ab and is perpendicular to the line A B as shown in Fig. 3.3 (b).
- We know that the velocity of the point B with respect to A

$$v_{BA} = \omega \times AB \ \ldots\ldots \ldots \ldots \ldots\ldots \ldots \ldots . . (i)$$

- Similarly the velocity of the point C on AB with respect to A

$$v_{CA} = \omega \times AC \ \ldots \ldots \ldots \ldots \ldots . . \ldots \ldots \ldots (ii)$$



(a) (b)

Fig. 3.3 Motion of a Link

– Form equation (i) and (ii),

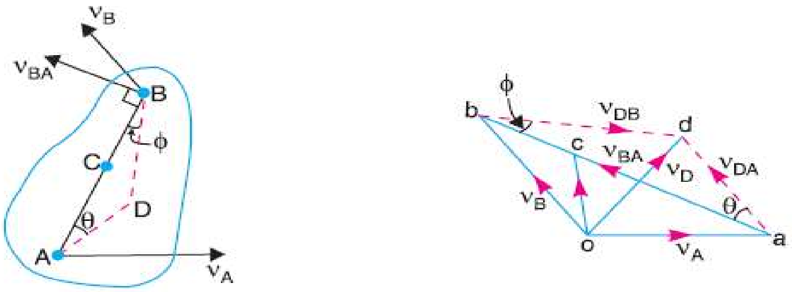
$$\frac{v_{CA}}{v_{BA}} = \frac{\omega \times AC}{\omega \times AB} = \frac{AC}{AB} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

– Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the link AB.

3.4 Velocity of A Point On A Link By Relative Velocity Method

– Consider two points A and B on a link as shown in Fig. 3.4 (a). Let the absolute velocity of the point A i.e. v_A is known in magnitude and direction and the absolute velocity of the point B i.e. v_B is known in direction only. Then the velocity of B may be determined by drawing the velocity diagram as shown in Fig. 3.4 (b). The velocity diagram is drawn as follows :

1. Take some convenient point o, known as the pole.
2. Through o, draw oa parallel and equal to v_A, to some suitable scale.
3. Through a, draw a line perpendicular to AB of Fig. 3.4 (a). This line will represent the velocity of B with respect to A, i.e. v_{BA}.
4. Through o, draw a line parallel to v_B intersecting the line of v_{BA} at b
5. Measure ob, which gives the required velocity of point B (v_B), to the scale

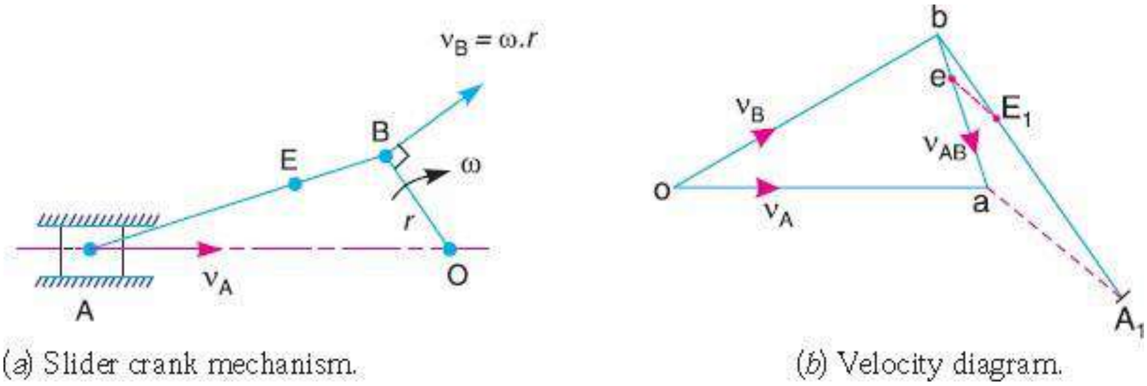


(a) Motion of points on a link. (b) Velocity diagram.

Fig. 3.4

3.5 Velocities In Slider Crank Mechanism

- In the previous article, we have discussed the relative velocity method for the velocity of any point on a link, whose direction of motion and velocity of some other point on the same link is known. The same method may also be applied for the velocities in a slider crank mechanism.
- A slider crank mechanism is shown in Fig. 3.5 (a). The slider A is attached to the connecting rod AB. Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O with uniform angular velocity ω rad/s. Therefore, the velocity of B i.e. v_B is known in magnitude and direction. The slider reciprocates along the line of stroke AO.



(a) Slider crank mechanism. (b) Velocity diagram.

Fig. 3.5

- The velocity of the slider A (i.e. v_A) may be determined by relative velocity method as discussed below :
  1. From any point o, draw vector ob parallel to the direction of v_B (or perpendicular to OB) such that ob = $v_B = \omega$.r, to some suitable scale, as shown in Fig. 3.5 (b).
  2. Since AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB. Now draw vector ba perpendicular to AB to represent the velocity of A with respect to B i.e. v_{AB}.
  3. From point o, draw vector oa parallel to the path of motion of the slider A (which is along AO only). The vectors ba and oa intersect at a. Now oa represents the velocity of the slider I.e. v_A, to the scale.
- The angular velocity of the connecting rod A B (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}$$

3.6 Rubbing Velocity at A Pin Joint

- The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.
- Consider two links OA and OB connected by a pin joint at O as shown in fig.

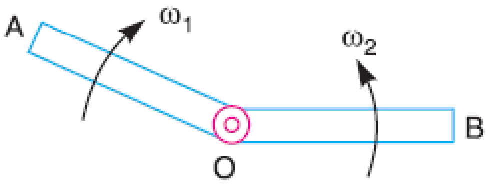


Fig. 3.6 Links connected by pin joints

- Let,
  ω1 = angular velocity of link OA
  ω2 = angular velocity of link OB
- According to the definition,
- Rubbing velocity at the pin joint O

$$= (\omega_1 - \omega_1) \times r \textbf{ if the links move in the same direction}$$
$$= (\omega_1 + \omega_1) \times r \textbf{ if the links move in the same direction}$$

3.7 Examples Based On Velocity

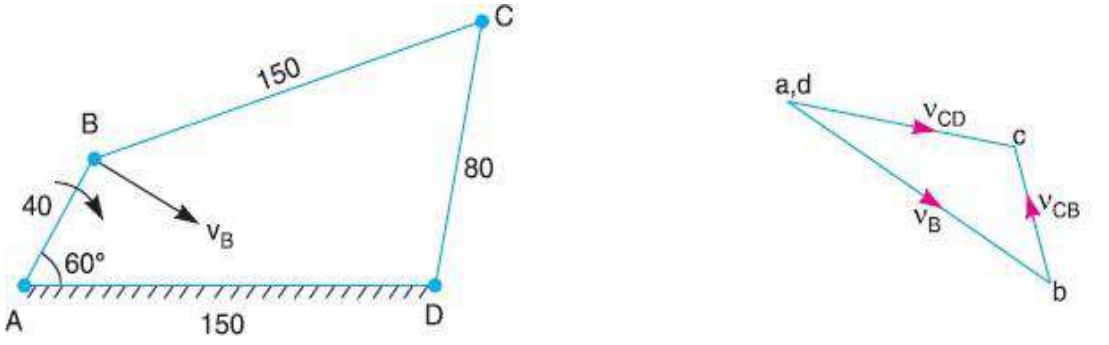
In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.

- **Given :** N_{BA} = 120 r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 12.568$ rad/s
- Since the length of crank A B = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),
- Since the length of crank A B = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$

- Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram. Draw vector ab perpendicular to B A, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (i.e. v_{BA} or v_B) such that

$$\text{Vector } ab = v_{BA} = v_B = 0.503 \text{ m/s}$$



(*a*) Space diagram (All dimensions in mm). (*b*) Velocity diagram.

Fig. 3.7

- Now from point b, draw vector bc perpendicular to CB to represent the velocity of C with respect to B (i.e. v_{CB}) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bc and dc intersect at c.

  By measurement, we find that

$$V_{CD} = v_C = \text{vector } dc = 0.385 \text{ m/s}$$

- Angular velocity of link CD,

$$\boldsymbol{\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8\ rad/s}$$

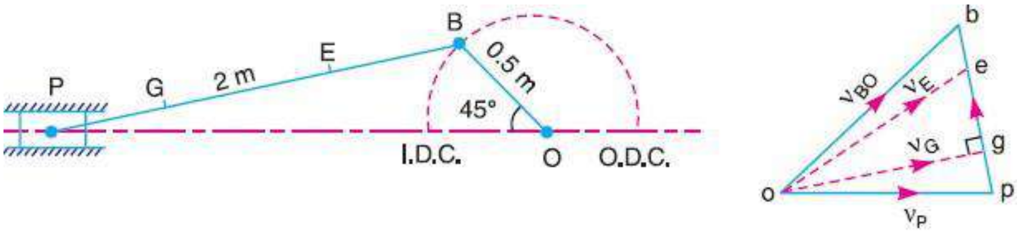
The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine:

1. Velocity of piston, 2. Angular velocity of connecting rod, 3. Velocity of point E on the connecting rod 1.5 m from the gudgeon pin, 4. velocities of rubbing at the pins of the crank shaft, crank and crosshead when the diameters of their pins are 50 mm, 60 mm and 30 mm respectively, 5. Position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft.

- **Given:**
- $N_{BO} = 180$ r.p.m. or $\omega_{BO} = 2\,\pi \times 180/60 = 18.852$ rad/s
- Since the crank length *OB* = 0.5 m, therefore linear velocity of *B* with respect to *O* or velocity of *B* (because *O* is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426 \text{ m/s}$$

- First of all draw the space diagram and then draw the velocity diagram as shown in fig.



(a) Space diagram. (b) Velocity diagram.

Fig. 3.8

- By measurement, we find that velocity of piston P,

$$v_P = vector\ op = 8.15\ m/s$$

- From the velocity diagram, we find that the velocity of P with respect to B

$$v_{PB} = \text{vector } bp = 6.8 \text{ m/s}$$

- Since the length of connecting rod PB is 2 m, therefore angular velocity of the connecting rod,

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4\ rad/s$$

$$v_E = vector\ oe = 8.5\ m/s$$

- We know that velocity of rubbing at the pin of crank-shaft

$$= \frac{d_0}{2} \times \omega_{BO} = 0.47\ m/s$$

- Velocity of rubbing at the pin of crank

$$= \frac{d_B}{2}(\omega_{BO} + \omega_{PB}) = 0.6675\ m/s$$

- Velocity of rubbing at the pin of crank

$$= \frac{d_c}{2} \times \omega_{PB} = 0.051\ m/s$$

- By measurement we find that

$$vector\ bg = 5\ m/s$$

- By measurement we find linear velocity of point G

$$v_G = vector\ og = 8\ m/s$$

In Fig. , the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are: OA = 28 mm; AB = 44 mm; BC 49 mm; and BD = 46 mm. The centre distance between the canters of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.

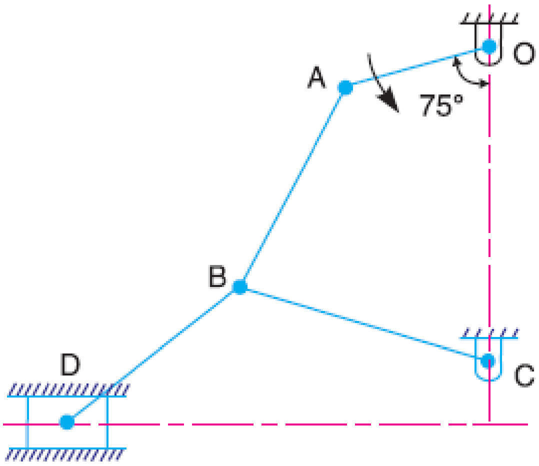


Fig. 3.9

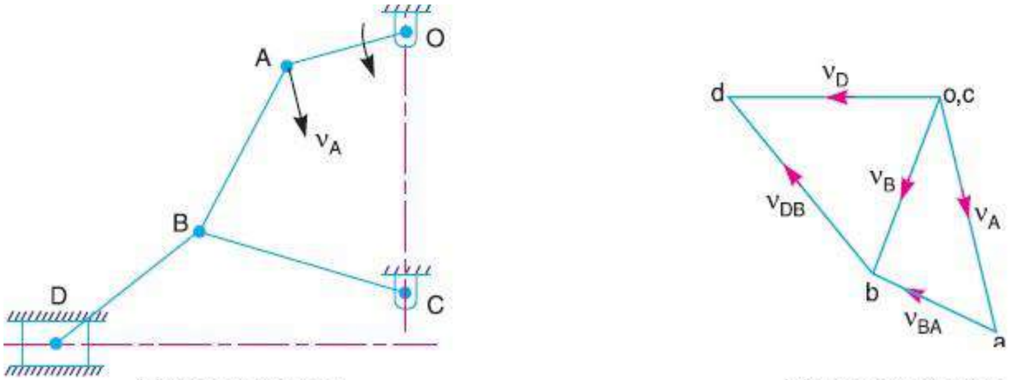
- **Given :**
- N_{AO} = 180 r.p.m. or ω_{BO} = 2 π × 180/60 = 18.852 rad/s
- OA = 28 mm

$$v_{OA} = v_A = \omega_{AO} \times AO = 1.76\ m/s$$

- Since the points O and C are fixed, therefore these points are marked as one point, in the velocity diagram. Now from point o, draw vector oa perpendicular to O A, to some suitable scale, to represent the velocity of A with respect to O or simply velocity of A such that

$$vector\ oa = v_{OA} = v_A = 1.76\ m/s$$

- From point a, draw vector ab perpendicular to A B to represent the velocity of B with respect A (i.e. v_{BA}) and from point c, draw vector cb perpendicular to CB to represent the velocity of B with respect to C or simply velocity of B (i.e. v_{BC} or v_B). The vectors ab and cb intersect at b.
- From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B (i.e. v_{DB}) and from point o, draw vector od parallel to the path of motion of the slider D which is horizontal, to represent the velocity of D (i.e. v_D). The vectors bd and od intersect at d.



(*a*) Space diagram. (*b*) Velocity diagram.

Fig.3.10

- By measurement, we find that velocity of slider D,

$$v_D = \text{vector } od = 1.6\ m/s$$

- By measurement from velocity diagram, we find that velocity of D with respect to B,

$$v_{DB} = \text{vector } bd = 1.7\ m/s$$

- Therefore angular velocity of link BD

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96\ rad/s$$

The mechanism, as shown in Fig. 3.11, has the dimensions of various links as follows: AB = DE = 150 mm; BC = CD = 450 mm; EF = 375 mm. The crank AB makes an angle of 45° with the horizontal and rotates about A in the clockwise direction at a uniform speed of 120 r.p.m. The lever DC oscillates about the fixed point D, which is connected to AB by the coupler BC. The block F moves in the horizontal guides, being driven by the link EF. Determine: 1. velocity of the block F, 2. angular velocity of DC, and 3. rubbing speed at the pin C which is 50 mm in diameter.

- Given :
- $N_{BA} = 120$ r.p.m. or $\omega_{BA} = 2\ \pi \times 120/60 = 4\ \pi$ rad/s
- Since the crank length A B = 150 mm = 0.15 m, therefore velocity of B with respect to A or simply velocity of B (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 4\ \pi \times 0.15 = 1.885\ m/s$$

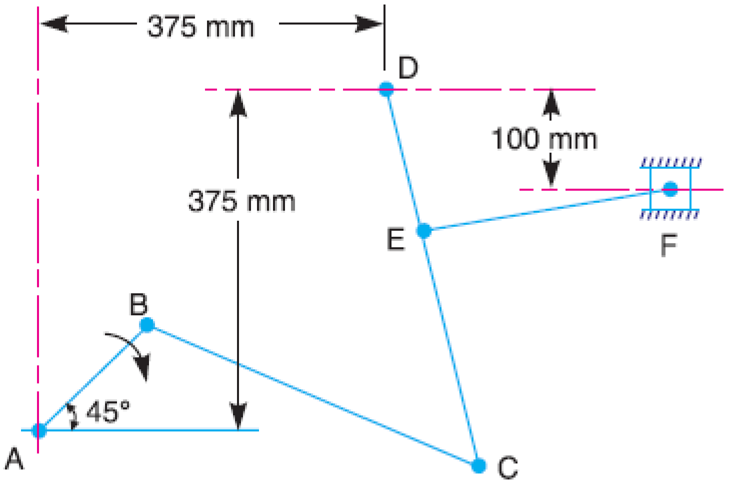
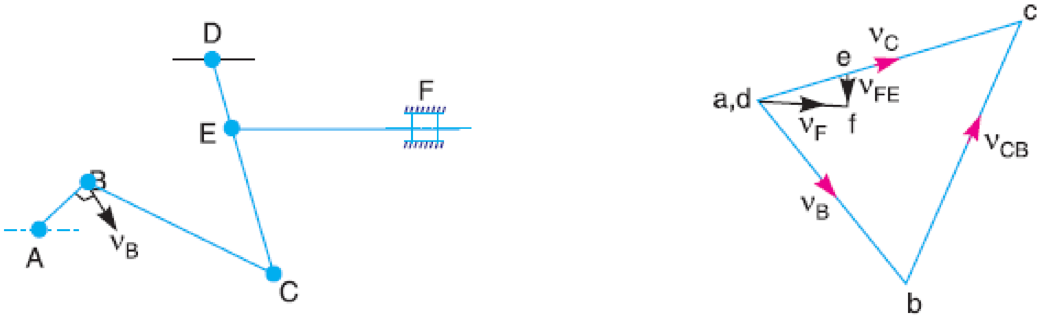


Fig.3.11

- Since the points A and D are fixed, therefore these points are marked as one point as shown in Fig. (b). Now from point a, draw vector ab perpendicular to A B, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B, such that

$$\text{Vector ab} = v_{BA} = v_B = 1.885 \text{ m/s}$$

- The point C moves relative to B and D, therefore draw vector bc perpendicular to BC to represent the velocity of C with respect to B (i.e. v_{CB}), and from point d, draw vector dc perpendicular to DC to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bc and dc intersect at c.



(a) Space diagram. (b) Velocity diagram.

Fig. 3.12

- Since the point E lies on DC, therefore divide vector dc in e in the same ratio as E divides CD in Fig. (a). In other words

$$ce/cd = CE/CD$$

- From point e, draw vector ef perpendicular to EF to represent the velocity of F with respect to E (i.e. v_{FE}) and from point d draw vector df parallel to the path of motion of F, which is horizontal, to represent the velocity of F i.e. v_F. The vectors ef and df intersect at f.

$$v_F = \text{vector df} = 0.7 \text{ m/s}$$

- By measurement from velocity diagram, we find that velocity of C with respect to D,

$$v_{CD} = \text{vector } dc = 2.25 \text{ m/s}$$

$$\omega_{DC} = \frac{v_{CD}}{DC} = 5 \frac{rad}{s}$$

- From velocity diagram, we find that velocity of C with respect to B,

$$v_{CB} = \text{vector } bc = 2.25 \text{ m/s}$$

- Angular velocity of BC,

$$\omega_{CD} = \frac{v_{CD}}{BC} = \frac{2.25}{0.45} = 5\, rad/s$$

3.8 Velocity Of A Point On A Link By Instantaneous Centre Method

- The instantaneous centre method of analyzing the motion in a mechanism is based upon the concept that any displacement of a body (or a rigid link) having motion in one plane, can be considered as a pure rotational motion of a rigid link as a whole about some centre, known as instantaneous centre or virtual centre of rotation.

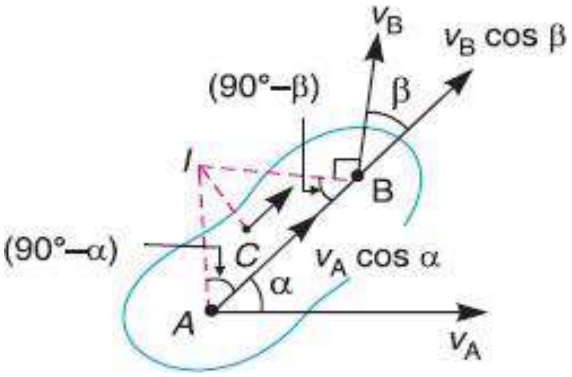


Fig. 3.13 velocity of a point on a link

- The velocities of points A and B, whose directions are given a link.by angles α and β as shown in Fig. If v_A is known in magnitude and direction and v_B in direction only, then the magnitude of v_B may be determined by the instantaneous centre method as discussed below :
- Draw AI and BI perpendiculars to the directions v_A and v_B respectively. Let these lines intersect at I, which is known as instantaneous centre or virtual centre of the link. The complete rigid link is to rotate or turn about the centre I.
- Since A and B are the points on a rigid link, therefore there cannot be any relative motion between them along line AB.
- Now resolving the velocities along AB,

$$\boldsymbol{v_A \times \cos\alpha = v_B \times \cos\beta}$$

$$\boldsymbol{\frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90-\beta)}{\sin(90-\alpha)}} \ldots \ldots\ldots \ldots \ldots \ldots \ldots \ldots . (\boldsymbol{i})$$

- Applying Lami’s theorem to triangle ABI,

$$\boldsymbol{\frac{AI}{\sin(90-\beta)} = \frac{BI}{\sin(90-\alpha)}}$$

$$\boldsymbol{\frac{AI}{BI} = \frac{\sin(90-\ \beta)}{\sin(90-\alpha)}} \ldots \ldots \ldots \ldots\ldots \ldots \ldots\ldots \ldots \ldots \ldots. \ldots (\boldsymbol{ii})$$

- Hence,

$$\boldsymbol{\frac{v_A}{v_B} = \frac{AI}{BI}}$$

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

- If C is any other point on link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

3.9 Properties of Instantaneous Method

- The following properties of instantaneous centre are important :
    1. A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.
    2. The two rigid links have no linear velocity relative to each other at the instantaneous centre. At this point (i.e. instantaneous centre), the two rigid links have the same linear velocity relative to the third rigid link. In other words, the velocity of the instantaneous centre relative to any third rigid link will be same whether the instantaneous centre is regarded as a point on the first rigid link or on the second rigid link

3.10 Number of Instantaneous Centre in A Mechanism:

- The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number 3 of instantaneous centres is the number of combinations of n links taken two at a time. Mathematically, number of instantaneous centres

$$N = \frac{n\,(n-1)}{2}, where\ n = Number\ of\ Link$$

3.11 Location of Instantaneous centres:

- The following rules may be used in locating the instantaneous centres in a mechanism :
    1. When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies on the centre of the pin as shown in Fig. (a). such an instantaneous centre is of permanent nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.
    2. When the two links have a pure rolling contact (i.e. link 2 rolls without slipping upon the fixed link 1 which may be straight or curved), the instantaneous centre lies on their point of contact, as shown in Fig.(b). The velocity of any point A on the link 2 relative to fixed link 1 will be perpendicular to I12 A and is proportional to I12 A.
    3. When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact. We shall consider the following three cases :

    a. When the link 2 (slider) moves on fixed link 1 having straight surface as shown in Fig.(c), the instantaneous centre lies at infinity and each point on the slider have the same velocity.

b. When the link 2 (slider) moves on fixed link 1 having curved surface as shown in Fig.(d),the instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

c. When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as shown in Fig. 3.14 (e), the instantaneous centre lies at the centre of curvature i.e. the centre of the circle, for all configuration of the links.

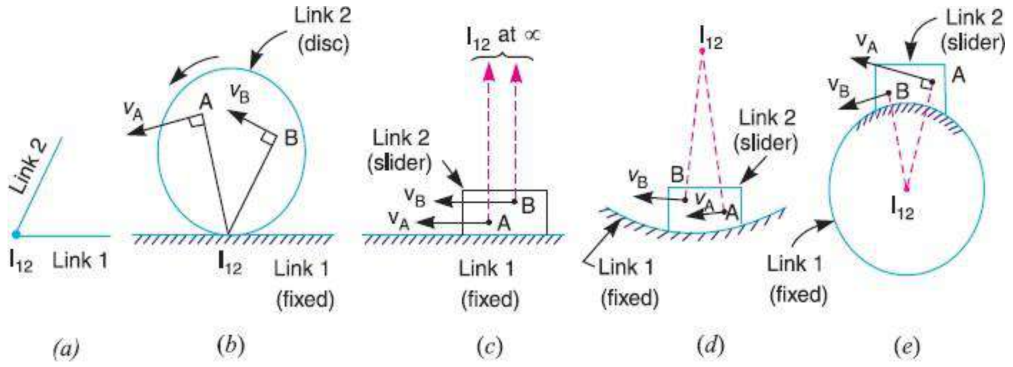


Fig. 3.14 Location of Instantaneous centres

3.12 Kennedy’s Theorem

- The Aronhold Kennedy’s theorem states that “if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.”
- Consider three kinematic links A, B and C having relative plane motion. The number of instantaneous centres (N) is given by

$$\boldsymbol{N = \frac{n\,(n-1)}{2} = \frac{3\,(3-1)}{2} = 3}$$

- The two instantaneous centres at the pin joints of B with A , and C with A (i.e. I_{ab} and I_{ac}) are the permanent instantaneous centre According to Aronhold Kennedy’s theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac}. In order to prove this let us consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in Fig. The point I_{bc} belongs to both the links B and C. Let us consider the point I_{bc} on the link B. Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc}. Now consider the point I_{bc} on the link C. Its velocity v_{BC} must be perpendicular to the line joining I_{ac} and I_{bc}.

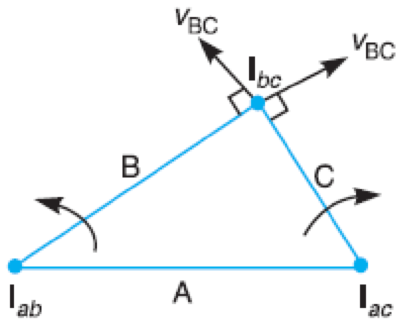
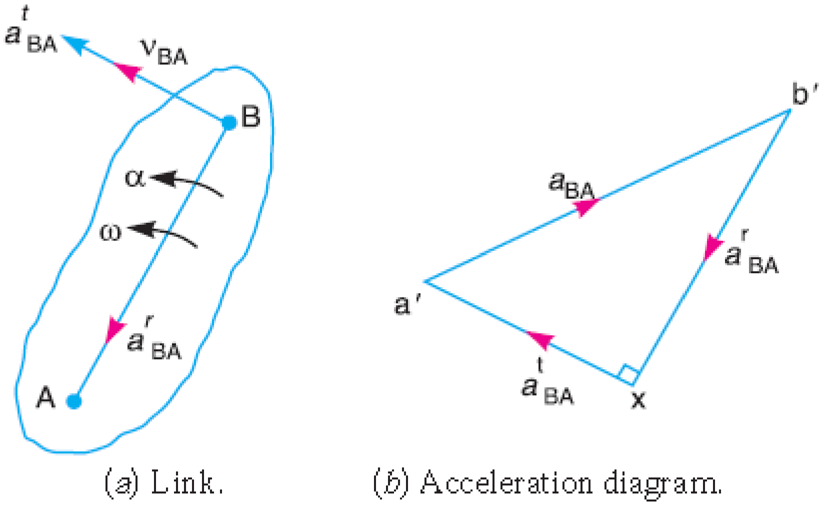


Fig. 3.15 Aronhold Kennedy's theorem

- We have already discussed that the velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point I_{bc} cannot be perpendicular to both lines I_{ab} I_{bc} and I_{ac} I_{bc} unless the point Ibc lies on the line joining the points I_{ab} and I_{ac}. Thus the three instantaneous centres (I_{ab}, I_{ac} and I_{bc}) must lie on the same straight line. The exact location of Ibc on line I_{ab} I_{ac} depends upon the directions and magnitudes of the angular velocities of B and C relative to A.

3.13 Acceleration Diagram for a Link

- Consider two points A and B on a rigid link as shown in Fig. (a). Let the point B moves with respect to A, with an angular velocity of **ω** rad/s and let **α** rad/s^2 be the angular acceleration of the link AB.



(*a*) Link. (*b*) Acceleration diagram.

Fig. 3.16 Acceleration of a link

- We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components.

1 The centripetal or radial component, which is perpendicular to the velocity of the particle at the given instant.

2 The tangential component, which is parallel to the velocity of the particle at the given instant.

- Thus for a link A B, the velocity of point B with respect to A (i.e. vBA) is perpendicular to the link A B as shown in Fig.(a). Since the point B moves with respect to A with an angular velocity of $\boldsymbol{\omega}$ rad/s, therefore centripetal or radial component of the

  acceleration of B with respect to A

$$a^r_{BA} = \omega^2 \times Length\ of\ link\ AB = \omega^2 \times AB = {v^2_{BA}}/{AB}$$

- This radial component of acceleration acts perpendicular to the velocity vBA, In other words, it acts parallel to the link AB.

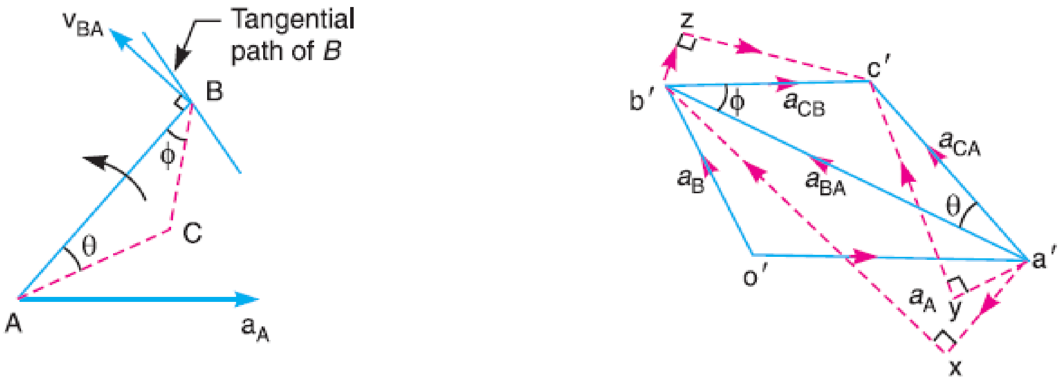
  We know that tangential component of the acceleration of B with respect to A ,

$$a^t_{BA} = \alpha \times Length\ of\ link\ AB = \alpha \times AB$$

- This tangential component of acceleration acts parallel to the velocity vBA. In other words, it acts perpendicular to the link AB.
- In order to draw the acceleration diagram for a link A B, as shown in Fig. 3.16 (b), from any point b', draw vector b'x parallel to BA to represent the radial component of acceleration of B with respect to A.

3.14 Acceleration of a Point on a Link

- Consider two points A and B on the rigid link, as shown in Fig. 3.17 (a). Let the acceleration of the point A i.e. a_A is known in magnitude and direction and the direction of path of B is given. The acceleration of the point B is determined in magnitude and direction by drawing the acceleration diagram as discussed below.



(*a*) Points on a Link. (*b*) Acceleration diagram.

Fig. 3.17 acceleration of a point on a link

- From any point o', draw vector o'a' parallel to the direction of absolute acceleration at point A i.e. a_A , to some suitable scale, as shown in Fig. 3.17 (b).
- We know that the acceleration of B with respect to A i.e. a_{BA} has the following two components:

1. Radial component of the acceleration of B with respect to A i.e. a^r_{BA}
2. Tangential component of the acceleration B with respect to A i.e. a^t_{BA}

- Draw vector a’x parallel to the link AB such that,

$$\boldsymbol{vector\ a'x = a^{r}_{BA} = v^{2}_{BA}/AB}$$

- From point x, draw vector xb’ perpendicular to AB or vector a’x and through o’ draw a line parallel to the path of B to represent the absolute acceleration of B i.e. a_B
- By joining the points a' and b' we may determine the total acceleration of B with respect to A i.e. a_{BA}. The vector a' b' is known as acceleration image of the link AB.
- For any other point C on the link, draw triangle a' b' c' similar to triangle ABC. Now vector b' c' represents the acceleration of C with respect to B i.e. a_{CB}, and vector a' c' represents the acceleration of C with respect to A i.e. a_{CA}. As discussed above, a_{CB} and a_{CA} will each have two components as follows :
  a. a_{CB} has two components; $\boldsymbol{a^{r}_{CB}}$ and $\boldsymbol{a^{t}_{CB}}$ as shown by triangle b’zc’ in fig.b
  b. a_{CA} has two components; $\boldsymbol{a^{r}_{CA}}$ and $\boldsymbol{a^{t}_{CA}}$ as shown by triangle a’yc’
- The angular acceleration of the link AB is obtained by dividing the tangential component of acceleration of B with respect to A to the length of the link.

$$\boldsymbol{\alpha_{AB} = a^{t}_{BA}/AB}$$

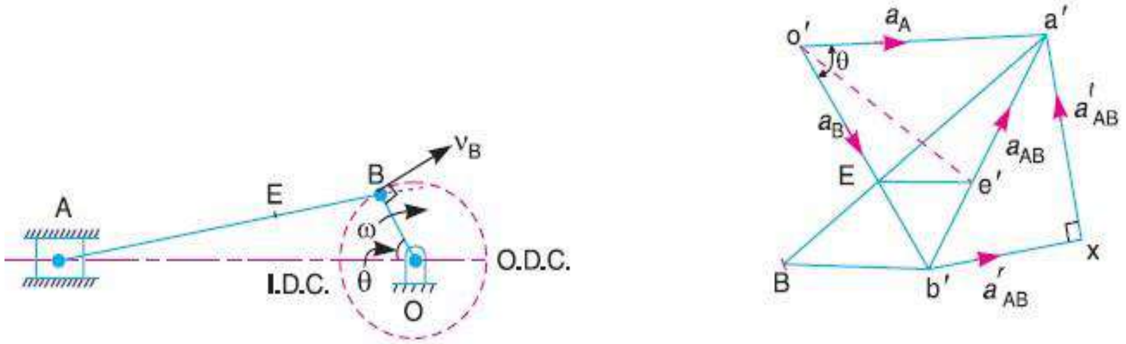
3.15 Acceleration in Slider Crank Mechanism

- A slider crank mechanism is shown in Fig. 3.18 (a). Let the crank OB makes an angle **θ** with the inner dead centre (I.D.C) and rotates in a clockwise direction about the fixed point O with uniform angular velocity $\boldsymbol{\omega}_{BO}$ rad/s
- Velocity of B with respect to O or velocity of B (because O is a fixed point),

$$\boldsymbol{v_{BO} = v_B = \omega_{BO} \times OB\ acting\ tangentially\ at\ B}$$

- We know that centripetal or radial acceleration of B with respect to O or acceleration of B (Because O is a fixed point)

$$\boldsymbol{a^{r}_{BO} = a_B = \omega^{2}_{BO} \times OB = \frac{v^{2}_{BO}}{BO}}$$



(a) Slider crank mechanism. (b) Acceleration diagram.

Fig. 3.18 acceleration in the slider crank mechanism

- The acceleration diagram, as shown in Fig. 3.18 (b), may now be drawn as discussed below:

1 Draw vector o' b' parallel to BO and set off equal in magnitude of a=a, to some BO suitable scale.
2 From point b', draw vector b'x parallel to BA. The vector b'x represents the radial component of the acceleration of A with respect to B whose magnitude is given by :

$$a^r_{AB} = v^2_{AB}/BA$$

3 From point x, draw vector xa’ perpendicular to b’x. The vector xa’ represents the tangential components of the acceleration of A with respect to B.
4 Since the point A reciprocates along AO, therefore the acceleration must be parallel to velocity. Therefore from o', draw o' a' parallel to A O, intersecting the vector xa' at a'.
5 The vector b' a', which is the sum of the vectors b' x and x a', represents the total acceleration of A with respect to B i.e. a_{AB}. The vector b'a' represents the acceleration of the connecting rod AB.
6 The acceleration of any other point on A B such as E may be obtained by dividing the vector b' a' at e' in the same ratio as E divides A B in Fig. 8.3 (a). In other words

$$a'e'/a'b' = AE/AB$$

7 The angular acceleration of the connecting rod A B may be obtained by dividing the tangential component of the acceleration of A with respect to B to the length of AB. In other words, angular acceleration of AB,

$$\alpha_{AB} = a^t_{AB}/AB$$

3.16 Examples Based on Acceleration

The crank of the slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine:

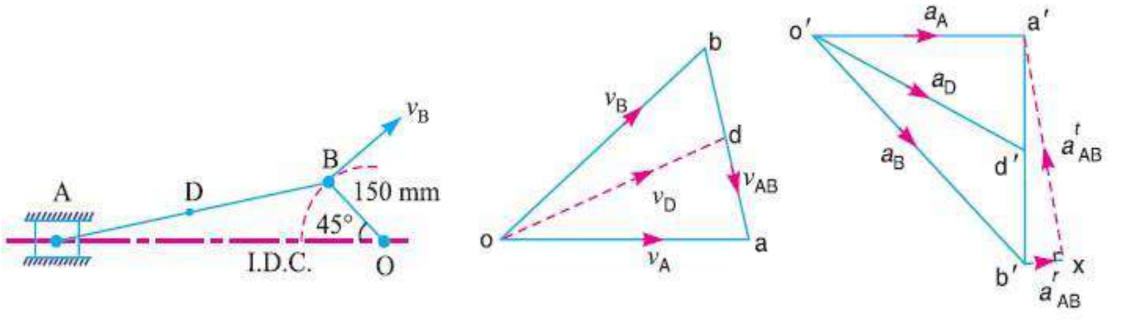
1. **Linear velocity and acceleration of the midpoint of the connecting rod, and**
2. **Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position**

- **Given:**
- N_{BO} = 300 r.p.m. or ω_{BO} = 2 π × 300/60 = 31.42 rad/s; OB = 150 mm = 0.15 m ; B A = 600 mm = 0.6 m
- We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713\ m/s$$

- Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B i.e. v_{BO} or vB, such that

vector ob = v_{BO} = v_B = 4.713 m/s



(a) Space diagram. (b) Velocity diagram. (c) Acceleration diagram.

Fig. 3.19

- From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B i.e. v_{AB} , and from point o draw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A i.e. v_A. The vectors ba and oa intersect at a.
- By measurement we find the velocity A with respect to B,

$$v_{AB} = vector\ ba = 3.4\ m/s$$

$$v_A = \text{vector oa} = 4\ \text{m/s}$$

- In order to find the velocity of the midpoint D of the connecting rod A B, divide the vector ba at d in the same ratio as D divides A B, in the space diagram. In other words,

$$bd/ba = BD/BA$$

- By measurement, we find that

$$\text{v}_\text{D} = \text{vector od} = 4.1\ \text{m/s}$$

- We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a^2_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1\ m/s^2$$

- And the radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3\ m/s^2$$

$$vector\ o'b' = a^r_{BO} = a_B = 148.1\ m/s^2$$

- By measurement, we find that

$$a = vector\ o'd' = 117\ m/s^2$$

- We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67\ rad/s^2$$

- From the acceleration diagram, we find that

$$\text{a}^\text{t}_\text{AB} = 103\ \text{m/s}^2$$

- We know that angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67\ rad/s^2$$

An engine mechanism is shown in Fig. 3.20. The crank CB = 100 mm and the connecting rod BA = 300 mm with centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s^2. Find:

1. **Velocity of G and angular velocity of AB, and**
2. **Acceleration of G and angular acceleration of AB.**

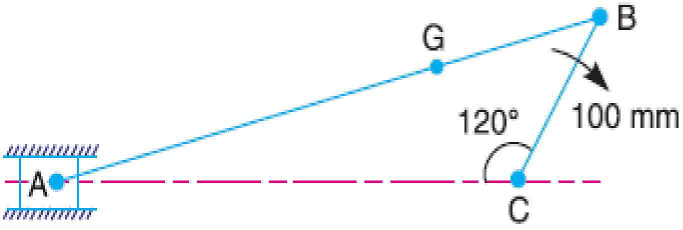


Fig. 3.20

- **Given :**
- ω_{BC} = 75 rad/s ; α_{BC} = 1200 rad/s^2, CB = 100 mm = 0.1 m; B A = 300 mm = 0.3 m
- We know that velocity of B with respect to C or velocity of B

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5\ m/s$$

- Since the angular acceleration of the crankshaft, α_{BC} = 1200 rad/s^2, therefore tangential component of the acceleration of B with respect to C,

$$a^t_{BC} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120\ m/s^2$$

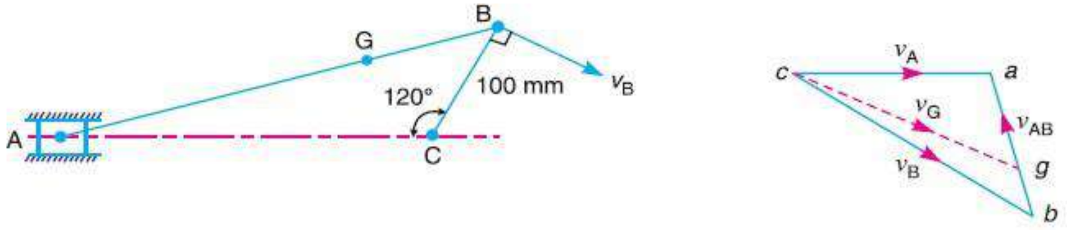
$$vector\ cb = v_{BC} = v_B = 7.5\ \frac{m}{s}$$

- By measurement, we find that velocity of G,

$$v_G = ector\ cg = 6.8\ m/s$$

- From velocity diagram, we find that the velocity of A with respect to B,

$$v_{AB} = vector\ ba = 4\ m/s$$

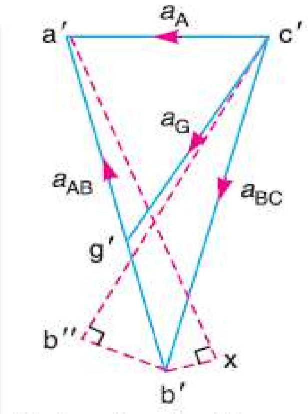


(a) Space diagram. (b) Velocity diagram.

Fig. 3.21

- We know that angular velocity of AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3\ rad/s$$



(c) Acceleration diagram.

Fig. 3.22

- We know that radial component of the acceleration of B with respect to C

$$a^r_{B_C} = \frac{v^2_{BC}}{CB} = \frac{(7.5)^2}{0.1} = 562.5\ m/s^2$$

- And radial component of the acceleration of A with respect to B,

$$a^r_{A_B} = \frac{v^2_A}{CB} = \frac{(4)^2}{0.3} = 53.3\ m/s^2$$

$$vector\ c'b'' = {}^r\ {}_{BC} = 562.5\ m/s^2$$

$$vecto\ ''b' = a^t\ {}_{BC} = 120\ m/s^2$$

$$vecto\ 'x = a^r\ {}_{AB} = 53.3\ m/s^2$$

- By measurement we find that acceleration of G,

$$a_G = vector\ xa' = 414\ m/s^2$$

- From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = ector\ xa' = 546 m/s^2$$

- Angular acceleration of AB

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820\ rad/s^2$$

In the mechanism shown in Fig. 8.7, the slider C is moving to the right with a velocity of 1 m/s and an acceleration of 2.5 m/s2.The dimensions of various links are AB = 3 m inclined at 45° with the vertical and BC = 1.5 m inclined at 45° with the horizontal. Determine: 1. the magnitude of vertical and horizontal component of the acceleration of the point B, and 2. the angular acceleration of the links AB and BC.

- **Given:**
- $v_C = 1$ m/s ; $a_C = 2.5$ m/s2; AB = 3 m ; BC = 1.5 m
- Here,

$$vector\ d = v_{CD} = v_c = 1 m/s$$

- By measurement, we find that velocity of B with respect to A

$$v_{BA} = vector\ ab = 0.72\ m/s$$

- Velocity of B with respect to C

$$v_{BC} = vector\ cb = 0.72\ m/s$$

- We know that radial component of acceleration of B with respect to C,

$$a^r_{B_C} = \frac{v^2_{BC}}{CB} = \frac{(0.72)^2}{1.5} = 0.346\ m/s^2$$

- And radial component of acceleration of B with respect to A,

$$a^r_{B_A} = \frac{v^2_{BA}}{AB} = \frac{(0.72)^2}{3} = 0.173\ m/s^2$$

$$vectro\ d'c' = a_{cd} = a_c = 2.5\ m/s^2$$

$$vecto\ 'x = a^r_{BC} = 0.346 \frac{m}{s^2}$$

$$vecto\ 'y = a^r_{BA} = 0.173\ m/s^2$$

- By measurement,

$$vector\ b'b'' = 1.13\ m/s^2$$

- By measurement from acceleration diagram, we find that tangential component of acceleration of the point B with respect to A

$$a^t_{BA} = ector\ yb' = 1.41\ m/s^2$$

- And tangential component of acceleration of the point B with respect to C,

$$a^t_{BC} = vector\ xb' = 1.94\ m/s^2$$

- we know that angular velocity of AB,

$$\alpha_{AB} = \frac{v^t_{BA}}{AB} = 0.47\ rad/s^2$$

- And aglular acceleration of BC,

$$\alpha_{BC} = \frac{a^t_{BC}}{CB} = \frac{1.94}{1.5}\ \mathrm{rad}/s^2$$

LECTURE 1

MOTION OF LINK IN MACHINE

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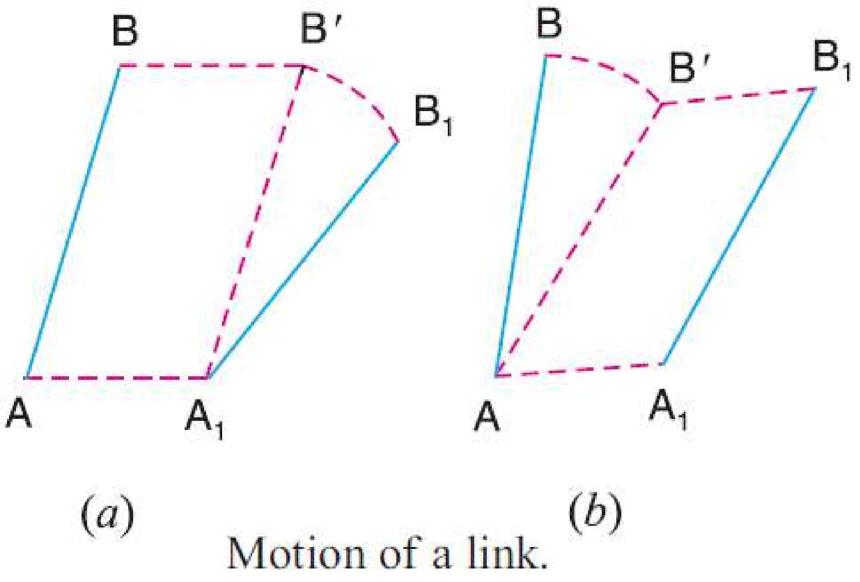
INTRODUCTION



(a) (b)

Motion of a link.

<u>Source : R. S. Khurmi</u>

Motion of link *AB to A*1*B*1 is an example of combined motion of rotation and translation, it being immaterial whether the motion of rotation takes first, or the motion of translation.

METHODS FOR DETERMINING THE VELOCITY OF A POINT ON A LINK

1. Relative velocity method

   Can be used in any configuration

2. Instantaneous centre method

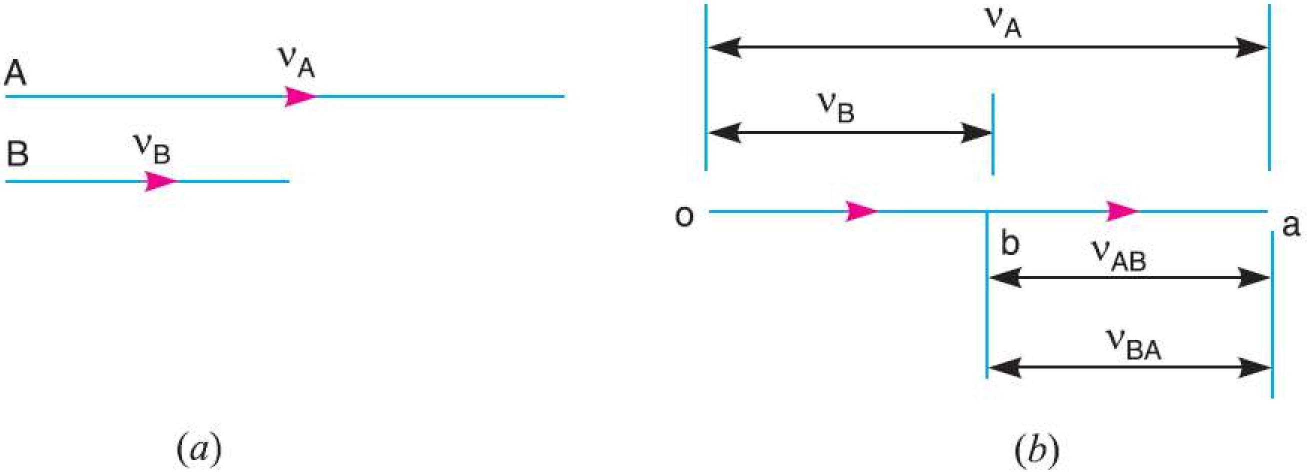
   convenient and easy to apply in simple mechanisms

RELATIVE VELOCITY METHOD

From Fig., the relative velocity of A with respect to B (*i.e.* v_{AB}) may be written in the vector form as follows :

$$\overline{ba} = \overline{oa} - \overline{ob}$$

Source : R. S. Khurmi



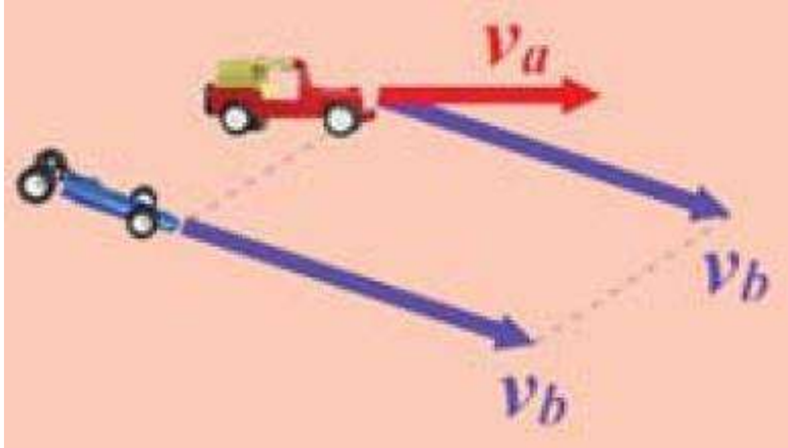
Relative velocity of two bodies moving along parallel lines.

Similarly, the relative velocity of B with respect to A ,

$$v_{BA} = \text{Vector difference of } v_B \text{ and } v_A = \overline{v_B} - \overline{v_A}$$

$$\overline{ab} = \overline{ob} - \overline{oa}$$

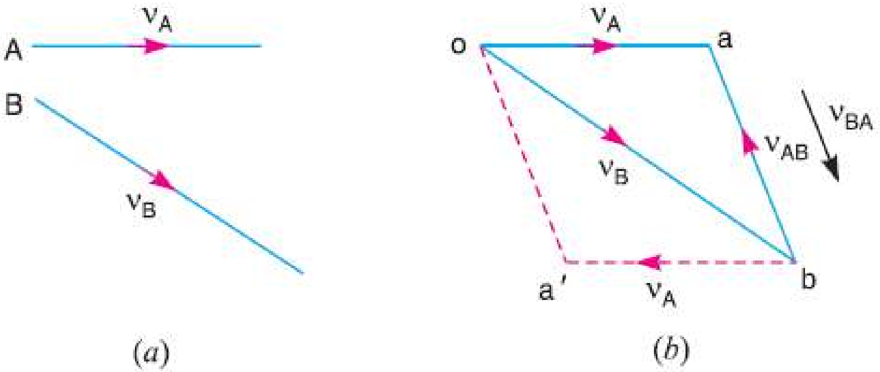
RELATIVE VELOCITY



v_{AB} = Vector difference of v_A and $v_B = \overline{v_A} - \overline{v_B}$

$\overline{ba} = \overline{oa} - \overline{ob}$

Source : R. S. Khurmi



Relative velocity of two bodies moving along inclined lines.

Similarly, the relative velocity of B with respect to A,

v_{BA} = Vector difference of v_B and $v_A = \overline{v_B} - \overline{v_A}$

$\overline{ab} = \overline{ob} - \overline{oa}$

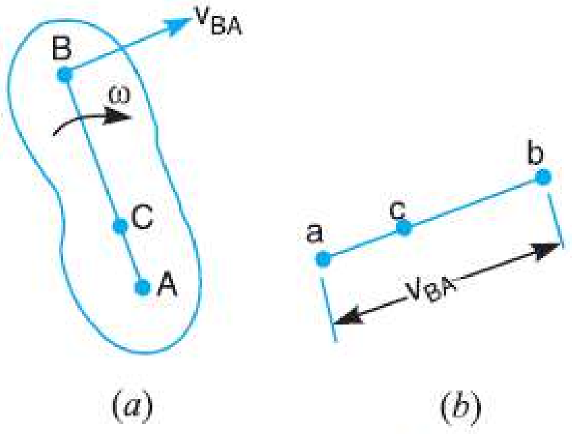
From above, we conclude that the relative velocity of point A with respect to B (v_{AB}) and the relative velocity of point B with respect A (v_{BA}) are equal in magnitude but opposite in direction, *i.e.*

$$v_{AB} = -v_{BA} \quad \text{or} \quad \overline{ba} = -\overline{ab}$$

Note: It may be noted that to find v_{AB}, start from point b towards a and for v_{BA}, start from point a towards b.

MOTION OF A LINK

Source : R. S. Khurmi



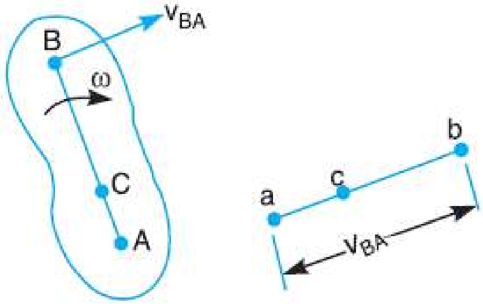
Motion of a Link.

- ➢Let one of the extremities (B) of the link move relative to A, in a clockwise direction.
- ➢No relative motion between A and B, along the line AB
- ➢relative motion of B with respect to A must be perpendicular to AB.

Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.

MOTION OF A LINK

Source : R. S. Khurmi



(*a*) (*b*)

Motion of a Link.

Let ω = Angular velocity of the link *A B* about *A*.

We know that the velocity of the point *B* with respect to *A*,

$$v_{BA} = \overline{ab} = \omega . AB \qquad ...(i)$$

Similarly, the velocity of any point *C* on *A B* with respect to *A*,

$$v_{CA} = \overline{ac} = \omega . AC \qquad ...(ii)$$

From equations (*i*) and (*ii*),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \qquad ...(iii)$$

Thus, we see from equation (*iii*), that the point *c* on the vector *ab* divides it in the same ratio as *C* divides the link *A B*.

LECTURE 2

VELOCITY AND ACCELERATION DIAGRAMS

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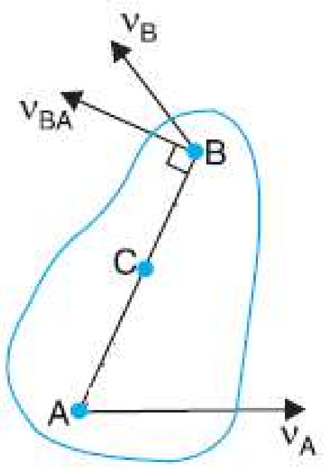
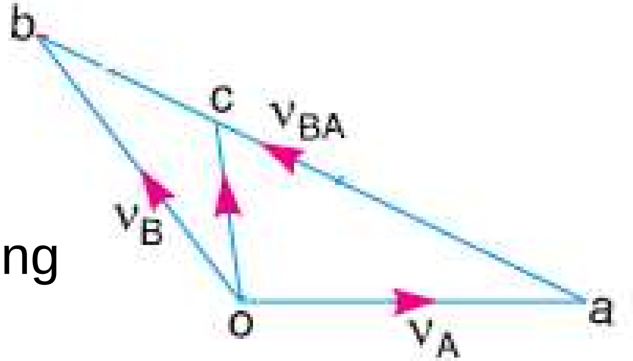
VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD



Motion of points on a link.

Source : R. S. Khurmi

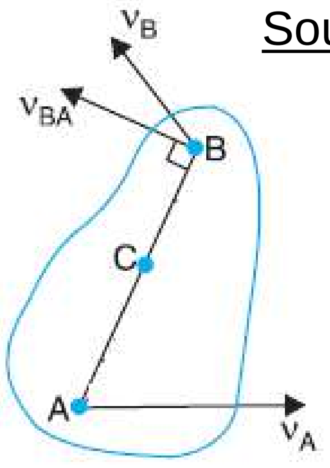
- V_A is known in magnitude and direction
- absolute velocity of the point B *i.e.* V_B is known in direction only
- V_B be determined by drawing the velocity diagram



Velocity diagram.

- With suitable scale, Draw oa = V_A
- Through a, draw a line perpendicular to AB
- Through o, draw a line parallel to V_B intersecting the line of V_{BA} at b
- Measure ob, which gives the required velocity of point B (V_B), to the scale
- ab = velocity of the link AB

VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD



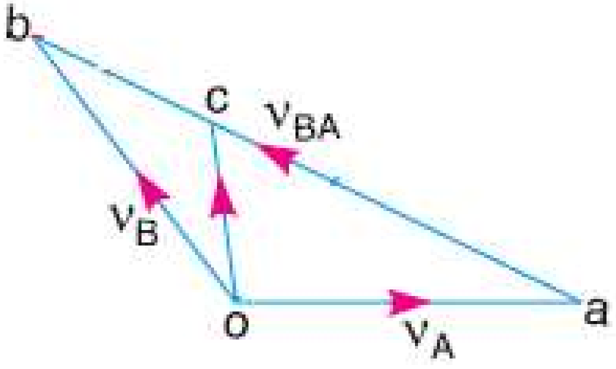
Motion of points on a link.

Source : R. S. Khurmi

- *How to find V_C ?*

Fix ‘c’ on the velocity diagram, using

$$\frac{ac}{ab} = \frac{AC}{AB}$$



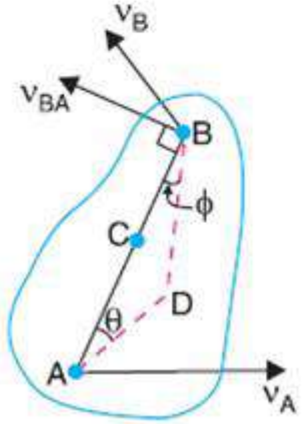
Velocity diagram.

- oc = V_C = Absolute velocity of C
- the vector ac represents the velocity of C with respect to A i.e. V_{CA}.

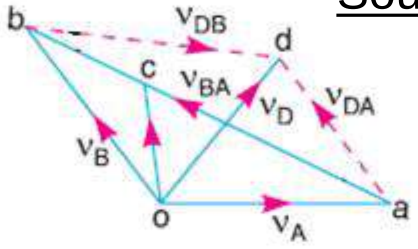
VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD

How to find the absolute velocity of any other point D outside AB?

Source : R. S. Khurmi



(*a*) Motion of points on a link.



(*b*) Velocity diagram.

Construct triangle ABD in the space diagram

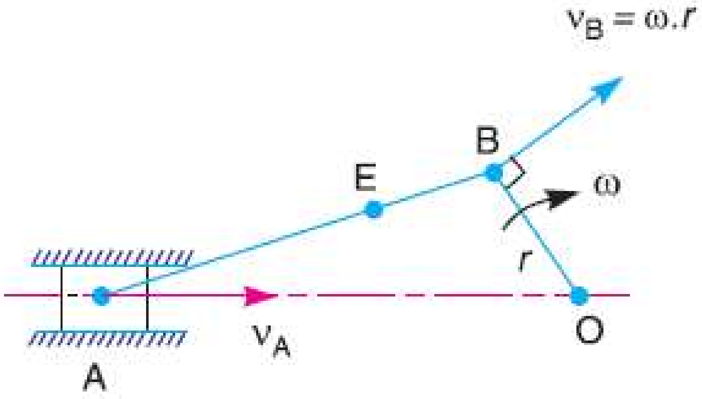
Completing the velocity triangle abd:

- ➢Draw VDA perpendicular to AD;
- ➢Draw VDB perpendicular to BD, intersection is 'd'.
- ➢od = absolute velocity of D.

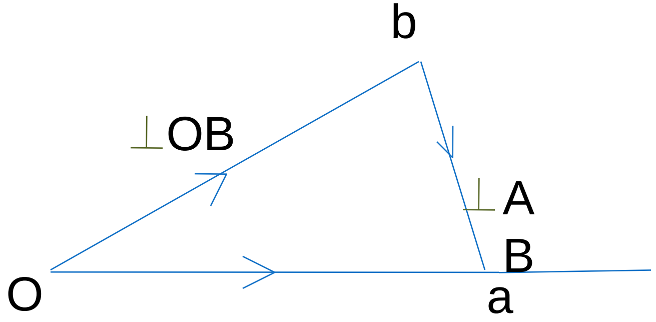
The angular velocity of the link AB $= \omega_{AB} = \dfrac{v_{BA}}{AB} = \dfrac{ab}{AB}$

VELOCITIES IN SLIDER CRANK MECHANISM

Source : R. S. Khurmi



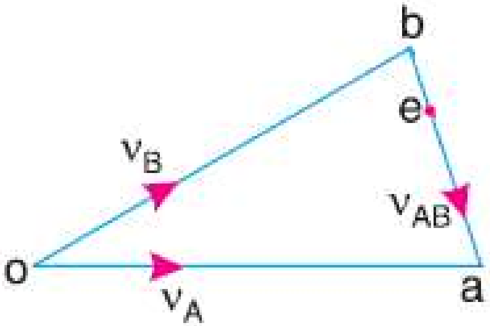
Slider crank mechanism.



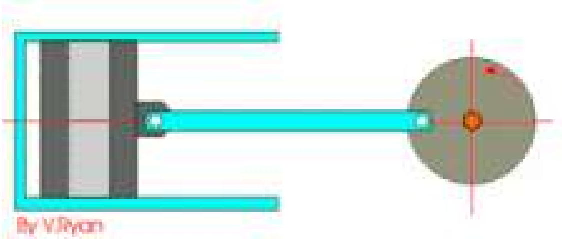
Fix ‘e’, based on the ratio

$$be/ba = BE/BA$$

V_E = length ‘oe’ = absolute vel. Of E



Velocity diagram.



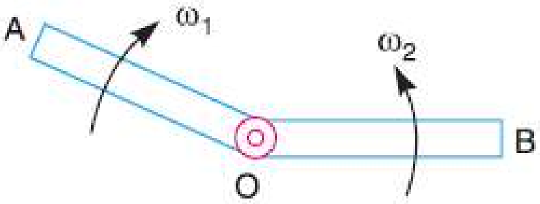
The angular velocity of the connecting rod AB (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \qquad \text{(Anticlockwise about A)}$$

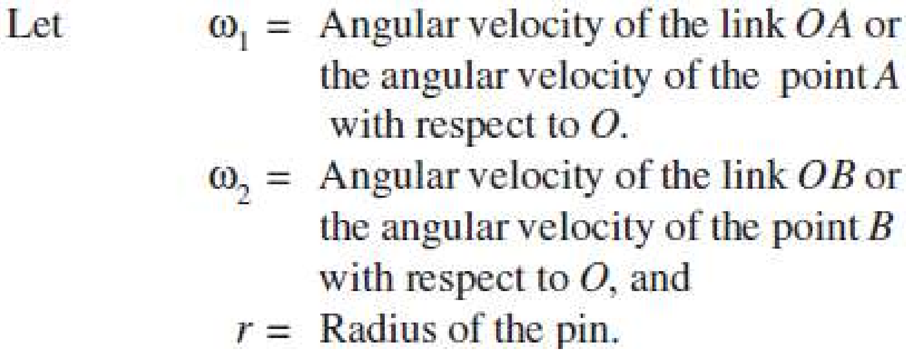
RUBBING VELOCITY AT A PIN JOINT

The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.

Source : R. S. Khurmi



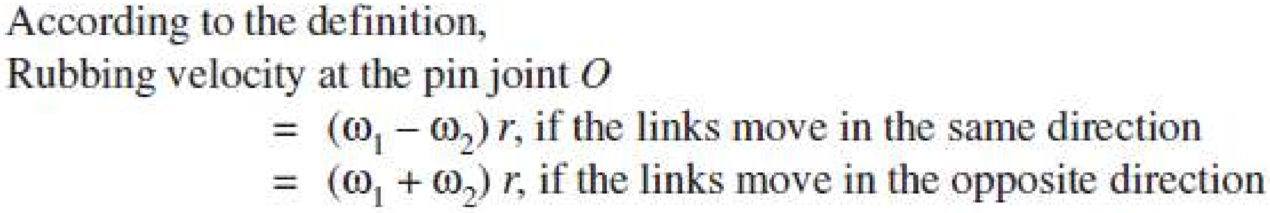
Links connected by pin joints.



Let ω_1 = Angular velocity of the link OA or the angular velocity of the point A with respect to O.

ω_2 = Angular velocity of the link OB or the angular velocity of the point B with respect to O, and

r = Radius of the pin.



According to the definition,

Rubbing velocity at the pin joint O

$= (\omega_1 - \omega_2)\, r$, if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$, if the links move in the opposite direction

LECTURE 3

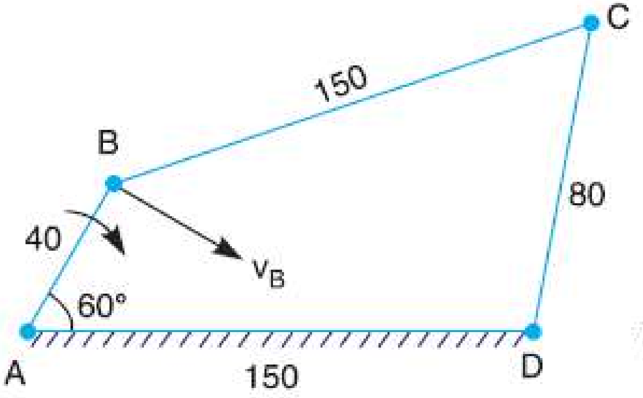
GRAPHICAL METHOD

DEPARTMENT OF MECHANICAL ENGINEERING

NUMERICAL EXAMPLE-1

In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.

Step-1 : Draw Space diagram with suitable scale



Space diagram (All dimensions in mm).

Source : R. S. Khurmi

Step-2 : Identify Given data & convert it into SI units

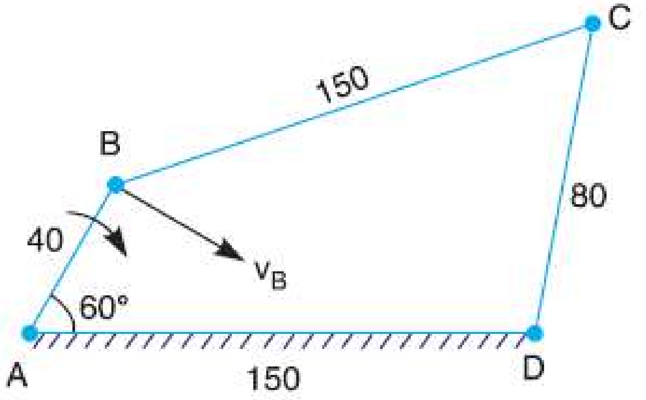
$N_{BA} = 120$ r.p.m. or $\omega_{BA} = 2\,\pi \times 120/60 = 12.568$ rad/s

AB = 0.04 m ; BC = 0.15 m; CD = 0.08 m; AD = 0.15 m

Step-3 : Calculate V_B

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$

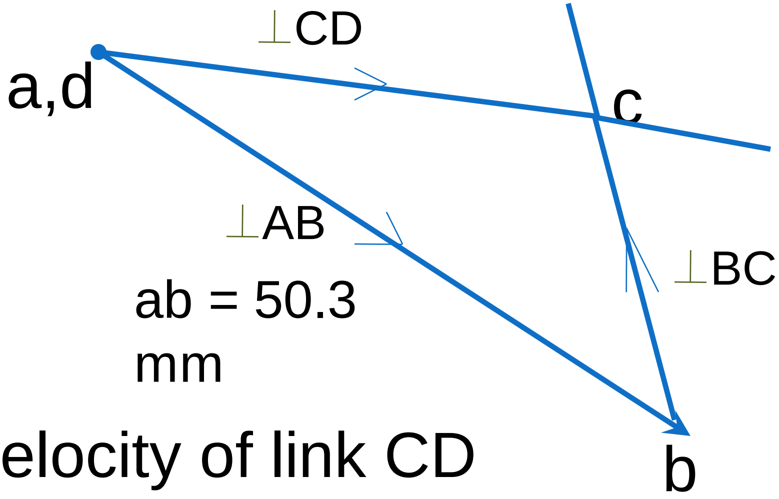
NUMERICAL EXAMPLE-1



Space diagram (All dimensions in mm).

Scale 1:100; i.e. $V_B = 0.503$ m/s = 50.3 mm

Source : R. S. Khurmi



ab = 50.3 mm

Question: Find the angular velocity of link CD

V_{CD} = cd = 38.5 mm (by measurement) = 0.385 m/s ,   CD = 0.08 m

∴ Angular velocity of link *CD*,

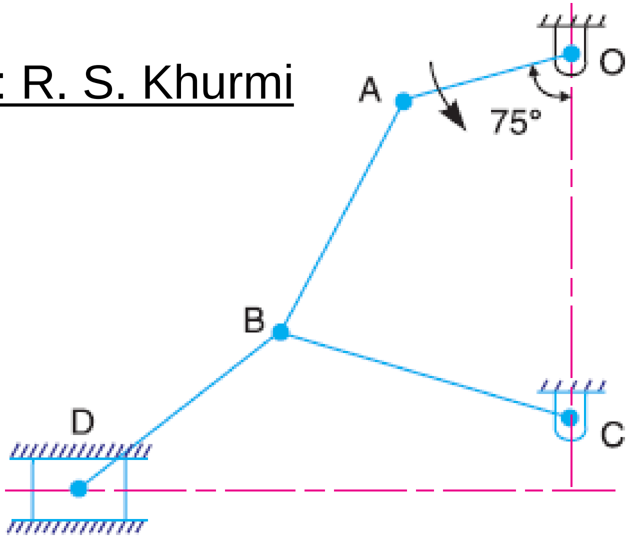
$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \text{ rad/s (clockwise about } D\text{)} \textbf{ Ans.}$$

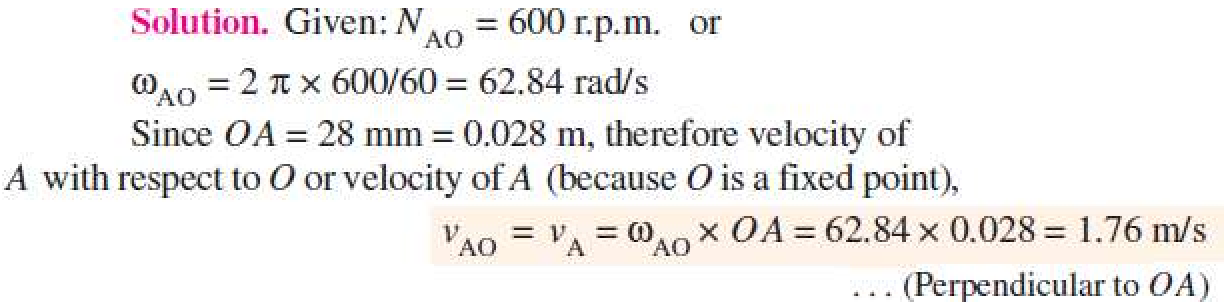
NUMERICAL EXAMPLE -2

In the given Fig., the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are : OA = 28 mm ; AB = 44 mm ; BC 49 mm ; and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider m
path and OC is vertical.

Source : R. S. Khurmi

Find: V_D, ω_{BD}





Solution. Given: N_{AO} = 600 r.p.m. or

$\omega_{AO} = 2\pi \times 600/60 = 62.84$ rad/s

Since OA = 28 mm = 0.028 m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

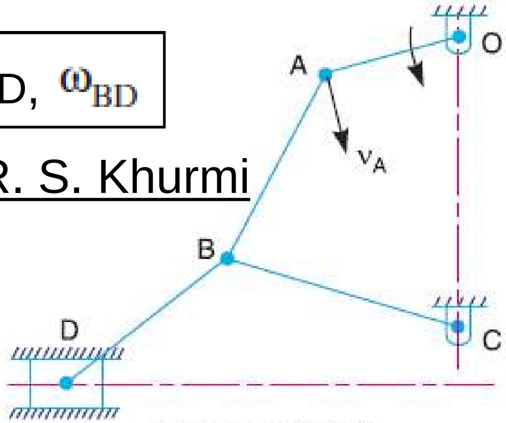
$$v_{AO} = v_A = \omega_{AO} \times OA = 62.84 \times 0.028 = 1.76 \text{ m/s}$$

. . . (Perpendicular to OA)

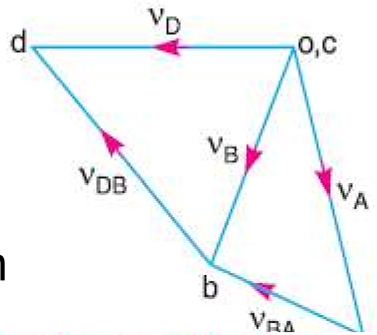
NUMERICAL EXAMPLE -2

Find: V_D, ω_{BD}

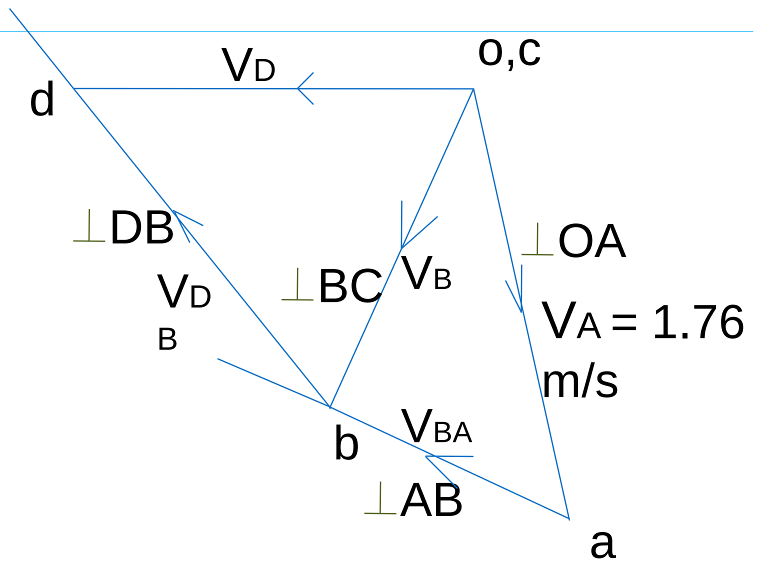
Source : R. S. Khurmi



(*a*) Space diagram.



Velocity diagram



Velocity diagram

By measurement, cd = od = V_D = 1.6 m/s

Angular velocity of the link BD

By measurement from velocity diagram, we find that velocity of *D* with respect to *B*,

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

Since the length of link *BD* = 46 mm = 0.046 m, therefore angular velocity of the link *BD*,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B) \quad \textbf{Ans.}$$

NUMERICAL EXAMPLE -3

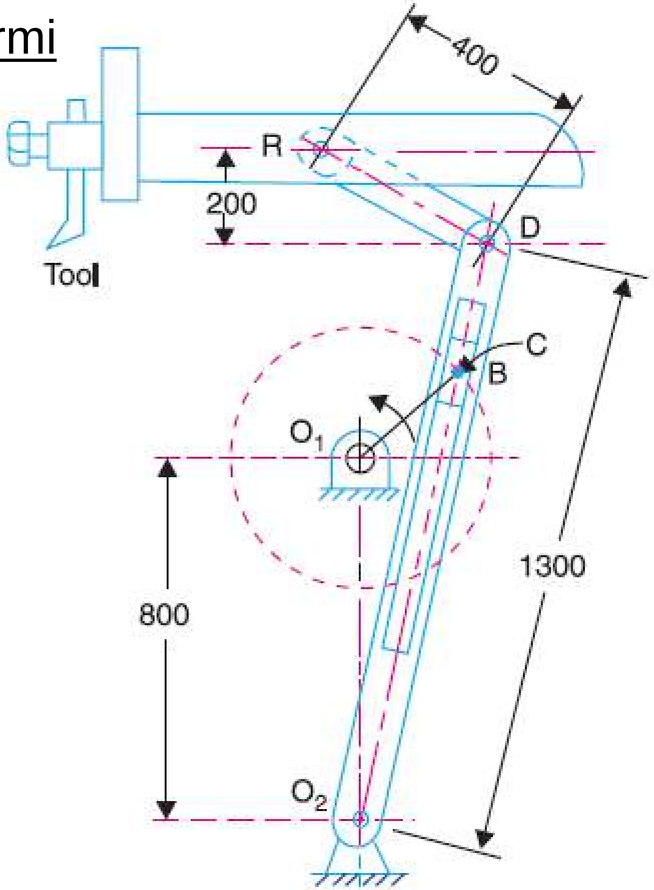
Source : R. S. Khurmi

A quick return mechanism of the crank and slotted lever type shaping machine is shown in the Fig. The dimensions of the various links are as follows :

O_1O_2 = 800 mm ; O_1B = 300 mm ; O_2D = 1300 mm ; DR = 400 mm.

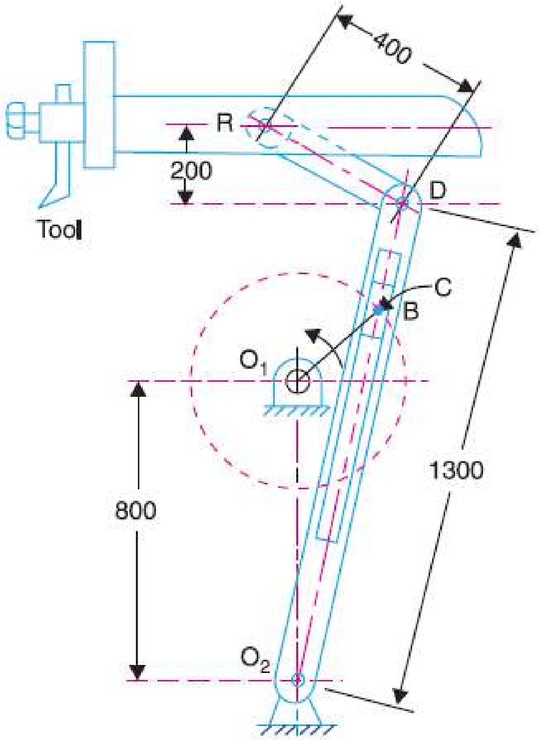
The crank O_1B makes an angle of 45° with the vertical and rotates at 40 r.p.m. in the counter clockwise direction.

Find : 1. velocity of the ram R, or the velocity of the cutting tool, and 2. angular velocity of link O_2D.

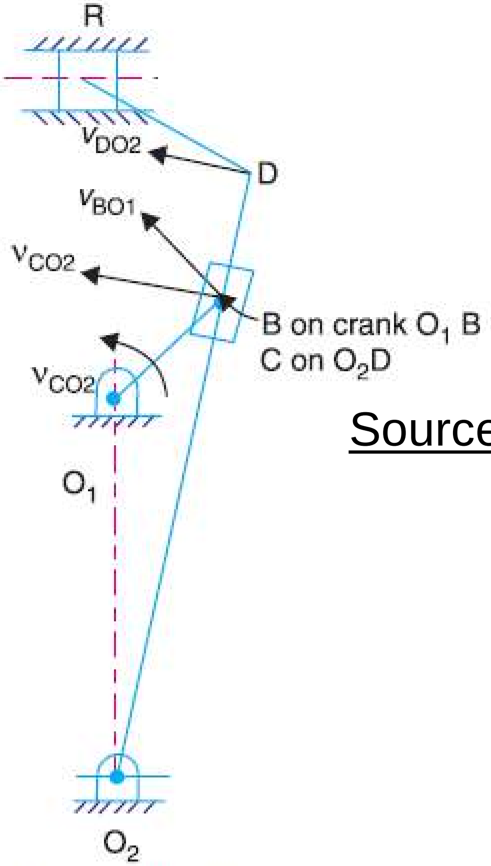


All dimensions in mm.

NUMERICAL EXAMPLE -3



All dimensions in mm.



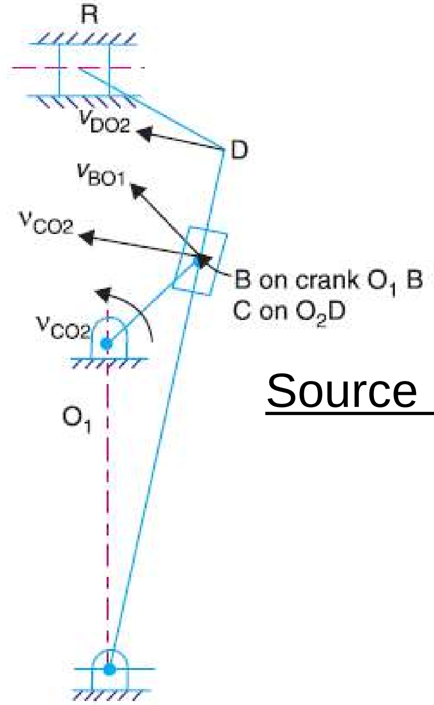
Space diagram

Source : R. S. Khurmi

Solution. Given: $N_{BO1} = 40$ r.p.m. or $\omega_{BO1} = 2\pi \times 40/60 = 4.2$ rad/s

$$v_{BO1} = v_B = \omega_{BO1} \times O_1B = 4.2 \times 0.3 = 1.26 \text{ m/s}$$

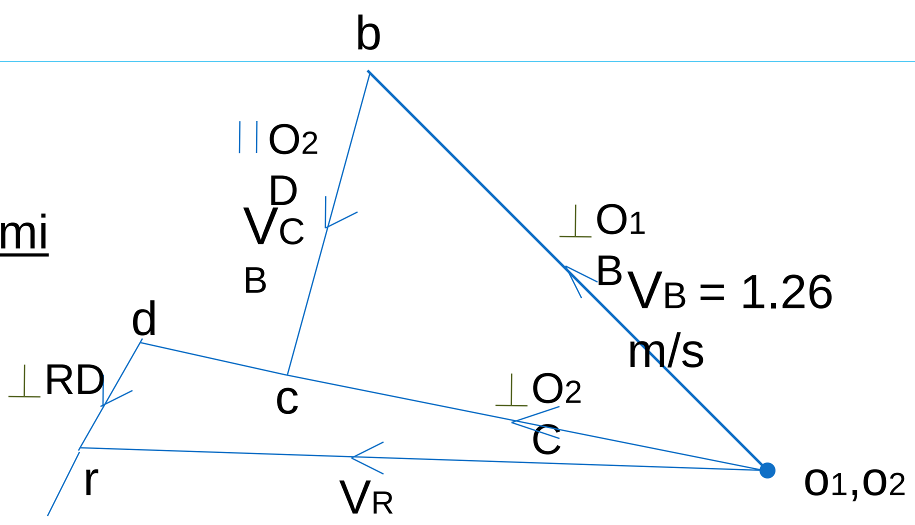
. . . (Perpendicular to O_1B)



Space diagram

AL EXAMPLE -3

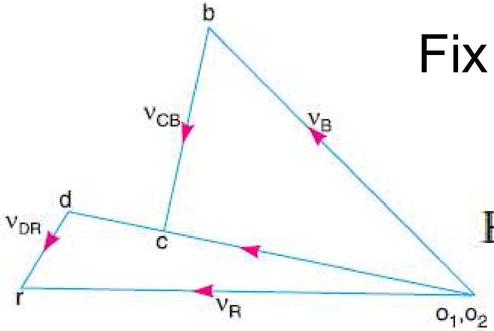
Source : R. S. Khurmi



Draw bc parallel to O_2D, to intersect at 'c'

Fix 'd' using the ratio $cd / o_2d = CD/O_2D$ ⟹ $\frac{cd}{cd + co_2} = \frac{CD}{O_2D}$

Find V_R & ω_{DO_2}



Velocity diagram.

By measurement, velocity of the ram R, v_R = vector $o_1r = 1.44$ m/s **Ans.**

Angular velocity of link O_2D

By measurement, $v_{DO2} = v_D$ = vector $o_2d = 1.32$ m/s

$$\omega_{DO2} = \frac{v_{DO2}}{O_2D} = \frac{1.32}{1.3} = 1.015 \text{ rad/s (Anticlockwise about } O_2) \quad \textbf{Ans.}$$

TUTORIAL PROBLEM

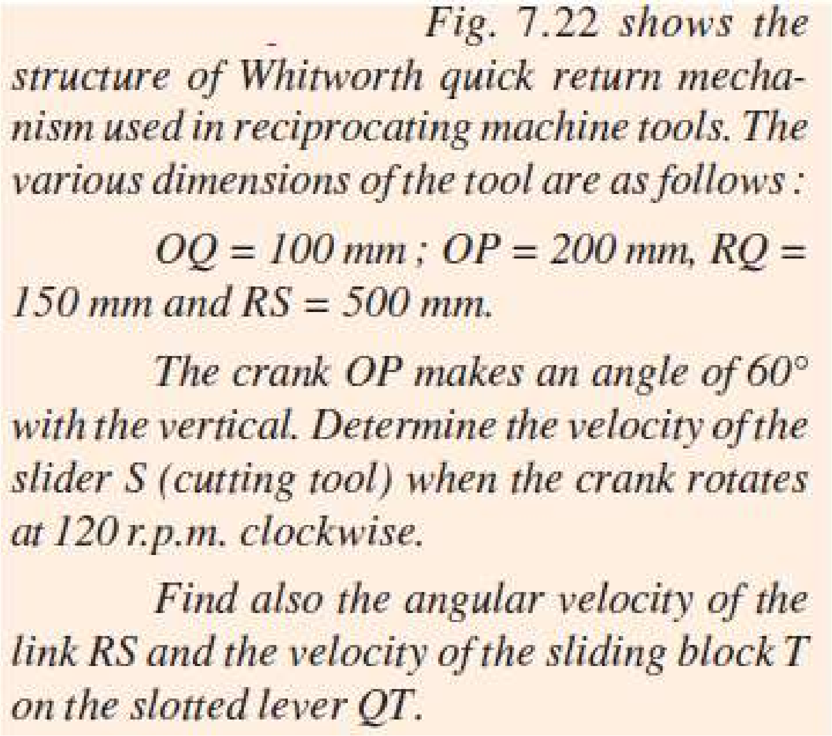


Fig. 7.22 shows the structure of Whitworth quick return mechanism used in reciprocating machine tools. The various dimensions of the tool are as follows :

OQ = 100 mm ; OP = 200 mm, RQ = 150 mm and RS = 500 mm.

The crank OP makes an angle of 60° with the vertical. Determine the velocity of the slider S (cutting tool) when the crank rotates at 120 r.p.m. clockwise.

Find also the angular velocity of the link RS and the velocity of the sliding block T on the slotted lever QT.

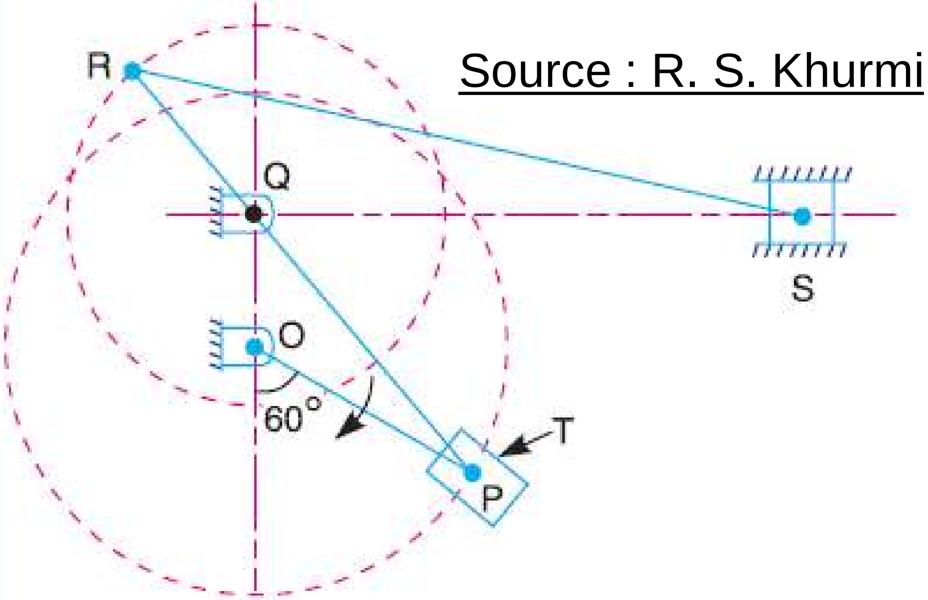
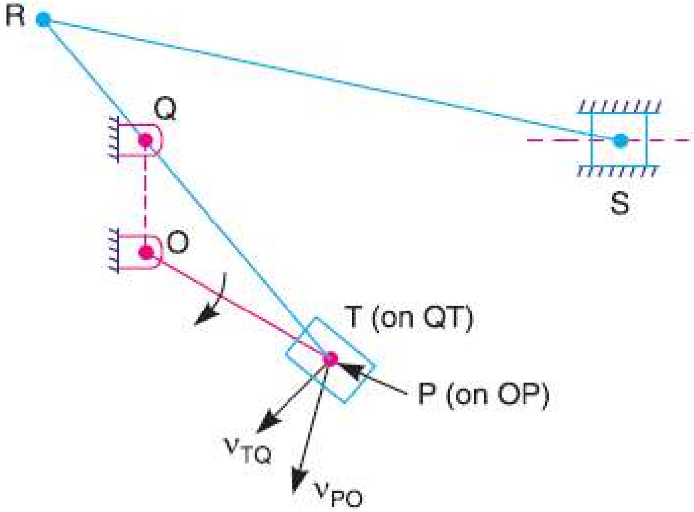


Fig. 7.22

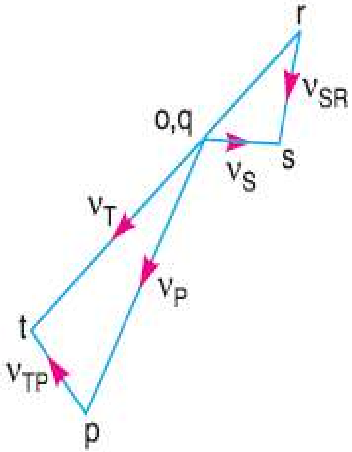
Source : R. S. Khurmi

TUTORIAL PROBLEM (SOLUTION)

Source : R. S. Khurmi



(a) Space diagram.



(b) Velocity diagram.

v_S = vector os = 0.8 m/s **Ans.**

Angular velocity of link RS

$$\omega_{RS} = \frac{v_{SR}}{RS} = \frac{0.96}{0.5} = 1.92 \text{ rad/s} \quad \textbf{Ans.}$$

Velocity of the sliding block T on the slotted lever QT

v_{TP} = vector pt = 0.85 m/s **Ans.**

LECTURE 4

INSTANTANEOUS CENTRE OF ROTATION

LIVE TO LEARN & LEARN TO SHARE

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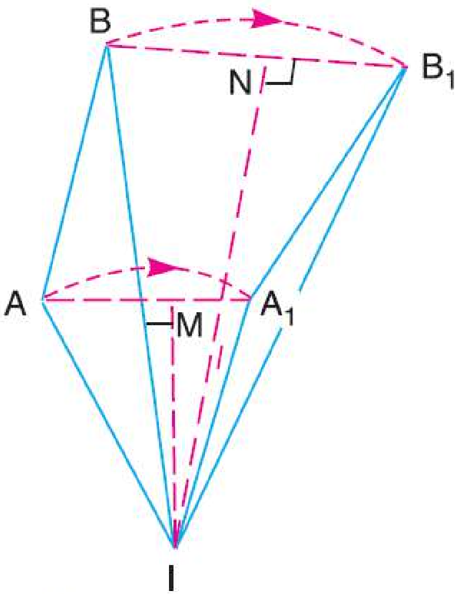
INSTANTANEOUS CENTRE METHOD

Translation of the link AB may be assumed to be a motion of pure rotation about some centre I, known as the instantaneous centre of rotation (also called centro or virtual centre).

The position of the centre of rotation must lie on the intersection of the right bisectors of chords *AA*1 and *BB*1. these bisectors intersect at *I as shown in Fig., which is the instantaneous centre of rotation or* virtual centre of the link *AB.*
(also called centro or virtual centre).



Instantaneous centre of rotation.

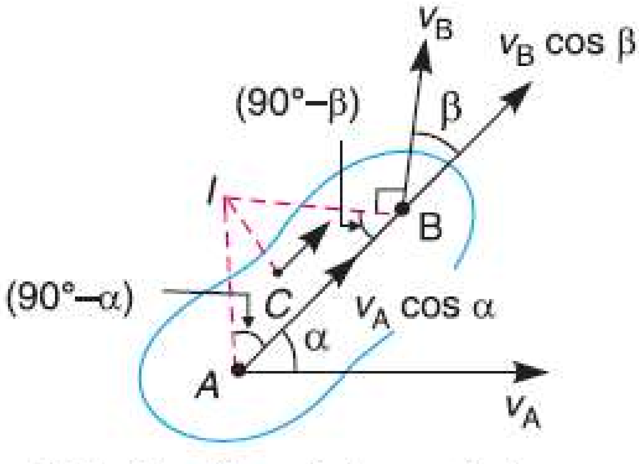
Source : R. S. Khurmi

VELOCITY OF A POINT ON A LINK BY INSTANTANEOUS CENTRE METHOD

V_A is known in Magnitude and direction V_B direction alone known How to calculate Magnitude of V_B using instantaneous centre method ?

Draw AI and BI perpendiculars to the directions V_A and V_B respectively to intersect at I, which is known as instantaneous centre of the link.

Source : R. S. Khurmi



Velocity of a point on a link.

VELOCITY OF A POINT ON A LINK BY INSTANTANEOUS CENTRE METHOD

Since A and B are the points on a rigid link, there cannot be any relative motion between them along the line AB.

Source : R. S. Khurmi

Now resolving the velocities along *A B*,

$$v_A \cos\alpha = v_B \cos\beta$$

or

$$\frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)} \quad \text{...}(i)$$

Applying Lami's theorem to triangle *ABI*,

$$\frac{AI}{\sin(90° - \beta)} = \frac{BI}{\sin(90° - \alpha)}$$

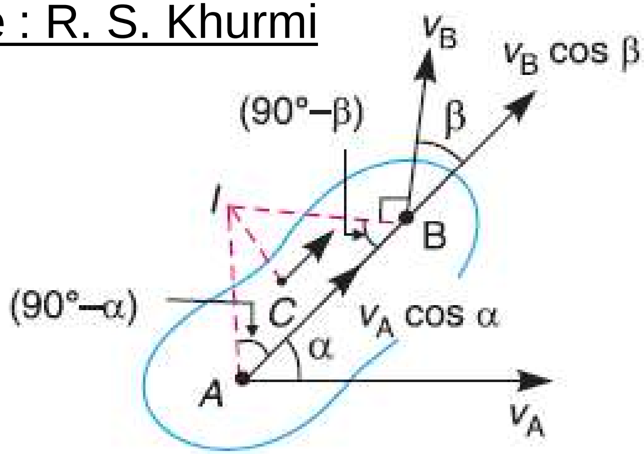
or

$$\frac{AI}{BI} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)} \quad \text{...}(ii)$$

From equation (*i*) and (*ii*),

$$\frac{v_A}{v_B} = \frac{AI}{BI} \quad \text{or} \quad \frac{v_A}{AI} = \frac{v_B}{BI} = \omega \quad \text{...}(iii)$$

where ω = Angular velocity of the rigid link.



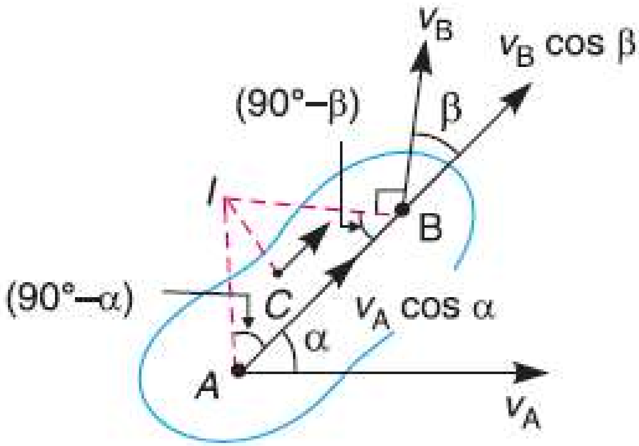
Velocity of a point on a link.

VELOCITY OF A POINT ON A LINK BY INSTANTANEOUS CENTRE METHOD

Source : R. S. Khurmi

If *C* is any other point on the link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \qquad \ldots(iv)$$



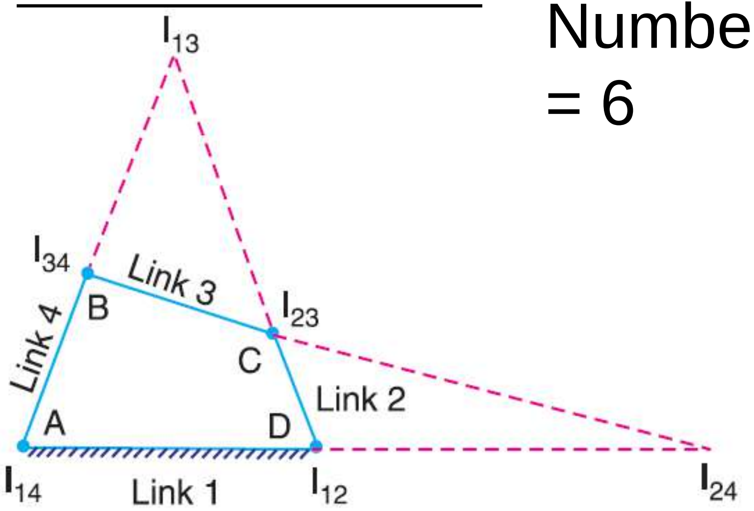
Velocity of a point on a link.

If V_A is known in magnitude and direction and V_B in direction only, then velocity of point B or any other point C lying on the same link may be determined (Using *iv*) in magnitude and direction.

TYPES OF INSTANTANEOUS CENTRES

Source : R. S. Khurmi

Number of Instantaneous Centres = N = 6



Types of instantaneous centres.

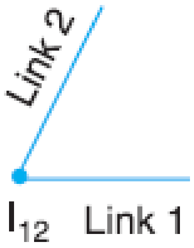
The instantaneous centres I_{12} and I_{14} *fixed instantaneous centres*

The instantaneous centres I_{23} and I_{34} *permanent instantaneous centres* as they move when the mechanism moves, but the joints are of permanent nature.

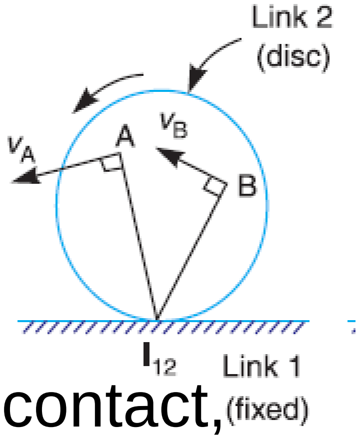
I_{13} and I_{24} are *neither fixed nor permanent instantaneous centres* as they vary with the configuration of the mechanism.

LOCATION OF INSTANTANEOUS CENTRES

Source : R. S. Khurmi

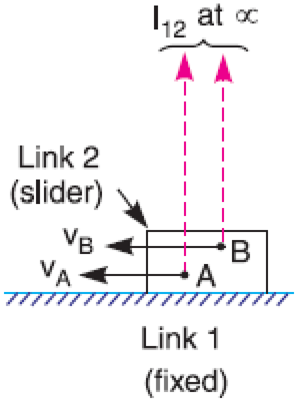


When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies on the centre of the pin



Pure rolling contact (i.e. link 2 rolls without slipping), the instantaneous centre lies on their point of contact.

$$\frac{v_A}{v_B}=\frac{I_{12}\,A}{I_{12}\,B}$$

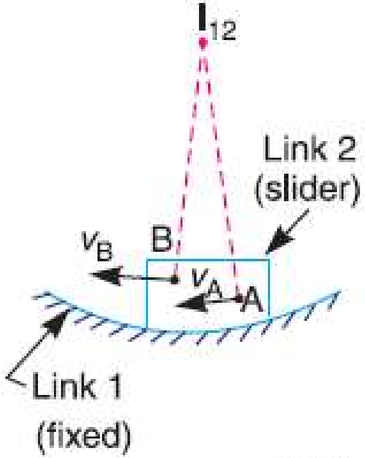


When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact.

The instantaneous centre lies at infinity and each point on the slider have the same velocity.

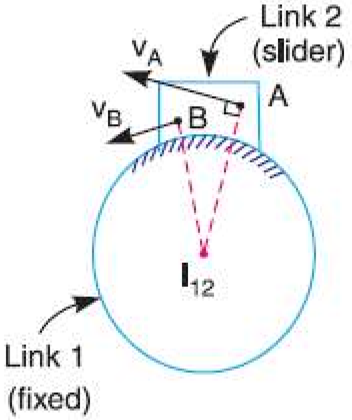
LOCATION OF INSTANTANEOUS CENTRES

When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact.



The instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

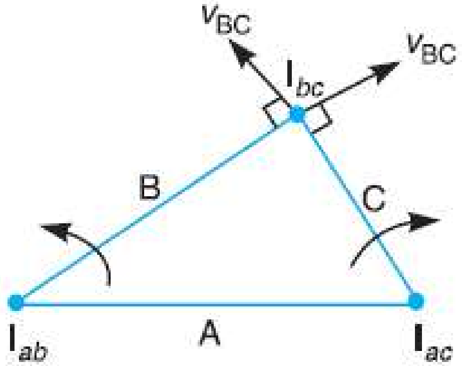
Source : R. S. Khurmi



When the link 2 (slider) moves on fixed link 1 having constant radius of curvature, the instantaneous centre lies at the centre of curvature i.e. the centre of the circle, for all configuration of the links.

ARONHOLD KENNEDY (OR THREE CENTRES IN LINE) THEOREM

It states that if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.



Aronhold Kennedy's theorem.

Source : R. S. Khurmi

the velocity of the point I_{bc} cannot be perpendicular to both lines $I_{ab}\, I_{bc}$ and $I_{ac}\, I_{bc}$ unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac}.

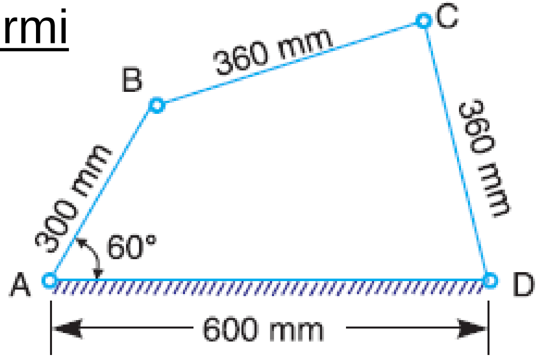
Thus the three instantaneous centres (I_{ab}, I_{ac} and I_{bc}) must lie on the same straight line.

The exact location of I_{bc} on line $I_{ab}\, I_{ac}$ depends upon the directions and magnitudes of the angular velocities of B and C relative to A.

NUMERICAL EXAMPLE-1

In a pin jointed four bar mechanism, as shown in Fig. AB = 300 mm, BC = CD = 360 mm, and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find the angular velocity of the link BC

Source : R. S. Khurmi



Solution. Given : $N_{AB} = 100$ r.p.m or

$$\omega_{AB} = 2\,\pi \times 100/60 = 10.47 \text{ rad/s}$$

Since the length of crank $AB = 300$ mm $= 0.3$ m,

therefore velocity of point B on link AB,

$$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141 \text{ m/s}$$

Location of instantaneous centres:

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

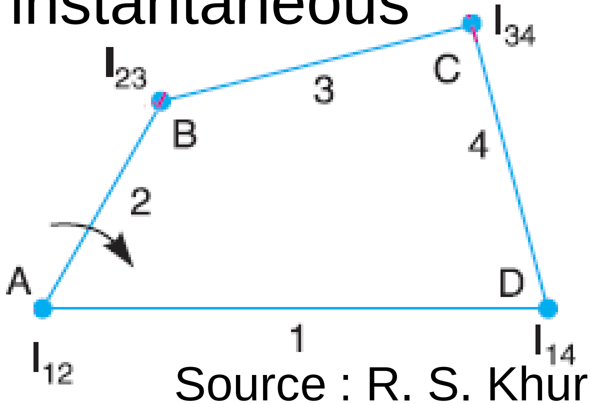
1. Find number of Instantaneous centres

NUMERICAL EXAMPLE-1

2. List the Ins. centres

Links	1	2	3	4
Ins. Centres	12	13	14	
		23	24	
			34	

3. Draw configuration (space) diagram with suitable scale. And, Locate the fixed and permanent instantaneous centres by inspection



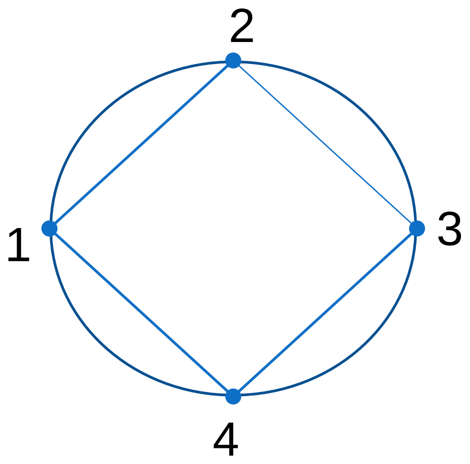
Source : R. S. Khurmi

I_{12}, I_{14} – Fixed centres;
I_{23}, I_{34} – Permanent centres

How to locate I_{13}, I_{24} – Neither fixed nor Permanent centres

NUMERICAL EXAMPLE-1

4. Locate the neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem.

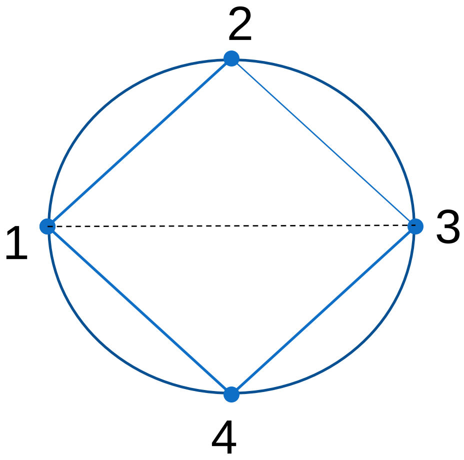


Source : R. S. Khurmi

Draw a circle with any arbitrary radius
At equal distance locate Links 1, 2, 3 & 4 as points on the circle.

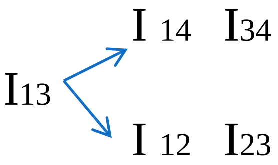
NUMERICAL EXAMPLE-1

Locating I_{13}



Source : R. S. Khurmi

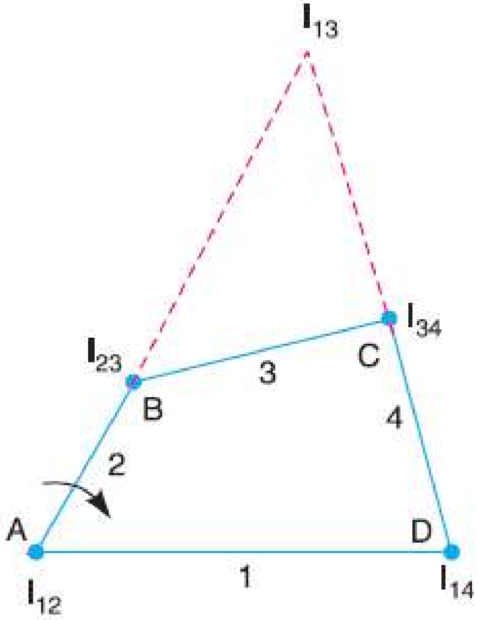
13 is common side to Triangle 134 & 123



I_{13} → I_{14} I_{34}

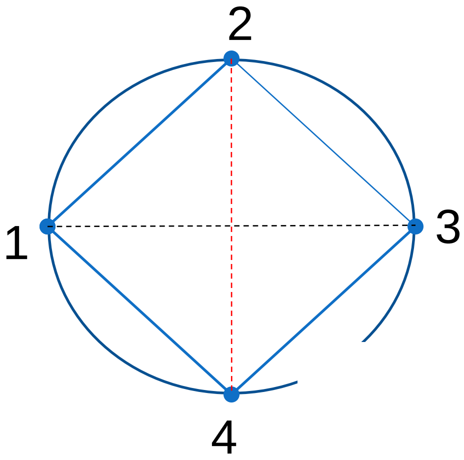
I_{13} → I_{12} I_{23}

Therefore, I_{13} lies on the intersection of the lines joining the points $I_{14}I_{34}$ & I_{12} I_{23}



NUMERICAL EXAMPLE-1

Locating I_{24}

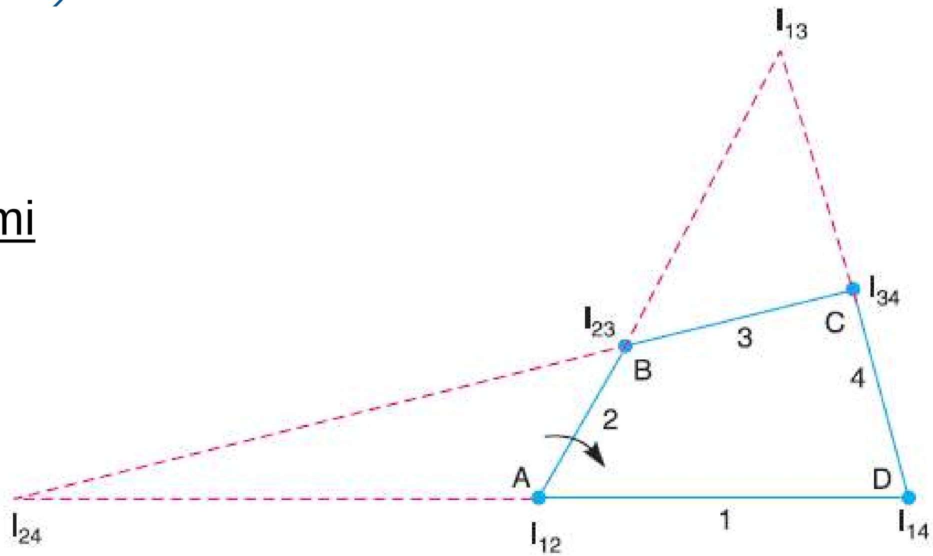


24 is common side to Triangle 124 & 234

I_{24}
I_{12} I_{14}
I_{34} I_{23}

Therefore, I_{24} lies on the intersection of the lines joining the points $I_{12}I_{14}$ & I_{34} I_{23}

Source : R. S. Khurmi



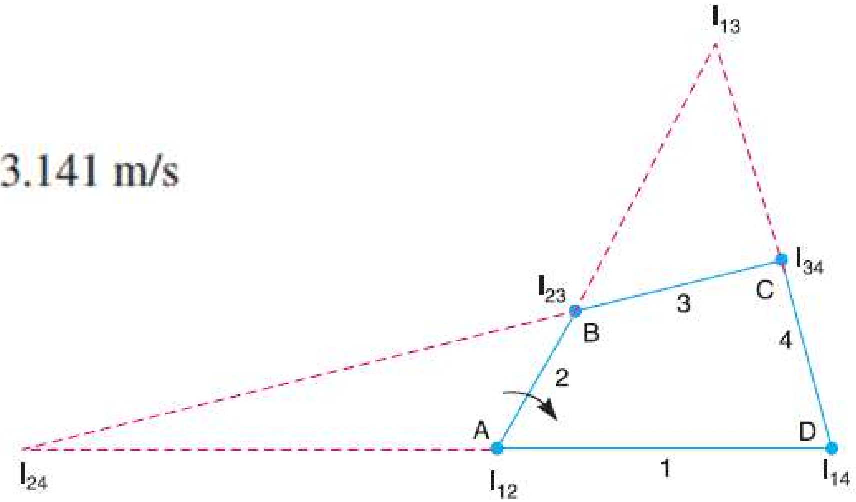
Thus all the six instantaneous centres are located.

NUMERICAL EXAMPLE-1

Find Angular velocity of the link BC

$$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141 \text{ m/s}$$

We know that:



Source : R. S. Khurmi

Let ω_{BC} = Angular velocity of the link *BC*.

Since *B* is also a point on link *BC*, therefore velocity of point *B* on link *BC*,

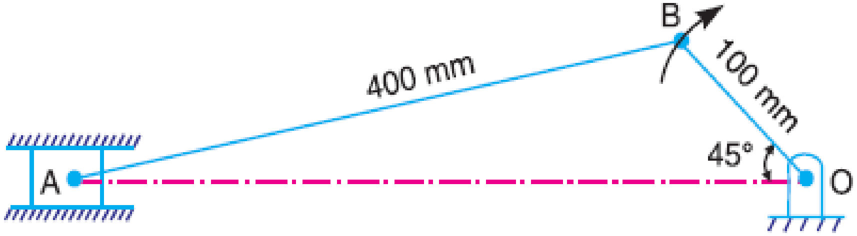
$$v_B = \omega_{BC} \times I_{13}B$$

By measurement, we find that $I_{13}B = 500 \text{ mm} = 0.5 \text{ m}$

$$\therefore \quad \omega_{BC} = \frac{v_B}{I_{13}B} = \frac{3.141}{0.5} = 6.282 \text{ rad/s} \quad \textbf{Ans.}$$

NUMERICAL EXAMPLE-2

Locate all the instantaneous centres of the slider crank mechanism as shown in the Fig. The lengths of crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s, find: 1. Velocity of the slider A, and 2. Angular velocity



Source : R. S. Khurmi

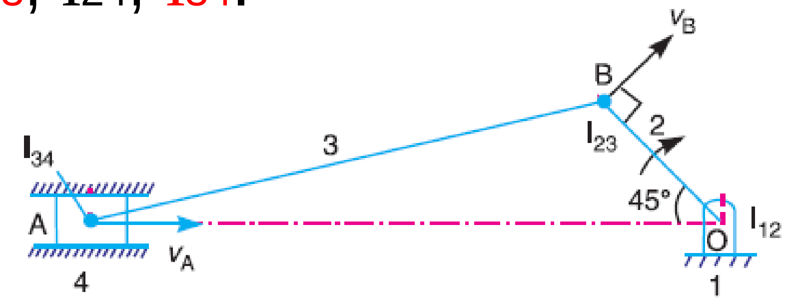
Solution. Given : $\omega_{OB} = 10$ rad/ s; $OB = 100$ mm $= 0.1$ m

We know that linear velocity of the crank OB,

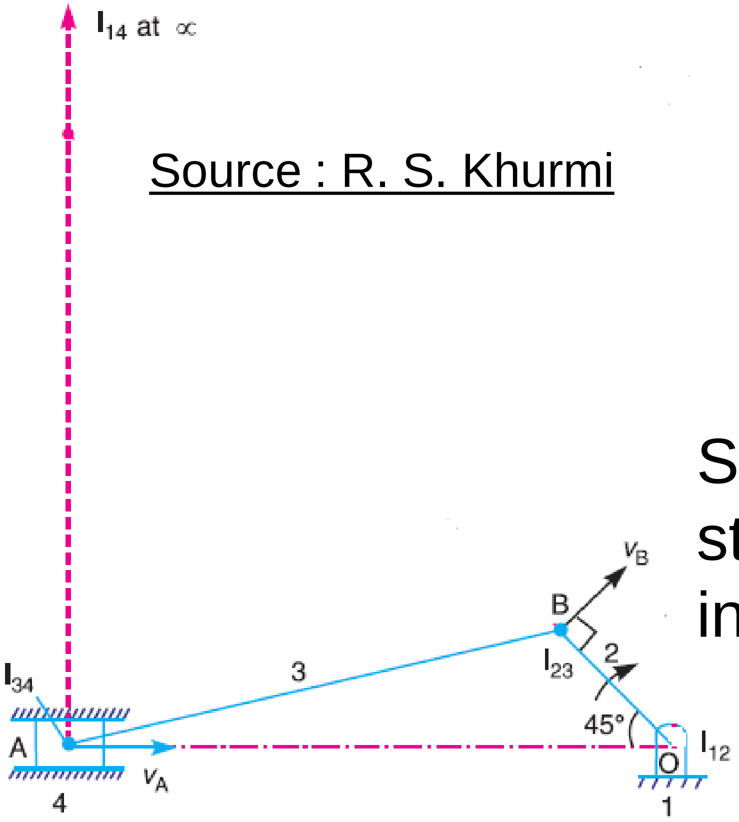
$$v_{OB} = v_B = \omega_{OB} \times OB = 10 \times 0.1 = 1 \text{ m/s}$$

NUMERICAL EXAMPLE-2

- Draw configuration diagram with suitable scale.
- Locate Ins. Centres (Here, n = 4; No. of Ins. Centres N = 6)
- Ins. Centers are I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}.



By inspection Locate I_{12}, I_{23} & I_{34}.



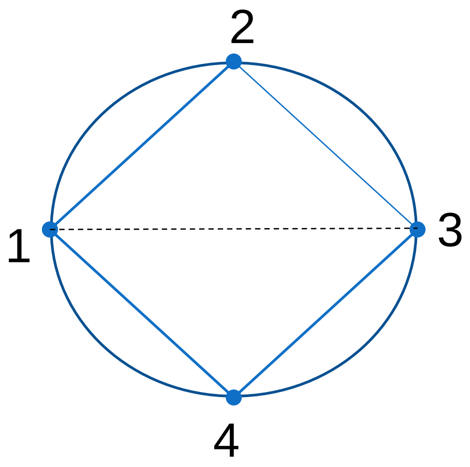
Source : R. S. Khurmi

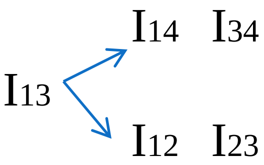
Since the slider (link 4) moves on a straight surface (link 1), I_{14}, will be at infinity.

NUMERICAL EXAMPLE-2

- Ins. Centers are I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}.
- Fixing I_{13}…… ?

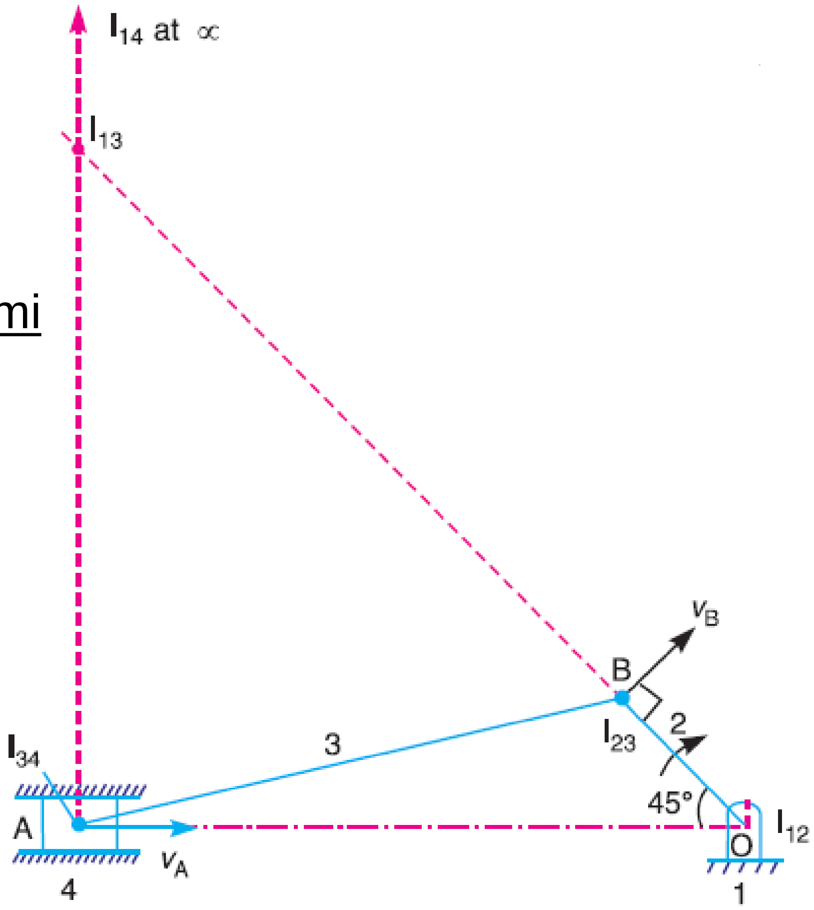
Source : R. S. Khurmi




I_{13}

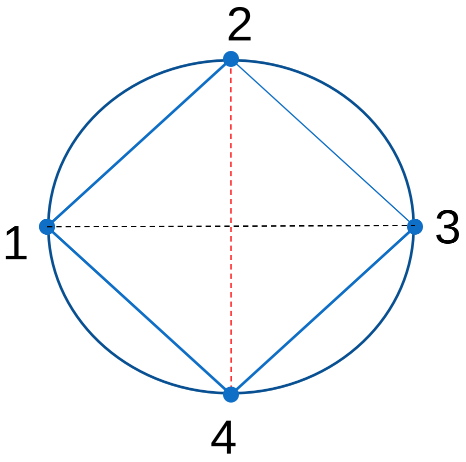
I_{14} I_{34}

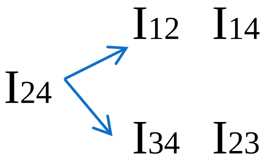
I_{12} I_{23}



NUMERICAL EXAMPLE-2

- Ins. Centers are I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}.
- Fixing I_{24}…… ?

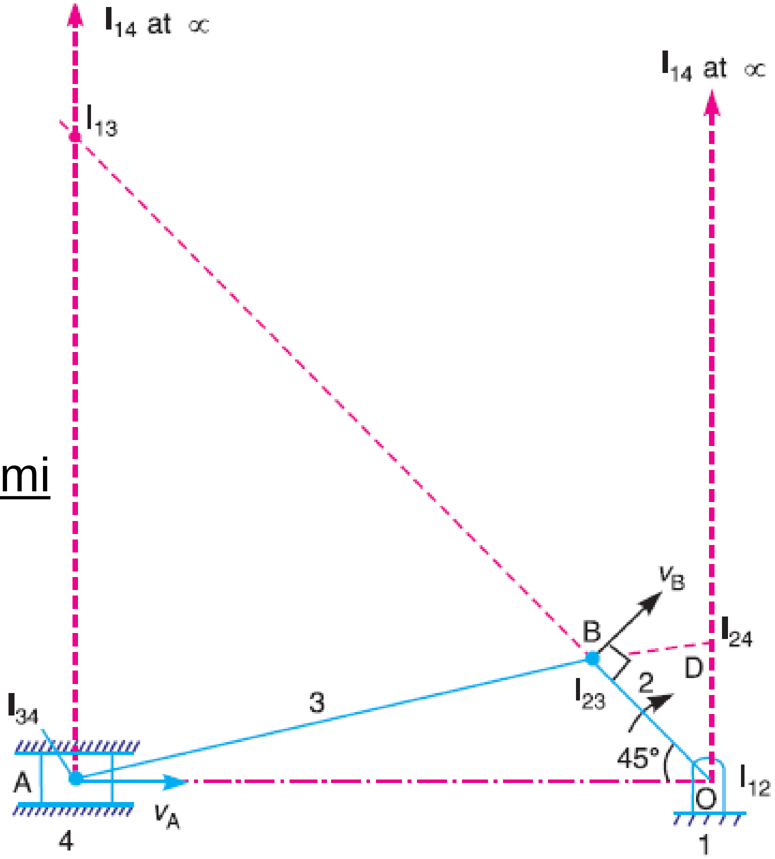




I_{24} → I_{12} I_{14}

I_{24} → I_{34} I_{23}

Source : R. S. Khurmi

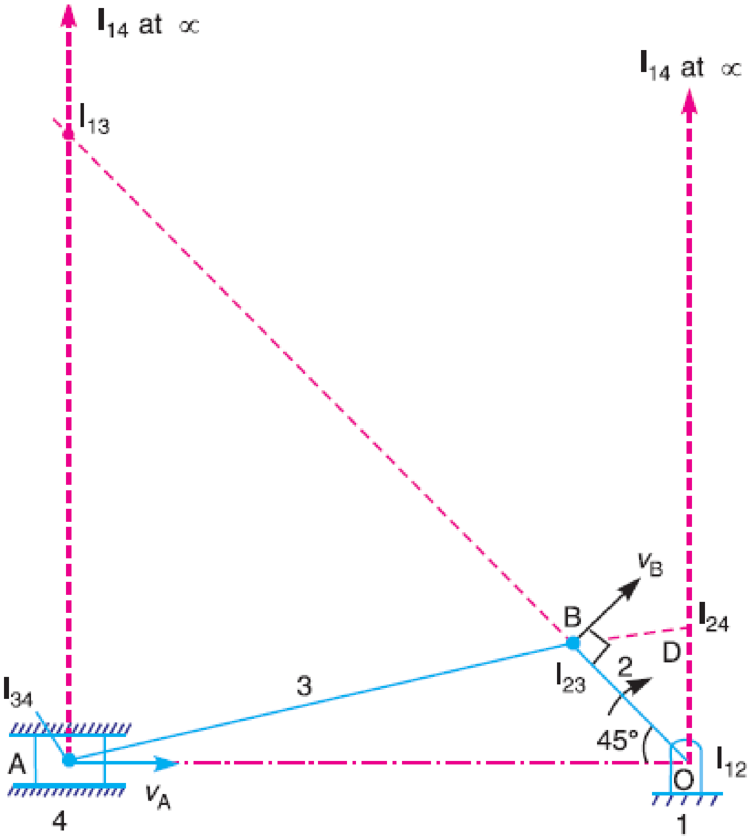


I_{14} can be moved to any convenient joint

NUMERICAL EXAMPLE-2

Solution:

Source : R. S. Khurmi



By measurement, we find that

$$I_{13}A = 460 \text{ mm} = 0.46 \text{ m} ; \text{ and } I_{13}B = 560 \text{ mm} = 0.56 \text{ m}$$

1. *Velocity of the slider A*

Let v_A = Velocity of the slider A.

We know that
$$\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B}$$

or
$$v_A = v_B \times \frac{I_{13}A}{I_{13}B} = 1 \times \frac{0.46}{0.56} = 0.82 \text{ m/s} \quad \textbf{Ans.}$$

2. *Angular velocity of the connecting rod AB*

Let ω_{AB} = Angular velocity of the connecting rod AB.

We know that
$$\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} = \omega_{AB}$$

$$\therefore \quad \omega_{AB} = \frac{v_B}{I_{13}B} = \frac{1}{0.56} = 1.78 \text{ rad/s} \quad \textbf{Ans.}$$

VISIT THE FOLLOWING VIDEOS IN YOUTUBE

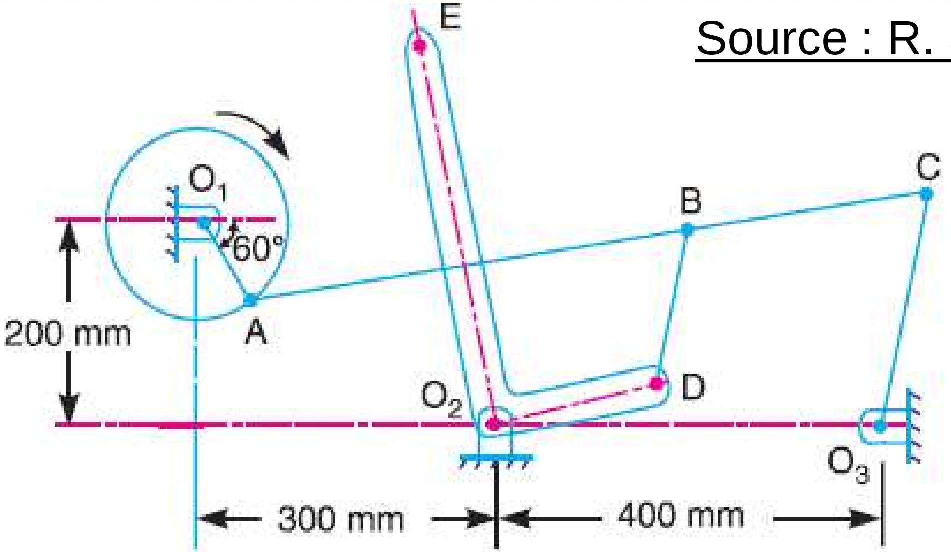
- https://www.youtube.com/watch?v=-tgruur8O0Q
- https://www.youtube.com/watch?v=WNh5Hp0lgms
- https://www.youtube.com/watch?v=ha2PzDt5SbE

EXERCISE-1

The mechanism of a wrapping machine, as shown in Fig. 6.18, has the following dimensions :

O_1A = 100 mm; AC = 700 mm; BC = 200 mm; O_3C = 200 mm; O_2E = 400 mm; O_2D = 200 mm and BD = 150 mm.

The crank O_1A rotates at a uniform speed of 100 rad/s. Find the velocity of the point E of the bell crank lever by instantaneous centre method.

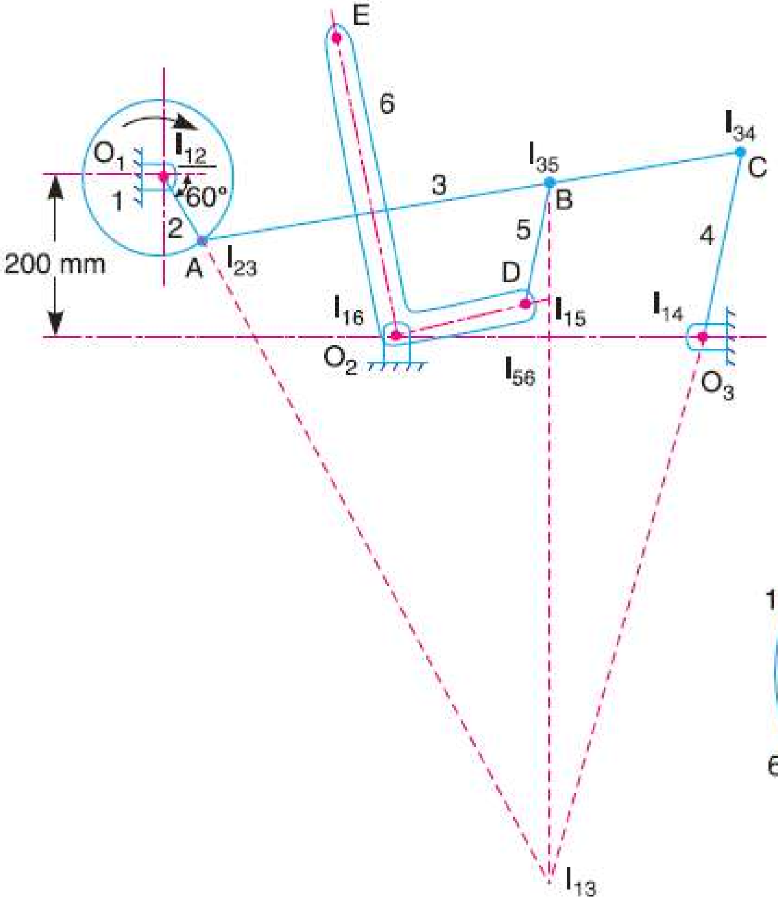


Source : R. S. Khurmi

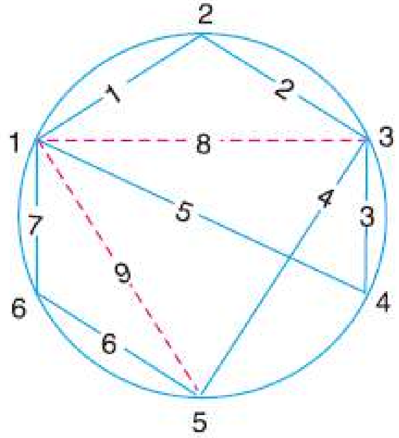
Fig. 6.18

EXERCISE-1: SOLUTION

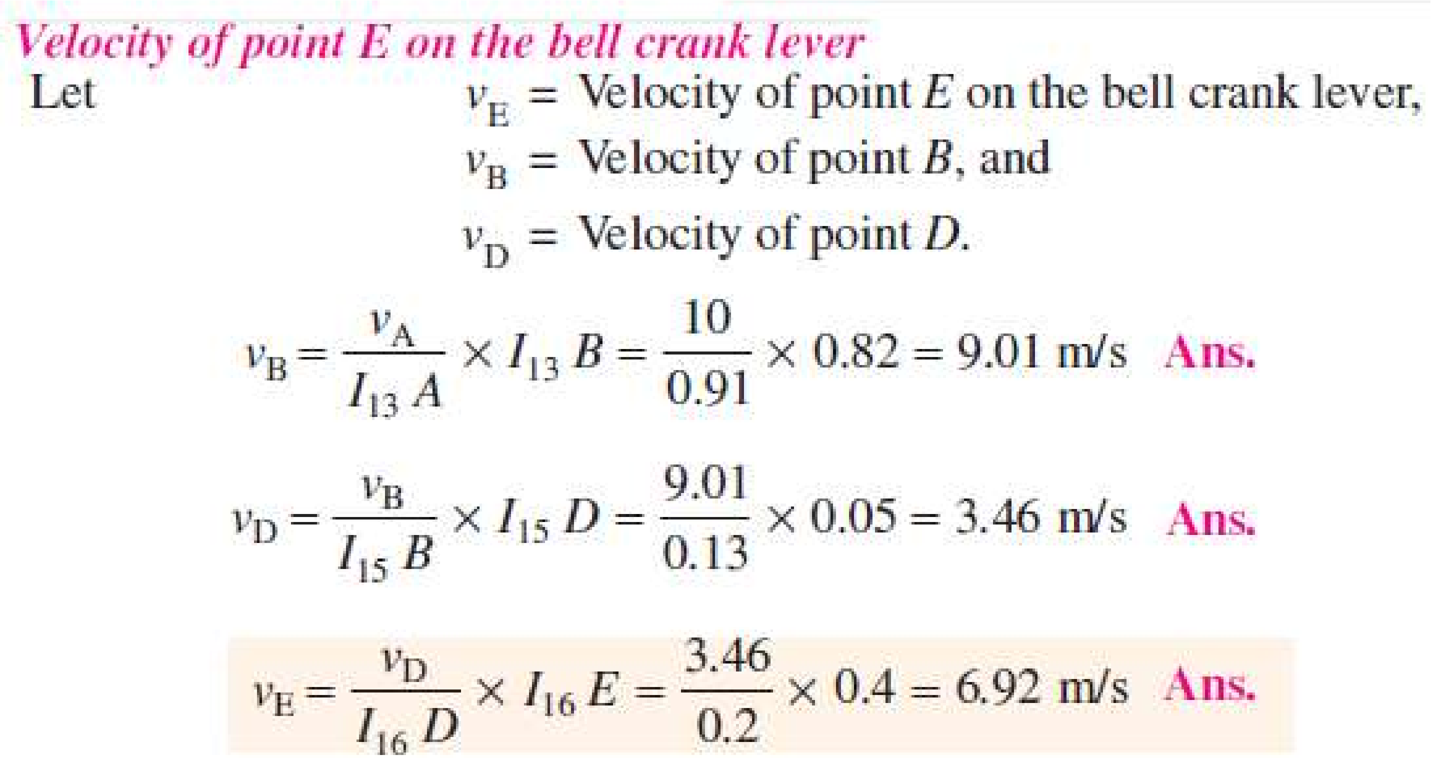
$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$



<u>Source : R. S. Khurmi</u>



EXERCISE-1: ANSWER



Velocity of point E on the bell crank lever

Let v_E = Velocity of point E on the bell crank lever,

v_B = Velocity of point B, and

v_D = Velocity of point D.

$$v_B = \frac{v_A}{I_{13}\,A} \times I_{13}\,B = \frac{10}{0.91} \times 0.82 = 9.01 \text{ m/s} \quad \textbf{Ans.}$$

$$v_D = \frac{v_B}{I_{15}\,B} \times I_{15}\,D = \frac{9.01}{0.13} \times 0.05 = 3.46 \text{ m/s} \quad \textbf{Ans.}$$

$$v_E = \frac{v_D}{I_{16}\,D} \times I_{16}\,E = \frac{3.46}{0.2} \times 0.4 = 6.92 \text{ m/s} \quad \textbf{Ans.}$$

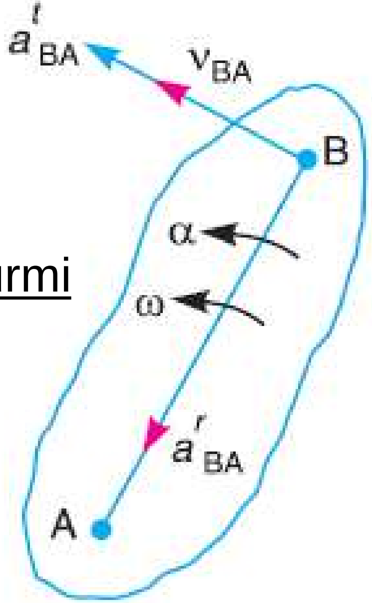
LECTURE 5

ACCELERATION IN MECHANISMS

DEPARTMENT OF MECHANICAL ENGINEERING

ACCELERATION IN MECHANISMS

Acceleration analysis plays a very important role in the development of machines and mechanisms



Acceleration for a link.

Let the point B moves with respect to A, with an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link AB.

Source : R. S. Khurmi

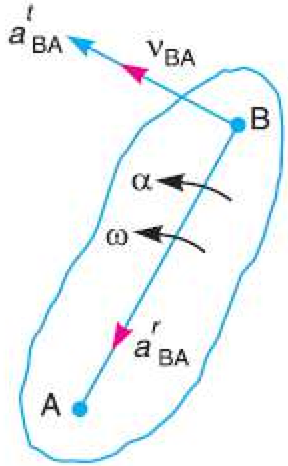
1. The centripetal or radial component of acceleration, which is perpendicular to the velocity (i.e. parallel to link AB) of the particle at the given instant.

$$a^r_{BA} = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v^2_{BA} / AB \qquad \ldots\left(\because\ \omega = \frac{v_{BA}}{AB}\right)$$

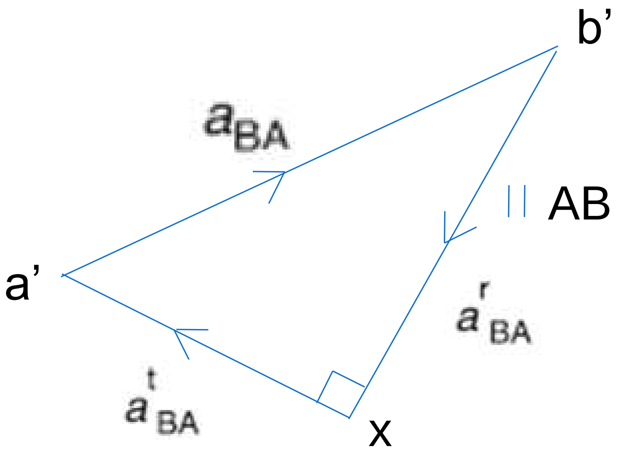
2. The tangential component, which is parallel to the velocity (i.e. Perpendicular to Link AB) of the particle at the given i

$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

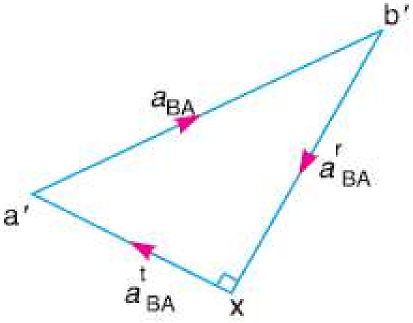
ACCELERATION DIAGRAM FOR A LINK



Acceleration for a link.



Source : R. S. Khurmi



Acceleration diagram.

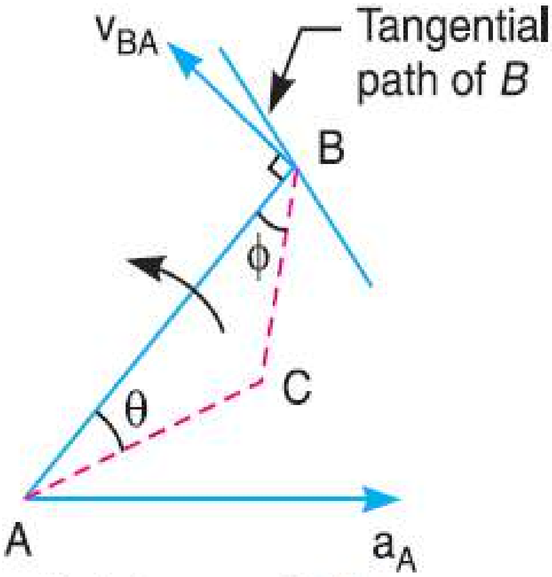
Total acceleration of B with respect to A is the vector sum of radial component and tangential component of acceleration

$$\overrightarrow{a_{BA}} = \overrightarrow{a^r_{BA}} + \overrightarrow{a^t_{BA}}$$

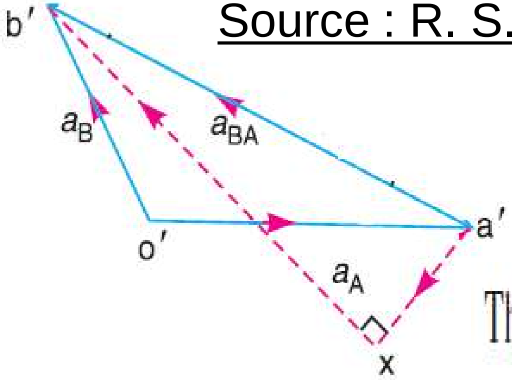
ACCELERATION OF A POINT ON A LINK

Let the acceleration of the point A i.e. a_A is known in magnitude and direction and the direction of path of B is given.

How to determine a_B ?
Draw acceleration diagram.

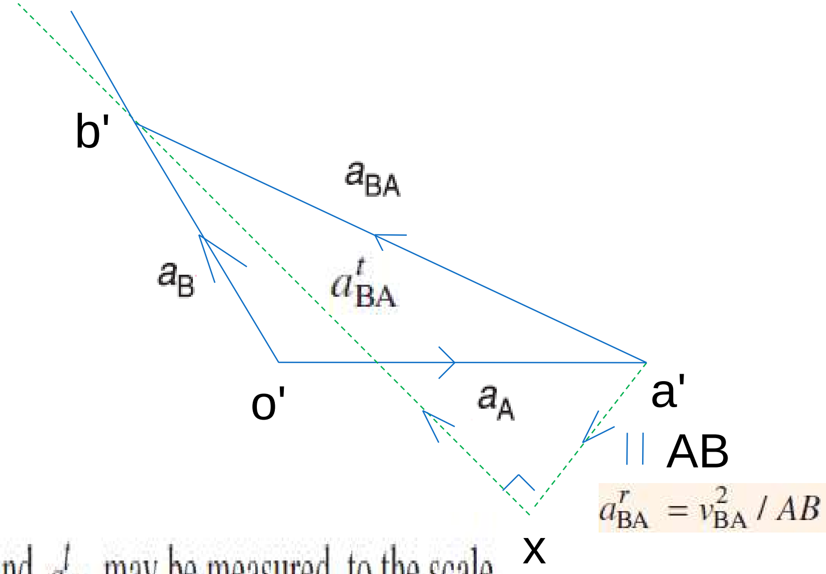


Points on a Link.



Source : R. S. Khurmi

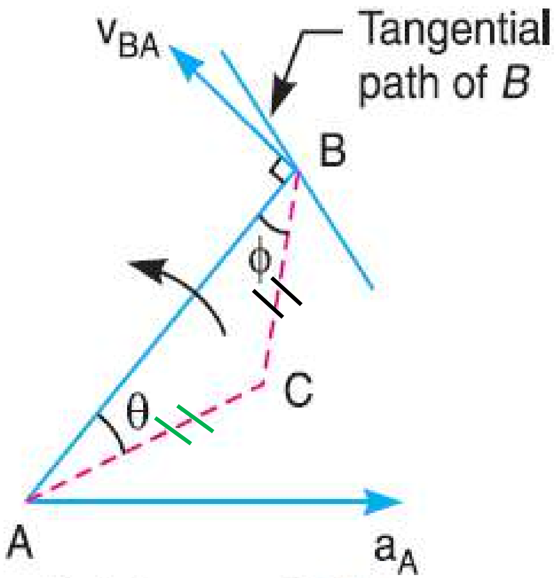
The values of a_B, a_{BA} and a^t_{BA} may be measured, to the scale.



$a^r_{BA} = v^2_{BA} / AB$

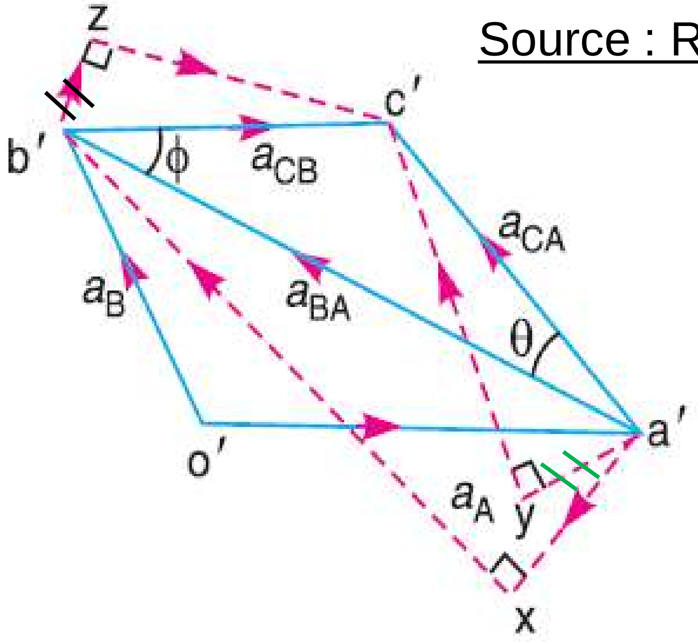
ACCELERATION OF A POINT ON A LINK

For any other point C on the link, draw triangle a'b'c' similar to triangle ABC.



Points on a Link.

Source : R. S. Khurmi

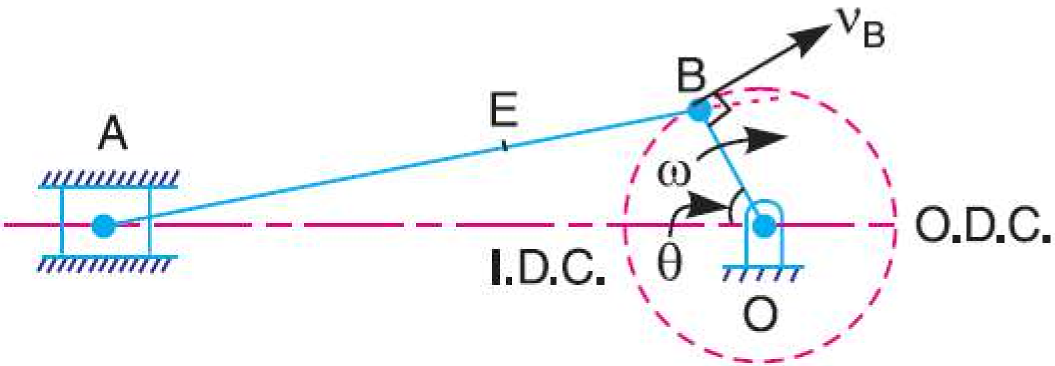


Acceleration diagram.

Mathematically, angular acceleration of the link A B,

$$\alpha_{AB} = a^{t}_{BA} / AB$$

ACCELERATION IN SLIDER CRANK MECHANISM



Slider crank mechanism.

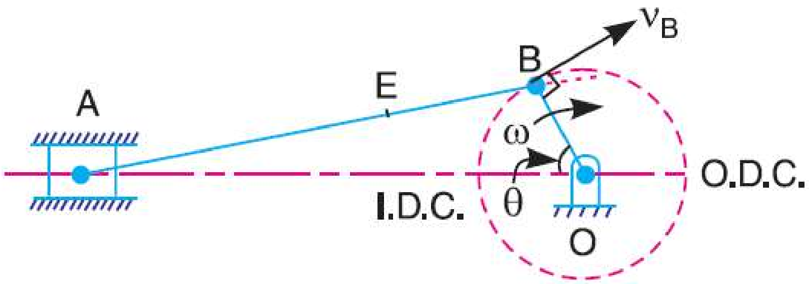
Source : R. S. Khurmi

$v_{BO} = v_B = \omega_{BO} \times OB$, acting tangentially at B.

$$a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{OB}$$

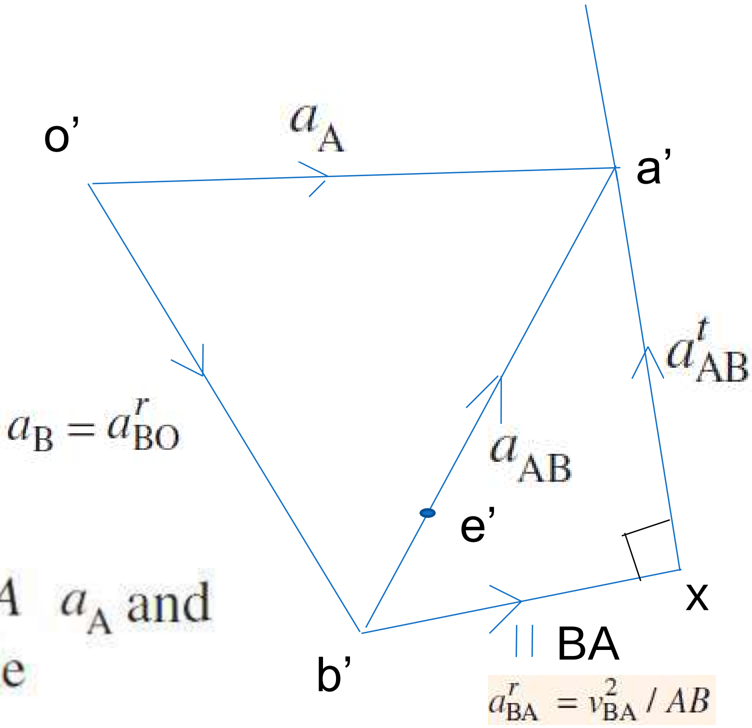
A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.

ACCELERATION IN SLIDER CRANK MECHANISM



Slider crank mechanism.

Source : R. S. Khurmi



acceleration of the piston or the slider A a_A and a^t_{AB} may be measured to the scale

Point e' can be fixed using $a'e'/a'b' = AE/AB$

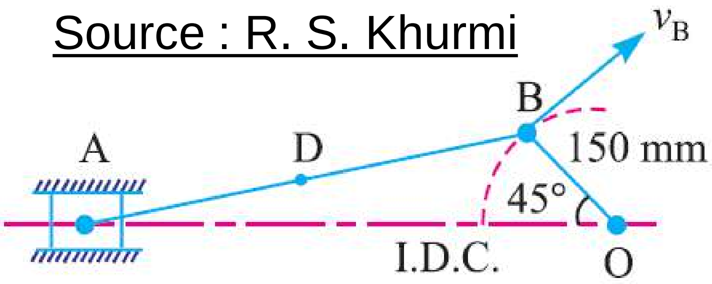
angular acceleration of A B, $\alpha_{AB} = a^t_{AB} / AB$

NUMERICAL EXAMPLE -1

The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : 1. linear velocity and acceleration of the midpoint of the connecting rod, and 2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

NUMERICAL EXAMPLE -1

Source : R. S. Khurmi



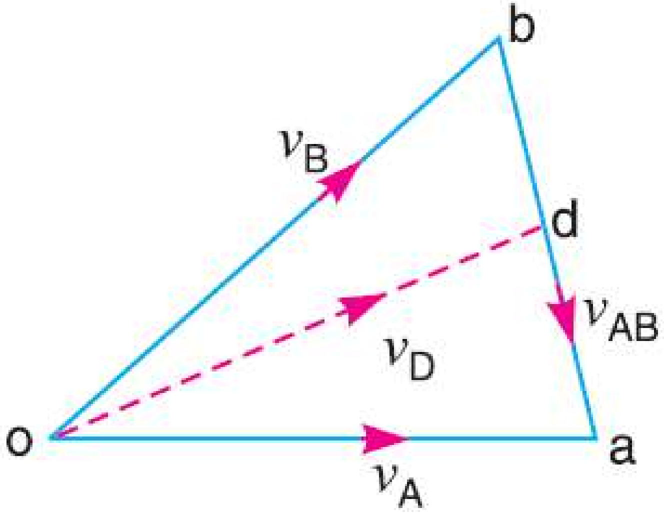
Space diagram.

Solution.

Given : $N_{BO} = 300$ r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s;

$OB = 150$ mm $= 0.15$ m ; $BA = 600$ mm $= 0.6$ m

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$



Velocity diagram.

By measurement, v_{AB} = vector ba = 3.4 m/s

v_A = vector oa = 4 m/s

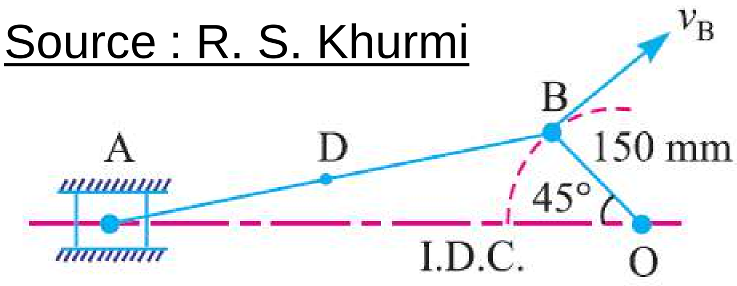
Since D is the midpoint of AB, d is also midpoint of vector ba.

velocity of the midpoint D

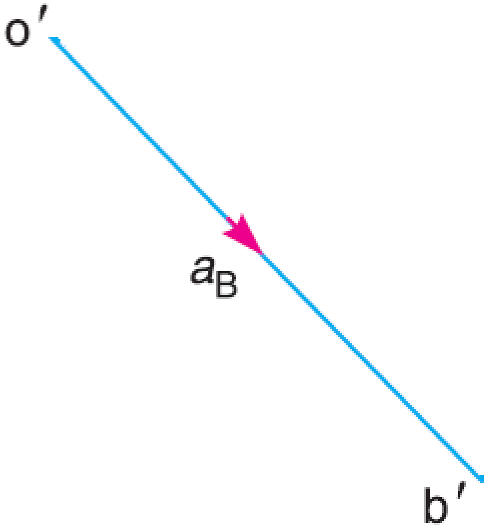
v_D = vector od = 4.1 m/s **Ans.**

NUMERICAL EXAMPLE -1

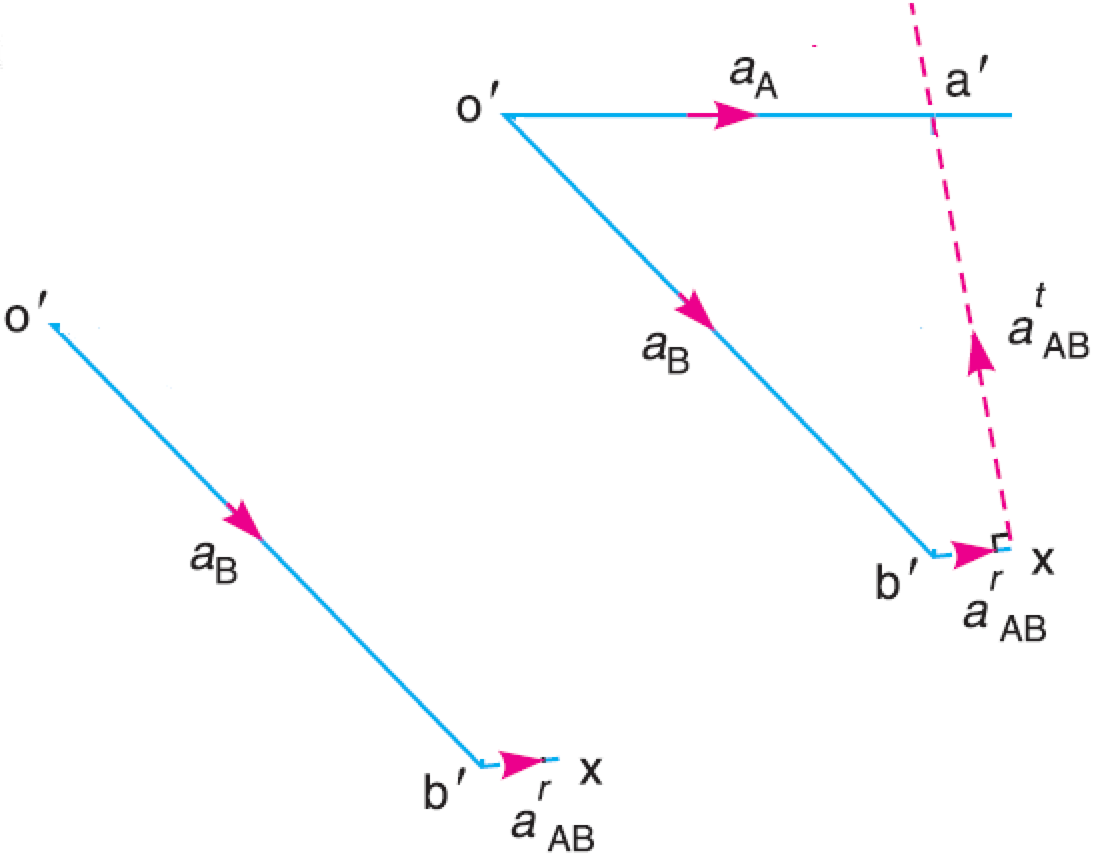
Source : R. S. Khurmi



Space diagram.

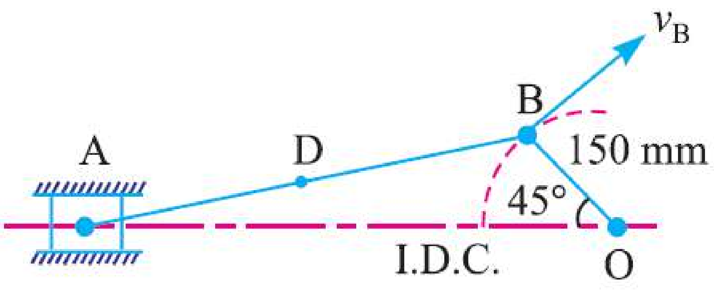


$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

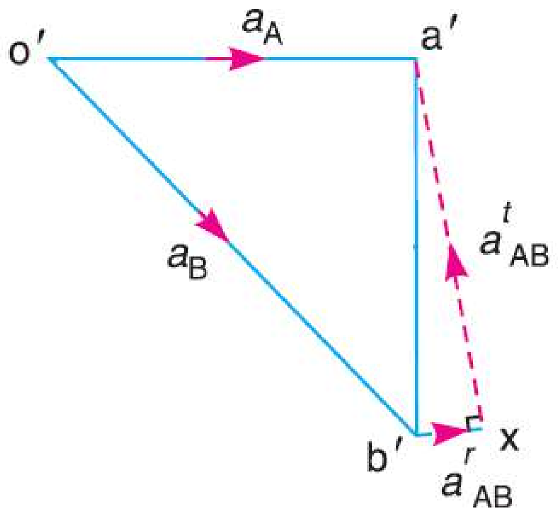


$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

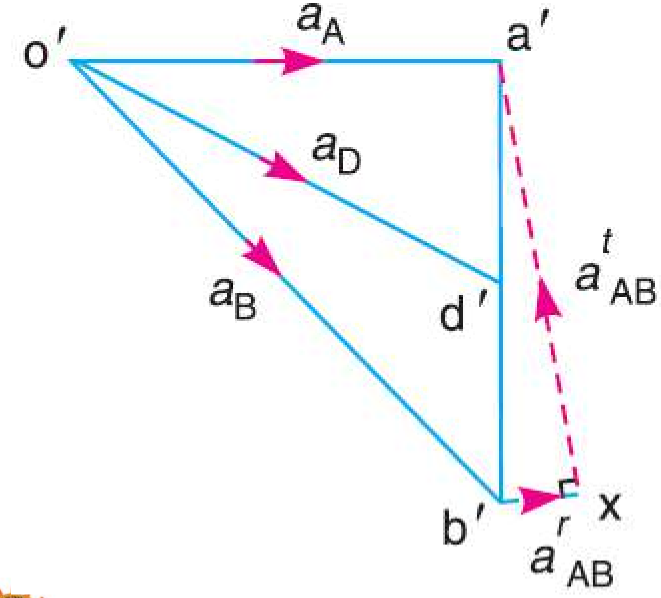
NUMERICAL EXAMPLE -1



Space diagram.



Source : R. S. Khurmi



By measurement, a_D = vector $o'\,d'$ = 117 m/s^2 **Ans.**

Angular velocity of the connecting rod

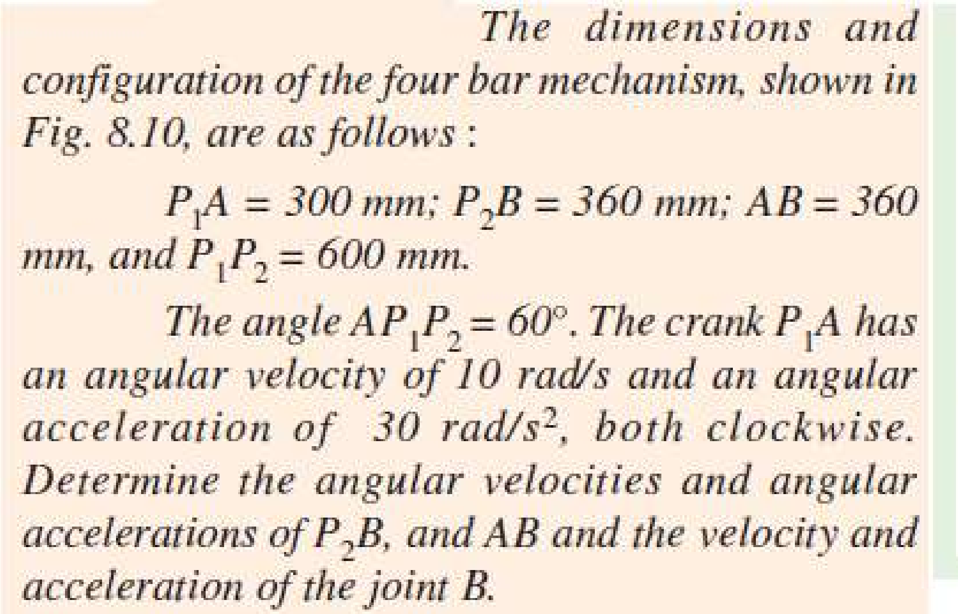
$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \quad \textbf{Ans.}$$

Angular acceleration of the connecting rod

From the acceleration diagram, $a^t_{AB} = 103$ m/s^2

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \quad \textbf{Ans.}$$

TUTORIAL PROBLEM-1



The dimensions and configuration of the four bar mechanism, shown in Fig. 8.10, are as follows :

$P_1A = 300$ mm; $P_2B = 360$ mm; $AB = 360$ mm, and $P_1P_2 = 600$ mm.

The angle $AP_1P_2 = 60°$. The crank P_1A has an angular velocity of 10 rad/s and an angular acceleration of 30 rad/s², both clockwise. Determine the angular velocities and angular accelerations of P_2B, and AB and the velocity and acceleration of the joint B.

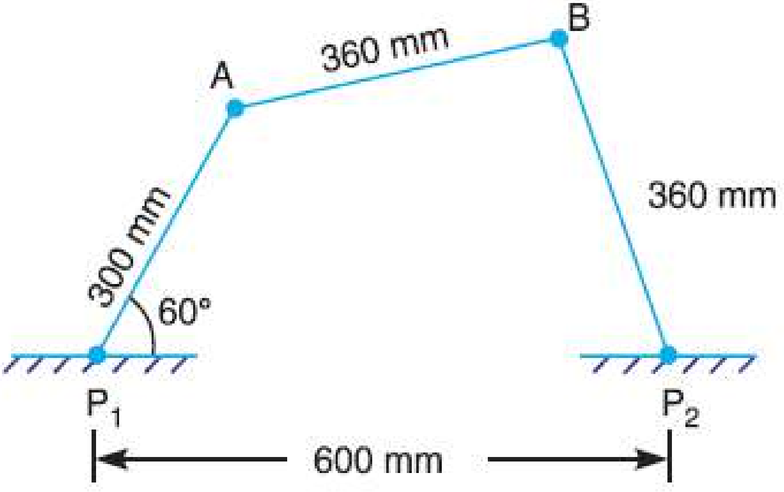


Fig. 8.10

Source : R. S. Khurmi

$$v_{BP2} = v_B = 2.2 \text{ m/s} \quad \textbf{Ans.}$$

$$\omega_{P2B} = \frac{v_{BP_2}}{P_2B} = \frac{2.2}{0.36} = 6.1 \text{ rad/s} \quad \textbf{Ans.}$$

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{2.05}{0.36} = 5.7 \text{ rad/s} \quad \textbf{Ans.}$$

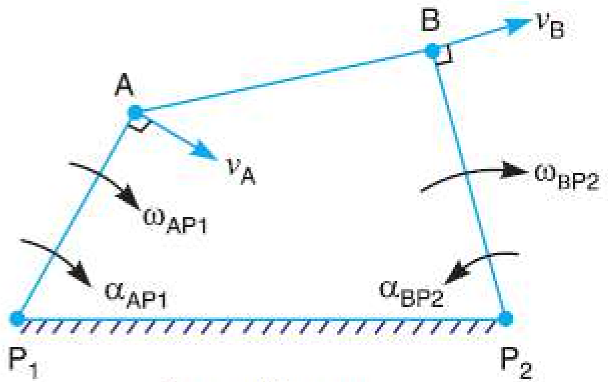
$$a_B = 29.6 \text{ m/s}^2 \ \textbf{Ans.}$$

$$\alpha_{P2B} = \frac{a^t_{BP_2}}{P_2B} = \frac{26.6}{0.36} = 73.8 \text{ rad/s}^2 \quad \textbf{Ans.}$$

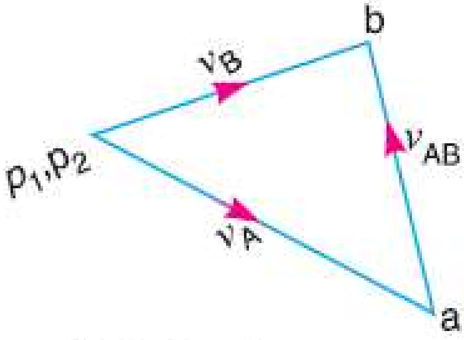
$$\alpha_{AB} = \frac{a^t_{BA}}{AB} = \frac{13.6}{0.36} = 37.8 \text{ rad/s}^2 \quad \textbf{Ans.}$$

TUTORIAL PROBLEM-1

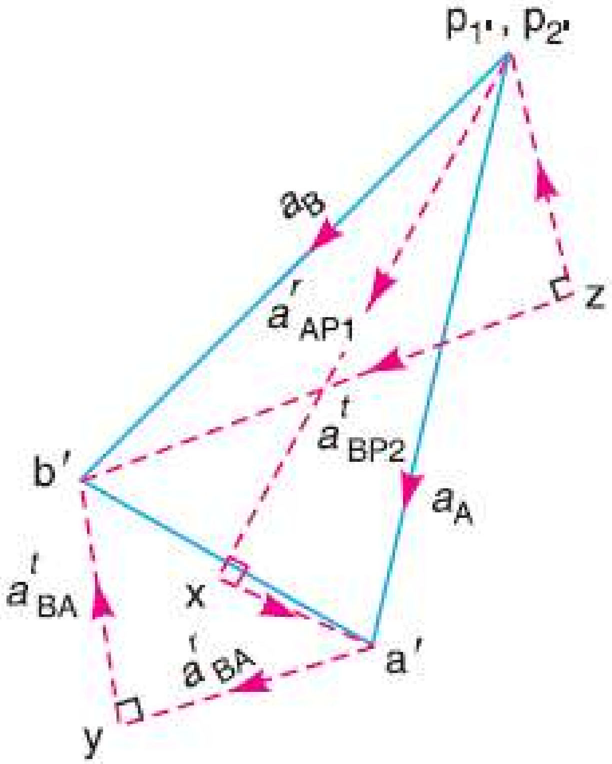
Source : R. S. Khurmi



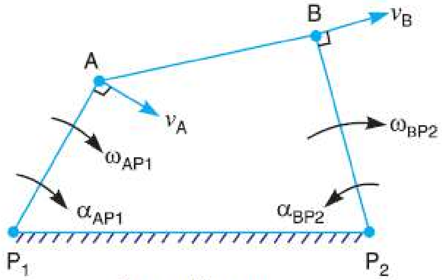
Space diagram.



Velocity diagram.



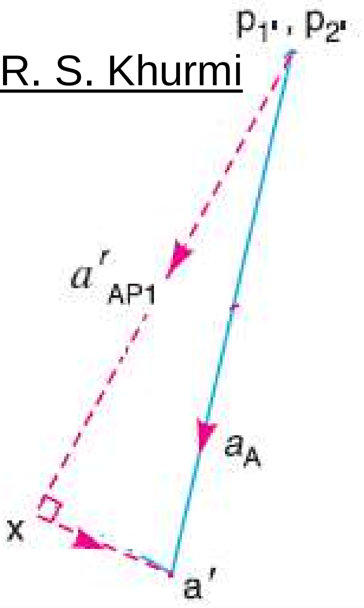
Acceleration diagram



Space diagram.

RIAL PROBLEM-1

Source : R. S. Khurmi

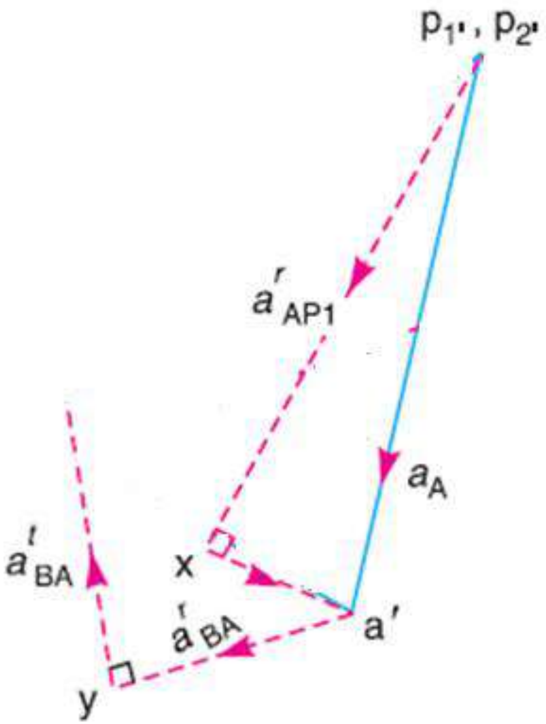


vector $p_1' x = a^r_{AP_1} = 30 \text{ m/s}^2$

vector $xa' = a^t_{AP_1} = 9 \text{ m/s}^2$

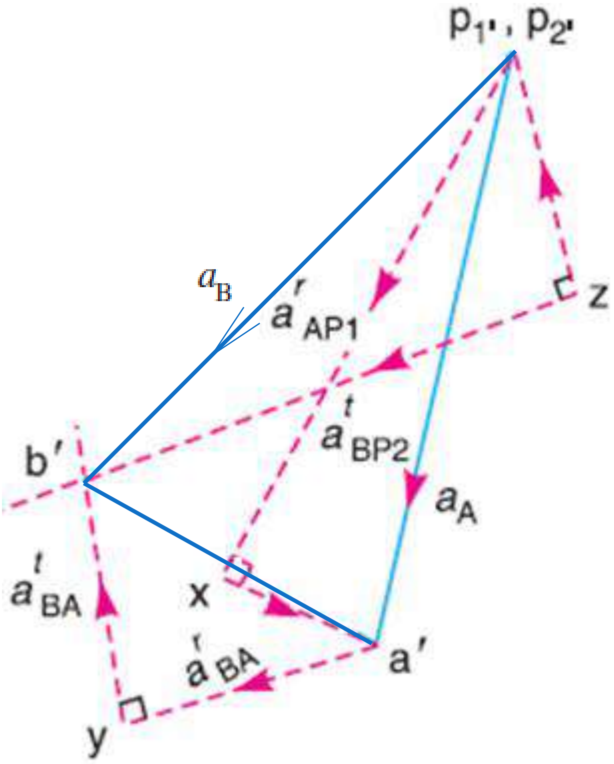
By measurement,

$a_A = a_{AP1} = 31.6 \text{ m/s}^2$



vector $a'y = a^r_{BA} = 11.67 \text{ m/s}^2$

a^t_{BA} magnitude is yet unknown

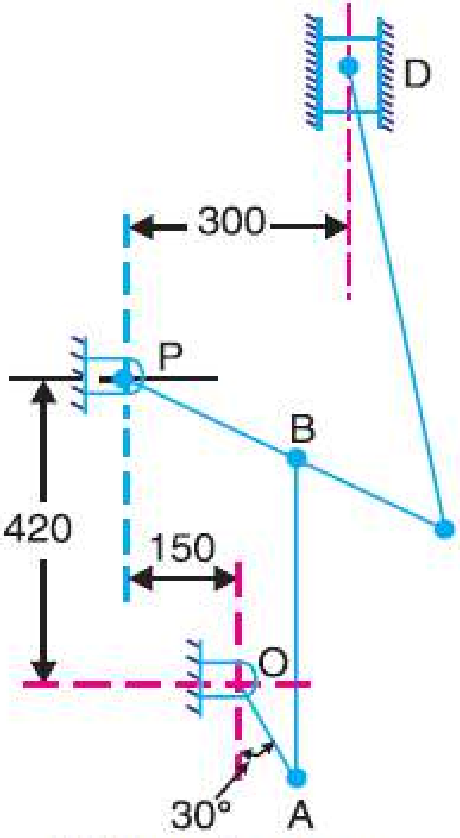


$p_2' z = a^r_{BP_2} = 13.44 \text{ m/s}^2$

EXERCISE-1

Find out the acceleration of the slider D and the angular acceleration of link CD for the engine mechanism shown in Fig. 8.14.

The crank OA rotates uniformly at 180 r.p.m. in clockwise direction. The various lengths are: OA = 150 mm ; AB = 450 mm; PB = 240 mm ; BC = 210 mm ; CD = 660 mm.



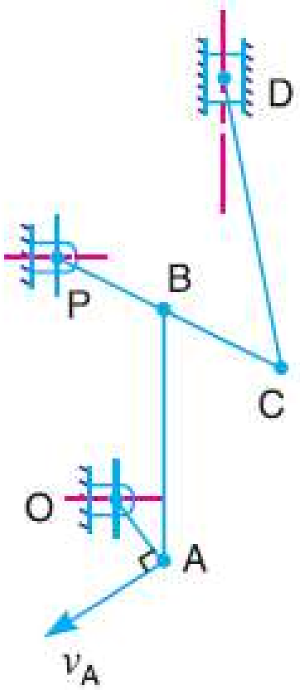
All dimensions in mm.

Fig. 8.14

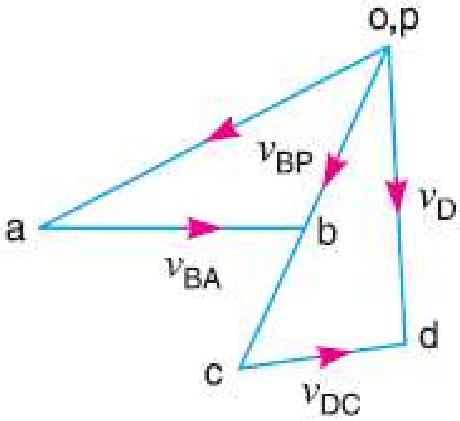
Source : R. S. Khurmi

ANSWER

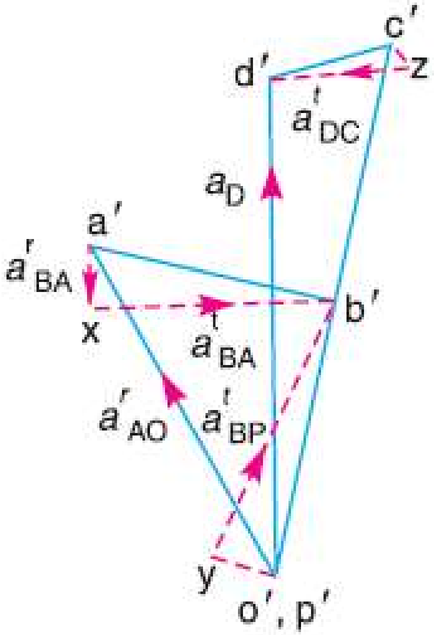
Source : R. S. Khurmi



(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.

a_D = vector $o'd'$ = 69.6 m/s^2 **Ans.**

$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{17.4}{0.66} = 26.3 \text{ rad/s}^2 \quad \textbf{Ans.}$$

LECTURE 6

CORIOLIS COMPONENT OF ACCELERATION

LIVE TO LEARN & LEARN TO SHARE

MRCET CAMPUS

UGC Autonomous

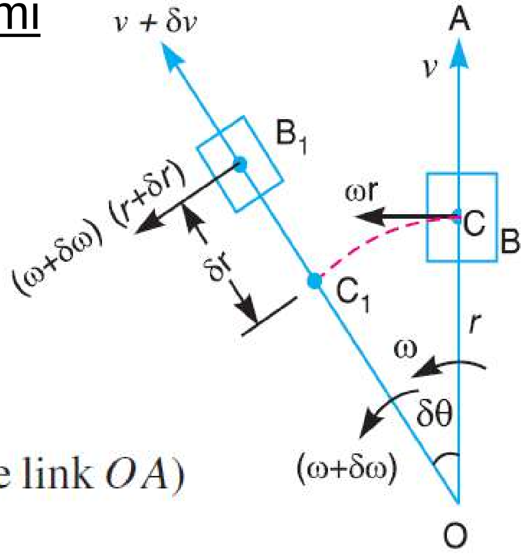
DEPARTMENT OF MECHANICAL ENGINEERING

CORIOLIS COMPONENT OF ACCELERATION

Where?

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism

Source : R. S. Khurmi



Let ω = Angular velocity of the link OA at time t seconds.

v = Velocity of the slider B along the link OA at time t seconds.

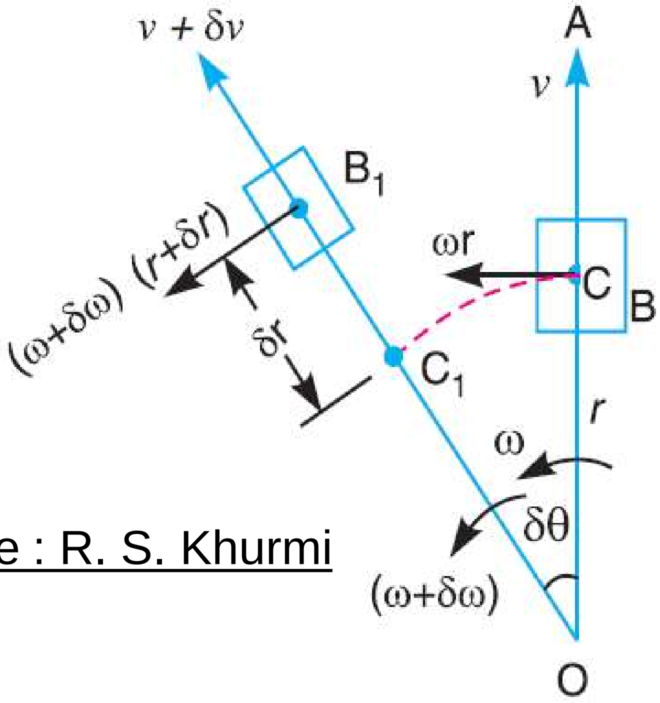
$\omega . r$ = Velocity of the slider B with respect to O (perpendicular to the link OA) at time t seconds, and

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega)\ (r + \delta r)$

= Corresponding values at time $(t + \delta t)$ seconds.

CORIOLIS COMPONENT OF ACCELERATION

The tangential component of acceleration of the slider B with respect to the coincident point C on the link is known as coriolis component of acceleration and is always perpendicular to the link.



Source : R. S. Khurmi

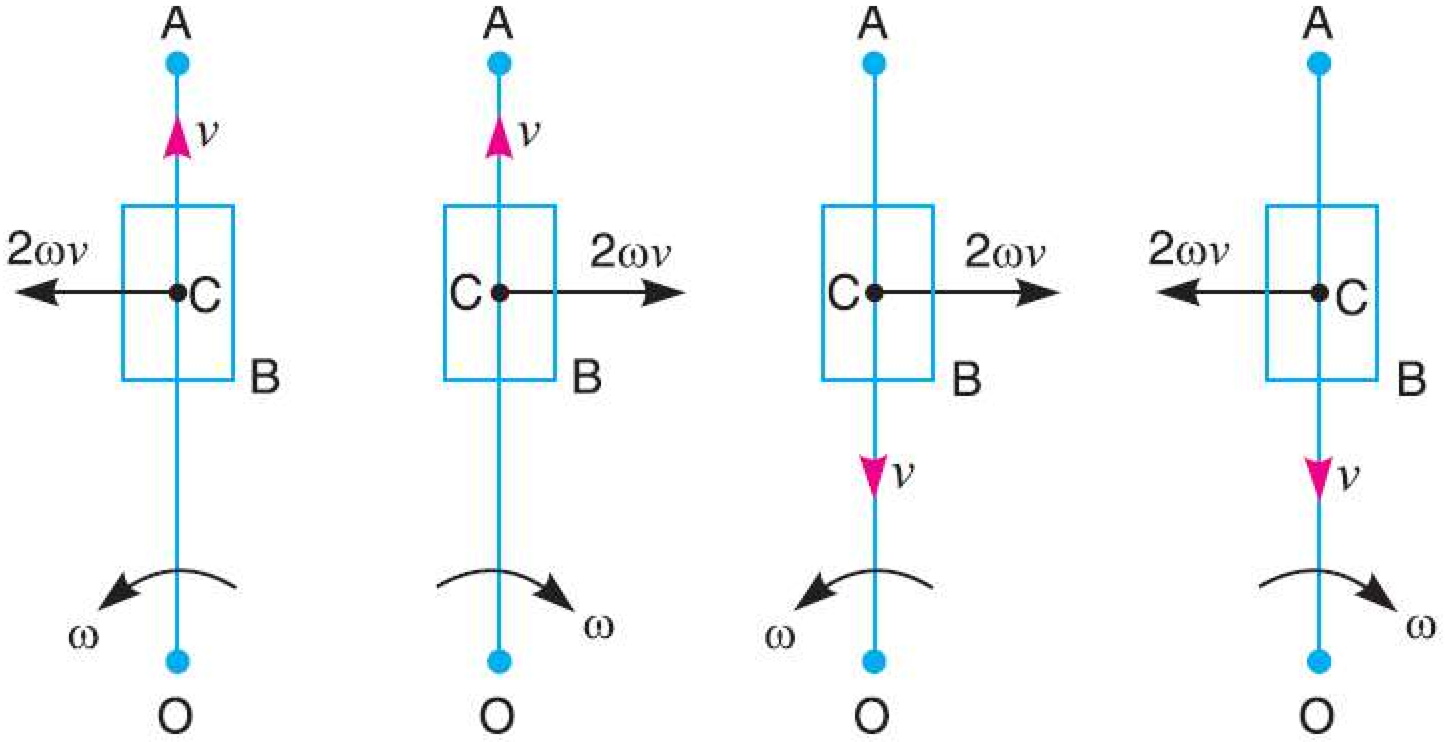
∴ Coriolis component of the acceleration of B with respect of C,

$$a_{BC}^{c} = a_{BC}^{t} = 2\,\omega.v$$

where ω = Angular velocity of the link OA, and

v = Velocity of slider B with respect to coincident point C.

CORIOLIS COMPONENT OF ACCELERATION



Direction of coriolis component of acceleration.

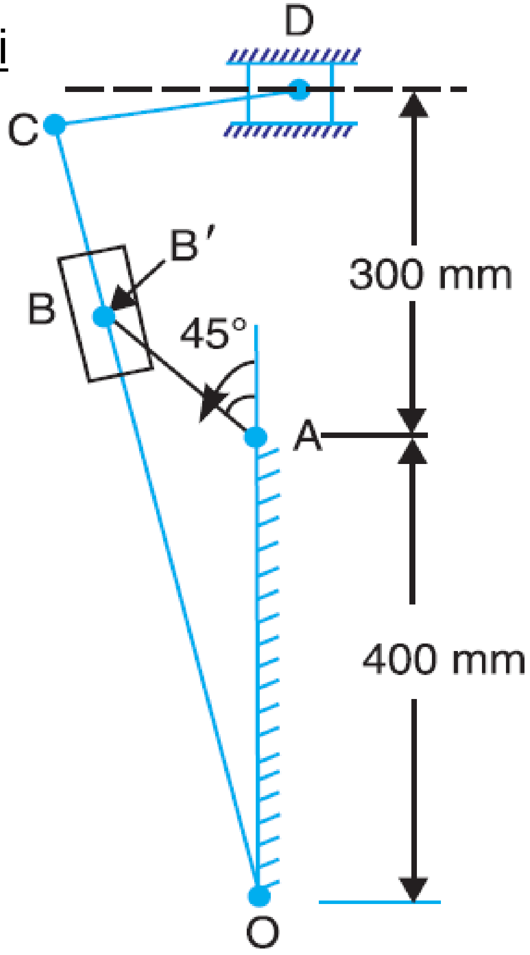
Source : R. S. Khurmi

NUMERICAL EXAMPLE -1

Source : R. S. Khurmi

A mechanism of a crank and slotted lever quick return motion is shown in the Fig. If the crank rotates counter clockwise at 120 r.p.m., determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever.

Crank, AB = 150 mm ; Slotted arm, OC = 700 mm and link CD = 200 mm.



NUMERICAL EXAMPLE -1 (CONSTRUCTION OF VELOCITY DIAGRAM)

Solution. Given : $N_{BA} = 120$ r.p.m or $\omega_{BA} = 2\,\pi \times 120/60 = 12.57$ rad/s ; $AB = 150$ mm $= 0.15$ m; $OC = 700$ mm $= 0.7$ m; $CD = 200$ mm $= 0.2$ m

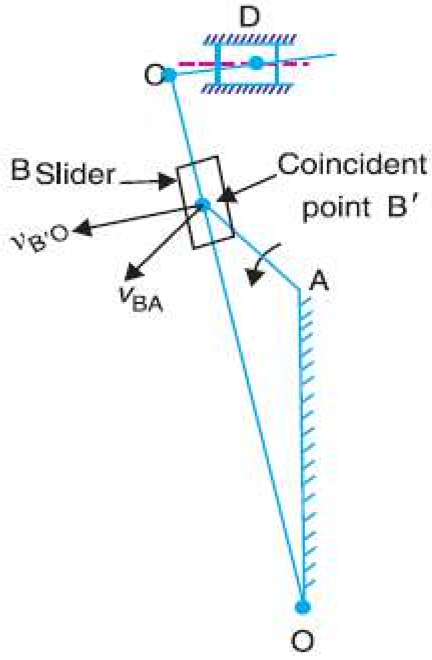
We know that velocity of B with respect to A,

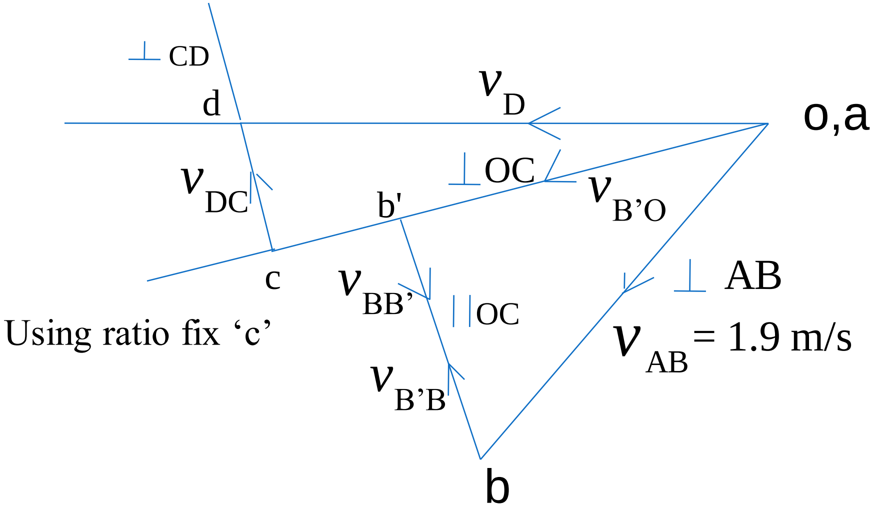
Source : R. S. Khurmi

$$v_{BA} = \omega_{BA} \times AB$$

$$= 12.57 \times 0.15 = 1.9 \text{ m/s}$$

...(Perpendicular to AB)





Using ratio fix ‘c’

NUMERICAL EXAMPLE -1

v_D = vector od = 2.15 m/s **Ans.**

From velocity diagram by measurement :

From velocity diagram, we also find that

Velocity of B with respect to B',

$v_{BB'}$ = vector $b'b$ = 1.05 m/s

Velocity of D with respect to C,

v_{DC} = vector cd = 0.45 m/s

Velocity of B' with respect to O

$v_{B'O}$ = vector ob' = 1.55 m/s

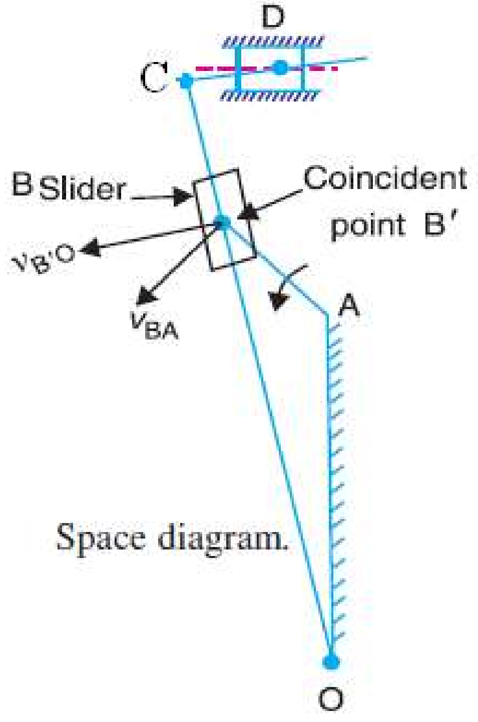
Velocity of C with respect to O,

v_{CO} = vector oc = 2.15 m/s

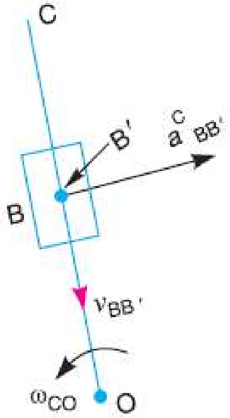
∴ Angular velocity of the link OC or OB',

$$\omega_{CO} = \omega_{B'O} = \frac{v_{CO}}{OC} = \frac{2.15}{0.7} = 3.07 \text{ rad/s}$$

NUMERICAL EXAMPLE -1 (CONSTRUCTION OF ACCELERATION DIAGRAM)



Space diagram.



Direction of coriolis component.

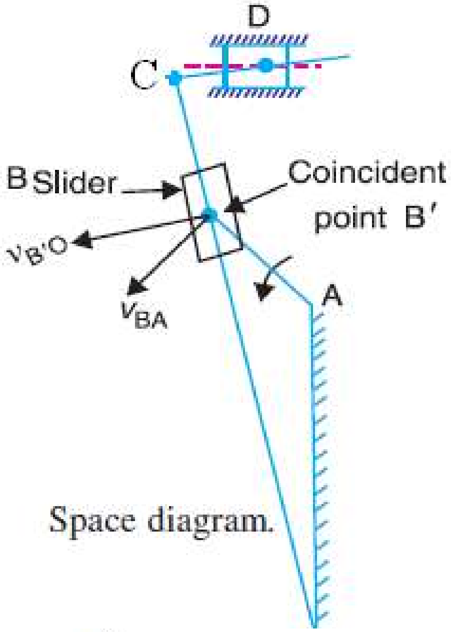
Link	Radial accel.	Tangen. accel.	Coriolis Accel.
AB	$a^r_{BA} = \omega^2_{BA} \times AB$ $= (12.57)^2 \times 0.15$ $= 23.7\ \text{m/s}^2$	Zero	Nil
BB’	Direction $a^r_{BB'}$ wn.	-	$a^c_{BB'} = 2\omega.v$ $= 2\omega_{CO}.v_{BB'}$ $= 2 \times 3.07 \times 1.05 = 6.45$ m/
DC	$a^r_{DC} = \frac{v^2_{DC}}{CD}$ $= \frac{(0.45)^2}{0.2} = 1.01\ \text{m/s}^2$	Direction known a^t_{DC}	Nil
B’O	$a^r_{B'O} = \frac{v^2_{B'O}}{B'O}$ $= \frac{(1.55)^2}{0.52} = 4.62\ \text{m/s}^2$	Direction known.	Nil

$a^t_{B'O}$

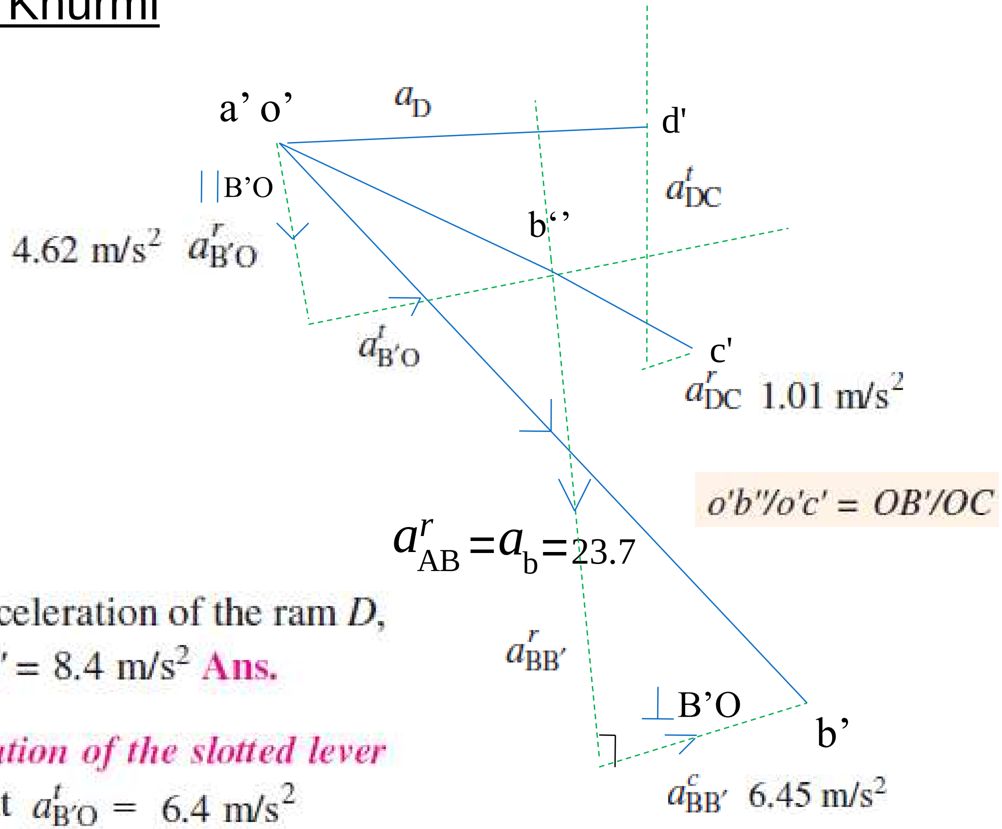
Source : R. S. Khurmi

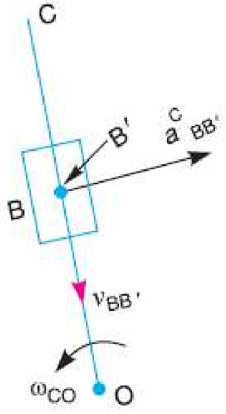
NUMERICAL EXAMPLE -1 (CONSTRUCTION OF ACCELERATION DIAGRAM)

Source : R. S. Khurmi



Space diagram.





Direction of coriolis component.

By measurement, acceleration of the ram *D*,

a_D = vector $o'd'$ = 8.4 m/s² **Ans.**

Angular acceleration of the slotted lever

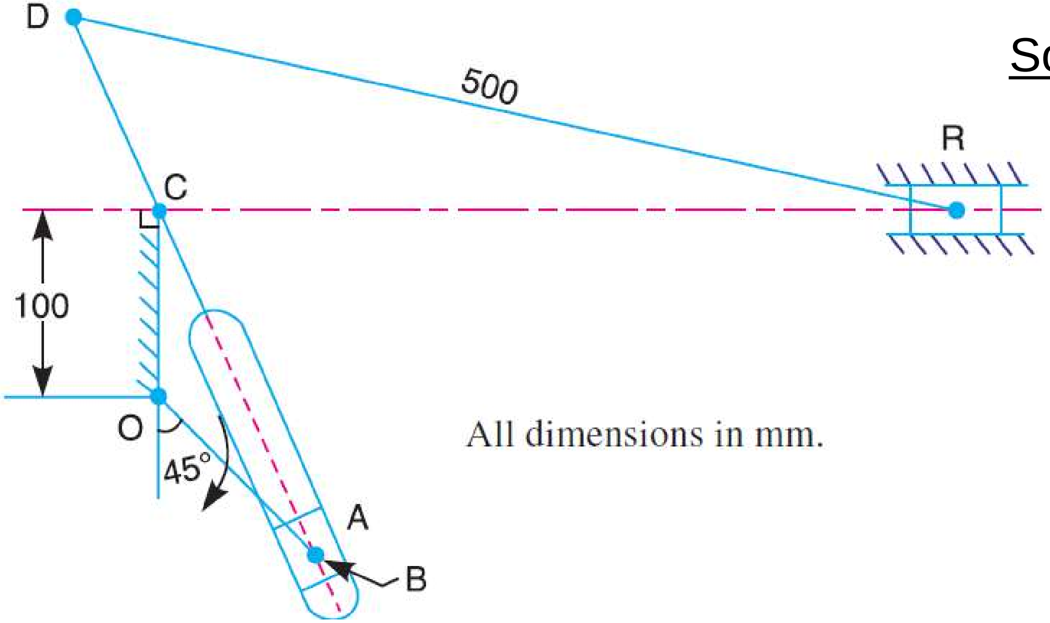
By measurement $a^t_{B'O}$ = 6.4 m/s²

angular acceleration of the slotted lever,

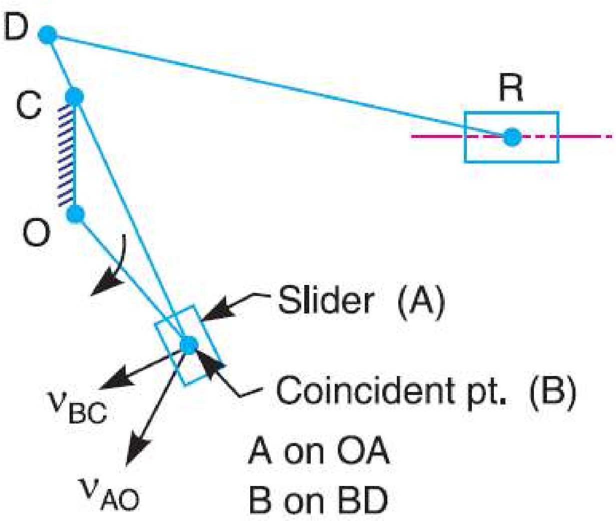
$$= \frac{a^t_{B'O}}{OB'} = \frac{6.4}{0.52} = 12.3 \text{ rad/s}^2 \quad \textbf{Ans.}$$

TUTORIAL PROBLEM

In a Whitworth quick return motion, as shown in the Fig., OA is a crank rotating at 30 r.p.m. in a clockwise direction. The dimensions of various links are : OA = 150 mm; OC = 100 mm; CD = 125 mm; and DR = 500 mm. Determine the acceleration of the sliding block R and the angular acceleration of the slotted lever CA.

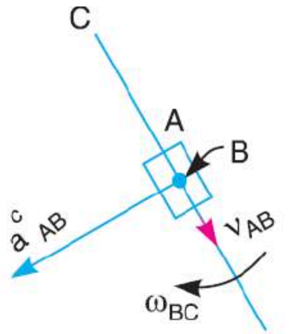


Source : R. S. Khurmi

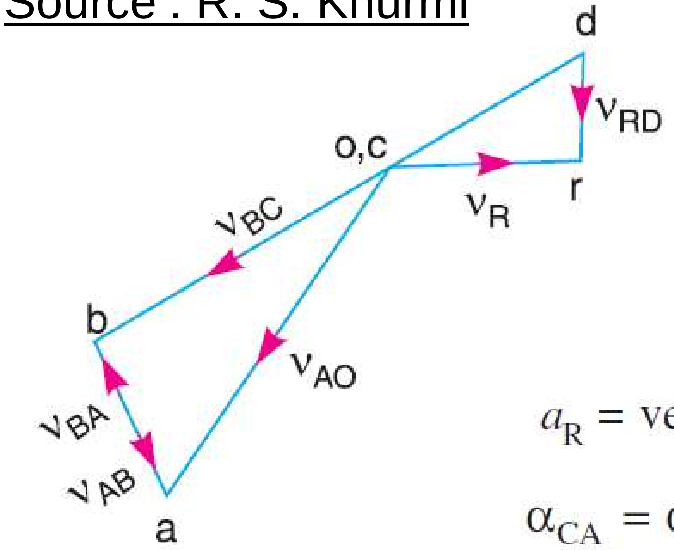


Space diagram.

Source : R. S. Khurmi



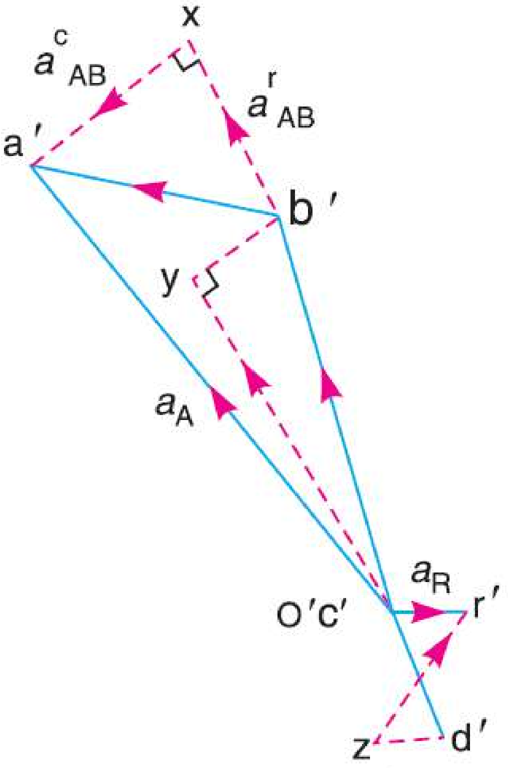
Direction of coriolis component.



Velocity diagram.

$$a_R = \text{vector } c'r' = 0.18 \text{ m/s}^2$$ **Ans.**

$$\alpha_{CA} = \alpha_{BC} = \frac{a^t_{CB}}{BC} = \frac{0.14}{0.24} = 0.583 \text{ rad/s}^2$$ **Ans.**



Acceleration diagram.

Industry applications

1. Four bars used to define the profile for non-circular gear
2. Railway engine wheels.
3. Double wishbone suspension.
4. Pantograph.
5. Pump jack.

Question Bank for Assignments

1. In a four bar chain ABCD, AD is fixed and is 150mm long. The crank AB is 40mm long and rotates at 120 rpm clockwise while the link CD=80mm oscillates about D. BC and AD are of equal length. Find the angular velocity and angular acceleration of link CD when angle BAD=60^0.
2. Locate all the instantaneous centres of the slider crank mechanism. The length of the crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates with an angular velocity of 10 rad/s, find: 1) Velocity of the slider A, 2. Angular velocity of the connecting rod AB.
3. In a small steam engine running at 600 rad/min clockwise, length of crank is 80 mm and ratio of connecting rod length to crank radius is 3. For the position when crank makes 45^0 to horizontal, Determine:

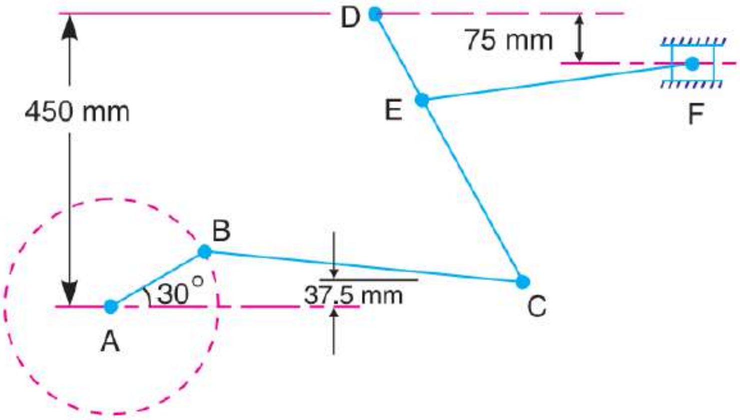
   A) The velocity and acceleration of piston

   B) The angular velocity and angular acceleration of the connecting rod.

Tutorial Questions

1. In a slider crank mechanism, the length of crank OB and connecting rod AB are 125 mm and 500 mm respectively. The centre of gravity G of the connecting rod is 275 mm from the slider A. The crank speed is 600 r.p.m. clockwise. When the crank has turned 45° from the inner dead centre position, determine: 1. velocity of the slider A, 2. velocity of the point G, and 3. angular velocity of the connecting rod AB.

2. In a link work, as shown in Fig. the crank AB rotates about A at a uniform speed of 150 r.p.m. The lever DC oscillates about the fixed point D, being connected to ABby the connecting link BC. The block F moves, in horizontal guides being driven by the link EF, when the crank AB is at 30°. The dimensions of the various links are: AB= 150 mm; BC= 450 mm; CE= 300 mm ; DE= 150 mm ; and EF= 350 mm. Find, for the given configuration, 1. velocity of slider F, 2. angular velocity of DC, and 3. Rubbing speed at pin C which is 50 mm in diameter.



3. In a four bar chain ABCD, link AD is fixed and the crank AB rotates at 10 radians per second clockwise. Lengths of the links are AB= 60 mm; BC= CD= 70 mm; DA= 120 mm. When angle DAB= 60° and both Band Clie on the same side of AD, find 1. angular velocities (magnitude and direction) of BC and CD; and 2. angular acceleration of BC and CD. 4. Describe the Watt's parallel mechanism for straight line motion and derive the condition under which the straight line is traced.

UNIT 4
CAMS

COURSE OBJECTIVE

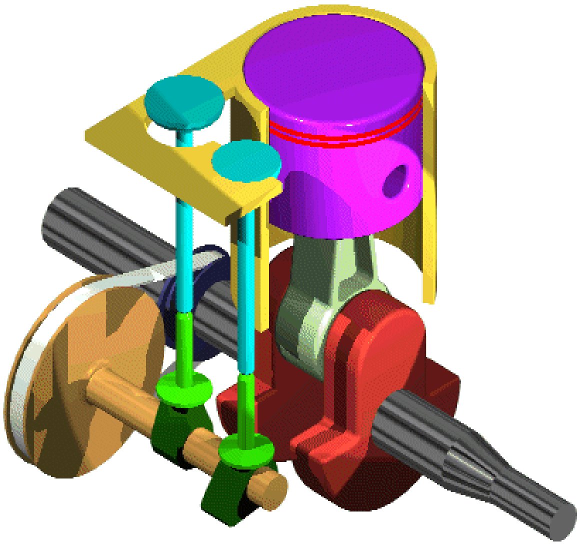
- To familiarize higher pairs like cams and principles of cams design

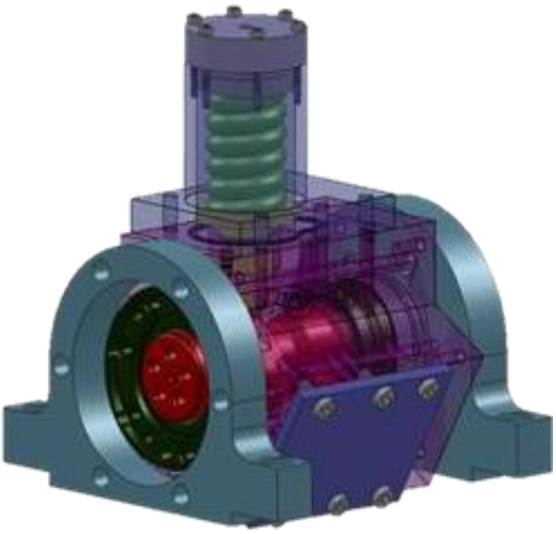
COURSE OUTCOME

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	Cams Terminology	• Classifications of CAMs • Cam nomenclature • Types of Followers	• Understanding what is cam (B2) • Analyse cam with examples (B4) • Evaluate tangent cam (B5)
2	Uniform velocity Simple harmonic motion	• Classification based on Motion of the Follower • Follower Moves with Simple Harmonic Motion	• Remember different types of follower (B1) • Evaluate motion to a knife edged follower (B5)
3	Uniform acceleration	• Follower Moves with Uniform Acceleration and Retardation • Follower Moves with Cycloidal Motion	• Understand line of stroke of the follower passes through the center of the cam shaft (B2) • Create profile of a cam operating a roller reciprocating follower (B6)
4	Maximum velocity during outward and return strokes	• Derive maximum velocity of the follower during its ascent and descent	• Evaluate the expression for maximum velocity during its ascent and descent (B5)
5	Maximum acceleration during outward and return strokes	• Derive maximum acceleration of the follower during its ascent and descent	• Evaluate the expression for maximum acceleration during its ascent and descent (B5)
6	Roller follower circular cam with straight	• Numerical Examples using Roller follower circular cam with straight	• Evaluate roller follower cam mechanism (B5) • Apply roller follower cam mechanism to generate cam profile (B3)
7	Concave and convex flanks	• Specified contour cams • Pressure angle and undercutting & sizing of cams	• Analyse undercutting in cam (B4)

4

Cams





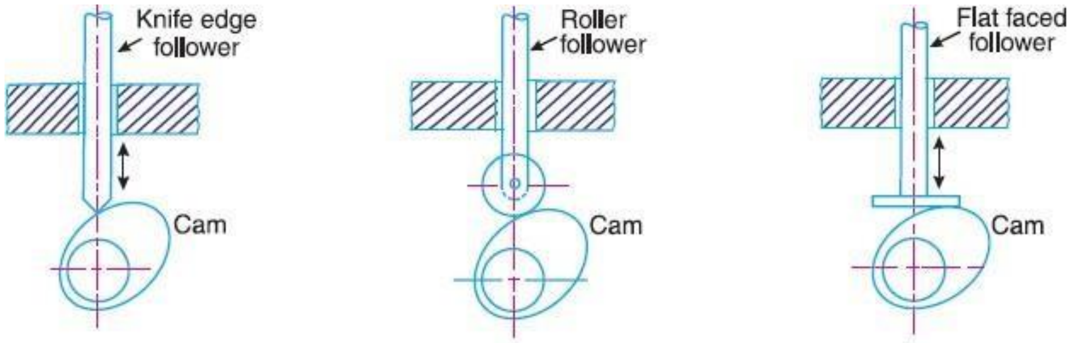
Course Contents

4.1 Introduction

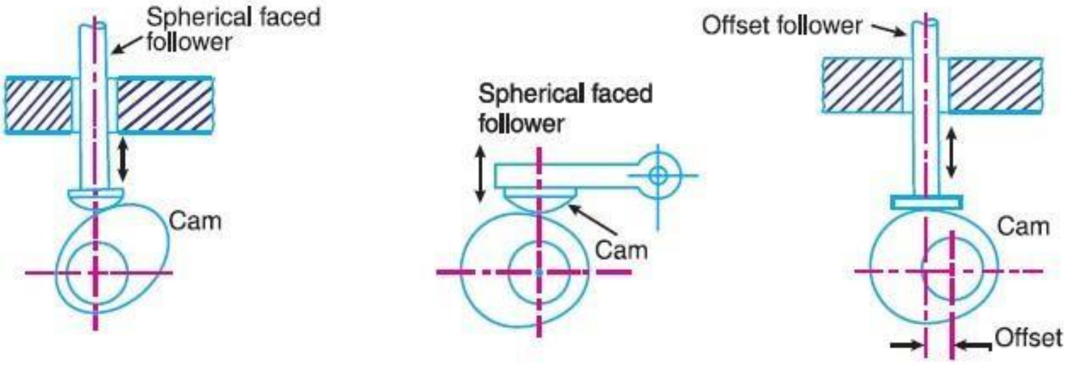
- A cam is a rotating machine element which gives reciprocating or oscillating motion to another element known as follower.
- The cam and the follower have a line contact and constitute a higher pair. The cams are usually rotated at uniform speed by a shaft, but the follower motion is pre-determined and will be according to the shape of the cam. The cam and follower is one of the simplest as well as one of the most important mechanisms found in modern machinery today.
- The cams are widely used for operating the inlet and exhaust valves of internal combustion engines, automatic attachment of machineries, paper cutting machines, spinning and weaving textile machineries, feed mechanism of automatic lathes etc.

4.2 Classification of Followers

The followers may be classified as discussed below :



(a) Cam with knife edge follower. (b) Cam with roller follower. (c) Cam with flat faced follower.



(d) Cam with spherical faced follower. (e) Cam with spherical faced follower. (f) Cam with offset follower.

Fig. 4.1 classification of follower

According to surface in contact

a Knife edge follower

- When the contacting end of the follower has a sharp knife edge, it is called a knife edge follower, as shown in Fig. 4.1 (a).
- The sliding motion takes place between the contacting surfaces (i.e. the knife edge and the cam surface). It is seldom used in practice because the small area of contacting surface results in excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide.

b Roller follower

- When the contacting end of the follower is a roller, it is called a roller follower, as shown in Fig. 4.1 (b). Since the rolling motion takes place between the contacting surfaces (i.e. the roller and the cam), therefore the rate of wear is greatly reduced.
- In roller followers also the side thrust exists between the follower and the guide. The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraft engines.

c Flat faced or mushroom follower

- When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower, as shown in Fig. 4.1 (c). It may be noted that the side thrust between the follower and the guide is much reduced in case of flat faced followers.
- The only side thrust is due to friction between the contact surfaces of the follower and the cam. The relative motion between these surfaces is largely of sliding nature but wear may be reduced by off-setting the axis of the follower, as shown in Fig. 4.1 (f) so that when the cam rotates, the follower also rotates about its own axis.
- The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobile engines.

d Spherical faced follower

- When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, as shown in Fig. 4.1 (d). It may be noted that when a flat-faced follower is used in automobile engines, high surface stresses are produced. In order to minimize these stresses, the flat end of the follower is machined to a spherical shape.

According to the motion of follower

a Reciprocating or Translating Follower

- When the follower reciprocates in guides as the cam rotates uniformly, it is known as reciprocating or translating follower. The followers as shown in Fig. 4.1 (a) to (d) are all reciprocating or translating followers.

b Oscillating or Rotating Follower

- When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower. The follower, as shown in Fig 4.1 (e), is an oscillating or rotating follower.

According to the path of motion of the follower

a Radial Follower

- When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial follower. The followers, as shown in Fig. 4.1 (a) to (e), are all radial followers.

b Off-set Follower

- When the motion of the follower is along an axis away from the axis ofthe cam centre, it is called off-set follower. The follower, as shown in Fig. 4.1 (f), is an off-set follower.

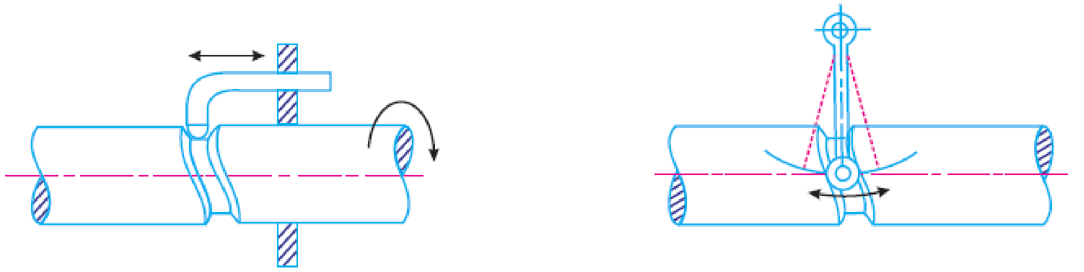
Classification of cams

a Radial or Disc cam

- In radial cams, the follower reciprocates or oscillates in a direction perpendicular to the cam axis. The cams as shown in Fig. 7.1 are all radial cams.

b Cylindrical cam

- In cylindrical cams, the follower reciprocates or oscillates in a direction parallel to the cam axis. The follower rides in a groove at its cylindrical surface. A cylindrical grooved cam with a reciprocating and an oscillating follower is shown in Fig. 7.2 (a) and (b) respectively.



(*a*) Cylindrical cam with reciprocating follower.

(*b*) Cylindrical cam with oscillating follower.

Fig. 4.2 cylindrical cam

4.3 Terms used in radial cams

a Base circle

- It is the smallest circle that can be drawn to the cam profile.

b Trace point

- It is a reference point on the follower and is used to generate the pitch curve. In case of knife edge follower, the knife edge represents the trace point and the pitch curve corresponds to the cam profile. In a roller follower, the centre of the roller represents the trace point.

c Pressure angle

- It is the angle between the direction of the follower motion and a normal to the pitch curve. This angle is very important in designing a cam profile. If the pressure angle is too large, a reciprocating follower will jam in its bearings.

d Pitch point

- It is a point on the pitch curve having the maximum pressure angle.

e Pitch circle

- It is a circle drawn from the center of the cam through the pitch points.

f Pitch curve

- It is the curve generated by the trace point as the follower moves relative to the cam. For a knife edge follower, the pitch curve and the cam profile are same whereas for a roller follower, they are separated by the radius of the roller.

g Prime circle

- It is the smallest circle that can be drawn from the center of the cam and tangent to the pitch curve. For a knife edge and a flat face follower, the prime circle and the base circle are identical. For a roller follower, the prime circle is larger than the base circle by the radius of the roller.

h Lift or Stroke

- It is the maximum travel of the follower from its lowest position to the topmost position.

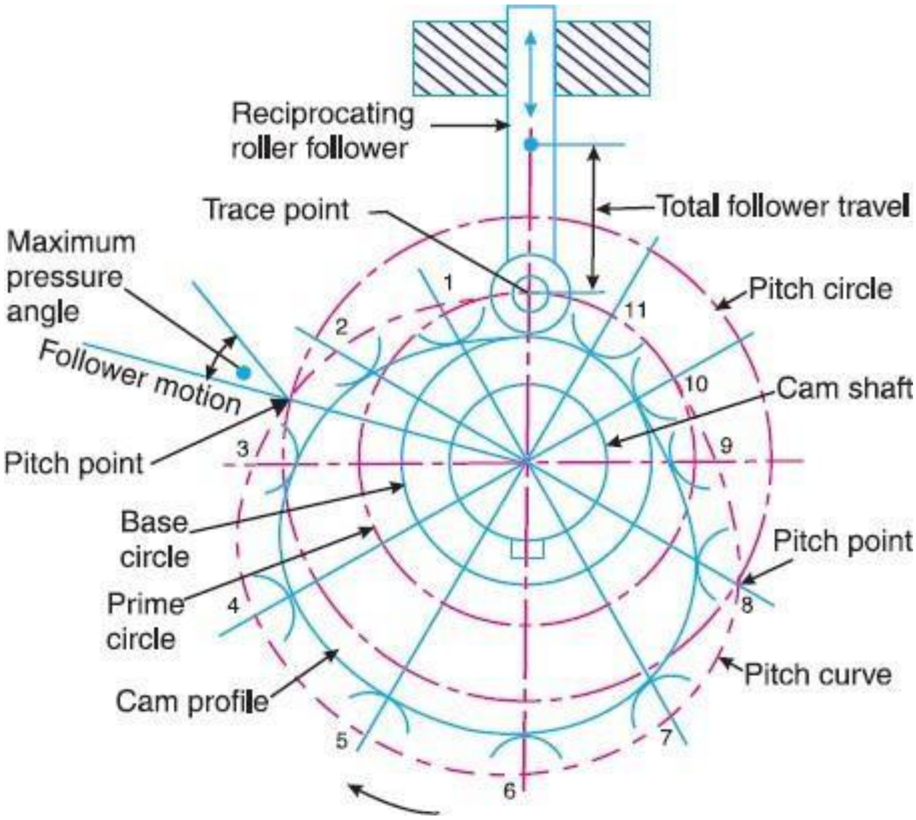


Fig. 4.3 terms used in radial cams

Motion of follower

The follower, during its travel, may have one of the following motions:

a Uniform velocity

b Simple harmonic motion
c Uniform acceleration and retardation
d Cycloidal motion

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Velocity

The displacement, velocity and acceleration diagrams when a knife-edged follower moves with uniform velocity are shown in Fig. 4.4 (a), (b) and (c) respectively. The abscissa (base) represents the time (i.e. the number of seconds required for the cam to complete one revolution) or it may represent the angular displacement of the cam in degrees. The ordinate represents the displacement, or velocity or acceleration of the follower. Since the follower moves with uniform velocity during its rise and return stroke, therefore the slope of the displacement curves must be constant. In other words, AB1 and C1D must be straight lines.

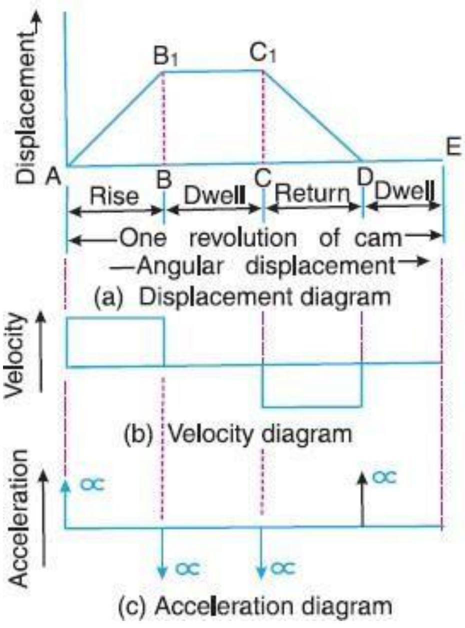


Fig. 4.4 displacement, velocity and acceleration diagrams

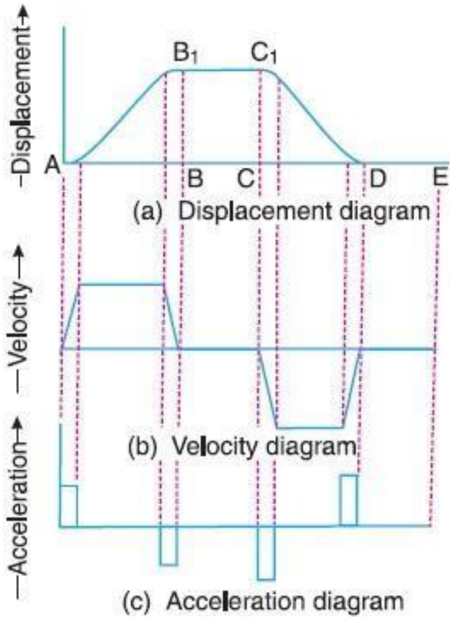


Fig. 4.5 modified displacement, velocity acceleration diagrams

A little consideration will show that the follower remains at rest during part of the cam rotation. The periods during which the follower remains at rest are known as dwell periods, as shown by lines B1C1 and DE in Fig. 4.4 (a). From Fig. **4.4** (c), we see that the acceleration or retardation of the follower at the beginning and at the end of each stroke is infinite. This is due to the fact that the follower is required to start from rest and has to gain a velocity within no time. This is only possible if the acceleration or retardation at the beginning and at the end of each stroke is infinite. These conditions are however, impracticable.

In order to have the acceleration and retardation within the finite limits, it is necessary to modify the conditions which govern the motion of the follower. This may be done by rounding off the sharp corners of the displacement diagram at the beginning and at the end of each stroke, as shown in Fig. 4.5 (a). By doing so, the velocity of the follower increases gradually to its maximum value at the beginning of each stroke and decreases gradually to zero at the end of each stroke as shown in Fig. 4.5 (b).

The modified displacement, velocity and acceleration diagrams are shown in Fig.4.5. The round corners of the displacement diagram are usually parabolic curves because the parabolic motion results in a very low acceleration of the follower for a given stroke and cam speed.

Displacement, Velocity and Acceleration Diagrams

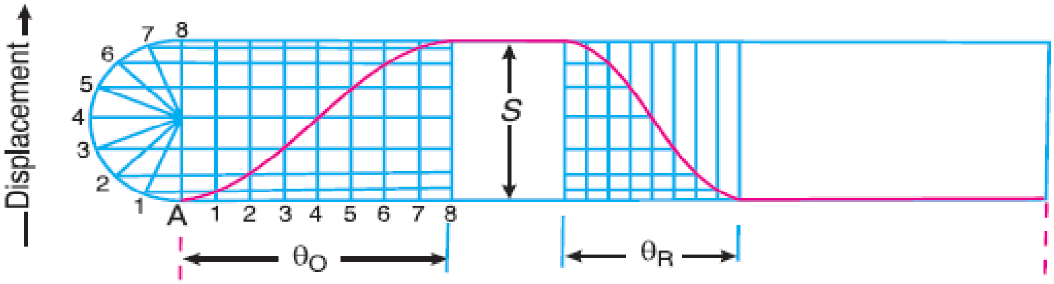
when the Follower Moves with Simple Harmonic Motion

The displacement, velocity and acceleration diagrams when the follower moves with simple harmonic motion are shown in Fig. 4.6 (a), (b) and (c) respectively. The displacement diagram is drawn as follows:

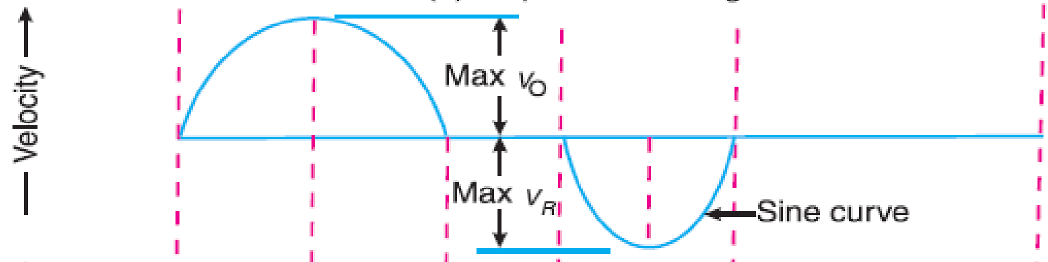
a Draw a semi-circle on the follower stroke as diameter.
b Divide the semi-circle into any number of even equal parts (say eight).
c Divide the angular displacements of the cam during out stroke and return stroke into the same number of equal parts.
d The displacement diagram is obtained by projecting the points as shown in Fig. 4.6 (a).

The velocity and acceleration diagrams are shown in Fig. 4.6 (b) and (c) respectively. Since the follower moves with a simple harmonic motion, therefore velocity diagram consists of a sine curve and the acceleration diagram is a cosine curve.

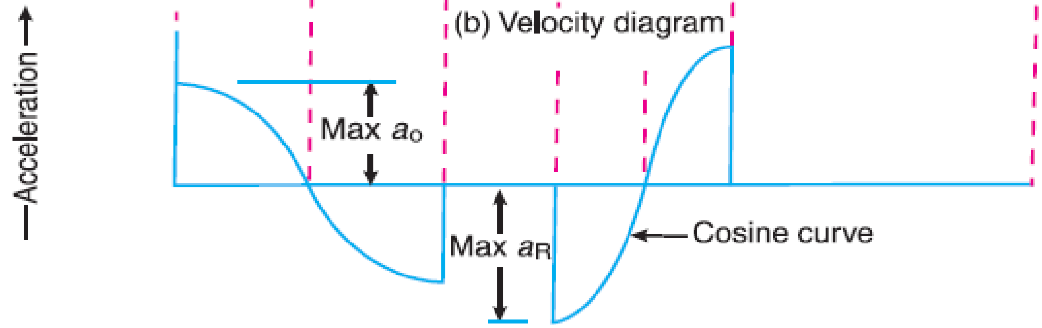
We see from Fig. 4.6 (b) that the velocity of the follower is zero at the beginning and at the end of its stroke and increases gradually to a maximum at mid-stroke. On the other hand, the acceleration of the follower is maximum at the beginning and at the ends of the stroke and diminishes to zero at mid-stroke.



(a) Displacement diagram



(b) Velocity diagram



(c) Acceleration diagram

Fig. 4.6 acceleration diagram

S= Stroke of the follower

Θ_0 and Θ_R = Angular displacement of the cam during out stroke and return stroke of the follower respectively

ω = angular velocity of cam

Time required for the outstroke of the follower in second

$$t_0 = \frac{0}{\omega}$$

Consider a point P moving at uniform speed ωp radians per sec round the circumference of a circle with the stroke S as diameter, as shown in Fig. 7.7 the point (which is the projection of a point P on the diameter) executes a simple harmonic motion as the point P rotates. The motion of the follower is similar to that of point P'.

Peripheral speed of the point P'

$$v_p = \frac{\pi \times s}{2} \times \frac{1}{t_0} = \frac{\pi \times s}{2} \times \frac{\omega}{\theta_0}$$

and maximum velocity of the follower on the outstroke,

$$v_0 = v_p = \frac{\pi \times s}{2} \times \frac{\omega}{\theta_0} = \frac{\pi \times \omega \times \mathrm{s}}{2\theta_0}$$

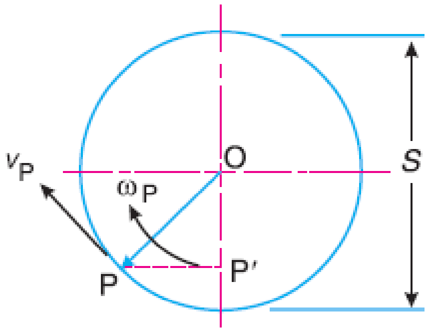


Fig. 7.7 motion of a point

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation

The displacement, velocity and acceleration diagrams when the follower moves with uniform acceleration and retardation are shown in Fig. 4.8 (a), (b) and (c) respectively. We see that the displacement diagram consists of a parabolic curve and may be drawn as discussed below:

Divide the angular displacement of the cam during outstroke (ϴ) into any even number of equal parts and draw vertical lines through these points as shown in fig.4.8 (a)

Divide the stroke of the follower (S) into the same number of equal even parts.

Join Aa to intersect the vertical line through point 1 at B. Similarly, obtain the other points C, D etc. as shown in Fig. 4.8 (a). Now join these points to obtain the parabolic curve for the out stroke of the follower.

In the similar way as discussed above, the displacement diagram for the follower during return stroke may be drawn.

We know that time required for the follower during outstroke,

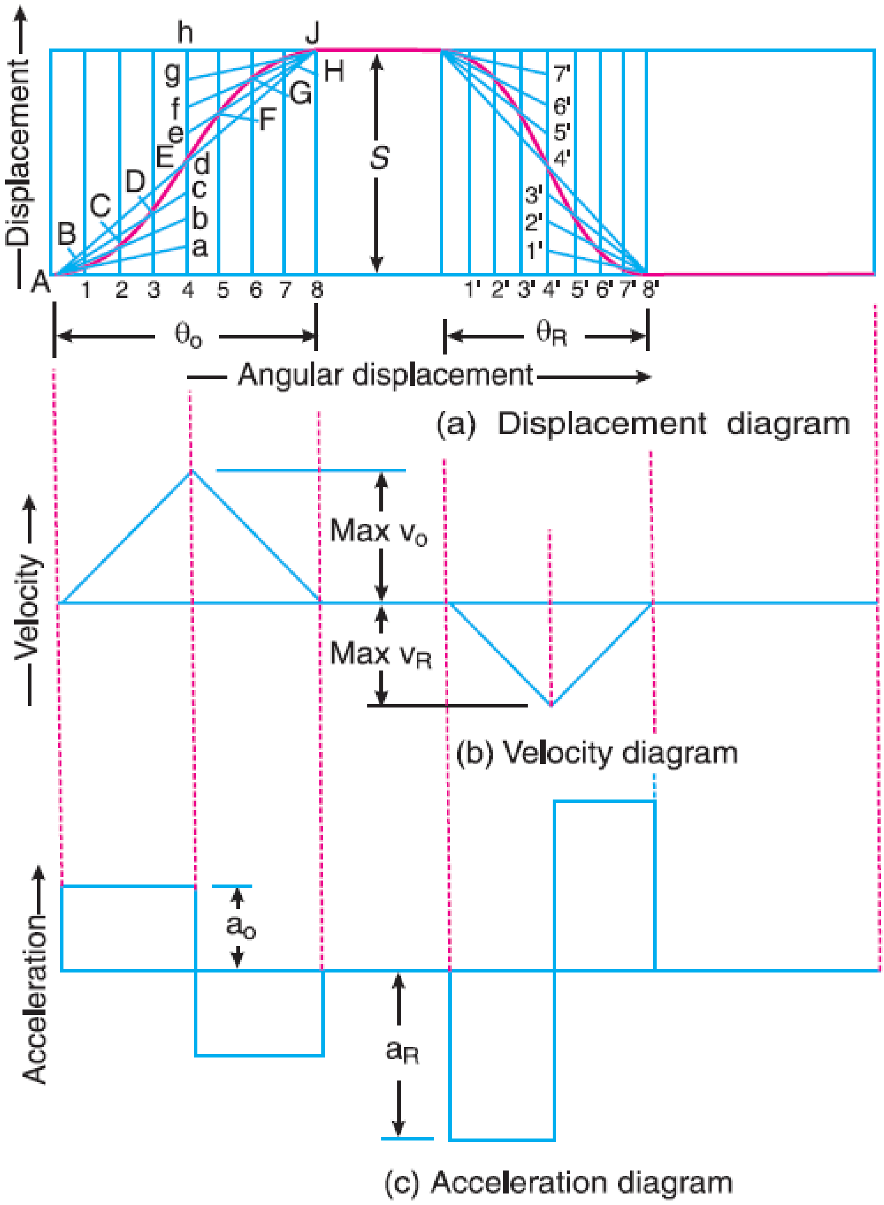
$$t_0 = \frac{0}{\omega}$$

and time required for the follower during return stroke,

$$t_R = \frac{\theta_R}{\omega}$$

Mean velocity of the follower during outstroke

$$v_0 = \frac{S}{t_0}$$



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

Fig. 4.8 Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation

Since the maximum velocity of follower is equal to twice the mean velocity, therefore maximum velocity of the follower during outstroke,

$$v_0 = \frac{2\,S}{t_0} = \frac{2\,\omega\,S}{\theta_0}$$

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\,\omega\,S}{\theta_R}$$

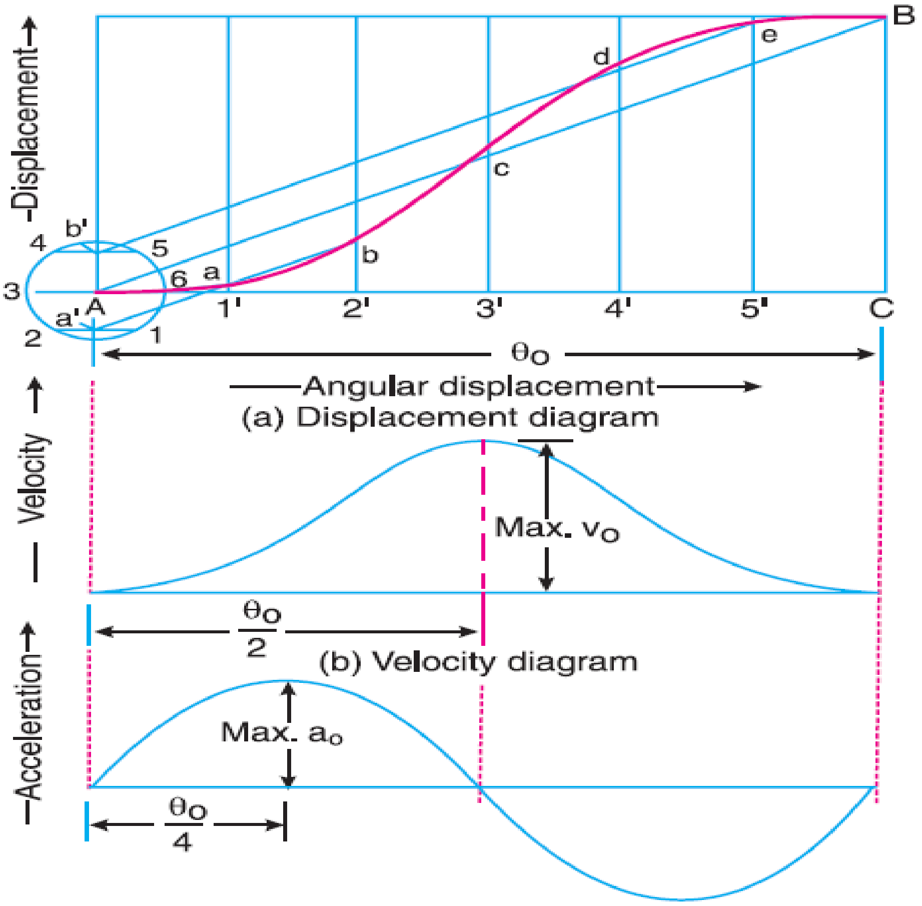
Maximum acceleration of the follower during outstroke,

$$a_0 = \frac{v_0}{t_0/2} = \frac{2 \times 2\,\omega s}{t_0\,\theta_0} = \frac{4\,\omega^2 .\, S}{(\theta_0)^2}$$

Similarly, maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\,\omega^2\,S}{(\theta_R)^2}$$

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with cycloidal Motion



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

- The displacement, velocity and acceleration diagrams when the follower moves with cycloidal motion are shown in Fig. (a), (b) and (c) respectively. We know that cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line.

We know that displacement of the follower after time t seconds,

$$x = S \left[\frac{\theta}{\theta_0} - \frac{1}{2\pi} \sin\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

Velocity of the follower after time t seconds,

$$\frac{dx}{dt} = S \left[\frac{1}{\theta_0} \times \frac{d\theta}{d} - \frac{2\pi\theta}{\theta_0} \cos\left(\frac{2\pi\theta}{\theta_0}\right) \frac{d\theta}{dt}\right]$$

$$= \frac{S}{\theta_0} \times \frac{\overset{t}{d\theta}}{dt} \left[1 - \cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

$$= \frac{\omega S}{\theta_0} \left[1 - \cos\left(\frac{2\pi\theta}{\theta_0}\right)\right]$$

The velocity is maximum, when

$$\cos\left(\frac{2\pi\theta}{\theta_0}\right) = -1$$

$$\frac{2\pi\theta}{\theta_0} = \pi$$

$$= \frac{\theta_0}{2}$$

Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega S}{\theta_R}$$

- The velocity is maximum, when

$$\cos\left(\frac{2\,\pi\,\theta}{\theta_0}\right) = -1$$

$$\frac{2\,\pi\,\theta}{\theta_0} = \pi$$

$$= \frac{\theta_0}{2}$$

- Similarly, maximum velocity of the follower during return stroke,

$$v_R = \frac{2\,\omega\,S}{\theta_R}$$

- Now, acceleration of the follower after time t sec,

$$\frac{d^2x}{dt^2} = \frac{\omega\,S}{\theta_0}\ \frac{2\,\pi}{\theta_0}[\ \sin\left(\frac{2\,\pi\,\theta}{\theta_0}\right)\frac{d\theta}{dt}]$$

$$= \frac{2\,\pi\,\omega^2\,S}{(\ )_0^2}\ \sin\left(\frac{2\,\pi\,\theta}{\theta_0}\right)$$

- The acceleration is maximum, when

$$\sin\left(\frac{2\,\pi\,\theta}{\theta_0}\right) = 1$$

$$= \frac{\theta_0}{4}$$

$$a_0 = \frac{2\,\pi\,\omega^2\,S}{(\theta_0)^2}$$

Construction of cam profile for a Radial cam

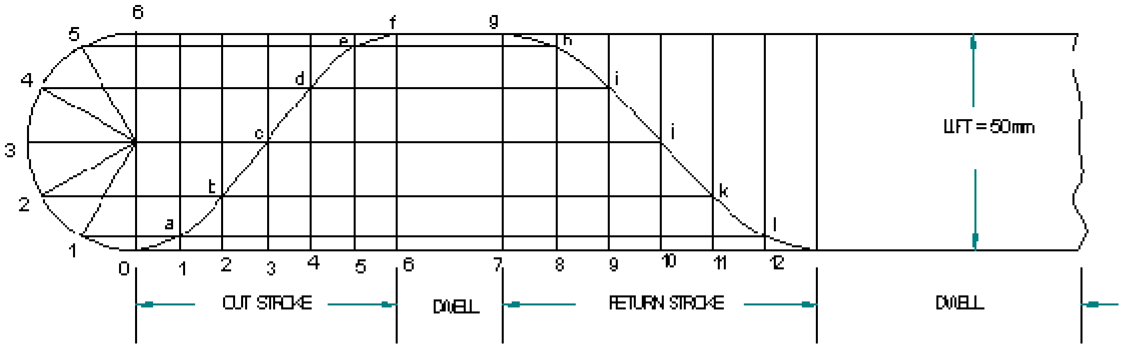
In order to draw the cam profile for a radial cam, first of all the displacement diagram for the given motion of the follower is drawn. Then by constructing the follower in its proper position at each angular position, the profile of the working surface of the cam is drawn.

In constructing the cam profile, the principle of kinematic inversion is used, i.e. the cam is imagined to be stationary and the follower is allowed to rotate in the opposite direction to the cam rotation.

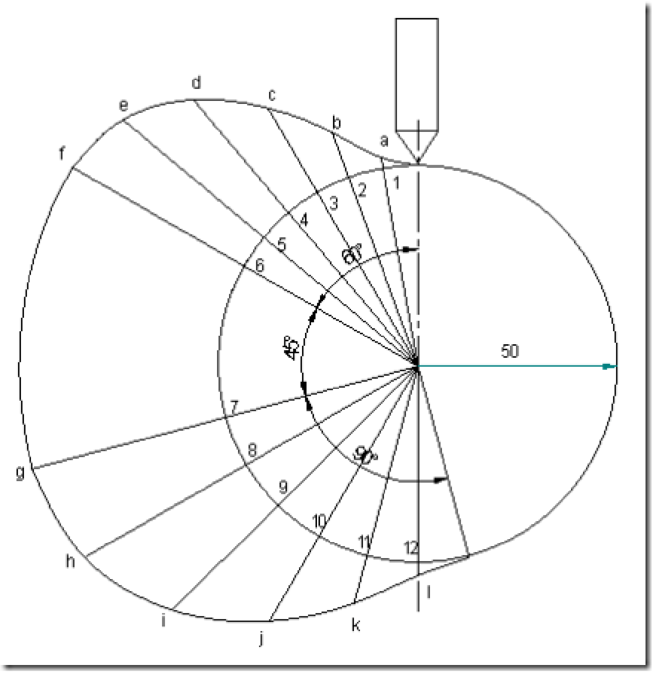
Examples based on cam profile

1. ***Follower type = Knife edged, in-line; lift = 50mm; base circle radius = 50mm; out stroke with SHM, for 60^0 cam rotation; dwell for 45^0 cam rotation; return stroke with SHM, for 90^0 cam rotation; dwell for the remaining period. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 1000 rpm in clockwise direction.***

Displacement diagram:



Cam profile: Construct base circle. Mark points 1,2,3…..in direction opposite to the direction of cam rotation. Transfer points a,b,c…..l from displacement diagram to the cam profile and join them by a smooth free hand curve. This forms the required cam profile.



Calculations:

Angular velocity of cam = $\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times1000}{60}$ =**104.76 rad/sec**

Max. velocity of follower during outstroke = vo_{max} = $\frac{\pi \omega s}{2\theta_o}$ =

= $\frac{\pi \times 104.76 \times 50}{2 \times \pi/3}$ =7857mm/sec =**7.857m/sec**

Similarly Max. velocity of follower during return stroke = , vr_{max} = $\frac{\pi \omega s}{2\theta_r}$ =

= $\frac{\pi \times 104.76 \times 50}{2 \times \pi/2}$ = 5238mm/sec = **5.238m/sec**

Max. acceleration during outstroke = ao_{max} = $r\omega^2{}_p$ (from d3) = $\frac{\pi^2 \omega^2 s}{2\theta_o{}^2}$ =

= $\frac{\pi^2 \times (104.76)^2 \times 50}{2 \times (\pi/3)^2}$ = 2469297.96mm/sec^2 = **2469.3m/sec^2**

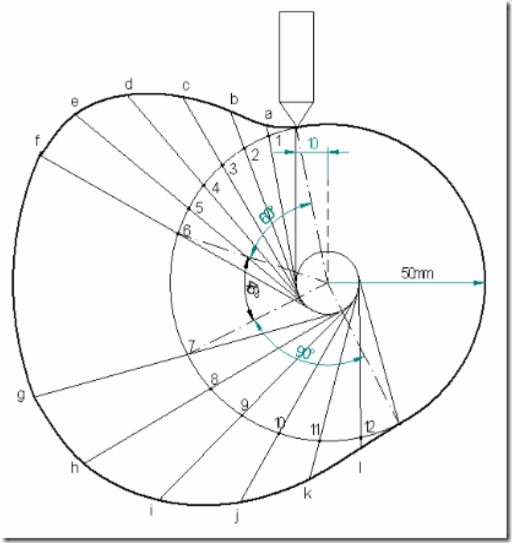
Similarly, Max. acceleration during return stroke = ar_{max} = $\frac{\pi^2 \omega^2 s}{2\theta^2{}_r}$ =

= $\frac{\pi^2 \times (104.76)^2 \times 50}{2 \times (\pi/2)^2}$ = 1097465.76mm/sec^2 = **1097.5m/sec^2**

2. ***Draw the cam profile for the same operating conditions of problem (1), with the follower off set by 10 mm to the left of cam center.***

Displacement diagram: Same as previous case.

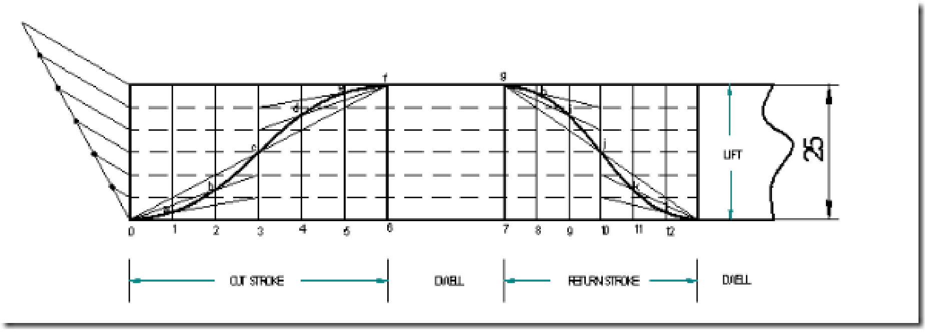
Cam profile: Construction is same as previous case, except that the lines drawn from 1,2,3…. are tangential to the offset circle of 10mm dia. as shown in the fig.



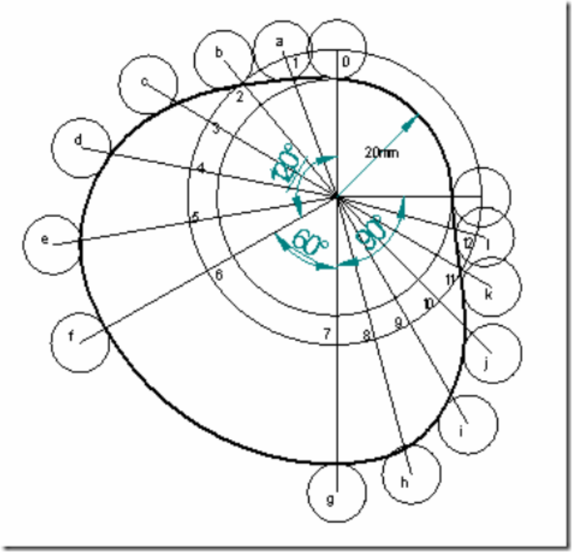
3. **Draw the cam profile for following conditions:**

Follower type = roller follower, in-line; lift = 25mm; base circle radius = 20mm; roller radius = 5mm; out stroke with UARM, for 120^0 cam rotation; dwell for 60^0 cam rotation; return stroke with UARM, for 90^0 cam rotation; dwell for the remaining period. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 1200 rpm in clockwise direction.

Displacement diagram:



Cam profile: Construct base circle and prime circle (25mm radius). Mark points 1,2,3…..in direction opposite to the direction of cam rotation, on prime circle. Transfer points a,b,c…..l from displacement diagram. At each of these points a,b,c… draw circles of 5mm radius, representing rollers. Starting from the first point of contact between roller and base circle, draw a smooth free hand curve, tangential to all successive roller positions. This forms the required cam profile.



Calculations:

Angular velocity of the cam = $\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times1200}{60} =$ **125.71rad/sec**

Max. velocity during outstroke = $vo_{\max} = \frac{2s}{t_o} = \frac{2\omega s}{\theta_o}$ =

= $\frac{2\times125.71\times25}{2\times\pi/3} =$ 2999.9mm/sec =**2.999m/sec**

Max. velocity during return stroke = $vr_{\max} = \frac{2s}{t_r} = \frac{2\omega s}{\theta_r} = \frac{2\times125.71\times25}{\pi/2} =$

= 3999.86mm/sec = **3.999m/sec**

Acceleration of the follower during outstroke = $a_o = \frac{vo_{\max}}{t_o/2} = \frac{4\omega^2 s}{\theta_o{}^2}$ =

= $\frac{4\times(125.71)^2\times25}{\left(2\times\pi/3\right)^2} =$ 359975mm/sec^2 = **359.975m/sec^2**

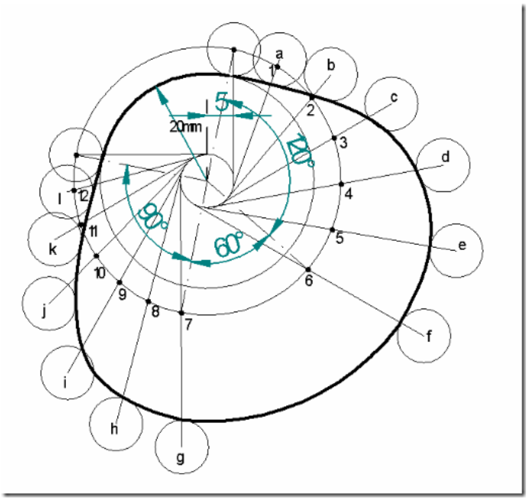
Similarly acceleration of the follower during return stroke = $a_r = \frac{4\omega^2 s}{\theta_r{}^2}$ =

$$= \frac{4\times(125.71)^2\times 25}{\left(\pi/2\right)^2} = 639956\text{mm/sec}^2 = \mathbf{639.956m/sec^2}$$

4. ***Draw the cam profile for conditions same as in (3), with follower off set to right of cam center by 5mm and cam rotating counter clockwise.***

Displacement diagram: Same as previous case.

Cam profile: Construction is same as previous case, except that the lines drawn from 1,2,3…. are tangential to the offset circle of 10mm dia. as shown in the fig.



5. **Draw the cam profile for following conditions:**

Follower type = roller follower, off set to the right of cam axis by 18mm; lift = 35mm; base circle radius = 50mm; roller radius = 14mm; out stroke with SHM in 0.05sec; dwell for 0.0125sec; return stroke with UARM, during 0.125sec; dwell for the remaining period. During return stroke, acceleration is 3/5 times retardation. Determine max. velocity and acceleration during out stroke and return stroke if the cam rotates at 240 rpm.

Calculations:

Cam speed = 240rpm. Therefore, time for one rotation = $\frac{60}{240} = 0.25\text{sec}$

Angle of out stroke = $\theta_o = \frac{0.05}{0.25}\times 360 = 72^0$

Angle of first dwell = $\theta_{w1} = \frac{0.0125}{0.25}\times 360 = 18^0$

Angle of return stroke = $\theta_r = \frac{0.125}{0.25}\times 360 = 180^0$

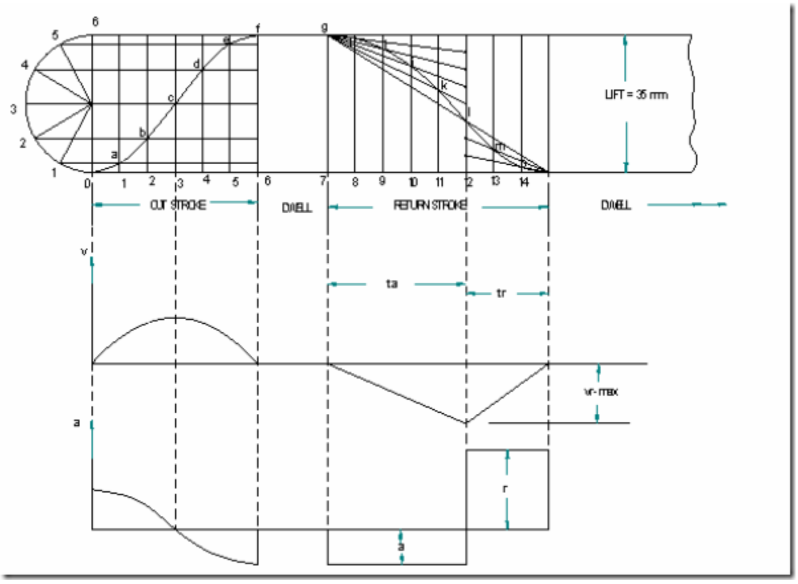
Angle of second dwell = $\theta_{w2} = 90^0$

Since acceleration is 3/5 times retardation during return stroke,

$a = \frac{3}{5}r$ (from acceleration diagram) $\therefore \frac{a}{r} = \frac{3}{5}$

But $a = \frac{v_{\max}}{t_a}; r = \frac{v_{\max}}{t_r} \therefore \frac{a}{r} = \frac{t_r}{t_a} = \frac{3}{5}$

Displacement diagram is constructed by selecting t_a and t_r accordingly.



Angular velocity of cam = $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 240}{60}$ =**25.14 rad/sec**

Max. velocity of follower during outstroke = $vo_{\max}$ = $\frac{\pi \omega s}{2\theta_o}$ =

= $\frac{\pi \times 25.14 \times 35}{2 \times \left(2 \times \pi/5\right)}$ = 1099.87mm/sec =**1.1m/sec**

Similarly Max. velocity during return stroke = $vr_{\max} = \frac{2\omega s}{\theta_r} = \frac{2 \times 25.14 \times 35}{\pi} =$

= 559.9 mm/sec = **0.56m/sec**

Max. acceleration during outstroke = $ao_{\max}$ = $r\omega^2_p$ (from d3) = $\frac{\pi^2 \omega^2 s}{2\theta_o^{\,2}}$ =

= $\frac{\pi^2 \times (25.14)^2 \times 35}{2 \times \left(2 \times \pi/5\right)^2} =$ 69127.14mm/sec^2 = **69.13m/sec^2**

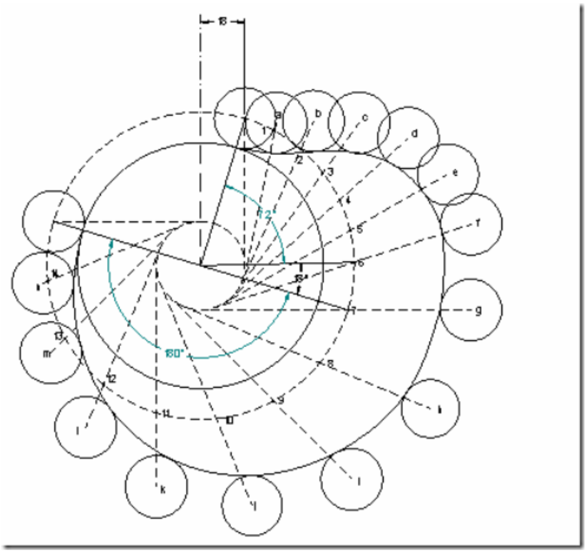
acceleration of the follower during return stroke

= $a_r = \frac{vr_{\max}}{t_a} = \frac{2\omega s/\theta_r}{5 \times \pi/8 \times \omega} = \frac{16 \times \omega^2 \times s}{5 \times \pi \times \theta_r} = \frac{16 \times (25.14)^2 \times 35}{5 \times \pi \times \pi}$ = 7166.37 mm/sec^2 = **7.17m/sec^2**

similarly retardation of the follower during return stroke

$$r_r = \frac{vr_{\max}}{t_r} = \frac{2\omega s/\theta_r}{3\times\pi/8\times\omega} = \frac{16\times\omega^2\times s}{3\times\pi\times\theta_r} = \frac{16\times(25.14)^2\times 35}{3\times\pi\times\pi}$$

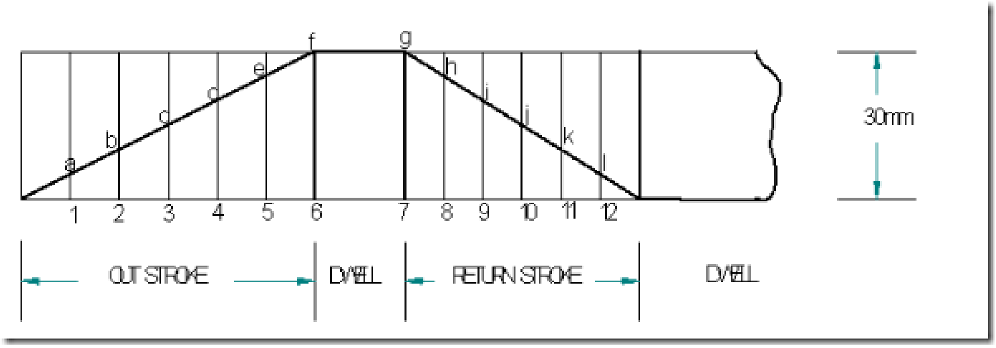
= = 11943.9 mm/sec^2 = **11.94m/sec^2**



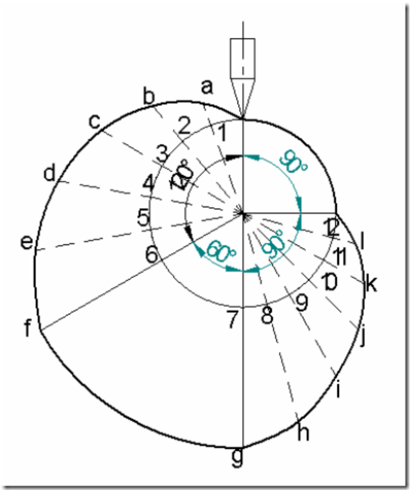
6. ***Draw the cam profile for following conditions:***

Follower type = knife edged follower, in line; lift = 30mm; base circle radius = 20mm; out stroke with uniform velocity in 120^0 of cam rotation; dwell for 60^0; return stroke with uniform velocity, during 90^0 of cam rotation; dwell for the remaining period.

Displacement diagram:



Cam profile:



LECTURE 1

CAMS TERMINOLOGY

LIVE TO LEARN & LEARN TO SHARE

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OUTLINE

- CAMS
- Classifications of CAMs
- Cam nomenclature
- Motion of the Follower
  - Uniform velocity
  - Simple harmonic motion
  - Uniform acceleration and retardation,
  - Cycloidal motion

CAMS

Cam - A mechanical device used to transmit motion to a follower by direct contact.

Cam – driver; Follower - driven

In a cam - follower pair, the cam normally rotates while the follower may translate or oscillate.

CLASSIFICATION OF CAMS (BASED ON SHAPE)

- **Disk or plate cams**
- Cylindrical Cam
- Translating cam

CLASSIFICATION OF CAMS
(BASED ON SURFACE IN CONTACT)

- Knife edge follower
- Roller follower
- Flat faced follower
- Spherical follower

CAM NOMENCLATURE

Base circle : smallest circle of the cam profile.

Trace point :

Reference point on the follower

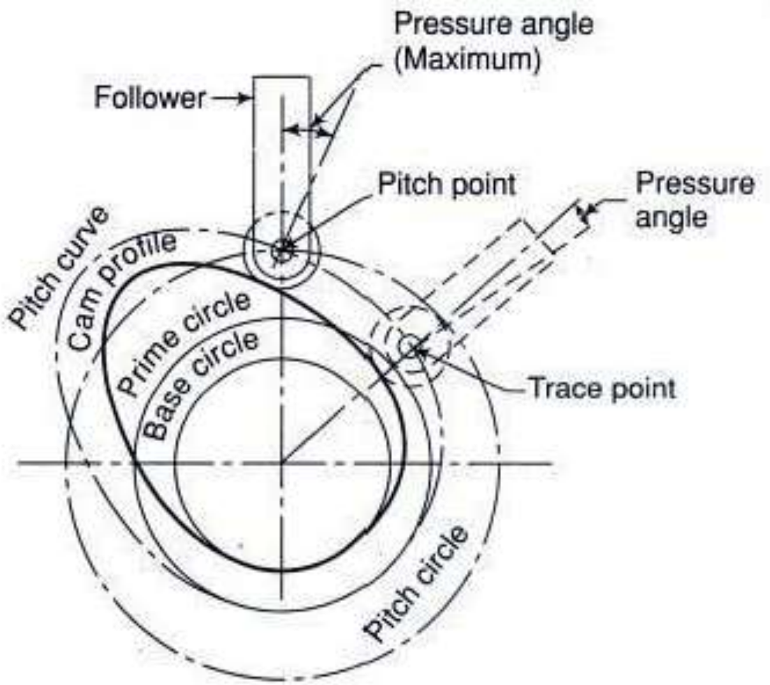
Which generates the pitch curve.

Pressure angle:

Angle between the direction of

the follower motion and a normal

to the pitch curve



CAM NOMENCLATURE

Pitch point: Point on the pitch curve having the maximum pressure angle.

Pitch circle: circle drawn through the pitch points.

Pitch curve: curve generated by the trace point

Prime circle: It is tangent to the pitch curve.

Lift or stroke: maximum travel of the follower from its lowest position to the Top most position.

MOTION OF THE FOLLOWER

1. Uniform velocity
2. Simple harmonic motion
3. Uniform acceleration and retardation,
4. Cycloidal motion

LECTURE 2

UNIFORM VELOCITY SIMPLE HARMONIC MOTION

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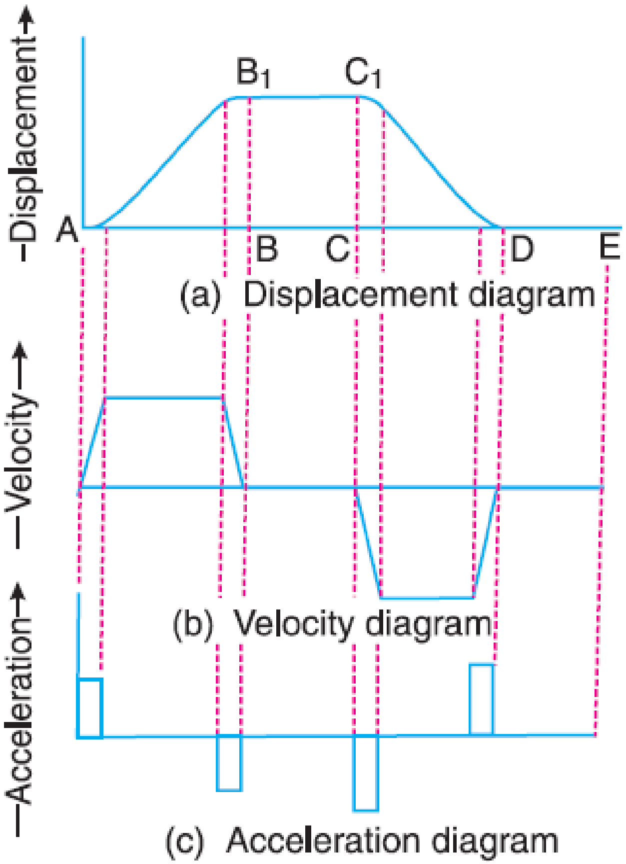
UNIFORM VELOCITY



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

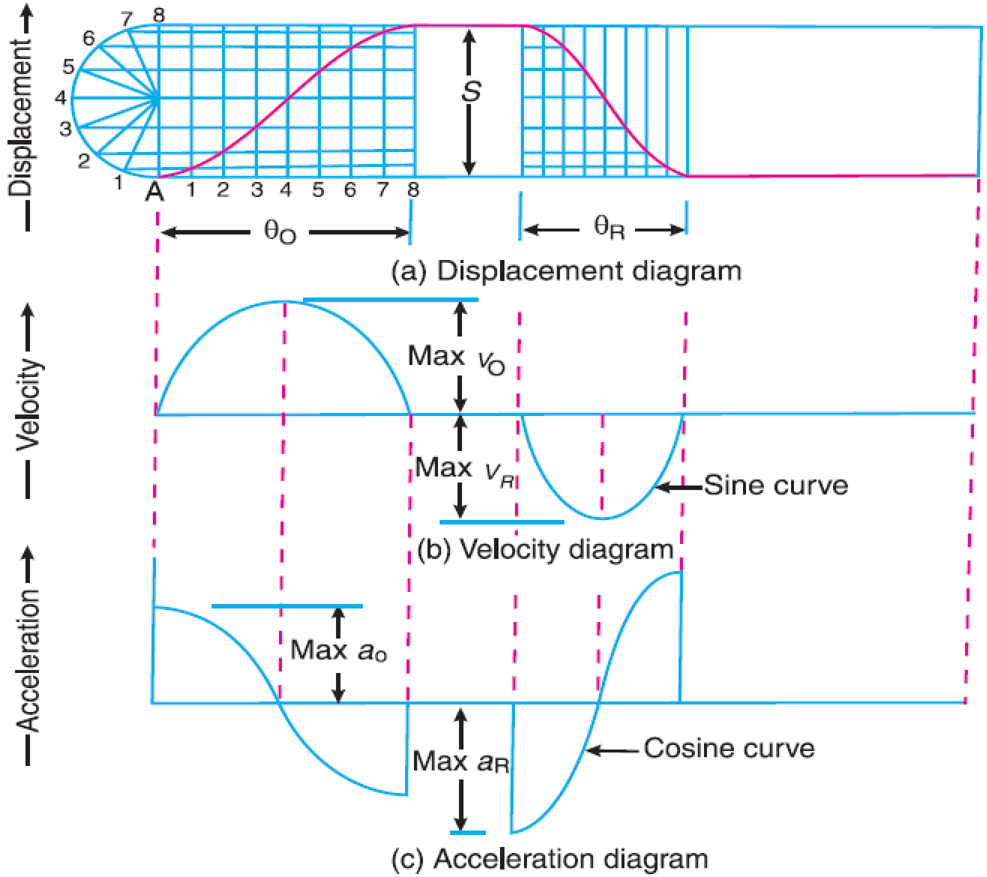
Modified displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

[This Figure is taken from Book authored by R S Khurmi]

- ➢The sharp corners at the beginning and at the end of each stroke are rounded off by the parabolic curves in the displacement diagram.

- ➢The parabolic motion results in a very low acceleration of the follower for a given stroke and cam speed.

FOLLOWER MOVES WITH SIMPLE HARMONIC MOTION (SHM)



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

[This Figure is taken from Book authored by R S Khurmi]

- ➢Draw a semi-circle on the follower stroke as diameter.
- ➢Divide the semi-circle into any number of even equal parts (say eight).

FOLLOWER MOVES WITH SIMPLE HARMONIC MOTION (SHM)

Outward stroke in SHM is equivalent to π ;
Meanwhile CAM is making θ
At any instant of time 't', angular disp. = θ = ωt

$$\text{SHM, } y = \frac{S}{2}\left(1 - \cos\frac{\pi\theta}{\theta_o}\right)$$

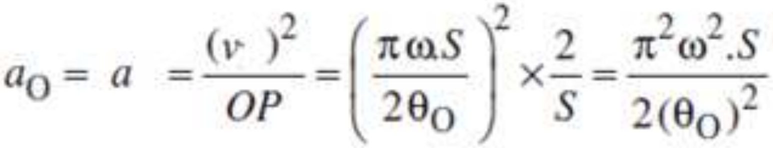
$$V = \frac{dy}{dt} = \frac{dy}{d\theta}\,\frac{d\theta}{dt} = \frac{dy}{d\theta}\,\omega = \frac{\pi\omega S}{2\theta_o}\,\text{Sin}\,\frac{\pi\theta}{\theta_o}$$

$$\text{For Max. outward velocity } V_o = \frac{\pi\omega S}{2\theta_o}$$

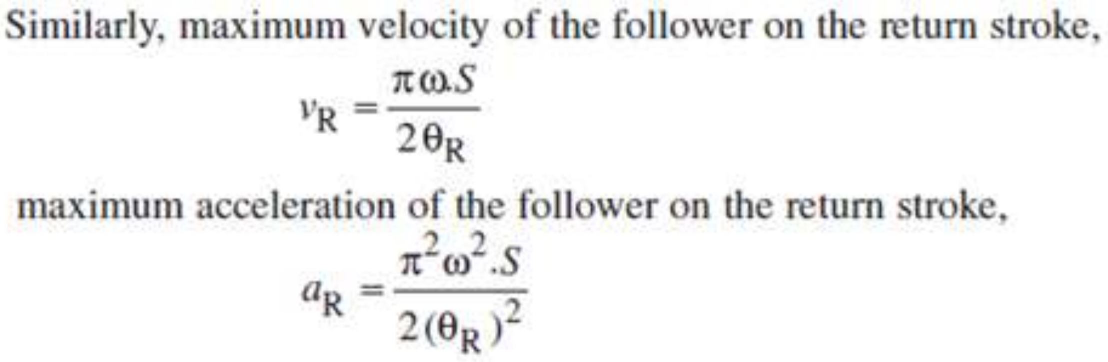
FOLLOWER MOVES WITH SIMPLE HARMONIC MOTION (SHM)

Similar manner, acceleration can be found by taking time derivative of velocity.

(OR)



$$a_O = a \;= \frac{(v\ )^2}{OP} = \left(\frac{\pi\omega.S}{2\theta_O}\right)^2 \times \frac{2}{S} = \frac{\pi^2\omega^2.S}{2(\theta_O)^2}$$



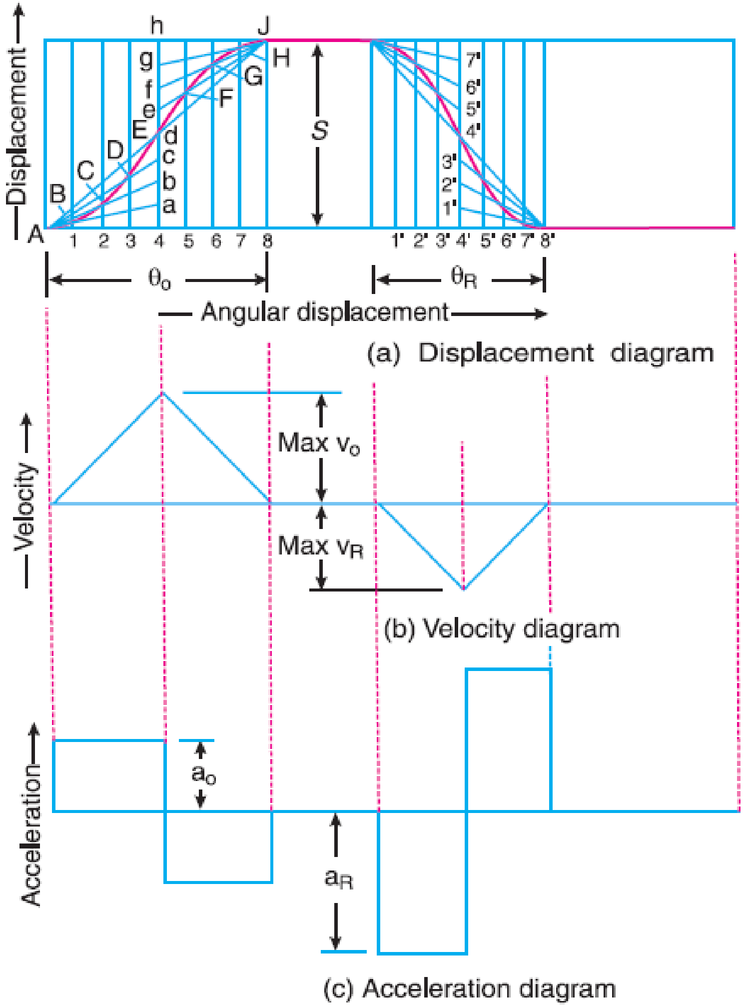
Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi\omega.S}{2\theta_R}$$

maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2\omega^2.S}{2(\theta_R)^2}$$

FOLLOWER MOVES WITH UNIFORM ACCELERATION AND RETARDATION



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

[This Figure is taken from Book authored by R S Khurmi]

maximum velocity of the follower during outstroke,

$$v_O = \frac{S}{t_O/2} = \frac{2\omega.S}{\theta_O}$$

maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega.S}{\theta_R}$$

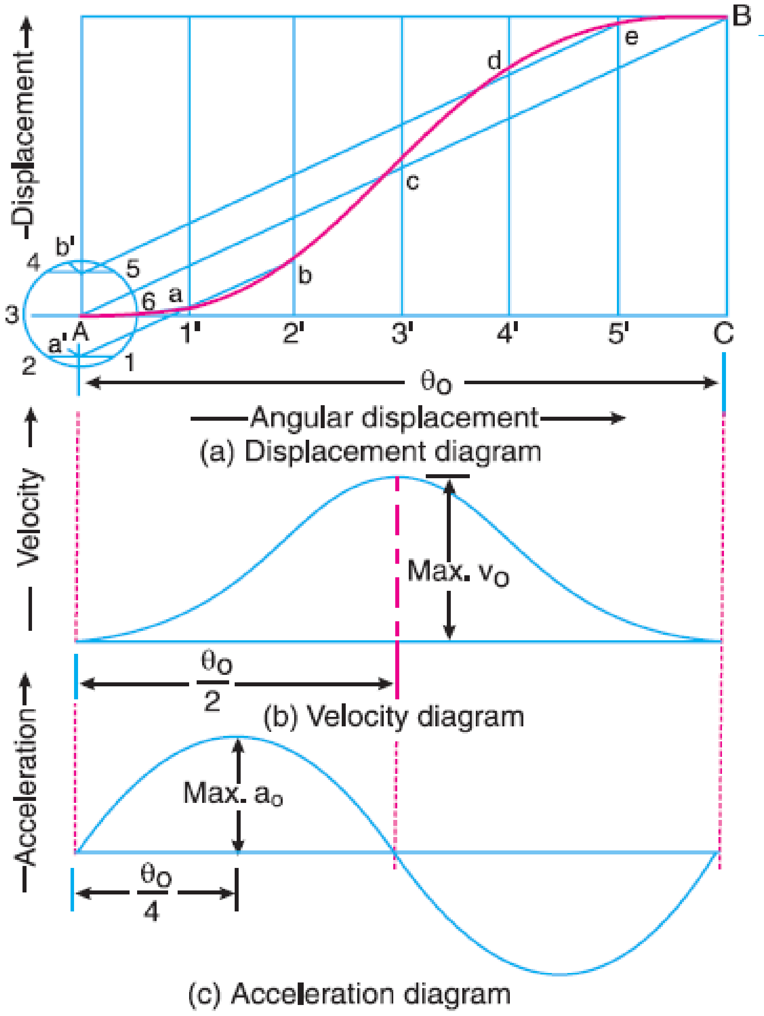
Maximum acceleration of the follower during outstroke,

$$a_O = \frac{v_O}{t_O/2} = \frac{2\times 2\,\omega.S}{t_O.\theta_O} = \frac{4\,\omega^2.S}{(\theta_O)^2}$$

maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\,\omega^2.S}{(\theta_R)^2}$$

FOLLOWER MOVES WITH CYCLOIDAL MOTION



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

[This Figure is taken from Book authored by R S Khurmi]

cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line

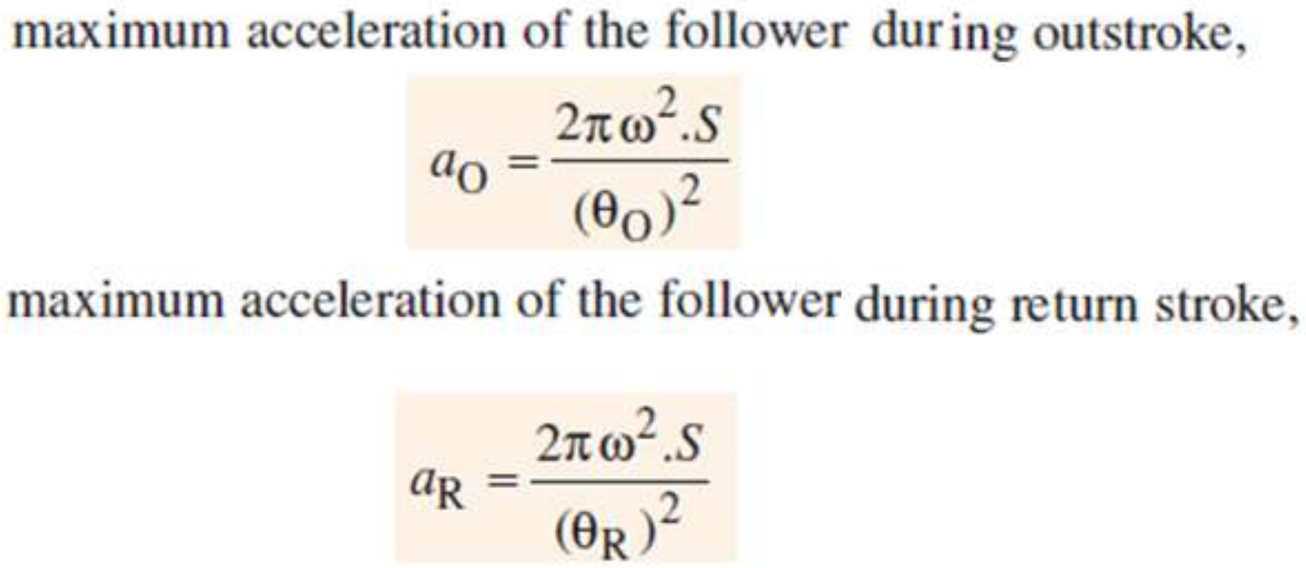
Radius of the circle r = S / 2π

Where S = stroke

Max. Velocity of the follower during outward stroke = $v_O = \frac{2\omega S}{\theta_O}$

Max. Velocity of the follower during return stroke = $v_R = \frac{2\omega . S}{\theta_R}$

FOLLOWER MOVES WITH CYCLOIDAL MOTION



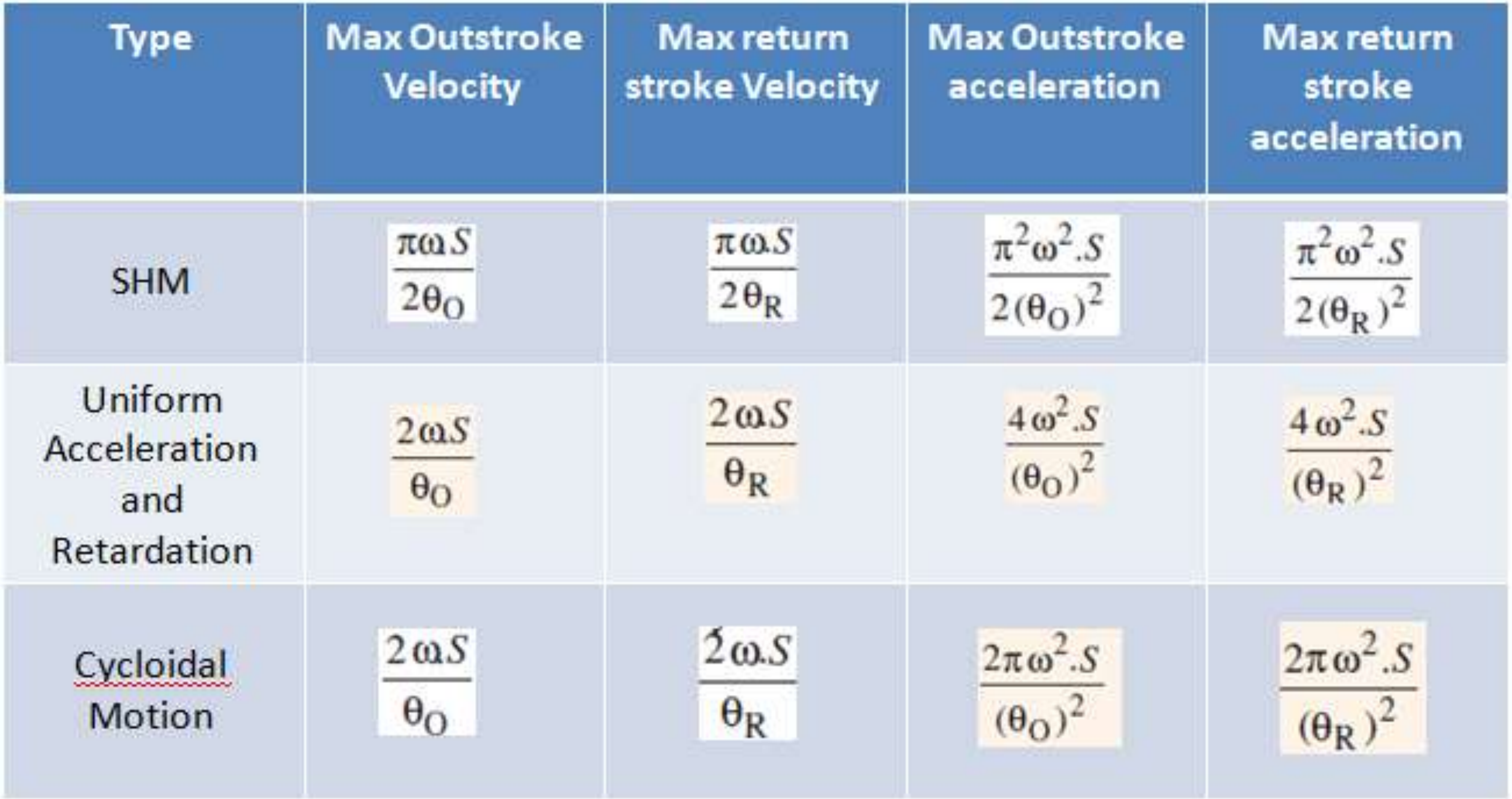
maximum acceleration of the follower during outstroke,

$$a_O = \frac{2\pi\omega^2.S}{(\theta_O)^2}$$

maximum acceleration of the follower during return stroke,

$$a_R = \frac{2\pi\omega^2.S}{(\theta_R)^2}$$

SUMMARY



Type	Max Outstroke Velocity	Max return stroke Velocity	Max Outstroke acceleration	Max return stroke acceleration
SHM	$\frac{\pi\omega S}{2\theta_O}$	$\frac{\pi\omega.S}{2\theta_R}$	$\frac{\pi^2\omega^2.S}{2(\theta_O)^2}$	$\frac{\pi^2\omega^2.S}{2(\theta_R)^2}$
Uniform Acceleration and Retardation	$\frac{2\omega S}{\theta_O}$	$\frac{2\omega S}{\theta_R}$	$\frac{4\omega^2.S}{(\theta_O)^2}$	$\frac{4\omega^2.S}{(\theta_R)^2}$
Cycloidal Motion	$\frac{2\omega S}{\theta_O}$	$\frac{2\omega.S}{\theta_R}$	$\frac{2\pi\omega^2.S}{(\theta_O)^2}$	$\frac{2\pi\omega^2.S}{(\theta_R)^2}$

LECTURE 3

UNIFORM ACCELERATION

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NUMERICAL EXAMPLE -1

A cam is to be designed for a knife edge follower with the following data :

1. Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.
2. Dwell for the next 30°.
3. During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion.
4. Dwell during the remaining 180°.

Draw the profile of the cam when

(a) the line of stroke of the follower passes through the axis of the cam shaft, and

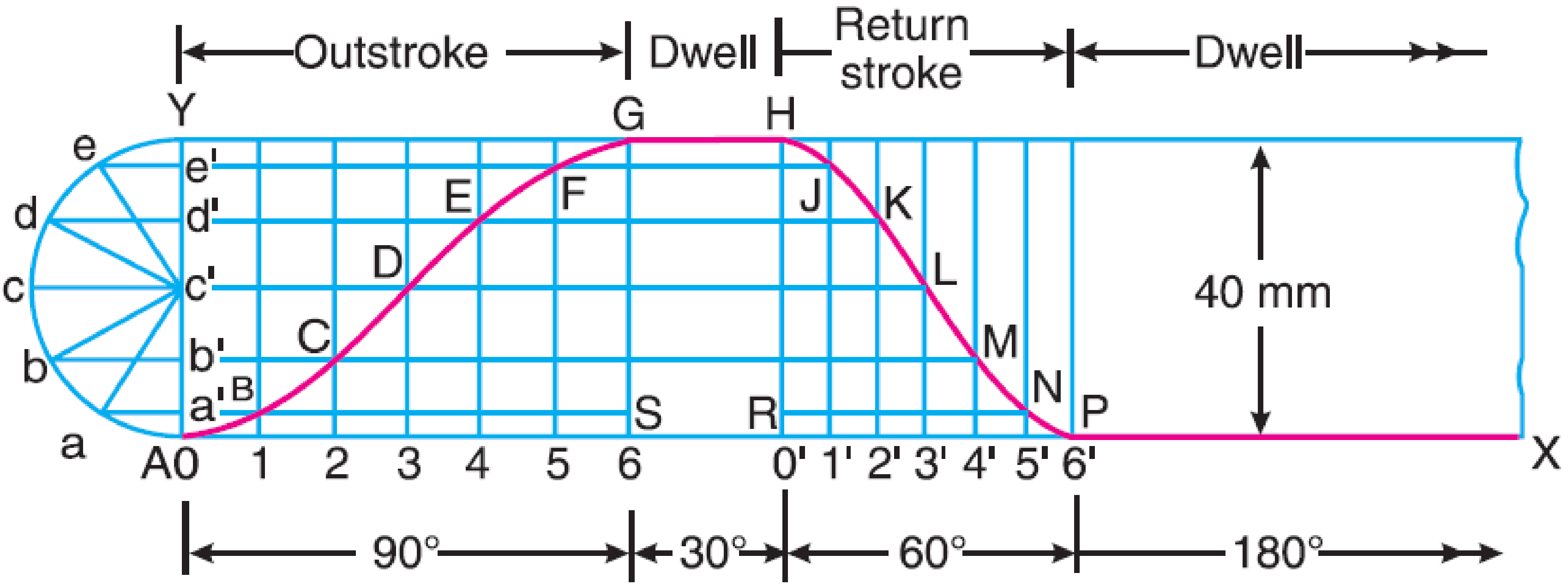
(b) the line of stroke is offset 20 mm from the axis of the cam shaft.

The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

NUMERICAL EXAMPLE -1

Given : S = 40 mm = 0.04 m; θ_O = 90° = $\pi/2$ rad = 1.571 rad

θ_R = 60° = $\pi/3$ rad = 1.047 rad ; N = 240 r.p.m.

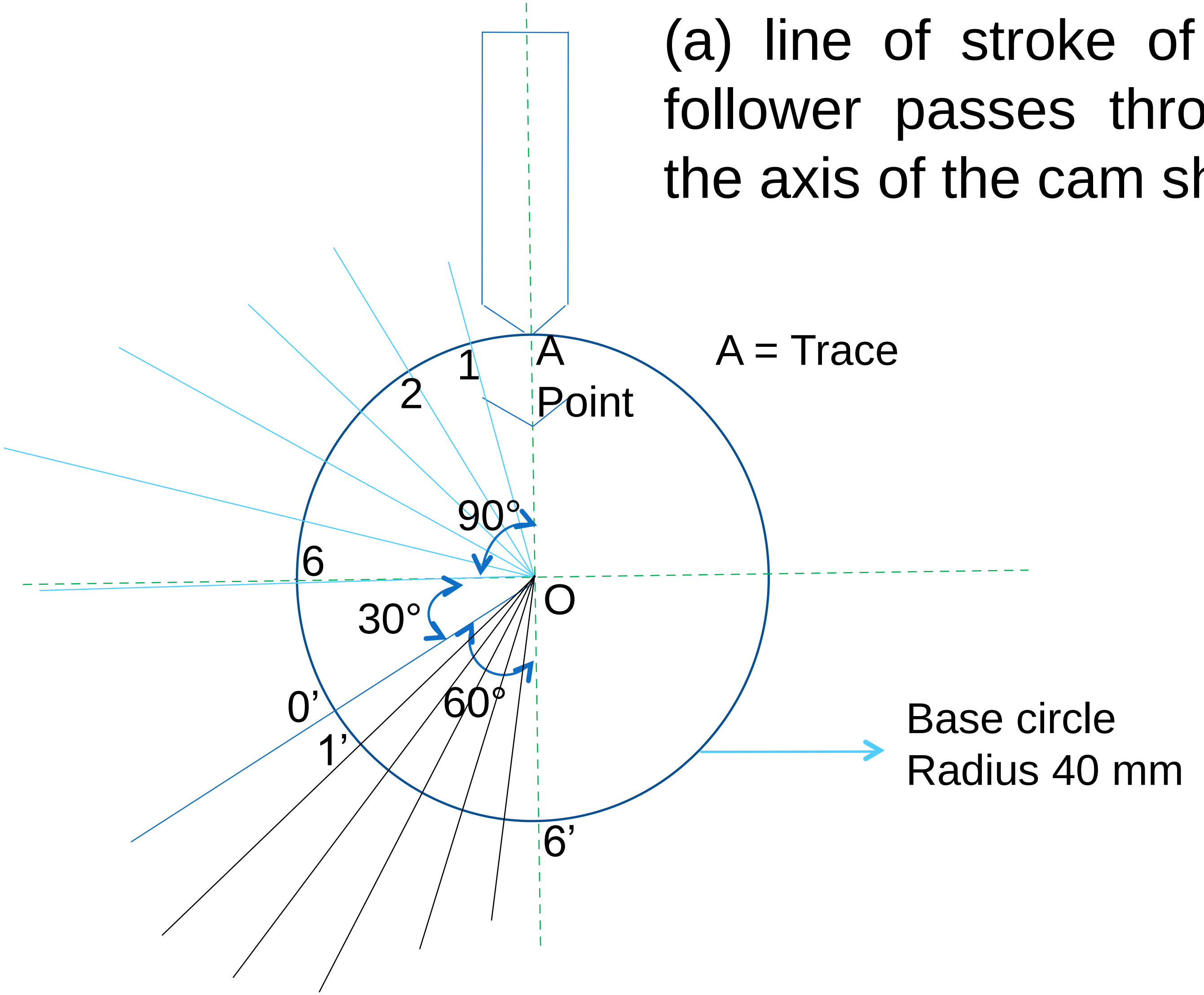


[This Figure is taken from Book authored by R S Khurmi]

Draw horizontal line AX = 360° to any convenient scale

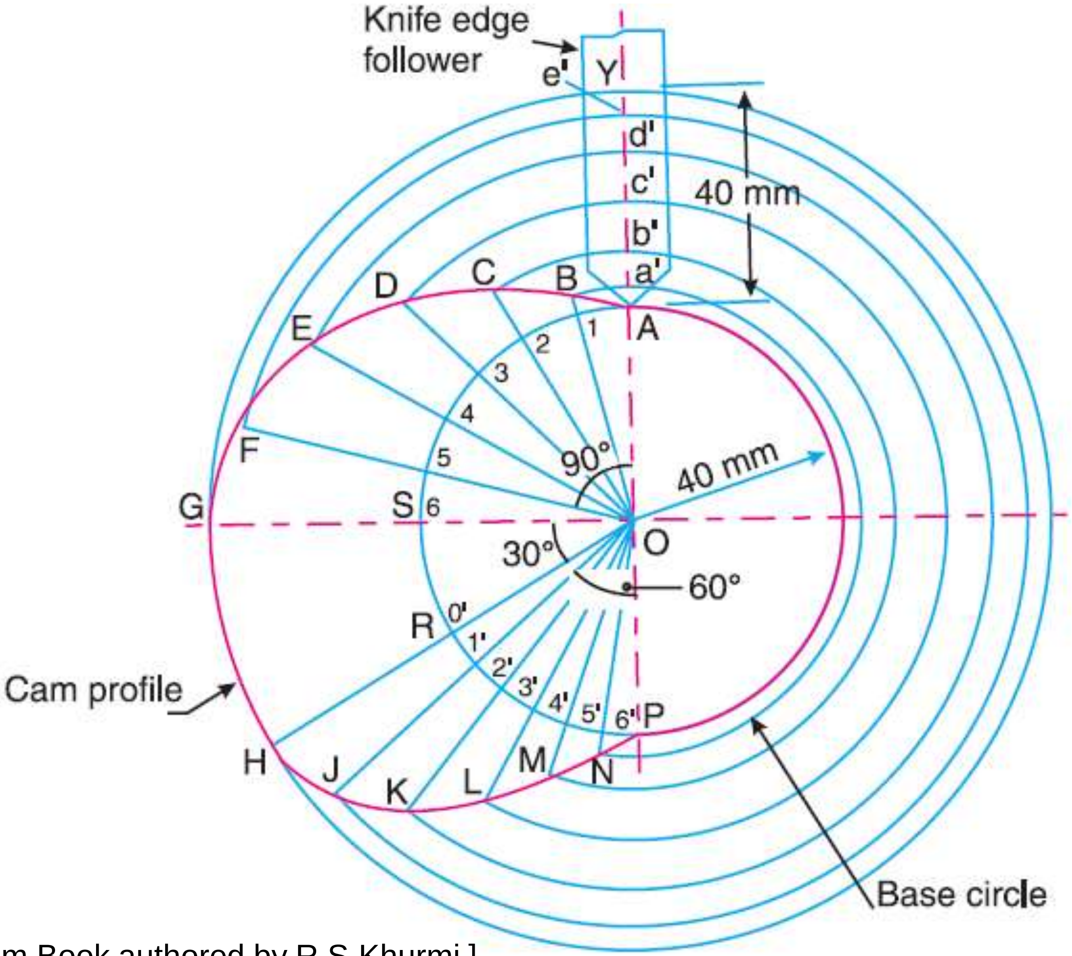
NUMERICAL EXAMPLE -1

(a) line of stroke of the follower passes through the axis of the cam shaft



NUMERICAL EXAMPLE -1

Line of stroke of the follower passes through the axis of the cam shaft



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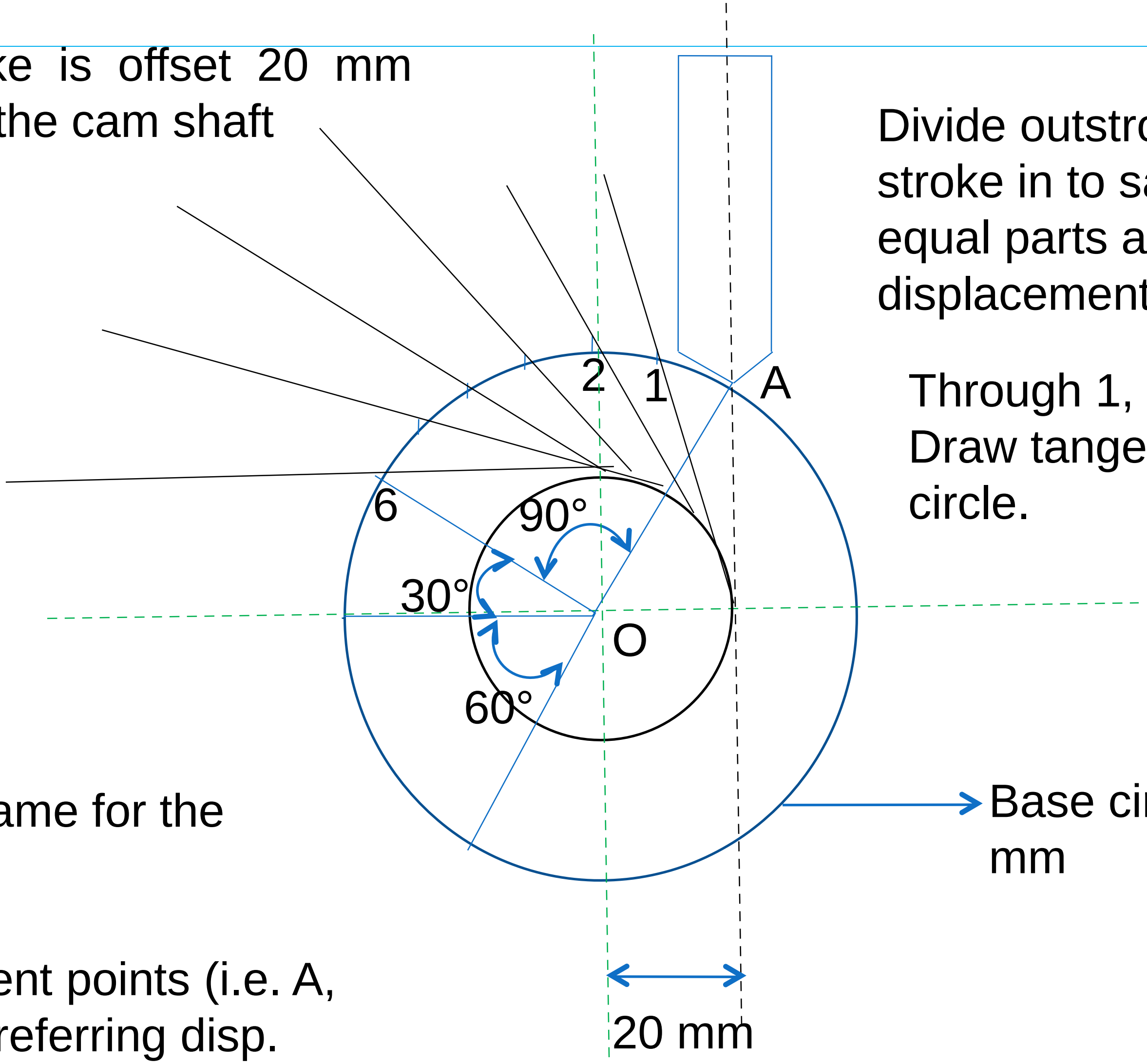
NUMERICAL EXAMPLE -1

(b) line of stroke is offset 20 mm from the axis of the cam shaft



Divide outstroke and return stroke in to same number of equal parts as in displacement diagram

Through 1, 2, .. & 6, Draw tangent to offset circle.

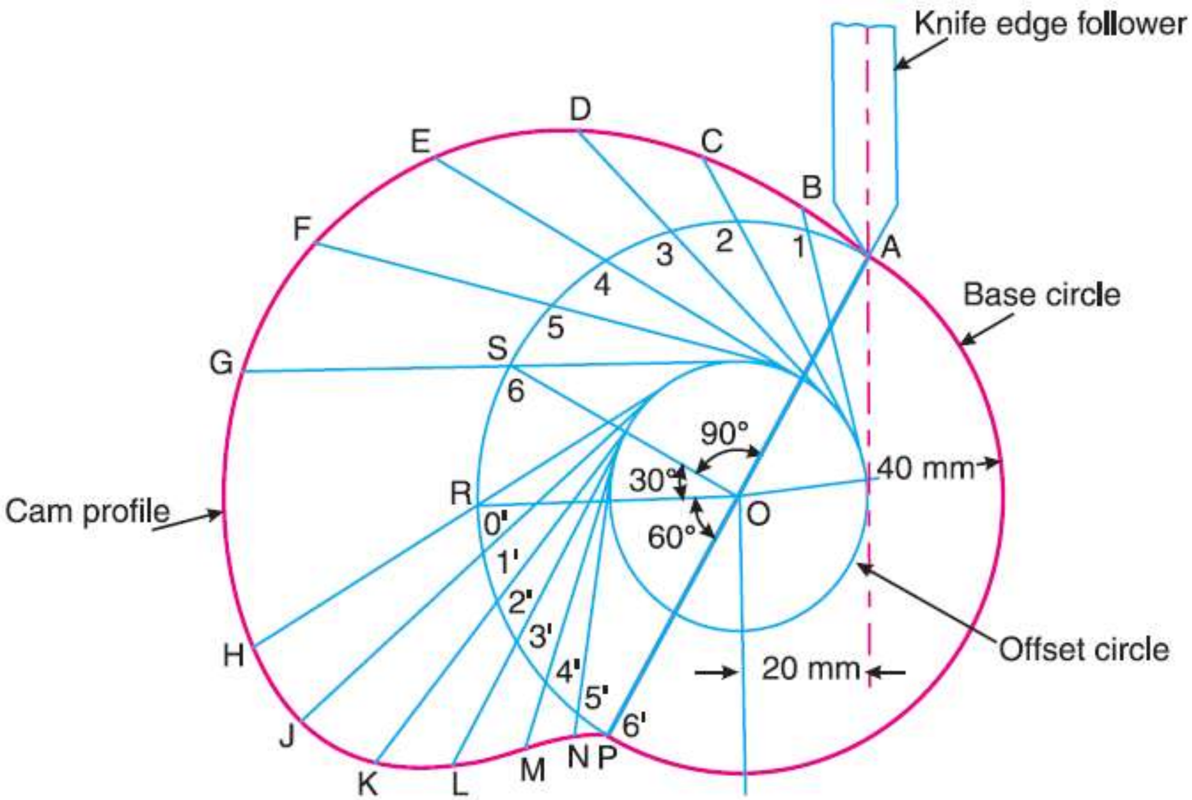
Base circle Radius 40 mm

Repeat the same for the return stroke.

Mark the salient points (i.e. A, B, C .. G) by referring disp. diagram

NUMERICAL EXAMPLE -1

line of stroke is offset 20 mm from the axis of the cam shaft



[This Figure is taken from Book authored by R S Khurmi]

NUMERICAL EXAMPLE -1

Maximum velocity of the follower during its ascent and descent

We know that $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14$ rad/s

$$v_O = \frac{\pi \omega . S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s}$$ **Ans.**

$$v_R = \frac{\pi \omega . S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s}$$ **Ans.**

Maximum acceleration of the follower during its ascent and descent

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.571)^2} = 50.6 \text{ m/s}^2$$ **Ans.**

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.047)^2} = 113.8 \text{ m/s}^2$$ **Ans.**

LECTURE 4

MAXIMUM VELOCITY DURING OUTWARD AND RETURN STROKES

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NUMERICAL EXAMPLE-2

A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower, at the end of a valve rod, motion described below :

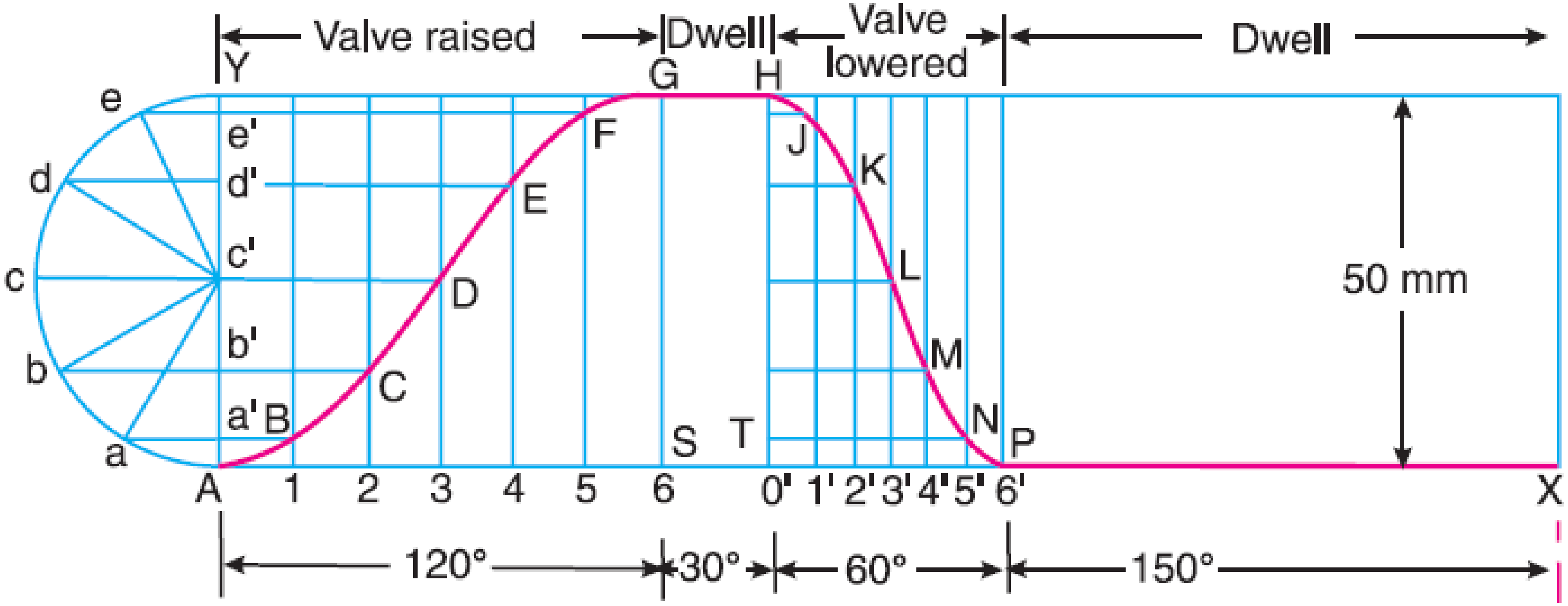
1. To raise the valve through 50 mm during 120° rotation of the cam ;
2. To keep the valve fully raised through next 30°;
3. To lower the valve during next 60°; and
4. To keep the valve closed during rest of the revolution i.e. 150° ;

The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm. Draw the profile of the cam when (a) the line of stroke of the valve rod passes through the axis of the cam shaft, and (b) the line of the stroke is offset 15 mm from the axis of the cam shaft. The displacement of the valve, while being raised and lowered, is to take place with simple harmonic motion. Determine the maximum acceleration of the valve rod when the cam shaft rotates at 100 r.p.m.

Draw the displacement, the velocity and the acceleration diagrams for one complete revolution of the cam.

NUMERICAL EXAMPLE-2

Given : $S = 50 \text{ mm} = 0.05 \text{ m}$; $\theta_O = 120° = 2\pi/3 \text{ rad} = 2.1 \text{ rad}$;
$\theta_R = 60° = \pi/3 \text{ rad} = 1.047 \text{ rad}$; $N = 100$ r.p.m.

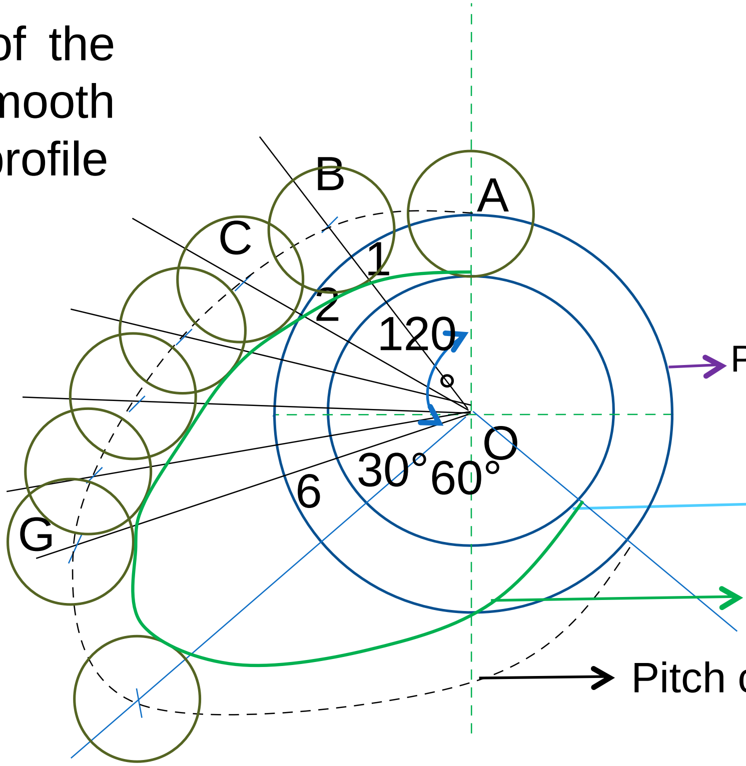


[This Figure is taken from Book authored by R S Khurmi]

NUMERICAL EXAMPLE-2

Draw circle by keeping B as center &
r = roller radius
Similarly from C, D, .. G.

Join the bottoms of the circles with a smooth curve to get CAM profile



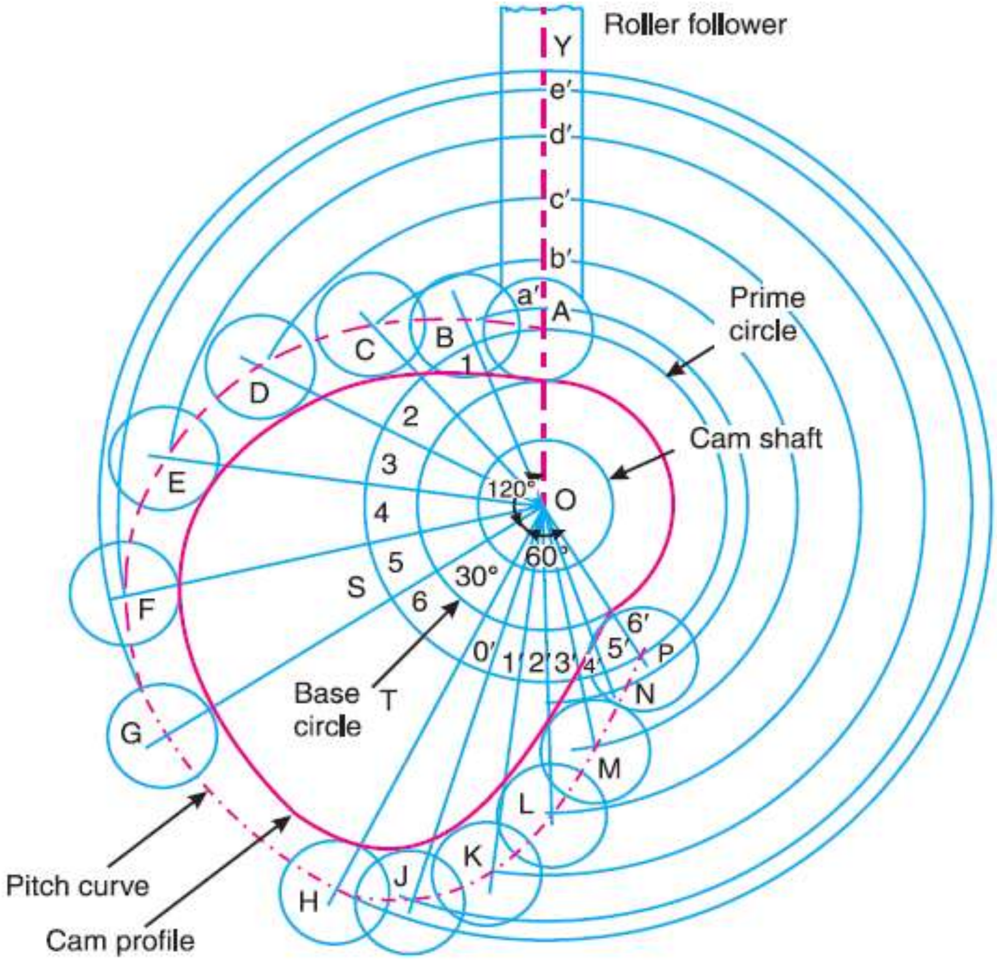
Prime circle R =25 + .5(20) = 35mm

Base circle R =25 mm

CAM Profile

Pitch curve

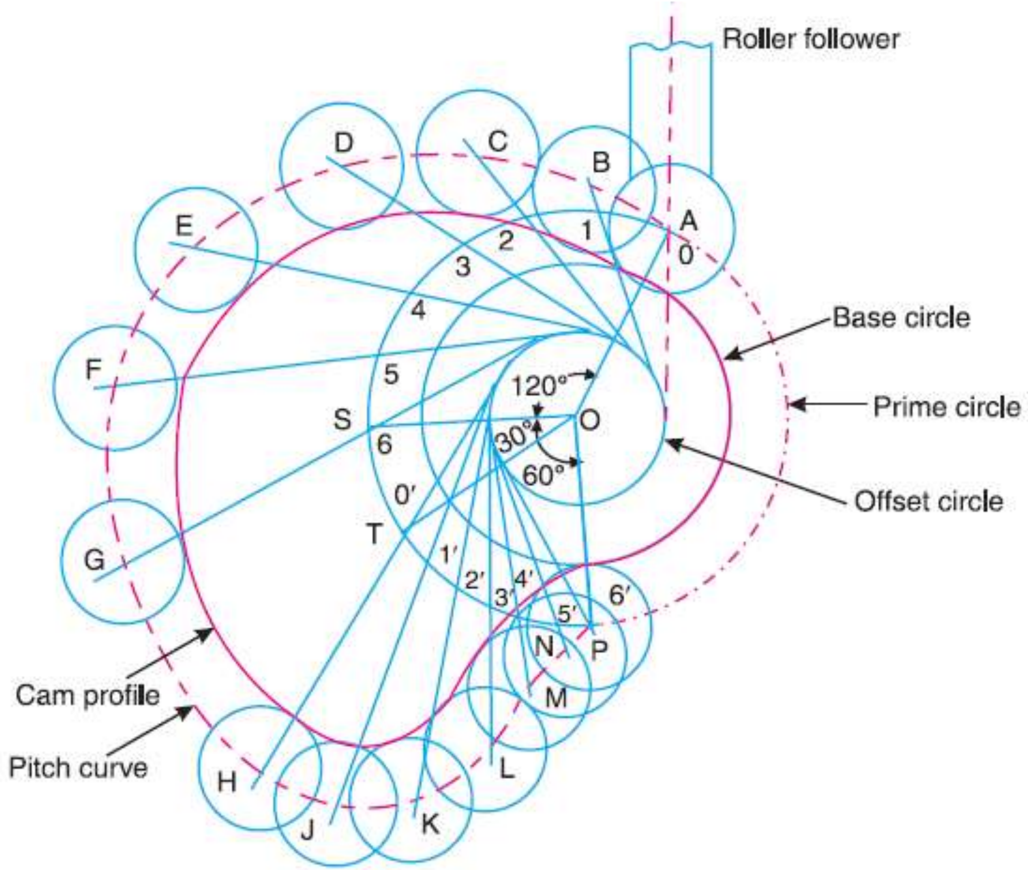
NUMERICAL EXAMPLE-2



[This Figure is taken from Book authored by R S Khurmi]

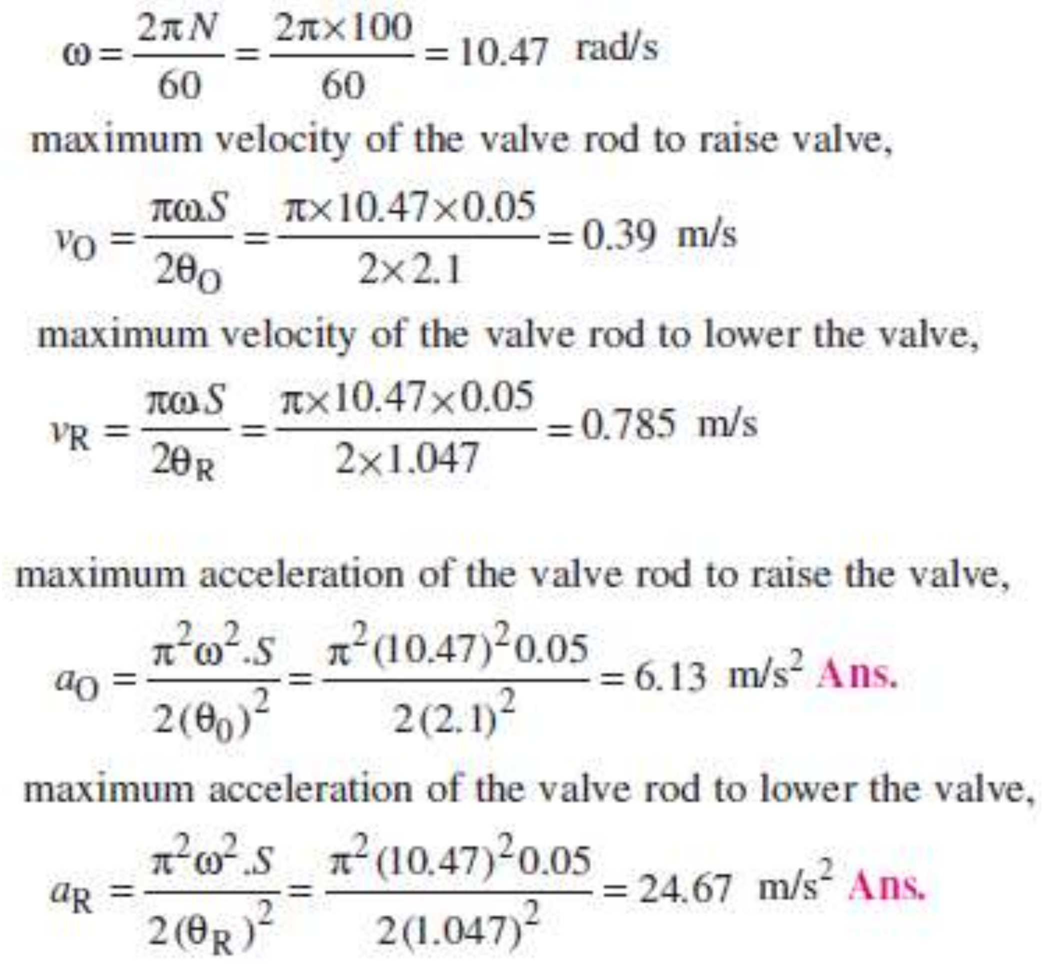
NUMERICAL EXAMPLE-2

When the line of the stroke is offset 15 mm from the axis of the cam shaft



[This Figure is taken from Book authored by R S Khurmi]

NUMERICAL EXAMPLE-2



$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} = 10.47 \text{ rad/s}$$

maximum velocity of the valve rod to raise valve,

$$v_O = \frac{\pi \omega . S}{2\theta_O} = \frac{\pi \times 10.47 \times 0.05}{2 \times 2.1} = 0.39 \text{ m/s}$$

maximum velocity of the valve rod to lower the valve,

$$v_R = \frac{\pi \omega . S}{2\theta_R} = \frac{\pi \times 10.47 \times 0.05}{2 \times 1.047} = 0.785 \text{ m/s}$$

maximum acceleration of the valve rod to raise the valve,

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_0)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(2.1)^2} = 6.13 \text{ m/s}^2 \text{ Ans.}$$

maximum acceleration of the valve rod to lower the valve,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(1.047)^2} = 24.67 \text{ m/s}^2 \text{ Ans.}$$

LECTURE 5

MAXIMUM ACCELERATION DURING OUTWARD AND RETURN STROKES

LIVE TO LEARN & LEARN TO SHARE

MRCET CAMPUS

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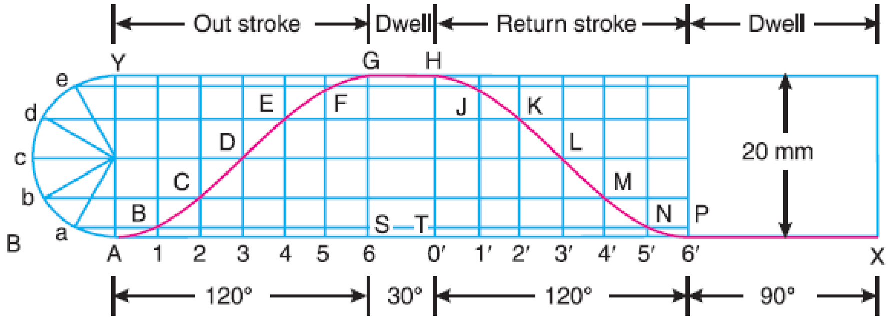
DEPARTMENT OF MECHANICAL ENGINEERING

NUMERICAL EXAMPLE -3

A cam drives a flat reciprocating follower in the following manner :

During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with simple harmonic motion. The follower dwells during next 30° of cam rotation. During next 120° of cam rotation, the follower moves inwards with simple harmonic motion. The follower dwells for the next 90° of cam rotation. The minimum radius of the cam is 25 mm. Draw the profile of the cam.

NUMERICAL EXAMPLE -3



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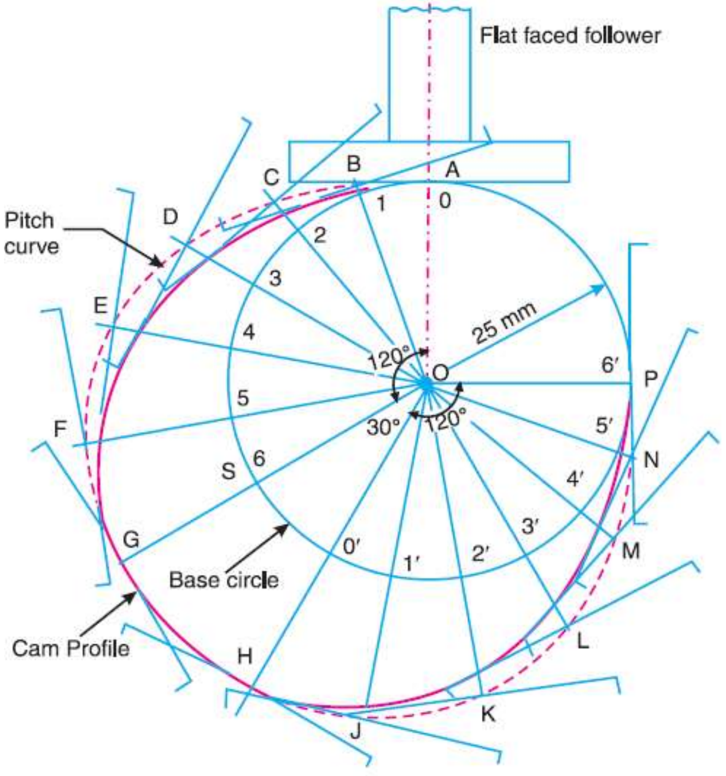


[This Figure is taken from Book authored by R S Khurmi]

- Construction procedure is Similar to knife edge / roller follower.

- Pitch circle is drawn by transferring distances 1B, 2C, 3D etc., from displacement diagram to the profile construction.

- Now at points B, C, D . . . M, N, P, draw the position of the flat-faced follower. The axis of the follower at all these positions passes through the cam centre.

- CAM profile is the curve drawn tangentially to the flat side of the follower .

NUMERICAL EXAMPLE -4

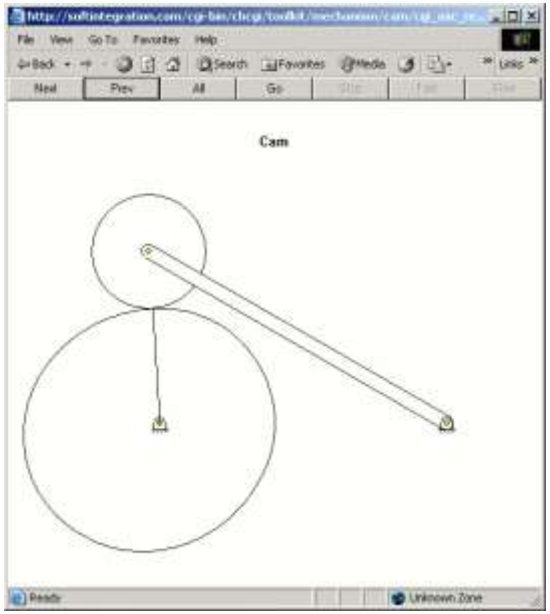
Draw a cam profile to drive an oscillating roller follower to the specifications given below :

(a).Follower to move outwards through an angular displacement of 20° during the first 120° rotation of the cam ;

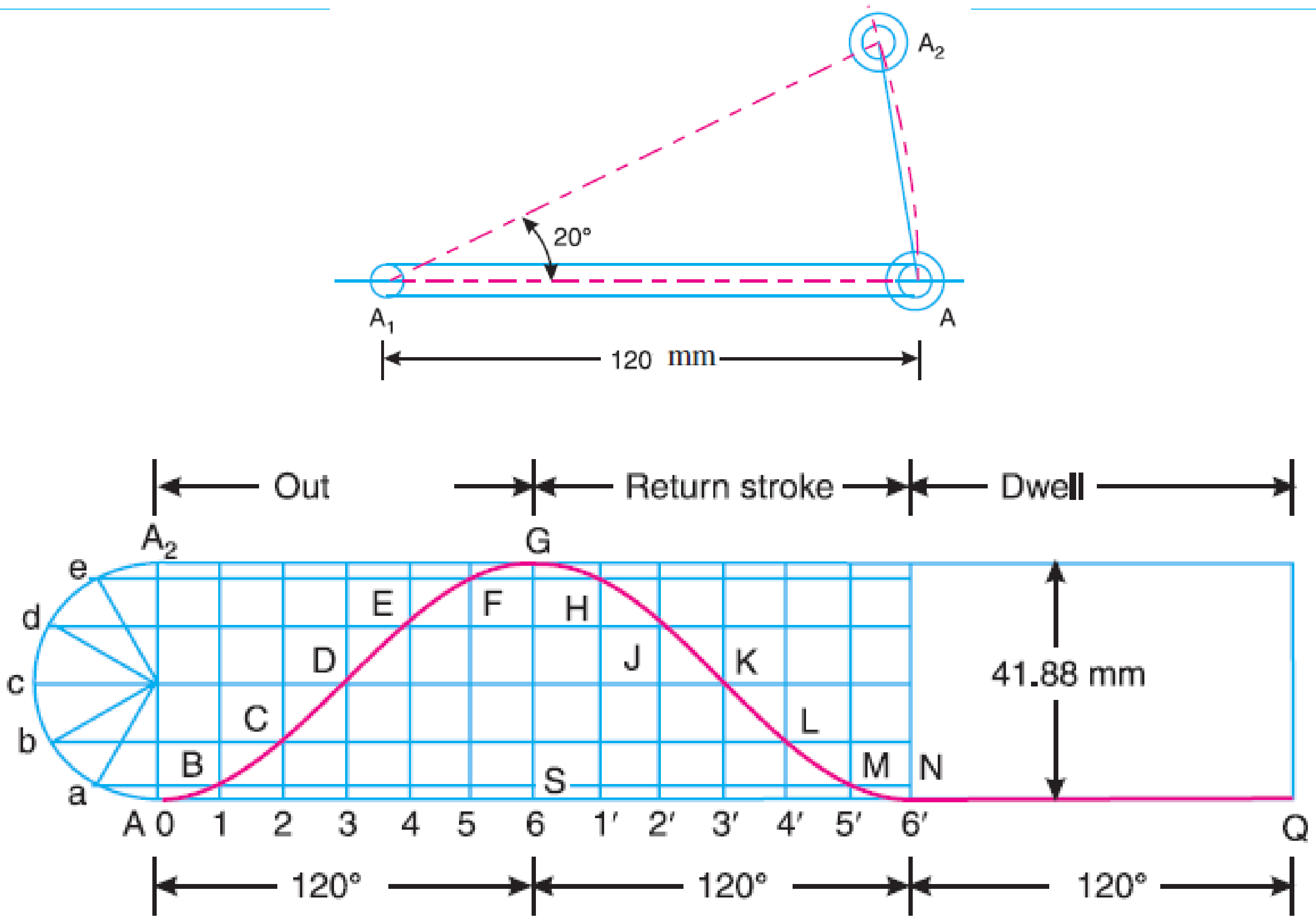
(b).Follower to return to its initial position during next 120° rotation of the cam ;

(c) Follower to dwell during the next 120° of cam rotation.

The distance between pivot centre and roller centre = 120 mm ; distance between pivot centre and cam axis = 130 mm; minimum radius of cam = 40 mm ; radius of roller = 10 mm ; inward and outward strokes take place with simple harmonic motion.



NUMERICAL EXAMPLE -4



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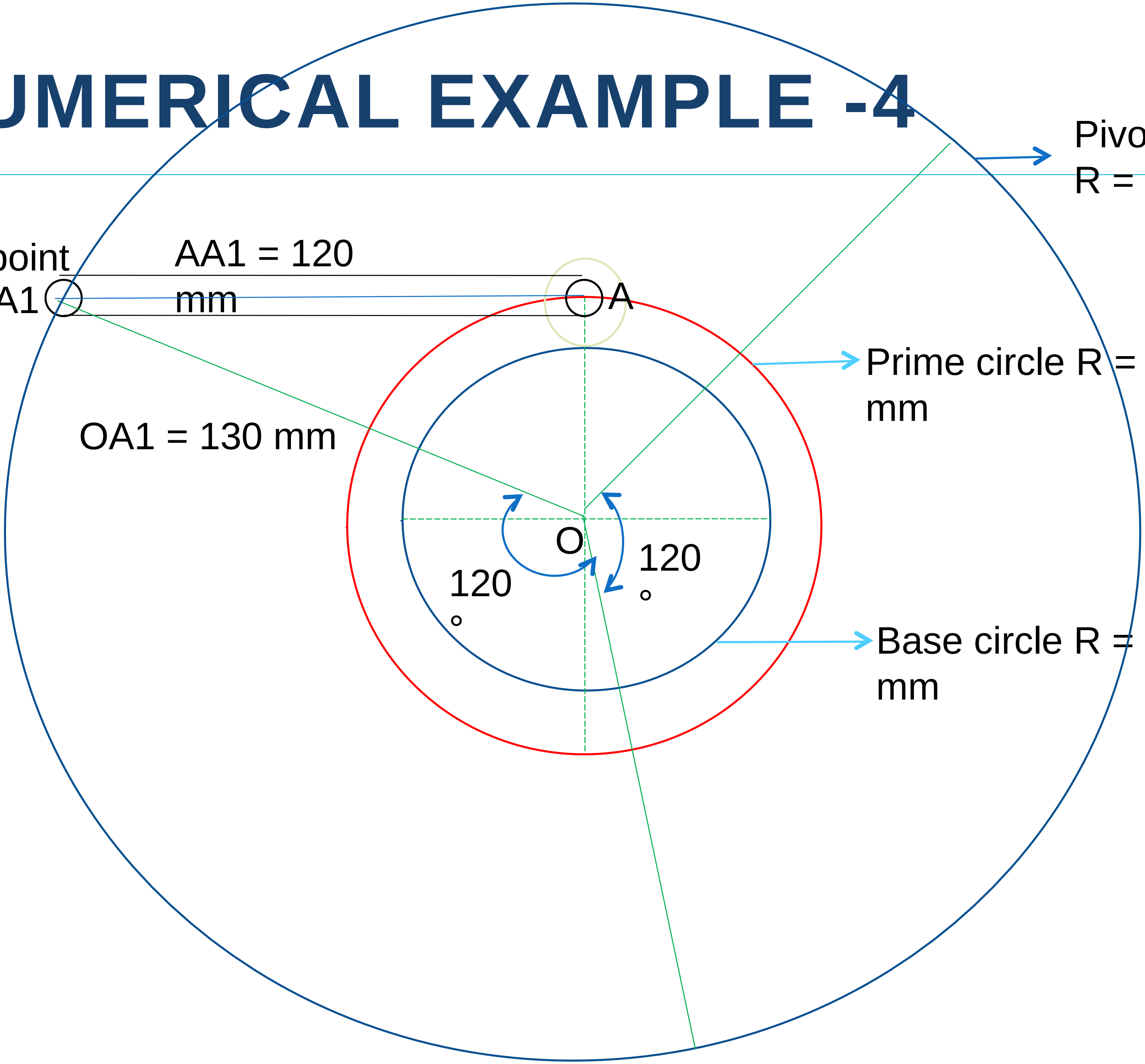
NUMERICAL EXAMPLE -4



Pivot circle R = OA1

Pivot point A1

AA1 = 120 mm

A

Prime circle R = 50 mm

OA1 = 130 mm

O

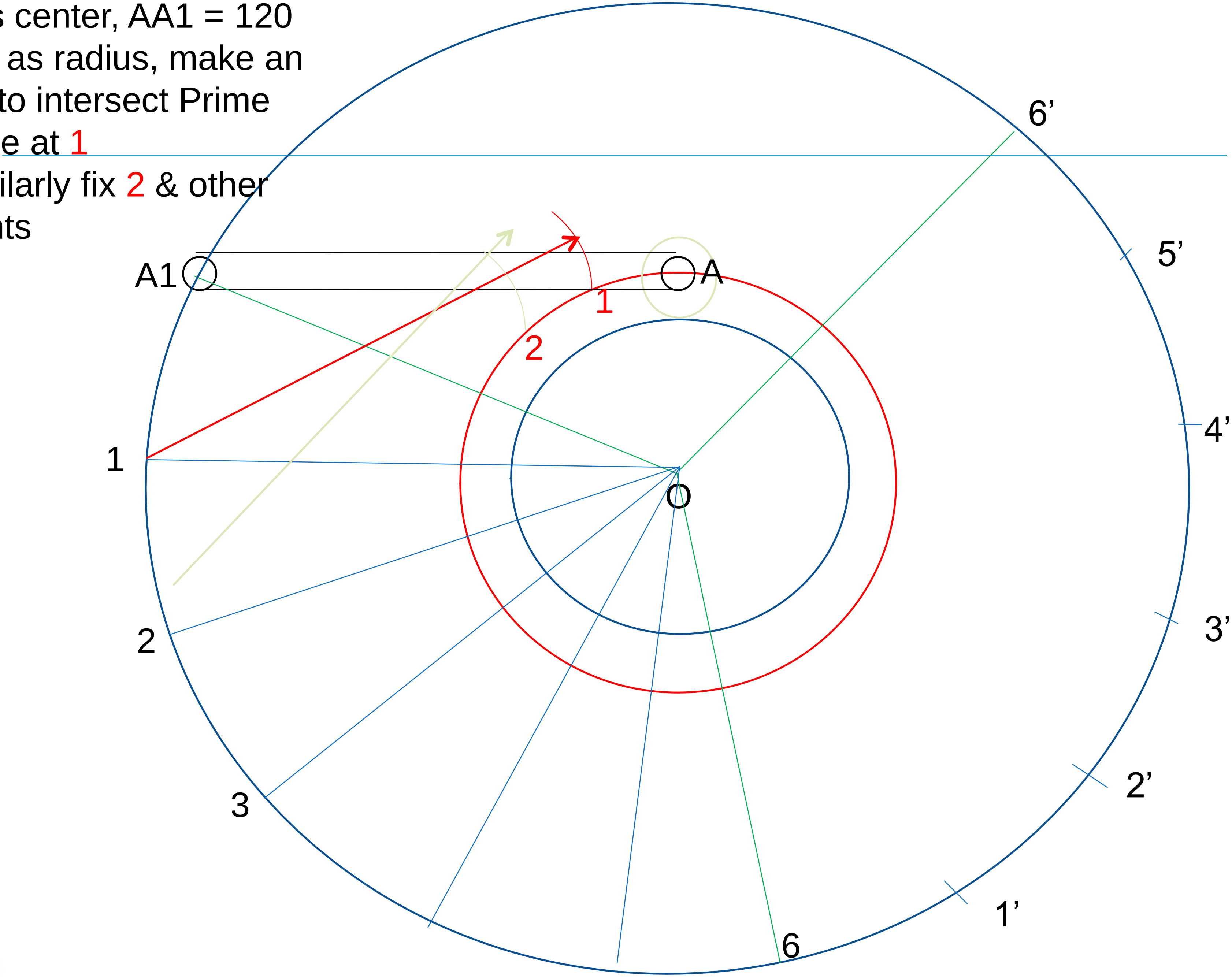
120 °

120 °

Base circle R = 40 mm

1 as center, AA1 = 120 mm as radius, make an arc to intersect Prime circle at 1

Similarly fix 2 & other points



NUMERICAL EXAMPLE -4

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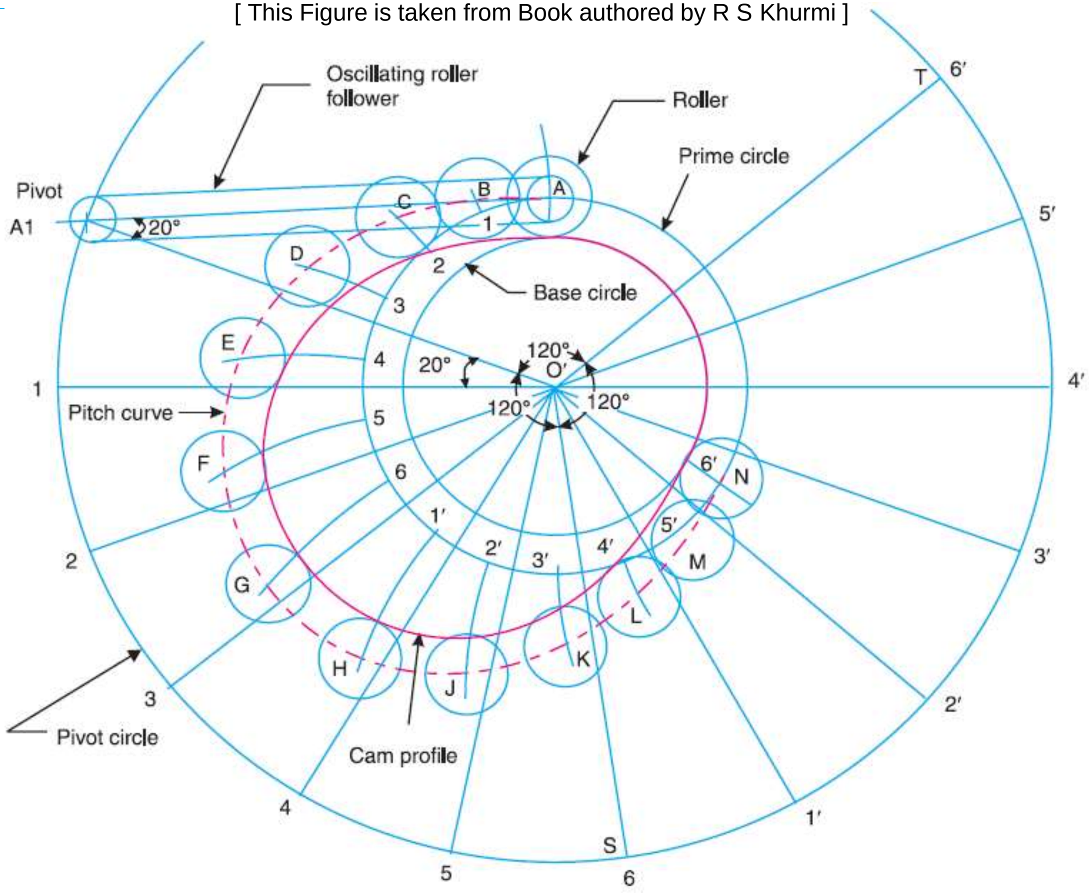


Set off the distances 1B, 2C, 3D... 4L, 5M along the arcs drawn equal to the distances as measured from the displacement diagram

NUMERICAL EXAMPLE -4

- The curve passing through the points A, B, C....L, M, N is known as pitch curve.
- Now draw circles with A, B, C, D....L, M, N as centre and radius equal to the radius of roller.
- Join the bottoms of the circles with a smooth curve as shown in Fig.
- This is the required CAM profile.

LECTURE 6

CONCAVE AND CONVEX FLANKS

DEPARTMENT OF MECHANICAL ENGINEERING

CAMS WITH SPECIFIED CONTOURS

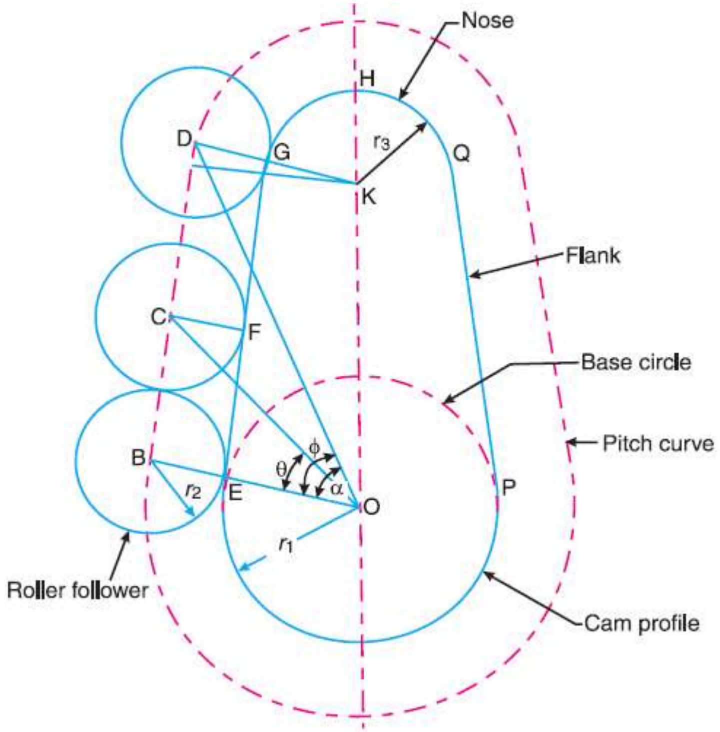
In the previous sessions, we have discussed the design of the profile of a cam when the follower moves with the specified motion - the shape of the cam profile obtained may be difficult and costly to manufacture.

In actual practice, the cams with specified contours (cam profiles consisting of circular arcs and straight lines are preferred) are assumed and then motion of the follower is determined.

CAMS WITH SPECIFIED CONTOURS

- When the flanks of the cam are straight and tangential to the base circle and nose circle, then the cam is known as a tangent cam.

- Used for operating the inlet and exhaust valves of IC engines



Tangent cam with reciprocating roller follower having contact with straight flanks.

[This Figure is taken from Book authored by R S Khurmi]

RADIUS OF CURVATURE

- It is a mathematical property of a function. No matter how complicated the a curve's shape may be, nor how high the degree of the function, it will have always an instantaneous radius of curvature at every point of the curve.

- When they are wrapped around their prime or base circle, they may be concave, convex or flat portions.

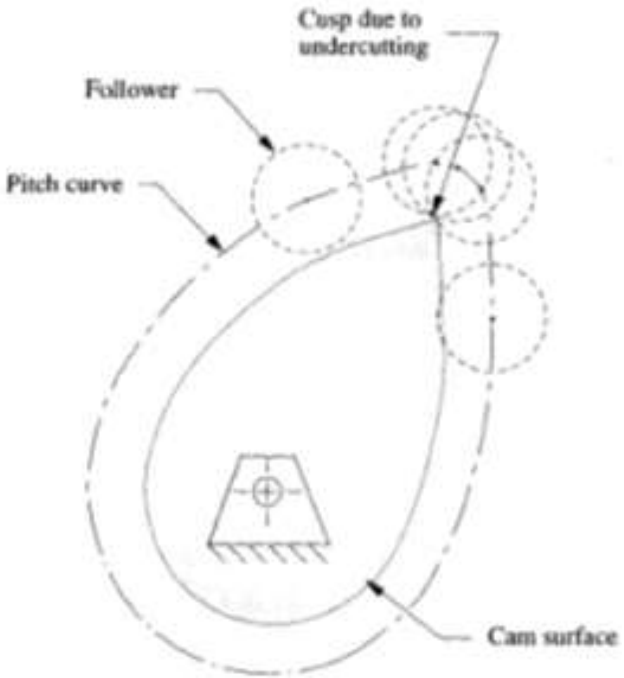
- Both, the pressure angle and the radius of curvature will dictate the minimum size of the cam and they must be checked.

RADIUS OF CURVATURE

- Undercutting: The roller follower radius R_f is larger than the smallest positive (convex) local radius. No sharp corners for an acceptable cam design.

- The golden rule is to keep the absolute value of the minimum radius of curvature of the cam pitch curve at least 2 or 3 times large as the radius of the follower.

- Radius of curvature can not be negative for a flat-faced follower.



MANUFACTURING CONSIDERATIONS

Materials: Hard materials as high carbon steels, cast iron. Sometimes made of brass, bronze and plastic cams (low load and low speed applications).

Production process: rotating cutters. Numerical control machinery. For better finishing, the cam can be ground after milling away most of the unneeded material. Heat treatments are usually required to get sufficient hardness to prevent rapid wear.

Geometric generation: actual geometries are far from been perfect. Cycloidal function can be generated. Very few other curves can.

Industry applications of cams

1. Cam and follower are widely used for operating inlet and exhaust valve of I C engine.
2. These are used in wall clock.
3. These are used in feed mechanism of automatic lathe Machine.
4. These are used in paper cutting machine.
5. Used in weaving textile machineries.
6. The cam mechanism is a versatile one. It can be designed to produce almost unlimited types of motioning the follower.
7. It is used to transform a rotary motion into a translating or oscillating motion.
8. On certain occasions, it is also used to transform one translating or oscillating motion into a different translating or oscillating motion.
9. Cams are used in a wide variety of automatic machines and instruments.
10. The certain usages of cam and followers that includes textile machineries, computers, printing presses, food processing machines, internal combustion engines, and countless other automatic machines, control systems and devices. The cam mechanism is indeed a very important component in modern mechanization.

Tutorial Questions

1. A cam is to give the following motion to a knife-edged follower : a) Outstroke during 60° of cam rotation ; b) Dwell for the next 30° of cam rotation ; c) Return stroke during next 60° of cam rotation, and d) Dwell for the remaining 210° of cam rotation.

The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes. Draw the profile of the cam when (a) the axis of the follower passes through the axis of the cam shaft, and (b) the axis of the follower is offset by 20 mm from the axis of the cam shaft.

2. A cam is to be designed for a knife edge follower with the following data :

1. Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion. 2. Dwell for the next 30°. 3. During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion. 4. Dwell during the remaining 180°.

Draw the profile of the cam when

(a) the line of stroke of the follower passes through the axis of the cam shaft, and

(b) the line of stroke is offset 20 mm from the axis of the cam shaft. The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

3. Draw the cam profile for the following data: Basic circle radius of cam = 50mm, Lift = 40mm, Angle of ascent with cycloidal = 60°, angle of dwell = 90°, angle of descent with uniform velocity = 90°, speed of cam = 300rpm, Follower offset = 10mm, Type of follower = knife – Edge.

4. Draw the cam profile for the following data: Basic circle radius of cam = 50mm, Lift = 40mm, Angle of ascent with SHM = 90°, Angle of Dwell = 90°, Angle of descent with uniform

acceleration and deceleration = 90°, speed of cam = 300 rpm, Type of follower = Roller follower (With roller radius = 10mm).

Question bank for Assignments

1. a) Explain the procedure to layout the cam profile for a reciprocating follower. (b) Derive relations for velocity and acceleration for a convex cam with a flat faced follower.

2. Draw a cam profile which would impart motion to a flat faced follower in the following desired way. The stroke of the follower being 5 cm. (i) The follower to move with uniform acceleration upward for 900 , dwell for next 900, (ii) The follower to return downward with uniform retardation for 1200 and dwell for next 600. The minimum radius of the cam being 3 cm.

3. Compare the performance of Knife –edge, roller and mushroom followers. (b) A knife edged follower for the fuel valve of a four stroke diesel engine has its centre line coincident with the vertical centre line of the cam. It rises 2.5 cm with SHM during 600 rotation of cam, then dwells for 200 rotation of cam and finally descends with uniform acceleration and deceleration during 450 rotation of cam, the deceleration period being half the acceleration period. The least radius of the cam is 5 cm. Draw the profile of the cam to full size.

4. A cam profile consists of two circular arcs of radii 24 mm and 12 mm joined by straight lines giving the follower a lift of 12 mm. The follower is a roller of 24 mm radius and its line of action is a straight line passing through the cam shaft axis. When the cam shaft has a uniform speed of 500 r.p.m., find the maximum velocity and acceleration of the follower while in contact with the straight flank of the cam.

5. A cam with 30mm as minimum diameter is rotating clockwise at a uniform speed of 1200rpm and has to give the following motion to a roller follower 10mm in diameter: (i) Follower to complete outward stroke of 25mm during 120° of cam rotation with equal uniform acceleration and retardation. (ii) Follower to dwell for 60°of cam rotation. (iii) Follower to return to its initial position during 90°of cam rotation with equal uniform acceleration and retardation. (iv) Follower to dwell for remaining 90°of cam rotation. Draw the cam profile if the axis of the roller follower passes through the axis of the cam.

UNIT 5
Gear and GearsTrains

COURSE OBJECTIVE

To study the relative motion analysis and design of gears, gear trains

COURSE OUTCOME

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	**Gears**	• Definition of Gear Drives • Classification of toothed wheel	• Understanding an angle of obliquity in gear (B2) • Remember the purpose of gears (B1)
2	Condition for constant velocity ratio	• Definition of Gearing Law • Number of Pairs of Teeth in Contact	• Evaluate arc of approach (B5) • State law of gearing (B1) • Remember arc of recess (B1)
3	Velocity of sliding phenomena	• Definition of normal and axial pitch in helical gears. • Definition of interference	• Evaluate the methods to avoid interference (B5) • Create normal and axial pitch in helical gears (B6)
4	Expressions for arc of contact and path of contact	• Minimum Number of Teeth on the Pinion in Order to Avoid Interference	• Analyze interference occur in an involute pinion and gear (B4) • Analyse arc of recess (B4)
5	Gear Trains	• Definition of gear trains • Types of gear trains	• Compare gear and gear train (B2) • Evaluate co-axial used in the gear train (B5)
6	Simple and reverted wheel train	• Definition of gear ratio. • Definition of train value • Numerical Examples to evaluate the number of teeth	• Understanding the uses of differential gear trains (B2) • Analyse compound gear train (B4)
7	Epicycle gear Train	• Definition of Epicycle gear Train • Methods to evaluate velocity ratio	• Remember the purpose of epicyclic gear trains (B1) • Find the speed and direction of gear (B3)
8	Differential gear for an automobile	• Applications of cycloidal tooth profile and involute tooth profile.	• Evaluate the uses of co-axial gear train (B5)

5

GEARS

Course Contents

5.1 Introduction

- If power transmitted between two shafts is small, motion between them may be obtained by using two plain cylinders or discs 1 and 2 as shown in fig.
- If there is no slip of one surface relative to the other, a definite motion of 1 can be transmitted to 2 and vice-versa. Such wheels are termed as "**friction wheels**". However, as the power transmitted increases, slip occurs between the discs and the motion no longer remains definite.
- Assuming no slipping of the two surfaces, the following kinematic relationship exists for their linear velocity:
- To transmit a definite motion of one disc to the other or to prevent slip between the surfaces, projection and recesses on the two discs can be made which can mesh with each other. This leads to formation of teeth on the discs and the motion between the surfaces changes from rolling to sliding. The discs with the teeth are known as **gears** or **gear wheels**.
- It is to be noted that if the disc I rotates in the clockwise direction, 2 rotates in the counter clockwise direction and vice-versa.

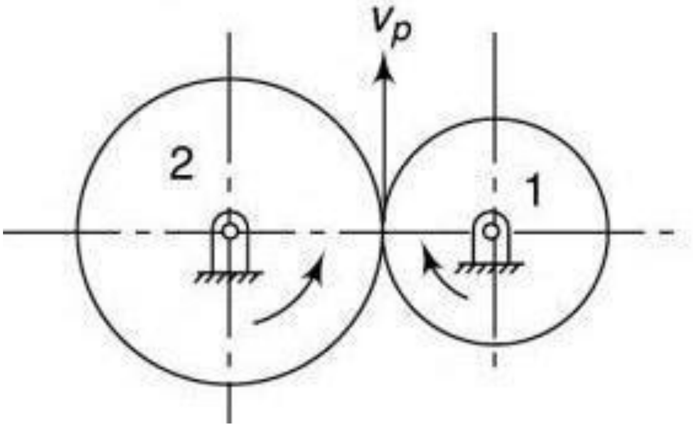


Fig. 5.1

5.2 Advantages and Disadvantages of Gear Drive

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears required special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.
3. They are costly.

5.3 Classification of Gears

5.3.1. According to the position of axes of the shafts

A. The axes of the two shafts between which the motion is to be transmitted, may be Parallel shaft,

B. Intersecting (Non parallel) shaft

C. Non-intersecting and non-parallel shaft.

A. Parallel shaft

- **Spur gear**

– The two parallel and co-planar shafts connected by the gears are called *spur gears.* These gears have teeth parallel to the axis of the wheel.

– They have straight teeth parallel to the axes and thus are not subjected to axial thrust due to tooth load.

– At the time of engagement of the two gears, the contact extends across the entire width on a line parallel to the axis of rotation. This results in sudden application of the load, high impact stresses and excessive noise at high speeds.

– If the gears have external teeth on the outer surface of the cylinders, the shaft rotate in the opposite direction.

– In an internal spur gear, teeth are formed on the inner surface of an annulus ring. An internal gear can mesh with an external pinion (smaller gear) only and the two shafts rotate in the same direction.

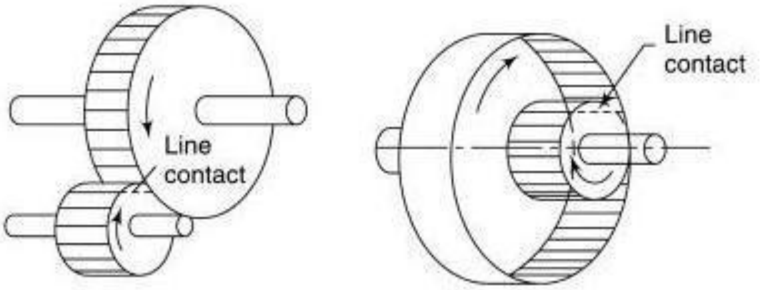


Fig.5.3 (a) Spur Gear

- *Spur rack and pinion*

– Spur rack is a special case of a spur gear where it is made of infinite diameter so that the pitch surface is plane.

– The spur rack and pinion combination converts rotary motion into translator motion, or vice-versa.

– It is used in a lathe in which the rack transmits motion to the saddle.

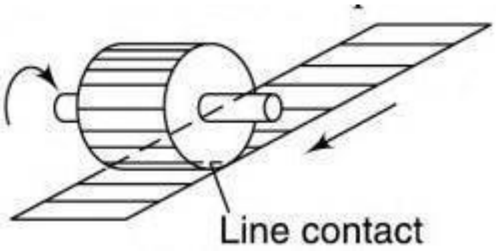


Fig. 5.3(b) Rack and pinion

- *Helical Spur Gears*

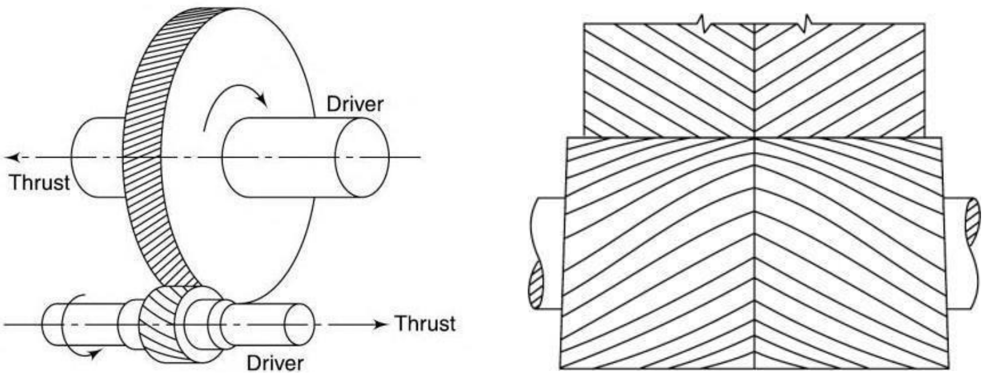
– In helical gears, the teeth are curved, each being helical in shape. Two mating gears have the same helix angle, but have teeth of opposite hands.

– At the beginning of engagement, contact occurs only at the point of leading edge of the curved teeth. As the gears rotate, the contact extends along a diagonal line across the teeth. Thus, the load application is gradual which results in low impact stresses and reduction in noise. Therefore, the helical gear can be used at higher velocities than the spur gears and have greater load-carrying capacity.

– Helical gears have the **disadvantage** of having end thrust as there is a force component along the gear axis. The bearing and assemblies mounting the helical gears must be able to withstand thrust loads.

– **Double helical:** A double-helical gear is equivalent to a pair of helical gears secured together, one having a right hand helix and other left hand helix.
  - The teeth of two rows are separated by groove used for tool run out.
  - Axial thrust which occurs in case of single-helical gears is eliminated in double-helical gears.
  - This is because the axial thrusts of the two rows of teeth cancel each other out. These can be run at high speeds with less noise and vibrations.

– **Herringbone gear**: If the left and the right inclinations of a double-helical gear meet at a common apex and there is no groove in between, the gear is known as Herringbone gear.



Helical gear *Herringbone gear*

(c) *(d)*

Fig. 5.3

B. *Intersecting Shafts*

- The two non-parallel or intersecting, but coplanar shafts connected by gears are called **bevel gears**
- When teeth formed on the cones are straight, the gears are known as bevel gears when inclined, they are known as **spiral** or **helical bevel**.

- **Straight Bevel Gears**

- The teeth are straight, radial to the point of intersection of the shaft axes and vary in cross section throughout their length.
- Usually, they are used to connect shafts at right angles which run at low speeds
- Gears of the same size and connecting two shafts at right angles to each other are known as “Mitre” gears.

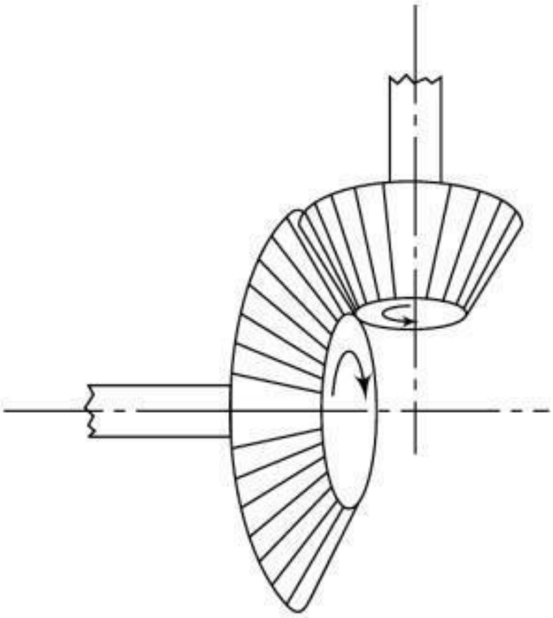


Fig. 5.3(e) Straight Bevel Gears

- *Spiral Bevel Gears*

- When the teeth of a bevel gear are inclined at an angle to the face of the bevel, they are known as spiral bevels or helical bevels.
- They are smoother in action and quieter than straight tooth bevels as there is gradual load application and low impact stresses. Of course, there exists an axial thrust calling for stronger bearings and supporting assemblies.
- These are used for the drive to the differential of an automobile.

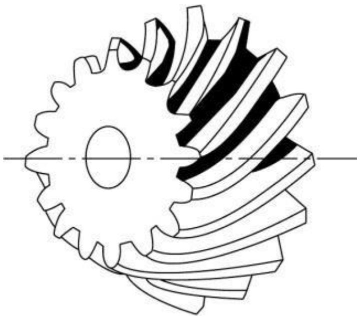


Fig. 5.3(f) Spiral Bevel Gear

- *Zero Bevel Gears*

– Spiral bevel gears with curved teeth but with a zero degree spiral angle are known as zero bevel gears.

– Their tooth action and the end thrust are the same as that of straight bevel gears and, therefore, can be used in the same mountings.

– However, they are quieter in action than the straight bevel type as the teeth are curved.

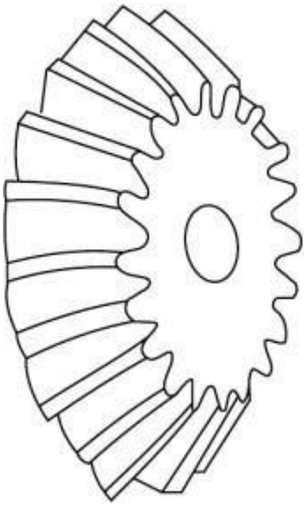


Fig. 5.3(g) Zero Bevel Gears

C. *Non-intersecting and non-parallel shaft(Skew shaft)*

– The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiral gearing.

– In these gears teeth have a point contact.

– These gears are suitable for transmitting small power.

– **Worm gear** is as special case of a spiral gear in which the larger wheel, usually, has a hollow shape such that a portion of the pitch diameter of the other gear is enveloped on it.

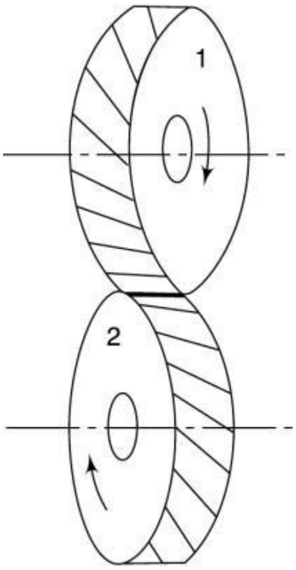


Fig.5.3 (h)Non-intersecting and non-parallel shaft

5.3.2. According to the peripheral velocity of the gears

(a) Low velocity	V < 3 m/sec
(b) Medium velocity	3 < V < 15 m/sec
(c) High velocity	V > 15 m/sec

5.3.3. According to position of teeth on the gear surface

(a) Straight,
(b) Inclined, and
(c) Curved.

5.4 Terms Used in Gears

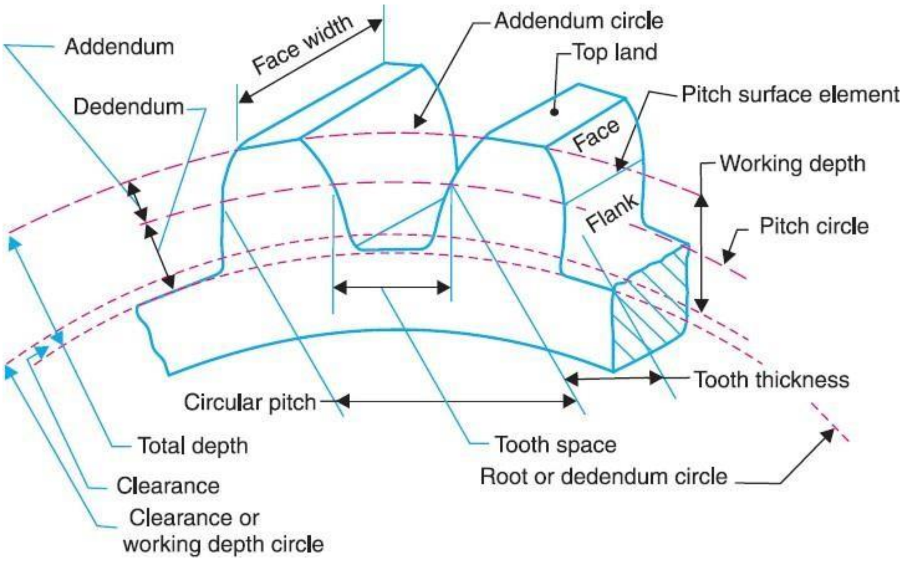


Fig.5.4 Terms used in gears.

1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as **pitch diameter**.

3. Pitch point. It is a common point of contact between two pitch circles.

4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point.

- For more power transmission lesser pressure on the bearing and pressure angle must be kept small.
- It is usually denoted by ø.
- The standard pressure angles are 20° and 25°.Gears with pressure anglehas become obsolete.

6. Addendum. It is the radial distance ofa tooth from the pitch circle to the top of the tooth.

- Standard value = 1 module

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

- Standard value = 1.157 module

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

10. Clearance. It is the radial difference between the addendum and the Dedendum of a tooth.

Addendum circle diameter $= d + 2m$

Dedendum circle diameter $= d - 2 \times 1.157m$

Clearance $= 1.157m - m$

$= 0.157m$

11. Full depth of Teeth It is the total radial depth of the toothspace.

Full depth= Addendum + Dedendum

12. Working Depth of Teeth The maximum depth to which a tooth penetrates into the tooth space of the mating gear is the working depth of teeth.

- Working depth = Sum of addendums of the two gears.

15. Working depth. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. Tooth thickness. It is the width of the tooth measured along the pitch circle.

17. Tooth space. It is the width of space between the two adjacent teeth measured along the pitch circle.

18. Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. Face of tooth. It is the surface of the gear tooth above the pitch surface.
20. Flank of tooth. It is the surface of the gear tooth below the pitch surface.
21. Top land. It is the surface of the top of the tooth.
22. Face width. It is the width of the gear tooth measured parallel to its axis.

20. Fillet It is the curved portion of the tooth flank at the root circle.

21. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from point of one tooth to the corresponding point on the next tooth.

- It is usually denoted by p_c.
  Mathematically,

$$\text{Circular pitch, } p_c = \frac{\pi d}{T}$$

Where d = Diameter of the pitch circle, and
T = Number of teeth on the wheel.

- The angle subtended by the circular pitch at the center of the pitch circle is known as the **pitch angle**.

22. Module (m). It is the ratio of the pitch diameter in mm to the number of teeth.

$$m = \frac{d}{T}$$

πd

$$\text{Also } p_c = \frac{}{T} = \pi m$$

- Pitch of two mating gear must be same.

23. Diametral Pitch (P) It is the number of teeth per unit length of the pitch circle diameter in inch.

OR

It is the ratio of no. of teeth to pitch circle diameter in inch.

$$P_d = \frac{T}{d}$$

- The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

24. Gear Ratio (G). It is the ratio of the number of teeth on the gear to that on the pinion.

$$G = \frac{T}{t}$$ Where T= No of teeth on gear

t = No. of teeth on pinion

25. Velocity Ratio (VR) The velocity ratio is defined as the ratio of the angular velocity of the follower to the angular velocity of the driving gear.

$$VR = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

26. Length of the path of contact. It is the length of the common normal cut-off by the Addendum circles of the wheel and pinion.

OR

The locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement is known as the contact.

a. **Path of Approach** Portion of the path of contact from the beginning of the engagement to the pitch point.
b. **Path of Recess** Portion of the path of contact from the pitch point to the end of engagement.

27. Arc of Contact The locus of a point on the pitch circle from the beginning to the end of engagement of two mating gears is known as the arc of contact.

a. **Arc of Approach** It is the portion of the arc of contact from the beginning of engagement to the pitch point.
b. **Arc of Recess** The portion of the arc of contact from the pitch point to the end of engagements the arc of recess.

28. Angle of Action ( ) It is the angle turned by a gear from the beginning of engagement to the end of engagement of a pair of teeth, i.e., the angle turned by arcs of contact of respective gear wheels.

$\delta = \alpha + \beta$ Where α =Angle of approach

β =Angle of recess

29. Contact ratio .It is the angle of action divided by the pitch angle

$$\text{Contact ratio} = \frac{\delta}{\gamma} = \frac{\alpha+\beta}{\gamma}$$

OR

$$\text{Contact ratio} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

5.5 Condition for Constant Velocity Ratio of Toothed Wheels –Law of Gearing

- To understand the theory consider the portions of two gear teeth gear 1 and gear 2 as shown in figure 1.5.
- The two teeth come in contact at point C and the direction of rotation of gear 1 is anticlockwise & gear 2 is clockwise.
- Let TT be the common tangent & NN be the common normal to the curve at the point of contact C. From points O, &O_2, draw O_1 A & O_2 B perpendicular to common normal NN.
- When the point D is consider on gear 1, the point C moves in the direction of "CD" & when it is consider on gear 2. The point C moves in direction of "CE".
- The relative motion between tooth surfaces along the common normal NN must be equal to zero in order to avoid separation.
- So, relative velocity

$$V_1 \cos\alpha = V_2 \cos\theta$$

$$(\omega_1 \times O_1\ C)\cos\alpha = (\omega_2 \times O_2\ C) \qquad (\because \nu = r\omega)\square(1)$$

$$\cos\alpha$$

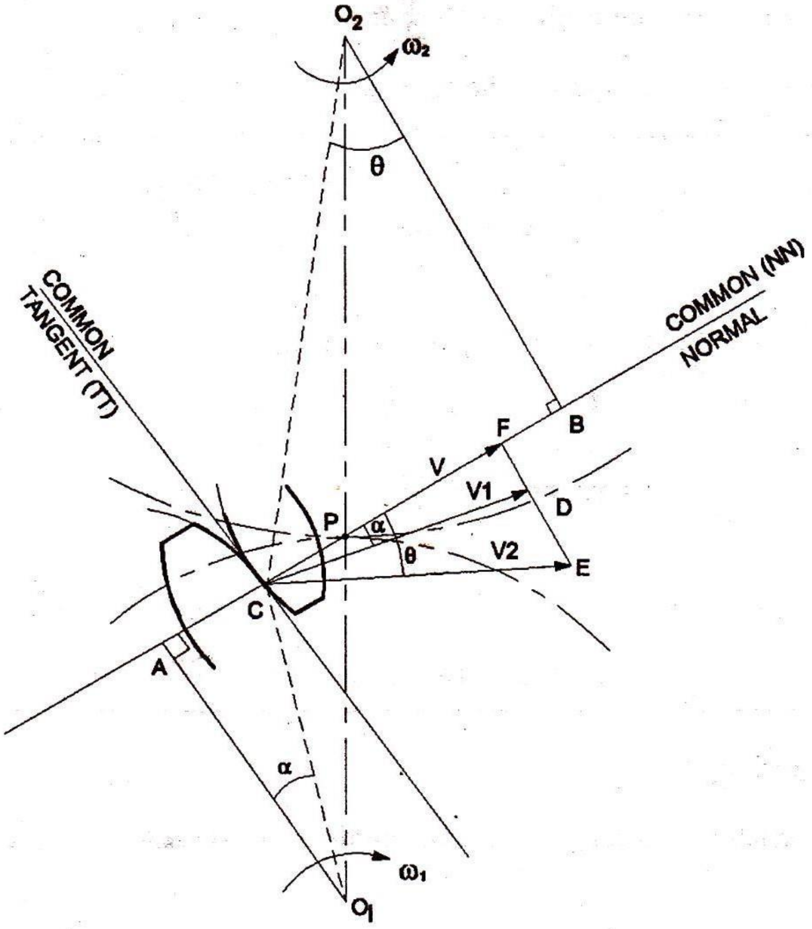


Fig.5.5 Law of gearing

- But from $\Delta\, O_1AC$, $\cos\alpha = \dfrac{O_1A}{O_1C}$

  and from $\Delta\, O_2BC$, $\cos\theta = \dfrac{O_2B}{O_2C}$
- Putting above value in equation (1) it become

$$(\omega_1 \times O_1C)\frac{O_1A}{O_1C} = (\omega_2 \times O_2C)\frac{O_2B}{O_2C}$$

$$\omega_1 \times O_1A = \omega_2 O_2B$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2B}{O_1A} \quad \text{...} \quad (2)$$

- From the similar triangle $\Delta\, O_1AP$ & $\Delta\, O_2BP$

$$\frac{O_2B}{O_1A} = \frac{O_2P}{O_1P} \quad \text{...} \quad (3)$$

- Now equating equation (2) & (3)

$$\frac{\omega_1}{\omega_2} = \frac{O_2B}{O_1A} = \frac{O_2P}{O_1P} = \frac{PB}{AP}$$

- From the above we can conclude that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the central O_1& O_2.
- If it is desired that the angular velocities of two gear remain constant, the common normal at the point of contact of two teeth always pass through a fixed point P. This fundamental condition is called as law of gearing. Which must be satisfied while designing the profiles of teeth for gears.

Systems of Gear Teeth

The following four systems of gear teeth are commonly used in practice:

1. 14 ½^O Composite system
2. 14 ½^O Full depth involute system
3. 20^O Full depth involute system
4. 20^O Stub involute system

The 14½^O *composite system* is used for general purpose gears.

It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion.

The teeth are produced by formed milling cutters or hobs.

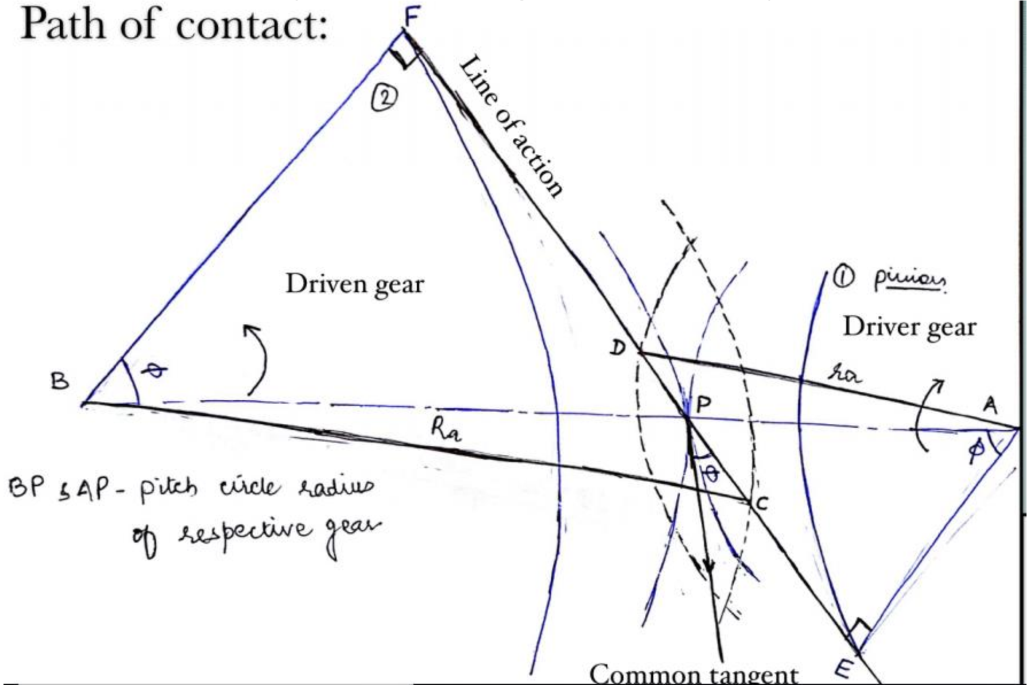
The tooth profile of the 14½^O *full depth involute system* was developed using gear hobs for spur and helical gears.

The tooth profile of the 20^o *full depth involute system* may be cut by hobs.

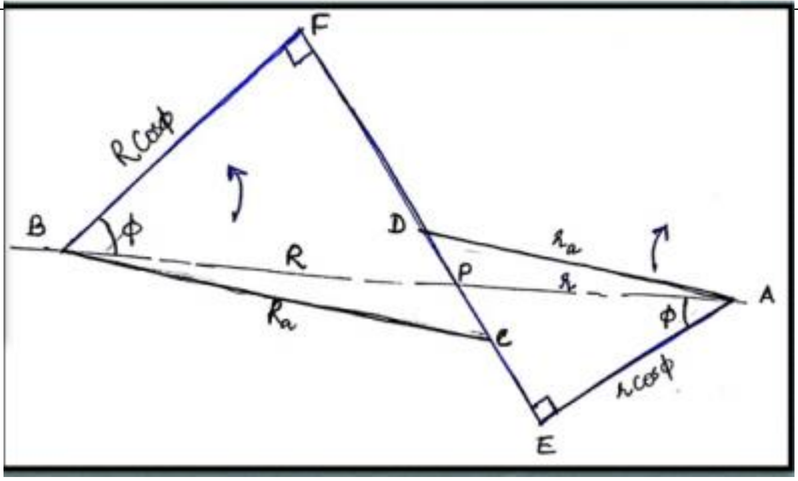
The increase of the pressure angle from 14½^o to 20^o results in a stronger tooth, because the tooth acting as a beam is wider at the base.

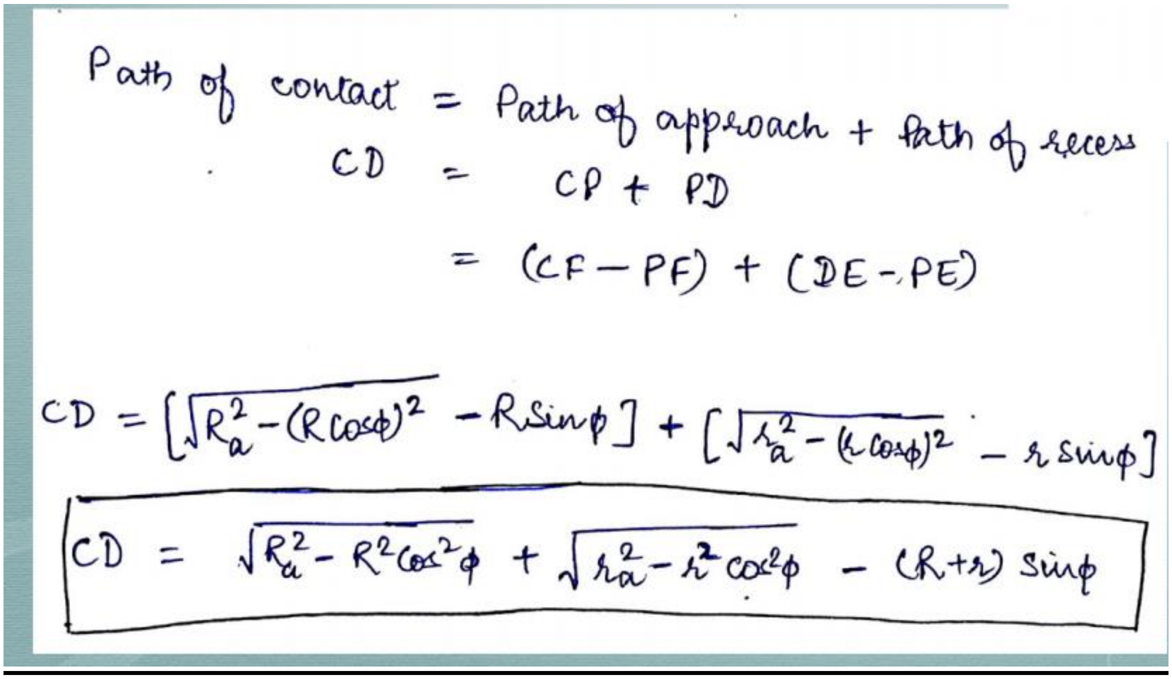
The *20^o stub involute system* has a strong tooth to take heavy loads.

Path of contact:



BP & AP - pitch circle radius of respective gear





Path of contact = Path of approach + Path of recess

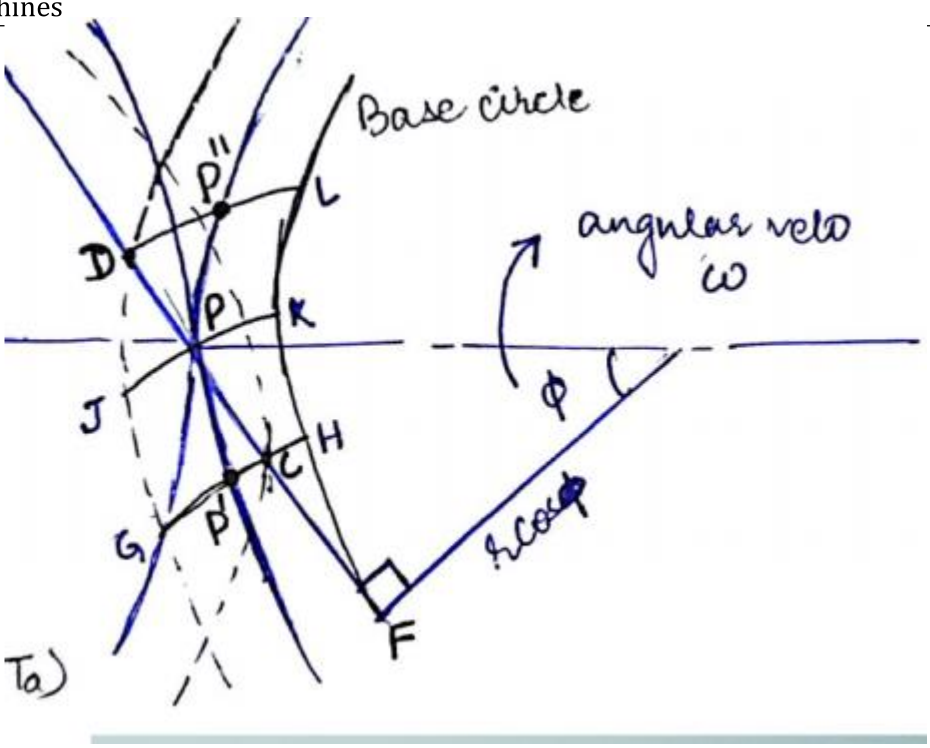
$$CD = CP + PD$$

$$= (CF - PF) + (DE - PE)$$

$$CD = \left[\sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi\right] + \left[\sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi\right]$$

$$\boxed{CD = \sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)\sin\phi}$$

Arc of contact:



Path of contact = CD = Path of approach + Path of recess

Arc of contact = P'P'' = Arc of approach (P'P) + Arc of recess (PP'')

Arc of approach (P'P) = Tangential velo of P' (Va) on pitch circle × Time of approach

(distance = speed × time)

$$P'P = r\,\omega_a \times T_a$$

$$= \underbrace{\boxed{\omega_a\ (r \times \cos\phi)}}_{\text{tangential velo on base circle}} \times \frac{1}{\cos\phi} \times T_a$$

$$= \frac{\text{tangential velo of H} \times T_a}{\cos\phi}$$

$$P'P = \frac{\text{Arc } HK}{\cos\phi} = \frac{\text{Arc } FK - \text{Arc } F}{\cos\phi}$$

$$P'P = \frac{FP - FC}{\cos\phi} = \frac{CP}{\cos\phi}$$

Backlash:

The gap between the non-drive face of the pinion tooth and the adjacent wheel tooth is known as ***backlash****. Backlash is the error in motion that occurs when gears change direction. The term "backlash" can also be used to refer to the size of the gap, not just the phenomenon it causes; thus, one could speak of a pair of gears as having, for example, "0.1 mm of backlash."*

Practise problems:

1) Two gears in mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 teeth while the pinion has 23 teeth. If the addenda on pinion and gear wheel are equal to one module (1*m*), find
   a. The number of pairs of teeth in contact and
   b. The angle of action of the pinion and the gear wheel.

Solution:

Data: *t* =23; *T*=57; addendum = 1*m*= 8*mm* and f=20°

$$\textit{Pitch circle radiusof the pinion} = r = \frac{mt}{2} = \frac{8\times 23}{2} = 92mm$$

$$\textit{Pitch circle radiusof the gear} = R = \frac{mT}{2} = \frac{8\times 57}{2} = 228mm$$

$$\textit{Addendumcircle radiusof the pinion} = r_a = r + \textit{addendum}$$

$$r_a = 92 + 8 = 100mm$$

$$\textit{Addendumcircle radiusof the gear} = R_A = R + \textit{addendum}$$

$$R_A = 228 + 8 = 236mm$$

$$\textit{Lengthof pathof contact} = KL = KP + PL$$

$$= \sqrt{(R_A)^2 - R^2\cos^2\varphi} + \sqrt{(r_a)^2 - r^2\cos^2\varphi} - (R+r)\sin\varphi$$

$$= \sqrt{(236)^2 - (228)^2\cos^2 20} + \sqrt{(100)^2 - (92)^2\cos^2 20}$$

$$- (228+92)\sin 20$$

$$= 39.76mm$$

$$\textit{Lengthof arcof contact} = \frac{\textit{Lengthof pathof contact}}{\cos\varphi}$$

$$= \frac{39.76}{\cos 20} = 42.31mm$$

$$\textit{Numberof pairsof teeth in contact} = \frac{\textit{Lengthof arcof contact}}{\textit{circular pitch}}$$

$$= \frac{\textit{Lengthof arcof contact}}{p_c} = \frac{42.31}{\pi m} = 1.684 \approx 2$$

$$Angleof\ actionof\ gear\ wheel = \frac{Lengthof\ arc\ of\ contact}{2\pi \times R} \times 360^o$$

$$= \frac{42.31}{2\pi \times 228} \times 360 = 10.637^o$$

$$Angleof\ \ actionof\ \ pinion = \frac{Lengthof\ arc\ of\ contact}{2\pi \times r} \times 360^o$$

$$= \frac{42.31}{2\pi \times 92} \times 360 = 26.36^o$$

2.) Two gear wheels mesh externally and are to give a velocity ratio of 3 to 1. The teeth are of involute form ; module = 6 mm, addendum = one module, pressure angle = 20°. The pinion rotates at 90 r.p.m. Determine: **1.** The number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel, **2.** The length of path and arc of contact, **3.**The number of pairs of teeth in contact, and **4.** The maximum velocity of sliding.

Solution:

Given : $G = T/t = 3$; $m = 6$ mm ; $A_P = A_W = 1$ module $= 6$ mm ; $\phi = 20°$; $N_1 = 90$ r.p.m. or $\omega_1 = 2\pi \times 90 / 60 = 9.43$ rad/s

We know that number of teeth on the pinion to avoid interference,

$$t = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1} = \frac{2 \times 6}{\sqrt{1 + 3(3+2)\sin^2 20°} - 1}$$

$$= 18.2 \text{ say } 19$$

and corresponding number of teeth on the wheel,

$$T = G.t = 3 \times 19 = 57$$

Length of path and arc of contact:

We know that pitch circle radius of pinion,

$$r = m.t/2 = 6 \times 19/2 = 57 \text{ mm}$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum on pinion } (A_P) = 57 + 6 = 63 \text{ mm}$$

and pitch circle radius of wheel,

$$R = m.T/2 = 6 \times 57/2 = 171 \text{ mm}$$

∴ Radius of addendum circle of wheel,

$$R_A = R + \text{Addendum on wheel } (A_W) = 171 + 6 = 177 \text{ mm}$$

We know that the path of approach (*i.e.* path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(177)^2 - (171)^2\cos^2 20°} - 171\sin 20° = 74.2 - 58.5 = 15.7 \text{ mm}$$

and the path of recess (*i.e.* path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2\phi} - r\sin\phi$$

$$= \sqrt{(63)^2 - (57)^2 \cos^2 20°} - 57\sin 20° = 33.17 - 19.5 = 13.67 \text{ mm}$$

∴ Length of path of contact,

$$KL = KP + PL = 15.7 + 13.67 = 29.37 \text{ mm}$$

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{29.37}{\cos 20°} = 31.25 \text{ mm}$$

Number of pairs of Teeth in contact:

We know that circular pitch,

$$p_c = \pi \times m = \pi \times 6 = 18.852 \text{ mm}$$

∴ Number of pairs of teeth in contact

$$= \frac{\text{Length of arc of contact}}{p_c} = \frac{31.25}{18.852} = 1.66$$

Maximum velocity of sliding:

Let ω_2 = Angular speed of wheel in rad/s.

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t}$ or $\omega_2 = \omega_1 \times \frac{t}{T} = 9.43 \times \frac{19}{57} = 3.14$ rad/s

∴ Maximum velocity of sliding,

$$v_S = (\omega_1 + \omega_2)\, KP$$

$$= (9.43 + 3.14)\, 15.7 = 197.35 \text{ mm/s}$$

Gear Train

Introduction

Definition

- When two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels.
- The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train
5. Compound epicyclic gear train

Simple gear train.

- When there is only one gear on each shaft, as shown in Fig. , it is known as **simple gear train.** The gears are represented by their pitch circles.
- When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig.
- Since the gear 1 drives the gear 2, therefore gear 1 is called the **driver** and the gear 2 is called the **driven** or **follower.** It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

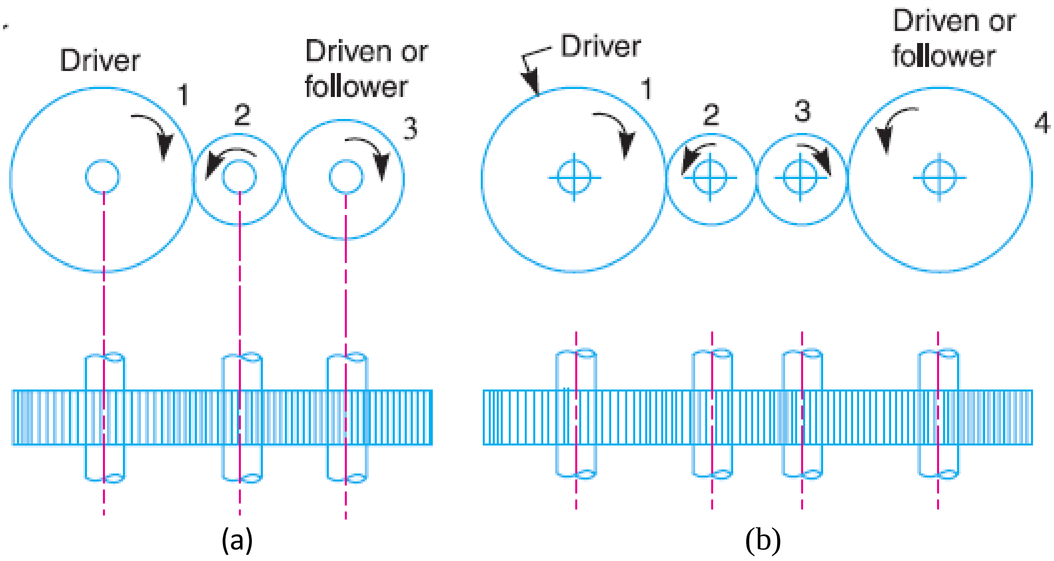


Fig.5.2.1 Simple gear train

Let

$N_1 =$ Speedof driver rpm

$N_2 =$ Speedof intermediatewheel rpm

N_3 = Speedof follower rpm
T_1 = Number of teethon driver
T_2 = Number of teethonintermediatewheel T_3
= Number of teethonfollower

- **speed ratio** (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth.

$$\text{Speedratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

- **Train value** of the gear train is the ratio of the speed of the driven or follower to the speed of the driver.

$$\text{Trainvalue} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

Compound Gear Train

– When there is more than one gear on a shaft, as shown in Fig., it is called a **compound train of gear.**

– The idle gears, in a simple train of gears do not affect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

– But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts.

– In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig.

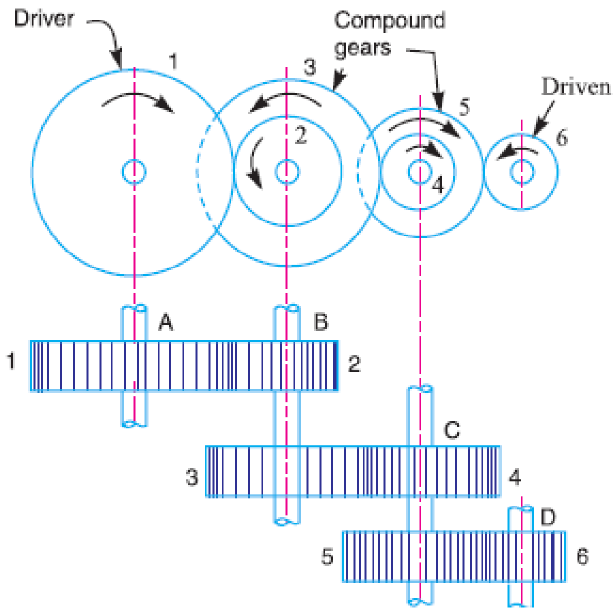


Fig. 5.2.2 compound gear train

- In a compound train of gears, as shown in Fig., the gear 1 is the driving gear mounted on shaft A; gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let

N1 = Speed of driving gear 1,
T1 = Number of teeth on driving gear 1,
N2 ,N3 ..., N6 = Speed of respective gears in r.p.m.,
and T2 ,T3..., T6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \cdots\cdots\cdots(1)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \cdots\cdots\cdots(2)$$

And for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \cdots\cdots\cdots(3)$$

The speed ratio of compound gear train is obtained by multiplying the equations (1), (2) and (3),

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

- The **advantage** of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.
- If a simple gear train is used to give a large speed reduction, the last gear has to be very large.
- Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Reverted Gear Train

- When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as **reverted gear train.**
- Gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is **like.**

Let

T1 Number of teeth on gear 1,
= Pitch circle radius of gear 1, and
r1 = Speed of gear 1 in r.p.m.
N1
=
Similarly,
T2, T3, = Number of teeth on respective gears,
T4
r2, r3, r4 = Pitch circle radii of respective gears, and
N2, N3, N4 = Speed of respective gears in r.p.m.

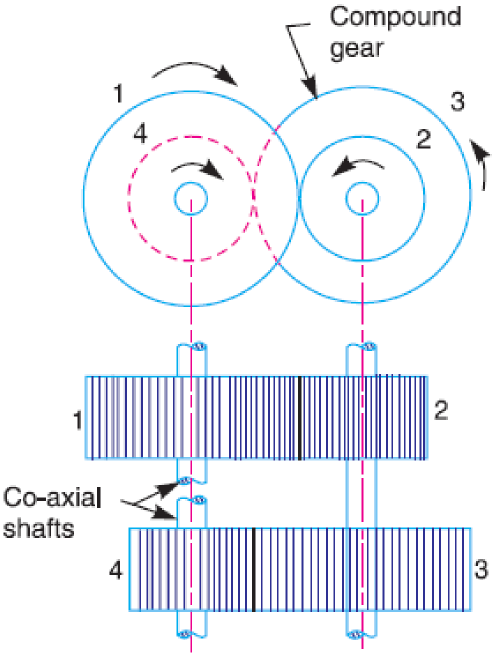


Fig. 5.2.3 Reverted gear train

- Since the distance between the centers of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4$$

- Also, the circular pitch or module of all the gears is assumed to be same; therefore number of teeth on each gear is directly proportional to its circumference orradius.

$$T_1 + T_2 = T_3 + T_4$$

$$\text{Speedratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

Application

- The reverted gear trains are used in automotive transmissions**,** lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

Epicyclic Gear Train

- In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. where a gear A and the arm C have a common axis at O1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate.
- If the arm is fixed, the gear train is simple and gear A can drive gear B or **vice- versa**, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate **upon** and **around** gear A. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member is known as **epicyclic gear trains** (**epi.** means upon and **cyclic** means around). The epicyclic gear trains may be **simple** or **compound.**

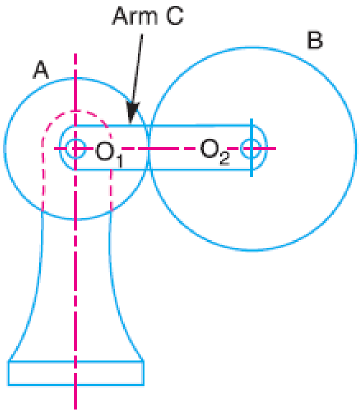


Fig. 5.2.4 Epicyclic gear train

Sr. No.	Condition of motion	Revolution of element		
		Arm C	Gear A	Gear B
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through + x revolutions	0	$+x$	$-x\frac{T_A}{T_B}$
3	Add + y revolutions to all elements	+ Y	+ y	+ y
4	Total motion	+ y	$x+y$	$y- x\frac{T_A}{T_B}$

Application

- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

Compound Epicyclic Gear Train—Sun and Planet Gear

- A compound epicyclic gear train is shown in Fig. It consists of two co-axial shafts S1 and S2, an annulus gear A which is fixed, the compound gear (or planet gear) B-C, the sun gear D and the arm H. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm H. The sun gear is co-axial with the annulus gear and the arm but independent of them.
- The annulus gear A meshes with the gear B and the sun gear D meshes with the gear C. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

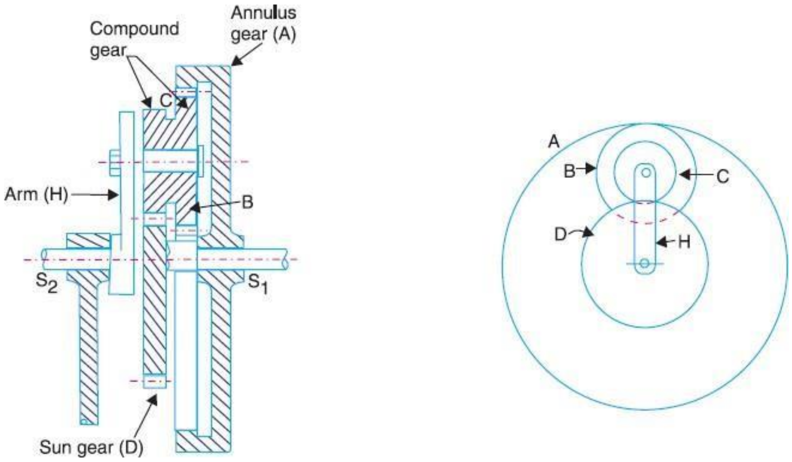


Fig. 5.2.5 Compound epicyclic gear train.

Note: The gear at the center is called the **sun gear** and the gears whose axes move are called **planet gears.**

Let T_A, T_B, T_C, and T_D be the teeth and N_A, N_B, N_C and N_D be the speeds for the gears *A*, *B*, *C* and *D* respectively. A little consideration will show that when the arm is fixed and the sun gear *D* is turned anticlockwise, then the compound gear *B*-*C* and the annulus gear A will rotate in the clockwise direction.

The motion of rotations of the various elements is shown in the table below.

Table of motions

Sr. No.	**Condition of motion**	**Revolution of motion**			
		Arm	**Gear D**	**Compound Gear (B-C)**	**Gear A**
1	Arm fixe, gear D rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_B} \times \frac{T_C}{T_A}$
2	Arm fixed gear D rotates through + *x* revolutions	0	+x	$x\frac{T_D}{T_C}$	$-x\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3	Add + *y* revolutions to all elements	+y	+y	+y	+y
4	Total motion	+y	x + y	$y - x\frac{T_D}{T_C}$	$y - x\frac{T_D}{T_C} \times \frac{T_B}{T_A}$

EXAMPLES

Example 5.1. The gearing of a machine tool is shown in Fig.2.1. The motor shaft is connected to gear A and rotates at 975 rpm. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F? The number of teeth on each gear is as given below:

Gear	A	B	C	D	E	F
No. of teeth	20	50	25	75	26	65

Solution:

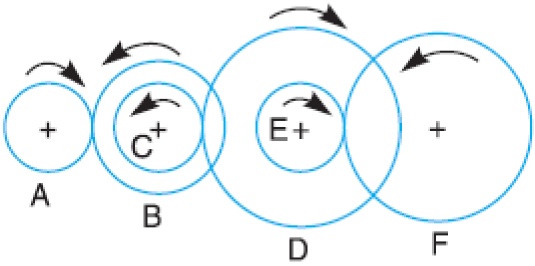


Fig. 6.1

Given data

$T_A = 20$ $N_F = ?$

$T_B = 50$

$T_C = 25$

$T_D = 75$

$T_E = 26$

$T_F = 65$

$N_A = 975$ rpm

$$\frac{N_F}{N_A} = \frac{T_A}{T_B} \times \frac{T_C}{T_D} \times \frac{T_E}{T_F}$$

$$\therefore \frac{N_F}{975} = \frac{20}{50} \times \frac{20}{75} \times \frac{26}{65}$$

$$\boxed{\therefore N_F = 52 \text{ rpm}}$$

Example 5.2 **In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 rpm in the anticlockwise direction about the center of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed makes 300 rpm in the clockwise direction, what will be the speed of gear B?**

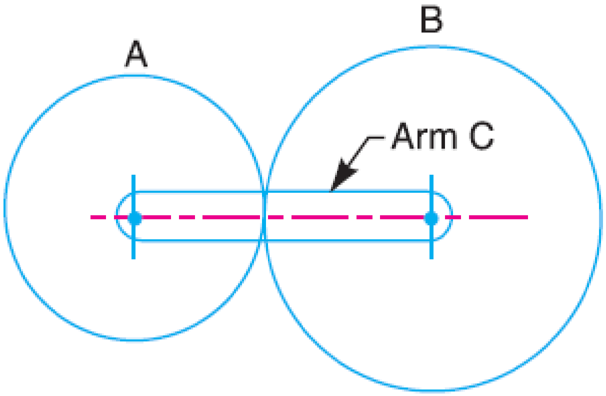


Fig.6.2

Solution :

Given data	Find
$T_A = 36$	Gear A fixed $\Rightarrow N_B = ?$
$T_B = 45$	$N_A = -300$(Clockwise) $\Rightarrow N_B =$
? $N_C = 150$(Anticlockwise)	

Sr. No.	Condition of motion	Revolution of element		
		Arm C	Gear A	Gear B
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through + x revolutions	0	$+x$	$-x\frac{T_A}{T_B}$
3	Add + y revolutions to all elements	+ Y	+ y	+ y
4	Total motion	+ y	$x + y$	$y - x\frac{T_A}{T_B}$

1. Speed of gear B (N_B) when gear A is fixed

Here, gear A fixed

$$\Rightarrow x + y = 0$$
$$\Rightarrow x + 150 = 0$$
$$\Rightarrow x = -150$$

Speed of gear B (N_B) $= y - x\frac{T_A}{T_B}$

$$= y - (-150)\frac{36}{45}$$

$$\boxed{= +270 \text{rpm (Anticlockwise)}}$$

2. Speed of gear B (N_B) when gear $N_A = -300$ (Clockwise)

   Here given

$$x + y = -300$$
$$\therefore x + 150 = -300$$
$$\therefore x = -450 \quad \text{rpm}$$

Speed of gear B (N_B)

$$= y - x\frac{T_A}{T_B}$$
$$= 150 - (-450)\frac{36}{45}$$
$$\boxed{= +510 \text{ rpm(Anti clockwise)}}$$

Example 5.3 In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 rpm clockwise.

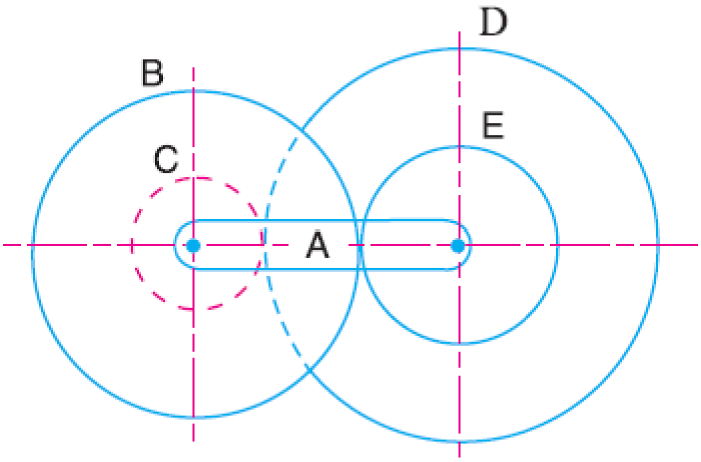


Fig. 6.3

Solution Given data find

$T_B = 75$ GearBfixed $\Rightarrow N_C = ?$

$T_C = 30$ $N_A = -100 \Rightarrow N_C = ?$

$T_D = 90$

$N_A = -100$(Clockwise)

Let $d_C + d_D = d_B + d_E$ $(r_C + r_D = r_B + r_E)$

$\therefore T_C + T_D = T_B + T_E$

$\therefore 30 + 90 = 75 + T_E$

$\therefore T_E = 45$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$-x\frac{T_E}{T_B}$	$-x\frac{T_D}{T_C}$
3	Add + *y* revolutions to all elements	$+ y$	$+ y$	$+ y$	$+ y$
4	Total motion	$+ y$	$x + y$	$y - x\frac{T_E}{T_B}$	$y - x\frac{T_D}{T_C}$

GearBis fixed $\Rightarrow y - x\frac{T_E}{T_B} = 0$

$\Rightarrow -100 - x\frac{45}{75} = 0$

$\Rightarrow x = -166.67$

Speedof gear C $(N_C) = y - x\frac{T_D}{T_C}$

$= -100 - (-166.67) \times \frac{90}{30}$

$= +400$ rpm(Anti clockwise)

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_B}{T_E}$	$+\frac{T_B}{T_E}\times\frac{T_D}{T_C}$
2	Arm fixed gear A rotates through + *x* revolutions	0	+*x*	$x\frac{T_B}{T_E}$	$+x\frac{T_B}{T_E}\times\frac{T_D}{T_C}$
3	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4	Total motion	+ *y*	$x+y$	$y-x\frac{T_B}{T_E}$	$y+x\frac{T_B}{T_E}\times\frac{T_D}{T_C}$

From fig $(r_C+r_D = r_B+r_E)$

$\therefore T_C + T_D = T_B + T_E$

$\therefore T_E = 90 + 30 - 75$

$\therefore T_E = 45$

When gear B is fixed

$\therefore x + y = 0$

$\therefore x + (-100) = 0$

$\therefore x = 100$

Now $N_C = y + x\frac{T_B}{T_E}\times\frac{T_D}{T_C}$

$= -100 + 100\times\frac{75}{45}\times\frac{90}{30}$

$\boxed{N_C = 400 \text{ rpm (Anticlockwise)}}$

Example 5.4 An epicyclic gear consists of three gears A, B and C as shown in Fig. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 rpm. If the gear A is fixed, determine the speed of gears B and C.

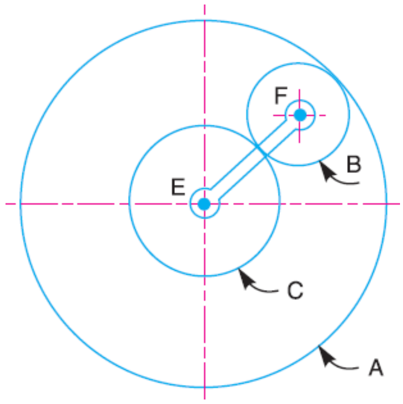


Fig. 6.4

Solution:

T_B =72 (Internal) Gear Afixed $\Rightarrow N_B = ?$
$T_C = 32$ $\Rightarrow N_C = ?$
(External) Arm
EF = 18rpm

From the geometry of fig.

$$r_A = r_C + 2r_B$$
$$\therefore T_A = T_C + 2T_B$$
$$\therefore T_B = 20$$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2	Arm fixed gear A rotates through + *x* revolutions	0	+x	$-x\frac{T_C}{T_B}$	$-x\frac{T_C}{T_A}$
3	Add + *y* revolutions to all elements	+y	+y	+y	+y
4	Total motion	y	x+y	$y - x\frac{T_C}{T_B}$	$y - x\frac{T_C}{T_A}$

1. Speed of gear C (N_c)

Gear Ais fixed $\Rightarrow y - x\frac{T_C}{T_A} = 0$

$$\Rightarrow -18 - x\frac{32}{72} = 0$$

$$\Rightarrow x = -40.5$$

Speed of gear C (N_C) = x + y

$$= 40.5 + 18$$

$$\boxed{= 58.5 \text{ rpm(inthedirectionof arm)}}$$

2. Speed of gear B (N_B)

Speedof gearB $= y - x\frac{T_C}{T_B}$

$$= -18 - 40.5 \times \frac{32}{20}$$

$$= -46.8 \text{ rpm}$$

$$\boxed{= 46.8 \text{ rpm (intheopposite direction of arm)}}$$

Example 5.5 **Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft A rotates at 110 rpm, find the speed of shaft B.**

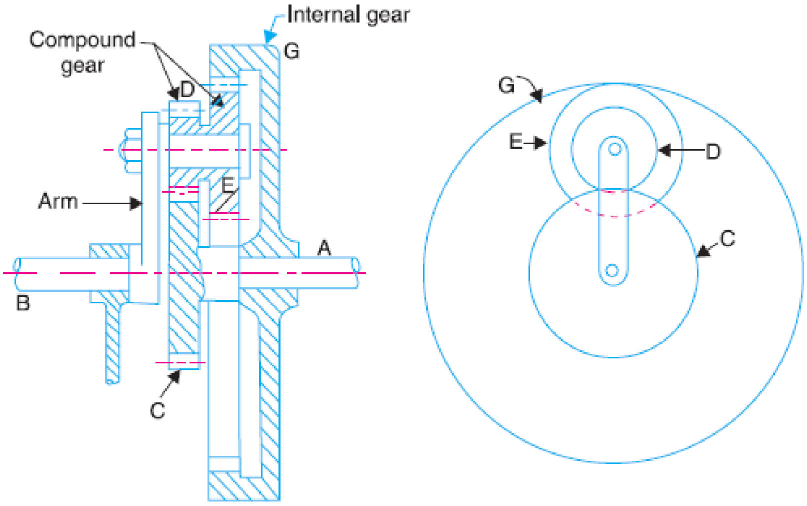


Fig 6.5

Solution:

$T_C = 50$ No.of teethon internal gear =?

$T_D = 20$ Speed ofshaft B = ?

$T_E = 35$

$N_C = 110$ (Rotationofshaft)

From the geometry of fig.

$$\frac{d_G}{2} = \frac{d_C}{2} + \frac{d_D}{2} + \frac{d_E}{2}$$

$$\therefore d_G = d_C + d_D + d_E$$

$$\therefore T_G = T_C + T_D + T_E$$

$$\therefore T_G = 50 + 20 + 35$$

$$\therefore T_G = 105$$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear C (Shaft A)	Compound Gear (D-E)	Gear G
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
2	Arm fixed gear A rotates through + *x* revolutions	0	+x	$-x\frac{T_C}{T_D}$	$-x\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
3	Add + *y* revolutions to all elements	+y	+y	+y	+y
4	Total motion	y	x+ y	$y-x\frac{T_C}{T_D}$	$y-x\frac{T_C}{T_D}\times\frac{T_E}{T_G}$

Speed of shaft B

Here given gear G is fixed

$$\therefore y - x\frac{T_C}{T_D}\times\frac{T_E}{T_G} = 0$$

$$\therefore y - x\frac{50}{20}\times\frac{35}{105} = 0$$

$$\therefore y - x\times\frac{5}{6} = 0 \qquad \cdots\cdots\cdots\cdots(1)$$

Also given gear C is rigidly mounted on shaft A

$$\therefore x + y = 110 \qquad \cdots\cdots\cdots\cdots(2)$$

Solving eq. (1) & (2)

$x = 60$

$y = 50$

$$\boxed{\text{Speed of shaft B} = \text{Speed of arm} = +y = 50 \text{ rpm}}$$

Example 6.6: In an epicyclic gear train, as shown in Fig.13.33, the number of teeth on wheels *A*, *B* and *C* are 48, 24 and 50 respectively. If the arm rotates at 400 rpm, clockwise,

Find: 1. Speed of wheel *C* when *A* is fixed, and

2. Speed of wheel *A* when *C* is fixed

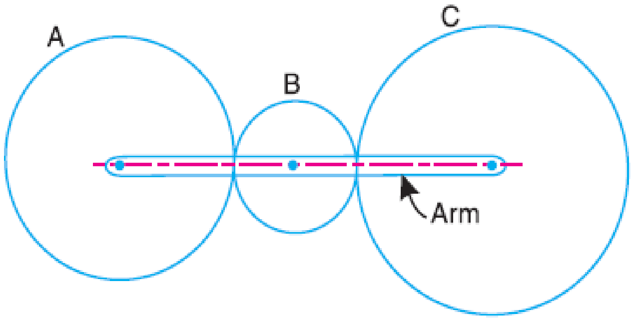


Fig. 6.6

Solution:

$T_A = 48$ Gear Afixed $\Rightarrow N_C = ?$

$T_B = 24$ GearC fixed $\Rightarrow N_A = ?$

$T_C = 50$ $y = -400$ rpm (Armrotationclockwise)

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$	$\left(-\frac{T_A}{T_B}\right)\times\left(-\frac{T_B}{T_C}\right)=+\frac{T_A}{T_C}$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$-x\frac{T_A}{T_B}$	$+x\frac{T_A}{T_C}$
3	Add + *y* revolutions to all elements	$+\ y$	$+\ y$	$+\ y$	$+\ y$
4	Total motion	y	$x+y$	$y-x\frac{T_A}{T_B}$	$y+x\frac{T_A}{T_C}$

1. Speed of wheel C when A is fixed

When A is fixed

$$\Rightarrow x + y = 0$$

$$\Rightarrow x - 400 = 0$$

$$\Rightarrow x = 0$$

$$N_C = y + x\frac{T_A}{T_C}$$

$$= -400 + 400 \times \frac{48}{50}$$

$$= -16 \text{ rpm}$$

$$\boxed{N_C = 16 \text{ rpm (Clockwise direction)}}$$

2. Speed wheel A when C is fixed

When C is fixed

$$\therefore N_C = 0$$

$$\therefore y + x\frac{T_A}{T_C} = 0$$

$$\therefore -400 + x\,\frac{48}{50} = 0$$

$$\therefore x = 416.67$$

$$N_A = x + y$$
$$= 416.67 - 400$$
$$\boxed{N_A = 16.67}$$

Example 5.7: An epicyclic gear train, as shown in Fig. 13.37, has a sun wheel *S* of 30 teeth and two planet wheels *P-P* of 50 teeth. The planet wheels mesh with the internal teeth of a fixed annulus *A*. The driving shaft carrying the sunwheel transmits 4 kW at 300 rpm. The driven shaft is connected to an arm which carries the planet wheels. Determine the speed of the driven shaft and the torque transmitted, if the overall efficiency is 95%.

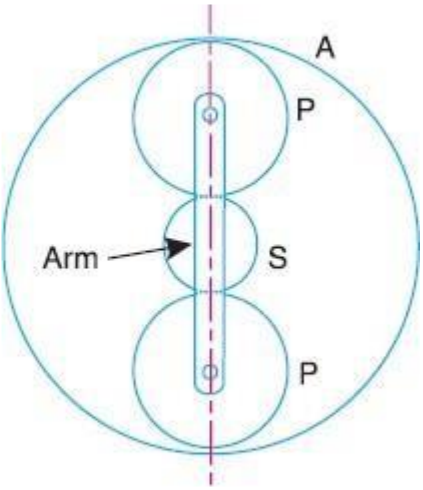


Fig.6.7

Solution

$$T_S = 30 \; T_P = 50 \; T_A = 130$$
$$N_S = 300 \text{ rpm } P = 4 \text{ KW}$$

From the geometry of fig.

$$r_A = 2r_P + r_S$$
$$\therefore T_A = 2T_P + T_S$$
$$= 2\times 50 + 30$$
$$= 130$$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$\left(-\frac{T_S}{T_P}\right)\times\left(\frac{T_P}{T_A}\right) = +\frac{T_S}{T_A}$
2	Arm fixed gear A rotates through + *x* revolutions	0	+*x*	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_A}$
3	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4	Total motion	*y*	$x + y$	$y - x\frac{T_S}{T_P}$	$y - x\frac{T_S}{T_A}$

Here,

$$N_S = 300 \text{rpm}$$

$$\therefore x + y = 300 \qquad \text{………}(1)$$

Also, Annular gear A is fixed

$$\therefore y - x\frac{T_S}{T_A} = 0$$

$$\therefore y - x \times \frac{30}{130} = 0$$

$$\therefore y = 0.23x \qquad \text{………}(2)$$

Solving equation eq. (1) & (2)

$$x = 243.75$$

$$y = 56.25$$

Speed of Arm = Speed of driven shaft = y = 56.25 rpm

Here, P = 4 KW $\qquad \eta = 95\%$

&

$$\therefore \eta = \frac{P_{out}}{P_{in}}$$

$$\therefore P_{out} = \eta \times P_{in}$$

$$= \frac{95}{100} \times 4$$

$$= 3.8 \text{ KW}$$

Also,

$$P_{out} = \frac{2\pi N \, T}{60}$$

$$\therefore 3.8 \times 10^3 = \frac{2\pi \times 56.30 T}{60}$$

$$\therefore T = 644.5 \text{ N}\cdot\text{m}$$

Example 6.8 An epicyclic gear train is shown In fig. Find out the rpm of pinion D if arm A rotate at 60 rpm in anticlockwise direction. No of teeth on wheels are given below.

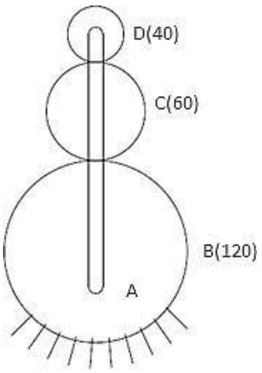


Fig.6.8

Solution:

$T_D = 40$ $N_D = ?$

$T_C = 60$

$T_B = 120$

$N_A = +60$ rpm(Anticlockwise)

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear B	Gear C	Gear D
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_B}{T_C}$	$\frac{T_B}{T_C} \times \frac{T_C}{T_D} = +\frac{T_B}{T_D}$
2	Arm fixed gear A rotates through + *x* revolutions	0	+*x*	$-x\frac{T_B}{T_C}$	$+x\frac{T_B}{T_D}$
3	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4	Total motion	*y*	*x* + *y*	$y - x\frac{T_B}{T_C}$	$y + x\frac{T_B}{T_D}$

From fig. Gear B is fixed

$\therefore x + y = 0$

$\therefore x + 60 = 0$ $(\because \text{rpm of arm} A = 60 = y)$

$\therefore x = -60$

Now motion of gear D

$= y + x\frac{T_B}{T_D}$

$= 60 - 60 \times \frac{120}{40}$

$= -120$ rpm

D rotates 120 rpm in clockwise direction.

Note: By fixing any gear C OR B this problem can be solved

Example 6.9 An epicyclic gear train for an electric motor is shown in Fig. The wheel *S* has 15 teeth and is fixed to the motor shaft rotating at 1450 rpm. The planet *P* has 45 teeth, gears with fixed annulus *A* and rotates on a spindle carried by an arm which is fixed to the output shaft. The planet *P* also gears with the sun wheel *S*. Find the speed of the output shaft. If the motor is transmitting 1.5 kW, find the torque required to fix the annulus *A*.

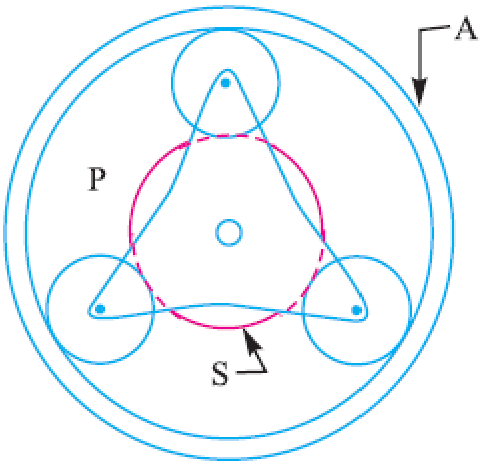


Fig. 6.9

Solution:

$T_S = 15$ Speedofoutputshaft = ?

$T_P = 45$ Torque =?

From fig.

$r_A = r_s + 2r_P$

$\therefore T_A = T_S + 2T_P$

$\therefore T_A = 105$

Sr. No.	Condition of motion	Revolution of element			
		Spindle	Gear S	Gear P	Gear A
1	Sector/Spindle fixed, gear S rotates +1 revolution (anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_P}\times\frac{T_P}{T_A}=-\frac{T_S}{T_A}$
2	Spindle fixed gear S rotates through + *x* revolutions	0	$+x$	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_A}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	y	$x+y$	$y-x\frac{T_S}{T_P}$	$y-x\frac{T_S}{T_A}$

Motor shaft is fixed with gear S

$$\therefore x + y = 1450 \qquad \ldots\ldots\ldots(1)$$

And Annular A is fixed

$$\therefore y - x\frac{T_S}{T_A} = 0$$

$$\therefore y - \frac{15}{105}x = 0$$

$$\therefore y = x\frac{15}{105} \qquad \ldots\ldots\ldots(2)$$

By solving equation (1) & (2)

$$x = 1268.76$$

$$y = 181.25$$

Speed of output shaft y = 181.25 rpm

- Torque on sun wheel (S) (input torque)

$$P = \frac{2\pi N T_i}{60}$$

$$\therefore T_i = \frac{P \times 60}{2\pi N}$$

$$= \left(\frac{2\times10^3}{1.35}\right) \times \frac{60}{2\pi\times1450} \qquad \left(\because 1.35\ \text{HP} = 1\ \text{KW} \Rightarrow 2 = \frac{2}{1.35}\ \text{KW}\right)$$

$$= 9.75 \text{N·m}$$

- Torque on output shaft (with 100% mechanical efficiency)

$$\therefore T_o = \frac{P \times 60}{2\pi N}$$

$$= \left(\frac{2\times10^3}{1.35}\right) \times \frac{60}{2\pi\times181.25}$$

$$= 78.05\ \text{N·m}$$

- Fixing torque

$$= T_o - T_i$$

$$= 78.05 - 9.75$$

$$= 68.3\ \text{N·m}$$

Example 6.10: **If wheel D of gear train as shown in fig. is fixed and the arm A makes 140 revolutions in a clockwise direction. Find the speed and direction of rotation of B & E. C is a compound wheel.**

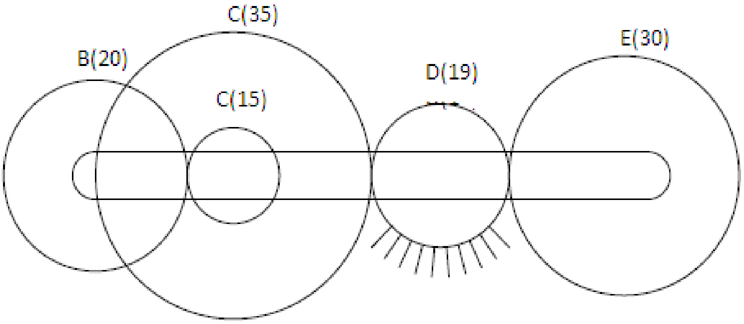


Fig.6.10

Solution:

$T_B = 30$ $T_C = 35$ $T_D = 19$ $T_E = 30$

Sr. No.	Condition of motion	Revolution of element			
		Spindle	Gear S	Gear P	Gear A
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{20}{15}$	$\left(-\frac{20}{15}\right)\times\left(-\frac{35}{19}\right)\times\left(-\frac{19}{30}\right)$
2	Arm fixed gear A rotates through + x revolutions	0	$+x$	$-1.33x$	$-1.555x$
3	Add + y revolutions to all elements	+ y	+ y	+ y	+ y
4	Total motion	y	$x+y$	$y-1.33x$	$y-1.555x$

- When gear D is fixed

$$y + 2.456x = 0$$

$$\therefore -140 + 2.456x = 0 \qquad (\because y = -140\text{rpm given})$$

$$\therefore x = 57$$

- Speed of gear B

$$N_B = x + y$$
$$= +57 - 140$$
$$= -83\text{rpm(Clockwise)}$$

- Speed of gear E

$$N_E = y - 1.555x$$
$$= -140 - 1.555(57)$$
$$= -228.63\text{rpm (Clockwise)}$$

Example 6.11: **The epicyclic train as shown in fig. is composed of a fixed annular wheel A having 150 teeth. Meshing with A is a wheel b which drives wheel D through an idle wheel C,D being concentric with A. Wheel B and C are carried on an arm which revolve clockwise at 100 rpm about the axis of A or D. If the wheels B and D are having 25 teeth and 40 teeth respectively, Find the no. of teeth on C and speed and sense of rotation of C.**

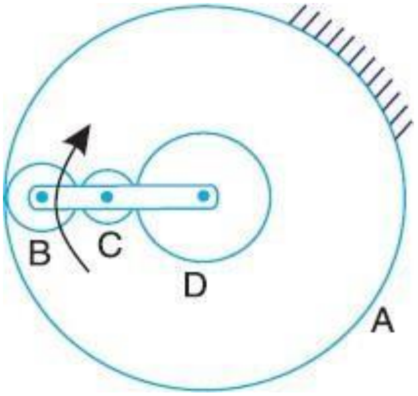


Fig. 6.11

Solution:

From the geometry of fig.

$$r_A = 2r_B + 2r_C + r_D$$
$$\therefore T_A = 2T_B + 2T_C + T_D$$
$$\therefore 150 = 50 + 2T_C + 40$$
$$\therefore T_C = 30$$

Sr. No.	Condition of motion	Revolution of element				
		Arm	Gear D	Gear C	Gear B	Gear A
1	Arm fixe, gear D rotates +1 revolution (anticlockwise)	0	+1	$-\frac{T_D}{T_C}$	$+\frac{T_D}{T_B}$	$+\frac{T_D}{T_A}$
2	Arm fixed gear D rotates through + *x* revolutions	0	$+x$	$-x\frac{T_D}{T_C}$	$+x\frac{T_D}{T_B}$	$+x\frac{T_D}{T_A}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_D}{T_C}$	$y+x\frac{T_D}{T_B}$	$y+x\frac{T_D}{T_A}$

Now

$$N_A = 0$$

$$\therefore y + x\frac{T_D}{T_A} = 0$$

$$\therefore -100 + x\times\frac{40}{150} = 0$$

$$\therefore x = 375$$

Let

$$N_C = y - x\frac{T_D}{T_C}$$

$$= -100 - 375\times\frac{40}{30}$$

$$= -600\text{rpm}$$

Example 6.12: Fig. 13.24 shows a differential gear used in a motor car. The pinion A on the propeller shaft has 12 teeth and gears with the crown gear B which has 60 teeth. The shafts P and Q form the rear axles to which the road wheels are attached. If the propeller shaft rotates at 1000 rpm and the road wheel attached to axle Q has a speed of 210 rpm. while taking a turn, find the speed of road wheel attached to axle P.

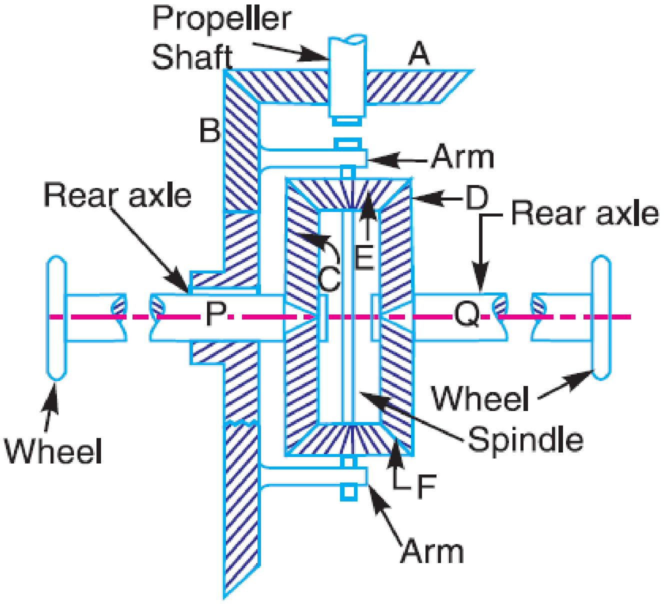


Fig. 6.12

Solution:

$$T_A = 12$$

$$T_B = 60$$

$$N_Q = N_D = 210\text{rpm}$$

$$N_A = 1000\text{rpm}$$

Let

$$N_A \times T_A = N_B T_B$$

$$\therefore N_B = N_A \times \frac{T_A}{T_B}$$

$$= 1000 \times \frac{12}{60}$$

$$= 200\text{rpm}$$

Sr. No.	Condition of motion	Revolution of element			
		Gear B	Gear C	Gear E	Gear D
1	Gear B is fixed, gear C rotates +1 revolution(anticlockwise)	0	+1	$+\frac{T_C}{T_E}$	-1
2	Gear B is fixed gear C rotates through + x revolutions	0	$+x$	$+x\frac{T_C}{T_E}$	$-x$
3	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y+x\frac{T_C}{T_E}$	$y-x$

Let here speed of gear B is 200 rpm

$$N_B = 200 = y$$

From table

$$N_D = y - x = 210$$

$$\therefore x = y - 210$$

$$\therefore x = 200 - 210$$

$$\therefore x = -10\text{rpm}$$

Let speed of road wheel attached to the axle P = Speed of gear C

$$= x + y$$

$$= -10 + 200$$

$$= 180\text{rpm}$$

Example 6.13: Two bevel gears A and B (having 40 teeth and 30 teeth) are rigidly mounted on two co-axial shafts X and Y. A bevel gear C (having 50 teeth) meshes with A and B and rotates freely on one end of an arm. At the other end of the arm is welded a sleeve and the sleeve is riding freely loose on the axes of the shafts X and Y. Sketch the arrangement. If the shaft X rotates at 100 rpm. clockwise and arm rotates at 100 rpm. anticlockwise, find the speed of shaft Y.

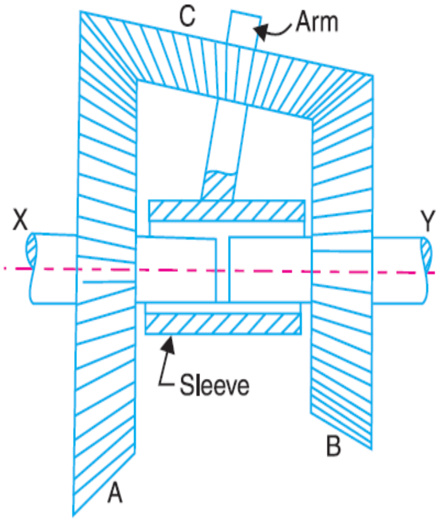


Fig. 6.13

Solution :

$$T_A = 40 \qquad T_C = 50 \qquad T_B = 30$$

$$N_X = N_A = -100\text{rpm(Clockwise)}$$

$$\text{Speedof arm} = 100\text{rpm}$$

Sr. No.	Condition of motion	Revolution of element			
		Arm	Gear A	Gear C	Gear B
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$\pm\frac{T_A}{T_C}$	$-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$\pm x\frac{T_A}{T_C}$	$-x\frac{T_A}{T_B}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y \pm x\frac{T_A}{T_C}$	$y - x\frac{T_A}{T_B}$

Here speed of arm = y =+100 rpm (given)

$$\text{Also given} \quad N_A = N_X = -100\text{rpm}$$

$$\therefore N_A = x + y$$

$$\therefore -100 = x + 100$$

$$\therefore x = -200$$

Speed of shaft Y = N_B

$$= y - x\frac{T_A}{T_B}$$

$$= 100 + 200 \times \frac{40}{30}$$

$$= +366.7\text{rpm (Anticlockwise)}$$

Example 6.14. An epicyclic train of gears is arranged as shown in Fig. How many revolutions does the arm, to which the pinions B and C are attached, make: 1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and 2. when A makes one revolution clockwise and D is stationary? The number of teeth on the gears A and D are 40 and 90 respectively.

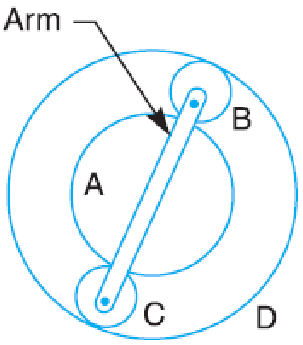


Fig. 6.14

Solution:

$T_A = 40$

$T_D = 90$

First of all, let us find the number of teeth on gear B and C (i.e. T_B and T_C). Let d_A, d_B, d_C, d_D be the pitch circle diameter of gears A, B, C, and D respectively. Therefore from the geometry of fig, $d_A + d_B + d_C = d_D$ or $d_A + 2d_B = d_D$ ...($d_B = d_C$)

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$T_A + 2\,T_B = T_D$ or $40 + 2\,T_B = 90$

$\therefore$ $T_B = 25$, and $T_C = 25$ ...($T_B = T_C$)

Sr. No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound Gear B-C	Gear D
1	Arm fixe, gear A rotates -1 revolution(clockwise)	0	-1	$+\frac{T_A}{T_B}$	$\left(+\frac{T_A}{T_B}\right)\times\left(+\frac{T_B}{T_D}\right)=+\frac{T_A}{T_D}$
2	Arm fixed gear A rotates through - *x* revolutions	0	$-x$	$+x\frac{T_A}{T_B}$	$+x\frac{T_A}{T_D}$
3	Add - *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4	Total motion	$-y$	$-x-y$	$x\frac{T_A}{T_B}-y$	$x\frac{T_A}{T_C}-y$

1. **Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise**
   Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table, $-x - y = -1$ or $x + y = 1$ ...(1)

Also, the gear D makes half revolution anticlockwise, therefore

$$x \times \frac{T_A}{T_D} - y = \frac{1}{2}$$

$$\therefore x \times \frac{40}{90} - y = \frac{1}{2}$$

$$\therefore 40x - 90y = 45$$

$$\therefore x - 2.25y = 1.125 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From equations (1) and (2),

$$x = 1.04 \qquad \text{and} \qquad y = -0.04$$

$$\text{Speed of arm} = -y = -(-0.04) = +0.04$$

$$\boxed{= 0.04 \text{ revolution(Anticlockwise)}}$$

2. **Speed of arm when A makes 1 revolution clockwise and D is stationary**

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x - y = -1$$

$$\therefore x + y = 1 \qquad \ldots(3)$$

Also the gear *D* is stationary, therefore

$$x \times \frac{T_A}{T_D} - y = 0$$

$$\therefore x \times \frac{40}{90} - y = 0$$

$$\therefore 40x - 90y = 0$$

$$\therefore x - 2.25y = 0 \qquad \ldots(4)$$

From equations (3) and (4),

$$\therefore \text{Speed of arm} = -y = -0.308$$

$$\boxed{\therefore \text{Speed of arm} = 0.308 \text{revolution (Clockwise)}}$$

Example 6.15. In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth is: TC = 28; TD = 26; TE = TF = 18. 1. Sketch the arrangement; 2. Find the number of teeth on A and B; 3. If the arm G makes 100 r.p.m. clockwise and A is fixed, find the speed of B; and 4. If the arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise; find the speed of Wheel B.

Solution:

Given: $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$

1. Sketch the arrangement

The arrangement is shown in Fig.

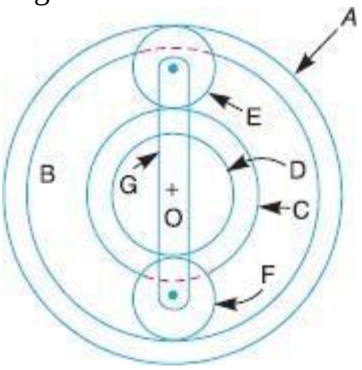


Fig. 6.15

2. Number of teeth on wheels A and B

*T*A = Number of teeth on wheel *A*, and

*T*B = Number of teeth on wheel *B*.

If d_A, d_B, d_C, d_D, d_E and d_F are the pitch circle diameters of wheels *A*, *B*, *C*, *D*, *E* and *F* respectively, then from the geometry of Fig.

$$d_A = d_C + 2\ d_E$$

$$\text{And}\quad d_B = d_D + 2\ d_F$$

Since the number of teeth are proportional to their pitch circle diameters, for the same module, therefore

$$T_A = T_C + 2\ T_E = 28 + 2 = 64$$

$$\text{And}\quad T_B = T_D + 2\ T_F = 26 + 2 = 62$$

3. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A is fixed

First of all, the table of motions is drawn as given below:

Sr. No	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1	Arm fixe, A rotates +1 revolution (Anti clockwise)	0	+1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E}\times\frac{T_E}{T_C}$ $=-\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}\times\frac{T_F}{T_B}$ $=+\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
2	Arm fixed A rotates through + *x* revolutions	0	$+x$	$+x\frac{T_A}{T_E}$	$-x\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
3	Add +*y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$x\frac{T_A}{T_E}+y$	$y-x\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$

Since the arm *G* makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$\mathbf{y = -100}$$

Also, the wheel *A* is fixed, therefore from the fourth row of the table,

$$\mathbf{x + y = 0 \quad or \quad x = -y = 100}$$

$$\text{Speedof wheel B} = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$$

$$= -100 + 100 \times \frac{64}{28} \times \frac{26}{62}$$

$$= -100 + 95.8 \text{ r.p.m.} = -\ 4.2 \text{r.p.m}$$

$$\boxed{\text{Speedof wheel B} = 4.2 \text{ r.p.m}}$$

4. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise

Since the arm *G* makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$y = -100 \qquad \text{...(3)}$$

Also the wheel *A* makes 10 r.p.m. counter clockwise, therefore from the fourth row of the table,

$$x + y = 10$$

$$\therefore x = 10 - y$$

$$\therefore x = 10 + 100$$

$$\therefore x = 110 \qquad \text{...(4)}$$

$$\therefore \text{Speed of wheel B} = +\ y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$$

$$= -100 + 110 \times \frac{64}{28} \times \frac{26}{62}$$

$$= -\ 100 + 105.4 \text{ r.p.m}$$

$$= +\ 5.4 \text{ r.p.m}$$

$$\boxed{\therefore \text{Speed of wheel B} = 5.4 \text{r.p.m}}$$

Example 6.16. Fig. shows diagrammatically a compound epicyclic gear train. Wheels A , D and E are free to rotate independently on spindle O, while B and C are compound and rotate together on spindle P, on the end of arm OP. All the teeth on different wheels have the same module. A has 12 teeth, B has 30 teeth and C has 14 teeth cut externally. Find the number of teeth on wheels D and E which are cut internally. If the wheel A is driven clockwise at 1 r.p.s. while D is driven counter clockwise at 5 r.p.s., determine the magnitude and direction of the angular velocities of arm OP and wheel E.

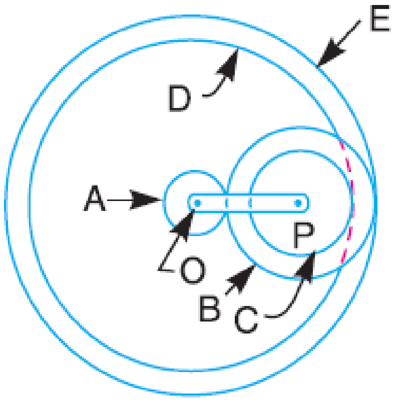


Fig. 6.16

Solution:

Given: $T_A=12$; $T_B=30$; $T_C=14$; $N_A=1$ r.p.s.; $N_D=5$r.p.s

Number of teeth on wheels D and E

Let T_D and T_E be the number of teeth on wheels D and E respectively. Let d_A, d_B, d_C, d_D and d_E be the pitch circle diameters of wheels A, B, C, D and E respectively. From the geometry of the figure,

$$d_E = d_A + 2d_B \quad \text{and} \quad d_D = d_E - (d_B - d_C)$$

Since the number of teeth are proportional to their pitch circle diameters for the same module, therefore

$$T_E = T_A + 2T_B \qquad T_D = T_E - (T_B - T_C)$$

$$\therefore T_E = 12 + 2 \times 30 \qquad \therefore T_D = 72 - (30 - 14)$$

$$\boxed{\therefore T_E = 72} \qquad \boxed{\therefore T_D = 56}$$

Magnitude and direction of angular velocities of arm OP and wheel

The table of motions is drawn as follows:

Sr. No.	Condition of motion	Revolutions of elements				
		Arm	Wheel A	Compound wheel B-C	Wheel D	Wheel E
1	Arm fixe, gear A rotates -1 revolution(clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_E}=+\frac{T_A}{T_E}$
2	Arm fixed gear A rotates through - x revolutions	0	$-x$	$+x\frac{T_A}{T_B}$	$+x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$+x\frac{T_A}{T_E}$
3	Add - y revolutions to all elements	$-y$	$-y$	$-y$	$-y$	$-y$
4	Total motion	$-y$	$-x-y$	$x\frac{T_A}{T_B}-y$	$x\frac{T_A}{T_B}\times\frac{T_C}{T_D}-y$	$x\frac{T_A}{T_E}-y$

Since the wheel *A* makes 1 r.p.s. clockwise, therefore from the fourth row of the table,

$$-x - y = -1$$

$$\therefore x + y = 1 \qquad (1)$$

Also, the wheel *D* makes 5 r.p.s. counter clockwise, therefore

$$x\frac{T_A}{T_B}\times\frac{T_C}{T_D} - y = 5$$

$$\therefore x\frac{T_A}{T_B}\times\frac{T_C}{T_D} - y = 5$$

$$\therefore x\frac{12}{30}\times\frac{14}{56} - y = 5$$

$$\therefore 0.1x - y = 5 \qquad (2)$$

From equations (1) and (2),

$$x = 5.45 \quad \text{and} \quad y = -4.45$$

Angular velocity of arm OP

$$= -y = -(-4.45) = 4.45 \text{ r.p.s}$$

$$\boxed{= 4.45\times 2\pi = 27.964 \text{rad/ sec (Anti clockwise)}}$$

And angular velocity of wheel E

$$= x\frac{T_A}{T_E} - y$$

$$= 5.45\times\frac{12}{72} - (-4.45)$$

$$= 5.36 \text{ r.p.s}$$

$$= 5.36\times 2\pi$$

$$\boxed{= 33.68 \text{ rad / sec (Anti}}$$

Example 6.17. Fig shows an epicyclic gear train known as Ferguson's paradox. Gear A is fixed to the frame and is, therefore, stationary. The arm B and gears C and D are free to rotate on the shaft S. Gears A, C and D have 100, 101 and 99 teeth respectively. The planet gear has 20 teeth. The pitch circle diameters of all are the same so that the planet gear P meshes with all of them. Determine the revolutions of gears C and D for one revolution of the arm B.

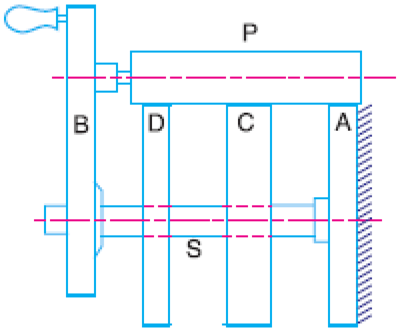


Fig. 6.17

Solution:

Given : $T_A = 100$; $T_C = 101$; $T_D = 99$;$T_P = 20$

The table of motions is given below:

Sr. No.	Condition of motion	Revolutions of elements			
		Arm B	Gear A	Gear C	Gear D
1	Arm B fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$+\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_C}{T_D}=\frac{T_A}{T_D}$
2	Arm B fixed gear A rotates through + *x* revolutions	0	$+x$	$+x\frac{T_A}{T_C}$	$x\frac{T_A}{T_D}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y+x\frac{T_A}{T_C}$	$y+x\frac{T_A}{T_D}$

The arm B makes one revolution, therefore

$$y = 1$$

Since the gear A is fixed, therefore from the fourth row of the table,

$$x + y = 0$$

$$\therefore x = -y = -1$$

Let N_C and N_D = Revolutions of gears *C* and *D* respectively.

From the fourth row of the table, the revolutions of gear *C*,

$$N_C = y + x\frac{T_A}{T_C}$$

$$=1-1\times\frac{100}{101}$$

$$\boxed{\therefore N_C = +\frac{1}{101}}$$

And the revolutions of gear *D*,

$$N_D = y + x\frac{T_A}{T_D} = 1-\frac{100}{99}$$

$$\boxed{\therefore N_D = -\frac{1}{99}}$$

From above we see that for one revolution of the arm B, the gear C rotates through 1/101 Revolution in the same direction and the gear D rotates through 1/99 revolutions in the opposite direction.

Example 6.18. Fig. shows an epicyclic gear train. Pinion A has 15 teeth and is rigidly fixed to the motor shaft. The wheel B has 20 teeth and gears with A and also with the annular

fixed wheel E. Pinion C has 15 teeth and is integral with B (B, C being a compound gear wheel). Gear C meshes with annular wheel D, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B,

B. If the motor runs at 1000 r.p.m., find the speed of the machine shaft. Find the torque exerted on the machine shaft, if the motor develops a torque of 100 N-m.

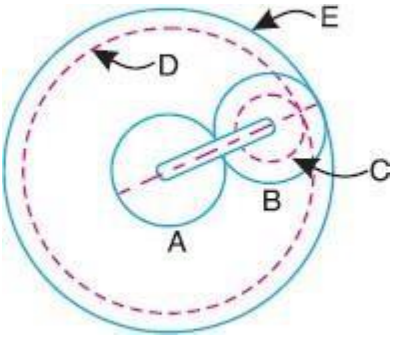


Fig. 6.18

Solution:

Given : $T_A = 15$; $T_B = 20$; $T_C = 15$; $N_A = 1000$ r.p.m.;

Torque developed by motor (or pinion A) = 100 N-m

1. Speed of the machine shaft

The table of motions is given below:

Sr. No.	Condition of motion	Revolution of element				
		Arm	Pinion A	Compound wheel D-C	Wheel D	Wheel E
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-\frac{T_A}{T_B}\times\frac{T_B}{T_E}=-\frac{T_A}{T_E}$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$-x\frac{T_A}{T_B}$	$-x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-x\frac{T_A}{T_E}$
3	Add + *y* revolutions to all elements	$+\ y$	$+\ y$	$+\ y$	$+y$	$+\ y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_A}{T_B}$	$y-x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$y-x.\frac{T_A}{T_E}$

First of all, let us find the number of teeth on wheels D and E. Let T_D and T_E be the number of teeth on wheels D and E respectively. Let d_A, d_B, d_C, d_D and d_E be the pitch circle diameters of wheels A, B, C, D and E respectively. From the geometry of the figure,

$$d_E = d_A + 2\, d_B \qquad \text{and} \qquad d_D = d_E - (d_B - d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_E = T_A + 2\,T_B = 15 + 2\times20 = 55$$

$$T_D = T_E - (T_B - T_C) = 55 - (20 - 15) = 50$$

We know that the speed of the motor or the speed of the pinion *A* is 1000 r.p.m.

Therefore

$$x + y = 1000 \qquad ...(1)$$

Also, the annular wheel *E* is fixed, therefore

$$y - x\frac{T_A}{T_E} = 0$$

$$\therefore\ y = x\frac{T_A}{T_E}$$

$$\therefore\ y = x\frac{15}{55}$$

$$\therefore\ y = 0.273x \qquad ...(2)$$

From equations (1) and (2),

$$x = 786 \qquad \text{and} \qquad y = 214$$

$\therefore$ Speedofmachineshaft =Speedof wheel D

$$N_D = y - x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$$

$$= 214 - 786\times\frac{15}{20}\times\frac{15}{50}$$

$$= +\ 37.15 \text{ r.p.m.}$$

$$\boxed{\therefore N_D = 37.15}$$

Torque exerted on the machine shaft

We know that

Torque developed by motor × Angular speed of motor

=Torque exerted on machine shaft ×Angular speed of machine shaft

$\therefore 100 \times\omega_A$ =Torque exerted on machine shaft $\times\omega_D$

$\therefore$ Torque exerted on machine shaft $= 100 \times \frac{\omega_A}{\omega_D}$

$$= 100 \times \frac{N_A}{N_D} = 100 \times \frac{1000}{37.5}$$

$$\boxed{\therefore \text{Torque exerted on machine shaft} = 2692 \text{ N·m}}$$

Example 6.19. An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star- shaped planet carrier C. The sizes of different toothed wheels are such that the planet carrier C rotates at 1/5th of the speed of

the sun wheel S. The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel is 100 N-m. Determine: 1. Number of teeth on different wheels of the train, and 2. torque necessary to keep the internal gear stationary.

Solution:

Given $N_C = \frac{N_S}{5}$

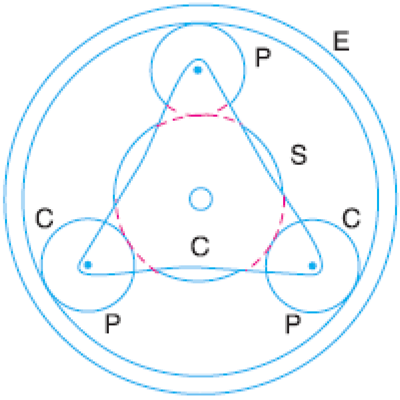


Fig. 6.19

1. Number of teeth on different wheels

The arrangement of the epicyclic gear train is shown in Fig.. Let T_S and T_E be the number of teeth on the sun wheel S and the internal gear E respectively. The table of motions is given below:

Sr. No.	Conditions of motion	Revolutions of elements			
		Plant carrier C	Sun wheel S	Planet Wheel P	Internal Gear E
1	Planet carrier *C* fixed, sun wheel *S* rotates through + 1 revolution (anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_P}\times\frac{T_P}{T_E}=-\frac{T_S}{T_E}$
2	Planet carrier *C* fixed, sun wheel *S* rotates through + *x* revolutions	0	$+x$	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_E}$
3	Add + *y* revolutions to all elements	$+ y$	$+ y$	$+ y$	$+ y$
4	Total motion	$+ y$	$x+y$	$y-x\frac{T_S}{T_P}$	$y-x\frac{T_S}{T_E}$

We know that when the sun wheel *S* makes 5 revolutions, the planet carrier *C* makes 1 revolution. Therefore from the fourth row of the table,

$$y = 1, \quad \text{and} \quad x+y=5 \therefore x=4$$

Since the gear *E* is stationary, therefore from the fourth row of the table,

$$y - x\frac{T_S}{T_E} = 0$$

$$\therefore 1 - 4\frac{T_S}{T_E} = 0$$

$$\therefore T_E = 4T_S$$

Since the minimum number of teeth on any wheel is 16, therefore let us take the number of teeth on sun wheel,

$$T_S = 16$$

$$\boxed{\therefore T_E = 4 \times 16 = 64}$$

Let d_S, d_P and d_E be the pitch circle diameters of wheels *S*, *P* and *E* respectively. Now from the geometry of Fig

$$d_S + 2\ d_P = d_E$$

Assuming the module of all the gears to be same, the number of teeth are proportional to their pitch circle diameters.

$$T_S + 2\ T_P = T_E$$

$$\therefore 16 + 2T_P = 64$$

$$\boxed{\therefore T_P = 24}$$

2. Torque necessary to keep the internal gear stationary

We know that

Torque on S× Angular speed of S = Torque on C× Angular speed of C

$$100 \times \omega_S = \text{Torque on C} \times \omega_C$$

$$\therefore \text{Torque on C} = 100 \times \frac{\omega}{\omega_C}$$

$$= 100 \times \frac{N_S}{N_C}$$

$$= 100 \times 5$$

$$\therefore \text{Torque on C} = 500\ \text{N·m}$$

$$\therefore \text{Torque necessary to keep the internal gear stationary}$$

$$= 500 - 100$$

$$\boxed{= 400\ \text{N·m}}$$

LECTURE 1

GEARS

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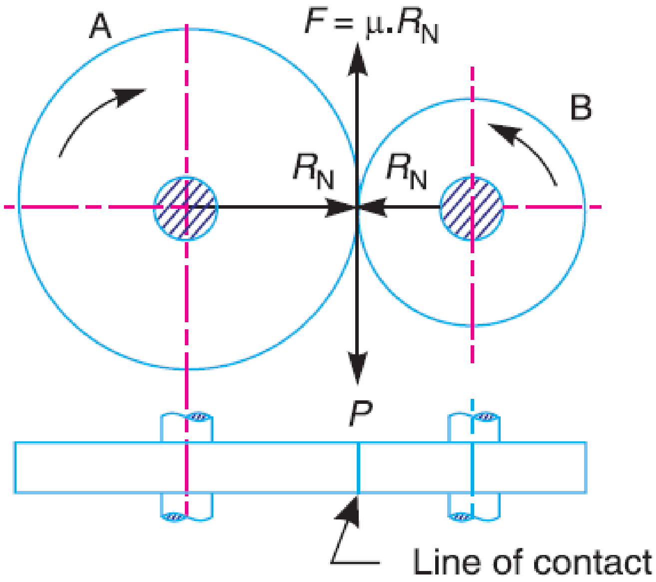
DEPARTMENT OF MECHANICAL ENGINEERING

POWER TRANSMISSION SYSTEMS

Belt/Rope Drives - Large center distance of the shafts

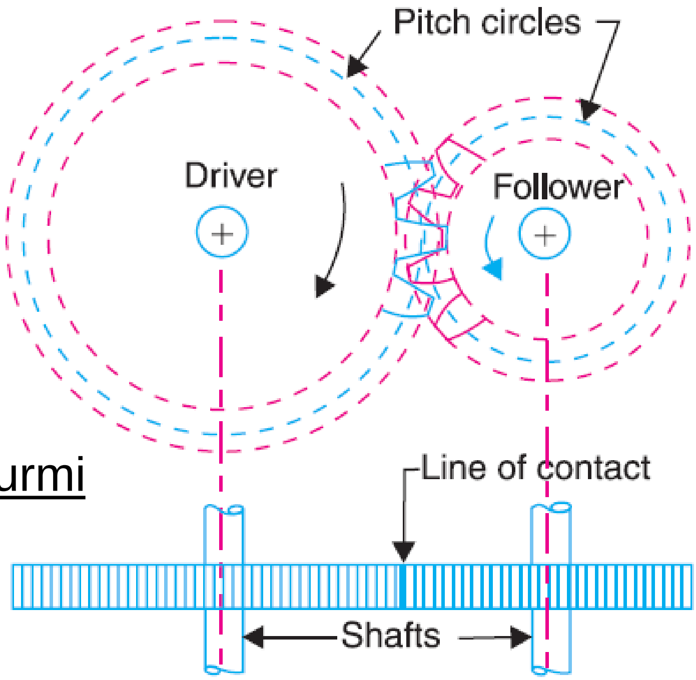
Chain Drives - Medium center distance of the shafts

Gear Drives - Small center distance of the shafts

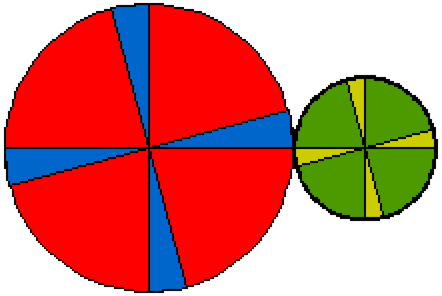


Friction wheels

Source: R. S. Khurmi



Toothed wheels.



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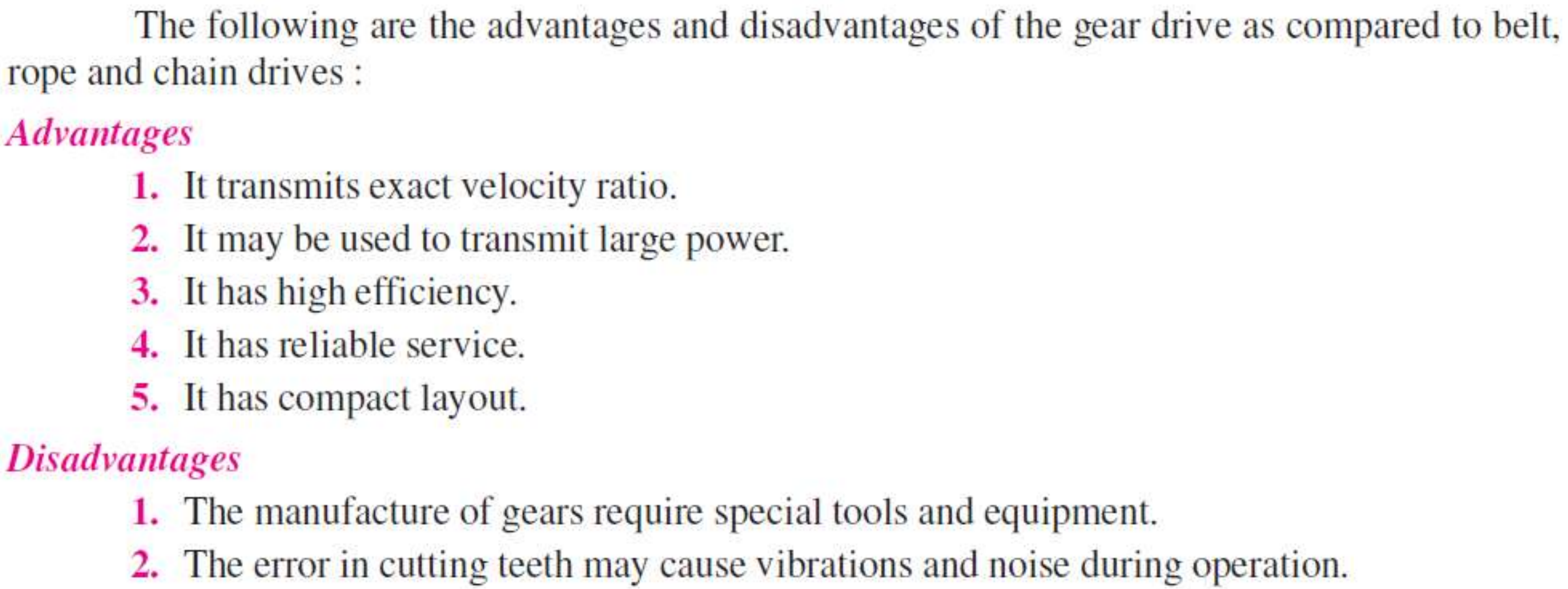
when the tangential force *(P)* *exceeds the frictional resistance (F), slipping will take place between the two* wheels. Thus the friction drive is not a positive drive.

ADVANTAGES AND DISADVANTAGES OF GEAR DRIVE



The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

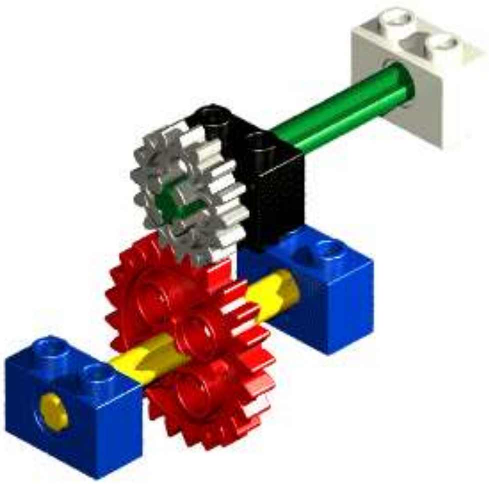
Disadvantages

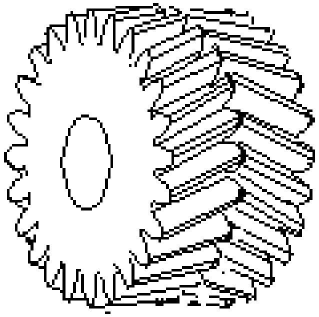
1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

CLASSIFICATION OF TOOTHED WHEELS

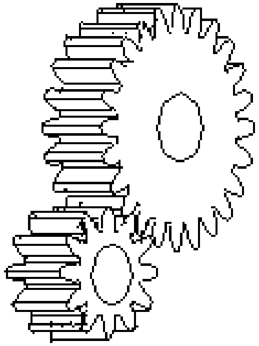
1. *According to the position of axes of the shafts*

(a) Parallel

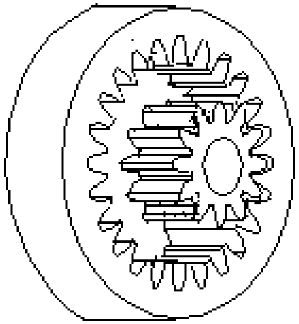




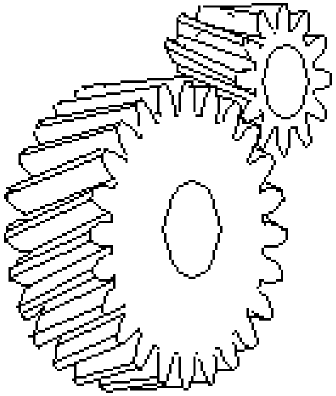
Heringbone gears (Double Helical gears)



External contact



Internal contact

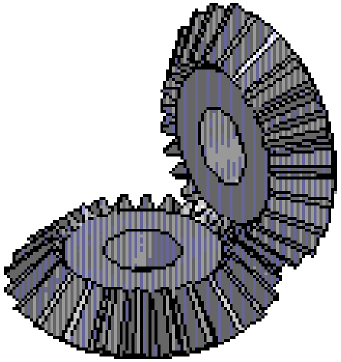


Parallel Helical gears

CLASSIFICATION OF TOOTHED WHEELS

1. *According to the position of axes of the shafts* **(b) Intersecting** *(Bevel Gears)*

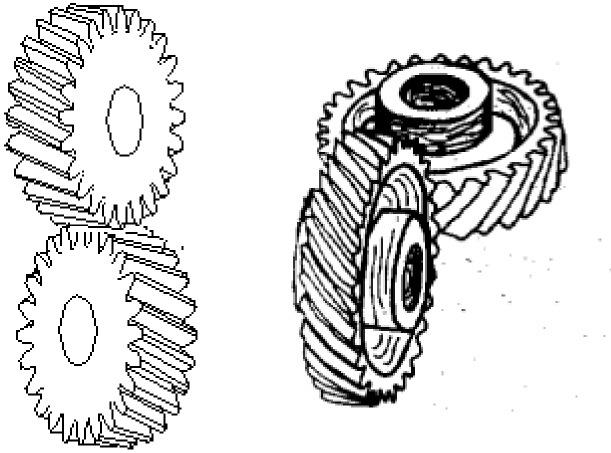
Spiral bevel gears



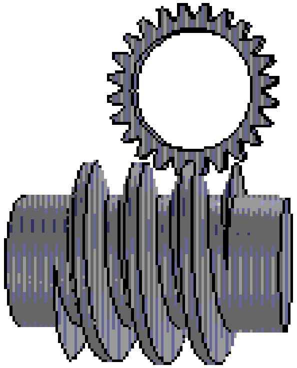
Straight bevel gears

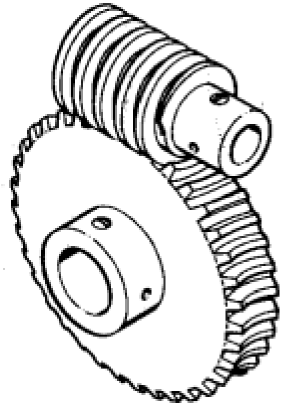
CLASSIFICATION OF TOOTHED WHEELS

1. *According to the position of axes of the shafts* **(c) Non-intersecting and non-parallel**



Crossed-helical gears





Worm & Worm Wheel

CLASSIFICATION OF TOOTHED WHEELS

2. According to the peripheral velocity of the gears

(a) Low velocity (velocity less than 3 m/s)

(b) Medium velocity (between 3 to 15 m/s)

(c) High velocity (More than 15 m/s)

CLASSIFICATION OF TOOTHED WHEELS

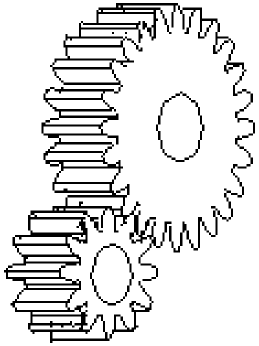
3. According to the type of gearing

(a) External gearing

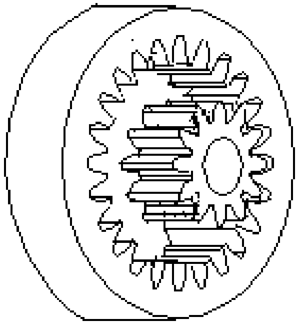
(b) Internal gearing

(c) Rack and pinion

Rack and pinion

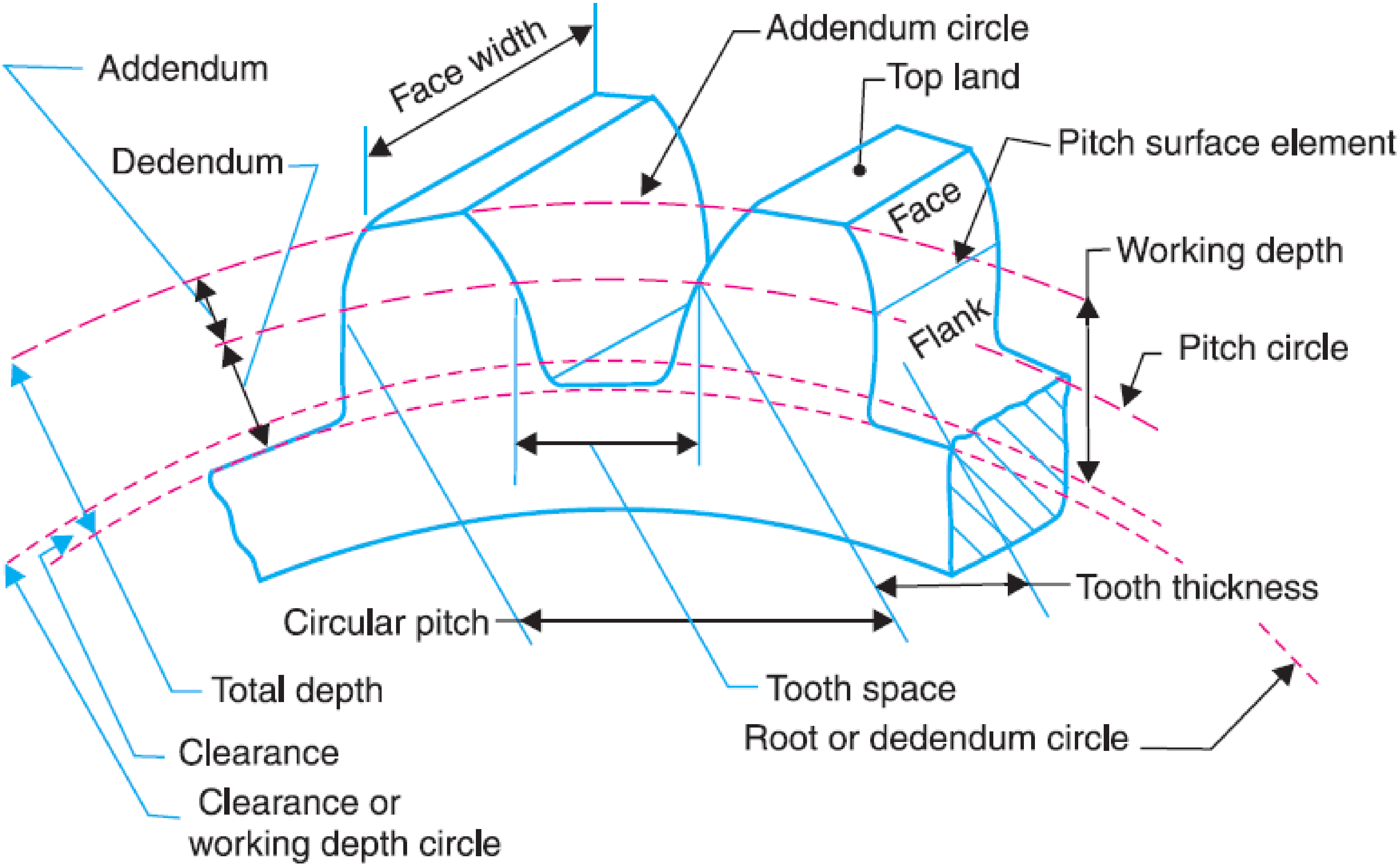


External gearing



Internal gearing

TERMINOLOGY : SPUR GEAR



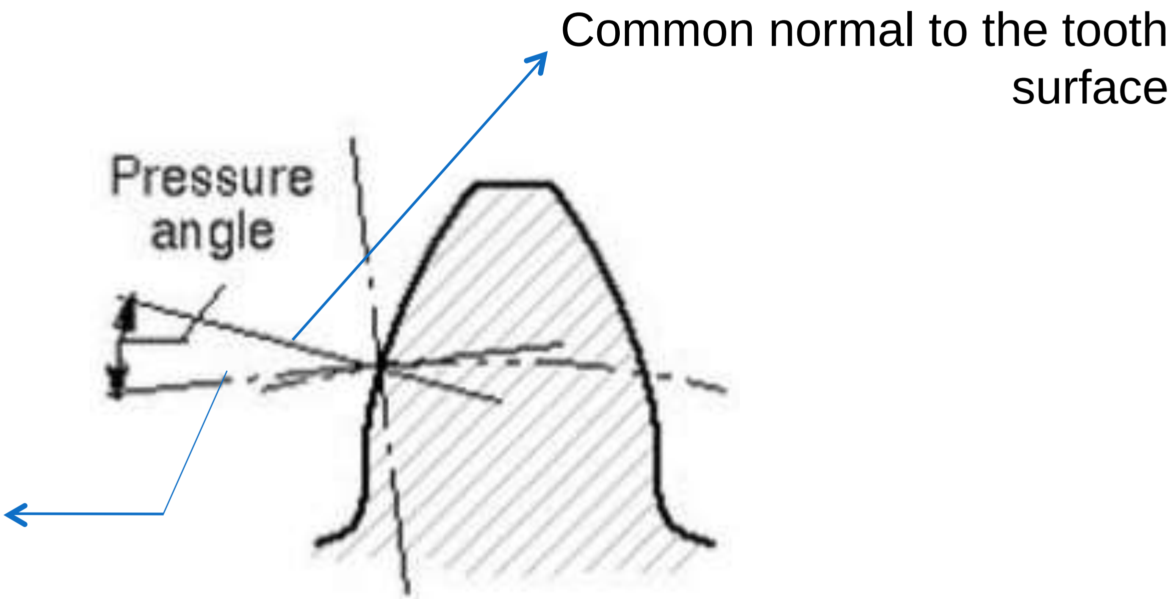
Source: R. S. Khurmi

TERMINOLOGY : SPUR GEAR

Pressure angle or angle of obliquity:

It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ø. The standard pressure angles are 14.5° and 20° .

Source: R. S. Khurmi



TERMINOLOGY : SPUR GEAR

Circular pitch: It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c .

$$p_c = \pi D/T$$

D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

Note: Two gears will mesh together correctly, if the two wheels have the same circular pitch.

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

TERMINOLOGY : SPUR GEAR

Diametral pitch.

It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d. Mathematically,

$$p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \cdots \left(\because \; p_c = \frac{\pi D}{T} \right)$$

T = Number of teeth, and

D = Pitch circle diameter.

Module.

It is the ratio of the pitch circle diameter in millimeters to the number of teeth.

Module, $m = D/T$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20.

TERMINOLOGY : SPUR GEAR

Backlash: It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle.

Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion

FORMULAE

$$Center\,dis\tan ce = \begin{pmatrix} Teeth\ on\ \ pinion \\ + \\ Teeth\ on\ Gear \end{pmatrix} \frac{Circular\ \ pitch}{2 \times \pi}$$

$$= \frac{(Teeth\ on\ \ pinion + Teeth\ on\ Gear)}{2 \times Diametral\ \ pitch}$$

$$\text{Base Circle Diameter} = \text{Pitch Diameter} \times \text{Cos}\,\phi$$

FORMULAE SPECIFIC TO GEARS WITH STANDARD TEETH

Addendum = 1 ÷ Diametral Pitch
= 0.3183 × Circular Pitch

Dedendum = 1.157 ÷ Diametral Pitch
= 0.3683 × Circular Pitch

Working Depth = 2 ÷ Diametral Pitch
= 0.6366 × Circular Pitch

Whole Depth = 2.157 ÷ Diametral Pitch
= 0.6866 × Circular Pitch

FORMULAE SPECIFIC TO GEARS WITH STANDARD TEETH

- **Clearance** = 0.157 ÷ Diametral Pitch
  = 0.05 × Circular Pitch

- **Outside Diameter** = (Teeth + 2) ÷ Diametral Pitch
  = (Teeth + 2) × Circular Pitch ÷ π

- **Diametral Pitch** = (Teeth + 2) ÷ Outside Diameter

GEAR MATERIALS

Selection of materials depends upon strength and service conditions like wear, noise etc.,

- Metallic materials (cast iron, steel (plain carbon steel or alloy steel) and bronze)

- Non- Metallic materials – reduces noise (wood, compressed paper and synthetic resins like nylon)

Note: phosphor bronze is widely used for worm gears in order to reduce wear of the worms

LECTURE 2

CONDITION FOR CONSTANT VELOCITY RATIO

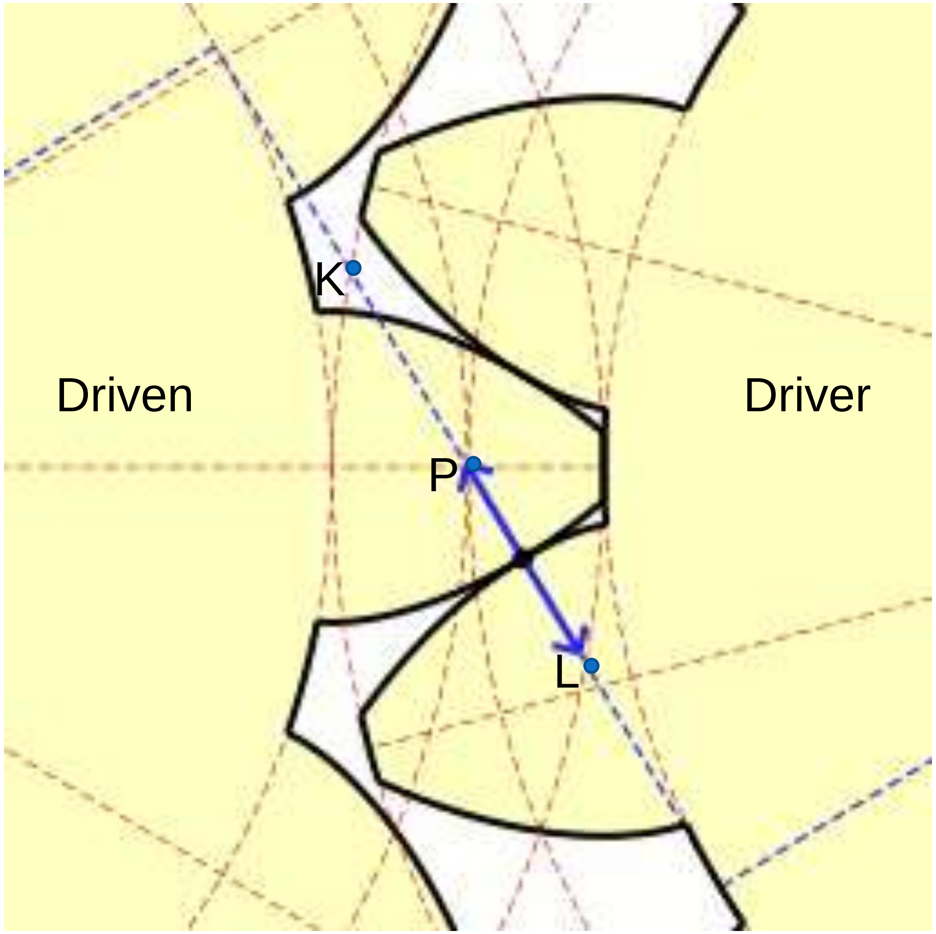
DEPARTMENT OF MECHANICAL ENGINEERING

LAW OF GEARING

Involute Gear

The moving point 'P' is Pitch point.

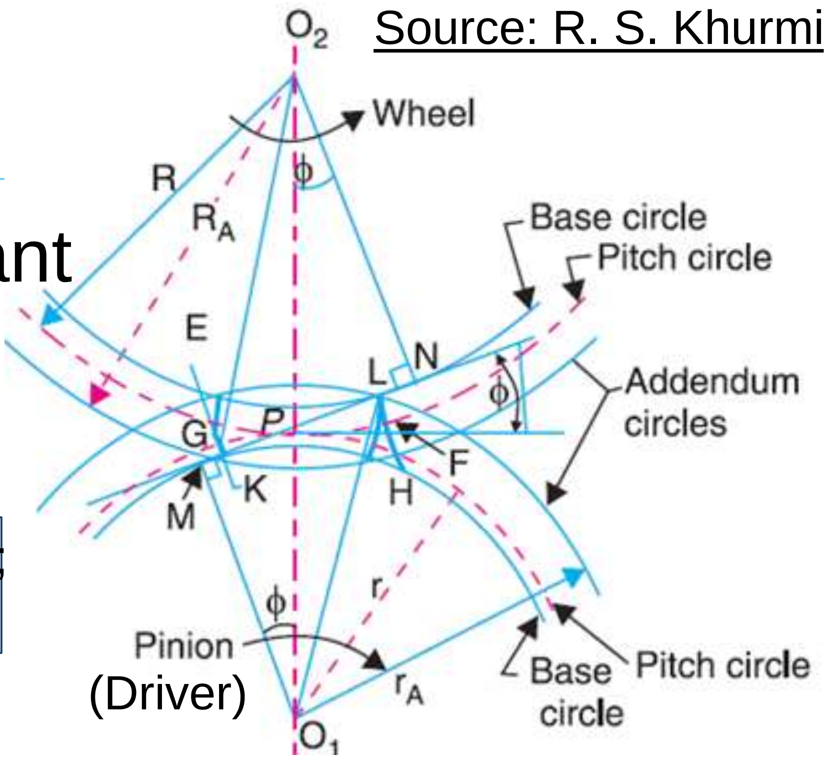
The profiles which give constant Velocity ratio & Positive drive is known as Conjugate profiles



KL – Length of path of contact
KP – Path of approach
PL – Path of recess

LAW OF GEARING

Source: R. S. Khurmi



Angular velocity Ratio is constant

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

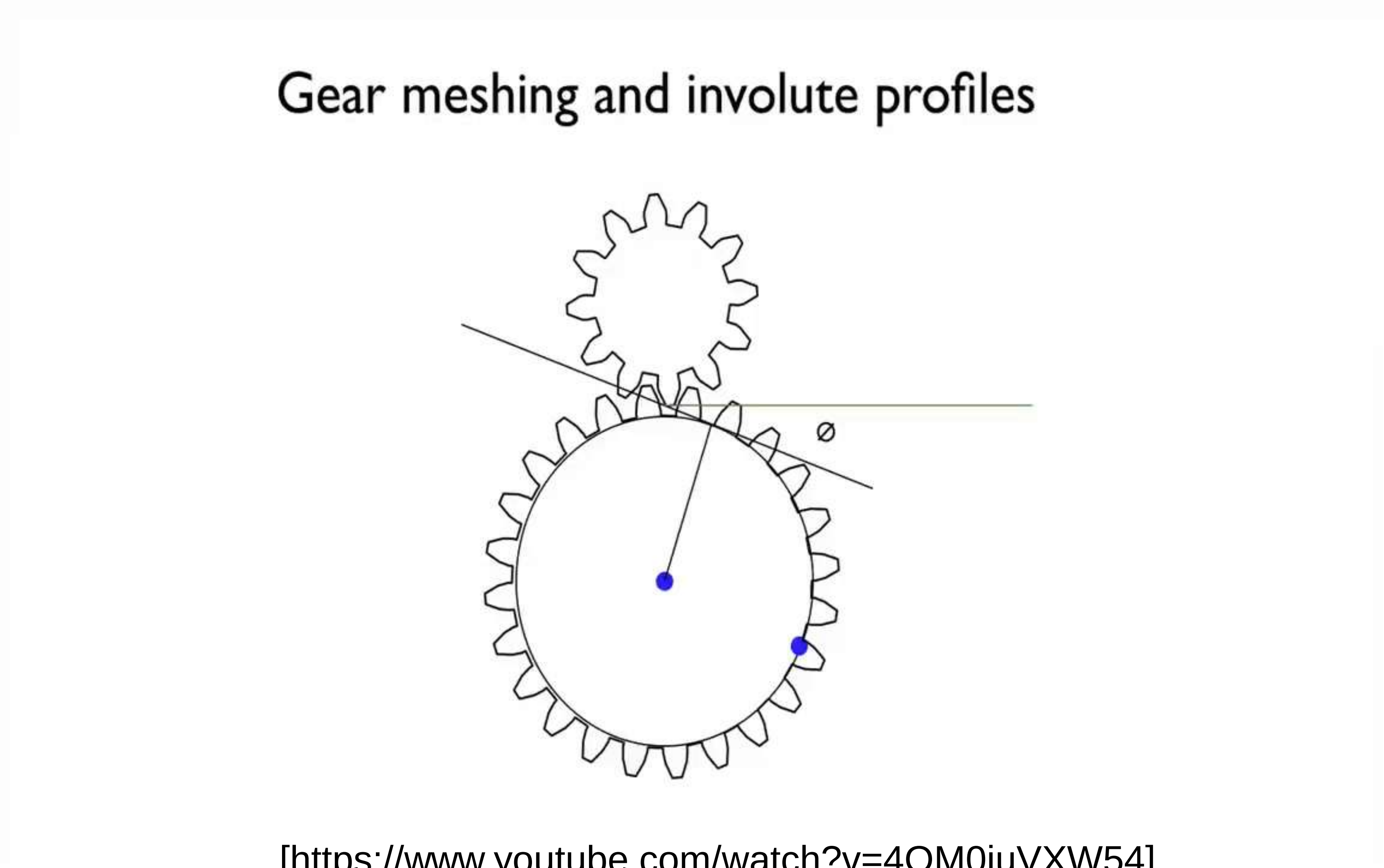
$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

D – pitch circle diameter ;
T – Number of Teeth

- In order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. i.e. the common normal at the point of contact between a pair of teeth must always pass through the pitch point.

- This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing

INVOLUTE TOOTH PROFILE



Gear meshing and involute profiles

[https://www.youtube.com/watch?v=4QM0juVXW54]

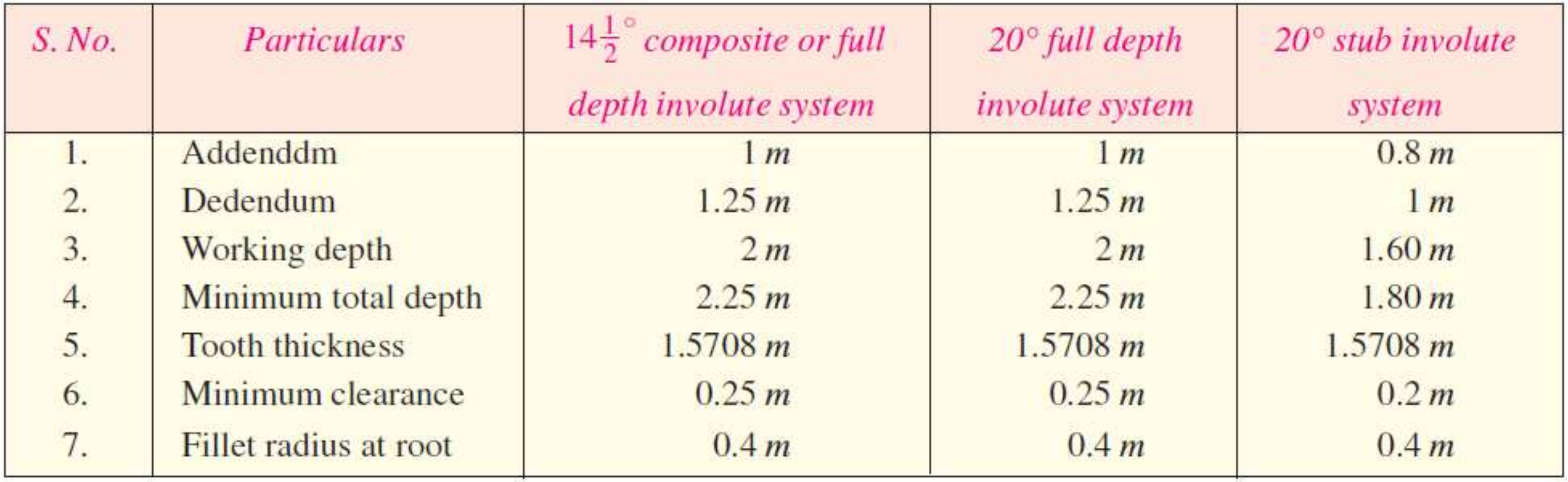
COMPARISON BETWEEN INVOLUTE AND CYCLOIDAL GEARS

S. No	Involute Gears	Cycloidal Gears
1.	Advantage of the involute gears is that the centre distance for a pair of involute gears can be varied within limits without affecting velocity ratio	Not true
2.	Pressure angle, from the start of the engagement of teeth to the end of the engagement, remains constant (smooth running and less wear of gears)	pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts decreasing and again becomes maximum at the end of engagement (less smooth running of gears)
3.	The face and flank of Involute teeth are generated by a single curve. Hence, easy to manufacture.	double curves (i.e. epi-cycloid and hypo-cycloid) . Hence, difficult to manufacture.

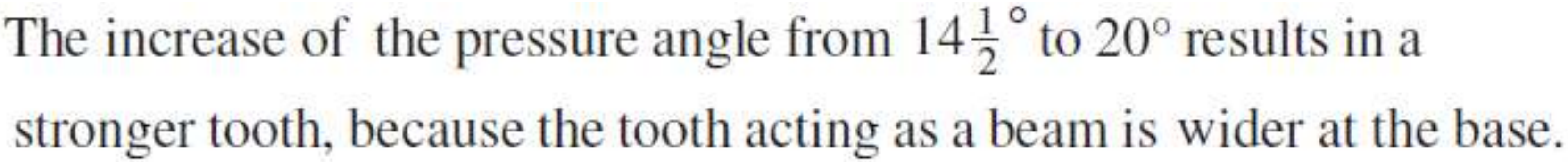
COMPARISON BETWEEN INVOLUTE AND CYCLOIDAL GEARS

S.No	Involute Gears	Cycloidal Gears
4.	Less strong	Cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears, for the same pitch
5.	Occurs	Interference does not occur
6.	Less weighted	outweighed

STANDARD PROPORTIONS OF GEAR SYSTEMS



S. No.	*Particulars*	$14\frac{1}{2}°$ *composite or full depth involute system*	*20° full depth involute system*	*20° stub involute system*
1.	Addenddm	1 m	1 m	0.8 m
2.	Dedendum	1.25 m	1.25 m	1 m
3.	Working depth	2 m	2 m	1.60 m
4.	Minimum total depth	2.25 m	2.25 m	1.80 m
5.	Tooth thickness	1.5708 m	1.5708 m	1.5708 m
6.	Minimum clearance	0.25 m	0.25 m	0.2 m
7.	Fillet radius at root	0.4 m	0.4 m	0.4 m



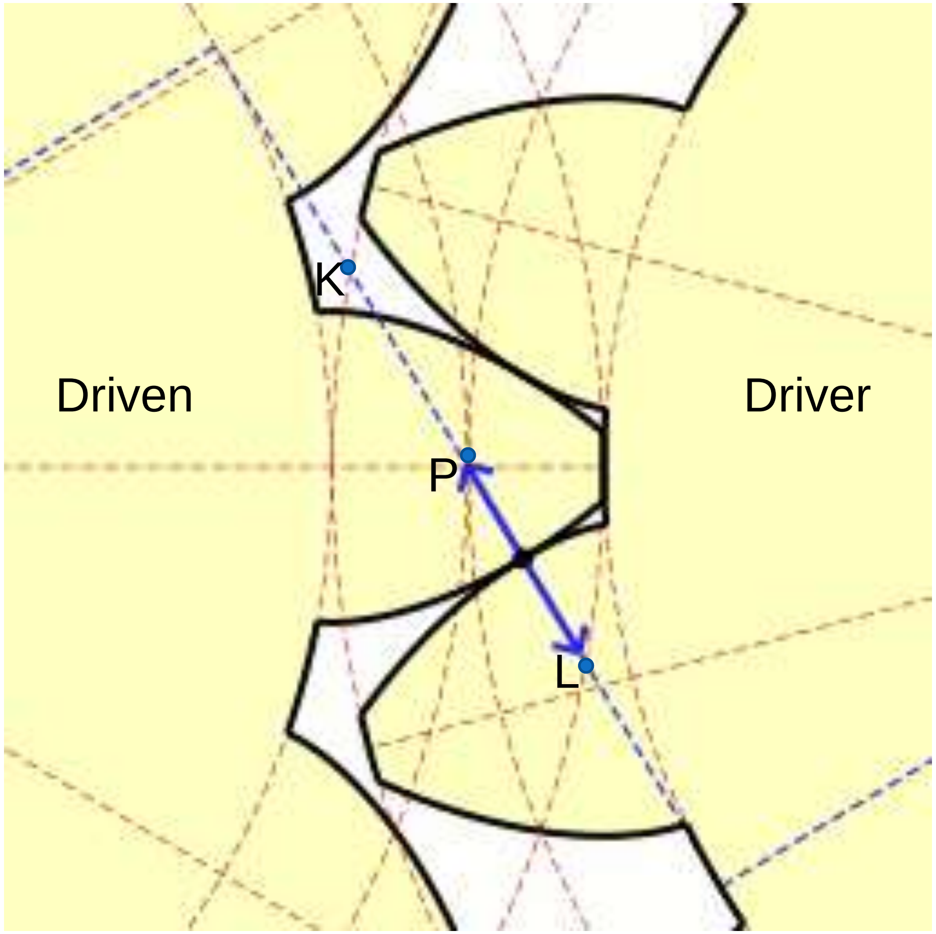
The increase of the pressure angle from $14\frac{1}{2}°$ to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base.

LECTURE 3

VELOCITY OF SLIDING PHENOMENA

LENGTH OF PATH OF CONTACT

KL – Length of path of contact
KP – Path of approach
PL – Path of recess



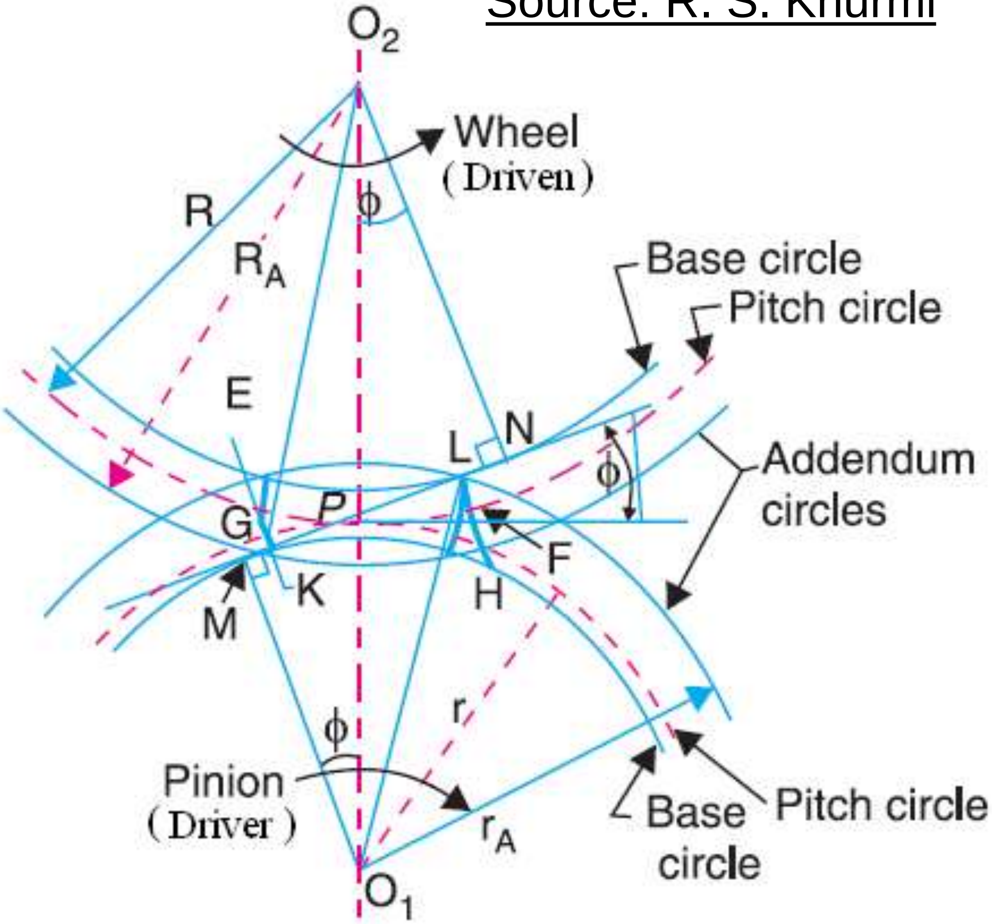
LENGTH OF PATH OF CONTACT

- contact between a pair of involute teeth begins at K ends at L

- MN is the common normal at the point of contact

- MN is also the common tangent to the base circles

KP – Path of approach
PL – Path of recess
KL – Length of path of contact



Source: R. S. Khurmi

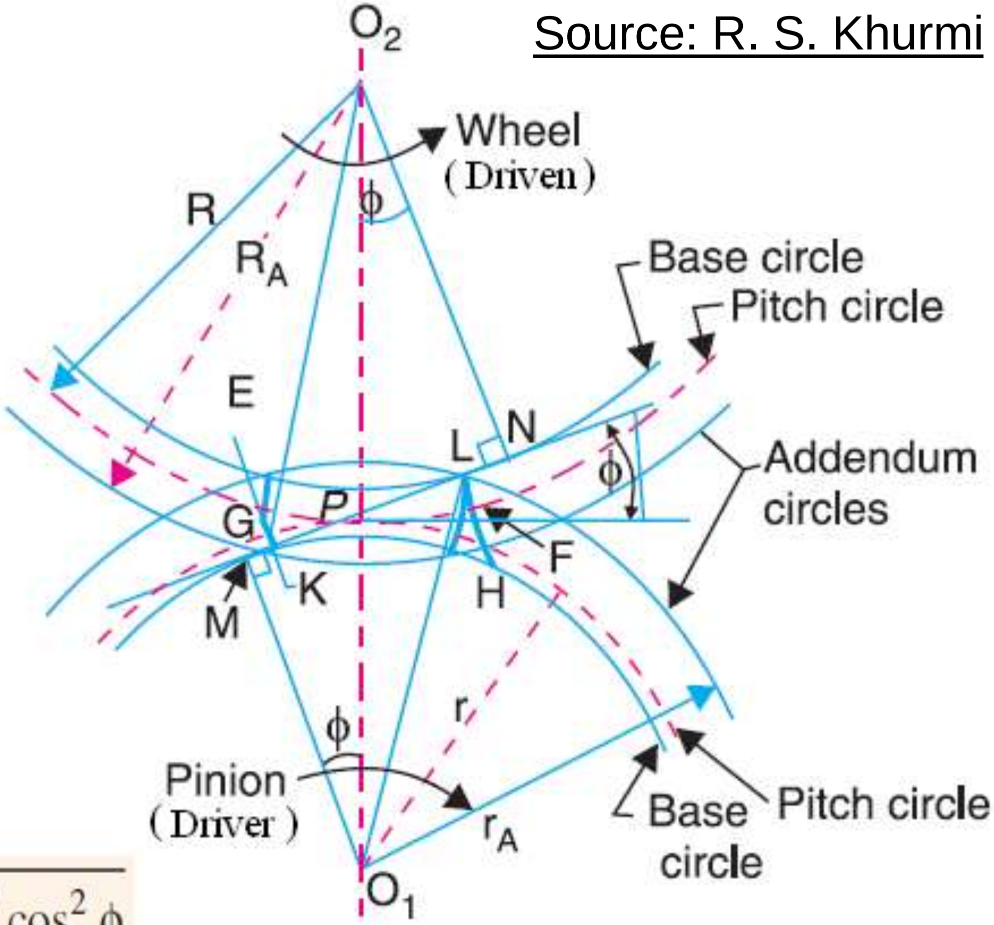
LENGTH OF PATH OF CONTACT

Let $r_A = O_1L =$ Radius of addendum circle of pinion,

$R_A = O_2K =$ Radius of addendum circle of wheel,

$r = O_1P =$ Radius of pitch circle of pinion, and

$R = O_2P =$ Radius of pitch circle of wheel.



Source: R. S. Khurmi

From Triangle O_1MP, $O_1M = r\cos\phi$

Triangle O_2NP, $O_2N = R\cos\phi$

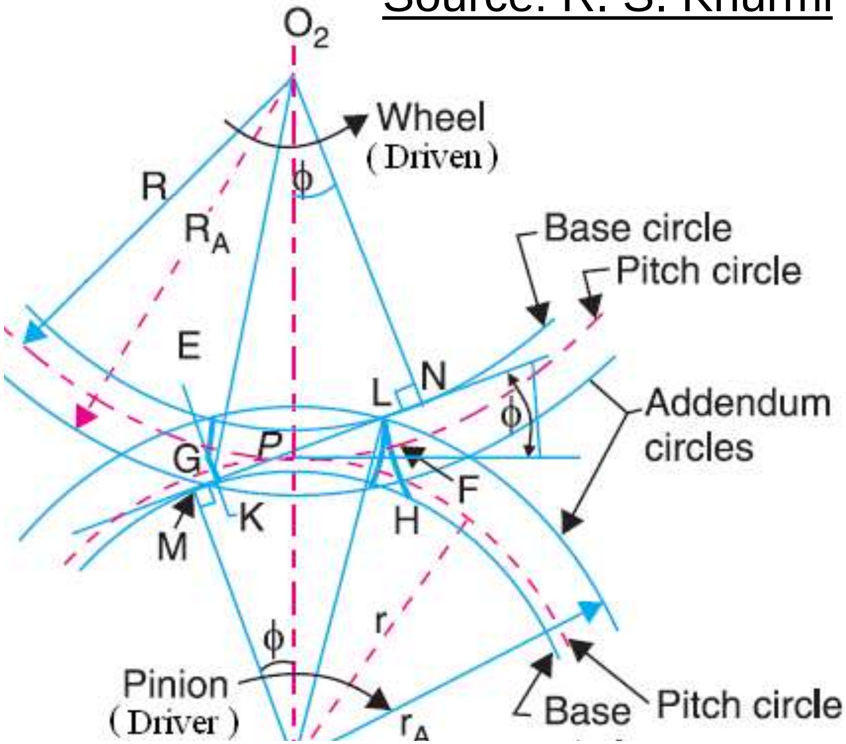
Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$PN = O_2P\sin\phi = R\sin\phi$$

LENGTH OF PATH OF CONTACT

Source: R. S. Khurmi



∴ Length of the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly from right angled triangle O_1ML,

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi} \quad \text{and}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

∴ Length of path of recess, PL

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

∴ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

LENGTH OF ARC OF CONTACT

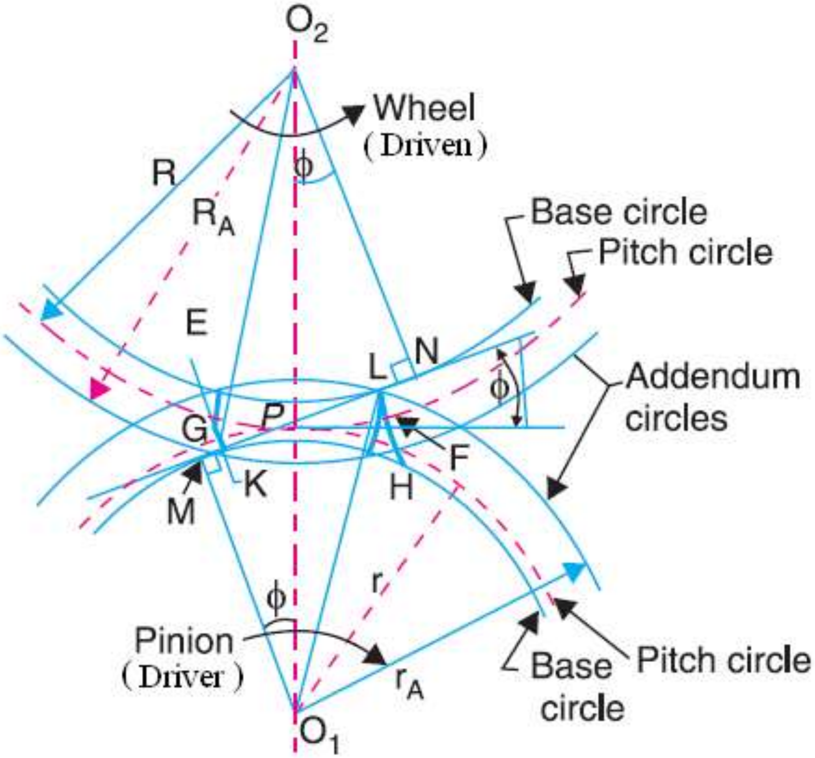
Source: R. S. Khurmi

- arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth

- Arc of contact is *EPF or GPH.*

- The arc *GP is known as arc of approach*

- The arc *PH is called arc of recess*



LENGTH OF ARC OF CONTACT

We know that the length of the arc of approach (arc *GP*)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

the length of the arc of recess (arc *PH*)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

Length of the arc of contact $= \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

CONTACT RATIO
(NUMBER OF PAIRS OF TEETH IN CONTACT)

- It is defined as the ratio of the length of the arc of contact to the circular pitch.

$$\text{Contact ratio} = \frac{\text{Length of the arc of contact}}{p_c}$$

$$p_c = \text{Circular pitch} = \pi m, \text{ and}$$

$$m = \text{Module.}$$

The contact ratio, usually, is not a whole number. For example, if the contact ratio is 1.6, it does not mean that there are 1.6 pairs of teeth in contact. It means that there are alternately one pair and two pairs of teeth in contact and on a time basis the average is 1.6

Larger the contact ratio, more quietly the gears will operate

LECTURE 4

EXPRESSIONS FOR ARC OF CONTACT AND PATH OF CONTACT

LIVE TO LEARN & LEARN TO SHARE

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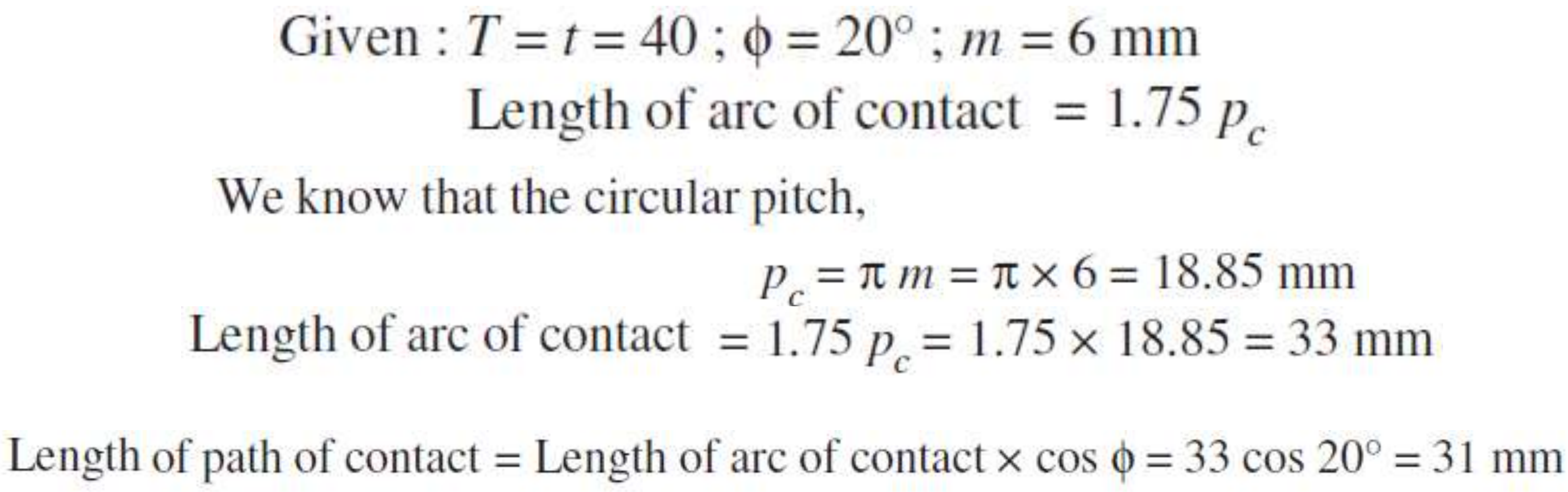
NUMERICAL EXAMPLE -1

The number of teeth on each of the two equal spur gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular pitch, find the addendum.



Given : $T = t = 40$; $\phi = 20°$; $m = 6$ mm

$$\text{Length of arc of contact} = 1.75\ p_c$$

We know that the circular pitch,

$$p_c = \pi\ m = \pi \times 6 = 18.85 \text{ mm}$$

$$\text{Length of arc of contact} = 1.75\ p_c = 1.75 \times 18.85 = 33 \text{ mm}$$

$$\text{Length of path of contact} = \text{Length of arc of contact} \times \cos\phi = 33 \cos 20° = 31 \text{ mm}$$

NUMERICAL EXAMPLE -1

We know that pitch circle radii of each wheel,

$$R = r = m.T / 2 = 6 \times 40/2 = 120 \text{ mm}$$

length of path of contact =31 $= \sqrt{(R_A)^2 - R^2 \cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$

$$= 2\left[\sqrt{(R_A)^2 - R^2 \cos^2\phi} - R\sin\phi\right] \quad ...(\because R = r, \text{ and } R_A = r_A)$$

$$R_A = 126.12 \text{ mm}$$

addendum of the wheel,

$= R_A - R = 126.12 - 120 = 6.12$ mm **Ans.**

NUMERICAL EXAMPLE -2

A pair of gears, having 40 and 20 teeth respectively, are rotating in mesh, the speed of the smaller being 2000 r.p.m.

Determine the velocity of sliding between the gear teeth faces at the point of engagement, at the pitch point, and at the point of disengagement if the smaller gear is the driver. Assume that the gear teeth are 20° involute form, addendum length is 5 mm and the module is 5 mm.

Also find the angle through which the pinion turns while any pairs of teeth are in contact.

NUMERICAL EXAMPLE -2

Solution. Given : $T = 40$; $t = 20$; $N_1 = 2000$ r.p.m. ; $\phi = 20°$; addendum = 5 mm ; $m = 5$ mm

We know that angular velocity of the smaller gear,

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 2000}{60} = 209.5 \text{ rad/s}$$

angular velocity of the larger gear, $\omega_2 = 104.75$ rad/s $\quad \ldots \left(\because \frac{\omega_2}{\omega_1} = \frac{t}{T} \right)$

Pitch circle radius of the smaller gear, $r = m.t / 2 = 5 \times 20/2 = 50$ mm

$R = m.T / 2 = 5 \times 40/2 = 100$ mm

Radius of addendum circle of smaller gear, $r_A = r + \text{Addendum} = 50 + 5 = 55$ mm

larger gear, $R_A = R + \text{Addendum} = 100 + 5 = 105$ mm

length of path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$= \sqrt{(105)^2 - (100)^2 \cos^2 20°} - 100 \sin 20°$$

$$= 12.65 \text{ mm}$$

length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(55)^2 - (50)^2 \cos^2 20°} - 50 \sin 20°$$

$$= 11.5 \text{ mm}$$

NUMERICAL EXAMPLE -2

Velocity of sliding at the point of engagement

We know that velocity of sliding at the point of engagement K,

$$v_{SK} = (\omega_1 + \omega_2)\ KP = (209.5 + 104.75)\ 12.65 = 3975 \text{ mm/s}$$ **Ans.**

Velocity of sliding at the pitch point

Since the velocity of sliding is proportional to the distance of the contact point from the pitch point, therefore the velocity of sliding at the pitch point is zero. **Ans.**

Velocity of sliding at the point of disengagement

We know that velocity of sliding at the point of disengagement L,

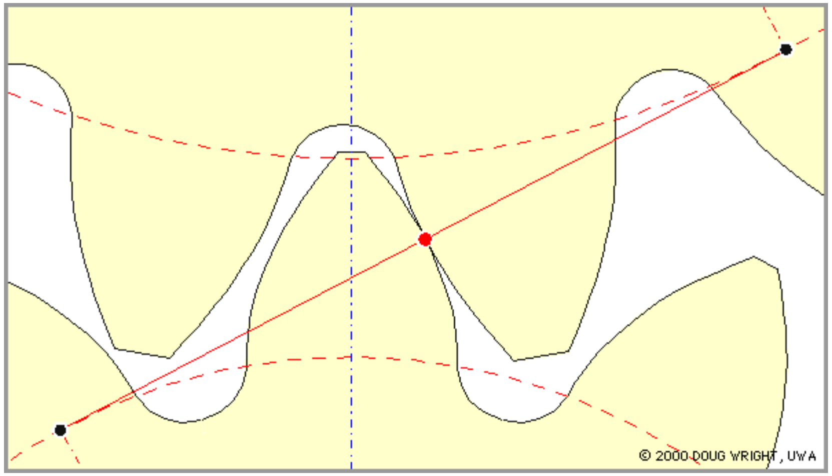
$$v_{SL} = (\omega_1 + \omega_2)\ PL = (209.5 + 104.75)\ 11.5 = 3614 \text{ mm/s}$$ **Ans.**

Angle through which the pinion turns

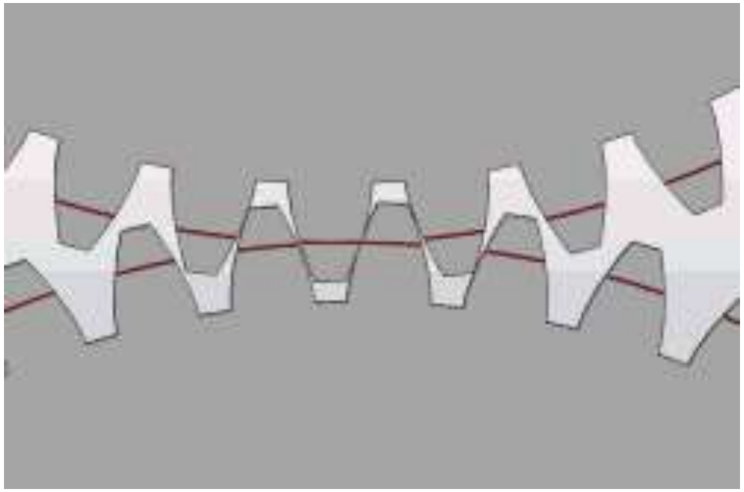
$$= \text{Length of arc of contact} \times \frac{360°}{\text{Circumference of pinion}}$$

$$= 25.7 \times \frac{360°}{314.2} = 29.45°$$ **Ans.**

INTERFERENCE AND UNDERCUTTING

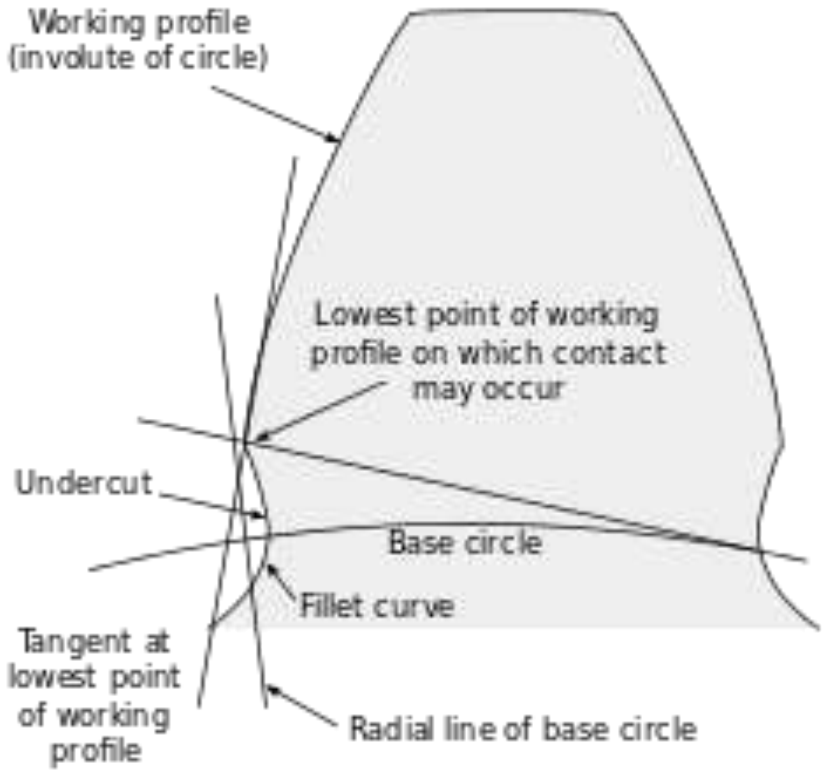


Full fit involute (Conjugate Profile)



Full fit involute (Conjugate Profile)

Source: R. S. Khurmi



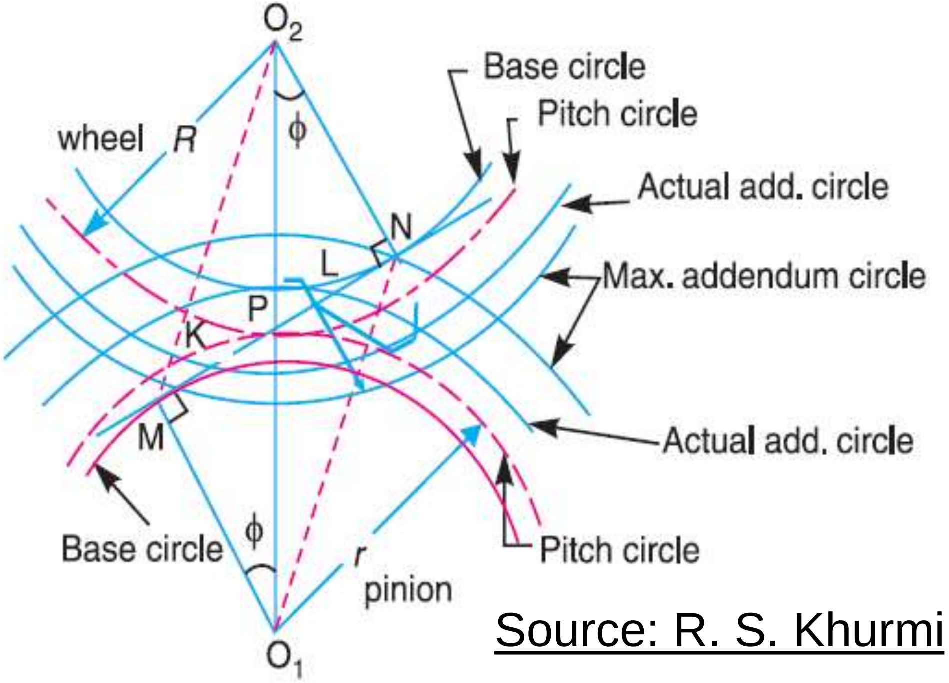
INTERFERENCE AND UNDERCUTTING

- if the radius of the addendum circle of pinion is increased to O1N, the point of contact L will move from L to N.

- When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel

The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as interference

The phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.



Source: R. S. Khurmi

Interference in involute gears.

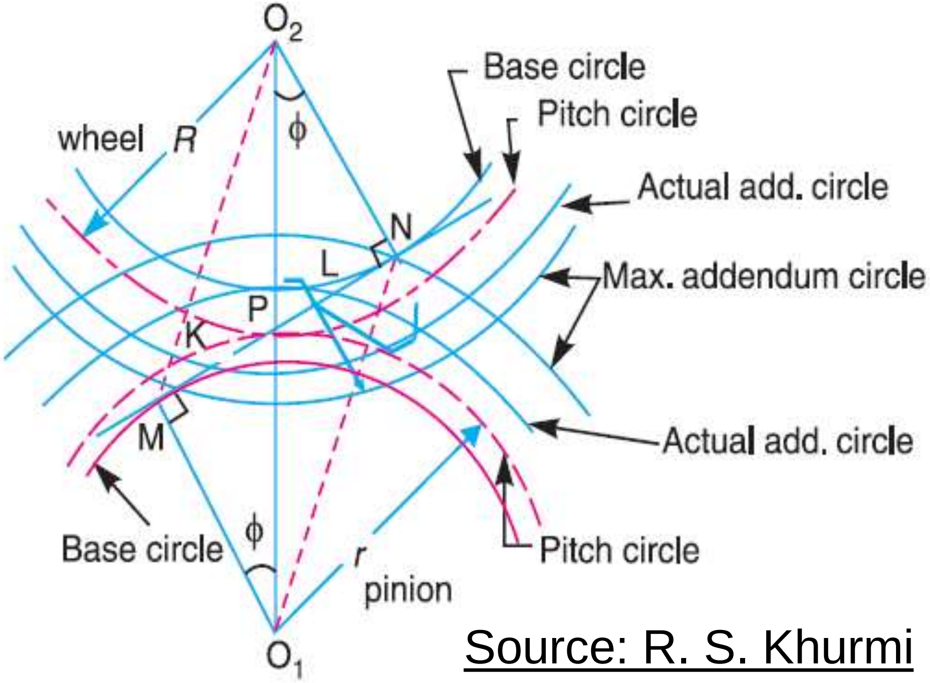
INTERFERENCE AND UNDERCUTTING

Similarly, if the radius of the addendum circle of the wheel increases beyond O2M, the tip of tooth on wheel will cause interference with the tooth on pinion.

The points M and N are called interference points.

Obviously, interference may be avoided if the path of contact does not extend beyond interference points.

The limiting value of the radius of the addendum circle of the pinion is O1N and of the wheel is O2M.



Source: R. S. Khurmi

Interference in involute gears.

INTERFERENCE AND UNDERCUTTING

To avoid interference: Maximum length of path of approach, $MP = r \sin \phi$

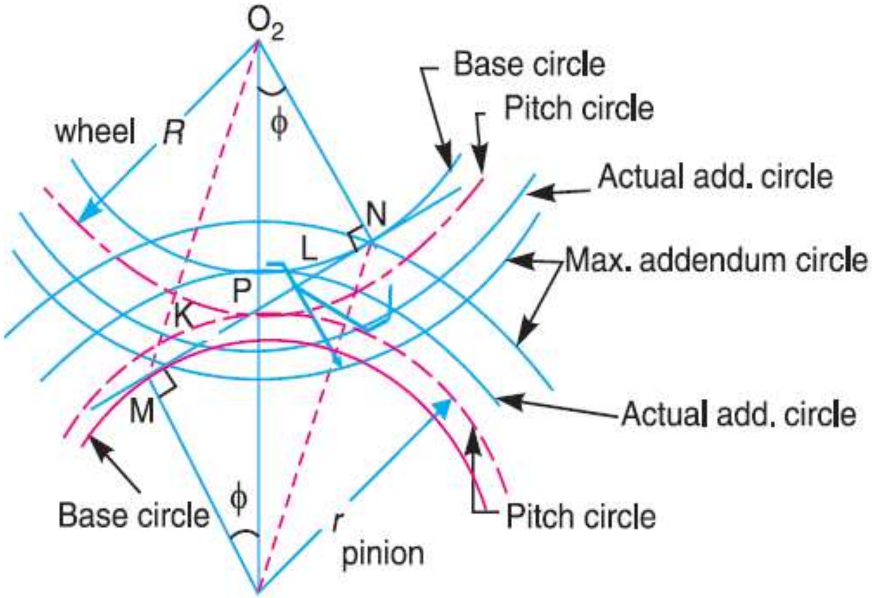
maximum length of path of recess, $PN = R \sin \phi$

$\therefore$ Maximum length of path of contact,

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

maximum length of arc of contact =

$$\frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$



Source: R. S. Khurmi

Interference in involute gears.

NUMERICAL EXAMPLE -3

Two mating gears have 20 and 40 involute teeth of module 10 mm and 20° pressure angle. The addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point has half the maximum possible length. Determine the addendum height for each gear wheel, length of the path of contact, arc of contact and contact ratio.

Solution. Given : $t = 20$; $T = 40$; $m = 10$ mm ; $\phi = 20°$

$r = 100$ mm

$R = 200$ mm

Find pitch circle radius using $r = m.t / 2$

the line of contact on each side of the pitch point (*i.e.* the path of approach and the path of recess) has half the maximum possible length, therefore

Path of approach, $KP = \frac{1}{2} MP$

$$\sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi = \frac{r.\sin \phi}{2} \Rightarrow R_A = 206.5 \text{ mm}$$

NUMERICAL EXAMPLE -3

$\therefore$ Addendum height for larger gear wheel

$$= R_A - R = 206.5 - 200 = 6.5 \text{ mm } \textbf{Ans.}$$

Now path of recess, $PL = \frac{1}{2} PN$

$$\sqrt{(r_A)^2 - r^2 \cos^2\phi} - r \sin\phi = \frac{R.\sin\phi}{2} \Rightarrow r_A = 116.2 \text{ mm}$$

Addendum height for smaller gear wheel $= r_A - r = 6.2$ mm **Ans.**

Length of the path of contact $= KP + PL = \frac{1}{2}MP + \frac{1}{2}PN = \frac{(r+R)\sin\phi}{2} = 51.3$ mm **Ans.**

Length of the arc of contact $= \frac{\text{Length of the path of contact}}{\cos\phi} = \frac{51.3}{\cos 20°} = 54.6$ mm **Ans.**

Contact ratio

circular pitch, $P_c = \pi m = \pi \times 10 = 31.42$ mm

Contact ratio $= \frac{\text{Length of the path of contact}}{p_c} = 1.74$ **Ans.**

MINIMUM NUMBER OF TEETH ON THE PINION IN ORDER TO AVOID INTERFERENCE

t = Number of teeth on the pinion,,

T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $m.t / 2$

G = Gear ratio = $T / t = R / r$

ϕ = Pressure angle or angle of obliquity.

From triangle O_1NP,

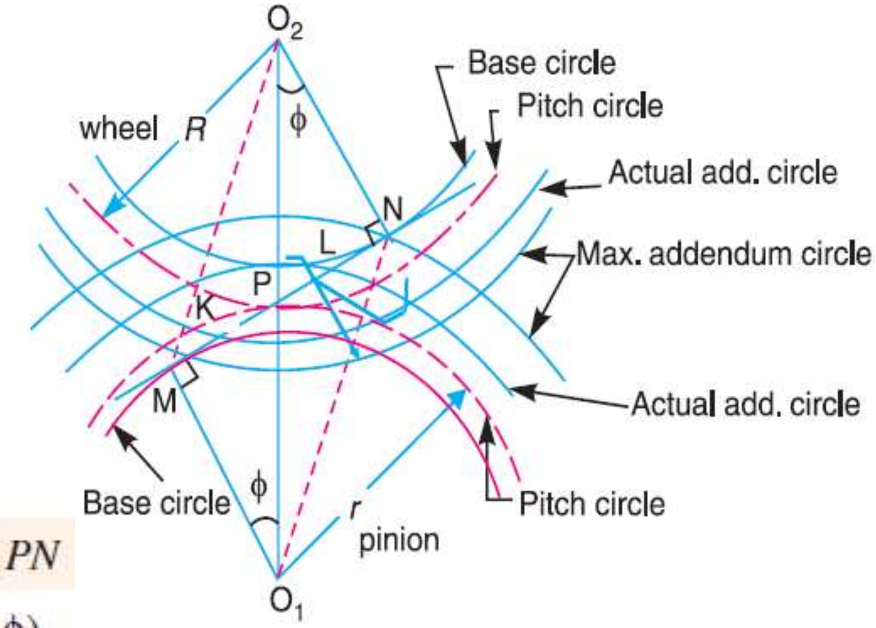
$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

$$= r^2 + R^2 \sin^2\phi + 2r.R \sin^2\phi$$

$$= r^2 \left[1 + \frac{R^2 \sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right]$$

$$= r^2 \left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$



Interference in involute gears.

Source: R. S. Khurmi

MINIMUM NUMBER OF TEETH ON THE PINION IN ORDER TO AVOID INTERFERENCE

$$(O_1N)^2 = r^2\left[1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi\right]$$

∴ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi} = \frac{m.t}{2}\sqrt{1+\frac{T}{t}\left[\frac{T}{t}+2\right]\sin^2\phi}$$

Let $A_P.m$ = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

addendum of the pinion $= O_1N - O_1P$

$$A_P.m = \frac{m.t}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - \frac{m.t}{2}$$

$$\ldots(\because O_1P = r = m.t/2)$$

$$= \frac{m.t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

$$A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

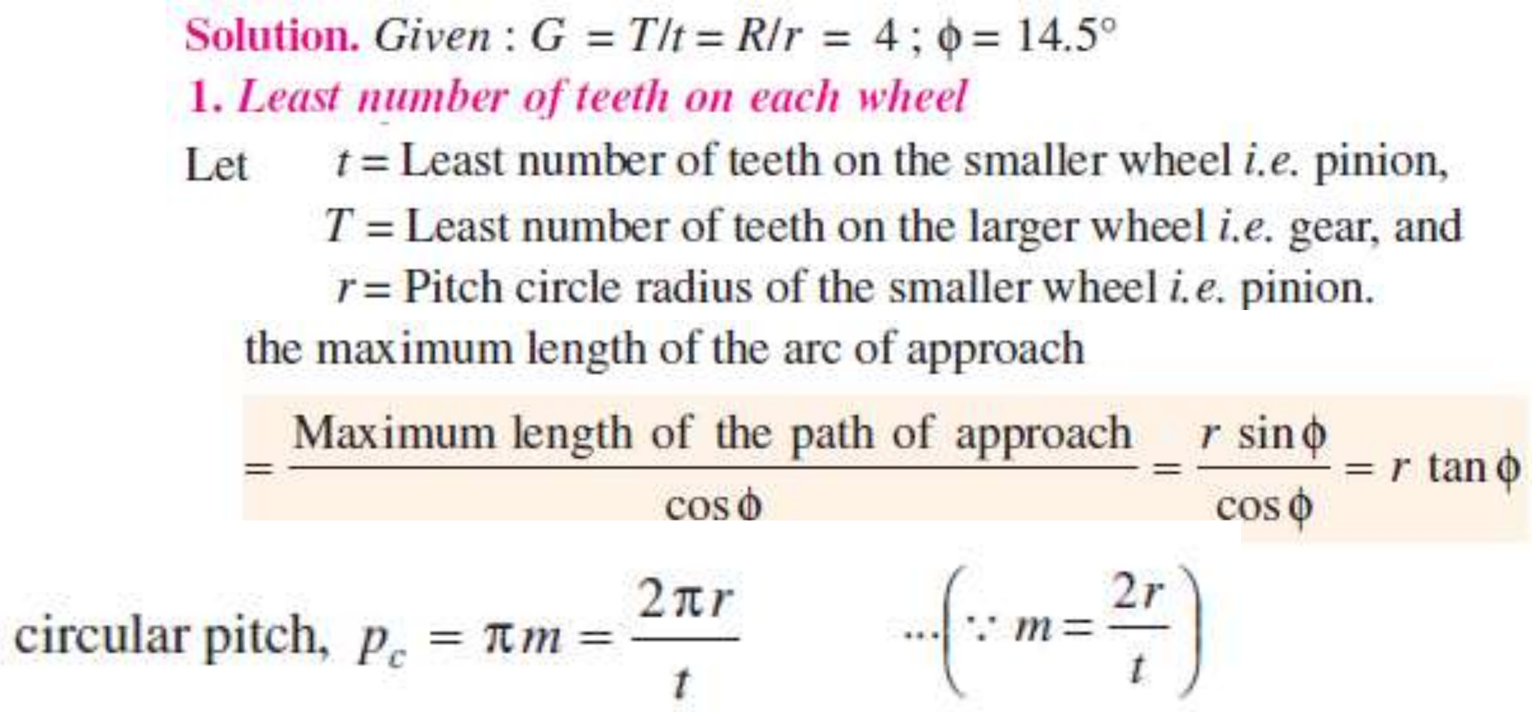
$$t = \frac{2A_P}{\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1} = \frac{2A_P}{\sqrt{1+G(G+2)\sin^2\phi} - 1}$$

MINIMUM NUMBER OF TEETH ON THE PINION IN ORDER TO AVOID INTERFERENCE

S. No.	*System of gear teeth*	*Minimum number of teeth on the pinion*
1.	$14\frac{1}{2}^\circ$ Composite	12
2.	$14\frac{1}{2}^\circ$ Full depth involute	32
3.	20° Full depth involute	18
4.	20° Stub involute	14

NUMERICAL EXAMPLE -4

A pair of spur gears with involute teeth is to give a gear ratio of 4 : 1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pressure is 14.5°. Find : 1. the least number of teeth that can be used on each wheel, and 2. the addendum of the wheel in terms of the circular pitch ?



Solution. *Given* : $G = T/t = R/r = 4$; $\phi = 14.5°$

1. ***Least number of teeth on each wheel***

Let t = Least number of teeth on the smaller wheel *i.e.* pinion,

T = Least number of teeth on the larger wheel *i.e.* gear, and

r = Pitch circle radius of the smaller wheel *i.e.* pinion.

the maximum length of the arc of approach

$$= \frac{\text{Maximum length of the path of approach}}{\cos\phi} = \frac{r\sin\phi}{\cos\phi} = r\tan\phi$$

$$\text{circular pitch, } p_c = \pi m = \frac{2\pi r}{t} \qquad \ldots\left(\because m = \frac{2r}{t}\right)$$

NUMERICAL EXAMPLE -4

Since the arc of approach is not to be less than the circular pitch, therefore

$$r\tan\phi = \frac{2\pi r}{t} \quad \text{or} \quad t = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 14.5°} = 24.3 \text{ say } 25 \textbf{ Ans.}$$

$$T = G.t = 4 \times 25 = 100 \textbf{ Ans.} \qquad ...(\because G = T/t)$$

2. ***Addendum of the wheel***

addendum of the wheel

$$= \frac{m.T}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$$

$$= \frac{m\times 100}{2}\left[\sqrt{1+\frac{25}{100}\left(\frac{25}{100}+2\right)\sin^2 14.5°} - 1\right]$$

$$= 0.85\ m = 0.85 \times p_c/\pi = 0.27\ p_c \textbf{ Ans.}$$

$$...(\because m = p_c/\pi)$$

GEAR TRAINS

Two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels.

Types of Gear Trains

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train

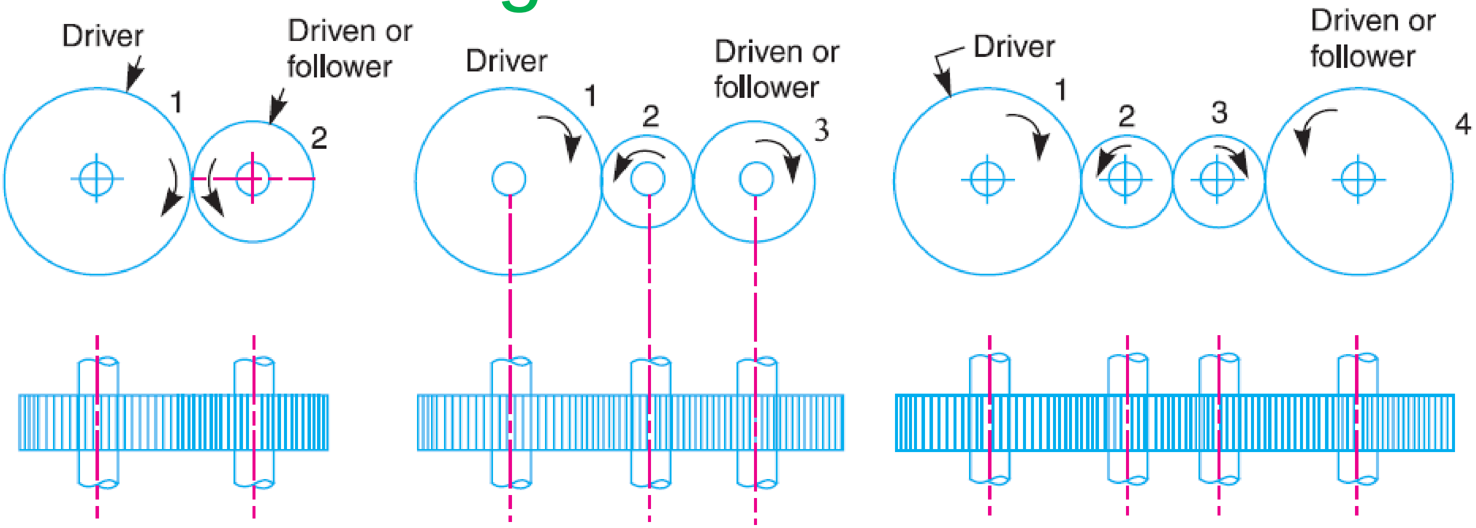
LECTURE 5

GEAR TRAIN

DEPARTMENT OF MECHANICAL ENGINEERING

SIMPLE GEAR TRAIN

One gear on each shaft



(*a*) (*b*) (*c*)

Source: R. S. Khurmi

If the distance between the two gears is large, intermediate gears employed. If the number of intermediate gears are odd, the motion of both the Gears is like. If Even - unlike direction

N_1 = Speed of gear 1(or driver) in r.p.m., N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,

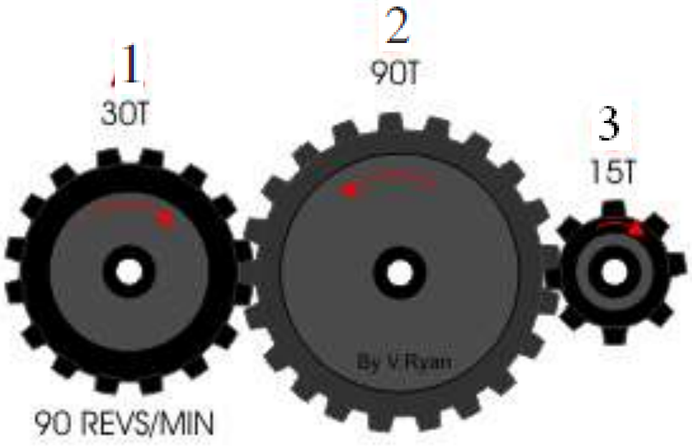
T_1 = Number of teeth on gear 1, and T_2 = Number of teeth on gear 2.

The speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower.

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

SIMPLE GEAR TRAIN

The ratio of the speed of the driven to the speed of the driver is known as train value of the gear train



$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

speed ratio for gear 1 & 2 $\frac{N_1}{N_2} = \frac{T_2}{T_1}$

speed ratio for gear 2 & 3 $\frac{N_2}{N_3} = \frac{T_3}{T_2}$

The speed ratio of the gear train is obtained by multiplying the above two equations

$$\frac{N_1}{\cancel{N_2}} \times \frac{\cancel{N_2}}{N_3} = \frac{\cancel{T_2}}{T_1} \times \frac{T_3}{\cancel{T_2}} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

SIMPLE GEAR TRAIN

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

The intermediate gears are called idle gears, as they do not effect the speed ratio or train value of the system.

The idle gears are used

- To connect gears where a large centre distance is required, and
- To obtain the desired direction of motion of the driven gear (i.e. clockwise or anticlockwise).

COMPOUND GEAR TRAIN

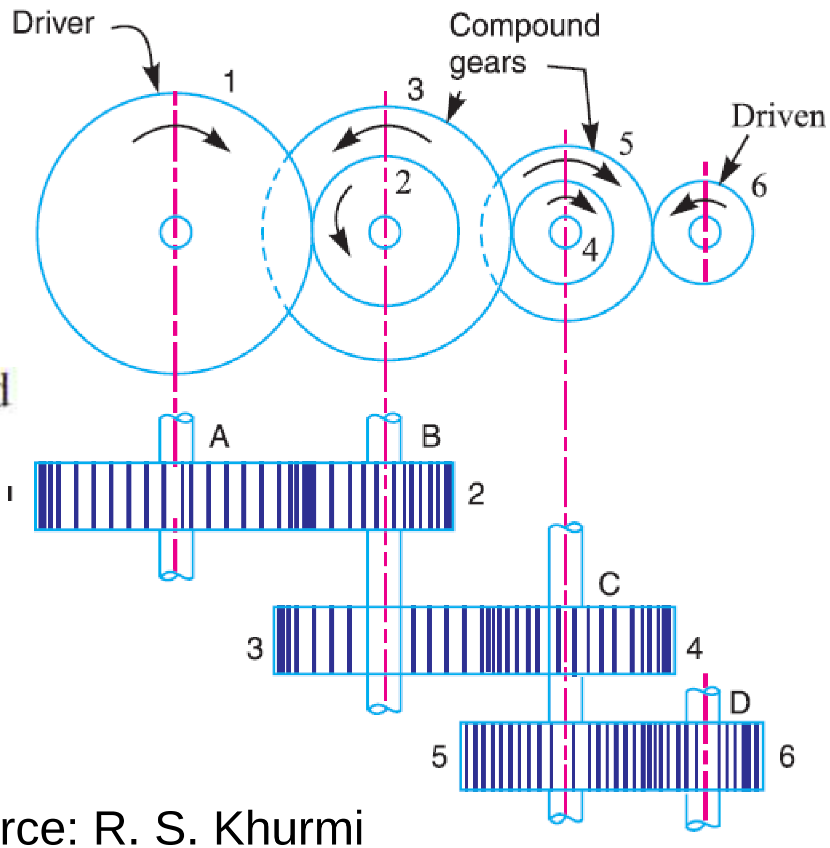
$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and
$T_2, T_3 ..., T_6$ = Number of teeth on respective gears.

speed ratio 1 and 2 $\frac{N_1}{N_2} = \frac{T_2}{T_1}$

speed ratio 3 and 4, $\frac{N_3}{N_4} = \frac{T_4}{T_3}$

gears 5 and 6, speed ratio $\frac{N_5}{N_6} = \frac{T_6}{T_5}$

More than one gear on a shaft



Source: R. S. Khurmi

COMPOUND GEAR TRAIN

The speed ratio of compound gear train is obtained by

$$\frac{N_1}{\cancel{N_2}} \times \frac{\cancel{N_3}}{\cancel{N_4}} \times \frac{\cancel{N_5}}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

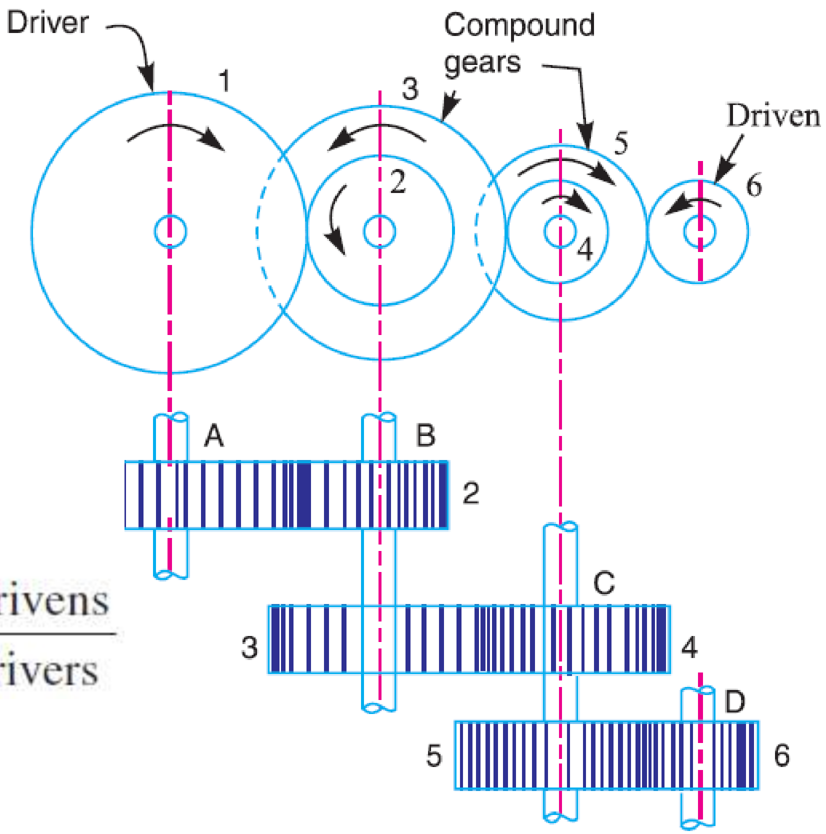
$$\text{or} \qquad \frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$



Source: R. S. Khurmi

COMPOUND GEAR TRAIN

Advantage of Compound Gear Train over simple gear train:

- a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.
- If a simple gear train is used to give a large speed reduction, the last gear has to be very large.

Design of Spur Gears

x = Distance between the centres of two shafts,

N_1 = Speed of the driver,

T_1 = Number of teeth on the driver,

d_1 = Pitch circle diameter of the driver,

N_2, T_2 and d_2 = Corresponding values for the driven

p_c = Circular pitch.

$$x = \frac{d_1 + d_2}{2}$$

speed ratio

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$$

NUMERICAL EXAMPLE -1

Two parallel shafts, about 600 mm apart are to be connected by spur gears. One shaft is to run at 360 r.p.m. and the other at 120 r.p.m. Design the gears, if the circular pitch is to be 25 mm.

Given : $x = 600$ mm ; $N_1 = 360$ r.p.m. ; $N_2 = 120$ r.p.m. ; $p_c = 25$ mm

d_1 = Pitch circle diameter of the first gear, and

d_2 = Pitch circle diameter of the second gear. $T_2 = 3\,T_1 = 114$ (∵ Speed ratio =3)

speed ratio, $\dfrac{N_1}{N_2} = \dfrac{d_2}{d_1} = \dfrac{360}{120} = 3$ or $d_2 = 3d_1$ ...(*i*)

$$x = 600 = \frac{1}{2}(d_1 + d_2) \quad \text{...}(ii)$$

From (*i*) and (*ii*), $d_1 = 300$ mm, and $d_2 = 900$ mm

Number of teeth on the first gear,

$$T_1 = \frac{\pi d_2}{p_c} = \frac{\pi \times 300}{25} = 37.7 \quad = 38$$

Now the exact pitch circle diameter of the first gear,

$$d_1' = \frac{T_1 \times p_c}{\pi} = \frac{38 \times 25}{\pi} = 302.36 \text{ mm}$$

the exact pitch circle diameter of the second gear,

$$d_2' = \frac{T_2 \times p_c}{\pi} = \frac{114 \times 25}{\pi} = 907.1 \text{ mm}$$

Exact distance between the two shafts,

$$x' = \frac{d_1' + d_2'}{2} = 604.73 \text{ mm}$$

LECTURE 6

SIMPLE AND REVERTED WHEEL TRAIN

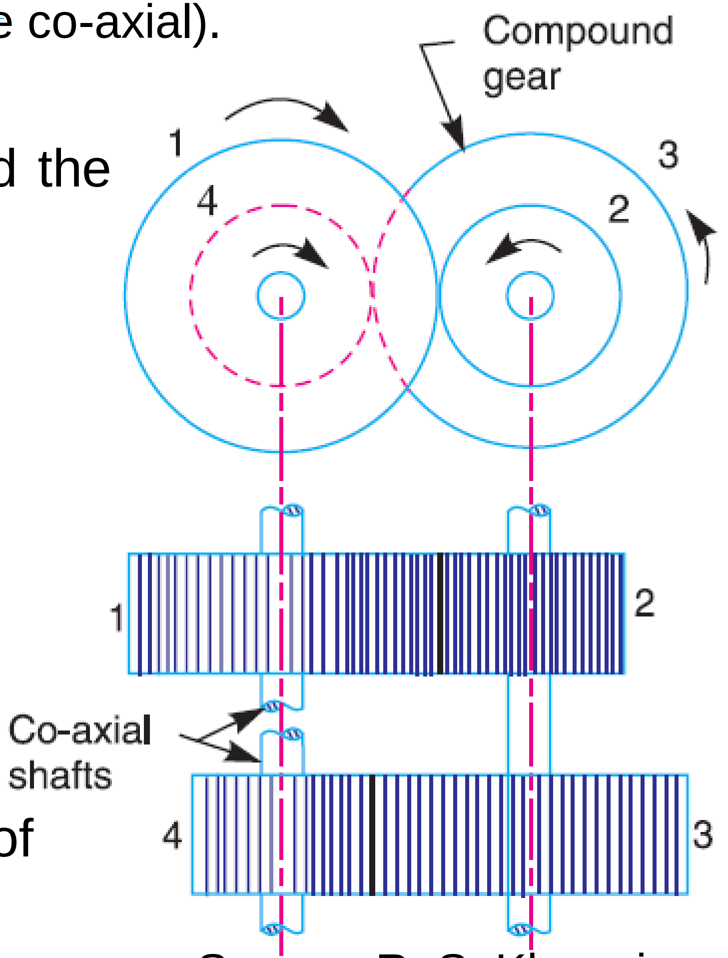
MRCET CAMPUS

UGC Autonomous

DEPARTMENT OF MECHANICAL ENGINEERING

REVERTED GEAR TRAIN

Used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).



Source: R. S. Khurmi

The axes of the first gear (i.e. first driver) and the last gear (i.e.Last driven are co-axial)

Let T_1 = Number of teeth on gear 1,

r_1 = Pitch circle radius of gear 1, and

N_1 = Speed of gear 1 in r.p.m.

Similarly,

T_2, T_3, T_4 = Number of teeth on respective gears,

r_2, r_3, r_4 = Pitch circle radii of respective gears, and

N_2, N_3, N_4 = Speed of respective gears in r.p.m.

The distance between the centres of the shafts of gears 1 and 2 and the gears 3 and 4 are same

$$r_1 + r_2 = r_3 + r_4 \quad \ldots(i)$$

REVERTED GEAR TRAIN

$$T_1 + T_2 = T_3 + T_4 \qquad ...(ii)$$

We know that circular pitch,

$$p_c = \frac{2\pi r}{T} = \pi m \quad \text{or} \quad r = \frac{mT}{2},$$

$$r_1 = \frac{mT_1}{2} \; ; \; r_2 = \frac{mT_2}{2} \; ; \; r_3 = \frac{mT_3}{2} \; ; \; r_4 = \frac{mT_4}{2}$$

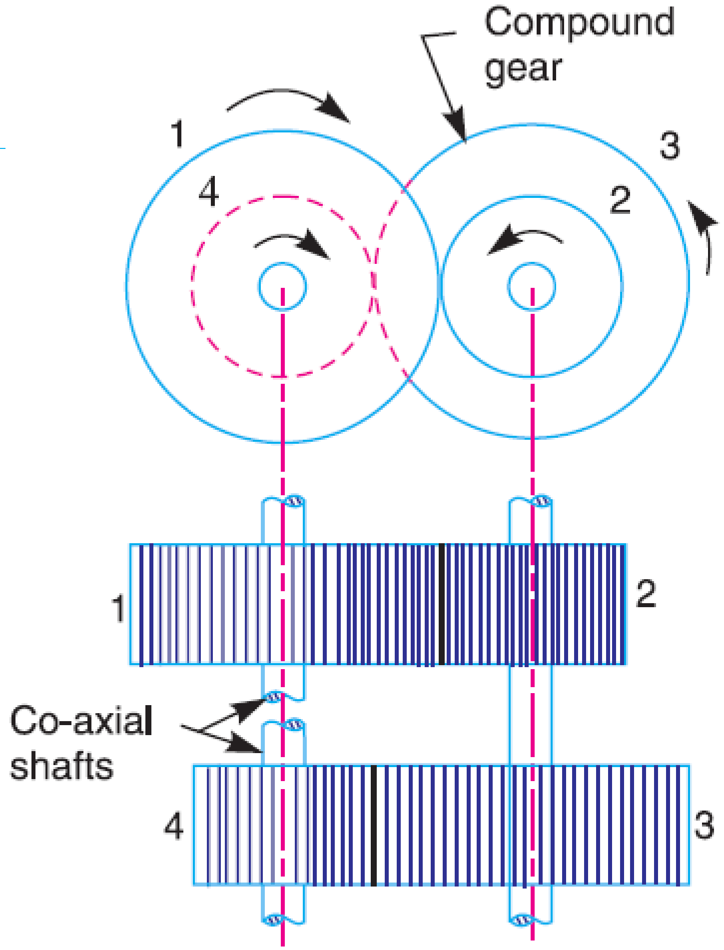
from equation $r_1 + r_2 = r_3 + r_4$

$$\frac{mT_1}{2} + \frac{mT_2}{2} = \frac{mT_3}{2} + \frac{mT_4}{2}$$

$$T_1 + T_2 = T_3 + T_4$$

$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \qquad ... (iii)$$

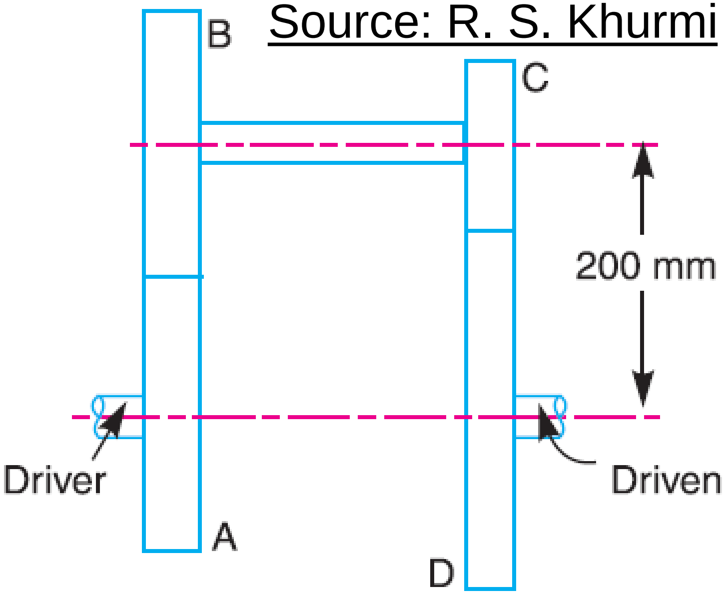


Source: R. S. Khurmi

From equations (i), (ii) and (iii), we can determine the number of teeth on each gear for the given centre distance, speed ratio and module only when the number of teeth on one gear is chosen arbitrarily

NUMERICAL EXAMPLE-2

The speed ratio of the reverted gear train, as shown in the figure is to be 12. The module of gears A and B is 3.125 mm and of gears C and D is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.



Source: R. S. Khurmi

Solution. Given : Speed ratio, $N_A/N_D = 12$;

$m_A = m_B = 3.125$ mm ; $m_C = m_D = 2.5$ mm

Let N_A = Speed of gear A,

T_A = Number of teeth on gear A,

r_A = Pitch circle radius of gear A,

N_B, N_C, N_D = Speed of respective gears,

T_B, T_C, T_D = Number of teeth on respective gears, and

r_B, r_C, r_D = Pitch circle radii of respective gears.

NUMERICAL EXAMPLE-2

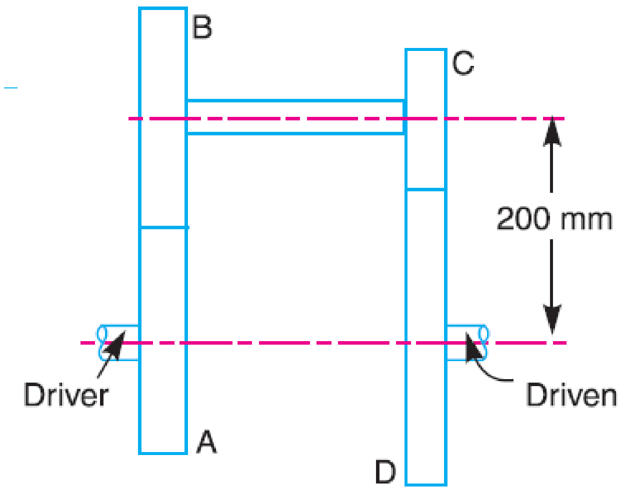
We know that speed ratio $= \dfrac{\text{Speed of first driver}}{\text{Speed of last driven}} = \dfrac{N_A}{N_D} = 12$

Also $\dfrac{N_A}{N_D} = \dfrac{N_A}{N_B} \times \dfrac{N_C}{N_D}$ ...($N_B = N_C$, being on the same shaft)

For $\dfrac{N_A}{N_B}$ and $\dfrac{N_C}{N_D}$ to be same, each speed ratio should be $\sqrt{12}$ so that

$$\frac{N_A}{N_D} = \frac{N_A}{N_B} \times \frac{N_C}{N_D} = \sqrt{12} \times \sqrt{12} = 12$$

therefore $\dfrac{N_A}{N_B} = \dfrac{N_C}{N_D} = \sqrt{12} = 3.464 \Rightarrow \dfrac{T_B}{T} = \dfrac{T_D}{T} = 3.464$ ...(*i*)



Source: R. S. Khurmi

We know that the distance between the shafts

$$x = r_A + r_B = r_C + r_D = 200 \text{ mm}$$

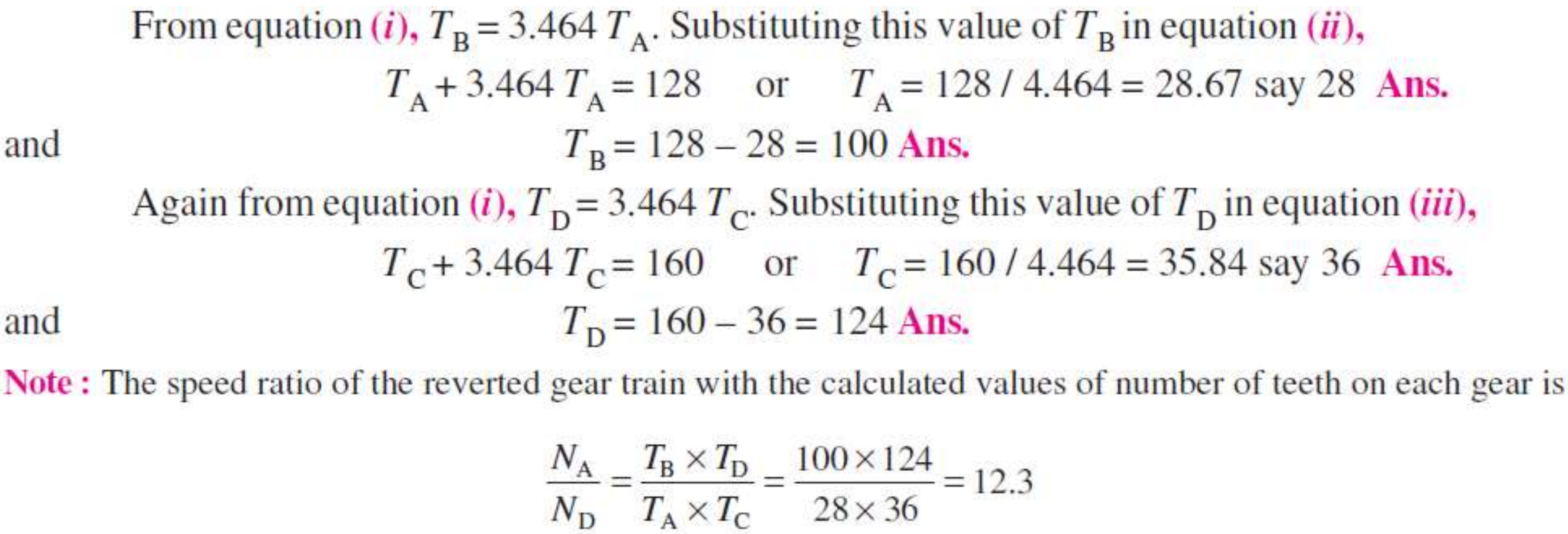
$$\frac{m_A \cdot T_A}{2} + \frac{m_B \cdot T_B}{2} = \frac{m_C \cdot T_C}{2} + \frac{m_D \cdot T_D}{2} = 200 \quad \ldots \left(\because r = \frac{m.T}{2}\right)$$

$3.125\,(T_A + T_B) = 2.5\,(T_C + T_D) = 400$ ...($\because m_A = m_B$, and $m_C = m_D$)

$\therefore$ $T_A + T_B = 400 / 3.125 = 128$ ...(*ii*)

$T_C + T_D = 400 / 2.5 = 160$ ...(*iii*)

REVERTED GEAR TRAIN



From equation *(i)*, $T_B = 3.464\ T_A$. Substituting this value of T_B in equation *(ii)*,

$$T_A + 3.464\ T_A = 128 \quad \text{or} \quad T_A = 128 / 4.464 = 28.67 \text{ say } 28 \quad \textbf{Ans.}$$

and

$$T_B = 128 - 28 = 100 \ \textbf{Ans.}$$

Again from equation *(i)*, $T_D = 3.464\ T_C$. Substituting this value of T_D in equation *(iii)*,

$$T_C + 3.464\ T_C = 160 \quad \text{or} \quad T_C = 160 / 4.464 = 35.84 \text{ say } 36 \quad \textbf{Ans.}$$

and

$$T_D = 160 - 36 = 124 \ \textbf{Ans.}$$

Note : The speed ratio of the reverted gear train with the calculated values of number of teeth on each gear is

$$\frac{N_A}{N_D} = \frac{T_B \times T_D}{T_A \times T_C} = \frac{100 \times 124}{28 \times 36} = 12.3$$

LECTURE 7

EPICYCLE GEAR TRAIN

DEPARTMENT OF MECHANICAL ENGINEERING

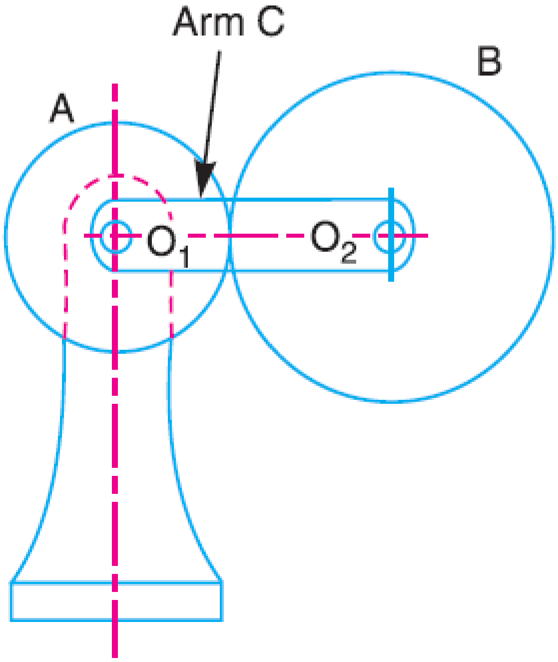
EPICYCLIC GEAR TRAIN

In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.

Gear A and the arm C have a common axis at O1 about which they can rotate

The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate.

Source: R. S. Khurmi



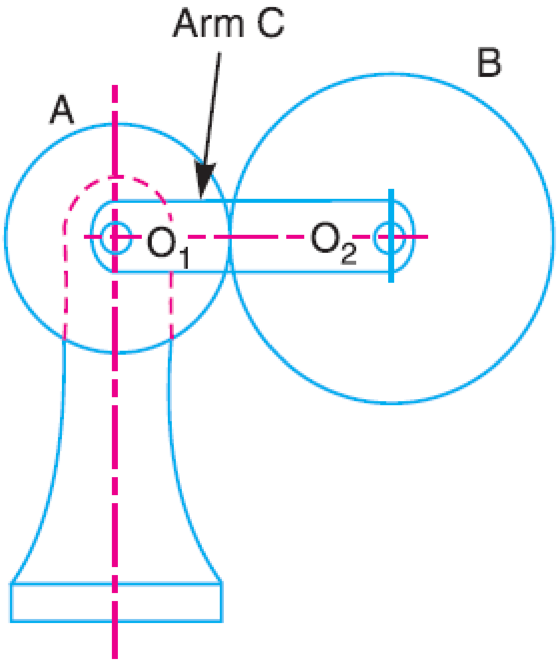
Epicyclic gear train.

Source: R. S. Khurmi

EPICYCLIC GEAR TRAIN

If the arm is fixed, the gear train is simple and gear A can drive gear B or vice- versa,.

If gear A is fixed and the arm is rotated about the axis of gear A (i.e. O_1), the gear B is forced to rotate upon and around gear A. Such a motion is called epicyclic.



Epicyclic gear train.

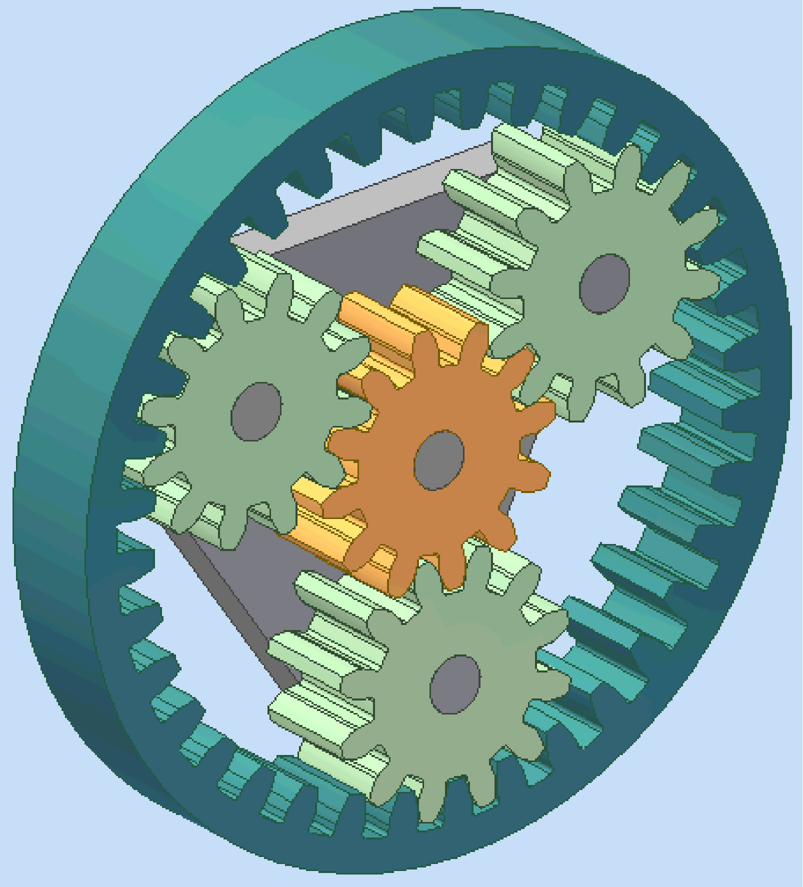
- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in comparatively lesser space.

- The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles,
- hoists, pulley blocks, wrist watches etc.,

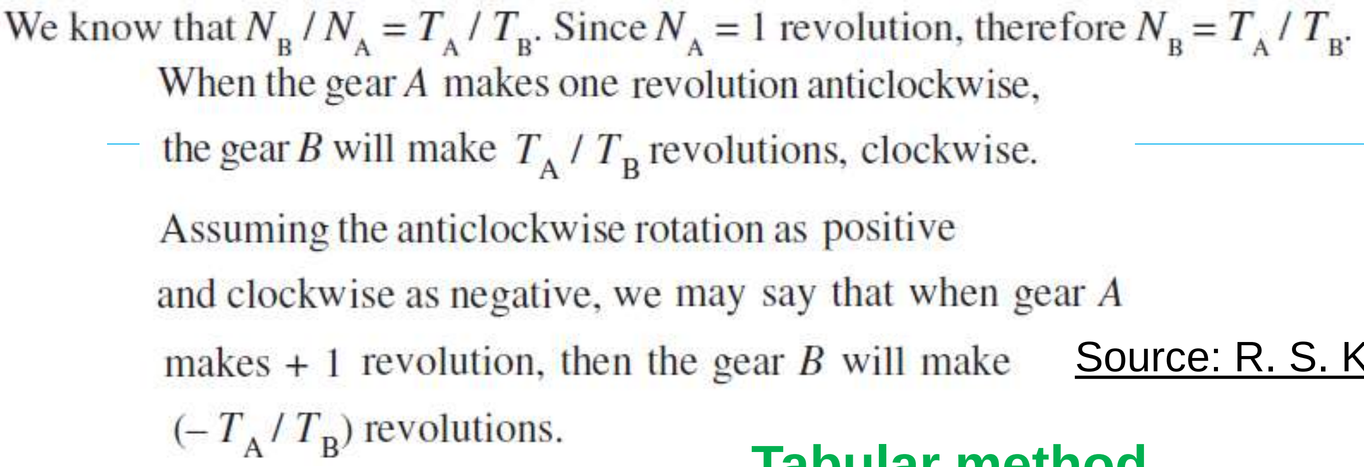
VELOCITY RATIOS IN EPICYCLIC GEAR TRAIN

The following two methods used for finding out the velocity ratio of an epicyclic gear train.

1. Tabular method
2. Algebraic method.



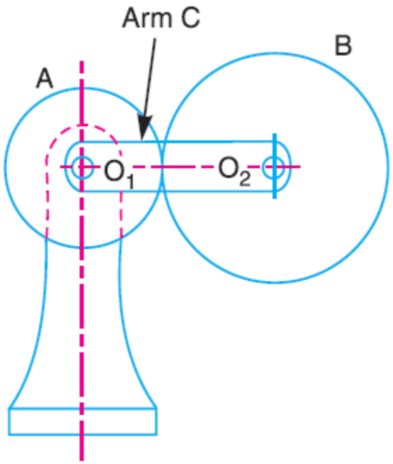
VELOCITY RATIOS IN EPICYCLIC GEAR



We know that $N_B / N_A = T_A / T_B$. Since $N_A = 1$ revolution, therefore $N_B = T_A / T_B$.

When the gear A makes one revolution anticlockwise, the gear B will make T_A / T_B revolutions, clockwise.

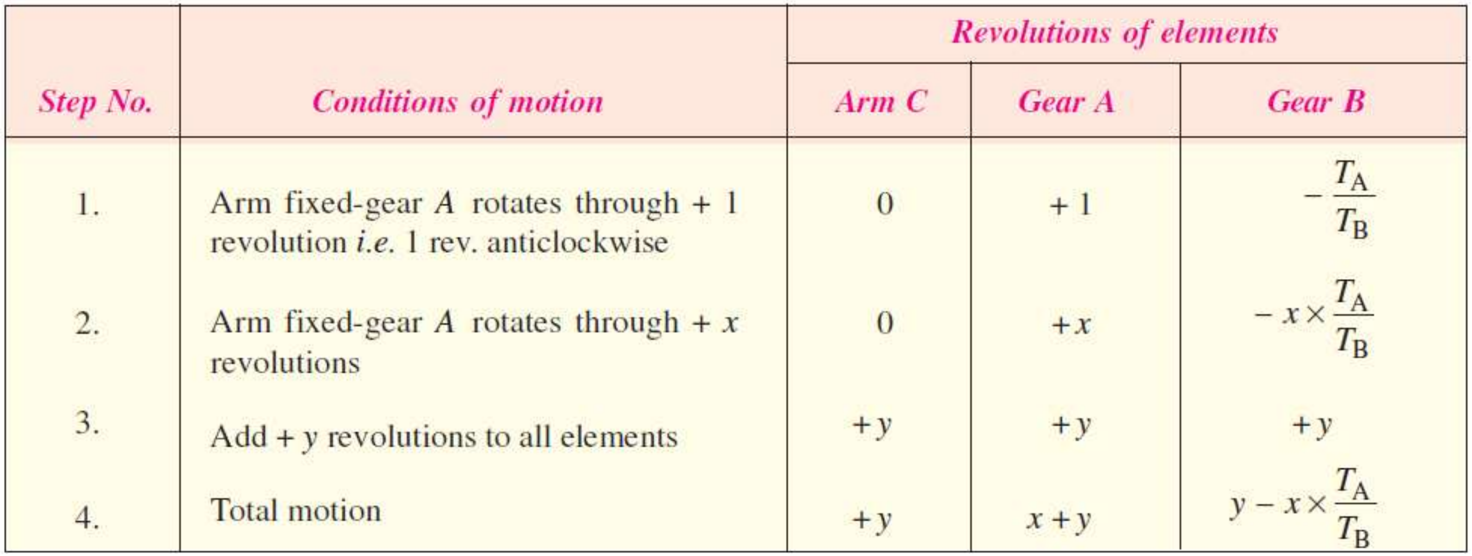
Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear A makes + 1 revolution, then the gear B will make $(- T_A / T_B)$ revolutions.



Epicyclic gear train.

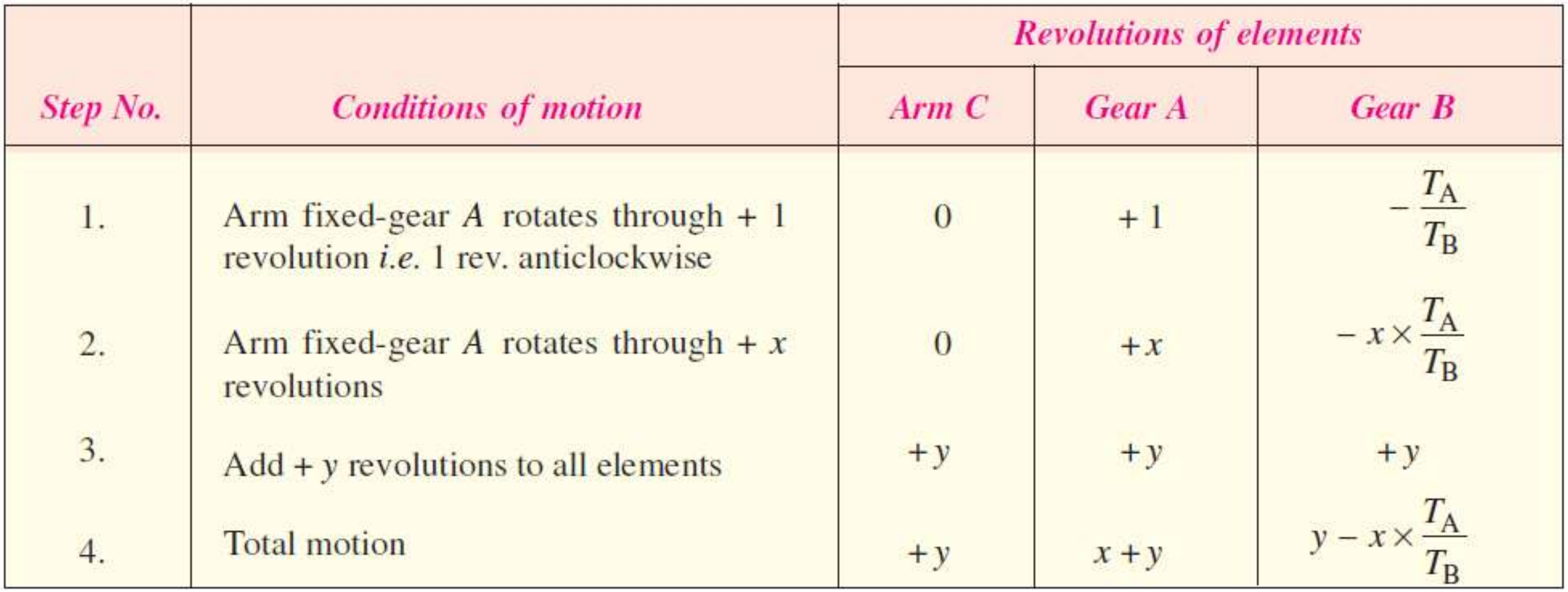
Source: R. S. Khurmi

Tabular method



Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear A rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through $+ x$ revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add $+ y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Velocity Ratios in Epicyclic Gear Train



Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	$+1$	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

when two conditions about the motion of rotation of any two elements are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row.

Velocity Ratios in Epicyclic Gear Train (Algebraic method)

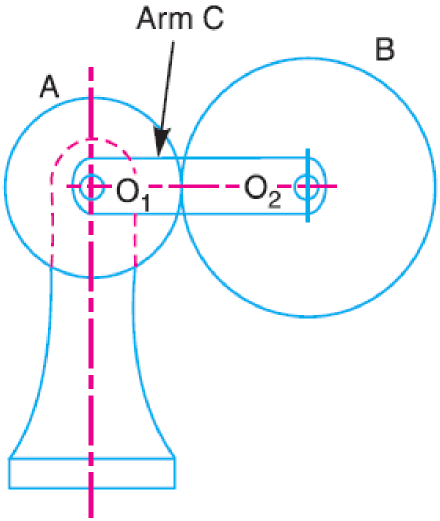
- The motion of each element of the epicyclic train relative to the arm is set down in the form of equations

- The number of equations depends upon the number of elements in the gear train

- But the two conditions are, usually, supplied in any epicyclic train *viz.* some element is fixed and the other has specified motion

- These two conditions are sufficient to solve all the equations

Source: R. S. Khurmi



Epicyclic gear train.

Velocity Ratios in Epicyclic Gear Train
(Algebraic method)

Let the arm C be fixed in an epicyclic gear train as shown in the figure

The speed of the gear A relative to the arm C $= N_A - N_C$

speed of the gear *B* relative to the arm *C* $= N_B - N_C$

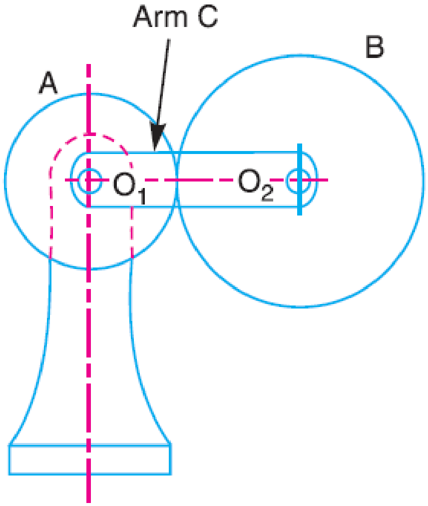
Since the gears *A* and *B* are meshing directly, they will revolve in ***opposite*** directions.

$$\therefore \quad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm *C* is fixed, $N_C = 0$. $\longrightarrow \dfrac{N_B}{N_A} = -\dfrac{T_A}{T_B}$

If the gear *A* is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \Longrightarrow \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$



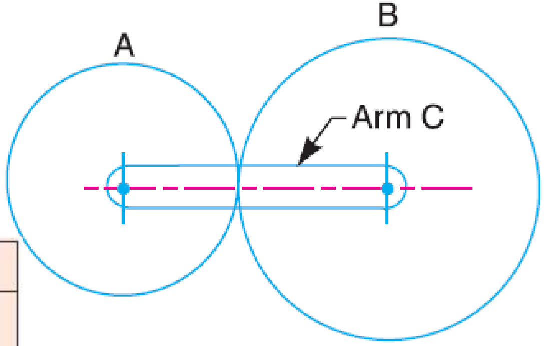
Epicyclic gear train.

Source: R. S. Khurmi

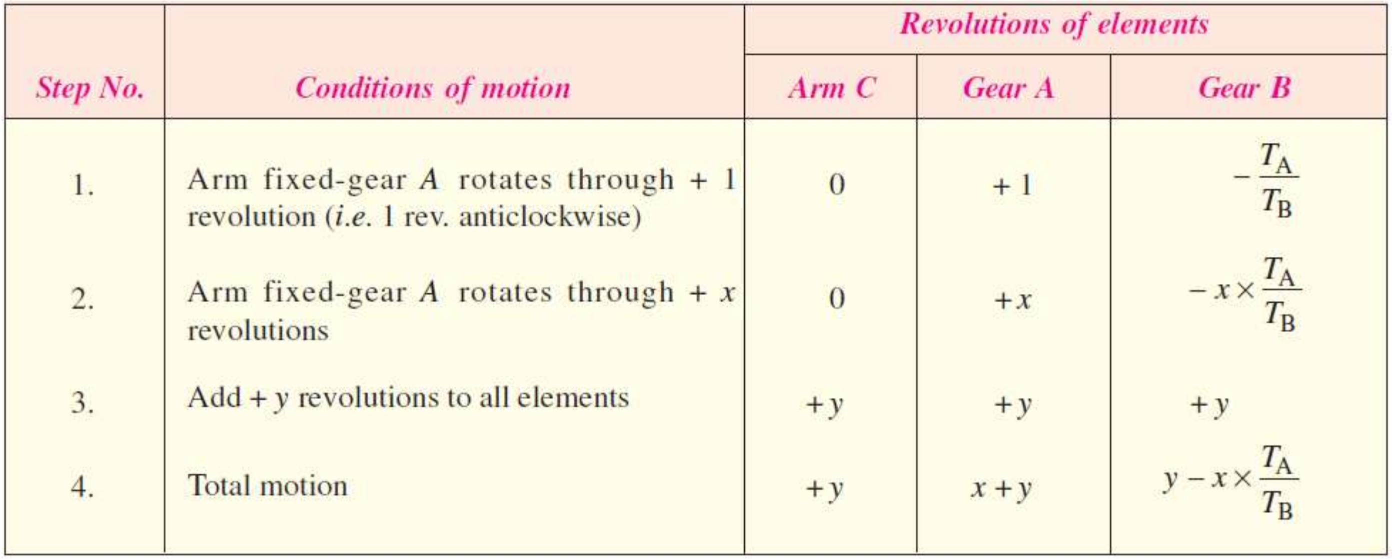
Note : The tabular method is easier and hence mostly used in solving problems on epicyclic gear train.

NUMERICAL EXAMPLE-3

In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?



Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)



Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Source: R. S. Khurmi

NUMERICAL EXAMPLE-3

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table, $y = +150$ r.p.m.

Also the gear A is fixed, therefore $x + y = 0$

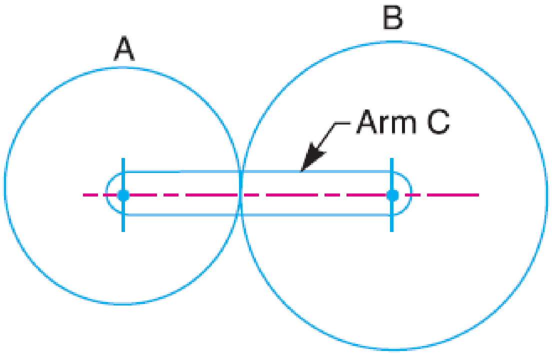
or $x = -y = -150$ r.p.m.

$$\therefore \text{ Speed of gear } B, \quad N_B = y - x \times \frac{T_A}{T_B}$$

$$= 150 + 150 \times \frac{36}{45} = +270 \text{ r.p.m.}$$

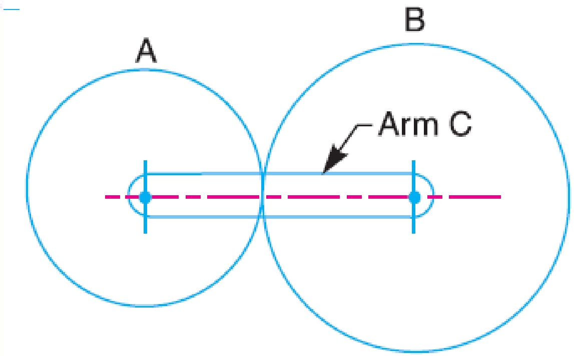
$$= 270 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

Source: R. S. Khurmi



NUMERICAL EXAMPLE-3

Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x\times\frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_A}{T_B}$



Source: R. S. Khurmi

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear *A* makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x+y=-300 \qquad \text{or} \qquad x=-300-y=-300-150=-450 \text{ r.p.m.}$$

∴ Speed of gear *B*,

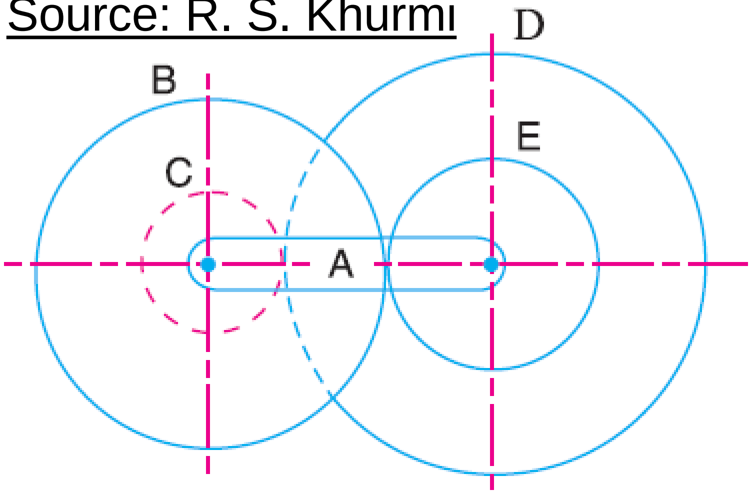
$$N_B = y-x\times\frac{T_A}{T_B}=150+450\times\frac{36}{45}=+510 \text{ r.p.m.}$$

$$= 510 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

NUMERICAL EXAMPLE-4

In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.

Source: R. S. Khurmi



Given : $T_B = 75$; $T_C = 30$; $T_D = 90$;

$N_A = 100$ r.p.m. (clockwise)

Find the number of teeth on gear (T_E)

$$T_B + T_E = T_C + T_D$$

$$\therefore \quad T_E = T_C + T_D - T_B = 30 + 90 - 75 = 45$$

NUMERICAL EXAMPLE-4

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D-E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D-E* rotated through + x revolutions	0	$+x$	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

Since the gear B is fixed, $y - x \times \frac{T_E}{T_B} = 0$

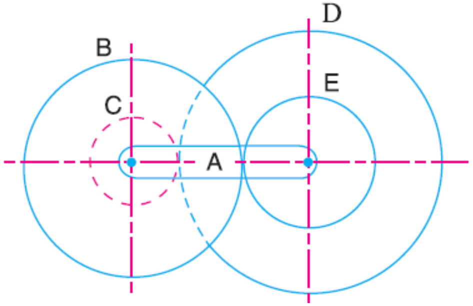
$$\therefore\ y - x \times \frac{45}{75} = 0 \longrightarrow y - 0.6x = 0 \quad ...(i)$$

Also the arm A makes 100 r.p.m. clockwise, therefore

$$y = -100 \quad ...(ii)$$

Substituting (*ii*) in equation (*i*), we get

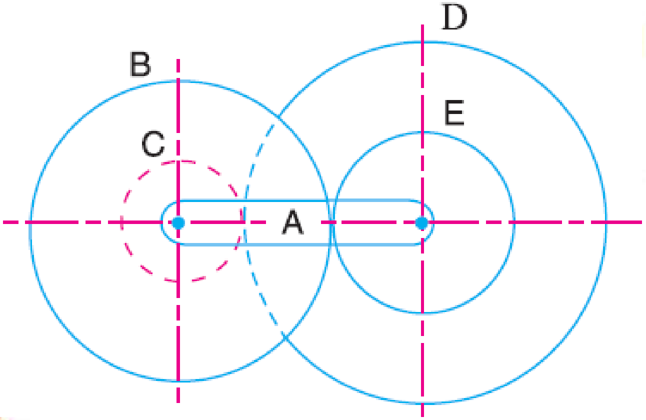
$$x = -100 / 0.6 = -166.67$$



Source: R. S. Khurmi

NUMERICAL EXAMPLE-4

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D-E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D-E* rotated through + *x* revolutions	0	$+x$	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$



From the fourth row of the table, speed of gear *C*,

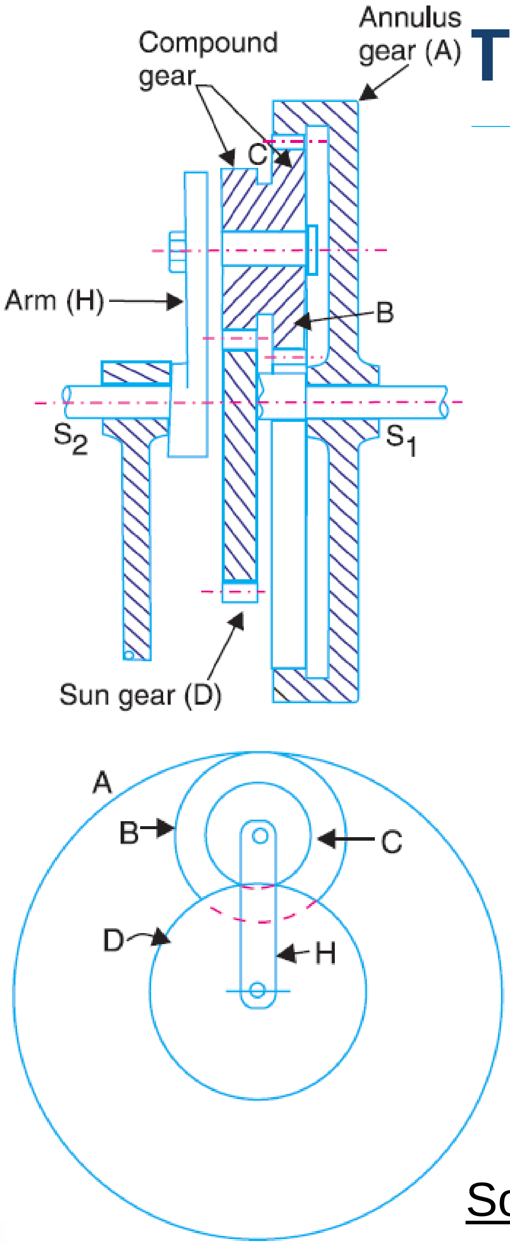
$$N_C = y - x \times \frac{T_D}{T_C} = -100 + 166.67 \times \frac{90}{30} = +400 \text{ r.p.m.}$$

$$= 400 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

Source: R. S. Khurmi

COMPOUND EPICYCLIC GEAR TRAIN: SUN AND PLANET GEAR

COMPOUND EPICYCLIC GEAR TRAIN: SUN T GEAR



- The annulus gear A meshes with the gear B

- the sun gear D meshes with the gear C.

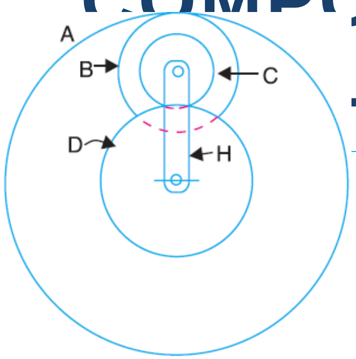
- when the annulus gear is fixed, the sun gear provides the drive

- when the sun gear is fixed, the annulus gear provides the drive.

- In both cases, the arm acts as a follower.

Source: R. S. Khurmi

COMPOUND EPICYCLIC GEAR TRAIN: SUN
LANET GEAR



Source: R. S. Khurmi

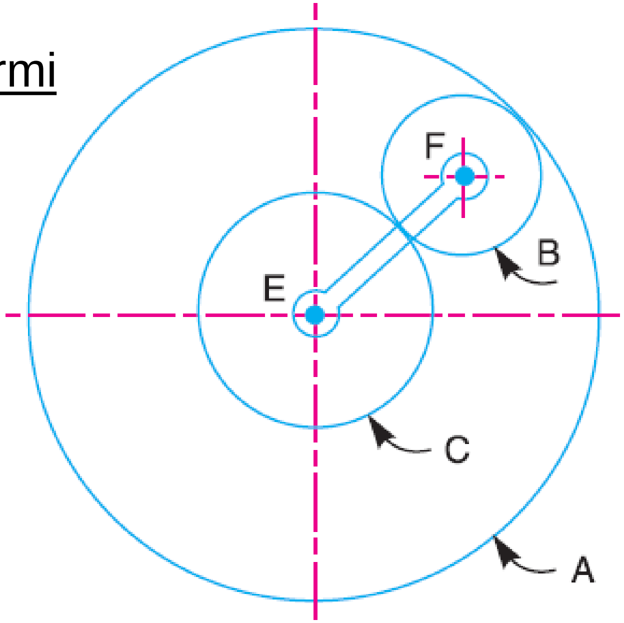
Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear D*	*Compound gear B-C*	*Gear A*
1.	Arm fixed-gear *D* rotates through + 1 revolution	0	+ 1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
2.	Arm fixed-gear *D* rotates through + *x* revolutions	0	$+x$	$-x\times\frac{T_D}{T_C}$	$-x\times\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_D}{T_C}$	$y-x\times\frac{T_D}{T_C}\times\frac{T_B}{T_A}$

NUMERICAL EXAMPLE-5

An epicyclic gear consists of three gears A, B and C as shown in the Figure. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.

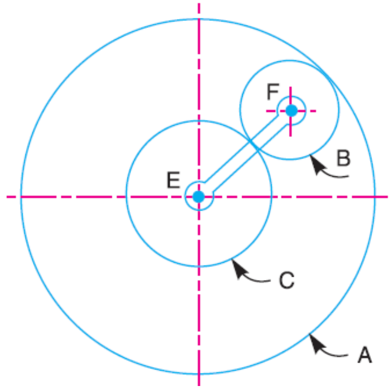
Source: R. S. Khurmi

Given : $T_A = 72$; $T_C = 32$; Speed of arm $EF = 18$ r.p.m.



NUMERICAL EXAMPLE-5

Step No.	Conditions of motion	Revolutions of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear *C* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B}\times\frac{T_B}{T_A}=-\frac{T_C}{T_A}$
2.	Arm fixed-gear *C* rotates through + *x* revolutions	0	$+x$	$-x\times\frac{T_C}{T_B}$	$-x\times\frac{T_C}{T_A}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_C}{T_B}$	$y-x\times\frac{T_C}{T_A}$



Source: R. S. Khurmi

Speed of gear C

the speed of the arm is 18 r.p.m. therefore, $y = 18$ r.p.m.

and the gear *A* is fixed, therefore

$$y - x\times\frac{T_C}{T_A} = 0 \longrightarrow 18 - x\times\frac{32}{72} = 0 \longrightarrow x = 40.5$$

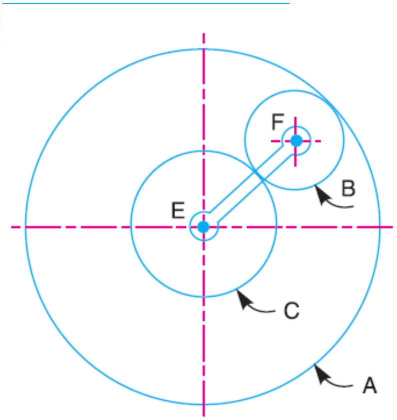
$$\therefore \text{ Speed of gear } C = x + y = 40.5 + 18$$

$$= +\,58.5 \text{ r.p.m.}$$

= 58.5 r.p.m. in the direction of arm. **Ans.**

NUMERICAL EXAMPLE-5

Step No.	Conditions of motion	Revolutions of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear *C* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	$+1$	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed-gear *C* rotates through $+x$ revolutions	0	$+x$	$-x \times \frac{T_C}{T_B}$	$-x \times \frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_C}{T_B}$	$y - x \times \frac{T_C}{T_A}$



Source: R. S. Khurmi

Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameters of gears

from the geometry of Fig. $d_B + \frac{d_C}{2} = \frac{d_A}{2}$ or $2\,d_B + d_C = d_A$

Since the number of teeth are proportional to their pitch circle diameters,

$2\,T_B + T_C = T_A$ or $2\,T_B + 32 = 72$ or $T_B = 20$

∴ Speed of gear *B* $= y - x \times \frac{T_C}{T_B} = 18 - 40.5 \times \frac{32}{20} = -46.8$ r.p.m.

= 46.8 r.p.m. in the opposite direction of arm. **Ans.**

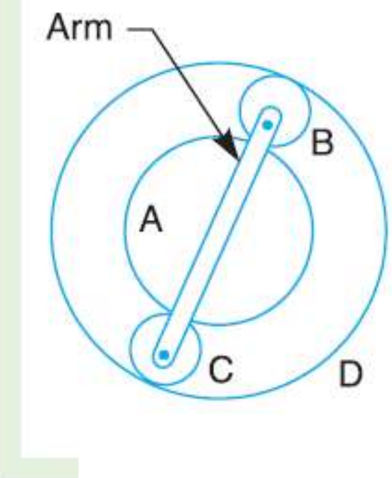
NUMERICAL EXAMPLE-6

An epicyclic train of gears is arranged as shown in Fig. How many revolutions does the arm, to which the pinions B and C are attached, make :

1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and

2. when A makes one revolution clockwise and D is stationary ?

The number of teeth on the gears A and D are 40 and 90 respectively.



Source: R. S. Khurmi

Given : $T_A = 40$; $T_D = 90$

find the number of teeth on gears B and C (*i.e.* T_B and T_C).

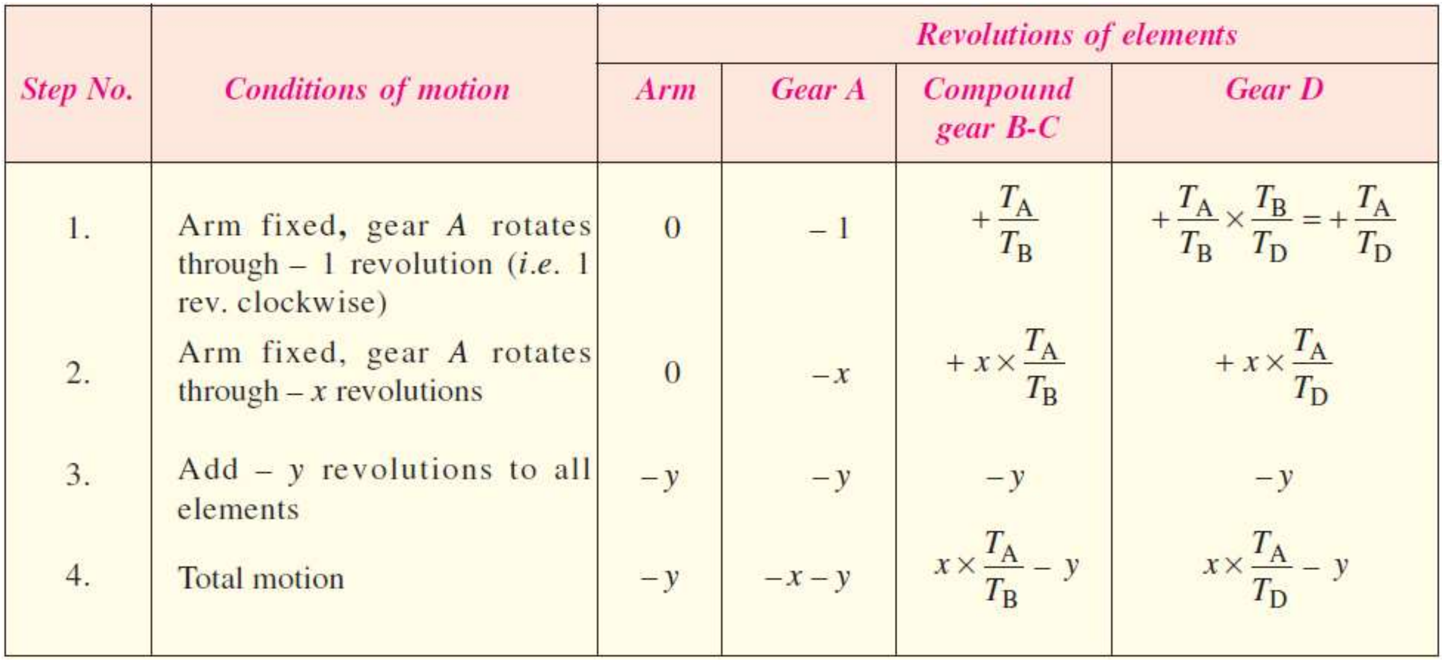
from the geometry of the figure, $d_A + d_B + d_C = d_D$ or $d_A + 2\,d_B = d_D$ ...($\because$ $d_B = d_C$)

Since the number of teeth are proportional to their pitch circle diameters,

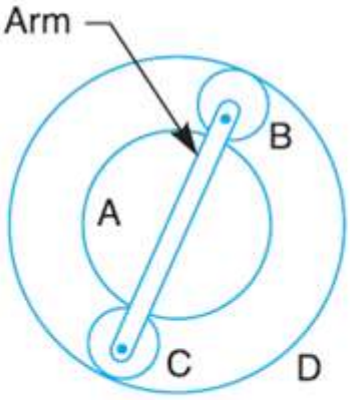
$$T_A + 2\,T_B = T_D \quad \text{or} \quad 40 + 2\,T_B = 90$$

$$T_B = 25, \quad \text{and} \quad T_C = 25 \qquad ...(\because\ T_B = T_C)$$

NUMERICAL EXAMPLE-6



Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear A*	*Compound gear B-C*	*Gear D*
1.	Arm fixed, gear *A* rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear *A* rotates through – *x* revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add – *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$



Source: R. S. Khurmi

1. ***Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise***

Since the gear *A* makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad \text{...}(i)$$

Also, the gear *D* makes half revolution anticlockwise, therefore

$$x\times\frac{T_A}{T_D}-y=\frac{1}{2} \quad \text{or} \quad x\times\frac{40}{90}-y=\frac{1}{2}$$

$$\therefore \quad 40\,x-90\,y=45 \quad \text{or} \quad x-2.25\,y=1.125 \qquad \text{...}(ii)$$

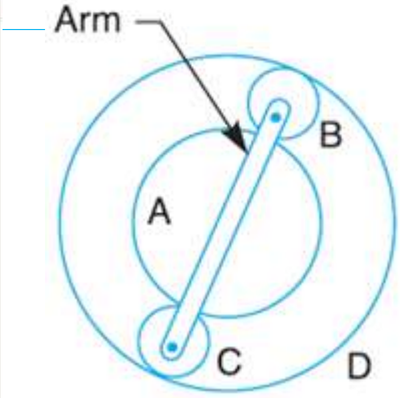
From equations (*i*) and (*ii*), $x=1.04$ and $y=-0.04$

$$\therefore \quad \text{Speed of arm} = -y = -(-0.04) = +0.04$$

$$= 0.04 \text{ revolution anticlockwise}$$ **Ans.**

NUMERICAL EXAMPLE-6

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear A*	*Compound gear B-C*	*Gear D*
1.	Arm fixed, gear A rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through – x revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add – y revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$



Source: R. S. Khurmi

2. *Speed of arm when A makes 1 revolution clockwise and D is stationary*

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad \text{...}(iii)$$

Also the gear D is stationary, therefore

$$x\times\frac{T_A}{T_D}-y=0 \quad \text{or} \quad x\times\frac{40}{90}-y=0$$

$$\therefore \quad 40\,x-90\,y=0 \quad \text{or} \quad x-2.25\,y=0 \qquad \text{...}(iv)$$

From equations (*iii*) and (*iv*),

$$x=0.692 \quad \text{and} \quad y=0.308$$

$\therefore$ Speed of arm $=-y=-0.308=0.308$ revolution clockwise **Ans.**

Industrial applications of gears

Type of Gear	Common Industries and Applications
Spur	• Clocks • Pumps • Watering systems • Household appliances • Clothes washing and drying machines • Power plants • Material handling systems • Aerospace and aircrafts • Railways and trains
Helical	• Same as spur gears but with greater loads and higher speeds (see above) • Automobiles (transmission systems)
Bevel	• Pumps • Power plants • Material handling systems • Aerospace and aircrafts • Railways and trains • Automobiles
Worm	• Instruments • Lifts and elevators • Material handling systems • Automobiles (steering systems)
Rack and Pinion	• Weighing scales • Material handling and transfer systems • Railways and trains • Automobiles (steering systems)

Assignment questions

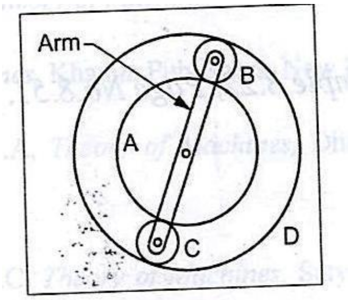
1. a)Make a comparison of cycloidal and involute tooth forms.b) Two 200 pressure angle involute gears in mesh have a module of 10mm. Addendum is module. Large gear has 50 teeth and the pinion has 13 teeth. Does interference occur? If it occurs, to what value should the pressure angle be changed to eliminate interference?
2. Sketch two teeth of a gear and show the following: face, flank, top land, bottom land, addendum, dedendum, tooth thickness, space width, face width and circular pitch. Derive a relation for minimum number of teeth on the gear wheel and the pinion to avoid interference

3. Two gears in mesh have a module of 10 mm and a pressure angle of 250. The pinion has 20 teeth and the gear has 52. The addendum on both the gears is equal to one module.
    a. Determine (i) The number of pairs of teeth in contact (ii) The angles of action of the pinion and the wheel (iii) The ratio of the sliding velocity to the rolling velocity at the pitch point and at the beginning and end of engagement.

4. What is a worm and worm wheel? Where is it used? Two 200 involute spur gears mesh externally and give a velocity ratio of 3. Module is 3 mm and the addendum is equal to 1.1 module. If the pinion rotates at 120 r.p.m. find: (i) The minimum number of teeth on each wheel to avoid interference. (ii) The number of pairs of teeth in contact

5. Two involute gears of 20° pressure angle are in mesh. The number of teeth on pinion is 20 and the gear ratio is 2. If the pitch expressed in module is 5 mm, and the pitch line speed is 1.2 m/s, assuming addendum as standard and equal to one module, find (i) the angle turned through by pinion when one pair of teeth is in mesh; and (ii) the maximum velocity of sliding

6. An epicyclic gear train shown in figure below.



The internal gear D has 90 teeth and the sun gear A has 40 teeth. The two planet gears B & C are identical and they are attached to an arm as shown. How many revolutions does the arm makes, (i) When'A' makes one revolution in clockwise and 'D' , makes one revolution in clockwise and 'D' makes ½ revolutions in opposite sense. (ii) When 'A' makes one revolution in clockwise and 'D' remains stationary.

7. Two mating gears have 20 and 40 involute teeth of module 10mm and 20° pressure angle .The addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point has half of the maximum possible length. Determine the addendum height for each gear wheel, length of the path of contact, arc of contact and contact ratio.

16/11/16

Code No: R15A0306-161

R15

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

B.Tech. III Semester Regular Examinations, NOV 2016

KINEMATICS OF MACHINERY

(ME)

Roll No			N	3						

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B

Part A is compulsory which carriers 25 marks and Answer all questions.

Part B Consists of 5 SECTIONS (One SECTION for each UNIT). Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

PART - A (25 Marks)

1.(a) What is the degrees of freedom of a mechanism? How it is determined?(2M)

(b) Explain Grubler's criterion for determining degrees of freedom for mechanisms?(3M)

(C)What Is a Double Hooke's Joint? Explain its significance.(2M)

(d) What is the general condition to maintain exact straight line motion of a point in a mechanism?(3M)

(e) Draw approximate acceleration diagram for a slider crank mechanism.(2M)

(f) Explain Coriolis component of acceleration and its significance.(3M)

(g) Derive the expression for velocity and acceleration during out stroke of follower with simple harmonic motion.(2M)

(h) Draw the displacement diagram for a follower with uniform velocity for cam rotation during rise.(3M)

(i) Define the terms module and circular pitch in gears.(2M)

(j) The distance between two parallel shafts is 600 mm are connected by spur gearing. If one shaft runs at 120 rpm and the other at 360 rpm. Find the number of teeth on each wheel, if the module is 8 mm.(3M)

PART - B (50 Marks)

SECTION – I

2. (a)Distinguish the machine and the mechanism? Explain how machines are classified?

(b)What is meant by inversion of a mechanism? Describe with the help of suitable sketches the inversion of double Slider crank chain.

(OR)

3. (a) Explain different types of constrained motions of a mechanism.

(b) How are kinematic pairs are classified? Explain with examples.

SECTION – II

4. (a) Determine the maximum permissible angle between the shaft axes of a universal joint if the driving shaft rotates at 800 rpm and the total fluctuation of speed does not exceed 60 rpm. Also find the maximum and the minimum speeds of the driven shaft.

(b) Sketch a Paucellier mechanism. Show that it can be used to trace a straight line.

(OR)

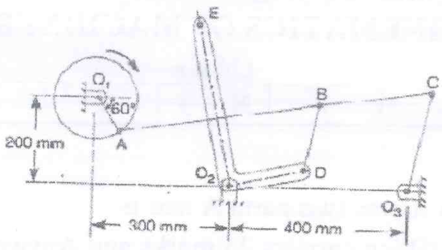
5. (a) What is a pantograph? Show that it can produce paths exactly similar to the ones traced out by a point on a link on an enlarged or reduced scale?

(b)What conditions must be satisfied by the steering mechanism of a car in order that the wheels may have a pure rolling motion when rounding a curve? Deduce the relationship connecting the inclinations of the front stub axles to the rear axle, the distance between the pivot centers for the front axles and wheelbase of the car.

SECTION – III

6. (a) Discuss the three types of instantaneous centres for a mechanisms.

(b) The mechanism of a wrapping machine as shown in figure below has the following dimensions O1A is 100 mm, AC is 700mm, BC is 200mm, O3C is 200mm, O2E is 400mm, O2D is 200mm and BD is 150mm. The crank O1A rotates at a uniform speed of 100 rad/sec. Find the velocity of the point E of the bell crank lever by instantaneous centre method.



(OR)

7. A crank and rocker mechanism ABCD has the following dimensions: AB =0.75m, BC =1.25 m, CD =1 m, AD =1.5 m and CF = 500 mm. AD is the fixed link. F lies on BC produced. Crank AB has an angular velocity of 30 rad/s counter clock-wise and a deceleration of 200 rad/s2 at the instant angle DAB= 300. Find
   (a) The instantaneous linear acceleration of C and F and
   (b) The instantaneous angular velocities and accelerations of links BC and CD.

SECTION – IV

8. A cam rotating clockwise at a uniform speed of 200 rpm is required to move an offset roller follower with a uniform and equal acceleration and retardation on both the outward and return strokes. The angle of ascent, the angle of dwell (between ascent and descent) and the angle of descent is 120, 60 and 90 degrees respectively. The follower dwells for the rest of cam rotation. The least radius of the cam is 50mm, the lift of the follower is 25mm and the diameter of the roller is 10mm. The line of stroke of the follower is offset by 20 mm from the axis of the cam. Draw the cam profile.

(OR)

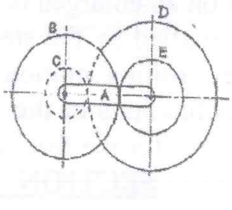
9. (a) Define the following terminology of cams.
   (i) base circle (ii) trace point (iii) pitch curve (IV) pressure angle.
   (b) Deduce expressions for the velocity and acceleration of the follower, when it moves with the simple harmonic motion.

SECTION – V

10. (a) Two gears in mesh have a module of 10 mm and a pressure angle of 25^{0}. The pinion has 20 teeth and the gear has 52. The addendum on both the gears is equal to one module. Determine
    i) The number of pairs of teeth in contact
    ii) The angles of action of the pinion and the wheel
    iii) The ratio of the sliding velocity to the rolling velocity at the pitch point and at the beginning and end of engagement.
    (b) Derive a relation for minimum number of teeth on the gear wheel and the pinion to avoid interference.

(OR)

11. (a)Explain with a neat sketch the sun and planet wheel.
    (b) In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D-E. The gear B meshes with gear E and the gear C meshes with gear D. The number teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 rpm clockwise.



(b) The driving shaft of a Hooke's joint has a uniform angular speed of 280 rpm. Determine the maximum permissible angle between the axis of the shafts to permit a maximum variation in speed of the driven shaft by 8% of the mean speed.

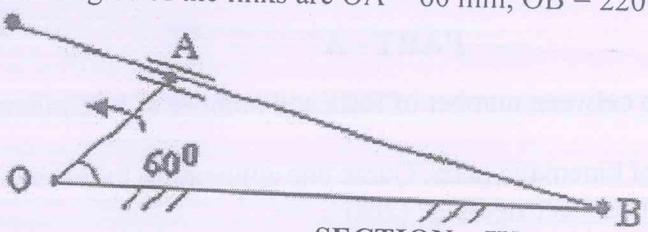
SECTION – III

6. (a)State and prove the Kennedy's theorem as applicable to instantaneous centres of rotation of three bodies. How is it helpful in locating various instantaneous centres of a mechanism?

(b)In a four bar chain ABCD, AD is the fixed link 12 cm long, crank AB is 3 cm long and rotates uniformly at 100 r.p.m. clockwise while the link CD is 6 cm long and oscillates about D. Link BC is equal to link AD. Find the angular velocity of link DC when angle BAD is 60^0.

(OR)

7. In the mechanism shown in figure, the crank OA makes 400 rpm in the counter clock wise direction. Find (i) Angular velocity of the link BA and (ii) Velocity of the slider at A. The lengths of the links are OA = 60 mm, OB = 220 mm and BC = 300 mm.



SECTION – IV

8. A cam rotating clockwise at a uniform speed is required to give a knife edge reciprocating follower a motion defined below.
(i) follower to move outwards a distance of 25mm during 120^0 of cam rotation
(ii) follower to dwell for 60^0 of cam rotation.
(iii) follower to return to its initial position during 90^0 of cam rotation.
(iv) follower to dwell for the remaining 90^0 of cam rotation.
The minimum radius of the cam is 50 mm, the line of stroke of the follower is offset 18.75 mm from the axis of the cam and the displacement of the follower is to takes place uniform and equal acceleration and retardation on both the outward and return strokes. Draw the profile of cam.

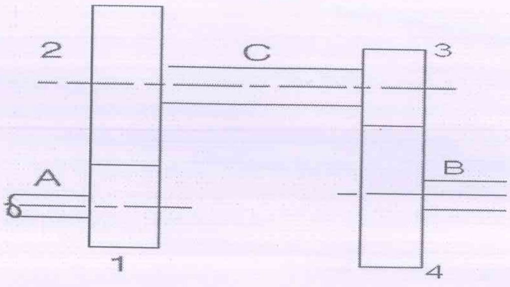
(OR)

9. (a) Name the different motions that a follower can have

(b) From the following data draw the profile of cam in which the follower moves with S.H.M during ascent while it moves with U.A.D motion during decent:
Least radius of cam is 50mm, Angle of ascent, $\theta_0 = 48^0$
Angle of dwell between ascent and descent, $\theta_{D0} = 42^0$
Angle of descent $\theta_r = 60^0$, the lift of follower = 40 mm
Diameter of roller = 30mm
Distance between line of action of the follower and axis of cam = 20mm
If the cam rotates at 360 r.p.m anti-clockwise, find the maximum velocity and acceleration of the follower during descent.

SECTION – V

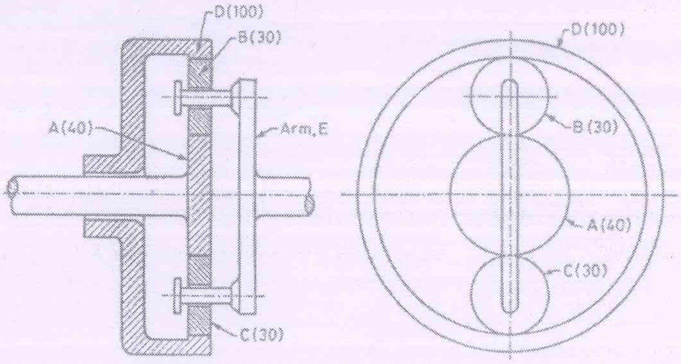
10. In a reverted gear train, as shown in Figure two shafts A and B are in the same straight line and are geared together through an intermediate parallel shaft C. The gears connecting the shafts A and C have a module of 2 mm and those connecting the shafts C and B have a module of 4.5 mm. The speed of shaft A is to be about but greater than 12 times the speed of shaft B, and the ratio at each reduction is same. Find suitable number of teeth for gears. The number of teeth of each gear is to be a minimum but not less than 16.



11. Figure shows an epicyclic gear train. Two planet gears B and C having 30 teeth each are attached to the arm E and Gear A is having 40 teeth instead of 50, then find the number of revolutions made by the arm when:

(i) gear A makes one revolution Clockwise and D makes half a revolution anticlockwise and

(ii) gear A makes one revolution clockwise and D is stationary.



Figure

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R17

Code No: R17A0306

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B. Tech I Semester Regular Examinations, November 2018

Kinematics of Machinery

(ME)

Roll No										

Time: 3 hours **Max. Marks: 70**

Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

***

SECTION-I

1 Sketch and explain any two inversions of a double slider crank chain. **[14M]**

OR

2 In a crank and slotted lever quick return motion mechanism, the distance between the fixed centers is 240 mm and the length of the driving crank is 120mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke. If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever. **[14M]**

SECTION-II

3 Sketch and describe the Peaucellier straight line mechanism indicating clearly the conditions under which the point P on the corners of the rhombus of the mechanism generates a straight line. **[14M]**

OR

4 What is the condition for correct steering? Sketch and show the two main types of steering gears and discuss their relative advantages. **[14M]**

SECTION-III

5 Draw and explain Klien's construction for determining the velocity and acceleration of the piston in a slider crank mechanism. **[14M]**

OR

6 The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine: 1. velocity of piston, 2. angular velocity of connecting rod, 3.Velocity of point E on the connecting rod 1.5 m from the gudgeon pin. **[14M]**

SECTION-IV

7 Construct the profile of a cam to suit the following specifications: Cam shaft diameter = 40mm, least radius of cam = 25mm, diameter of roller=25mm, angle of lift=120^0, Angle of fall = 150^0, Lift of the follower = 40mm, Number of pauses are two of equal interval between motions, During the lift the motion is SHM. During the fall the motion is uniform acceleration and deceleration. The speed of the cam shaft is uniform. The line of stroke of the follower is off-set 12.5mm from the center of cam. **[14M]**

OR

8 A cam is to be designed for a knife edge follower with the following data: 1. Cam lift = 40mm during 90^0 of cam rotation with simple harmonic motion. 2. Dwell for the next 30^0. 3. During the next 60^0 of cam rotation, the follower returns to its original position with simple harmonic motion 4. Dwell during the remaining 180^0. Draw the profile of the cam when (a) the line of stroke of the follower passes through the axis of the cam shaft, and (b) the line of stroke is offset 20 mm from the axis of the cam shaft. The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 rpm. **[14M]**

SECTION-V

9 Two involutes gears of 20° pressure angle are in mesh. The number of teeth on pinion is 20 and the gear ratio is 2. If the pitch expressed in module is 5 mm and the pitch line speed is 1.2 m/s, assuming addendum as standard and equal to one module, find: **[14M]**

(i) The angle turned through by pinion when one pair of teeth is in mesh; and

(ii) The maximum velocity of sliding

OR

10 Classify and explain gear trains with neat sketches. Also determine the velocity ratio in each case. **[14M]**

*****

R17

Code No: R17A0306

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B. Tech I Semester Regular Examinations, November 2018

Kinematics of Machinery

(ME)

Roll No										

Time: 3 hours **Max. Marks: 70**

Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

***

SECTION-I

1 Sketch and explain any two inversions of a double slider crank chain. **[14M]**

OR

2 In a crank and slotted lever quick return motion mechanism, the distance between the fixed centers is 240 mm and the length of the driving crank is 120mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke. If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever. **[14M]**

SECTION-II

3 Sketch and describe the Peaucellier straight line mechanism indicating clearly the conditions under which the point P on the corners of the rhombus of the mechanism generates a straight line. **[14M]**

OR

4 What is the condition for correct steering? Sketch and show the two main types of steering gears and discuss their relative advantages. **[14M]**

SECTION-III

5 Draw and explain Klien's construction for determining the velocity and acceleration of the piston in a slider crank mechanism. **[14M]**

OR

6 The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine: 1. velocity of piston, 2. angular velocity of connecting rod, 3.Velocity of point E on the connecting rod 1.5 m from the gudgeon pin. **[14M]**

SECTION-IV

7 Construct the profile of a cam to suit the following specifications: Cam shaft diameter = 40mm, least radius of cam = 25mm, diameter of roller=25mm, angle of lift=120^0, Angle of fall = 150^0, Lift of the follower = 40mm, Number of pauses are two of equal interval between motions, During the lift the motion is SHM. During the fall the motion is uniform acceleration and deceleration. The speed of the cam shaft is uniform. The line of stroke of the follower is off-set 12.5mm from the center of cam. **[14M]**

OR

8 A cam is to be designed for a knife edge follower with the following data: 1. Cam lift = 40mm during 90^0 of cam rotation with simple harmonic motion. 2. Dwell for the next 30^0. 3. During the next 60^0 of cam rotation, the follower returns to its original position with simple harmonic motion 4. Dwell during the remaining 180^0. Draw the profile of the cam when (a) the line of stroke of the follower passes through the axis of the cam shaft, and (b) the line of stroke is offset 20 mm from the axis of the cam shaft. The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 rpm. **[14M]**

SECTION-V

9 Two involutes gears of 20° pressure angle are in mesh. The number of teeth on pinion is 20 and the gear ratio is 2. If the pitch expressed in module is 5 mm and the pitch line speed is 1.2 m/s, assuming addendum as standard and equal to one module, find: **[14M]**

(i) The angle turned through by pinion when one pair of teeth is in mesh; and

(ii) The maximum velocity of sliding

OR

10 Classify and explain gear trains with neat sketches. Also determine the velocity ratio in each case. **[14M]**

*****

Code No: R15A0306 **R15**

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B.Tech I Semester supplementary Examinations, May 2019

Kinematics of Machinery

(ME)

Roll No										

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B
Part A is compulsory which carriers 25 marks and Answer all questions.
Part B Consists of 5 SECTIONS (One SECTION for each UNIT). Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

****

PART-A (25 Marks)

1). a What is the expression that gives the relation between number of links and number of joints, which constitute a kinematic chain? **[2M]**

b Write a short notes on Scotch Yoke mechanism. **[3M]**

c What is fundamental equation of steering gears. **[2M]**

d When a double Hooks joint provide constant speed of driving and driven shafts. **[3M]**

e What is coriolis component of acceleration. **[2M]**

f What is meant by Space centrode and body centrode. **[3M]**

g Define pitch curve, pressure angle and base circle with reference to cams. **[2M]**

h What are the different types of motion with which a follower can move? **[3M]**

i State the fundamental law of gearing **[2M]**

j What do you mean by gear train? **[3M]**

PART-B (50 MARKS)

SECTION-I

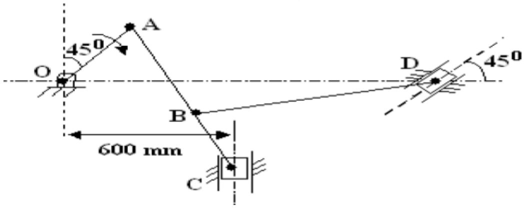
2 (a) Define a kinematic pair. Explain the various types of kinematic pairs giving at least one distinguishing feature of each. **[10M]**
(b) Define kinematic chain. How does it differ from a mechanism?

OR

3 Define inversion of mechanism. Explain with the help of suitable sketches the inversion of double slides crank mechanism. **[10M]**

SECTION-II

4 In a Watt mechanism of the type shown in figure. The links OA and DB are perpendicular to the link AB in the mean position. If OA =40mm, DB =80mm and AB=55mm, find the point C on the link AB produced for approximate straight line motion of point C **[10M]**



OR

5 (a) What conditions must be satisfied by the steering mechanism of a car in order that the wheels may have a pure rolling motion when rounding a curve? Deduce the relationship **[10M]**

connecting the inclinations of the front stub axles to the rear axle, the distance between the pivot centers for the front axles and wheelbase of the car.

(b) A Hooke's joint connects a shaft running at a uniform speed of 1200 rpm to a second shaft, the angle between their axes being 20 degrees. Find the velocity and acceleration of the driven shaft at the instant when the fork of the driving shaft has turned through an angle of 15^0 from the plane containing the shaft axes. At what positions of the driving shaft during a revolution, the angular velocity of the driven shaft is the same as that determined above?

SECTION-III

6 Explain the Klein construction for finding the velocity and acceleration in the following mechanisms. **[10M]**

(a) Single slider crank chain

(b) Four bar chain.

OR

7 In a four link mechanism, the dimensions of the links are as under: **[10M]**

AB=50 mm, BC=66 mm, CD= 56 mm and AD=100 mm.

At the instant when angle DAB= 60^0 , the link AB has angular velocity of 10.5 radians/s in the counter clockwise direction. Determine the

(a) Velocity of the point C

(b) Velocity of the point E on the link BC when BE=40 mm

(c) Angular velocities of the links BC and CD

SECTION-IV

8 Draw the profile of cam operating a knife edge follower having a lift of 30 mm. The cam raises the follower with SHM for 150^o of rotation followed by a period of dwell for 60^o . The follower descends for the next 100^o rotation of the cam with uniform velocity, again followed by a dwell period. The cam rotates at a uniform velocity of 120 rpm and has a least radius of 20 mm. What will be the maximum velocity and acceleration of the follower during the lift and the return **[10M]**

OR

9 A symmetrical cam has a base circle 60 mm radius, arc of action 110^0, straight flanks and tip is a circular arc. The line of action of the follower passes through the centre line of cam shaft. The follower which has 40 mm diameter roller has a lift of 26 mm. Calculate the velocity and acceleration of the follower when moving outward and contact is just reaching the end of straight flank. The cam rotates at 500 rpm **[10M]**

SECTION-V

10 Two mating in volute spur gears with module pitch of 5 mm have 20 and 40 teeth of 20^0 pressure angle and 5 mm addendum. Determine the maximum velocity sliding and the angle turned through by pinion, when one pair of teeth is in mesh and pitch line speed is 1.2 m/s **[10M]**

OR

11 In an epicyclic train an annular wheel A having 54 teeth meshes with a planet wheel B which gears with a sun wheel C, the wheels A and C being co-axial. The wheel B is carried on a pin fixed on one end of arm P which rotates about the axis of the wheels A and C. if the wheel A makes 20 r.p.m. in a clockwise sense and the arm P rotates at 100 r.p.m. in the anticlockwise direction and the wheel C has 24 teeth, determine r. p. m and sense of rotation of the wheel C **[10M]**

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R17

Code No: R17A0306

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B.Tech I Semester Supplementary Examinations, May 2019

Kinematics of Machinery

(ME)

Roll No										

Time: 3 hours **Max. Marks: 70**

Note: This question paper Consists of 5 Sections. Answer **FIVE** Questions, Choosing ONE Question from each SECTION and each Question carries 14 marks.

***

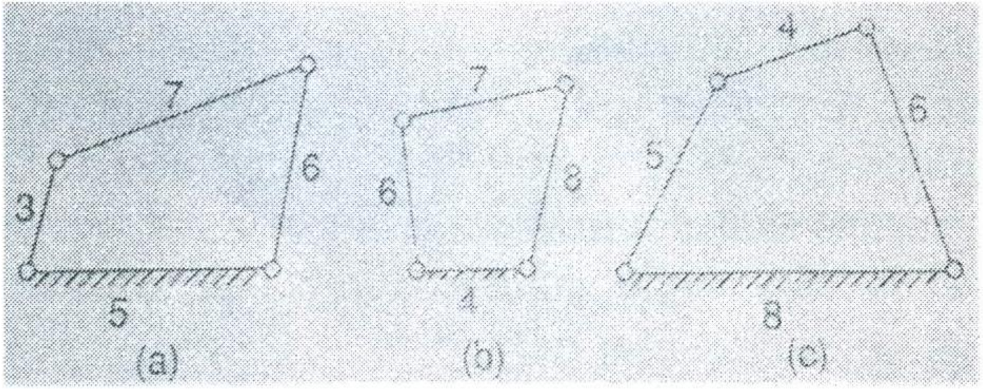
SECTION-I

1 a) What is the significance of degrees of freedom of a kinematic chain when it functions as a mechanism.(7M) **[14M]**

b) Sketch and explain any two inversions of a slider crank chain. (7M)

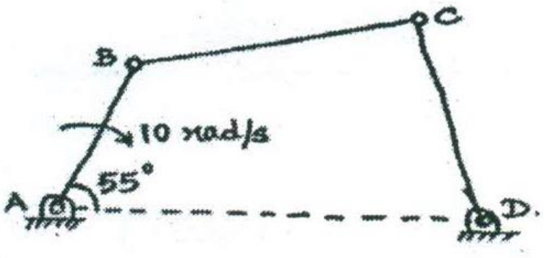
OR

2 Find the maximum and minimum transmission angles for the mechanism as shown in figure. The figure indicates the dimensions in standard units of length. **[14M]**



SECTION-II

3 In the four bar mechanism shown in fig., AB = 190mm, BC=CD=280mm, AD=500mm, Determine i. Acceleration of C, ii. Angular acceleration of links Bc and CD. **[14M]**



OR

4 Explain Scott Russel mechanism for tracing a straight line. **[14M]**

SECTION-III

5 State and prove the Kennedy's theorem as applicable to instantaneous centers of rotation of three bodies. How is it helpful in locating various instantaneous centers of a mechanism? **[14M]**

OR

6 Explain in detail about the various methods to find velocity and acceleration of links of a mechanism. **[14M]**

SECTION-IV

7 Explain the construction of cam profile for simple harmonic motion to the roller follower of the cam. Also derive the expression for maximum velocity and maximum acceleration of the follower. **[14M]**

OR

8 A cam with 30mm as minimum diameter is rotating clockwise at a uniform speed of 1200rpm and has to give the following motion to a roller follower 10mm in diameter: **[14M]**

(i) Follower to complete outward stroke of 25mm during 120° of cam rotation with equal uniform acceleration and retardation.

(ii) Follower to dwell for 60°of cam rotation.

(iii) Follower to return to its initial position during 90°of cam rotation with equal uniform acceleration and retardation.

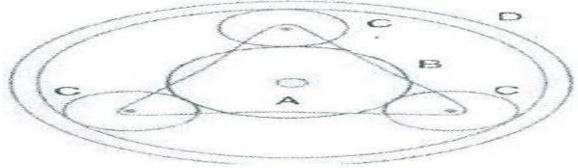
(iv) Follower to dwell for remaining 90°of cam rotation.

SECTION-V

9 A pair of gears, having 40 and 20 teeth respectively, are rotating in mesh, the speed of the smaller being 2000 rpm. Determine the velocity of sliding between the gear teeth faces at the point of engagement, at the pitch point, and at the point of disengagement if the smaller gear is the driver. Assume that the gear teeth are 20^0 involute form, addendum length is 5 mm and the module is 5 mm. Also find the angle through which the pinion turns while any pairs of teeth are in contact. **[14M]**

OR

10 In an epi cyclic gear train of the sun and planet type as shown in Fig, the pitch circle diameter of the internally toothed ring D is to be 216 mm and the module 4 mm. When the ring D is stationary, the spider A, which carries three planet wheels C of equal size, is to make one revolution in the same sense as the sun wheel B for every five revolutions of the driving spindle carrying the sun wheel B, Determine suitable number of teeth for all the wheels and the exact diameter of pitch circle of the ring. **[14M]**



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R15

Code No: R15A0306

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

II B.Tech I Semester supplementary Examinations, November 2018

Kinematics of Machinery

(ME)

Roll No										

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B
Part A is compulsory which carriers 25 marks and Answer all questions.
Part B Consists of 5 SECTIONS (One SECTION for each UNIT). Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

PART-A (25 Marks)

1). a Explain giving examples the following terms: **[2M]**
i. Kinematic link ii. Kinematic pair

b Define the term 'Inversion of a mechanism **[3M]**

c What is the main disadvantage of Scott Russell's mechanism **[2M]**

d What are the major differences between Ackerman and Davis steering mechanisms. **[3M]**

e A car starts from rest and accelerates uniformly to a speed of 80Km/hr over a distance of 250m. Calculate the acceleration and time taken to attain the speed. **[2M]**

f Define the term instantaneous centre of rotation. **[3M]**

g Differentiate between Pitch point and Trace point with reference to cams. **[2M]**

h Why a roller follower is preferred to that of a knife-edged follower **[3M]**

i Define the terms: path of approach and path of recess and path of contact between two mating gears **[2M]**

j What are the various types of gear trains **[3M]**

PART-B (50 MARKS)

SECTION-I

2 (a) What is the difference between an element and a kinematic link of a mechanism? How do you classify links of a mechanism? **[10M]**
(b) What do you mean by degree of freedom of a kinematic pair? How the pairs are classified? Give examples.

OR

3 Discuss in detail atleast two inversions of single slider crank chain mechanisms which give quick return motion. Give examples of their applications. **[10M]**

SECTION-II

4 (a) Describe mechanisms used by approximate copying. **[10M]**
(b) Describe working of Watt.T. Chebicheff mechanism.

OR

5 (a) For an Ackermann steering gear, derive the expression for the angle of inclination of the track arms to longitudinal axis of the vehicle. **[10M]**
(b) A Hooke's joint connects two shafts whose axes intersect at 150^0. The driving shaft rotates uniformly at 120rpm. The driven shaft operates against a steady torque of 150 Nm and carries a flywheel whose mass is 45 Kg and radius of gyration 150mm. Find the maximum torque which will be exerted by the driving shaft.

SECTION-III

6 In a four bar chain ABCD, AB = 10 cm, BC = 28 cm, CD = 24 cm and DA =40 cm. When AB is inclined to horizontal at 45^0 and rotates at 30 r.p.m, find the velocity and acceleration of points E, F and G. E is on BC produced at 5 cm, BF = 20 cm and G is 30 cm from B and 15 cm from C outside the chain. Also find the angular velocity and angular acceleration of BC. **[10M]**

OR

7 Sketch a quick return motion of the crank and slotted lever type and explain the procedure of drawing the velocity and acceleration diagram, for any given configuration of the mechanism by instantaneous centre method. **[10M]**

SECTION-IV

8 A cam, with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to giver a knife-edged follower the motion as described below: (a) To move outwards through 40 mm during 100° rotation of the cam; (b) to dwell for next 80° (c) To return to its starting position during next 90 ° and (d) To dwell for the rest period of revolution. Draw the profile of the cam (i) When the line of stroke of the follower passes through the centre of the cam shaft and (ii) When the line of stroke of the follower is to take place with Uniform acceleration and uniform retardation. Determine the maximum velocity and acceleration of the follower when the cam shaft rotates at 900 r.p.m. **[10M]**

OR

9 A cam has straight working faces which are tangential to a base circle of diameter 90 mm. The follower is a roller of diameter 40 mm and the centre of roller moves along a straight line passing through the centre line of the cam shaft. The angle between the tangential faces of the cam is 90° and the faces are joined by a nose circle of 10 mm radius. The speed of rotation of the cam is 120 revolutions per min. Find the acceleration of the roller centre (i) When during the lift; the roller is just about to leave the straight flank. (ii) When the roller is at the outer end of its lift? **[10M]**

SECTION-V

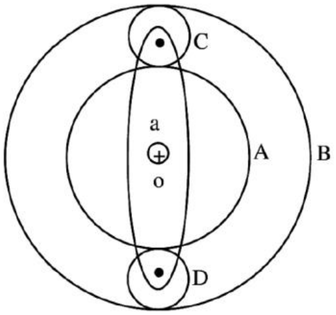
10 Two 20^0 involute spur gears have a module of 10mm. The addendum is one module. The larger gear has 50 teeth and the pinion 13 teeth. Does the interference occur? If it occurs, to what value should the pressure angle be changed to eliminate interference? **[10M]**

OR

11 An epicycle gear train is shown in figure, the number of teeth on A and B are 80 and 200. Determine the speed of the arm A. **[10M]**

(a) if `A' rotates at 100 rpm clock wise and B at 50 rpm counter clockwise.

(b) if `A' rotates at 100 rpm clockwise and B is stationary



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