UNIT-I
Introduction to Electrical Circuits

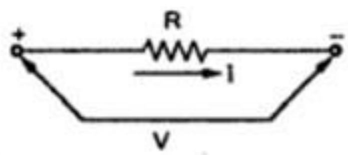
1.1 OHM'S LAW

This law gives relationship between the potential differences (V), the current (I) and the resistance (R) of a d.c. circuit. Dr. Ohm in 1827 discovered a law called Ohm's law. It states,

Ohm's law: The current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provide the temperature remains constant.

Mathematically, $$I \propto \frac{V}{R}$$

Where I is the current flowing in amperes, the V is voltage applied and R is the resistance of the conductor, as shown in the fig 1.



Now $$I = \frac{V}{R}$$

The unit of potential difference is defined in such a way such the constant of proportionality is unity.

Ohm's law is,

$I = \frac{V}{R}$		amperes
$V = IR$		volts
$\frac{V}{I}$ = constant = R		ohms

The Ohm's law can be defined as, at constant temperature, the potential difference (V) between any two points of a conductor is directly proportional to current (I) flowing through the conductor.

Limitations of Ohm's law:

The limitations of Ohm's law are

1. It is not applicable to the non linear devices such as diodes, zener diodes, voltage regulators etc.
2. It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by,

$$V = K I^m$$ Where K, m are constants.

1.2. Classification of Electrical Networks

The behaviour of the entire network depends on the behaviour and characteristics of its elements. Based on such characteristics electrical network can be classified as below :

i) Linear Network : A circuit or network whose parameters i.e. elements like resistances, inductances and capacitances are always constant irrespective of the change in time, voltage, temperature etc. is known as **linear network.** The Ohm's law can be applied to such network. The mathematical equations of such network can be obtained by using the law of superposition. The response of the various network elements is linear with respect to the excitation applied to them.

ii) Non linear Network : A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as **non linear network** . The Ohm's law may not be applied to such network. Such network does not follow the law of superposition. The response of the various elements is not linear with respect to their excitation. The best example is a circuit consisting of a diode where diode current does not vary linearly with the voltage applied to it.

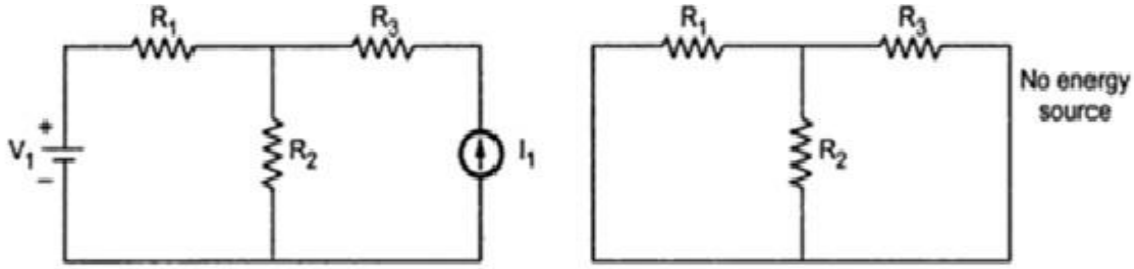
iii) Bilateral Network : A circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it, is called **bilateral network.** Network consisting only resistances is good example of bilateral network.

iv) Unilateral Network : A circuit whose operation, behaviour is dependent on the direction of the current through various elements is called **unilateral network.** Circuit

consisting diodes, which allows flow of current only in one direction is good example of unilateral circuit.

v) Active Network : A circuit which contains at least one source of energy is called **active.** An energy source may be a voltage or current source.

vi) Passive Network : A circuit which contains no energy source is called **passive circuit.** This is shown in the Fig. 1.7.



(a) Active network (b) Passive network

Fig. 1.7

vii) Lumped Network : A network in which all the network elements are physically separable is known as **lumped network**. Most of the electric networks are lumped in nature, which consists elements like R, L, C, voltage source etc.

viii) Distributed Network : A network in which the circuit elements like resistance, inductance etc. cannot be physically separable for analysis purposes, is called **distributed network**. The best example of such a network is a transmission line where resistance, inductance and capacitance of a transmission line are distributed all along its length and cannot be shown as a separate elements, any where in the circuit.

The classification of networks can be shown as,

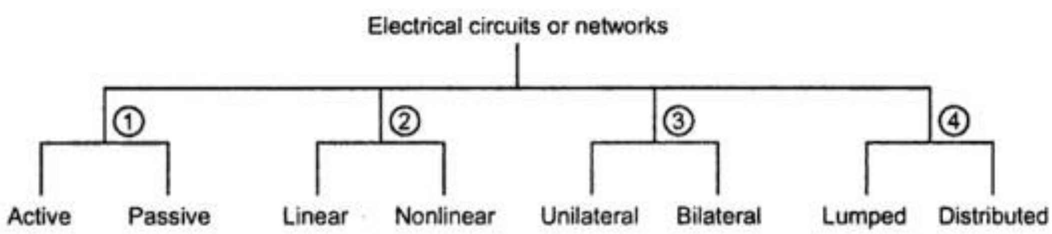


Fig. 1.8 Classification of networks

1.3. Basic Circuit Components

Let us take a brief review of three basic elements namely resistance, capacitance and inductance.

Resistance

It is the property of the material by which it opposes the flow of current through it. The resistance of element is denoted by the symbol 'R'. Resistance is measured in ohms (Ω).

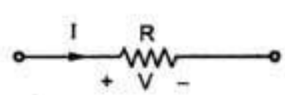


Fig. 1.9

The resistance of a given material depends on the physical properties of that material and given by,

$$R = \frac{\rho l}{a}$$

where l = Length in metres

a = Cross-sectional area in square metres

ρ = Resistivity in ohms-metres

R = Resistance in ohms

ρ = Resistivity in ohms-metres

R = Resistance in ohms

We can define unit ohm as below.

Key Point : 1 Ohm : *The resistance of a circuit, in which a current of 1 ampere generates the heat at the rate of 1 joules per second is said to be 1 ohm.*

Now 4.186 Joules = 1 Calorie

hence 1 Joule = 0.24 Calorie

Thus unit 1 ohm can be defined as that resistance of the circuit if it develops 0.24 calories of heat, when one ampere current flows through the circuit for one second.

The unit ohm also can be defined as, one ohm resistance is that which allows one ampere current to flow through it when one volt voltage is impressed across it.

The relation between voltage and current for a resistance is given by **Ohm's law** as,

$$v = R\,i$$

$$\therefore \quad R = \frac{v}{i}$$

The power absorbed by a resistance is given by,

$$\therefore \quad p(t) = vi = \frac{v^2}{R} = i^2 R \text{ watts}$$

while the amount of energy converted to heat energy in time t is given by,

$$\therefore \quad w = \int_{-\infty}^{t} p\,dt = \int_{-\infty}^{t} i^2 R\,dt = \int_{-\infty}^{t} vi\,dt$$

Key Point : *As i^2 term is always positive, the energy absorbed by the resistance is always positive.*

If the voltage across resistance is constant V and the currrent through it is constant I then the energy for t ≥ 0 is given by,

$$W = \int_0^t VI\,dt = VI\,t \text{ joules}$$

while, $$P = VI = \frac{V^2}{R} = I^2R \text{ watts}$$

1.10.2 Inductance

An inductance is the element in which energy is stored in the form of electromagnetic field. The inductance is denoted as 'L' and is measured in henries (H).

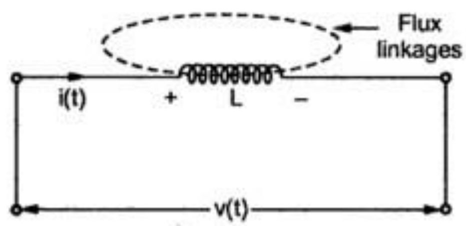


Fig. 1.10 Inductance

The Fig. 1.10 shows an inductance.

The time varying voltage v(t) is the voltage across it. It carries a current i(t) which is also time varying.

Key Point: *For an inductance, the voltage across it is proportional to the rate of change of current passing through it.*

$$\therefore \quad v(t) \;\alpha\; \frac{di(t)}{dt}$$

The constant of proportionality in the above equation is the inductance L.

$$\therefore \quad \boxed{v(t) = L\frac{di(t)}{dt}}$$

If the voltage v(t) is known across an inductor then the current is given by,

$$\therefore \quad \boxed{i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt}$$

If the inductance has N turns and the flux ϕ produced by the current i(t) entirely links with the coil of N turns then according to Faraday's law,

$$v(t) = N\frac{d\phi}{dt}$$

The total flux linkages $N\phi$ are thus proportional to the current through the coil.

$$\therefore \quad N\phi = Li$$

$$\therefore \quad \boxed{L = \frac{N\phi}{i}}$$

The power in the inductor is given by,

$$p(t) = vi = L\,i(t)\frac{di(t)}{dt}$$

The energy stored in the inductor in the form of an electromagnetic field is,

$$w = \int p(t)\,dt = \int L\,i(t)\frac{d\,i(t)}{dt}\,dt$$

$$w = \int L\,i(t)\,di(t) = L\frac{i^2(t)}{2}$$

$$\therefore \quad w = \frac{1}{2}L\,i^2(t) \text{ joules}$$

1.10.3 Capacitance

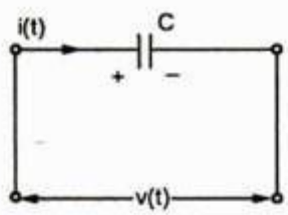


Fig. 1.11 Capacitor

An element in which energy is stored in the form of an electrostatic field is known as capacitance. It is made up of two conducting plates separated by a dielectric material. It is denoted as 'C' and is measured in farads (F).

The Fig. 1.11 shows a capacitor. The voltage across it is time varying v(t) and current through it is also time varying i(t).

Key Point: *For a capacitor, the current through it is proportional to the rate of change of voltage across it.*

$$i(t) \propto \frac{dv(t)}{dt}$$

The constant of proportionality is the capacitor C.

$$\therefore \quad i(t) = C\frac{dv(t)}{dt}$$

While the ratio of the charge stored to the voltage across the capacitor is known as the capacitance C.

$$\therefore \quad C = \frac{q}{v}$$

The voltage across the capacitor is given by,

$$v(t) = \frac{1}{C} \int_{-\infty}^{t} i(t)\, dt$$

The power in the capacitor is given by,

$$p(t) = v\, i = C\, v(t) \frac{dv(t)}{dt}$$

The energy stored in the capacitor is given by,

$$w = \int p(t)\, dt = \int C\, v(t) \frac{dv(t)}{dt}\, dt$$

$$w = \int C\, v(t)\, dv(t) = C \frac{v^2(t)}{2}$$

$$\therefore \quad w = \frac{1}{2} C\, v^2(t) \text{ joules}$$

1.11 Voltage-Current Relationships for Passive Elements

The three passive elements are resistance (R), inductance (L) and capacitance (C). The behaviour of these three elements alongwith the respective voltage-current relationship is given in the Table 1.3.

The behaviour of the three elements can be summarized as,

Element	Basic relation	Voltage across, if current known	Current through, if voltage known	Energy
R	$R = \frac{v}{i}$	$v_R(t) = R\, i_R(t)$	$i_R(t) = \frac{1}{R} v_R(t)$	$w = \int_{-\infty}^{t} i_R(t)\, v_R(t)\, dt$
L	$L = \frac{N\phi}{i}$	$v_L(t) = L \frac{d\, i_L(t)}{dt}$	$i_L(t) = \frac{1}{L} \int_{-\infty}^{t} v_L(t)\, dt$	$w = \frac{1}{2} L\, i(t)^2$
C	$C = \frac{q}{v}$	$v_C(t) = \frac{1}{C} \int_{-\infty}^{t} i_C(t)\, dt$	$i_C(t) = C \frac{dv_C(t)}{dt}$	$w = \frac{1}{2} C\, v^2(t)$

Table 1.3

Note that in the Table 1.3, v_R, v_L and v_C are the voltages across R, L and C respectively while i_R, i_L and i_C are the currents through R, L and C respectively.

1.4. Energy Sources

There are basically two types of energy sources; voltage source and current source. These are classified as i) Ideal source and ii) Practical source.

Let us see the difference between ideal and practical sources.

1.4.1 Voltage Source

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. The symbol for ideal voltage source is shown in the Fig. 1.15 (a). This is connected to the load as shown in Fig. 1.15 (b). At any time the value of voltage at load terminals remains same. This is indicated by V- I characteristics shown in the Fig. 1.15 (c).

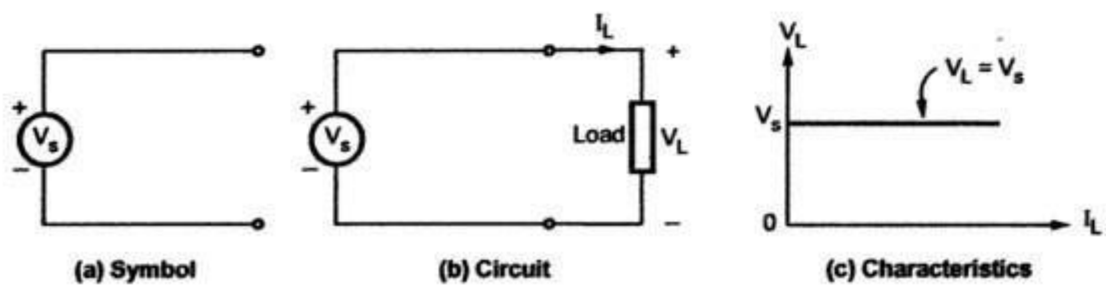


Fig. 1.15 Ideal voltage source

Practical voltage source :

But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by R_{se} as shown in the Fig. 1.16.

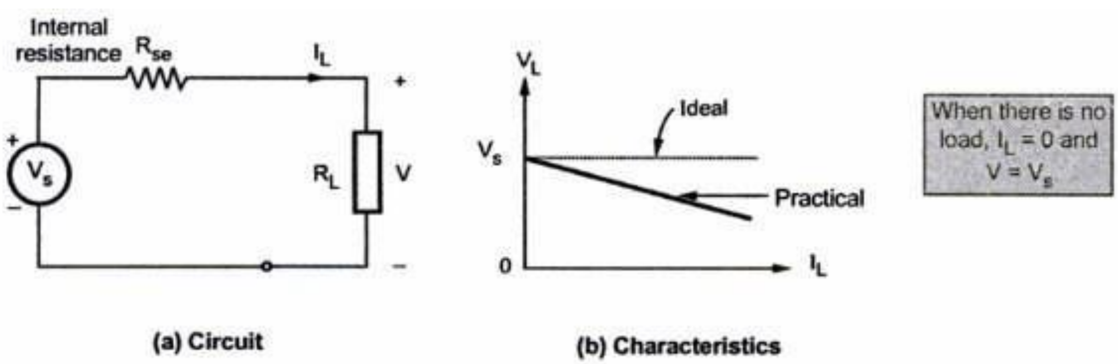


Fig. 1.16 Practical voltage source

Because of the R_{se}, voltage across terminals decreases slightly with increase in current and it is given by expression,

$$V_L = -(R_{se})\, I_L + V_s = V_s - I_L R_{se}$$

Key Point : *For ideal voltage source,* $R_{se} = 0$.

1.4.2 Current Source

Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminals. The symbol for ideal current source is shown in the Fig. 1.18 (a). This is connected to the load as shown in the Fig. 1.18 (b). At any time, the value of the current flowing through load I_L is same i.e. is irrespective of voltage appearing across its terminals. This is explained by V-I characteristics shown in the Fig. 1.18 (c).

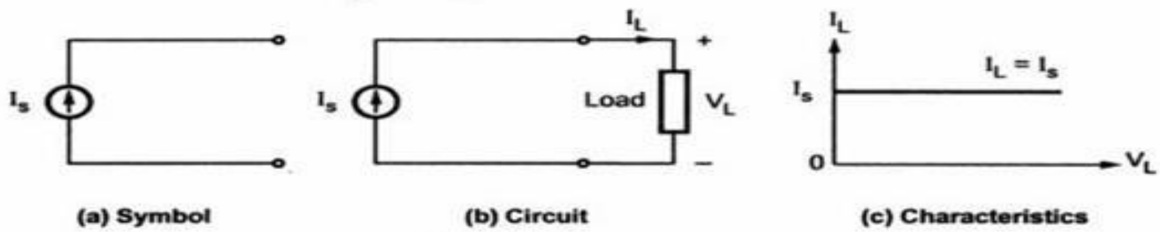


Fig. 1.18 Ideal current source

But practically, every current source has high internal resistance, shown in parallel with current source and it is represented by R_{sh}. This is shown in the Fig. 1.19.

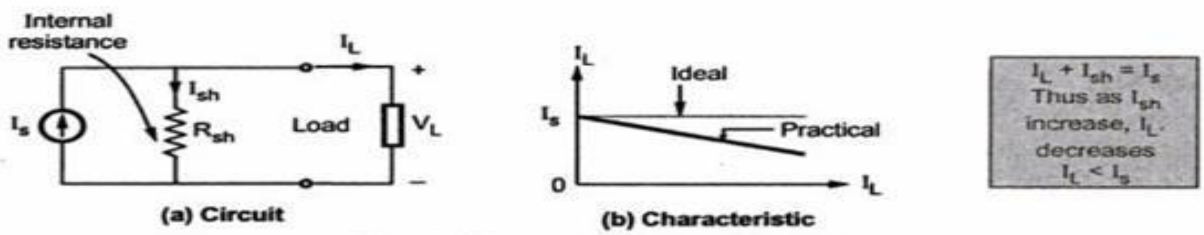


Fig. 1.19 Practical current source

1.16.3 Dependent Sources

Dependent sources are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the Fig. 1.21 and further classified as,

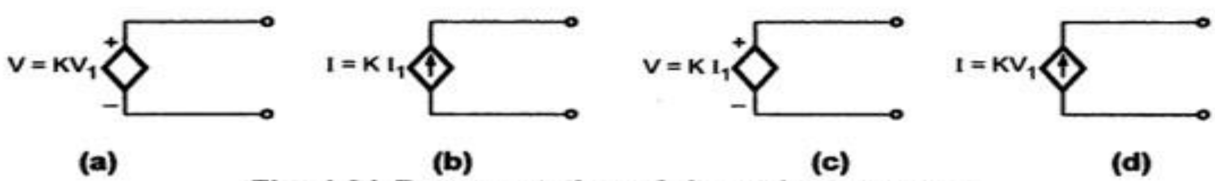


Fig. 1.21 Representation of dependent sources

i) Voltage Dependent Voltage Source : It produces a voltage as a function of voltages elsewhere in the given circuit. This is called **VDVS.** It is shown in the Fig. 1.21 (a).

ii) Current Dependent Current Source : It produces a current as a function of currents elsewhere in the given circuit. This is called **CDCS.** It is shown in the Fig. 1.21 (b).

iii) Current Dependent Voltage Source : It produces a voltage as a function of current elsewhere in the given circuit. This is called **CDVS.** It is shown in the Fig. 1.21 (c).

iv) Voltage Dependent Current Source : It produces a current as a function of voltage elsewhere in the given circuit. This is called **VDCS.** It is shown in the Fig. 1.21 (d).

1.17 Combinations of Sources

In a network consisting of many sources, series and parallel combinations of sources exist. If such combinations are replaced by the equivalent source then the network simplification becomes much more easy. Let us consider such series and parallel combinations of energy sources.

1.17.1 Voltage Sources in Series

If two voltage sources are in series then the equivalent is dependent on the polarities of the two sources.

Consider the two sources as shown in the Fig. 1.24.

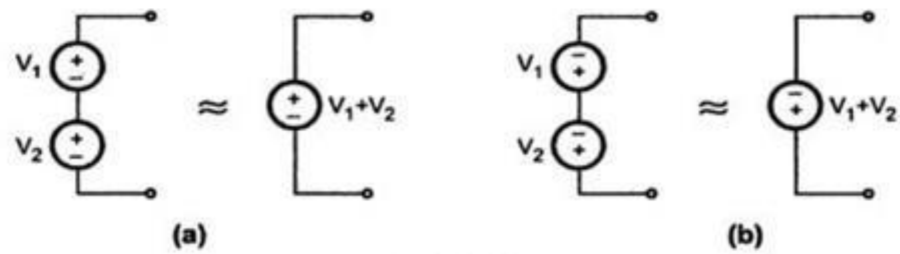


Fig. 1.24

Thus if the polarities of the two sources are same then the equivalent single source is the addition of the two sources with polarities same as that of the two sources.

Consider the two sources as shown in the Fig. 1.25.

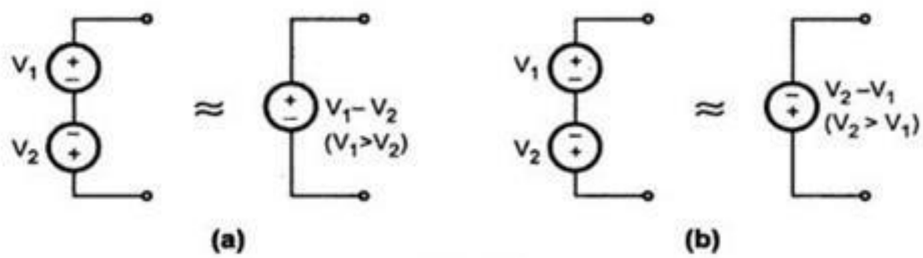


Fig. 1.25

1.17.2 Voltage Sources in Parallel

Consider the two voltage sources in parallel as shown in the Fig. 1.26.

The equivalent single source has a value same as V_1 and V_2.

It must be noted that at the terminals open circuit voltage provided by each source must be equal as the sources are in parallel.

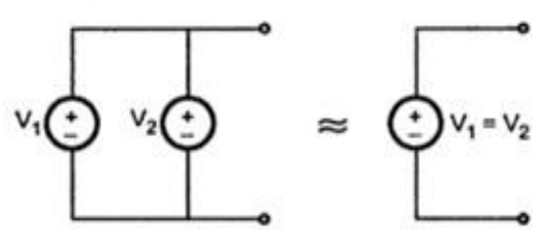


Fig. 1.26

1.17.3 Current Sources in Series

Consider the two current sources in series as shown in the Fig. 1.27.

The equivalent single source has a value same as I_1 and I_2.

Key Point: *The current through series circuit is always same hence it must be noted that the current sources to be connected in series must have same current ratings though their voltage ratings may be same or different.*

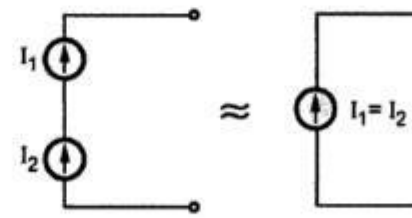


Fig. 1.27

1.17.4 Current Sources in Parallel

Consider the two current sources in parallel as shown in the Fig. 1.28.

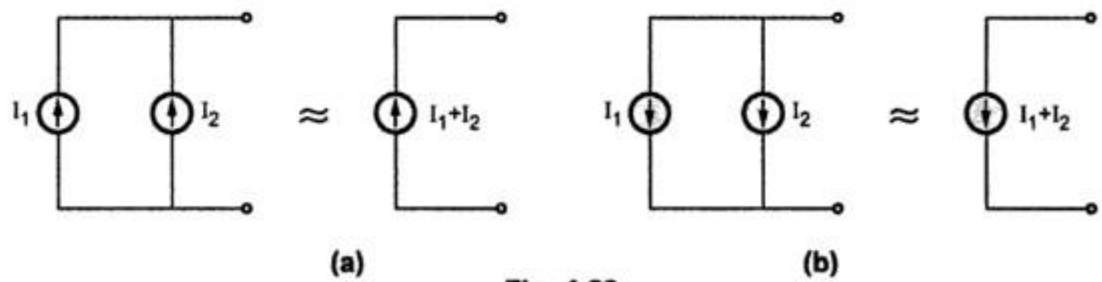


Fig. 1.28

Thus if the directions of the currents of the sources connected in parallel are same then the equivalent single source is the addition of the two sources with direction same as that of the two sources.

Consider the two current sources with opposite directions connected in parallel as shown in the Fig. 1.29.

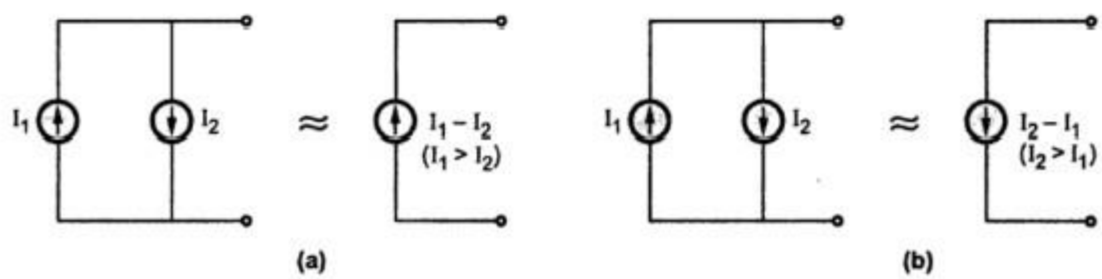


Fig. 1.29

Thus if the directions of the two sources are different then the equivalent single source has a direction same as greater of the two sources with a value equal to the difference between the two sources.

1.18 Source Transformation

Consider a practical voltage source shown in the Fig. 1.30 (a) having internal resistance R_{se}, connected to the load having resistance R_L.

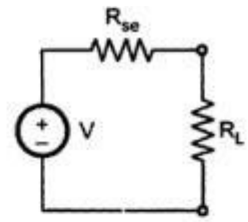


Fig. 1.30 (a) Voltage source

Now we can replace voltage source by equivalent current source.

Key Point: *The two sources are said to be* ***equivalent****, if they supply equal load current to the load, with same load connected across its terminals .*

The current delivered in above case by voltage source is,

$$I = \frac{V}{(R_{se}+R_L)}, \qquad R_{se} \text{ and } R_L \text{ in series} \qquad \ldots(1)$$

If it is to be replaced by a current source then load current must be $\frac{V}{(R_{se}+R_L)}$

Consider an equivalent current source shown in the Fig. 1.30 (b).

The total current is ' I '.

Both the resistances will take current proportional to their values.

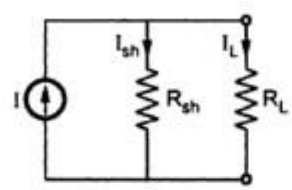


Fig. 1.30 (b) Current source

From the current division in parallel circuit we can write,

$$I_L = I\times\frac{R_{sh}}{(R_{sh}+R_L)} \qquad \ldots(2)$$

Now this I_L and $\frac{V}{R_{se}+R_L}$ must be same, so equating (1) and (2),

$$\therefore \quad \frac{V}{R_{se}+R_L} = \frac{I\times R_{sh}}{R_{sh}+R_L}$$

The internal resistance, $\boxed{R_{se} = R_{sh}}$ ... Equating denominator

Then, $V = I\times R_{sh} = I\times R_{se}$

or $I = \frac{V}{R_{sh}}$

$$\therefore \quad \boxed{I = \frac{V}{R_{se}}} \qquad \ldots R_{sh} = R_{se}$$

Key Point : *If voltage source is converted to current source, then current source* $I = \frac{V}{R_{se}}$ *with parallel internal resistance equal to* R_{se}.

Key Point : *If current source is converted to voltage source, then voltage source* $V = I R_{sh}$ *with series internal resistance equal to* R_{sh}.

The direction of current of equivalent current source is always from **–ve to +ve, internal to the source.** While converting current source to voltage source, polarities of voltage is always as +ve terminal at top of arrow and –ve terminal at bottom of arrow, as direction of current is from –ve to +ve, internal to the source. **This ensures that current flows from positive to negative terminal in the external circuit.**

Note the directions of transformed sources, shown in the Fig. 1.31 (a), (b), (c) and (d).

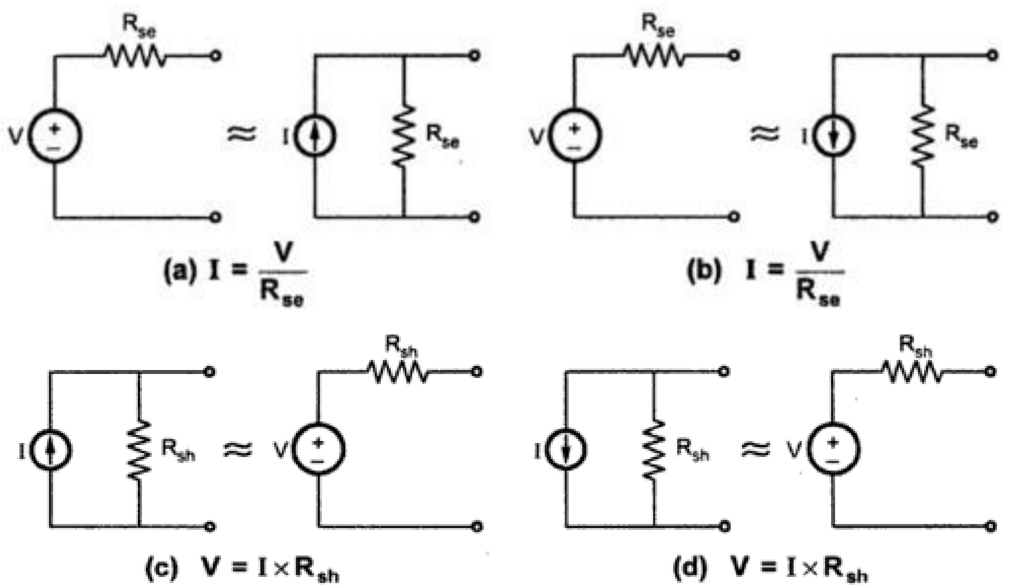


Fig. 1.31 Source transformation

Example 1.4 : *Transform a voltage source of 20 volts with an internal resistance of 5 Ω to a current source.*

Solution : Refer to the Fig. 1.32 (a).

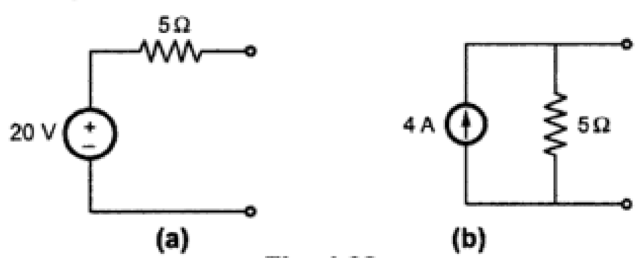


Fig. 1.32

Then current of current source is, $I = \frac{V}{R_{se}} = \frac{20}{5} = 4$ A with internal parallel resistance same as R_{se}.

∴ Equivalent current source is as shown in the Fig. 1.32 (b).

Example 1.5 : *Convert the given current source of 50 A with internal resistance of 10 Ω to the equivalent voltage source.*

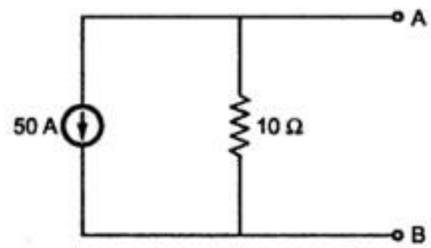


Fig. 1.33

Solution : The given values are, I = 50 A and R_{sh} = 10 Ω .

For the equivalent voltage source,

$$V = I \times R_{sh} = 50 \times 10 = 500 \text{ V}$$

$$V = I \times R_{sh} = 50 \times 10 = 500 \text{ V}$$

$$R_{se} = R_{sh} = 10\ \Omega \text{ in series.}$$

The equivalent voltage source is shown in the Fig. 1.33 (a).

Note the polarities of voltage source, which are such that +ve at top of arrow and –ve at bottom.

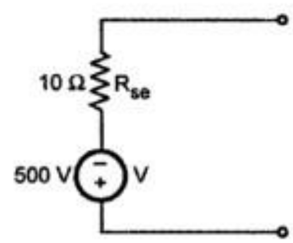


Fig. 1.33 (a)

1.19 Series Circuit

A **series** circuit is one in which several resistances are connected one after the other. Such connection is also called **end to end** connection or **cascade** connection. There is only one path for the flow of current.

1.19.1 Resistors in Series

Consider the resistances shown in the Fig. 1.34.

The resistance R_1, R_2 and R_3 are said to be in series. The combination is connected across a source of voltage V volts. Naturally the current flowing through all of them is same indicated as I amperes. e.g. the chain of small lights, used for the decoration purposes is good example of series combination.

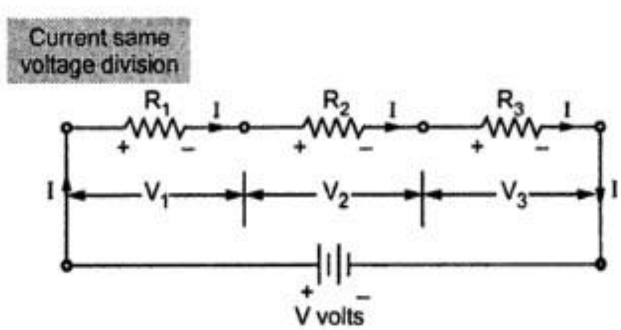


Fig. 1.34 A series circuit

Now let us study the **voltage distribution.**

Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively

Then, $$V = V_1 + V_2 + V_3$$

Now according to Ohm's law, $$V_1 = I R_1,\ V_2 = I R_2,\ V_3 = I R_3$$

Current through all of them is same i.e. I

$$\therefore \quad V = I R_1 + I R_2 + I R_3 = I(R_1 + R_2 + R_3)$$

Applying Ohm's law to overall circuit,

$$V = I R_{eq}$$

where R_{eq} = Equivalent resistance of the circuit. By comparison of two equations,

$$R_{eq} = R_1 + R_2 + R_3$$

i.e. total or **equivalent resistance** of the series circuit is arithmetic sum of the resistances connected in series.

For n resistances in series,	$R = R_1 + R_2 + R_3 + \ldots\ldots + R_n$

1.19.1.1 Characteristics of Series Circuits

1) The same current flows through each resistance.
2) The supply voltage V is the sum of the individual voltage drops across the resistances.

$$V = V_1 + V_2 + \ldots.. + V_n$$

3) The equivalent resistance is equal to the sum of the individual resistances.
4) The equivalent resistance is the largest of all the individual resistances.

i.e $\quad R > R_1,\ R > R_2,\ \ldots.. R > R_n$

1.19.2 Inductors in Series

Consider the Fig. 1.35 (a). Two inductors L_1 and L_2 are connected in series. The currents flowing through L_1 and L_2 are i_1 and i_2 while voltages developed across L_1 and L_2 are V_{L1} and V_{L2} respectively. The equivalent circuit is shown in the Fig. 1.35 (b).

We have, $\quad V_{L1} = L_1 \frac{di_1}{dt} \quad$ and $\quad V_{L2} = L_2 \frac{di_2}{dt} \quad$ while $\quad V_L = L_{eq} \frac{di}{dt}$

For series combination,

$$i = i_1 = i_2$$

(a) Fig. 1.35 (b)

and $V_L = V_{L1} + V_{L2}$

$$\therefore \quad L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$

$$\therefore \quad L_{eq}\frac{di}{dt} = (L_1 + L_2)\frac{di}{dt}$$

$$\therefore \quad L_{eq} = L_1 + L_2$$

That means, equivalent inductance of the series combination of two inductances is the sum of inductances connected in series.

The total equivalent inductance of the series circuit is sum of the inductances connected in series.

For n inductances in series,

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \ldots + L_n}$$

1.19.3 Capacitors in Series

Consider the Fig. 1.36 (a). Two capacitors C_1 and C_2 are connected in series. The currents flowing through and voltages developed across C_1 and C_2 are i_1, i_2 and V_{C1} and V_{C2} respectively. The equivalent circuit is shown in the Fig. 1.36 (b).

(a) Fig. 1.36 (b)

We have, $V_{C1} = \frac{1}{C_1}\int_{-\infty}^{t} i_1 dt$, $V_{C2} = \frac{1}{C_2}\int_{-\infty}^{t} i_2 dt$ while $V = \frac{1}{C_{eq}}\int_{-\infty}^{t} i dt$

For series combination,

$$i = i_1 = i_2 \quad \text{and}$$

$$V_C = V_{C1} + V_{C2}$$

$$\frac{1}{C_{eq}}\int_{-\infty}^{t} i dt = \frac{1}{C_1}\int_{-\infty}^{t} i_1 dt + \frac{1}{C_2}\int_{-\infty}^{t} i_2 dt$$

But $$i = i_1 = i_2$$

$$\therefore \quad \frac{1}{C_{eq}} \int_{-\infty}^{t} i\,dt = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) \int_{-\infty}^{t} i\,dt$$

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\therefore \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

That means, reciprocal of equivalent capacitor of the series combination is the sum of the reciprocal of individual capacitances.

The reciprocal of the total equivalent capacitor of the series combination is the sum of the reciprocols of the individual capacitors, connected in series.

For n capacitors in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \frac{1}{C_n}$$

1.20 Parallel Circuits

The **parallel circuit** is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point.

1.20.1 Resistors in Parallel

Consider a parallel circuit shown in the Fig. 1.38.

In the parallel connection shown, the three resistances R_1, R_2 and R_3 are connected in parallel and combination is connected across a source of voltage 'V'.

In parallel circuit current passing through each resistance is different. Let total current drawn is say ' I ' as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. Depending upon the values of R_1, R_2 and R_3 the appropriate fraction of total current passes through them. These individual currents are shown as I_1, I_2 and I_3. While the voltage across the two ends of each resistances R_1, R_2 and R_3 is the same and equals the supply voltage V.

Now let us study current distribution. Apply Ohm's law to each resistance.

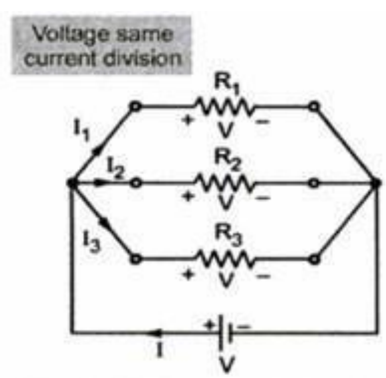


Fig. 1.38 A parallel circuit

$$V = I_1 R_1 , \quad V = I_2 R_2 , \quad V = I_3 R_3$$

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad \ldots (1)$$

For overall circuit if Ohm's law is applied,

$$V = I R_{eq}$$

and $$I = \frac{V}{R_{eq}} \quad \ldots (2)$$

where R_{eq} = Total or **equivalent resistance** of the circuit

Comparing the two equations,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

where R is the equivalent resistance of the parallel combination.

In general if 'n' resistances are connected in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$$

Conductance (G) :

It is known that, $\frac{1}{R} = G$ (conductance) hence,

$$\therefore \quad G = G_1 + G_2 + G_3 + \ldots\ldots + G_n \quad \ldots \text{ For parallel circuit}$$

Important result :

Now if n = 2, two resistances are in parallel then,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

This formula is directly used hereafter, for two resistances in parallel.

1.20.1.1 Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.

2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.

$$I = I_1 + I_2 + I_3 + \ldots\ldots + I_n$$

3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.

4) The equivalent resistance is the smallest of all the resistances.

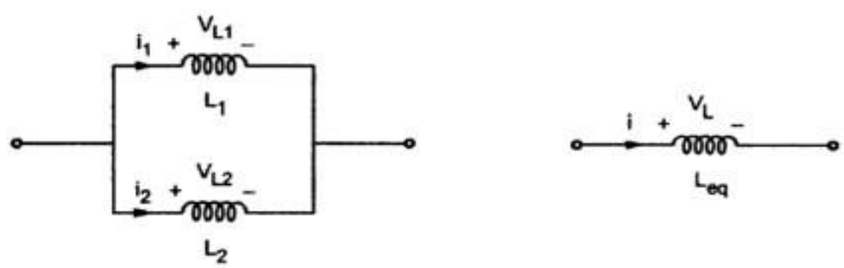
$$R < R_1, \quad R < R_2, \ldots\ldots, R < R_n$$

5) The equivalent conductance is the arithmetic addition of the individual conductances.

Key Point : *The equivalent resistance is smaller than the smallest of all the resistances connected in parallel.*

1.20.2 Inductors in Parallel

Consider the Fig. 1.39 (a). Two inductors L_1 and L_2 are connected in parallel. The currents flowing through L_1 and L_2 are i_1 and i_2 respectively. The voltage developed across L_1 and L_2 are V_{L1} and V_{L2} respectively. The equivalent circuit is shown in Fig. 1.39 (b).



(a) Fig. 1.39 (b)

For inductor we have,

$$i_1 = \frac{1}{L_1}\int_{-\infty}^{t} V_{L1}\, dt, \qquad i_2 = \frac{1}{L_2}\int_{-\infty}^{t} V_{L2}\, dt, \quad \text{while } i = \frac{1}{L_{eq}}\int_{-\infty}^{t} V_L\, dt$$

For parallel combination,

$$V_L = V_{L1} = V_{L2} \quad \text{and}$$

$$i = i_1 + i_2$$

$$\therefore \quad \frac{1}{L_{eq}}\int_{-\infty}^{t} V_L dt = \frac{1}{L_1}\int_{-\infty}^{t} V_L\, dt + \frac{1}{L_2}\int_{-\infty}^{t} V_L\, dt = \left(\frac{1}{L_1}+\frac{1}{L_2}\right)\int_{-\infty}^{t} V_L\, dt$$

$$\therefore \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

That means, reciprocal of equivalent inductance of the parallel combination is the sum of reciprocals of the individual inductances.

That means, reciprocal of equivalent inductance of the parallel combination is the sum of reciprocals of the individual inductances.

For n inductances in parallel,

$$\therefore \quad \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots \frac{1}{L_n}}$$

1.20.3 Capacitors in Parallel

Consider the Fig. 1.40 (a). Two capacitors C_1 and C_2 are connected in parallel. The currents flowing through C_1 and C_2 are i_1 and i_2 respectively and voltages developed across C_1, C_2 are V_{C1} and V_{C2} respectively.

The equivalent circuit is shown in the Fig. 1.40 (b).

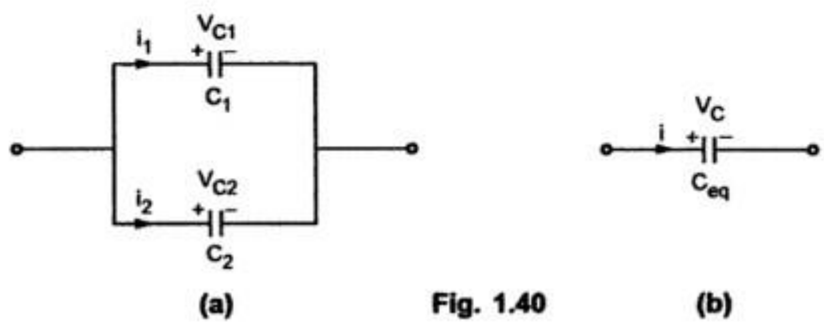


Fig. 1.40

For capacitor we have, $i_1 = C_1 \frac{dV_{C1}}{dt}$, $i_2 = C_2 \frac{dV_{C2}}{dt}$, while $i = C_{eq} \frac{dV_C}{dt}$

For parallel combination,

$$V_{C1} = V_{C2} = V_C \quad \text{and}$$

$$i = i_1 + i_2$$

$$C_{eq} \frac{dV_C}{dt} = C_1 \frac{dV_{C1}}{dt} + C_2 \frac{dV_{C2}}{dt}$$

$$\therefore \quad C_{eq} \frac{dV_C}{dt} = (C_1 + C_2) \frac{dV_C}{dt}$$

$$\therefore \quad C_{eq} = C_1 + C_2$$

That means, equivalent capacitance of the parallel combination of the capacitances is the sum of the individual capacitances connected in series.

For n capacitors in parallel,

$$\therefore \quad \boxed{C_{eq} = C_1 + C_2 + \ldots + C_n}$$

The Table 1.4 gives the equivalent at 'n' basic elements is series,

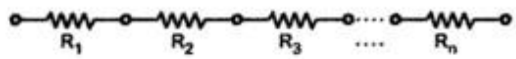
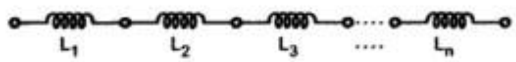
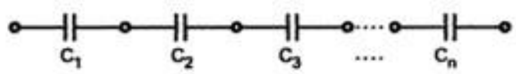
Element	Equivalent
'n' Resistances in series 	$R_{eq} = R_1 + R_2 + R_3 + ... + R_n$
'n' Inductors in series 	$L_{eq} = L_1 + L_2 + L_3 + ... + L_n$
'n' Capacitors in series 	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$

Table 1.4 Series combinations of elements

The Table 1.5 gives the equivalent of 'n' basic elements in parallel,

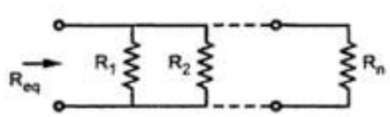
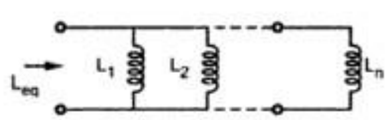
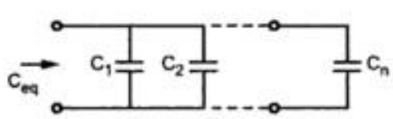
Element	Equivalent
'n' Resistances in parallel 	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$
'n' Inductors in parallel 	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$
'n' Capacitors in parallel 	$C_{eq} = C_1 + C_2 + ... + C_n$

Table 1.5 Parallel combinations of elements

Key Point : *The current through series combination remains same and voltage gets divided while in parallel combination voltage across combination remains same and current gets divided.*

Example 1 : *Find the equivalent resistance between the two points A and B shown in the Fig. 1.42.*

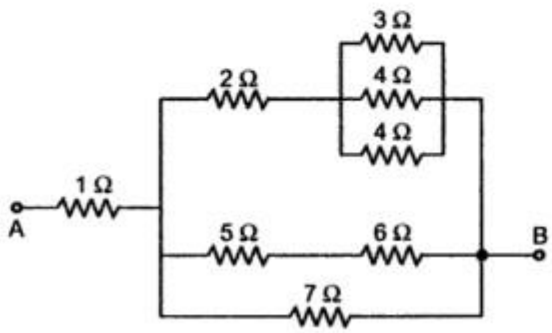


Fig. 1.42

Solution : Identify combinations of series and parallel resistances.

The resistances 5 Ω and 6 Ω are in series, as going to carry same current.

So equivalent resistance is 5 + 6 = 11 Ω

While the resistances 3 Ω , 4 Ω, and 4 Ω are in parallel, as voltage across them same but current divides.

∴ Equivalent resistance is, $\frac{1}{R} = \frac{1}{3}+\frac{1}{4}+\frac{1}{4} = \frac{10}{12}$

∴ $R = \frac{12}{10} = 1.2\ \Omega$

Replacing these combinations redraw the figure as shown in the Fig. 1.42 (a).

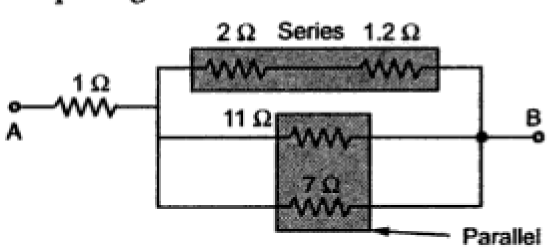


Fig. 1.42 (a)

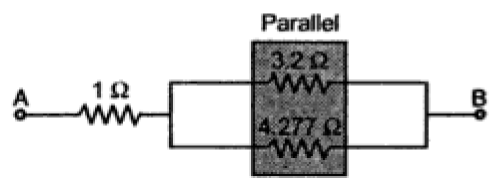


Fig. 1.42 (b)

Now again 1.2 Ω and 2 Ω are in series so equivalent resistance is 2 + 1.2 = 3.2 Ω while 11 Ω and 7 Ω are in parallel.

Using formula $\frac{R_1 R_2}{R_1 + R_2}$ equivalent resistance is $\frac{11\times7}{11+7} = \frac{77}{18} = 4.277\Omega$.

Replacing the respective combinations redraw the circuit as shown in the Fig. 1.42 (b).

Now 3.2 and 4.277 are in parallel.

∴ Replacing them by $\frac{3.2\times4.277}{3.2+4.277} = 1.8304\ \Omega$

∴ $R_{AB} = 1 + 1.8304 = \mathbf{2.8304}$

1.21 Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the network plays an important role.

1.21.1 Short Circuit

When any two points in a network are joined directly to each other with a thick metalic conducting wire, the two points are said to be short circuited. **The resistance of such short circuit is zero.**

The part of the network, which is short circuited is shown in the Fig. 1.43. The points A and B are short circuited. The resistance of the branch AB is $R_{sc} = 0\ \Omega$.

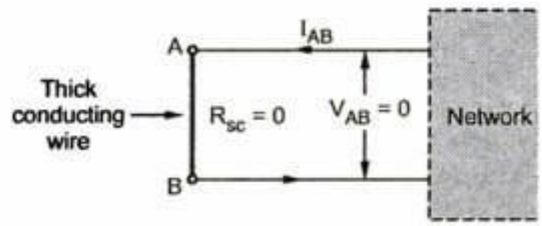


Fig. 1.43 Short circuit

The curent I_{AB} is flowing through the short circuited path.

According to Ohm's law,

$$V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0\ V$$

Key Point : *Thus, voltage across short circuit is always zero though current flows through the short circuited path.*

1.21.2 Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

As there is no direct connection in an open circuit, **the resistance of the open circuit is ∞.**

The part of the network which is open circuited is shown in the Fig. 1.44. The points A and B are said to be open circuited. The resistance of the branch AB is $R_{oc} = \infty\ \Omega$.

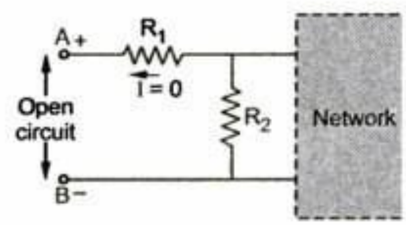


Fig. 1.44 Open circuit

There exists a voltage across the points AB called open circuit voltage, V_{AB} but $R_{oc} = \infty\ \Omega$.

According to Ohm's law,

$$I_{oc} = \frac{V_{AB}}{R_{oc}} = \frac{V_{AB}}{\infty} = 0\ A$$

Key Point : *Thus, current through open circuit is always zero though there exists a voltage across open circuited terminals.*

1.22 Voltage Division in Series Circuit of Resistors

Consider a series circuit of two resistors R_1 and R_2 connected to source of V volts.

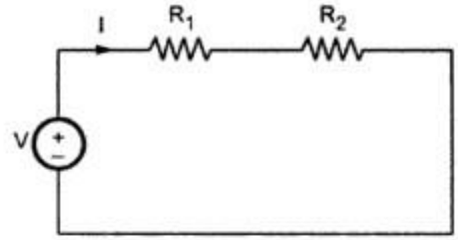


Fig. 1.46

As two resistors are connected in series, the current flowing through both the resistors is same, i.e. I. Then applying KVL, we get,

$$V = I R_1 + I R_2$$

$$\therefore \quad I = \frac{V}{R_1 + R_2}$$

Total voltage applied is equal to the sum of voltage drops V_{R1} and V_{R2} across R_1 and R_2 respectively.

$$\therefore \quad V_{R1} = I \cdot R_1$$

$$\therefore \quad \boxed{V_{R1} = \frac{V}{R_1 + R_2} \cdot R_1 = \left[\frac{R_1}{R_1 + R_2}\right] V}$$

Similarly, $V_{R2} = I \cdot R_2$

$$\therefore \quad \boxed{V_{R2} = \frac{V}{R_1 + R_2} \cdot R_2 = \left[\frac{R_2}{R_1 + R_2}\right] V}$$

So this circuit is a **voltage divider circuit.**

Example 1 : *Find the voltage across the three resistances shown in the Fig. 1.47.*

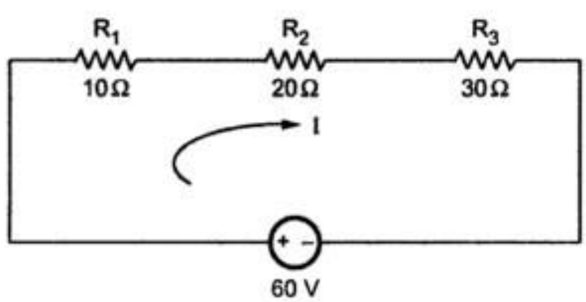


Fig. 1.47

Solution : $I = \frac{V}{R_1 + R_2 + R_3}$ ... series circuit

$$= \frac{60}{10+20+30} = 1 \text{ A}$$

$$\therefore \quad V_{R1} = IR_1 = \frac{V \times R_1}{R_1 + R_2 + R_3} = 1 \times 10 = \mathbf{10\ V}$$

$$\therefore \quad V_{R2} = IR_2 = \frac{V \times R_2}{R_1 + R_2 + R_3} = 1 \times 20 = \mathbf{20\ V}$$

and $$V_{R3} = IR_3 = \frac{V \times R_3}{R_1 + R_2 + R_3} = 1 \times 30 = \mathbf{30\ V}$$

Key Point : *It can be seen that voltage across any resistance of series circuit is ratio of that resistance to the total resistance, multiplied by the source voltage.*

1.23 Current Division in Parallel Circuit of Resistors

Consider a parallel circuit of two resistors R_1 and R_2 connected across a source of V volts.

Current through R_1 is I_1 and R_2 is I_2, while total current drawn from source is I_T.

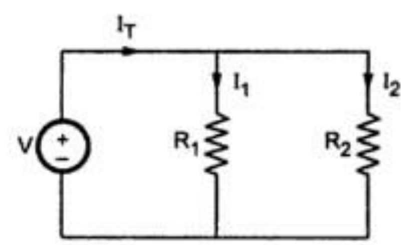


Fig. 1.48

$$\therefore \quad I_T = I_1 + I_2$$

But $I_1 = \frac{V}{R_1}$, $\quad I_2 = \frac{V}{R_2}$

i.e. $V = I_1 R_1 = I_2 R_2$

$$\therefore \quad I_1 = I_2 \left(\frac{R_2}{R_1}\right)$$

Substituting value of I_1 in I_T,

$$\therefore \quad I_T = I_2 \left(\frac{R_2}{R_1}\right) + I_2 = I_2 \left[\frac{R_2}{R_1} + 1\right] = I_2 \left[\frac{R_1 + R_2}{R_1}\right]$$

$$\therefore \quad \boxed{I_2 = \left[\frac{R_1}{R_1 + R_2}\right] I_T}$$

Now $$I_1 = I_T - I_2 = I_T - \left[\frac{R_1}{R_1 + R_2}\right] I_T$$

$$\therefore \quad I_1 = \left[\frac{R_1 + R_2 - R_1}{R_1 + R_2}\right] I_T$$

$$\therefore \quad \boxed{I_1 = \left[\frac{R_2}{R_1 + R_2}\right] I_T}$$

Key Point : *In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.*

Example 1 : *Find the magnitudes of total current, current through R_1 and R_2 if, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $V = 50$ V.*

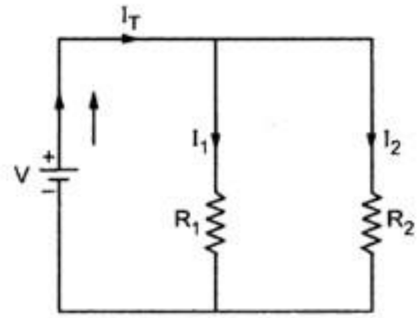


Fig. 1.49

Solution : The equivalent resistance of two is,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 20}{10 + 20} = 6.67\ \Omega$$

$$\therefore \quad I_T = \frac{V}{R_{eq}} = \frac{50}{6.67} = \mathbf{7.5\ A}$$

As per the current distribution in parallel circuit,

$$I_1 = I_T\left(\frac{R_2}{R_1 + R_2}\right) = 7.5 \times \left(\frac{20}{10+20}\right)$$

$$= \mathbf{5\ A}$$

and

$$I_2 = I_T\left(\frac{R_1}{R_1 + R_2}\right) = 7.5 \times \left(\frac{10}{10+20}\right)$$

$$= \mathbf{2.5\ A}$$

It can be verified that $I_T = I_1 + I_2$

1.26 Loop Analysis or Mesh Analysis

This method of analysis is specially useful for the circuits that have many nodes and loops. The difference between application of Kirchhoff's laws and loop analysis is, in loop analysis instead of branch currents, the loop currents are considered for writing the equations. The another difference is, in this method, each branch of the network may carry more than one current. The total branch current must be decided by the algebraic sum of all currents through that branch. While in analysis using Kirchhoff's laws, each branch carries only one current. The advantage of this method is that for complex networks the number of unknowns reduces which greatly simplifies calculation work.

Consider following network shown in the Fig. 1.55. There are two loops. So assuming two loop currents as I_1 and I_2.

Key Point: *While assume loop currents, consider the loops such that each element of the network will be included at least once in any of the loops.*

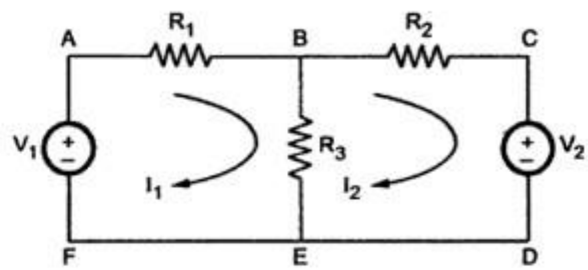


Fig. 1.55

Now branch B-E carries two currents; I_1 from B to E and I_2 from E to B. So net current through branch B-E will $(I_1 - I_2)$ and corresponding drop across R_3 must be as shown below in the Fig. 1.56.

Consider loop A - B - E - F - A,

For branch B-E, polarities of voltage drops will be B +ve, E –ve for current I_1 while E +ve, B –ve for current I_2 flowing through R_3.

Now while writing loop equations assume main loop current as positive and remaining loop current must be treated as negative for common branches.

Writing loop equations for the network shown in the Fig. 1.57.

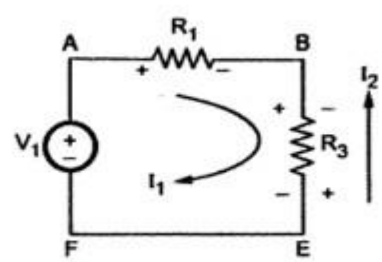


Fig. 1.57

For loop A - B - E - F - A,

$$-I_1 R_1 - I_1 R_3 + I_2 R_3 + V_1 = 0$$

For loop B - C - D - E - B

$$-I_2 R_2 - V_2 - I_2 R_3 + I_1 R_3 = 0$$

By solving above simultaneous equations any unknown branch current can be determined.

1.26.1 Points to Remember for Loop Analysis

1. While assuming loop currents make sure that at least one loop current links with every element.
2. No two loops should be identical.
3. Choose minimum number of loop currents.
4. Convert current sources if present, into their equivalent voltage sources for loop analysis, whenever possible.
5. If current in a particular branch is required, then try to choose loop current in such a way that only one loop current links with that branch.

1.26.2 Supermesh

Key Point: *If there exists a current source in any of the branches of the network then a loop can not be defined through the current source as drop across the current source is unknown, from KVL point of view.*

For example, consider the network shown in the Fig. 1.58. In this circuit, branch B-E consists of a current source. So loop ABEFA can not be defined as loop from KVL point of view, as drop across the current source is not known.

In such case, **to get the required equation interms of loop currents, analyse the branch consisting of a current source independently. Express the current source interms of the assumed loop currents**. For example, in the Fig. 1.58 analyse the branch BE. The current source is of I A in the direction of loop current I_2. So I_2 is more than I_1 and we can write an equation,

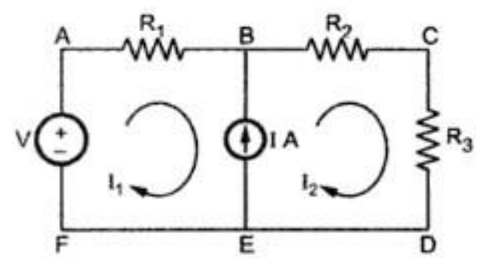


Fig. 1.58

$$I = I_2 - I_1$$

So all such branches, consisting current sources must be analysed independently. Get the equations for current sources interms of loop currents. Then apply KVL to the remaining loops which are existing without involving the branches consisting of current sources. The loop existing, around a current source which is common to the two loops is called **supermesh**. In the Fig. 1.58, the loop ABCDEFA is supermesh.

1.27 Node Analysis

This method is mainly based on Kirchhoff's Current Law (KCL). This method uses the analysis of the different nodes of the network. We have already defined a node. Every junction point in a network, where two or more branches meet is called a node. One of the nodes is assumed as reference node whose potential is assumed to be zero. It is also called zero potential node or datum node. At other nodes the different voltages are to be measured with respect to this reference node. The reference node should be given a

number zero and then the equations are to be written for all other nodes by applying KCL. The advantage of this method lies in the fact that we get (n – 1) equations to solve if there are 'n' nodes. This reduces calculation work.

Consider the following network shown in the Fig. 1.61.

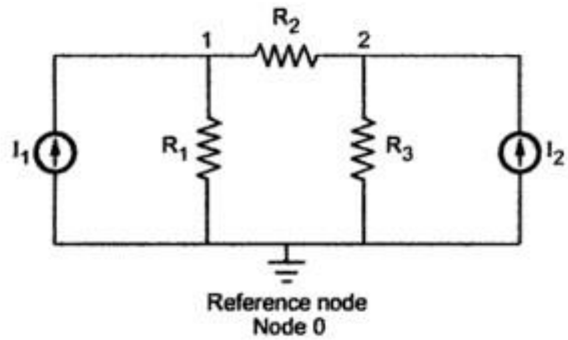


Fig. 1.61

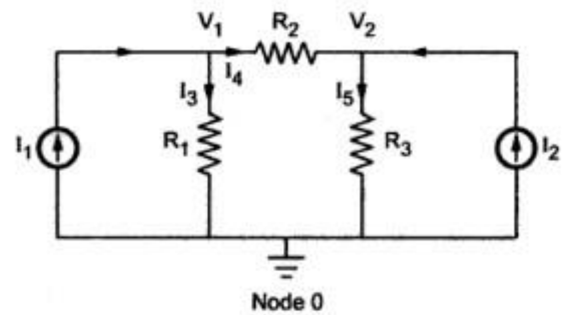


Fig. 1.62

Let voltages at node 1 and node 2 be V_1 and V_2. Mark various branch currents as shown in the Fig. 1.62. Now analyse each node using **KCL independently**.

Now applying KCL at node 1,

$$I_1 - I_3 - I_4 = 0 \quad \text{... (1)}$$

At node,

$$I_2 + I_4 - I_5 = 0 \quad \text{... (2)}$$

The currents in these equations can be expressed interms of node voltages as,

$$I_1 - \frac{V_1}{R_1} - \frac{(V_1 - V_2)}{R_2} = 0 \quad \text{... (3)}$$

and

$$I_2 + \frac{(V_1 - V_2)}{R_2} - \frac{V_2}{R_3} = 0 \quad \text{... (4)}$$

As I_1 and I_2 are known, we get two equations (3) and (4) with the two unknowns V_1 and V_2. Solving these equations simultaneously, the node voltages V_1 and V_2 can be determined. Once V_1 and V_2 are known, current through any branch of the network can be determined. If there exists a voltage source in any of the branches as shown in the Fig. 1.63 then that must be considered while writing the equation for the current through that branch.

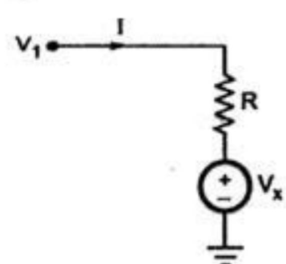


Fig. 1.63

Now V_1 is at higher potential with respect to base, forcing current I downwards. While polarity of V_x is such that it tries to force current upwards. So in such a case equation of current becomes,

$$I = \frac{V_1 - V_x}{R} \qquad \text{... I leaving the node}$$

If the direction of current I is assumed entering the node then it is assumed that V_x is more than V_1 and hence equation for current I becomes,

$$I = \frac{V_x - V_1}{R} \qquad \text{... I entering the node}$$

1.27.1 Points to Remember for Nodal Analysis

1. While assuming branch currents, make sure that each unknown branch current is considered at least once.
2. Convert the voltage source present into their equivalent current sources for node analysis, wherever possible.
3. Follow the same sign convention, currents entering at node are to be considered positive, while currents leaving the node are to be considered as negative.
4. As far as possible, select the directions of various branch currents leaving the respective nodes.

1.27.2 Supernode

Consider a circuit shown in the Fig. 1.64. In this circuit, the nodes labelled V_2 and V_3 are connected directly through a voltage source, without any circuit element. The region surrounding a voltage source which connects the two node directly is called **supernode**.

In such a case, the nodes in supernode region can be analysed seperately and the relation between such node voltages and a source voltage connecting them can be separately obtained. In the circuit shown in the Fig. 1.64 we can write,

$$V_2 = V_3 + V_x$$

In addition to this equation, apply KCL to all the nodes assuming different branch currents at the nodes. The current through voltage source, connecting supernodes must be expressed interms of node voltages, using these KCL equations. Then the resulting equations and supernode equation are to be solved simultaneously to obtain the required unknown.

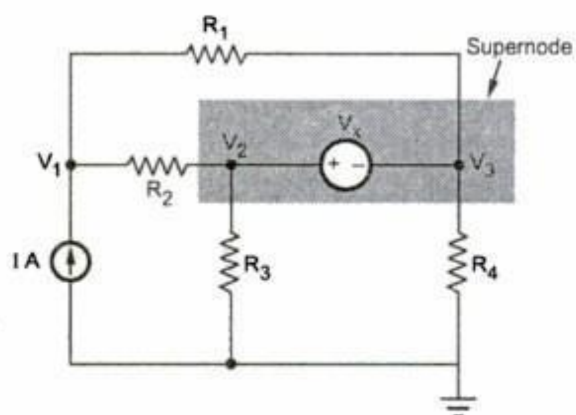


Fig. 1.64 Region of Supernode

1.27.3 Steps for the Node Analysis

Step 1 : Choose the nodes and node voltages to be obtained.

Step 2 : Choose the currents preferably leaving the node at each branch connected to each node.

Step 3 : Apply KCL at each node with proper sign convention.

Step 4 : If there are supernodes, obtain the equations directly interms of node voltages which are directly connected through voltage source.

Step 5 : Obtain the equation for the each branch current interms of node voltages and substitute in the equations obtained in step 3.

Step 6 : Solve all the equations obtained in step 4 and step 5 simultaneously to obtain the required node voltages.

Key Point: *If there are many number of branches in parallel in a network then node method is advantageous for the network analysis.*

➡ **Example 1.12 :** *Find the current through each resistor of the circuit shown in the Fig. 1.65, using nodal analysis.*

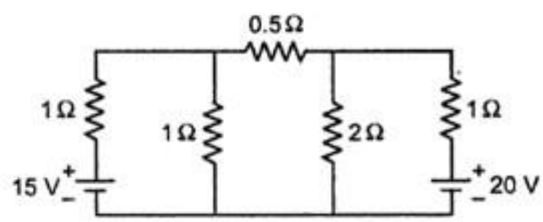


Fig. 1.65

Solution : The various node voltages and currents are shown in the Fig. 1.65 (a).

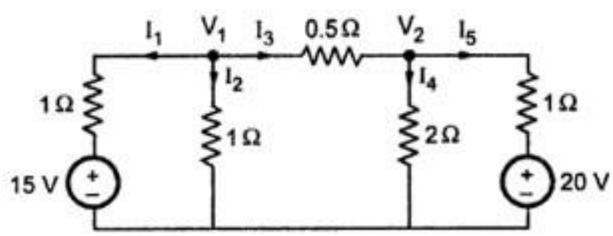


Fig. 1.65 (a)

At node 1, $$-I_1 - I_2 - I_3 = 0$$

$$\therefore \quad -\left[\frac{V_1 - 15}{1}\right] - \left[\frac{V_1}{1}\right] - \left[\frac{V_1 - V_2}{0.5}\right] = 0$$

$\therefore \quad -V_1 + 15 - V_1 - 2V_1 + 2V_2 = 0$

$\therefore \quad 4V_1 - 2V_2 = 15$ ... (1)

At node 2, $\quad I_3 - I_4 - I_5 = 0$

$$\therefore \quad \frac{V_1 - V_2}{0.5} - \frac{V_2}{2} - \frac{V_2 - 20}{1} = 0$$

$\therefore \quad 2V_1 - 2V_2 - 0.5\ V_2 - V_2 + 20 = 0$

$\therefore \quad 2V_1 - 3.5\ V_2 = -20$ ...(2)

Multiplying (2) by 2 and subtracting from (1) we get,

$$5V_2 = 55$$

$$\therefore \quad V_2 = 11\ V$$

and $\quad V_1 = 9.25\ V$

Hence the various currents are,

$$I_1 = \frac{V_1 - 5}{1} = 9.25 - 15 = -5.75A \text{ i.e. } \mathbf{5.75\ A} \uparrow$$

$$I_2 = \frac{V_1}{1} = \mathbf{9.25\ A}$$

$$I_3 = \frac{V_1 - V_2}{0.5} = -3.5A \text{ i.e. } \mathbf{3.5\ A} \leftarrow$$

$$I_4 = \frac{V_2}{2} = \mathbf{5.5\ A}$$

Example 1.41 : *For the circuit shown in Fig. 1.94, determine the voltages (i) V_{df} and (ii) V_{ag}*

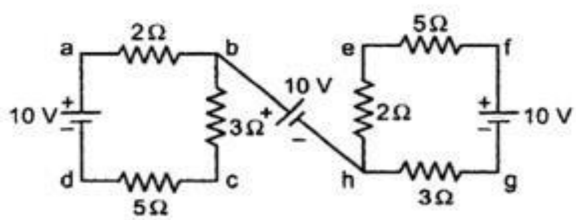


Fig. 1.94

Solution : Assume the two currents as shown in Fig. 1.94 (a).

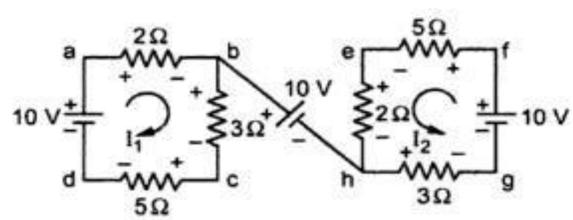


Fig. 1.94 (a)

Apply KVL to loop abcda,

$$-2 I_1 - 3 I_1 - 5 I_1 + 10 = 0$$

$$\therefore \quad -10 I_1 = -10$$

$$\therefore \quad I_1 = 1 \text{ A}$$

Apply KVL to loop efghe,

$$5 I_e - 10 + 3 I_2 + 2 I_2 = 0$$

$$\therefore \quad 10 I_2 = 10$$

$$\therefore \quad I_2 = 1 \text{ A}$$

Due to I_1 and I_2, various drops are shown in Fig. 1.94 (b).

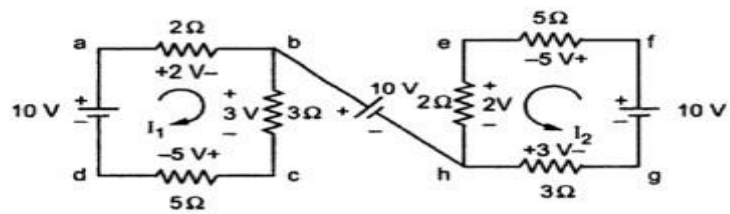


Fig. 1.94 (b)

1) We want V_{df} :

Trace the path df as shown n the Fig 1.95.

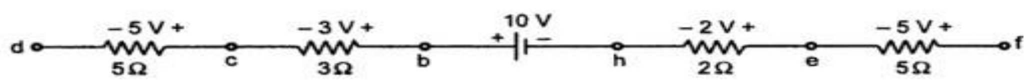


Fig. 1.95

$$\therefore \quad V_{df} = 10 - 5 - 3 - 2 - 5$$

$= -5$ **V** with – ve on d side and +ve on f side

$\therefore \quad V_{df} = 5$ V with d negative with respect to f

2) We want V_{ag} :

Trace the path ag as shown in Fig. 1.96.

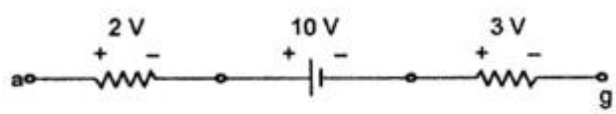


Fig. 1.96

$$\therefore \quad V_{ag} = 10 + 2 + 3$$

$V_{ag} = 10 + 2 + 3 =$ **15 V** with a positive with respect to g.

Example 1.25 : *Assuming ammeter has zero resistance, calculate its reading when connected in the circuit as shown in the Fig. 1.78.*

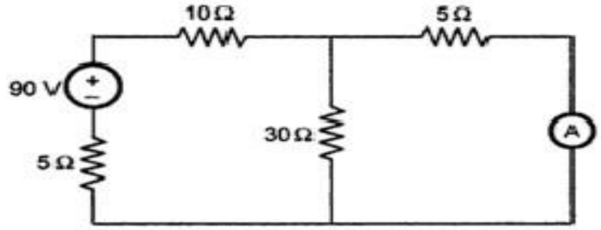


Fig. 1.78

Solution : Let us use loop current method.

Consider the loop currents and corresponding polarities in each loop assuming respective loop current positive.

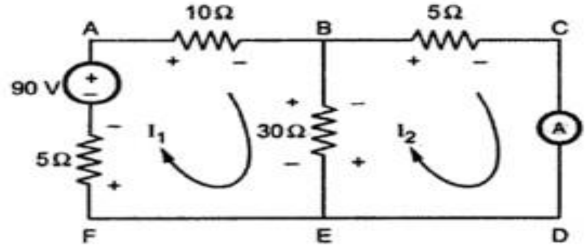


Fig. 1.78 (a)

Consider loops and apply KVL

Loop A-B-E-F-A,

$$-10\,I_1 - 30(I_1 - I_2) - 5\,I_1 + 90 = 0$$

$$45\,I_1 - 30\,I_2 = 90$$

$$3\,I_1 - 2\,I_2 = 6 \qquad \dots (1)$$

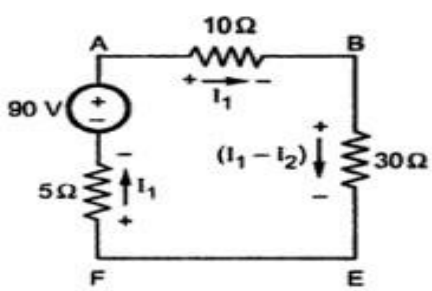


Fig. 1.78 (b)

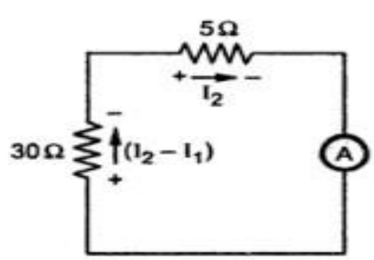


Fig. 1.78 (c)

Loop B-C-D-E-B as shown in Fig. 1.78 (c),

$$-5\,I_2 + 0 - 30(I_2 - I_1) = 0$$

(Drop across ammeter is zero as its resistance is given to be zero.)

i.e. $$6\,I_1 - 7\,I_2 = 0 \qquad \dots (2)$$

Apply Cramer's rule,

$$D = \begin{vmatrix} 3 & -2 \\ 6 & -7 \end{vmatrix} = -9, \quad D_1 = \begin{vmatrix} 6 & -2 \\ 0 & -7 \end{vmatrix} = -42$$

$$D_2 = \begin{vmatrix} 3 & 6 \\ 6 & 0 \end{vmatrix} = -36$$

$$I_1 = \frac{D_1}{D} = \frac{-42}{-9} = 4.67\text{ A}$$

$$I_2 = \frac{D_2}{D} = \frac{-36}{-9} = +4\text{ A}$$

∴ Current through the ammeter is 4 A, hence ammeter reading = **4 A.**

Example 1.26 : *In the circuit shown in the Fig. 1.79, use the loop analysis to find the power delivered to the 4 Ω resistor.*

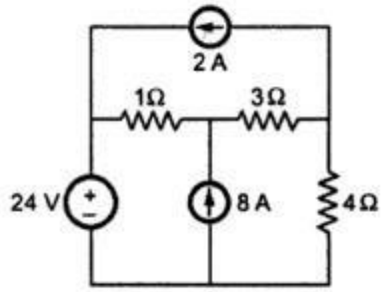


Fig. 1.79

Solution : The various loop currents are shown in the Fig. 1.79 (a). The problem consists of current sources hence follow supermesh steps.

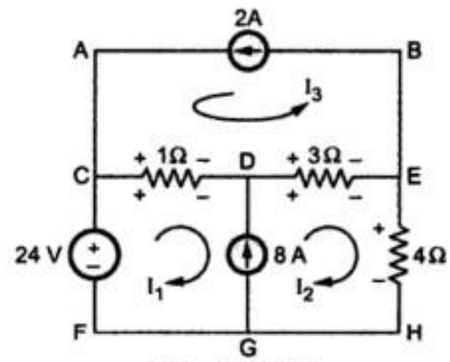


Fig. 1.79 (a)

Loops can not be defined through current sources. So analyse the branches consisting of current sources first.

From branch A-B we can write,

$$I_3 = 2 \text{ A} \qquad \dots (1)$$

From branch D-G we can write,

$$I_2 - I_1 = 8 \text{ A} \qquad \dots (2)$$

Now apply KVL to the loop **without current source** i.e. loop C-D-E-H-G-F-C,

$$-1 \times I_3 - 1 \times I_1 - 3I_3 - 3I_2 - 4I_2 + 24 = 0$$

$$\therefore \quad 4 I_3 + 7 I_2 + I_1 = 24 \qquad \dots (3)$$

Using (1) and (2) in (3) we get,

$$8 + 7 I_2 + (I_2 - 8) = 24$$

$$\therefore \quad 8I_2 = 24$$

$$\therefore \quad I_2 = 3 \text{ A}$$

This is current through 4 Ω resistor. So power delivered to the 4 Ω resistor is,

$$P = I_2^2 \times 4$$

$$= 3^2 \times 4 = 36 \text{ W}$$

Example 1.29 : *Using mesh analysis, calculate the current I_1 shown in the Fig. 1.82.*

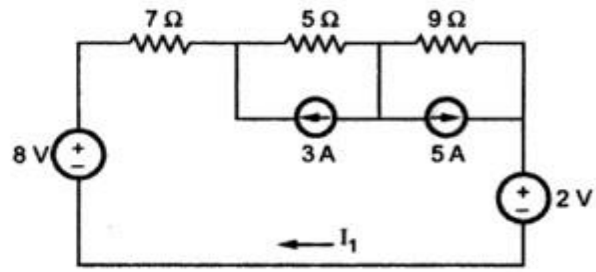


Fig. 1.82

Solution : The various loop currents are shown in the Fig. 1.82 (a).

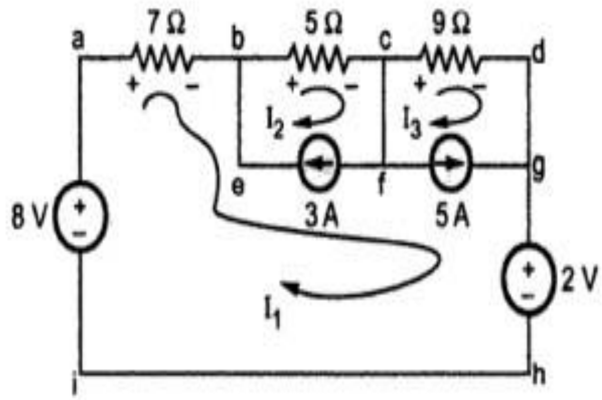


Fig. 1.82 (a)

Analyse the branch e-f,

$$3 = I_2 - I_1 \quad \text{... (1)}$$

Analyse the branch f-g,

$$5 = I_1 - I_3 \quad \text{... (2)}$$

Applying KVL to the loop a-b-c-d-g-h-i-a,

$$-7I_1 - 5\,I_2 - 9\,I_3 - 2 + 8 = 0$$

$$\therefore \quad +7\,I_1 + 5\,I_2 + 9\,I_3 = +6 \quad \text{... (3)}$$

Analyse the branch e-f,

$$3 = I_2 - I_1 \quad \dots (1)$$

Analyse the branch f-g,

$$5 = I_1 - I_3 \quad \dots (2)$$

Applying KVL to the loop a-b-c-d-g-h-i-a,

$$-7I_1 - 5\,I_2 - 9\,I_3 - 2 + 8 = 0$$

$$\therefore \quad +7\,I_1 + 5\,I_2 + 9\,I_3 = +6 \quad \dots (3)$$

Substituting from (1) and (2),

$$7\,I_1 + 5\,(3 + I_1) + 9\,(I_1 - 5) = 6$$

$$\therefore \quad 7\,I_1 + 15 + 5\,I_1 + 9\,I_1 - 45 = 6$$

$$\therefore \quad 21\,I_1 = 36$$

$$\therefore \quad I_1 = 1.7142 \text{ A}$$

Example 1.30 : *For the circuit given in the Fig. 1.83, find the branch currents I_1, I_2, I_3 and node voltages V_1 and V_2.*

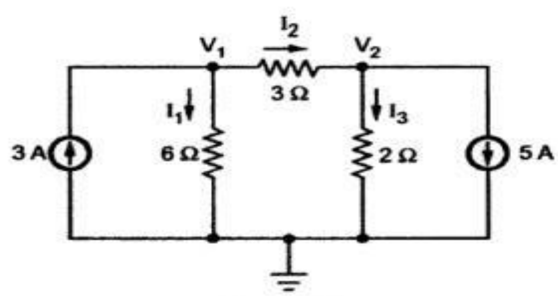


Fig. 1.83

Solution : Use node analysis method.

At node 1, $$+3 - I_1 - I_2 = 0 \quad \dots (1)$$

At node 2, $$I_2 - I_3 - 5 = 0 \quad \dots (2)$$

Now $$I_1 = \frac{V_1}{6}, \quad I_2 = \frac{V_1 - V_2}{3}, \quad I_3 = \frac{V_2}{2}$$

$$\therefore \quad 3 - \frac{V_1}{6} - \left[\frac{V_1 - V_2}{3}\right] = 0$$

$$\therefore \quad 0.5\ V_1 - 0.333\ V_2 = 3 \quad \dots (3)$$

And $$\frac{V_1 - V_2}{3} - \frac{V_2}{2} = 5$$

$$\therefore \quad 0.333\ V_1 - 0.8333\ V_2 = 5 \quad \dots (4)$$

$$\therefore \quad D = \begin{vmatrix} 0.5 & -0.333 \\ 0.333 & -0.8333 \end{vmatrix} = -0.30556$$

$$D_1 = \begin{vmatrix} 3 & -0.333 \\ 5 & -0.8333 \end{vmatrix} = -0.8334$$

$$D_2 = \begin{vmatrix} 0.5 & 3 \\ 0.333 & 5 \end{vmatrix} = 1.5$$

$$\therefore \quad V_1 = \frac{D_1}{D} = 2.7274 \text{ V} \quad \text{and} \quad V_2 = \frac{D_2}{D} = -4.909 \text{ V}$$

$$\therefore \quad I_1 = \frac{V_1}{6} = 0.4545 \text{ A}$$

$$\therefore \quad I_2 = \frac{V_1 - V_2}{3} = 2.5454 \text{ A}$$

$$\therefore \quad I_3 = \frac{V_2}{2} = -2.4545 \text{ A} = 2.4545 \uparrow$$

Example 1.35 : *What is the supply voltage V in order to get 1 ampere in 3 Ω resistor.*

(Nov./Dec.-2003)

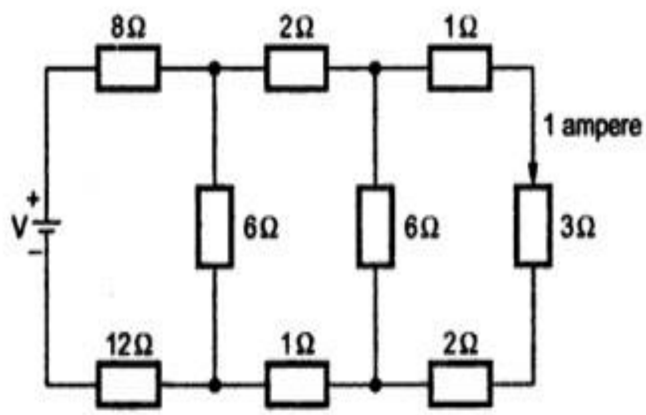


Fig. 1.88

Solution :

Step 1 :

Show the various loop currents.

Step 2 :

Show the polarities of drops due to these loop currents.

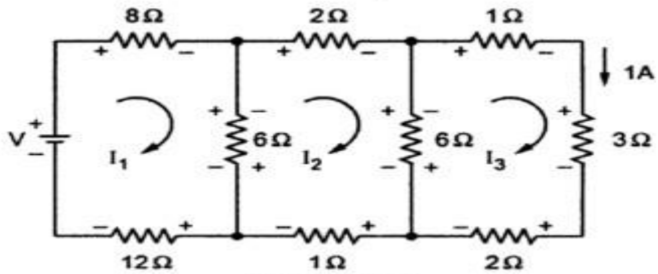


Fig. 1.88 (a)

Step 3 :

Apply KVL to various loops,

$$-8I_1 - 6I_1 + 6I_2 - 12I_1 + V = 0$$

$\therefore$ $$26 I_1 - 6 I_2 = V \quad \text{... (1)}$$

$$-2I_2 - 6I_2 + 6I_3 - I_2 - 6I_2 + 6I_1 = 0$$

$\therefore$ $$6I_1 - 15I_2 + 6I_3 = 0 \quad \text{... (2)}$$

$$-I_3 - 3I_3 - 2I_3 - 6I_3 + 6I_2 = 0$$

$\therefore$ $$6I_2 - 12I_3 = 0 \quad \text{... (3)}$$

But I_3 = Current through 3 Ω = 1 A given

So from (3), $$I_2 = \frac{12 I_3}{6} = 2\text{ A}$$

From (2), $$6I_1 - 15 \times 2 + 6 \times 1 = 0$$

$\therefore$ $$I_1 = 4\text{ A}$$

From (1), $$26 \times 4 - 6 \times 2 = V$$

$\therefore$ $$V = 92\text{ V}$$ ... Supply voltage

Example 1.36 : *Calculate the current in the 50 ohms resistor in the network shown in the Fig. 1.89 using Mesh analysis.* **(April/May-2005)**

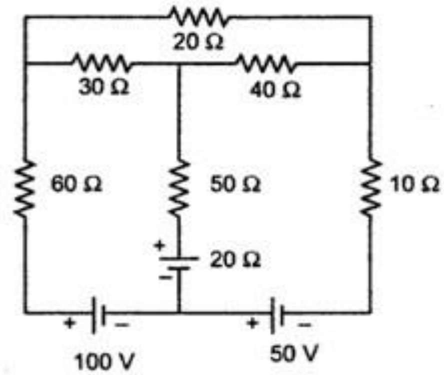


Fig. 1.89

Solution : The mesh currents are shown in the Fig. 1.89 (a).

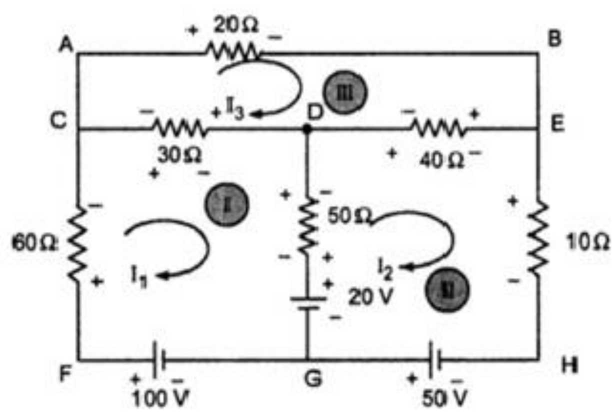


Fig. 1.89 (a)

Applying KVL to the three loops,

$- 30\, I_1 + 30\, I_3 - 50\, I_1 + 50\, I_2 - 20 + 100 - 60\, I_1 = 0$ ... Loop I

$$140\, I_1 - 50\, I_2 - 30\, I_3 = 80 \quad \text{... (1)}$$

$- 40\, I_2 + 40\, I_3 - 10\, I_2 + 50 + 20 - 50\, I_2 + 50\, I_1 = 0$ ... Loop II

$- 20\, I_3 - 40\, I_3 + 40\, I_2 - 30\, I_3 + 30\, I_1 = 0$ ... Loop III

$$30\, I_1 + 40\, I_2 - 90\, I_3 = 0 \quad \text{... (3)}$$

Solving equations (1), (2) and (3),

$$I_1 = 1.6489 \text{ A}, \; I_2 = 2.1214 \text{ A}, \; I_3 = 1.4925 \text{ A}$$

Thus the current through 50 Ω is,

$$I_{50} = I_1 - I_2 = 1.6489 - 2.121 = -\mathbf{0.4721\ A} \quad \text{i.e. } \mathbf{0.4721\ A} \uparrow$$

Example 1.37 : *Calculate the voltage across the 15 ohms resistor in the network shown in Fig. 1.90 using Nodal analysis.* **(April/May-2005)**

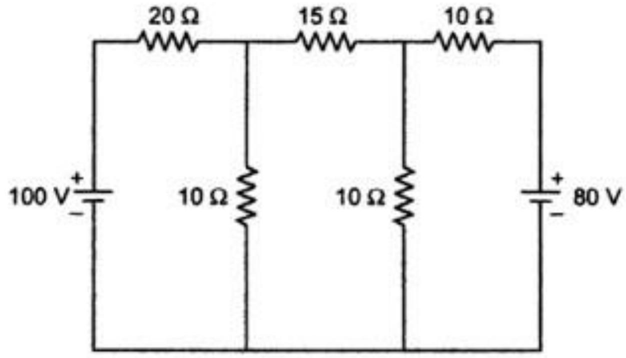


Fig. 1.90

Solution : The node voltages are shown in the Fig. 1.90 (a).

Applying KCL at node A and B,

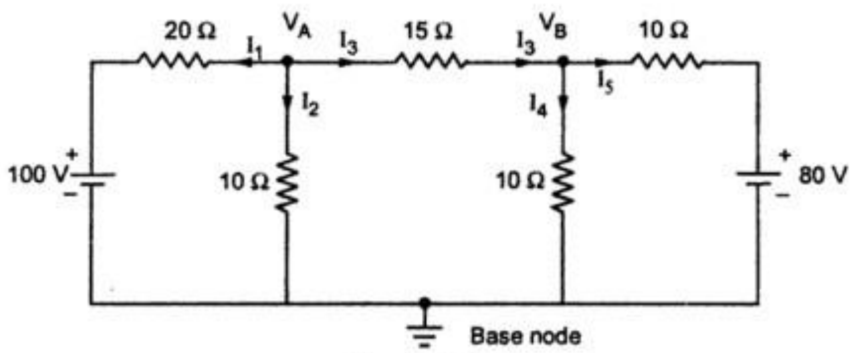


Fig. 1.90 (a)

$$-I_1 - I_2 - I_3 = 0 \quad \text{i.e.} \quad I_1 + I_2 + I_3 = 0 \qquad \dots (1)$$

$$I_3 - I_4 - I_5 = 0 \qquad \dots (2)$$

The expressions for the various currents are,

$$I_1 = \frac{V_A - 100}{20} = 0.05\ V_A - 5\ , \quad I_2 = \frac{V_A}{10} = 0.1\ V_A$$

$$I_3 = \frac{V_A - V_B}{15} = 0.066\ V_A - 0.066\ V_B\ , \quad I_4 = \frac{V_B}{10} = 0.1\ V_B$$

$$I_5 = \frac{V_B - 80}{10} = 0.1\ V_B - 8$$

Using in (1) and (2) ,

$0.05\ V_A - 5 + 0.1\ V_A + 0.0666\ V_A - 0.0666\ V_B = 0$

i.e. $\quad 0.21666\ V_A - 0.0666\ V_B = 5$ ... (3)

$0.0666\ V_A - 0.0666\ V_B - 0.1\ V_B - 0.1\ V_B + 8 = 0$

i.e. $\quad -0.0666\ V_A + 0.2666\ V_B = 8$ ... (4)

Solving (3) and (4) simultaneously,

$\therefore \quad V_A = 35\ V$ and $V_B = 38.75\ V$

$\therefore \quad V_{AB} = 38.75 - 35 =$ **3.75 V with B positive**

Example 1.38 : *Find the currents through various resistors in the circuit. Determine V_{BC}, V_{CD} and power across 1.5 k Ω resistor.* **(Nov./Dec.-2005)**

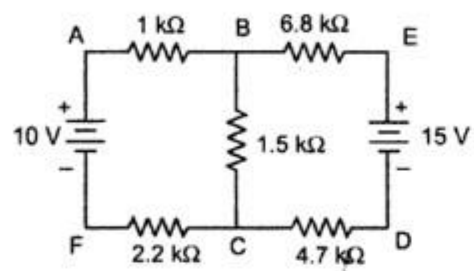


Fig. 1.91

Solution : Let us use **node voltage method.**

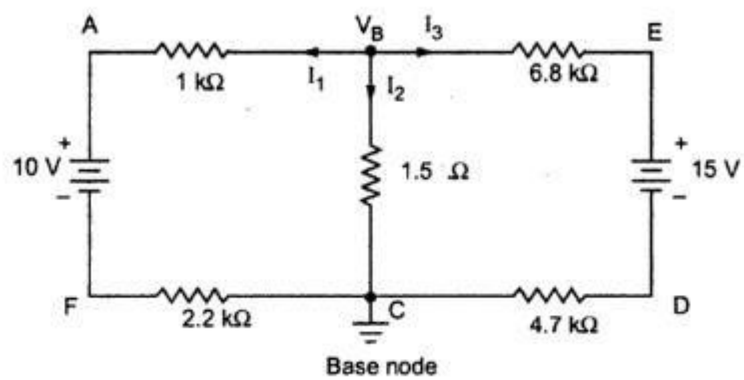


Fig. 1.91 (a)

Applying KCL at B, $-I_1 - I_2 - I_3 = 0$ i.e. $I_1 + I_2 + I_3 = 0$ ...(1)

Considering entire path from B to C through A and F,

$$I_1 = \frac{V_B - 10}{(1+2.2)\times 10^3} = 3.125\times 10^{-4}\ V_B - 3.125\times 10^{-3}$$

$$I_2 = \frac{V_B - 0}{1.5\times 10^3} = 6.666\times 10^{-4}\ V_B \qquad \text{... } V_C = 0 \text{ as C is base node}$$

$$I_3 = \frac{V_B - 15}{(6.8+4.7)\times 10^3} = 8.695\times 10^{-5}\ V_B - 1.3043\times 10^{-3}$$

... considering path B-E-D-C

Using I_1, I_2 and I_3 in equation (1),

$$3.125\times 10^{-4}\ V_B - 3.125\times 10^{-3} + 6.666\times 10^{-4}\ V_B + 8.695\times 10^{-5}\ V_B - 1.3043\times 10^{-3} = 0$$

$$\therefore \quad V_B = \frac{4.4293\times 10^{-3}}{1.06605\times 10^{-3}} = 4.1548 \text{ V}$$

Thus $V_{BC} = \mathbf{4.154\ V}$

The current through 1 k Ω and 2.2 k Ω is I_1,

$\therefore \quad I_1 = -1.8266\times 10^{-3}$ A i.e. **1.8266 mA** flowing from C-F-A-B

The current through 1.5 kΩ is I_2 = **2.7695 mA** flowing from B to C.

The current through 6.8 kΩ and 4.7 kΩ is I_3,

$\therefore \quad I_3 = -0.9431\times 10^{-3}$A i.e. **0.9431 mA** flowing from C-D-E-B

And $V_{CD} = I_3 \times 4.7\times 10^3 = \mathbf{-4.4322\ V}$ i.e. **4.4322 V with C positive.**

Example 1.39 : *Find equivalent resistance between points A-B.*

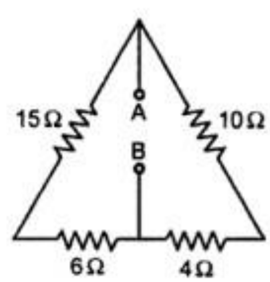


Fig. 1.92

Solution : Redrawing the circuit,

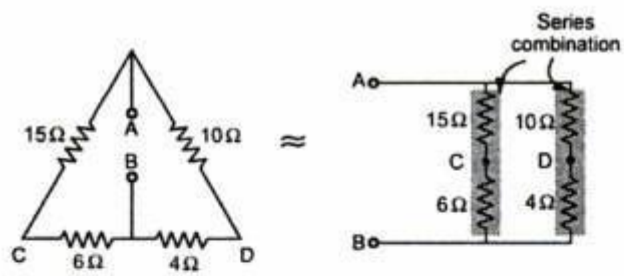


Fig. 1.92 (a)

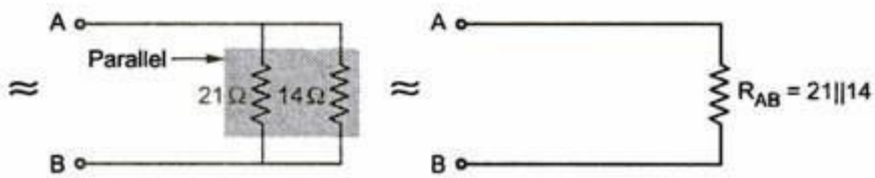


Fig. 1.92 (b)

$$R_{AB} = \frac{21 \times 14}{21 + 14} = 8.4\ \Omega$$

Example 1.40 : *Find equivalent resistance between points A-B.*

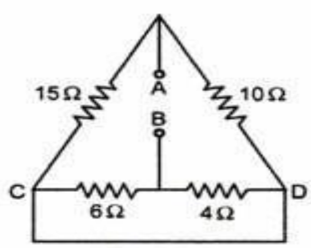


Fig. 1.93

Solution : Redraw the circuit,

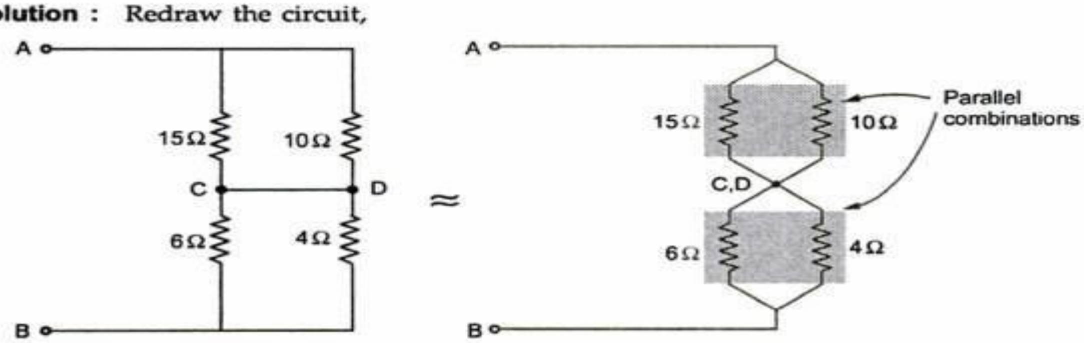


Fig. 1.93 (a)

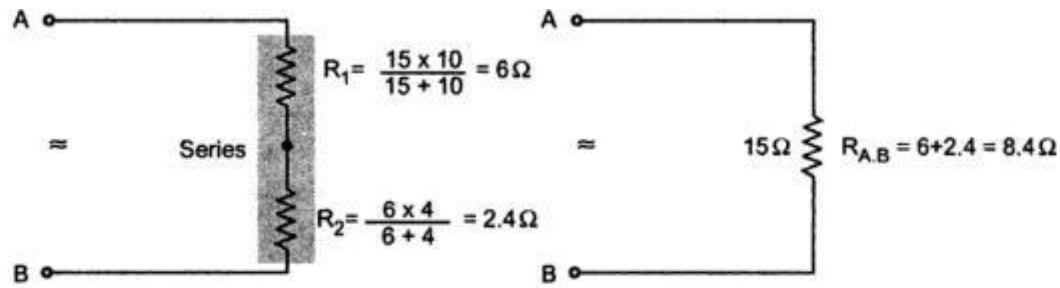


Fig. 1.93 (b)

∴ $R_{AB} = \mathbf{8.4\ \Omega}$

Fundamentals of Alternating Circuits

Let us discuss in brief, the a.c. fundamentals necessary to analyse the networks consisting of various alternating current and voltage sources, resistances and inductive, capacitive reactances.

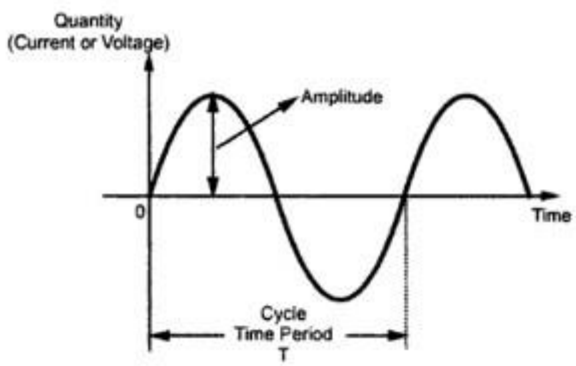


Fig. 0.28 Waveform of an alternating quantity

An alternating current or voltage is the one which changes periodically both in magnitude and direction. Such change in magnitude and direction is measured interms of cycles. Each cycle of a.c. consists of two half cycles namely positive cycle and negative cycle. Current increases in magnitude, in one particular direction, attains maximum and starts decreasing, passing through zero it increases in opposite direction and behaves similarly. The Fig. 0.28 shows a graph of alternating quantity against time.

Instantaneous Value

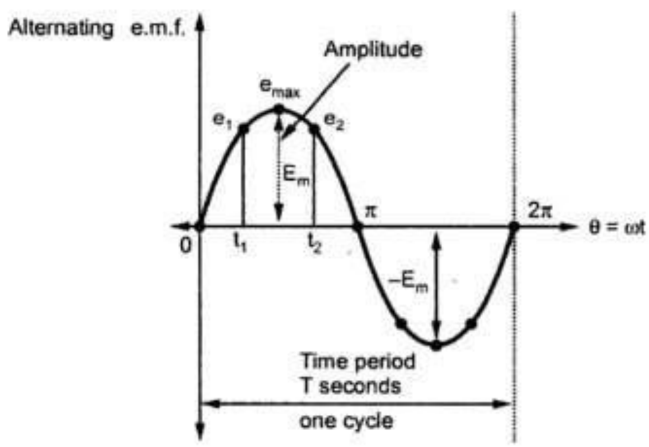


Fig. 0.29 Waveform of an alternating e.m.f.

The value of an alternating quantity at a particular instant is known as its **instantaneous value.** e.g. e_1 and e_2 are the instantaneous values of an alternating e.m.f. at the instants t_1 and t_2 respectively shown in the Fig. 0.29.

Waveform

The graph of instantaneous values of an alternating quantity plotted against time is called its **waveform.**

Cycle

Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a **cycle.**

Such repetition occurs at regular interval of time. Such a waveform which exhibits variations that reoccur after a regular time interval is called **periodic waveform.**

A **cycle** can also be defined as that interval of time during which a complete set of non–repeating events or wave form variations occur (containing positive as well as negative loops). One such cycle of the alternating quantity is shown in the Fig. 0.29.

Key Point: *One cycle corresponds to 2π radians or 360°.*

Time Period (T)

The time taken by an alternating quantity to complete its one cycle is known as its **time period** denoted by **T** seconds. After every T seconds, the cycle of an alternating quantity repeats. This is shown in the Fig. 0.29.

Frequency (f)

The number of cycles completed by an alternating quantity per second is known as its **frequency.** It is denoted by **f** and it is measured in **cycles / second** which is known as **Hertz,** denoted as **Hz.** As time period T is time for one cycle i.e. seconds / cycle and frequency is cycles/second, we can say that frequency is reciprocal of the time period.

$$f = \frac{1}{T} \quad \text{Hz}$$

Amplitude

The maximum value attained by an alternating quantity during positive or negative half cycle is called its **amplitude.** It is denoted as E_m or I_m.

Thus E_m is called peak value of the voltage while I_m is called peak value of the current.

Angular Frequency (ω)

It is the frequency expressed in electrical radians per second. As one cycle of an alternating quantity corresponds to 2π radians, the angular frequency can be expressed as (2 π × cycles/sec.) It is denoted by 'ω' and its unit is radians/second. Now, cycles/sec. means frequency. Hence the relation between frequency 'f' and angular frequency 'ω' is,

$$\omega = 2\pi f \quad \text{radians/sec.} \qquad \text{or} \qquad \omega = \frac{2\pi}{T} \quad \text{radians/sec.}$$

Equation of an Alternating Quantity

As alternating quantity is sinusoidal in nature, its equation is expressed using sin θ where θ is angle expressed in radians. Hence alternating voltage is expressed as,

$$e = E_m \sin\theta$$

While alternating current is expressed as,

$$i = I_m \sin\theta$$

This equation gives instantaneous values of an alternating quantity, at any time t.

Now $\theta = \omega t$ in radians

Thus various forms of the equation of an alternating quantity are,

$$e = E_m \sin(\omega t) = E_m \sin(2\pi f t) = E_m \sin\left(\frac{2\pi}{T} t\right)$$

and

$$i = I_m \sin(\omega t) = I_m \sin(2\pi f t) = I_m \sin\left(\frac{2\pi}{T} t\right)$$

Important Note : *In all the above equations, the angle θ is expressed in* ***radians****. Hence, while calculating the instantaneous value of the e.m.f., it is necessary to calculate the sine of the angle expressed in radians.* ***Mode of the calculator should be converted to radians, to calculate the sine of the angle expressed in radians, before substituting in any of the above equations****.*

In practice the alternating quantities are expressed interms of their r.m.s. values. The relation between r.m.s value and the maximum value is,

$$V_{r.m.s} = \frac{V_m}{\sqrt{2}} \qquad \text{and} \qquad I_{r.m.s} = \frac{I_m}{\sqrt{2}}$$

The r.m.s values are denoted by the capital letters as V or I.

Phasor Representation of an Alternating Quantity

In the analysis of a.c. circuits, it is very difficult to deal with alternating quantities interms of their waveforms and mathematical equations. The job of adding, subtracting, etc. of the two alternating quantities is tedious and time consuming interms of their mathematical equations. Hence, it is necessary to study a method which gives an easier way of representing an alternating quantity. Such a representation is called **phasor representation** of an alternating quantity.

The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow in this method. The length of the line represents the magnitude of the quantity and arrow indicates its direction. This is similar to a vector representation. Such a line is called a **phasor.**

Key Point: *The phasors are assumed to be rotated in anticlockwise direction.*

One complete cycle of a sine wave is represented by one complete rotation of a phasor. The anticlockwise direction of rotation is purely a conventional direction which has been universally adopted.

Consider a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting position 'a' as shown in the Fig. 0.30. If the projections of this phasor on Y-axis are plotted against the angle turned through 'θ', (or time as $\theta = \omega t$), we get a sine waveform.

Consider the various positions shown in the Fig. 0.30.

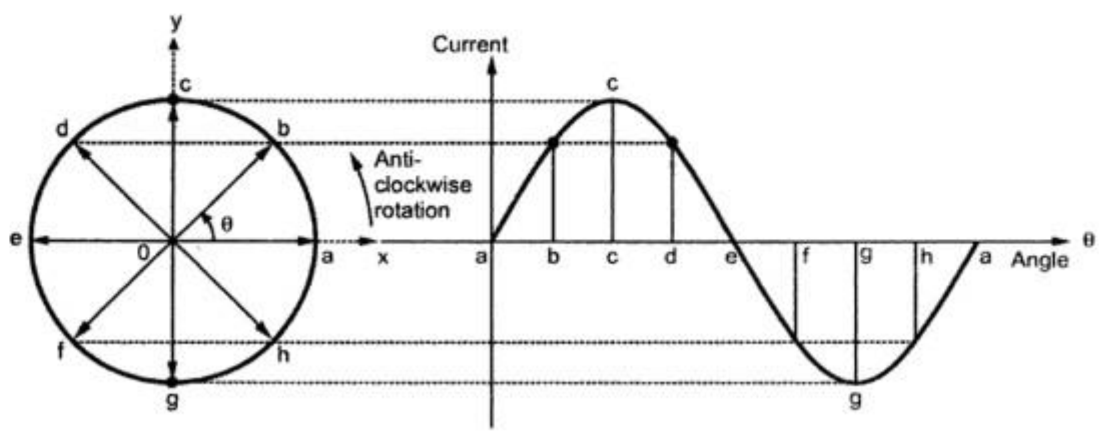


Fig. 0.30 Phasor representation of an alternating quantity

1. At point 'a', the Y-axis projection is zero. The instantaneous value of the current is also zero.
2. At point 'b', the Y-axis projection is [l (ob) sin θ)]. The length of the phasor is equal to the maximum value of an alternating quantity. So, instantaneous value of the current at this position is $i = I_m \sin\theta$, represented in the waveform.
3. At point 'c', the Y-axis projection 'oc' represents entire length of the phasor i.e. instantaneous value equal to the maximum value of current I_m.
4. Similarly, at point d, the Y-axis projection becomes $I_m \sin\theta$ which is the instantaneous value of the current at that instant.
5. At point 'e', the Y-axis projection is zero and instantaneous value of the current is zero at this instant.
6. Similarly, at points f, g, h the Y-axis projections give us instantaneous values of the current at the respective instants and when plotted, give us negative half cycle of the alternating quantity.

Thus, if the length of the phasor is taken equal to the maximum value of the alternating quantity, then its rotation in space at any instant is such that the length of its projection on the Y-axis gives the instantaneous value of the alternating quantity at that particular instant. The angular velocity 'ω' in an anticlockwise direction of the phasor

Concept of Phase of an Alternating Quantity

Phase : The phase of an alternating quantity at any instant is the angle ϕ (in radians or degrees) travelled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference.

Let us consider three cases;

Case 1 : $\phi = 0°$

When phase of an alternating quantity is zero, it is standard pure sinusoidal quantity having instantaneous value zero at t = 0. This is shown in the Fig. 0.31 (a).

Case 2 : Positive phase ϕ

When phase of an alternating quantity is positive it means that quantity has **some positive instantaneous value at t = 0.** This is shown in the Fig. 0.31 (b).

Case 3 : Negative phase ϕ

When phase of an alternating quantity is negative it means that quantity has **some negative instantaneous value at t = 0.** This is shown in the Fig. 0.31 (c).

1. The **phase** is measured with respect to **reference direction** i.e. positive X-axis direction.
2. The **phase** measured in **anticlockwise** direction is **positive** while the **phase** measured in **clockwise** direction is **negative**.

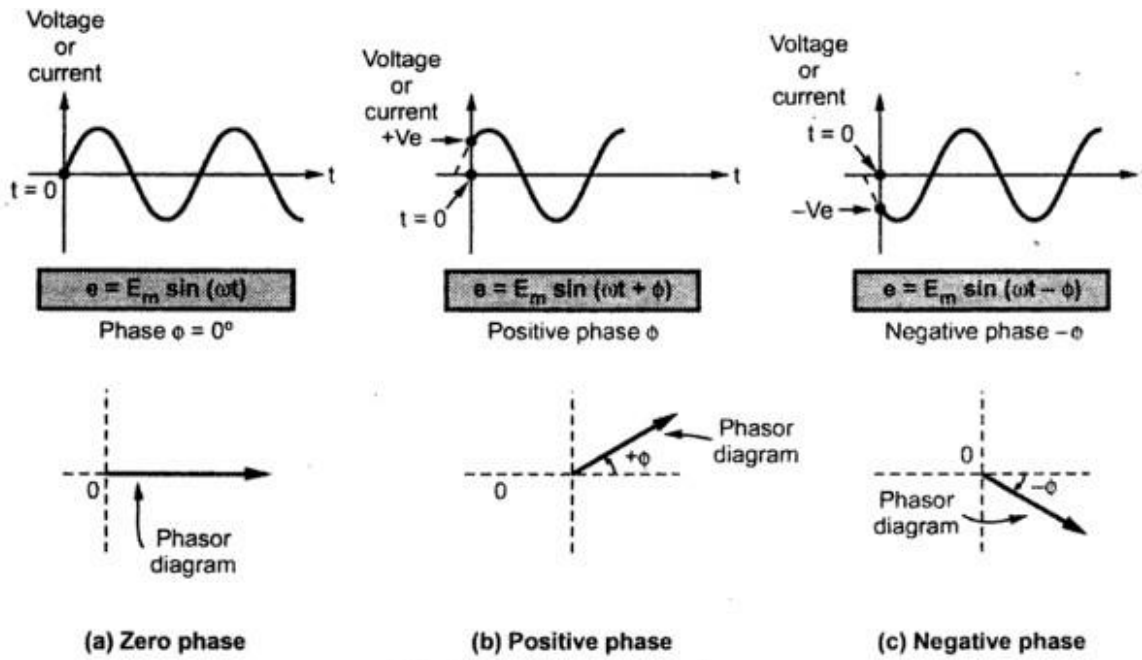


Fig. 0.31 Concept of phase

Phase Difference

Consider the two alternating quantities having same frequency f Hz having different maximum values.

$$e = E_m \sin(\omega t)$$

and $$i = I_m \sin(\omega t)$$

where $E_m > I_m$

The phasor representation and waveforms of both the quantities are shown in the Fig. 0.32.

The phasors $OA = E_m$

and $OB = I_m$

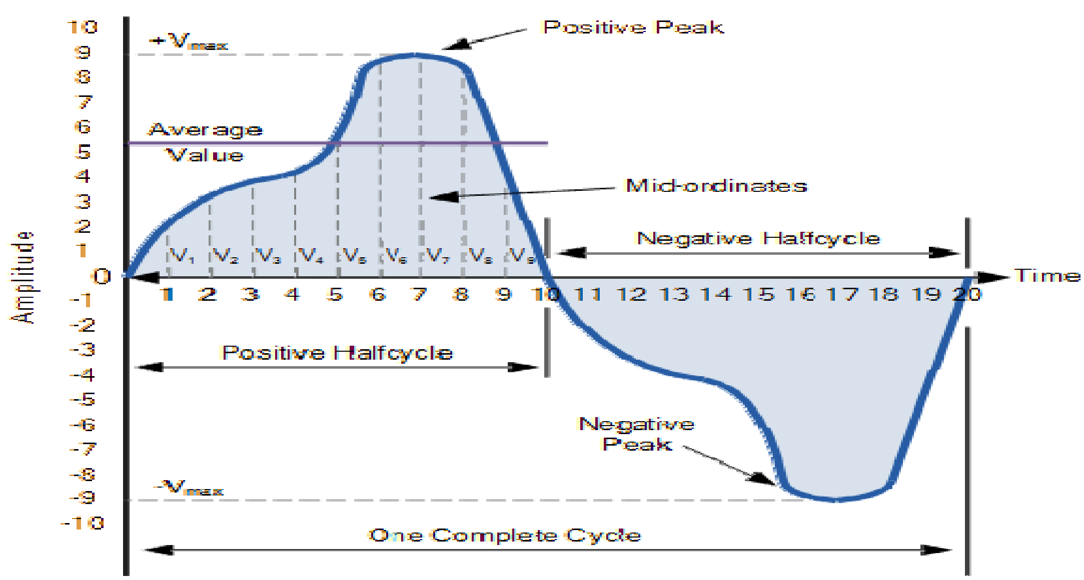
After $\theta = \frac{\pi}{2}$ radians, the OA phasor achieves its maximum E_m while at the same instant, the OB phasor achieves its maximum I_m. As the frequency of both is same, the angular velocity ω of both is also the same. So, they rotate together in synchronism.

So, at any instant, we can say that the phase of voltage e will be same as phase of i. Thus, the angle travelled by both within a particular time is always the same. So, the difference between the phases of the two quantities is zero at any instant. **The difference between the phases of the two alternating quantities is called the phase difference** which is nothing but the angle difference between the two phasors representing the two alternating quantities.

The Average Value of an AC Waveform

The average or mean value of a continuous DC voltage will always be equal to its maximum peak value as a DC voltage is constant. This average value will only change if the duty cycle of the DC voltage changes. In a pure sine wave if the average value is calculated over the full cycle, the average value would be equal to zero as the positive and negative halves will cancel each other out. So the average or mean value of an AC waveform is calculated or measured over a half cycle only and this is shown below.

Average Value of a Non-sinusoidal Waveform



- To find the average value of the waveform we need to calculate the area underneath the waveform using the mid-ordinate rule, trapezoidal rule or the Simpson's rule

- found commonly in mathematics. The approximate area under any irregular waveform can easily be found by simply using the mid-ordinate rule.
- The zero axis base line is divided up into any number of equal parts and in our simple example above this value was nine, (V_1 to V_9). The more ordinate lines that are drawn the mor
- accurate will be the final average or mean value. The average value will be the addition of all the instantaneous values added together and then divided by the total number. This is given as.

Average Value of an AC Waveform:

Where: n equals the actual number of mid-ordinates used.

For a pure sinusoidal waveform this average or mean value will always be equal 0.637*V_{max} and this relationship also holds true for average values of current.

The RMS Value of an AC Waveform:

- The average value of an AC waveform that we calculated above as being: 0.637*V_{max} is NOT the same value we would use for a DC supply. This is because unlike a DC supply which is constant and and of a fixed value, an AC waveform is constantly changing over time and has no fixed value. Thus the equivalent value for an alternating current system that provides the same amount of electrical power to a load as a DC equivalent circuit is called the "effective value".
- The effective value of a sine wave produces the same I^2*R heating effect in a load as we would expect to see if the same load was fed by a constant DC supply. The effective value of a sine wave is more commonly known as the **Root Mean Squared** or simply **RMS** value as it is calculated as the square root of the mean (average) of the square of the voltage or current.
- That is V_{rms} or I_{rms} is given as the square root of the average of the sum of all the squared mid-ordinate values of the sine wave. The RMS value for any AC waveform can be found from the following modified average value formula as shown.

RMS Value of an AC Waveform:

- For a pure sinusoidal waveform this effective or R.M.S. value will always be equal too: $1/\sqrt{2}*V_{max}$ which is equal to $0.707*V_{max}$ and this relationship holds true for RMS values of current. The RMS value for a sinusoidal waveform is always greater than the average value except for a rectangular waveform. In this case the heating effect remains constant so the average and the RMS values will be the same.
- One final comment about R.M.S. values. Most multimeters, either digital or analogue unless otherwise stated only measure the R.M.S. values of voltage and current and not the average. Therefore when using a multimeter on a direct current system the reading will be equal to I = V/R and for an alternating current system the reading will be equal to Irms = Vrms/R.
- Also, except for average power calculations, when calculating RMS or peak voltages, only use V_{RMS} to find I_{RMS} values, or peak voltage, Vp to find peak current, Ip values. Do not mix them together as Average, RMS or Peak values of a sine wave are completely different and your results will definitely be incorrect.

Form Factor and Crest Factor

Although little used these days, both **Form Factor** and **Crest Factor** can be used to give information about the actual shape of the AC waveform. Form Factor is the ratio between the average value and the RMS value and is given as.

For a pure sinusoidal waveform the Form Factor will always be equal to 1.11. Crest Factor is the ratio between the R.M.S. value and the Peak value of the waveform and is given as. For a pure sinusoidal waveform the Crest Factor will always be equal to 1.414.

AC Waveform Example No2

- A sinusoidal alternating current of 6 amps is flowing through a resistance of 40Ω. Calculate the average voltage and the peak voltage of the supply.

❖ The use and calculation of Average, R.M.S, Form factor and Crest Factor can also be use with any type of periodic waveform including Triangular, Square, Saw-toothed or any other irregular or complex voltage/current waveform shape. Conversion between the various sinusoidal values can sometimes be confusing so the following table gives a convenient way of converting one sine wave value to another.