

I B. Tech II Semester Supplementary Examinations, July/August - 2021
MATHEMATICS-III

(Com. to CE, EEE, ECE, CSE, Chem. E, EIE, IT, Auto E, Min E, Pet E)

Time: 3 hours

Max. Marks: 75

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

1. a) ϕ, ψ be two scalar functions then prove that $div(\phi \nabla \psi) = \phi \nabla^2 \psi + \nabla \phi \cdot \nabla \psi$ (8M)

b) Compute $\iiint_S (a^2 x^2 + b^2 y^2 + c^2 z^2)^{1/2} ds$ over the ellipsoid $ax^2 + by^2 + cz^2 = 1$. Using Gauss divergence theorem. (7M)

Or

2. a) Prove that $Curl(\bar{A} \times \bar{B}) = \bar{A} div \bar{B} - \bar{B} div \bar{A} + (\bar{B} \cdot \nabla) \bar{A} - (\bar{A} \cdot \nabla) \bar{B}$ (8M)

b) Evaluate $\int_C \bar{F} \cdot d\bar{r}$ where $\bar{F} = xy \bar{i} + (x^2 + y^2) \bar{j}$ and C is the axes from $x = 2$ to $x = 4$ and the line from $y = 0$ to $y = 12$. (7M)

3. a) Find $L\{e^t(t \sinh t)\}$ (7M)

b) Using Convolution theorem find the inverse Laplace transform of $\frac{1}{s^2(1+s)^2}$ (8M)

Or

4. a) Solve the differential equations by using Laplace transforms method (8M)

$$(D^2 + 4D + 3)y = e^{-t} \text{ if } y(0) = 1, y'(0) = 1$$

b) Find Laplace transform of $f(t)$ where $f(t) = \begin{cases} e^t, & 0 < t < 1 \\ 3, & t > 1 \end{cases}$ (7M)

5. a) Find the Half range sine series of $f(x) = x$ in $[0, \pi]$ (8M)

Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$

b) Find the Fourier Cosine transform of the function $f(x) = \begin{cases} \sin ax & \text{if } x < a \\ 0 & \text{if } x > a \end{cases}$ (7M)

Or

6. a) Obtain the Fourier series of $f(x) = x^2 - 2$ $-2 \leq x \leq 2$ (8M)

b) Using Fourier integral show that (7M)

$$e^{-ax} - e^{-bx} = \frac{2(b^2 - a^2)}{\pi} \int_0^{\infty} \frac{\lambda \sin \lambda x d\lambda}{(\lambda^2 + a^2)(\lambda^2 + b^2)}, a, b > 0$$

7. a) Form the partial differential equation by eliminating arbitrary constants from $z = (\sqrt{x+a})(\sqrt{y+b})$ (8M)

b) Solve the PDE $y^2p - xyq = x(z - 2y)$ (7M)

Or

8. a) Solve the PDE $x^2p^2 = yq^2$ (8M)

b) Form the partial differential equation by eliminating arbitrary function from $z = f(\sin x - \cos y)$ (7M)

9. a) Solve the steady state equation of a rectangular plate of sides 'a' and 'b' insulated on the lateral surfaces subject to $u(0, y) = 0 = u(a, y) = u(x, b)$ and $u(x, 0) = x(a-x)$. (8M)

b) Solve the PDE $(D^2 - 7DD^1 + 12D^1)^2 z = e^{x-y}$ (7M)

Or

10. a) A tightly stretched string with fixed end points $x = 0$ and $x = 1$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{1}$ if it is released from rest from this position, find displacement $y(x, t)$. (8M)

b) Solve the PDE $(D^2 - DD^1)z = \sin x \cos 2y$ (7M)