

II B. Tech I Semester Regular Examinations, March - 2021
MATHEMATICS - III
 (Agricultural Engineering)

Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
 All Questions carry **Equal** Marks

- 1 a) Evaluate Curl of $\vec{V} = e^{xyz} (i + j + k)$ at the point (1, 2, 3). [8M]
 b) Find the total work done in moving a particle in the force field [7M]
 $\vec{F} = 3xy \vec{i} - 5z \vec{j} + 10x \vec{k}$ along the curve
 $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.

Or

- 2 State Green's theorem and Verify Green's theorem in plane for
 $\oint_C [(3x^2 - 8y^2) dx + (4y - 6xy) dy]$, where C is boundary of the region
 defined by $y = \sqrt{x}$ and $y = x^2$.
 3 a) Find i) $L\{e^{-2t} \sin^3 t\}$ ii) $L\{t \cos at\}$ [6M]
 b) Find i) $L^{-1}\left\{\frac{8s+20}{s^2-12s+32}\right\}$, ii) $L^{-1}\left\{\cot^{-1}\left(\frac{s+3}{2}\right)\right\}$. [4M]
 c) Using Laplace transform, solve $y'' - 2y' - 8y = 0, y(0) = 3, y'(0) = 6$. [5M]

Or

- 4 a) Find i) $L\{\cos^3 2t\}$ ii) $L\{\sinh at \cdot \sin at\}$. [6M]
 b) Find $L^{-1}\left\{\frac{1}{(s-a)(s-b)}\right\}$ using Convolution theorem. [4M]
 c) Using Laplace transform, solve $y'' + y = 2e^t, y(0) = 0, y'(0) = 2$. [5M]
 5 a) Obtain the Fourier series of $f(x) = e^{ax}$ in the interval $(0, 2\pi)$. [8M]
 b) Find the Fourier transform of $f(x) = \begin{cases} 1, & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases}$ [7M]

and hence deduce that $\int_0^{\infty} \frac{\sin ax}{x} dx$.

Or

- 6 a) Obtain the Fourier series for $f(x) = \begin{cases} -\pi, & -\pi < x \leq 0 \\ x, & 0 < x \leq \pi \end{cases}$. [8M]
 Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$.
 b) Find the Fourier sine integral of $f(x) = e^{-ax} - e^{-bx}, a > 0, b > 0$. [7M]

- 7 a) Derive a partial differential equation from $xyz = f(x + y + z)$ [5M]
b) Solve the partial differential equation $p + 3q = 5z + \tan(y - 3x)$. [5M]
c) Solve the partial differential equation $p^2z^2 + q^2 = 1$. [5M]

Or

- 8 a) Derive the partial differential equation by eliminating the constants from the equation $z = ax + by + a^2 + b^2$. [5M]
b) Solve the partial differential equation $xp + yq = z$. [5M]
c) Solve the partial differential equation $p^2 + q^2 = m^2$. [5M]
- 9 a) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cdot \cos 2y$. [8M]
b) Solve by the method of separation of variables $y^3 z_x + x^2 z_y = 0$. [7M]

Or

- 10 A string of length L is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = bx(L-x)$ Find displacement $y(x, t)$. [15M]

