II B. Tech I Semester Regular Examinations, March - 2021
VECTOR CALCULUS & FOURIER TRANSFORMS
(Mechanical Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
All Questions carry **Equal** Marks
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1 a) Find the directional derivative of $f(x,y,z) = xy^3 + yz^3$ at $(2,\ -1,\ 1)$ in the direction of a vector $i + 2j + 2k$. [6M]

b) Prove that $div(r^n \bar{r}) = (n+3)r^n$. Hence show that $\frac{\bar{r}}{r^3}$ is solenoidal. [9M]

Or

2 Verify Green's theorem for $\int_C \left[\left(xy + y^2\right)dx + x^2 dy \right]$, where C is bounded by $y = x$ and $y = x^2$. [15M]

3 a) Find the laplace transform of (i) . $\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$. [8M]

(ii). $2^t + \frac{\cos 2t - \cos 3t}{t} + t \sin t$.

b) Find the inverse transforms of $\frac{5s+3}{(s-1)(s^2+2s+5)}$. [7M]

Or

4 a) Evaluate (i) $\int_0^\infty t\, e^{-2t} \sin t\, dt$ (ii). $L\left\{ \int_0^t \frac{e^t \sin t}{t} dt \right\}$. [8M]

b) Using Laplace transform ,solve $\left(D^2 + n^2\right)x = a \sin\left(nt + \alpha\right)$, $x = Dx = 0\ at\ t = 0$. [7M]

5 a) Obtain the Fourier series of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval 0 < x < 2π. Deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\cdots = \frac{\pi^2}{6}$. [8M]

b) Find the Fourier sine and cosine transform of $f(x) = e^{-ax}$, $a > 0, x > 0$. [7M]

Or

6 a) Express $f(x) = \begin{cases} 1 & for\ 0 \le x \le \pi \\ 0 & for\ \ x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate $\int_0^\infty \frac{1 - \cos(\pi\lambda)}{\lambda} \sin(x\lambda)\, d\lambda$. [8M]

b) Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$. [7M]
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7 a) Find one independent solution of the partial differential equation $(mz - ny)p + (nx - lz)q = ly - mx$. [8M]

b) Find the complete solution of $p(1+q) = qz$. [7M]

Or

8 a) Form the partial differential equation by eliminating the arbitrary function from the relation $z = f_1(x) f_2(y)$. [5M]

b) Find the general solution of the partial differential equation $\left(x^2 - y^2 - z^2\right)p + 2xyq = 2zx$. [5M]

c) Solve the partial differential equation: $p^2 + q^2 = x^2 + y^2$. [5M]

9 a) Solve $2\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 5\sin(2x+y)$. [8M]

b) Solve the by the method of separation of variables $4u_x + u_y = 3u \text{ and } u(0, y) = e^{-5y}$. [7M]

Or

10 An insulated rod of length L has its ends A and B maintained at 0^o C and 100^oc respectively until steady state conditions prevail. If B is suddenly reduced to 0^oc and maintained at 0^o C, find the temperature at a distance x from A at time t. [15M]

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1 a) Prove that $div(\phi\overline{F}) = grad\phi \cdot \overline{F} + \phi\, div\overline{F}$. [7M]

b) Prove that $curl(\overline{F}\times\overline{G}) = \overline{F}\cdot div\overline{G} - \overline{G}\cdot div\overline{F} + (\overline{G}\cdot\nabla)\overline{F} - (\overline{F}\cdot\nabla)\overline{G}$. [8M]

Or

2 Verify Stoke's theorem for $\overline{F} = (x^2+y^2)i - 2xyj$ taken around the rectangle bounded by the lines $x = \pm a$ and $y = \pm b$. [15M]

3 a) Find the Laplace transform of $i)$. $e^{-3t}\left(2\cos 5t - 3\sin 5t\right)$ $ii)\ t\cos at$ $ii)$. $\dfrac{e^t\sin^2 t}{t}$ [8M]

b) $If\ L\{f(t)\} = F(s)$ then prove that $L\{f(t-a)H(t-a)\} = e^{-as}F(s)$. [7M]

Or

4 a) Find the inverse transforms of $\dfrac{s}{s^4+4a^4}$. [8M]

b) Solve the D.E. $y''+2y'+5y = e^{-t}\sin t,\ y(0)=0,\ y'(0)=1.$ using Laplace transforms. [7M]

5 a) Find the Fourier series for the function $f(x) = \begin{cases} x & , \ 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$. [8M]

Also deduce that $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\dfrac{1}{7^2}+\cdots\cdots = \dfrac{\pi^2}{8}$.

b) Obtain the Fourier expansion of $f(x) = x\sin x$ as a cosine series in $(0,\pi)$. [7M]

Or

6 a) Express the function $f(x) = \begin{cases} 1 & for\ |x| \le 1 \\ 0 & for\ |x| \ge 1 \end{cases}$ as a Fourier integral. Hence evaluate $\displaystyle\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda$. [8M]

b) Find the Fourier cosine transform of the function $f(x) = \begin{cases} x & , 0 < x < 1 \\ 2-x & , 1 < x < 2 \\ 0 & , x > 2 \end{cases}$. [7M]

7 a) Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$. [8M]

b) Solve $p^2+q^2 = x+y$. [7M]

Or
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8 a) Derive the partial differential equation by eliminating the constants from the equation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$. [5M]

b) Solve the partial differential equation $\left(x^2 - yz\right)p + \left(y^2 - \text{zx}\right)\text{q} = \text{z}^2 - \text{xy}$. [5M]

c) Solve the partial differential equation $z^2 = 1 + p^2 + q^2$. [5M]

9 a) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. [8M]

b) Solve the by the method of separation of variables $3u_x + 2u_y = 0$ *and* $u(x,0) = 4e^{-x}$. [7M]

Or

10 A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{L}\right)$ if it is released from rest from this position, find the displacement $y(x,t)$. [15M]

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1 a) Find a unit normal vector to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1, 2, -1). [6M]

b) Show that $r^n \bar{r}$ is an irrotational vector for any value of n but is solenoidal if $n+3=0$, where $\bar{r} = xi + yj + zk$ and r is the magnitude of $\bar{r}$. [9M]

Or

2 Verify divergence theorem for $\bar{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ taken over the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. [15M]

3 a) Find the Laplace transform of $i)\ .\cos 3t \cos 2t \cos t$ $\quad ii)\ .e^{3t}\ t^{\frac{7}{2}}$ [8M]

$ii)\ .\dfrac{\cos 6t - \cos 4t}{t}$.

b) Find the inverse Laplace transform of i). $\dfrac{5s-2}{s^2+4s+8}$ ii). $\log\left(\dfrac{s+b}{s+a}\right)$. [7M]

Or

4 a) $If\ L\{f(t)\} = F(s)$ then prove that $L\left\{\int_0^t f(u)\,du\right\} = \dfrac{F(s)}{s}$. [8M]

b) Using Laplace transform ,solve $y'' + 7y' + 10y = 4e^{-3t}$, $y(0)=0$, $y'(0) = -1$. [7M]

5 a) Find a Fourier series to represent $f(x) = x - x^2$ in $-\pi \le x \le \pi$. Hence show that $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \cdots\cdots = \dfrac{\pi^2}{12}$. [8M]

b) Expand $f(x) = \begin{cases} \dfrac{1}{4} - x, & if\ 0 < x < \dfrac{1}{2} \\ x - \dfrac{3}{4}, & if\ \dfrac{1}{2} < x < 1 \end{cases}$ as a Fourier series of sine terms. [7M]

Or

6 a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & for\ |x| \le 1 \\ 0 & for\ |x| > 1 \end{cases}$ hence evaluate [8M]

$\int_0^\infty \left(\dfrac{x\cos x - \sin x}{x^3}\right)\cos\dfrac{x}{2}\,dx$.

b) Find the Fourier sine transform of the function $f(x) = \begin{cases} x & , 0 < x < 1 \\ 2 - x & , 1 < x < 2 \\ 0 & , x > 2 \end{cases}$. [7M]
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7 a) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$. [8M]

b) Solve $z^2(p^2 + q^2) = x^2 + y^2$. [7M]

Or

8 a) Form the partial differential equation by eliminating the arbitrary constants a, b from z = a $\log\left[\frac{b(y-1)}{1-x}\right]$. [5M]

b) Solve the partial differential equation $\left(\frac{y^2 z}{x}\right)p + xzq = y^2$. [5M]

c) Find the complete integral of $p + q = pq$. [5M]

9 a) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = \cos(2x + y)$. [8M]

b) Solve the partial differential equation by the method of separation of variables $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ *and* $u(0, y) = 8e^{-3y}$. [7M]

Or

10 Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy-plane, $0 \le x \le a$ *and* $0 \le y \le b$ satisfying the following boundary condition $u(0, y) = 0$, $u(a, y) = 0$, $u(x,b) = 0$ *and* $u(x,0) = f(x)$. [15M]

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1 a) What is the directional derivative of $\phi = xy^2 + yz^3$ at (*2, -1, 1*) in the direction of the normal to the surface $x\log z - y^2 = -4$ at (*-1, 2, 1*)? [8M]

b) Prove that $curl(\phi\overline{F}) = grad\phi \times \overline{F} + \phi\, curl\overline{F}$. [7M]

Or

2 Verify Green's theorem for $\int_C \left[(3x - 8y^2)dx + (4y - 6xy)dy\right]$ where C is the boundary of the region bounded by $x = 0,\ y = 0$ and $x + y = 1$. [15M]

3 a) Find the Laplace transform of *i*) $e^{-2t}\sin^3 t$ *ii*) $e^2 \cos 2t$ *iii*) $f(t) = \begin{cases} \sin t, & t > \pi \\ cost, & t < \pi \end{cases}$. [8M]

b) Find the inverse Laplace transform of i). $\frac{3s-14}{s^2-4s+8}$ ii). $\tan^{-1}\left(\frac{2}{s}\right)$. [7M]

Or

4 a) Find the Laplace transform of $e^{-4t}\int_0^t \frac{\sin 3u}{u}du$. [7M]

b) Using Laplace transform, solve $y'' - 8y' + 15y = 9te^{2t}$, $y(0) = 5$, $y'(0) = 10$. [8M]

5 a) Given that $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. Find the Fourier series for $f(x)$. Also deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\cdots = \frac{\pi^2}{8}$. [8M]

b) Obtain the half range sine series for $f(x) = e^x$ *in* $0 < x < 1$. [7M]

Or

6 a) Find the Fourier transform of f(x) = $f(x) = \begin{cases} 1 & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{cases}$ hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$. [8M]

b) Find the finite Fourier sine and cosine transforms of $f(x) = 2x,\ 0 < x < 4$. [7M]
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7 a) Form the partial differential equation by elimination the arbitrary function f from the relation $z = y^2 + 2f\left(\frac{1}{x} + logy\right)$. [5M]

b) Solve the partial differential equation $x^2\left(y-z\right)p + y^2\left(z-x\right)q = z^2(x-y)$. [5M]

c) Find the complete integral of $pe^y = qe^x$. [5M]

Or [5M]

8 a) Form the partial differential equation by eliminating the arbitrary function ϕ from the $\phi\left(x^2 + y^2 + z^2, lx + my + nz\right) = 0$. [5M]

b) Solve the partial differential equation $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = \left(z^2 - xy\right)$. [5M]

c) Find the complete integral of $pqz = p^2\left(xq + p^2\right) + q^2\left(yp + q^2\right)$. [5M]

9 a) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$. [8M]

b) Solve the by the method of separation of variables $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0$. [7M]

Or

10 Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy-plane, $0 \le x \le a$ *and* $0 \le y \le b$ satisfying the following boundary condition $u(x,0) = 0$, $u(x,b) = 0$, $u(0,y) = 0$ *and* $u(a,y) = f(y)$. [15M]