

II B. Tech I Semester Regular Examinations, March - 2021
COMPLEX VARIABLES AND STATISTICAL METHODS
 (Civil Engineering)

Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
 All Questions carry **Equal** Marks

1 a) Find k such that $u(x, y) = x^3 + 3kxy^2$ is harmonic and hence find its conjugate. [8M]

b) Evaluate $\int_C \frac{e^{3z}}{(z+1)^4} dz$ where $C: |z| = 3$ using Cauchy integral formula. [7M]

Or

2 a) Discuss the continuity of $f(z)$ at $z = 0$ where [8M]

$$f(z) = \begin{cases} \frac{z}{|z|} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

b) Find the analytic function $f(z)$ whose real part is $u = \sin x \cosh y$. [7M]

3 a) Expand $f(z) = \frac{1}{z(z^2 - 3z + 2)}$ in $0 < |z| < 1$. [8M]

b) Apply Cauchy residue theorem to evaluate $\oint_C \frac{z-3}{z^2+2z+5}$ where C is the circle [7M]

(i) $|z| = 1$, (ii) $|z+1-i| = 2$.

Or

4 a) Show that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\pi}{3}$ [8M]

b) Evaluate $\int_C \frac{(2z+1)^2}{4z^3+z} dz$ where C is the circle $|z| = 1$ using residue theorem. [7M]

5 a) A manufacturer firm produces steel pipes in three plants, with daily production volumes of 500, 1000 and 2000 units respectively. According to past experience; it is known that the fraction of defective outputs produced by these plants are respectively 0.005, 0.008 and 0.010. If a pipe is selected from day's total production and found to be defective, find out (i) from which plant the pipe came (ii) what is the probability that it came from the first plant? [8M]

b) If the probability density of a random variable is given by [7M]
 trouble shooting capability of an IC chip in a circuit is a random variable X whose distribution function is given by

$$f(x) = \begin{cases} k(1-x^2) & \text{for } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k and the probabilities that random variable having this probability density will take on a value (i) between 0.1 and 0.2 (ii) greater than 0.5

Or

- 6 a) Calculate expectation and variance of X, if the probability distribution of the random variable X is given by [8M]

X	-1	0	1	2	3
f	0.3	0.1	0.1	0.3	0.2

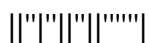
- b) In a sample of 1000 cases, the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find [7M]
- (i) How many students score between 12 and 15?
- (ii) In how many score above 18?
- In how many score below 18?
- 7 a) Find mean and standard deviation of sampling distribution of variances for the population 2,3,4,5 by drawing samples of size two(a) with replacement (b) without replacement. [8M]
- b) A random sample of size 100 is taken from a population with $\sigma = 5.1$. Given that sample the sample mean $\bar{x} = 21.6$. construct a 95% confidence interval for the population mean μ . [7M]

Or

- 8 a) A random sample of 400 items is found to have mean 82 and standard deviation 16. What are the mean and standard deviation of the sample mean for random samples of size 4 drawn with replacement? [8M]
- b) The mean of certain normal population is equal to standard error of the mean of the samples of 64 from that distribution. Find the probability that the mean of the sample size 36 will be negative. [7M]
- 9 a) Two independent samples of 8 and 7 items respectively had the following values [8M]

Sample-I	11	11	13	11	15	9	12	14
Sample-II	9	11	10	13	9	8	10	--

- b) A trucking firm suspects the claim that average life of certain tyres is at least 28,000 miles. To check the claim the firm puts 40 of these tyres on its trucks and gets a mean life time of 27463 miles with a standard deviation of 1348 miles. Can the claim be true? [7M]
- Or
- 10 a) A machine is designed to produce installing washers for electrical devices of average thickness of 0.025cm. A random sample of 10 washers was found to have a thickness of 0.024cm with a standard deviation of 0.002 cm. Test the significance of the deviation. Value of t for 9 degrees of freedom at 5% level is 2.262 [8M]
- b) A sample of 64 students have a mean weight of 70kgs. Can this be regarded as a sample from a population with mean weight 56 kg. and standard deviation 25kg [7M]



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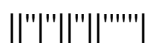
Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
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- 1 a) Find harmonic conjugate of $u = e^{x^2-y^2} \cos 2xy$. Hence find $f(z)$ in terms of z . [8M]
 b) Evaluate $\int_c \frac{1}{z^3(z+4)} dz$ where c is $|z| = 2$ using Cauchy integral formula. [7M]
- Or
- 2 a) Find the regular function whose imaginary part is $\log(x^2 + y^2) + x - 2y$ [8M]
 b) Evaluate $\int_c \bar{z} dz$ from $z = 0$ to $z = 4 + 2i$ along the curve defined by $z = t^2 + it$. [7M]
- 3 a) Find the Taylor's or Laurent's series which represents the function $f(z) = \frac{1}{(z^2+1)(z+2)}$ in the regions (i) when $|z| < 1$ (ii) when $1 < |z| < 2$ (iii) when $|z| > 2$. [8M]
 b) Evaluate $\oint_c \frac{z}{(z-1)(z-2)^2} dz$ where $c : |z-2| = \frac{1}{2}$ using residue theorem [7M]
- Or
- 4 a) Write Laurent's expansion for $f(z) = \frac{1}{(z+2)(1+z)^2}$ in (i) $|z| < 2$ (ii) $|1+z| > 1$ [8M]
 b) Evaluate by Contour integration $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. [7M]
- 5 a) In a factory, a machine A produces 40% of the output and machine B produces 60%. On the average, 9 items in 1000 produced by A are defective and 1 item in 250 produced by B is defective. An item drawn at random from a day's output is defective. What is the probability that it was produced by A or B? [8M]
 b) The average number of phone calls per minute coming into a switch board between 2.p.m and 4.p.m is 2.5. Determine the probability that during one particular minute there will be (i) 4 or fewer (ii) more than six calls. [7M]
- Or
- 6 a) The mean and variance of a binomial variable X with parameters n and p are 16 and 18. Find $P(X \geq 1)$ and $P(X > 2)$. [8M]
 b) In a normal distribution 7% of the items are under 35 and 89% are under 63. Determine the mean and variance of the distribution. [7M]
- 7 a) Samples of size 2 are taken from the population 1,2,3,4,5,6 with replacement. Find (a) the mean of the population (b) the standard deviation of the population (c) the mean of the sampling distribution of means and (d) the standard deviation of the sampling distribution of means [8M]
 b) A random sample of 100 teachers in a large metropolitan area revealed a mean weekly salary of Rs. 487 with a standard deviation Rs.48. With what degree of confidence can we assert that the average weekly salary of all teachers in the metropolitan area is between 472 and 502. [7M]

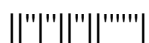
Or



- 8 a) Construct 99% confidence interval for the true mean weight loss if 16 persons on diet control after one month had a mean weight loss of 3.42 kgs with standard deviation of 0.68kgs. [8M]
- b) Determine the expected number of random samples of having their means (a) between 22.39 and 22.41 (b) Greater than 22.42 (c) less than 22.37 (d) less than 22.38 or more than 22.41 for the following data [7M]
N= size of the population =1500, n= size of the sample= 36, Number of samples $N_s=300$, σ = population SD=0.48, μ = Population mean = 22.4.
- 9 a) A random sample of size 16 values from a normal population showed a mean of 53 and a sum of squares of deviations from the mean equals to 150. Can this sample be regarded as taken from the population having 56 as mean? Obtain 95% confidence limits of the mean of the population. [8M]
- b) An oceanographer wants to check whether the depth of the ocean in a certain region is 57.4 fathoms, as had previously been recorded. What can he conclude at the 0.05 level of significance, if readings taken at 40 random locations in the given region yielded a mean of 59.1 fathoms with a standard deviation 5.2 fathoms. [7M]

Or

- 10 a) A sample of 100 iron bars is said to be drawn from a large number of bars whose lengths are normally distributed with mean 4 ft and standard deviation 6ft. if the sample mean is 4.2ft, can the sample be regarded as a truly random sample. [8M]
- b) In a sample of 600 students of a certain college 400 are found to use ball pens. In another college from a sample of 900 students 450 were found to use ball pens. Test whether 2 colleges are significantly different with respect to the habit of using ball pens. [7M]



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1 a) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate and the corresponding analytic function $f(z)$ in terms of z . [8M]

b) Evaluate $\int_0^{1+i} (x - y + ix^2) dz$ along the [7M]

i) straight line $z = 0$ to $z = 1 + i$

ii) the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$.

Or

2 a) If $f(z) = \begin{cases} \frac{x^3y(y-ix)}{x^6+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ prove that $\frac{f(z)-f(0)}{z} \rightarrow 0$ as $z \rightarrow 0$ along any [8M]

radius vector but not as $z \rightarrow 0$ along the curve $= ax^3$.

b) Use Cauchy's Integral formula to evaluate [7M]

$$\int_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz \text{ where } C \text{ is the circle } |z| = 3.$$

3 a) Find Laurent's series of $f(z) = \frac{1}{(z+2)(z^2+1)}$ in (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$. [8M]

b) Evaluate $\oint_C \frac{\sin z}{z^6} dz$ where $C: |z| = 2$ using Cauchy residue theorem. [7M]

Or

4 a) Show that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx = \frac{\pi}{a+b}$ ($a > 0, b > 0, a \neq b$). [8M]

b) Using residue theorem evaluate $\oint_C \frac{z+1}{z^2(z-2)} dz$ where $C: |z| = 3$ using Cauchy residue theorem [7M]

5 a) A manufacturing firm produces steel pipes in three plants, with daily production volumes of 500, 1000 and 2000 units respectively. According to past experience; it is known that the fraction of defective outputs produced by these plants are respectively 0.005, 0.008 and 0.010. If a pipe is selected from day's total production and found to be defective, find out (i) from which plant the pipe came (ii) what is the probability that it came from the first plant? [8M]

b) The diameter of an electrical cable is assumed to be continuous random variable with probability density function $f(x) = 6x(1-x), 0 \leq x \leq 1$. Justify. Find mean and variance of the distribution. [7M]

Or

6 a) A, B, C are aiming to a shoot a balloon, A will succeed 4 times out of 5 attempts. The chance of B to shoot the balloon is 3 out of 4 and that of C is 2 out of 3. If the three aim the balloon simultaneously, then find the probability that at least two of them hit the balloon. [8M]

b) Suppose that the weights of 800 male students are normally distributed with mean 140 pounds and standard distribution 10 pounds. Find the number of students whose weights are (i) between 138 and 148 pounds (ii) more than 152 pounds. [7M]

- 7 a) If the population is 3,6,9,15,27 [8M]
 (a) List all possible samples of size 3 that can be taken without replacement from the finite population
 (b) Calculate the mean of each of the sampling distribution of means
 (c) Find the standard deviation of the sampling distribution of means
 b) Measurements of the weights of a random sample of 200 ball bearings made by a certain machine during one week showed a mean of 0.824 and standard deviation of 0.042. Find maximum error at 95% confidence interval. Find confidence limits of the mean if $x = 32$. [7M]

Or

- 8 a) The mean life time of a light bulb produced by a company is 1500 hours and S D of 150 hours. Find the probability that lighting will take place for (a) at least 5000h (b) at most 4200h if three bulbs are connected such that when one bulb burns out, another bulb will go on. Assume that life times are normally distributed [8M]
 b) In a study of automobile insurance a random sample of 80 body repair costs had a mean of Rs. 472.36 and the SD of Rs. 62.35. If \bar{x} is used as a point estimate to the true average repair costs, with what confidence we assert that the maximum error does not exceed Rs.10 [7M]
 9 a) A die is thrown 264 times with the following results. Show that the die is biased. [8M]
 [Given $\chi^2_{0.05} = 11.07$ for 5 degrees of freedom]

No. Appeared on the die	1	2	3	4	5	6
Frequency	40	32	28	58	54	52

- b) The mean height of 50 male students who participated in sports is 68.2 inches with standard deviation of 2.5. The mean height of 50 male students who have not participated in sports is 67.2 inches with standard deviation of 2.8. Test the hypothesis that the height of the students who participated in sports is more than the students who have not participated in sports. [7M]

Or

- 10 a) It is known that the mean diameter of rivets produced by two firms A and B are practically the same. But the standard deviations may differ. For 22 rivets produced by firm A, standard deviation is 2.9 mm while for 16 rivets manufactured by firm B, the standard deviation is 3.8 mm compute the statistic you would use to test whether the products of firm A have the same variability as those of firm B and test its significance [8M]
 b) A machine puts out 9 imperfect articles in a sample of 200 articles. After the machine is overhauled it puts out 5 imperfect articles in a sample of 700 articles. Test at 5% level whether the machine is improved. [7M]

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1 a) Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin even though the Cauchy-Riemann equations are satisfied at that point. [8M]

b) Evaluate $\int_C |z|^2 dz$ around the square with vertices (0,0), (1,0), (1,1), (0,1), taken in positive sense. [7M]

Or

2 a) Find the analytic function $f(z)$ whose analytic part is $u = \sin x \cosh y$. [8M]

b) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ where $C: |z| = 3$ using Cauchy integral formula [7M]

3 a) Represent the function $f(z) = \frac{4z+3}{z(z-3)(z+2)}$ as Laurent's series (i) within $|z| = 1$ (ii) in the annular region between $|z| = 2$ and $|z| = 3$ (iii) exterior to $|z| = 3$. [8M]

b) Evaluate $\oint_C \frac{\tan z}{z^2-1} dz$ where $C: |z| = 1.5$ using Cauchy residue theorem [7M]

Or

4 a) Prove that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2} = \frac{\pi}{2}$ [8M]

b) Using Residue theorem evaluate $\int_C \frac{dz}{z^8(z+4)}$ where C is the circle (i) $|z| = 2$, (ii) $|z+2| = 3$. [7M]

5 a) In a certain college, 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body. (a) What is the probability that mathematics is being studied? (b) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl? (c) a boy. [8M]

b) The trouble shooting capability of an IC chip in a circuit is a random variable X whose distribution function is given by [7M]

$$F(X) = \begin{cases} 0 & \text{for } x \leq 3 \\ 1 - \frac{9}{x^2}, & \text{for } x > 3 \end{cases}$$

where x denote the number of years.

Find the probability that the IC chip will work properly (i) Less than 8 years (ii) Beyond 8 years (iii) Between 5 to seven years (iv) Anywhere from 2 to 5 years.

Or

6 a) Three electronic lamps are fitted in a room. Three bulbs are chosen at random from 10 bulbs having 6 good bulbs. What is the chance that the room is lighted? [8M]

b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution [7M]

- 7 a) Let $S=\{1,5,6,8\}$, find the probability distribution of the sample mean for random [8M]
sample of size 2 drawn without replacement.
b) Construct 99% confidence interval for the mean of contents of soft drink bottles [7M]
if contents of 7 such soft drink bottles are 10.2, 10.4, 9.8, 10.0, 9.8, 10.2, 9.6 ml.

Or

- 8 a) A Professor's feelings about the mean mark in the final examination in [8M]
'probability' of a large group of students is expressed subjectively by normal
distribution with $\mu_0 = 67.2$ and $\sigma_0 = 1.5$
(a) if the mean mark lies in the interval (65.0 70.0) determine the prior
probability the professor should assign to the mean mark
(b) find the Professor mean μ_1 and the posterior SD σ_1 if the examinations
are conducted on a random sample of 40 students yielding mean 74.9 and
S D 7.4 . Use $S = 7.4$ as an estimate σ
(c) determine the posterior probability which he will thus assign to the
mean mark being in the interval (65, 70) using results obtained in (b)
(d)construct a 95% Bayesian interval for μ .
b) Sample of size 2 are taken from the population 3, 6, 9, 15, 27 with replacement. [7M]
Find (a) the mean of the population (b) the standard deviation of the population
(c) the mean of the sampling distribution of means and (d) the standard deviation
of the sampling distribution of means
9 a) The nicotine contents in milligrams in two samples of tobacco were found to be [8M]
as follows

Sample A	24	27	26	21	25	--
Sample B	27	30	28	31	22	36

Can it be said that the two samples have come from the same normal population?

- b) Among the items produced by a factory out of 800, 65 were defective in another [7M]
sample out of 300, 40 were defective. test the significance between the
differences of two proportions at 1% level of significance

Or

- 10 a) Four coins were tossed 160times and the following results were obtained [8M]

No. of Heads	0	1	2	3	4
Frequency	17	52	54	31	6

- b) The mean life time of a sample of 100 light tubes produced by a company is [7M]
found to be 1560 hours with a population S D of 90hrs. Test the hypothesis for
 $\alpha = 0.05$ that the mean life time of the tube produced by the company is
1580hrs.