UNIT-4

Design of FIR Digital filter & Realizations

FIR filter :- (finite impulse response filter) :-

The filter designed by using finite number of samples of impulse response are called "FIR filter".

The following are the main advantages of the FIR filter over IIR filter.

1. FIR filters are always stable.
2. FIR filters with exactly linear phase can easily be designed
3. FIR filters can be realized in both recursive and non-recursive structures.
4. FIR filters are free of limit cycle oscillations, when implemented on a finite word length digital system.
5. Excellent design methods are available for various kinds of FIR filter.

Disadvantages of FIR filters :-

1. The implementation of narrow transition band FIR filter are very costly, as it requires considerably more arithmetic operations and hardware components. such as multipliers, adders and delay elements.
2. Memory requirement and execution time are very high.

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Characteristics of FIR filters with linear phase :-

The transfer function of a FIR causal filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n) \cdot z^{-n}$$

where $h(n)$ is the impulse response of the filter.

The fourier transform of $h(n)$ is

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) \cdot e^{-j\omega n}$$

which is Periodic in frequency with Period 2π

$$H(e^{j\omega}) = \pm |H(e^{j\omega})| \, e^{j\theta(\omega)}$$

where $|H(e^{j\omega})|$ is magnitude response and $\theta(\omega)$ is Phase response.

we define the phase delay and group delay of a filter as

$$\tau_p = \frac{-\theta(\omega)}{\omega} \quad \& \quad \tau_g = \frac{-d\theta(\omega)}{d\omega} \longrightarrow (1)$$

For FIR filters with linear phase we can define

$$\theta(\omega) = -\alpha \cdot \omega \; ; \; -\pi \le \omega \le \pi \longrightarrow (2)$$

where α is a constant phase delay in samples.

$\therefore$ Substituting Eq (2) in Eq (1), we have $\tau_p = \tau_g = \alpha$, which means that α is independent frequency.

we can write $\sum_{n=0}^{N-1} h(n) \, e^{-j\omega n} = \pm |H(e^{j\omega})| \, e^{j\theta(\omega)} \longrightarrow (3)$

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which gives us

$$\sum_{n=0}^{N-1} h(n)\cos\omega n = \pm |H(e^{j\omega})| \cdot \cos\theta(\omega) \longrightarrow (4)$$

and $$-\sum_{n=0}^{N-1} h(n)\sin\omega n = \pm |H(e^{j\omega})| \cdot \sin\theta(\omega) \longrightarrow (5)$$

By taking ratio Eq (5) & (4) we have

$$\frac{\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\sin\alpha\cdot\omega}{\cos\alpha\cdot\omega} \quad [\because \theta(\omega) = -\alpha\cdot\omega] \longrightarrow (6)$$

After simplifying Eq (6). we have

$$\sum_{n=0}^{N-1} h(n)\sin(\alpha - n)\cdot\omega = 0 \longrightarrow (7)$$

Eq (7) will be zero when

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$$h(n) = h(N-1-n) \longrightarrow (8)$$

and $$\alpha = \frac{N-1}{2} \longrightarrow (9)$$

∴ FIR filter will have constant phase and group delay when the impulse response is symmetrical about $\alpha = \frac{N-1}{2}$

The impulse response satisfying Eq (8) & Eq (9) for odd and Even values of N is ashow in fig. when N=7 the centre of symmetry of the sequence occurs at third sample and when N=6, the filter delay is $2\frac{1}{2}$ samples

if only constant group delay is required, and not the phase delay we can write $\theta(\omega) = \beta - \alpha\cdot\omega \longrightarrow (10)$

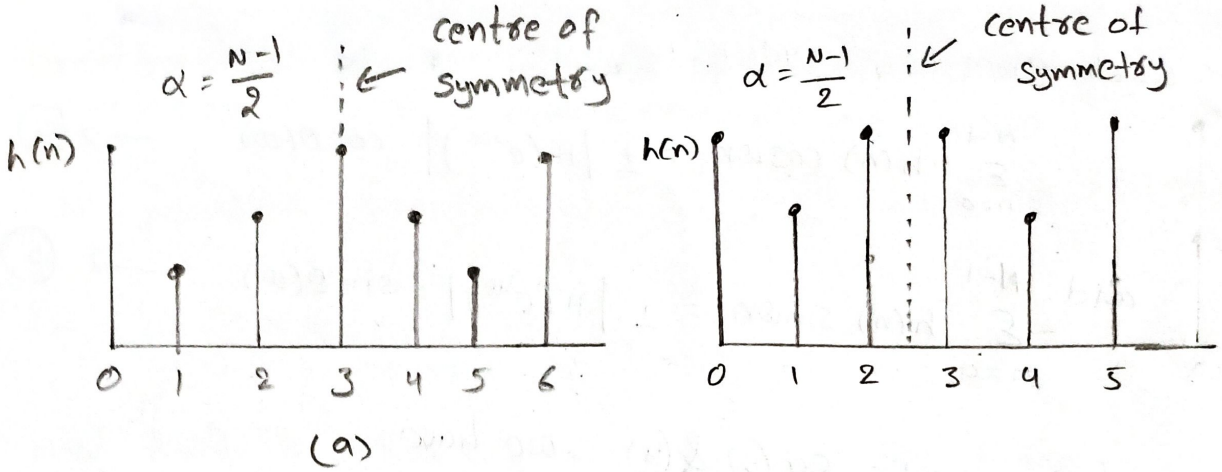


fig:- impulse - response of symmetric sequences for (a) N-odd (b) N - Even

Now, we have

$$H(e^{j\omega}) = \pm |H(e^{j\omega})| \, e^{j(\beta - \alpha\omega)} \quad \rightarrow (11)$$

Eq (11) can be Expreesion as

$$\sum_{n=0}^{N-1} h(n) e^{-j\omega n} = \pm |H(e^{j\omega})| \, e^{j(\beta - \alpha\omega)} \quad \rightarrow (12)$$

which give us

$$\sum_{n=0}^{N-1} h(n) \cos\omega n = \pm |H(e^{j\omega})| \cos(\beta - \alpha\omega) \quad \rightarrow (13)$$

and

$$-\sum_{n=0}^{N-1} h(n) \sin\omega n = \pm |H(e^{j\omega})| \sin(\beta - \alpha\omega) \quad \rightarrow (14)$$

By taking ratio of Equation (14) to Eq (13), we get

$$\frac{-\sum_{n=0}^{N-1} h(n) \sin\omega n}{\sum_{n=0}^{N-1} h(n) \cos\omega n} = \frac{\sin(\beta - \alpha\omega)}{\cos(\beta - \alpha\omega)}$$

from which we obtain

$$\sum_{n=0}^{N-1} h(n) \sin[\beta - (\alpha - n)\omega] = 0 \quad \rightarrow (15)$$

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if $\beta = \frac{\pi}{2}$ Eq (15) can be written as

$$\sum_{n=0}^{N-1} h(n) \cos\left(\alpha - n\right)\omega = 0 \quad \longrightarrow (16)$$

The Eq (16) will be satisfied when

$$h(n) = -h(N-1-n) \quad \longrightarrow (17)$$

and $$\alpha = \frac{N-1}{2} \quad \longrightarrow (18)$$

$\therefore$ FIR filters have constant group delay T_g, and not constant phase delay when the impulse response is Antisymmetrical about $\alpha = \frac{N-1}{2}$

The impulse response satisfying Eq (17) & (18) is shown in below fig. when $N=7$ the centre of antisymmetric occurs at third sample, and when $N=6$ the centre of Antisymmetry occurs at $2.\frac{1}{2}$ Samples. Below fig (a) we find that $h\left(\frac{N-1}{2}\right) = 0$ for Antisymmetry odd sequence.

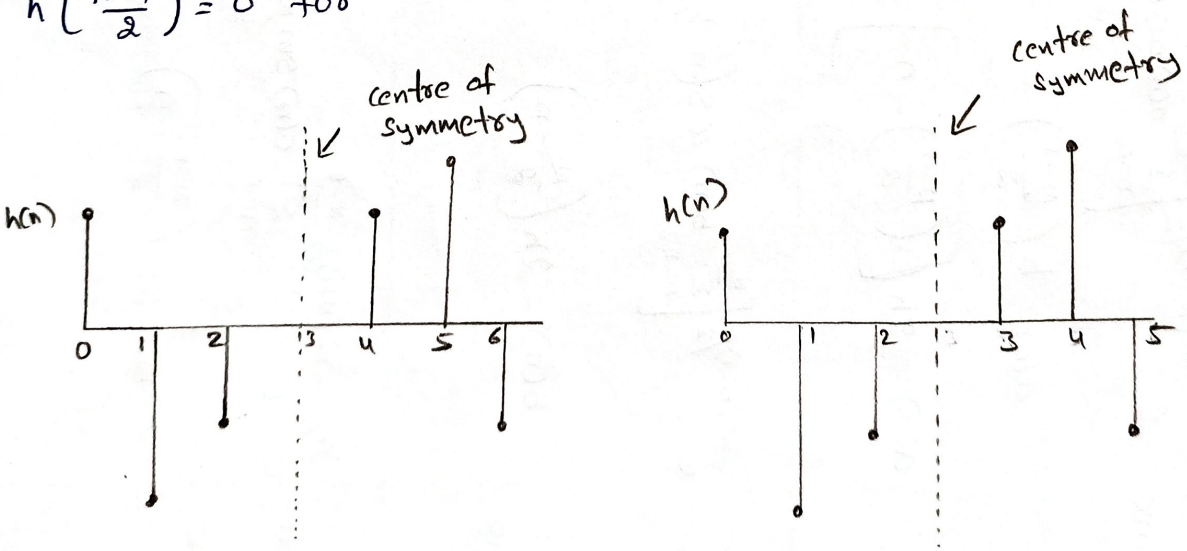


fig:- impulse response sequences satisfying Eq (17) & Eq (18) for $N=7$ & (b) $N=6$

Type	Frequency response $H(e^{j\omega})$	Magnitude response $\lvert \bar{H}(e^{j\omega}) \rvert$	Phase response $\angle H(e^{j\omega})$	Applications
Symmetrical impulse response N - odd	$e^{-j\omega(N-1)/2}\left[\sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n\right]$ $a(0) = h\left(\frac{N-1}{2}\right)$ $a(n) = 2h\left[\left(\frac{N-1}{2}\right) - n\right]$	$\left\lvert \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n \right\rvert$	$-\alpha\omega+\theta$ where $\theta=0$; $\lvert \bar{H}(e^{j\omega}) > 0$ $\theta=\pi$; $\bar{H}(e^{j\omega}) < 0$	low pass, high pass Band pass Band stop
Symmetrical impulse response N - Even	$e^{-j\omega(N-1)/2}\left[\sum_{n=1}^{\frac{N}{2}} b(n)\cos\left(n-\frac{1}{2}\right)\omega\right]$ $b(n) = 2h\left(\frac{N}{2} - n\right)$	$\left\lvert \sum_{n=1}^{\frac{N}{2}} b(n)\cos\left(n-\frac{1}{2}\right)\omega \right\rvert$	$-\alpha\cdot\omega+\theta$ where $\theta=0$ for $\bar{H}(e^{j\omega}) > 0$ $\theta=\pi$ for $\bar{H}(e^{j\omega}) < 0$	Low Pass Band Pass
Anti symmetrical impulse response N-Odd	$e^{j\pi/2}\, e^{-j\omega(N-1)/2} \sum_{n=1}^{\frac{N-1}{2}} c(n)\sin\omega n$ $c(n) = 2h\left(\frac{N-1}{2} - n\right)$	$\left\lvert \sum_{n=1}^{\frac{N-1}{2}} c(n)\sin\omega n \right\rvert$	$-\alpha\cdot\omega + \frac{\pi}{2} + \theta$ where $\theta=0$ for $\bar{H}(e^{j\omega}) > 0$ $\theta=\pi$ for $\bar{H}(e^{j\omega}) < 0$	differentiator, Hilbert - transformer

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Type	$H(e^{j\omega})$	$\lvert H(e^{j\omega})\rvert$	$\angle H(e^{j\omega})$	Application
Anti-symmetrical impulse response N - Even	$e^{j\pi/2} \cdot e^{-j\omega(N-1)/2} \sum_{n=1}^{\frac{N}{2}} d(n) \sin\omega\left(n-\frac{1}{2}\right)$ $d(n) = 2h\left(\frac{N}{2}-n\right)$	$\left\lvert \sum_{n=1}^{\frac{N}{2}} d(n) \sin\omega\left(n-\frac{1}{2}\right) \right\rvert$	$-\alpha\omega + \frac{\pi}{2} + \theta$ where $\theta = 0$ for $H(e^{j\omega}) > 0$ $\theta = \pi$ for $H(e^{j\omega}) < 0$	differentiator, Hilbert - transforme

Design of FIR filters using windows :-

1. Rectangular window :- The rectangular window sequence is given by

$$w_R(n) = 1 \quad \text{for} \quad -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

$$= 0 \quad \text{for} \quad \text{otherwise}$$

An Example is shown in fig for $N = 25$

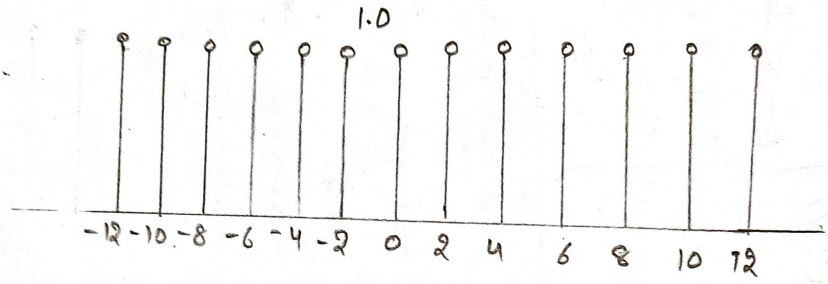


fig :- Rectangular window.

The spectrum of the rectangular window is given by

$$w_R(e^{j\omega}) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} e^{-j\omega n}$$

$$= e^{j\omega\left(\frac{N-1}{2}\right)} + \cdots + e^{j\omega} + 1 + e^{-j\omega} + \cdots + e^{-j\omega\left(\frac{N-1}{2}\right)}$$

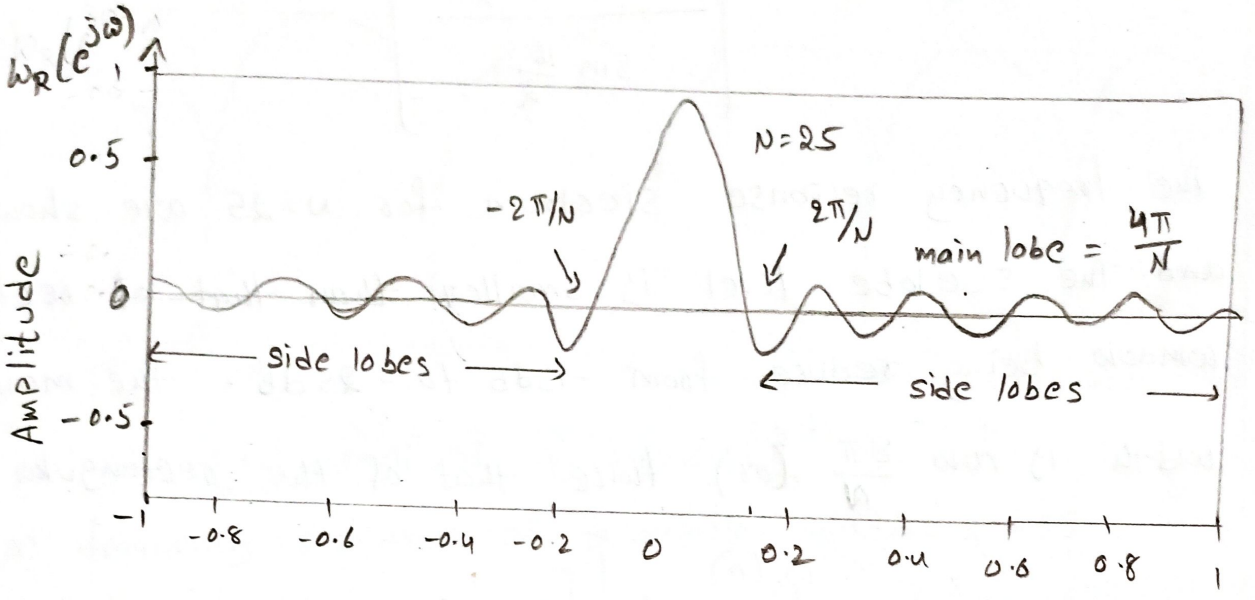
$$= e^{j\omega\left(\frac{N-1}{2}\right)} \left[1 + e^{-j\omega} + \cdots + e^{-j\omega(N-1)}\right]$$

$$= e^{j\omega\left(\frac{N-1}{2}\right)} \left[\frac{1 - e^{j\omega N}}{1 - e^{-j\omega}}\right]$$

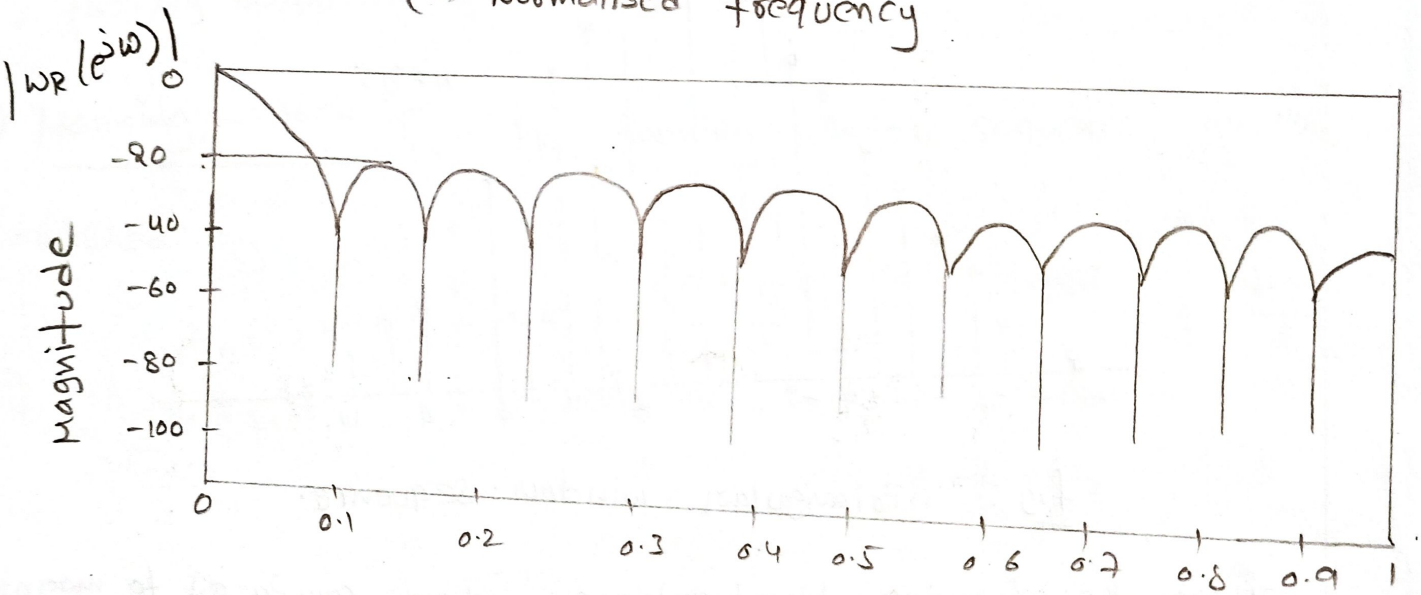
$$1 + a + a^2 \cdots + a^{N-1} = \frac{1-a^N}{1-a}$$

$$= \frac{e^{j\omega N/2}\left(1 - e^{-j\omega N}\right)}{e^{j\omega/2}\left(1 - e^{-j\omega}\right)} = \frac{e^{j\omega N/2} - e^{-j\omega N/2}}{e^{j\omega/2} - e^{-j\omega/2}} = \frac{\sin \omega N/2}{\sin \omega/2}$$

The frequency spectrum for N = 25 as shown in fig.



(a) Normalised frequency



(b) Normalized frequency.

fig:- (a) frequence response of rectangular window N=25 (b) log magnitude response of rectangular window N=25.

② The Triangular (or) Bartlett window :-

The N-Point triangular window is given by

$$W_T(n) = 1 - \frac{2|n|}{N-1} \quad \text{for} \quad -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

the fourier transform of triangular window is.

$$W_T(e^{j\omega}) = \left[\frac{\sin\left(\frac{N-1}{4}\right)\omega}{\sin\frac{\omega}{2}}\right]^2$$

The frequency response spectrum for $N=25$ are shown in fig ~~and~~ The side lobe level is smaller than that of rectangular window. being reduce from $-13db$ to $-25db$. the main lobe width is now $\frac{8\pi}{N}$ for) twice that of the rectangular window.

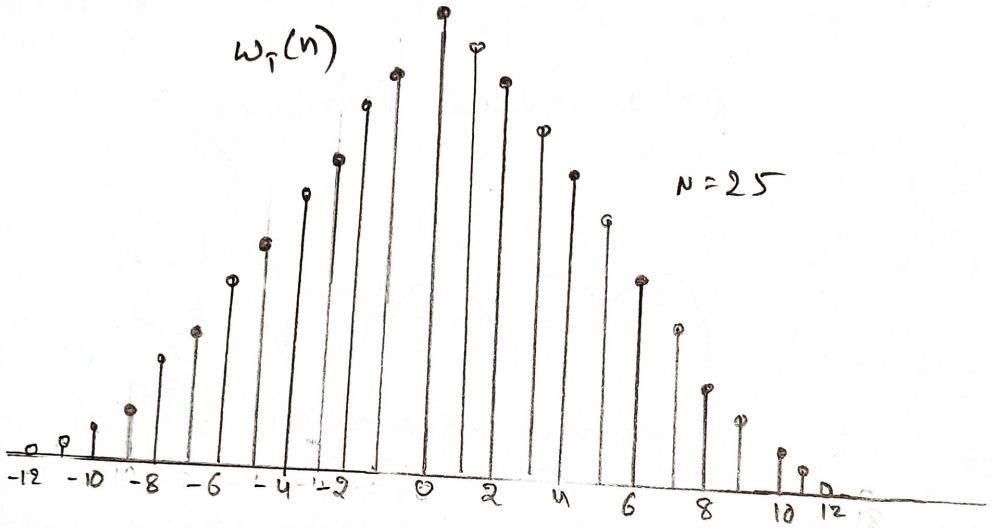


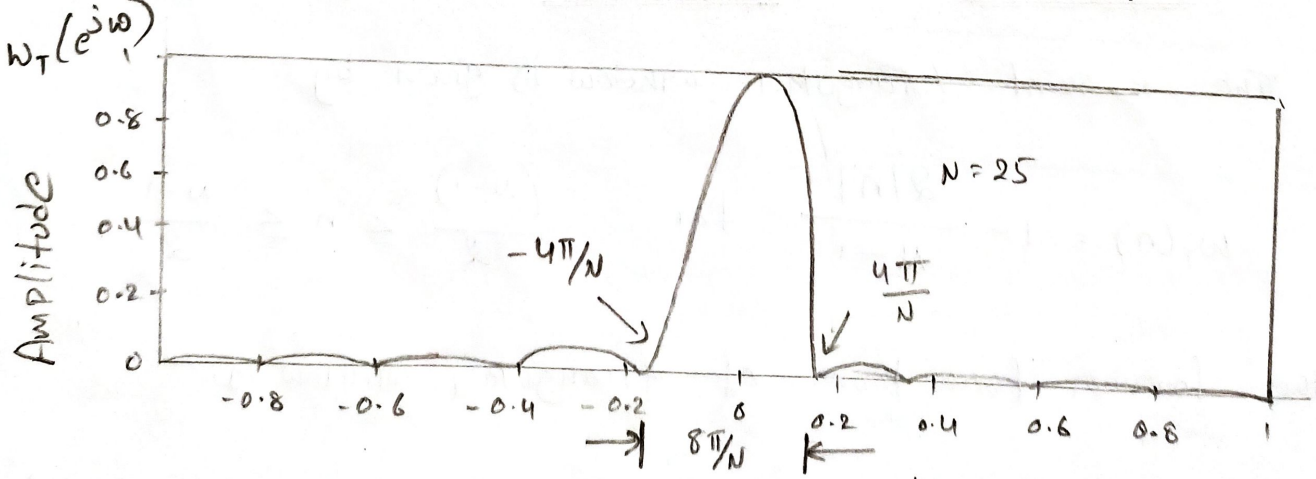
fig :- Triangular window sequence.

It has the following disadvantages, when compared to magnitude response obtained by using rectangular window.

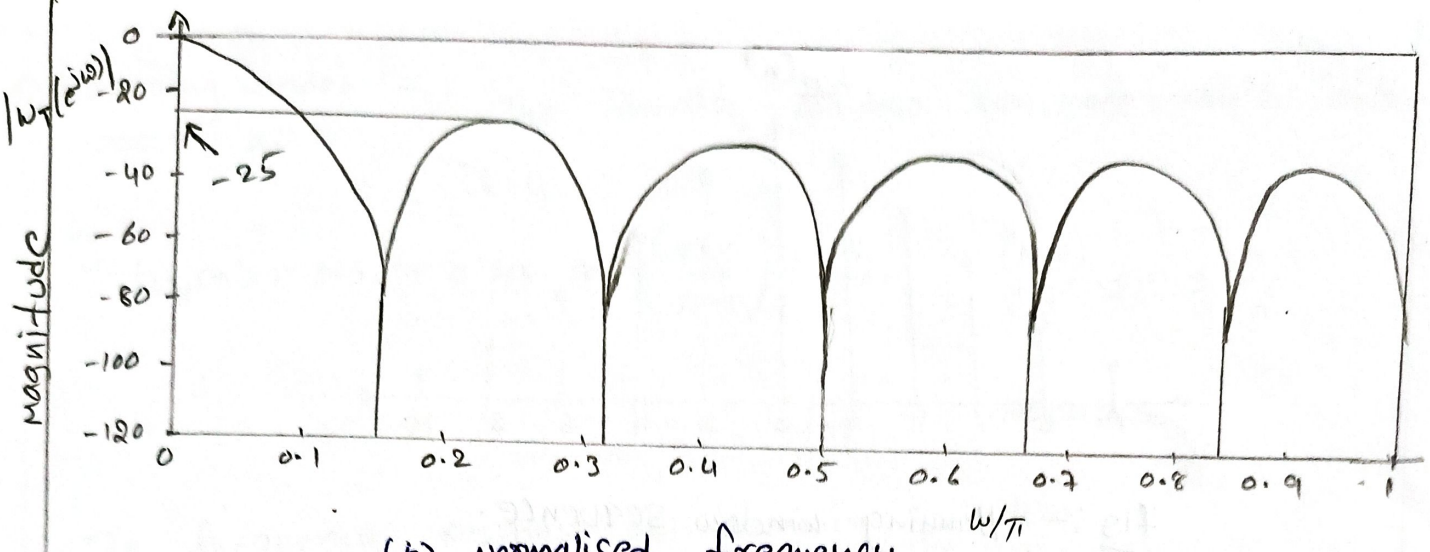
1- The transition region is more

2- The attenuation is stop band is less.

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(a) Normalised frequency.



(b) normalised frequency

fig:- (a) frequency response of triangular window $N = 25$

(b) log magnitude response of triangular window $N = 25$.

③ **Hanning window :-** The Hanning window sequence can be obtained by

$$w_H(n) = 0.5 + 0.5 \cos\left(\frac{2\pi n}{N-1}\right) \quad ; \quad -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

$$= 0 \quad ; \quad \text{otherwise}$$

The frequency response of Hanning window is

$$W_H(n)(e^{j\omega}) = 0.5 \frac{\sin\frac{\omega N}{2}}{\sin\frac{\omega}{2}} + 0.25 \frac{\sin\left(\omega N/2 - \pi N/(N-1)\right)}{\sin\left(\omega/2 - \pi/(N-1)\right)} +$$

$$0.25 \frac{\sin\left(\omega N/2 + \pi N/(N-1)\right)}{\sin\left(\omega/2 + \pi/(N-1)\right)}$$

The main lobe width of Hanning window is twice that of rectangular window, which is results in a doubling of the transition region of the filter. The magnitude of the side lobe level is -31db.

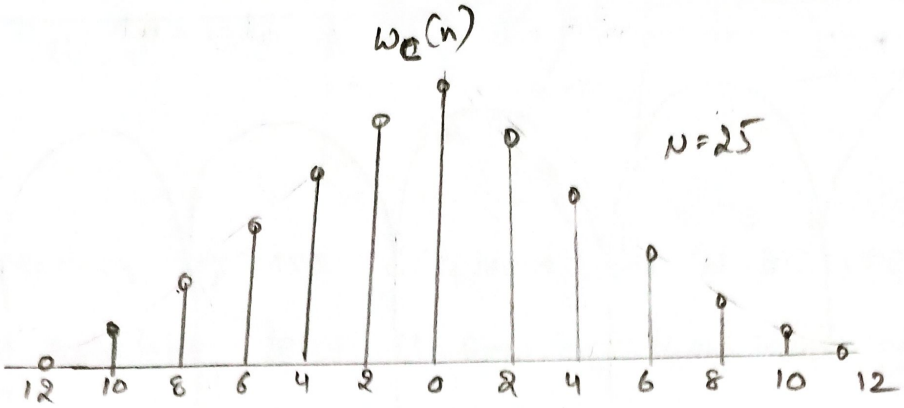
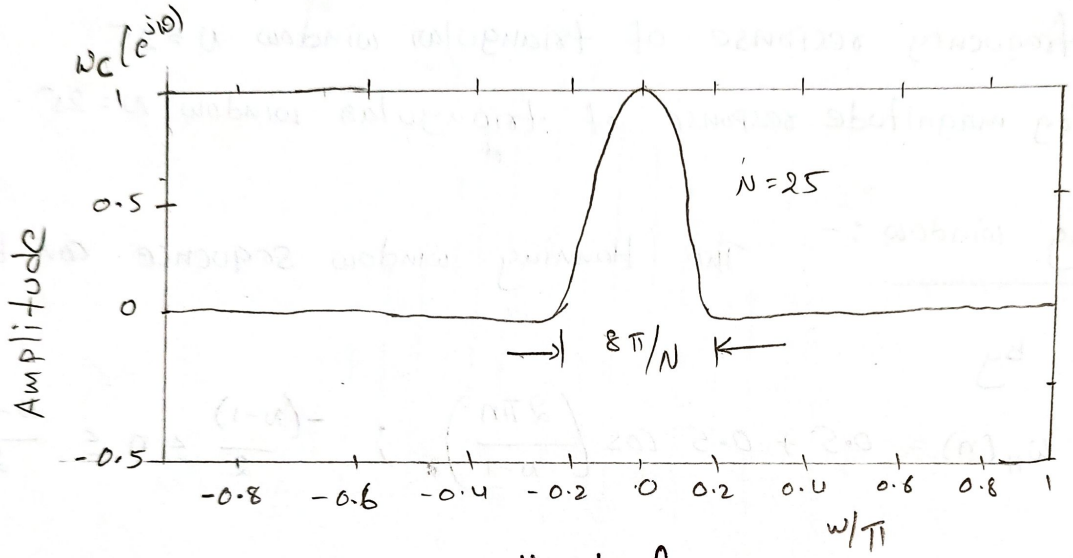
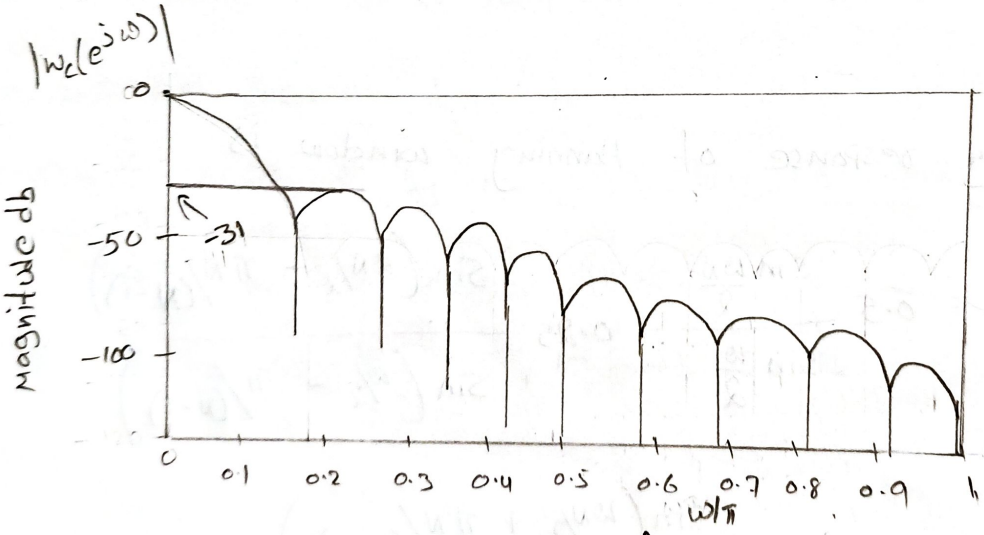


fig :- Hanning window sequence.



(a) Normalised frequency



(b) Normalised frequency

fig:- (a) frequency response of Hanning window for N = 25

(b) log Magnitude response of Hanning window for N = 25

④ Hamming window :- The Hamming window sequence can be obtained by

$$w_H(n) = 0.54 + 0.46 \cos\left(\frac{2\pi n}{N-1}\right) \quad \text{for} \quad -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

$$= 0 \quad \text{for otherwise}$$

The frequency response of Hamming window is

$$W_H(e^{j\omega}) = 0.54 \frac{\sin \omega N/2}{\sin \omega/2} + 0.23 \frac{\sin\left(\omega N/2 - \pi N/(N-1)\right)}{\sin\left(\omega/2 - \pi/(N-1)\right)}$$

$$+ 0.23 \frac{\sin\left(\omega N/2 + \pi N/(N-1)\right)}{\sin\left(\omega/2 - \pi/(N-1)\right)}$$

The Peak side lobe level is down about 41 db from the main lobe Peak, an improvement of 10db relative to the Hanning window. The Magnitude and log magnitude response of low Pass filter designed using hamming window as shown in below fig's.

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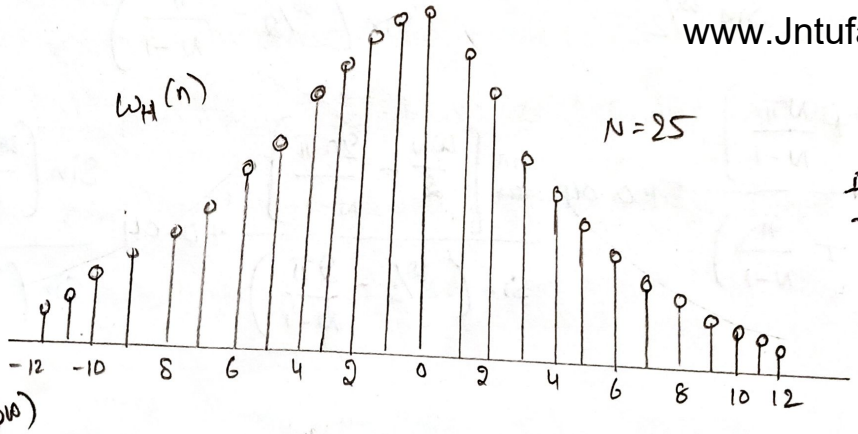


fig:- Hamming window

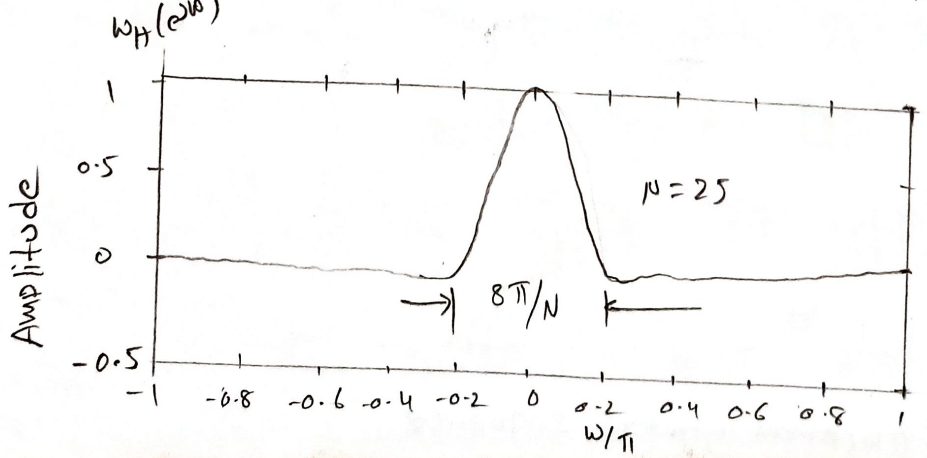
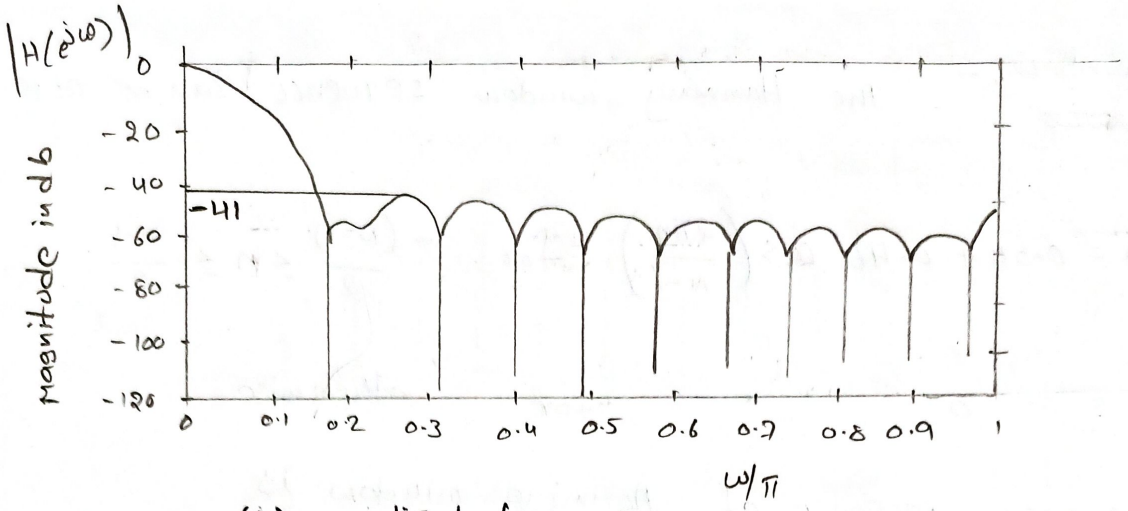


fig:- (a) Normalised frequency



(b) normalised frequency

⑤ **Blackman window :-** The blackman window sequence is given by

$$w_B(n) = 0.42 + 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$$

$$\text{for} \quad \frac{-(N-1)}{2} \le n \le \frac{N-1}{2}$$

$$= 0 \quad \text{for otherwise}$$

The frequence response of Blackman window is

$$w_B(e^{j\omega}) = 0.42\,\frac{\sin\frac{\omega N}{2}}{\sin\omega/2} + 0.25\,\frac{\sin\left(\frac{\omega N}{2} - \frac{N\pi}{N-1}\right)}{\sin\left(\omega/2 - \frac{\pi}{N-1}\right)} +$$

$$0.25\,\frac{\sin\left(\frac{\omega N}{2} + \frac{N\pi}{N-1}\right)}{\sin\left(\omega/2 + \frac{\pi}{N-1}\right)} + 0.04\,\frac{\sin\left[\frac{\omega N}{2} - \frac{2n\pi}{N-1}\right]}{\sin\left(\omega/2 - \frac{2\pi}{N-1}\right)} + 0.04\,\frac{\sin\left(\frac{\omega N}{2} + 2N\pi\right]}{\sin\left(\omega/2 + \frac{2\pi}{N-1}\right)}$$

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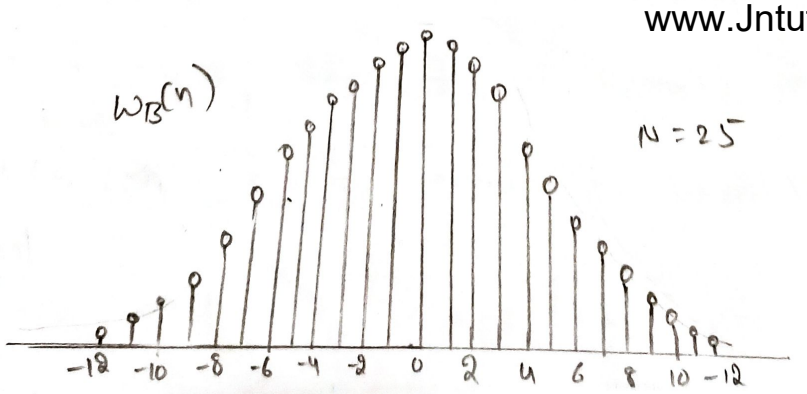
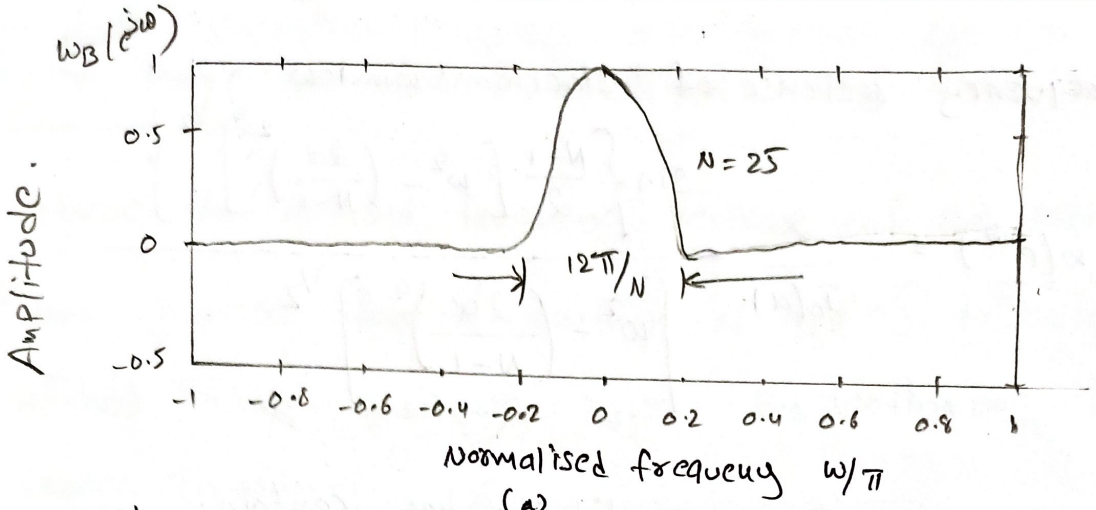
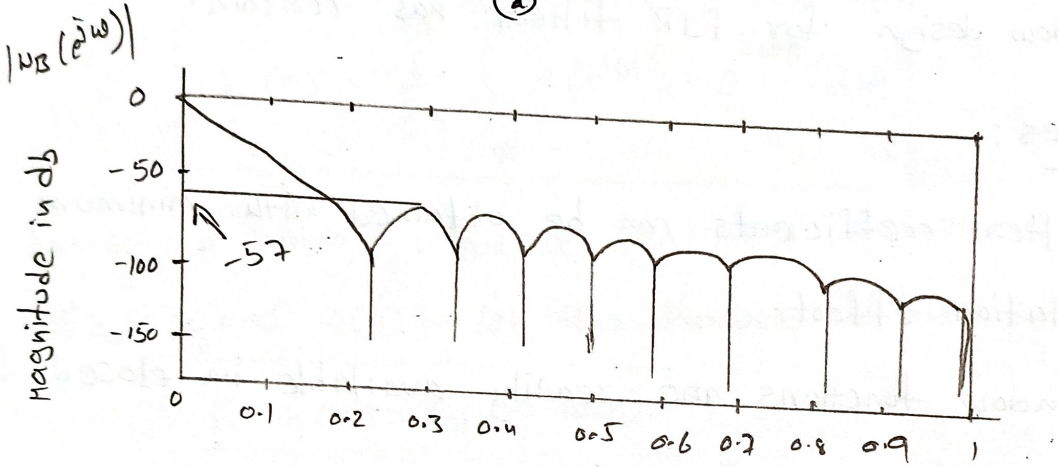


fig : Blackman window sequence



(a)



(b)

fig (a) :- frequency response of Blackman window for N = 25

(b) :- log magnitude response of Blackman window for N = 25

→ compared with the Hanning & Hamming windows reduces the sidelobes, but increase the main lobes width to $\frac{12\pi}{N}$. The frequency response of Blackman window as shown in above fig. The Peak side lobe level is down about 57 db from the main lobe peak, an improvement of 16 db relative to the Hamming window.

⑥ **kaiser window:-** The kaiser window is given by

$$W_K(n) = \frac{I_0\left[\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right]}{I_0(\alpha)} \quad \text{for } |n| \le \frac{N-1}{2}$$

$$= 0 \quad \text{for otherwise.}$$

The frequency response of Blackman window

$$w(e^{j\omega}) = \frac{2}{I_0(\alpha)} \frac{\sin\left\{\frac{N-1}{2}\left[\omega^2 - \left(\frac{2\alpha}{N-1}\right)^2\right]^{1/2}\right\}}{\left[\omega^2 - \left(\frac{2\alpha}{N-1}\right)^2\right]^{1/2}}$$

The window design for FIR filter has certain.

Advantages :-

1. The filter coefficients can be obtained with minimum computation effort.
2. The window functions are readily available in closed-form Expression.
3. The ripples in both Pass band and stop band are almost completely removed.

Disadvantages :-

1- It is always not possible to obtain a closed form Expression for the fourier series coefficients $h(n)$.
2. windows Provides limited flexibility in the design.
3- It is some what difficult to determine, In advance, the type of window and the duration N required to meet a given Prescribed frequency specification.

Design Procedure for FIR FILTER using windows :-

1. choose the desired frequency response of the filter $H(e^{j\omega})$
2. Take Inverse fourier transform of $H(e^{j\omega})$ to obtain the desired impulse response $h_d(n)$. By defination of Inverse fourier Transform.

$$\therefore h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) \cdot e^{j\omega n} \cdot d\omega$$

3. choose a window sequence $w(n)$. and determine the Product of $h_d(n)$ and $w(n)$. let this Product be $h(n)$

$$\therefore h(n) = h_d(n) \cdot w(n)$$

4. The transfer function $H(z)$ of the filter is obtained by taking z-Transform of $h(n)$. Realize the filter by a suitable structure.
5. choose a linear phase magnitude function $|H(e^{j\omega})|$. using $h(n)$, obtain an equation of $|H(e^{j\omega})|$. calculate $|H(\omega)|$. for various value of ω in the range $0 \le \omega \le \pi$ and sketch the graph b/w $|H(e^{j\omega})|$ and ω, which is the frequency response of the filter.

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** Problems (IMH)

Ideal high Pass filter

(1) Design an ~~linear phase~~ ~~FIR filter~~ with a frequence response

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad \frac{\pi}{4} \le |\omega| \le \pi$$

$$= 0 \quad \text{for} \quad |\omega| \le \pi/4$$

using (a) Hanning window (b) Hamming window.

Find the value of $h(n)$ for $N=11$. Find $H(z)$. Plot the magnitude response.

Sol:-

1. Given the desired frequency response

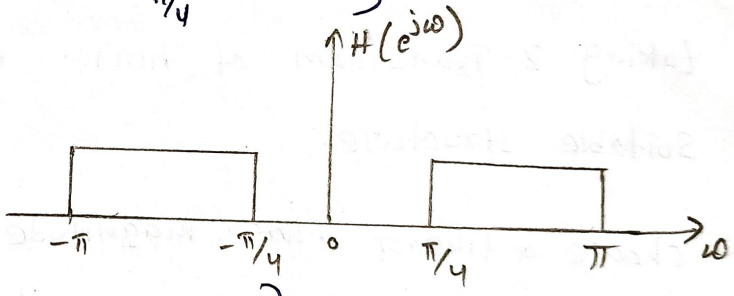
$$H(e^{j\omega}) = 1 \quad \text{for} \quad \frac{\pi}{4} \le |\omega| \le \pi \rightarrow \text{high pass condition.}$$

$$= 0 \quad \text{for} \quad |\omega| \le \pi/4$$

2. By defination of Inverse fourier transform

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) \cdot e^{j\omega n} \cdot d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{-\pi/4} 1 \cdot e^{j\omega n} \cdot d\omega + \int_{\pi/4}^{\pi} 1 \cdot e^{j\omega n} \cdot d\omega \right]$$



$$= \frac{1}{2\pi} \left[\left[\frac{e^{j\omega n}}{jn} \right]_{-\pi}^{-\pi/4} + \left[\frac{e^{j\omega n}}{jn} \right]_{\pi/4}^{\pi} \right]$$

$$= \frac{1}{2\pi jn} \left[e^{-j\frac{\pi}{4}n} - e^{-j\pi n} + e^{j\pi n} - e^{j\pi/4 n} \right]$$

$$= \frac{1}{\pi n} \left[-\left[\frac{e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}}{2j} \right] + \left[\frac{e^{j\pi n} - e^{-j\pi n}}{2j} \right] \right]$$

$$= \frac{1}{\pi n} \left[-\sin\frac{\pi}{4}n + \sin\pi n \right]$$

$\because \frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta$

$$\therefore h_d(n) = \frac{1}{\pi n}\left[\sin \pi n - \sin \frac{\pi}{4} n\right] \longrightarrow (1)$$

$\therefore$ Impulse response $\boxed{h(n) = h_d(n) \cdot w_c(n)} \longrightarrow (2)$

(a) Hanning window sequence

$$w_c(n) = 0.5 + 0.5 \cos \frac{2\pi n}{N-1} \quad \text{for} \quad \frac{-(N-1)}{2} \le n \le \frac{N-1}{2}$$

$$= 0 \quad \text{for otherwise.} \longrightarrow (3)$$

The filter coefficient are symmetrical about $n=0$ satisfying the condition $h(n) = h(-n)$

From Eq (1) ; $N = 11$

$$h_d(n) = \frac{\sin \pi n - \sin \pi/4\, n}{\pi n} =$$

L' Hospital rule

$\lim_{\theta \to 0} \frac{\sin A\theta}{\theta} = A$

when $n=0 \Rightarrow h_d(0) = \lim_{n\to 0} \frac{\sin n\pi}{\pi n} - \lim_{n\to 0} \frac{\sin \pi/4\, n}{\pi n}$

$$= 1 - \frac{1}{4} = \frac{3}{4} = 0.75$$

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when $n=1 \Rightarrow h_d(1) = h_d(-1) \Rightarrow \frac{\sin \pi - \sin \pi/4}{\pi} = -0.225$

when $n=2 \Rightarrow h_d(2) = h_d(-2) = \frac{\sin 2\pi - \sin \pi/2}{2\pi} = -0.159$

when $n=3 \Rightarrow h_d(3) = h_d(-3) = \frac{\sin 3\pi - \sin \frac{3\pi}{4}}{3\pi} = -0.075$

when $n=4 \Rightarrow h_d(4) = h_d(-4) = \frac{\sin 4\pi - \sin \pi}{4\pi} = 0$

when $n=5 \Rightarrow h_d(5) = h_d(-5) = \frac{\sin 5\pi - \sin \frac{5\pi}{4}}{5\pi} = 0.045$

From Eq (3) ; $N = 11$

$$w_H(n) = 0.5 + 0.5\cos\left(\frac{2\pi n}{10}\right) \quad ; \quad -\frac{(11-1)}{2} \le n \le \frac{(11-1)}{2}$$

$$w_H(n) = 0.5 + 0.5\cos\left(\frac{\pi n}{5}\right) \quad ; \quad -5 \le n \le 5$$

when $n = 0$; $w_H(0) = 0.5 + 0.5 = 1$

when $n = 1$; $w_H(1) = w_H(-1) = 0.5 + 0.5\cos\left(\pi/5\right) = 0.9045$

$n = 2$; $w_H(2) = w_H(-2) = 0.5 + 0.5\cos\left(\frac{2\pi}{5}\right) = 0.655$

$n = 3$; $w_H(3) = w_H(-3) = 0.5 + 0.5\cos\left(\frac{3\pi}{5}\right) = 0.345$

$n = 4$; $w_H(4) = w_H(-4) = 0.5 + 0.5\cos\left(\frac{4\pi}{5}\right) = 0.0945$

$n = 5$; $w_H(5) = w_H(-5) = 0.5 + 0.5\cos(\pi) = 0$

From Eq (2) $h(n) = h_d(n) \cdot w_H(n)$ for $-5 \le n \le 5$

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$h(0) = h_d(0) \cdot w_H(0) = 0.75(1) = 0.75$

$h(1) = h(-1) = h_d(1) \cdot w_H(1) = -0.225(0.905) = -0.204$

$h(2) = h(-2) = h_d(2) \cdot w_H(2) = -0.159(0.655) = -0.104$

$h(3) = h(-3) = h_d(3) \cdot w_H(3) = -0.075(0.345) = -0.026$

$h(4) = h(-4) = h_d(4) \cdot w_H(4) = 0(0.814) = 0$

$h(5) = h(-5) = h_d(5) \cdot w_H(5) = 0.045(0) = 0$

∴ The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{-n} + z^{n}\right]$$

$$= h(0) + \sum_{n=1}^{5} h(n)\left[z^{-n} + z^{n}\right]$$

$$= h(0) + h(1)\left[z^{-1} + z\right) + h(2)\left(z^{-2} + z^{2}\right) + h(3)\left(z^{-3} + z^{3}\right) +$$

$$h(4)\left(z^{-4} + z^{4}\right) + h(5)\left(z^{-5} + z^{5}\right)$$

$$= 0.75 - 0.204\left(z + z^{-1}\right) - 0.104\left(z^{2} + z^{-2}\right) - 0.026\left(z^{3} + z^{-3}\right) +$$

$$0\left(z^{4} + z^{-4}\right) + 0\left(z^{5} + z^{-5}\right)$$

$$H(z) = 0.75 - 0.204\left(z + z^{-1}\right) - 0.104\left(z^{2} + z^{-2}\right) - 0.026\left(z^{3} + z^{-3}\right)$$

$$H(z) = 0.75 - 0.204z - 0.204z^{-1} - 0.104z^{2} - 0.104z^{-2} - 0.026z^{3} - 0.026z^{-3}$$

The transfer function of the realizable filter is

$$H'(z) = z^{-5} H(z) \rightarrow (4)$$

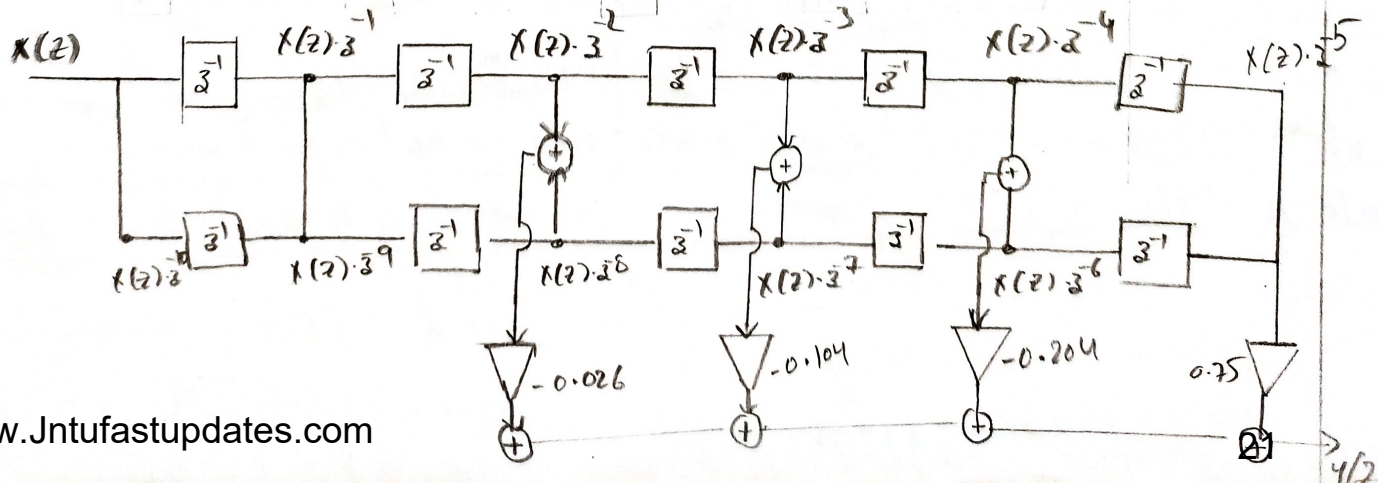
$$= 0.75z^{-5} - 0.204z^{-4} - 0.204z^{-6} - 0.104z^{-3} - 0.104z^{-7} - 0.026z^{-2}$$
$$- 0.026z^{-8}$$

$$H'(z) = -0.026z^{-2} - 0.104z^{-3} - 0.204z^{-4} + 0.75z^{-5} - 0.204z^{-6} - 0.104z^{-7}$$
$$- 0.026z^{-8}$$

from Eq(4) $H'(z) = z^{-5}\left[0.75 - 0.204\left[z + z^{-1}\right] - 0.104\left(z^{2} + z^{-2}\right) - 0.026\left(z^{3} + z^{-3}\right)\right]$

$$\frac{Y(z)}{X(z)} = 0.75z^{-5} - 0.204\left[z^{-4} + z^{-6}\right] - 0.104\left[z^{-3} + z^{-7}\right] - 0.026\left[z^{-2} + z^{-8}\right]$$

$$Y(z) = 0.75X(z)z^{-5} - 0.204\left(X(z)\cdot z^{-4} + X(z)\cdot z^{-6}\right) - 0.104\left[X(z)\cdot z^{-3} + X(z)\cdot z^{-7}\right]$$
$$- 0.026\left(X(z)\cdot z^{-2} + X(z)\cdot z^{-8}\right)]$$



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fig:- Linear phase realization structure of high pass FIR filter

Magnitude response of

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n) \cos \omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.75$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$n=1$; $a(1) = 2h(5-1) = 2h(4) = -0.408$

$n=2$, $a(2) = 2h(5-2) = 2h(3) = -0.208$

$n=3$; $a(3) = 2h(5-3) = 2h(2) = -0.052$

$n=4$, $a(4) = 2h(5-4) = 2h(1) = 0$

$n=5$, $a(5) = 2h(5-5) = 2h(0) = 0$

$$\therefore \bar{H}(e^{j\omega}) = a(0)\cdot\cos\omega(0) + a(1)\cos\omega + a(2)\cos 2\omega + a(3)\cos 3\omega + a(4)\cos 4\omega + a(5)\cos 5\omega$$

$$\therefore \bar{H}(e^{j\omega}) = 0.75 - 0.408\cos\omega - 0.208\cos 2\omega - 0.052\cos 3\omega$$

$$\therefore |H(e^{j\omega})|_{dB} = 20\log\left(\bar{H}(e^{j\omega})\right)$$

The causal filter coefficients are

$h(0) = h(10) = 0$

$h(1) = h(9) = 0$

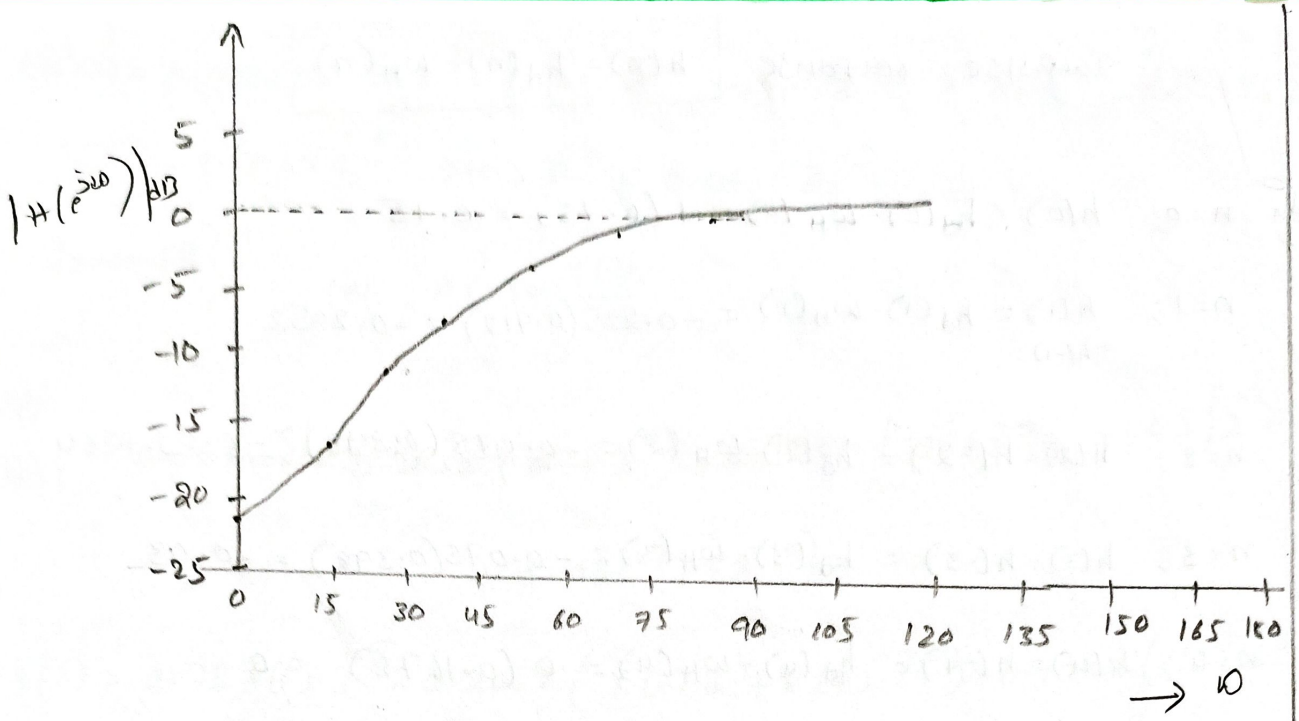
$h(2) = h(8) = -0.026$

$h(3) = h(7) = -0.104$

$h(4) = h(6) = -0.204$

$h(5) = 0.75$

ω	0	15	30	45	60	75	90	105	120	135	150	165	180
$\bar{H}(e^{j\omega})$	0.062	0.139	0.292	0.498	0.702	0.86	0.96	0.99	1.006	1.0012	1	1	1.002
$\lvert H(e^{j\omega})\rvert$	-21.72	-17.14	-10.67	-6.05	-3.07	-1.29	-0.37	-0.0087	0.052	0.015	0	0	0.017



Hamming window :- The Hamming window sequence is given by

$$w_H(n) = 0.54 + 0.46 \cos\left(\frac{2n\pi}{N-1}\right) ; \quad -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

$$= 0 \quad ; \quad \text{otherwise.}$$

when $n=0 \Rightarrow w_H(0) = 0.54 + 0.46 = 1$

$n=1 \Rightarrow w_H(1) = w_H(-1) = 0.54 + 0.46\cos\left(\frac{\pi}{5}\right) = 0.912$

$n=2 \Rightarrow w_H(2) = w_H(-2) = 0.54 + 0.46\cos\left(\frac{2\pi}{5}\right) = 0.682$

$n=3 \Rightarrow w_H(3) = w_H(-3) = 0.54 + 0.46\cos\left(\frac{3\pi}{5}\right) = 0.398$

$n=4 \Rightarrow w_H(4) = w_H(-4) = 0.54 + 0.46\cos\left(\frac{4\pi}{5}\right) = 0.1678$

$n=5 \Rightarrow w_H(5) = w_H(-5) = 0.54 + 0.46\cos(\pi) = 0.08$

from the above Problem eq(1) is $\boxed{h_d(n) = \frac{\sin\pi n - \sin\pi/4\, n}{\pi n}}$

$h_d(0) = 0.75$

$h_d(1) = h_d(-1) = -0.225$

$h_d(2) = h_d(-2) = -0.159$

$h_d(3) = h_d(-3) = -0.075$

$h_d(4) = h_d(-4) = 0$

$h_d(5) = h_d(-5) = 0.045$

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∴ Impulse response $\boxed{h(n) = h_d(n) \cdot w_H(n)}$

When $n=0$, $h(0) = h_d(0) \cdot w_H(0) = 1(0.75) = 0.75$

$n=1$; $h(1) = h_d(1) \cdot w_H(1) = -0.225(0.912) = -0.2052$

$= h(-1)$

$n=2$; $h(2) = h(-2) = h_d(2) \cdot w_H(2) = -0.159(0.682) = -0.1084$

$n=3$; $h(3) = h(-3) = h_d(3) \cdot w_H(3) = -0.075(0.398) = -0.03$

$n=4$; $h(4) = h(-4) = h_d(4) \cdot w_H(4) = 0(0.1678) = 0$

$n=5$; $h(5) = h(-5) = h_d(5) \cdot w_H(5) = -0.045(0.08) = 0.0036$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^n + z^{-n}\right] = h(0) + \sum_{n=1}^{5} h(n)\left[z^n + z^{-n}\right]$$

$$= h(0) + h(1)\left[z^1 + z^{-1}\right] + h(2)\left[z^2 + z^{-2}\right] + h(3)\left[z^3 + z^{-3}\right]$$
$$+ h(4)\left[z^4 + z^{-4}\right] + h(5)\left[z^5 + z^{-5}\right]$$

$$= 0.75 - 0.2052\left(z + z^{-1}\right) - 0.1084\left[z^2 + z^{-2}\right] - 0.03\left[z^3 + z^{-3}\right]$$
$$+ 0.0036\left[z^5 + z^{-5}\right]$$

The transfer function of the realizable filter is

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$$H'(z) = z^{-5} \cdot H(z)$$

$$= z^{-5}\left[0.75 - 0.2052\left(z + z^{-1}\right)\right] - 0.1084\left[z^2 + z^{-2}\right] - 0.03\left[z^3 + z^{-3}\right]$$
$$+ 0.0036\left[z^5 + z^{-5}\right]$$

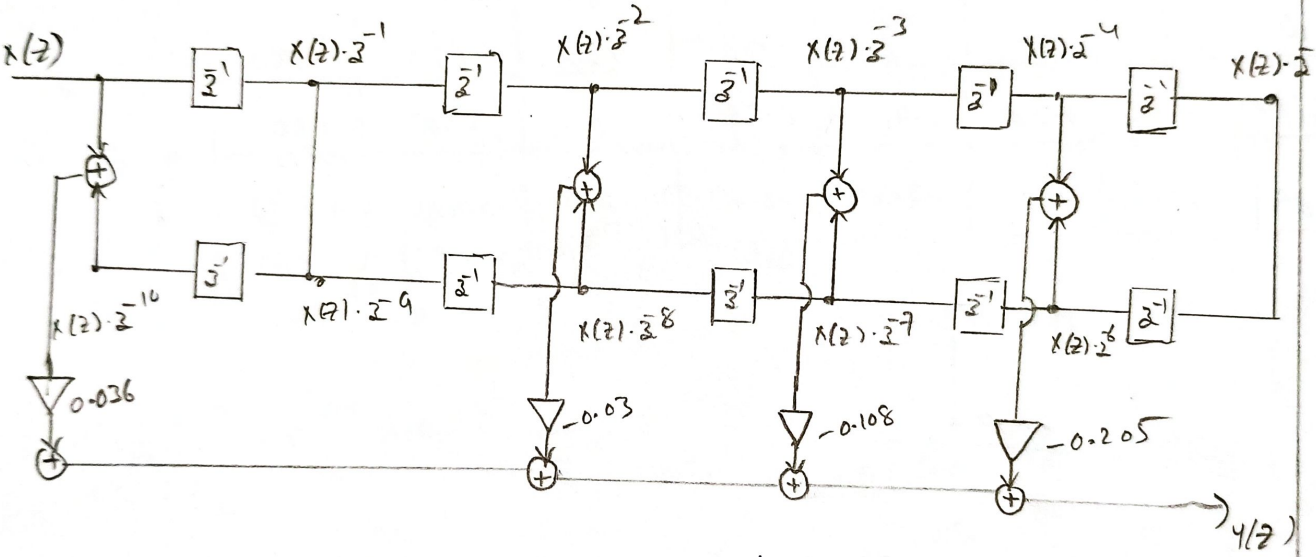
$$= 0.75z^{-5} - 0.2052\left(z^{-4} + z^{-6}\right) - 0.1084\left[z^{-3} + z^{-7}\right] - 0.03\left[z^{-2} + z^{-8}\right]$$
$$+ 0.0036\left[1 + z^{-10}\right]$$

$$\therefore H'(z) = 0.0036 - 0.03z^{-2} - 0.1084z^{-3} - 0.205z^{-4} + 0.75z^{-5} - 0.2052z^{-6} - 0.1084z^{-7} - 0.03z^{-8} + 0.0036z^{-10}$$

from eq (1) let $H(z) = \frac{Y(z)}{X(z)}$

$$\frac{Y(z)}{X(z)} = 0.75z^{-5} - 0.2052\left(z^{-4} + z^{-6}\right) - 0.1084\left(z^{-3} + z^{-7}\right) - 0.03\left[z^{-2} + z^{-8}\right] + 0.036\left(1 + z^{-10}\right)$$

$$\therefore Y(z) = 0.75X(z)\cdot z^{-5} - 0.2052\left(X(z)\cdot z^{-4} + X(z)\cdot z^{-6}\right) - 0.1084\left(X(z)\cdot z^{-3} + X(z)\cdot z^{-7}\right) - 0.03\left(X(z)\cdot z^{-2} + X(z)\cdot z^{-8}\right) + 0.036\left(X(z) + X(z)\cdot z^{-10}\right)$$



Magnitude response of FIR filter is

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.75$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$n=1$; $a(1) = 2h(5-1) = 2h(4) = -0.205$

$n=2$; $a(2) = 2h(5-2) = 2h(3) = -0.1084$

$n=3$; $a(3) = 2h(5-3) = 2h(2) = -0.03$

$n=4$; $a(4) = 2h(5-4) = 2h(1) = 0$

The filter coefficient of causal filter is

$h(0) = h(10) = 0.036$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.03$

$h(3) = h(7) = -0.1084$

$h(4) = h(6) = -0.205$

$h(5) = 0.75$

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$n=5;\quad a(5) = 2h(5-5) = 2h(0) = 0.0072$

$$\therefore \bar{H}(e^{j\omega}) = \sum_{n=0}^{5} a(n)\cos\omega n$$

$$= a(0)\cos\omega(0) + a(1)\cos\omega + a(2)\cos 2\omega + a(3)\cos 3\omega + a(4)\cos 4\omega + a(5)\cos 5\omega$$

$$\therefore \bar{H}(e^{j\omega}) = 0.75 - 0.4104\cos\omega - 0.2168\cos 2\omega - 0.06\cos 3\omega + 0.0072\cos 5\omega$$

$$\therefore |H(e^{j\omega})|_{dB} = 20\log\left(\bar{H}(e^{j\omega})\right)$$

ω	0	30	60	90	120	150	180
$\bar{H}(e^{j\omega})$	0.07	0.28	0.716	0.966	1	1.003	1.001
$\lvert H(e^{j\omega})\rvert$	-23.1	-11	-2.89	-0.29	0	0.028	0.093

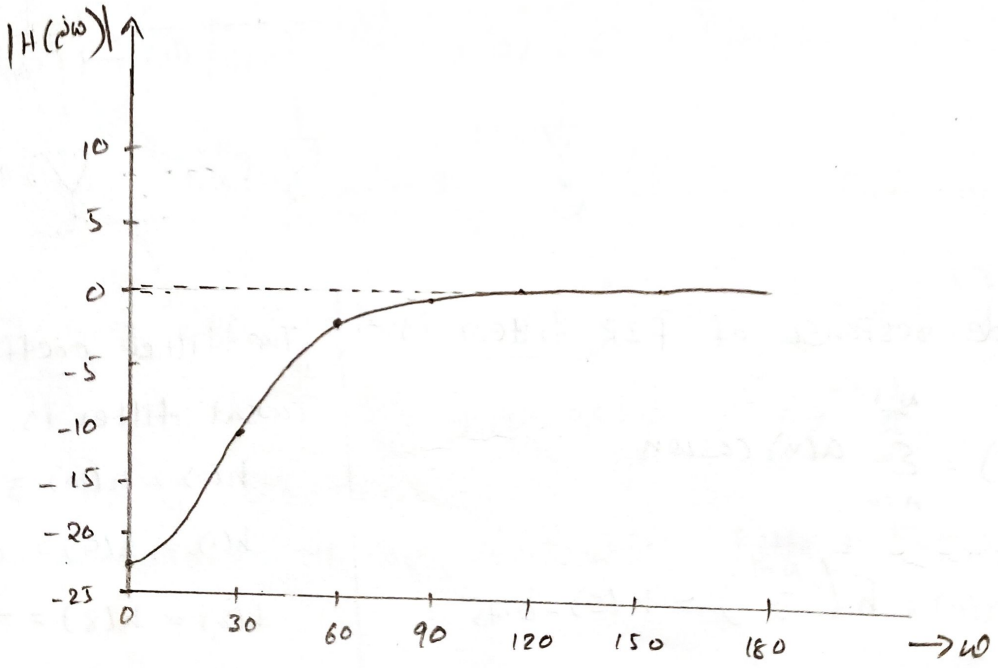


fig: frequency response of High Pass filter.

(2) Design an ideal lowpass filter with a frequency response

$$H(e^{j\omega}) = 1 \quad \text{for} \quad -\pi/2 \le \omega \le \pi/2$$
$$= 0 \quad \text{for} \quad \pi/2 \le |\omega| \le \pi$$

using rectangular window.

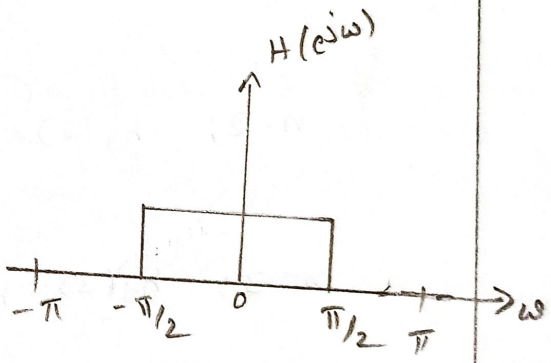
Find the value of $h(n)$ for $N=11$. Find $H(z)$. Plot the magnitude response.

Sol:- 1. Given frequency response of low pass filter is

$$H(e^{j\omega}) = 1 \quad \text{for} \quad -\pi/2 \le \omega \le \pi/2$$
$$= 0 \quad \text{for} \quad \pi/2 \le |\omega| \le \pi$$

2. Take inverse fourier transform

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} h(e^{j\omega}) \cdot e^{j\omega n} \cdot d\omega$$



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1 \cdot e^{j\omega n} \cdot d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi/2}^{\pi/2} = \frac{1}{2\pi jn}\left[e^{j\pi/2 n} - e^{-j\pi/2 n}\right]$$

$$\boxed{\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta}$$

$$= \frac{1}{\pi n}\left[\frac{e^{j\pi/2 n} - e^{-j\pi/2 n}}{2j}\right]$$

$$h_d(n) = \frac{\sin \pi/2\, n}{\pi n} \longrightarrow (1)$$

∴ Impulse response $\boxed{h(n) = h_d(n) \cdot w_R(n)}$ $\longrightarrow (2)$

Rectangular window sequence is given by

$$w_R(n) = 1 \quad \text{for} \quad \frac{-(N-1)}{2} \le n \le \frac{N-1}{2} \longrightarrow (3)$$

$$= 0 \quad \text{for} \quad \text{otherwise}$$

From the given frequency response of the filter coefficient are satisfying the condition $h(n) = h(-n)$

from Eq (1); $h_d(n) = \dfrac{\sin \pi/2\, n}{\pi n}$ [L'Hospital rule $\lim_{\theta \to 0} \frac{\sin A\theta}{\theta} = A$]

when $n=0$; $h_d(0) = \lim_{n \to 0} \dfrac{\sin \frac{1}{2}\pi n}{\pi n} = 1/2 = 0.5$

when $n=1$; $h_d(1) = h_d(-1) = \dfrac{\sin \pi/2}{\pi} = \dfrac{1}{\pi} = 0.3183$

$n=2$; $h_d(2) = h_d(-2) = \dfrac{\sin \pi}{2\pi} = 0$

$n=3$; $h_d(3) = h_d(-3) = \dfrac{\sin \pi/2\,(3)}{3\pi} = -\dfrac{1}{3\pi} = -0.106$

$n=4$; $h_d(4) = h_d(-4) = \dfrac{\sin 2\pi}{4\pi} = 0$

$n=5$; $h_d(5) = h_d(-5) = \dfrac{\sin \frac{5\pi}{2}}{5\pi} = \dfrac{1}{5\pi} = 0.0636$

From Eq (3) $N = 11$

$w_R(n) = 1$; $-5 \le n \le 5$

when $n=0$; $w_R(0) = 1$

$n=1$; $w_R(1) = w_R(-1) = 1$

$n=2$; $w_R(2) = w_R(-2) = 1$

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$n=3$; $w_R(3) = w_R(-3) = 1$

$n=4$; $w_R(4) = w_R(-4) = 1$

$n=5$; $w_R(5) = w_R(-5) = 1$

from Eq (2) : $h(n) = h_d(n) \cdot w_R(n)$

when $n=0$; $h(0) = h_d(0) \cdot w_R(0) = 0.5(1) = 0.5$

$n=1$; $h(1) = h(-1) = h_d(1) \cdot w_R(1) = 0.3183(1) = 0.318$

$n=2$; $h(2) = h(-2) = h_d(2) \cdot w_R(2) = 0(1) = 0$

$n=3$; $h(3) = h(-3) = h_d(3) \cdot w_R(3) = -0.106(1) = -0.106$

$n=4$; $h(4) = h(-4) = h_d(4) \cdot w_R(4) = 0(1) = 0$

$n=5$; $h(5) = h(-5) = h_d(5) \cdot w_R(5) = 0.0636(1) = 0.0636$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^n + z^{-n}\right]$$

$$= h(0) + \sum_{n=1}^{5} h(n)\left[z^n + z^{-n}\right]$$

$$= h(0) + h(1)\left[z + z^{-1}\right] + h(2)\left[z^2 + z^{-2}\right] + h(3)\left[z^3 + z^{-3}\right] + h(4)\left[z^4 + z^{-4}\right] + h(5)\left[z^5 + z^{-5}\right]$$

$$H(z) = 0.5 + 0.318\left[z + z^{-1}\right] - 0.106\left[z^3 + z^{-3}\right] + 0.0636\left[z^5 + z^{-5}\right]$$

The transfer function of realizable filter is

$$H'(z) = z^{-5} \cdot H(z).$$

$$H'(z) = z^{-5}\left[0.5 + 0.318\left[z + z^{-1}\right] - 0.106\left[z^3 + z^{-3}\right] + 0.0636\left[z^5 + z^{-5}\right]\right]$$

$$H'(z) = 0.5z^{-5} + 0.318\left[z^{-4} + z^{-6}\right] - 0.106\left[z^{-2} + z^{-8}\right] + 0.0636\left[1 + z^{-10}\right] \longrightarrow (4)$$

$$H'(z) = 0.0636 - 0.106z^{-2} + 0.5z^{-5} + 0.318z^{-4} + 0.318z^{-6} - 0.106z^{-8} + 0.0636z^{-10}$$

let $H(z) = \dfrac{Y(z)}{X(z)}$

$$\therefore \frac{Y(z)}{X(z)} = 0.5z^{-5} + 0.318\left[z^{-4} + z^{-6}\right] - 0.106\left[z^{-2} + z^{-8}\right] + 0.0636\left[1 + z^{-10}\right]$$

$$\therefore Y(z) = 0.5X(z)\cdot z^{-5} + 0.318\left[X(z)\cdot z^{-4} + X(z)z^{-6}\right] - 0.106\left[X(z)\cdot z^{-2} + X(z)\cdot z^{-8}\right] + 0.0636\left[X(z) + X(z)\cdot z^{-10}\right]$$

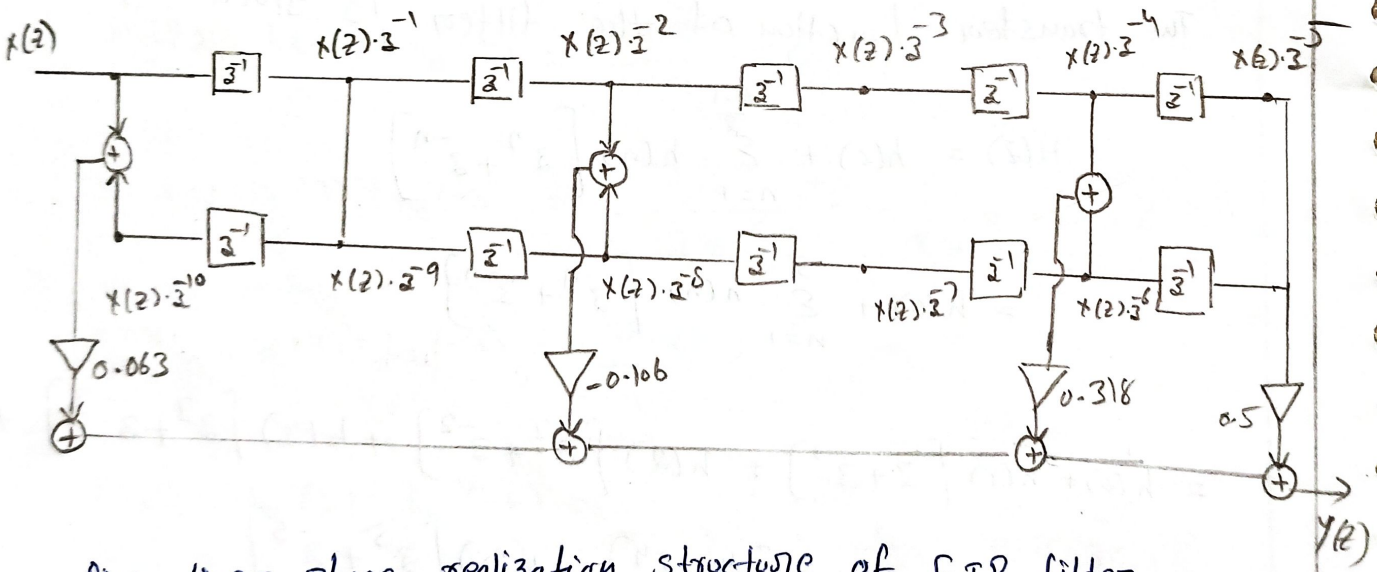


fig:- Linear phase realization structure of FIR filter.

→ The filter coefficient of causal filter is

$h(0) = h(10) = 0.0636$ $\quad h(4) = h(6) = 0.318$

$h(1) = h(9) = 0$ $\quad h(5) = 0.5$

$h(2) = h(8) = -0.106$

$h(3) = h(7) = 0$

Magnitude response of FIR filter is

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a(n)\cos\omega n = \sum_{n=0}^{5} a(n)\cos\omega n$$

when $n=0$; $a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.5$

n; $a(n) = 2h\left(\frac{N-1}{2} - n\right)$

$n=1$; $n=1$; $a(1) = 2h(5-1) = 2h(4) = 2(0.318) = 0.636$

$n=2$; $a(2) = 2h(5-2) = 2h(3) = 2(0) = 0$

$n=3$; $a(3) = 2h(5-3) = 2h(2) = 2(-0.106) = -0.212$

$n=4$; $a(4) = 2h(5-4) = 2h(1) = 2(0) = 0$

$n=5$; $a(5) = 2h(5-5) = 2h(0) = 2(0.0636) = 0.1272$

$$\therefore \bar{H}(e^{j\omega}) = \sum_{n=0}^{5} a(n)\cos\omega n$$

$$= a(0)\cos\omega(0) + a(1)\cos\omega + a(2)\cos 2\omega + a(3)\cos 3\omega + a(4)\cos 4\omega + a(5)\cos 5\omega$$

$$\bar{H}(e^{j\omega}) = 0.5 + 0.636\cos\omega - 0.212\cos 3\omega + 0.1272\cos 5\omega$$

$$\therefore |H(e^{j\omega})| = 20\log\left(\bar{H}(e^{j\omega})\right)$$

ω	0	30	60	90	120	150	180
$\bar{H}(e^{j\omega})$	1.051	0.940	1.09	0.05	-0.09	0.059	-0.051
$\lvert H(e^{j\omega})\rvert$	0.4	-0.517	0.707	-6	-20.6	-24.7	-26

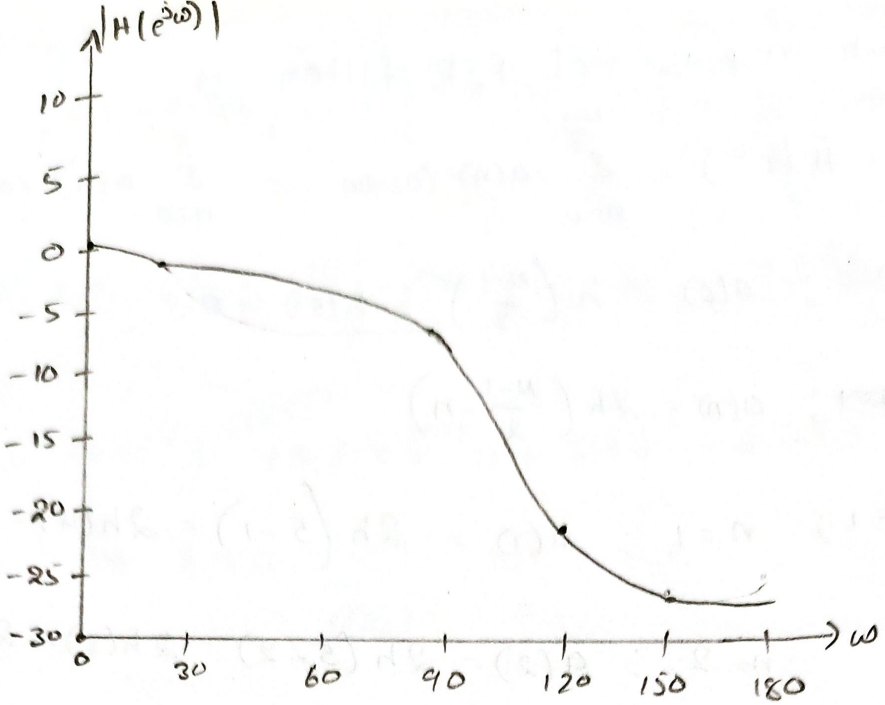


fig :- frequency response of low pass filter.

(1) what are the Properties of FIR filter?

Sol:- 1. FIR filter is always stable

2. A realizable filter can always be obtained

3. FIR filter has a linear phase response.

(2) state the condition for a digital filter to be causal and stable.

Sol:- A digital filter is causal if its impulse response

$$h(n) = 0 \quad \text{for } n < 0$$

A digital filter is stable if its impulse response is absoutely Summable i.e

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

③ what is Gibbs phenomenon?

sol: one possible way of finding an FIR filter that approximation $H(e^{j\omega})$ would be to truncate the infinite fourier series at $n = \pm\left(\frac{N-1}{2}\right)$. Abrupt truncation of the series will lead to oscillation both in Passband and in stopband. This phenomenon is known as Gibbs phenomenon.

④ comparison between IIR & FIR filters

sol:-

FIR filter	IIR filter
1. The impulse response of this filter is restricted to finite number of samples	1. The impulse response of this filter extends over an infinite duration
2. FIR filter can have precisely linear phase	2. These filter do not have linear phase.
3. closed-form design equations do not exist	3. A variety of frequency selective filters can be designed using closed-form design formulas.
4. Most of the design iterative procedures, requiring powerful computational facilities for their implementation	4. These filter can be designed using only a hand oseil calculator and tables of analog filter design parameters.
5. Greater flexibility to control the shape of their magnitude response.	5. less flexibility specially for obtaining non-standard frequency response.
6. Always stable	6. Not always stable.
7. Errors due to round off noise are less severe.	7. IIR filter are more susceptible to errors due to round off noise.

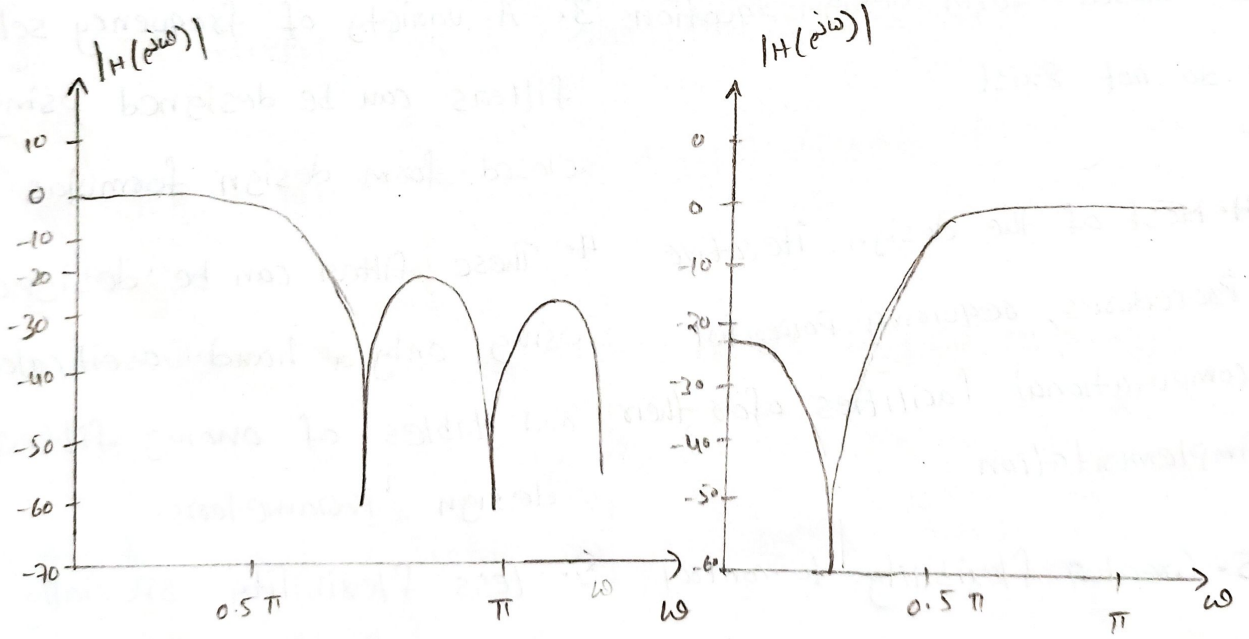
8- In these filter, the poles are fixed at the origin, high selectivity can be achieved by using a relatively high order for the transfer function.	8. The poles are placed anywhere inside the unit circle, high slectivity can be achieved with low-order transfer function.

(5) what are the characteristics of FIR digital filter?

Sol: characteristic of FIR filter :-

1. It Possess linear phase characteristics
2. It Possess higher stability as Poles are at origin and have exact linear phases.
3. It is used for higher order filter circuits.

(6) Draw the frequency of digital low Pass and high Pass filter?



(a) low Pass filter (b) high Pass filter

(7) Compare the types of windows ?

Rectangular window	Hanning window	Hamming window	Blackman window
The width of Mainlobe in window spectrum is $4\pi/N$	The width of Mainlobe in window spectrum is $8\pi/N$	The width of main lobe in window spectrum is $8\pi/N$	The width of main lobe in window spectrum is $12\pi/N$
The maximum side lobe magnitude in window spectrum is -13dB	The maximum side lobe magnitude in window spectrum is -31dB	The maximum side lobe magnitude in window spectrum is -41dB	The maximum side lobe magnitude in window spectrum is -58dB
In FIR filter designed using rectangular window the minimum stopband attenuation is 22dB	In FIR filter designed using hanning window the minimum stopband attenuation is 44dB	In FIR filter designed using hamming window the minimum stop band attenuation is 51dB	In FIR filter designed using Blackman window the minimum stop band attenuation is 78dB
In window spectrum the side lobe magnitude slightly decreases with increasing 'w'	In window spectrum the side lobe magnitude decreases with increasing 'w'	In window spectrum side lobe magnitude remains constant	In window spectrum the side lobe magnitude decreases rapidly with increasing 'w'