UNIT - III

DESIGN OF IIR DIGITAL FILTERS & REALIZATIONS

* Realizations of discrete time systems (or) Block diagram representation of linear constant coefficients of the ~~DE~~ D.E
* We are designing the two systems

1. IIR / Recursive system:- A system is said to recursive system then the present output of the system depends upon the present and past inputs and also on past outputs.

   Eg: $y(n) = \{x(n), x(n-1), x(n-2) \text{ - - - - - } y(n-1), y(n-2) \text{ - - - - }\}$

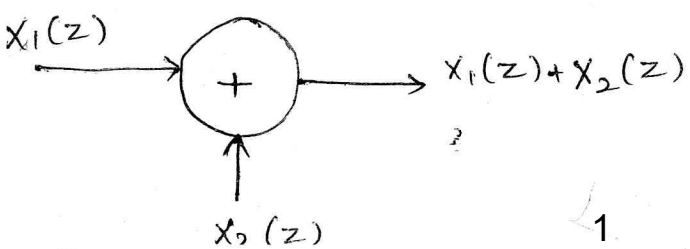
2. FIR / Non-Recursive system:- A system is said to be non-recursive system then the present output of the system depends upon the present and past inputs.

   Eg: $y(n) = \{x(n), x(n-1), x(n-2) \text{ - - - - }\}$

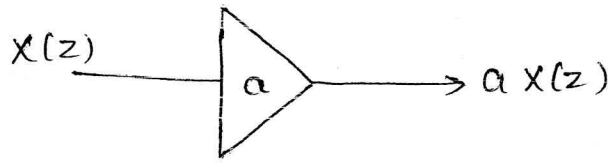
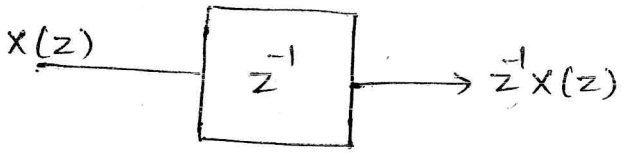
Different methods for realizing the discrete system is given by

IIR system	FIR system
1. ~~Discrete~~ Direct form-I realization	1. ~~Discrete~~ Direct form realization
2. ~~Direct~~ 2. ~~Discrete~~ Direct form-II realization	2. Cascade Realization
3. Cascade form realization	3. Linear Phase Realization
4. parallel form realization	4. Lattice structure
	5. Lattice ladder structure

Elements used to represent the Block diagram is given by

Elements	Block diagram representation
1 Adder	

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Element	Block diagram representation
2. Constant multiplier	
3. Delay element	

IIR system Realization:

In general, the time domain representation is given by

$$y(n) = -\sum_{k=1}^{M} a_k y(n-k) + \sum_{k=0}^{N} b_k x(n-k)$$

Apply z-T/F on Both sides

$$Y(z) = -\sum_{k=1}^{M} a_k z^{-k} Y(z) + \sum_{k=0}^{N} b_k z^{-k} X(z)$$

$$Y(z)\left[1 + \sum_{k=1}^{M} a_k z^{-k}\right] = \sum_{k=0}^{N} b_k z^{-k} X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

Direct form-I Realization: (DFI)

We have, the frequency domain representation of the given IIR system is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{N} b_k z^{-k}}{1+\sum_{k=1}^{M} a_k z^{-k}}$$

$$\frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \cdots + a_N z^{-M}}$$

$$Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) + \cdots + a_N z^{-N} Y(z) =$$

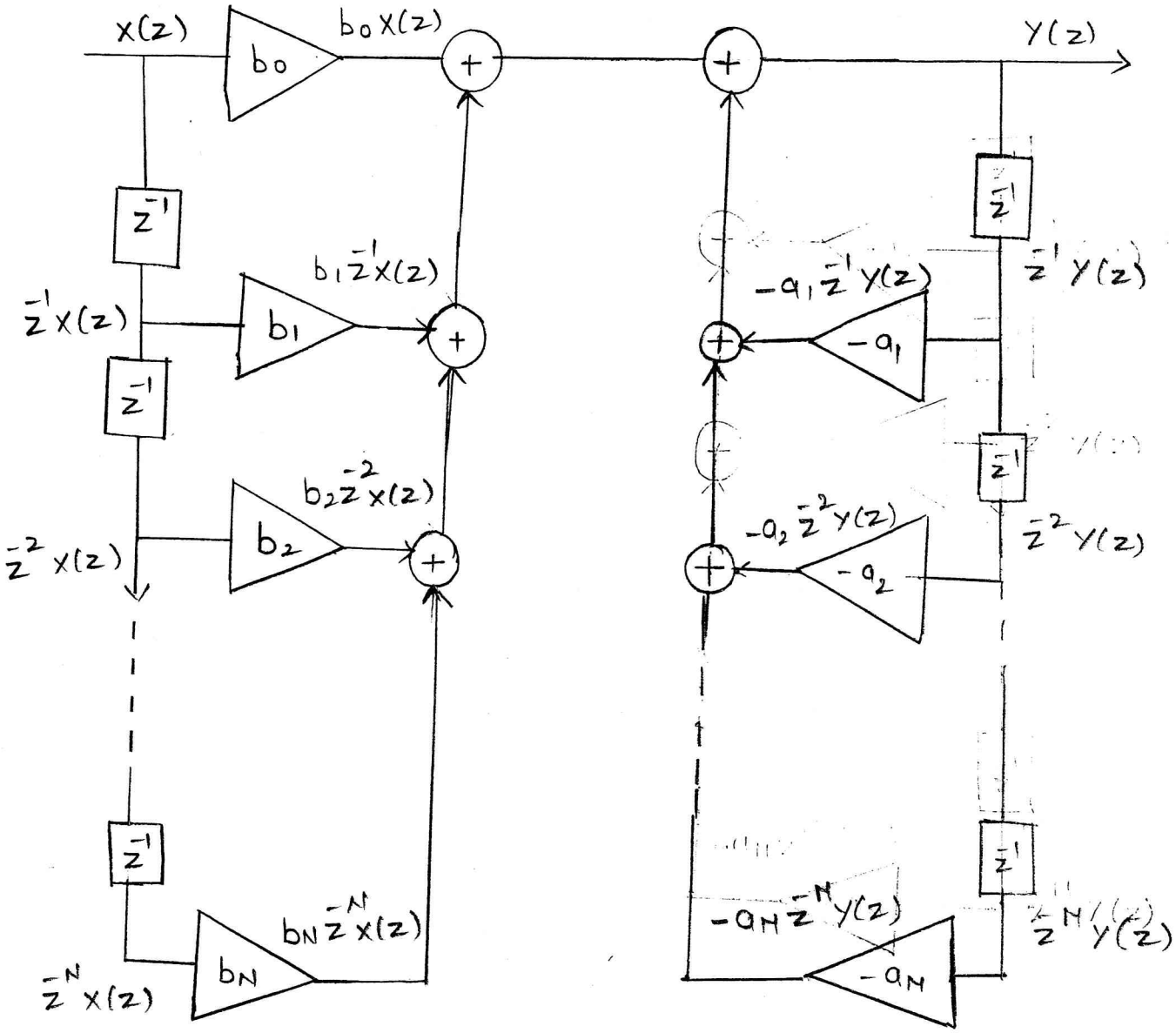
$$b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + \cdots + b_N z^{-N} X(z)$$

$$Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + \cdots + b_N z^{-N} X(z) - a_1 z^{-1} Y(z)$$

$$- a_2 z^{-2} Y(z) - \cdots - a_N z^{-N} Y(z) \rightarrow (1)$$

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Block diagram representation of Eq (1)



Drawback:- Here we using delay element in input section as well as output section. So memory is wasted. To overcome this drawback using form-II realization.

Direct form-II Realization:

In direct form-I, Delay elements are used in both the input and output section. so that the memory is wasted. To avoid that problem we entered into Direct form-II realization

we have

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_N z^{-N}}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

Assume $\frac{Y(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)}$

Consider

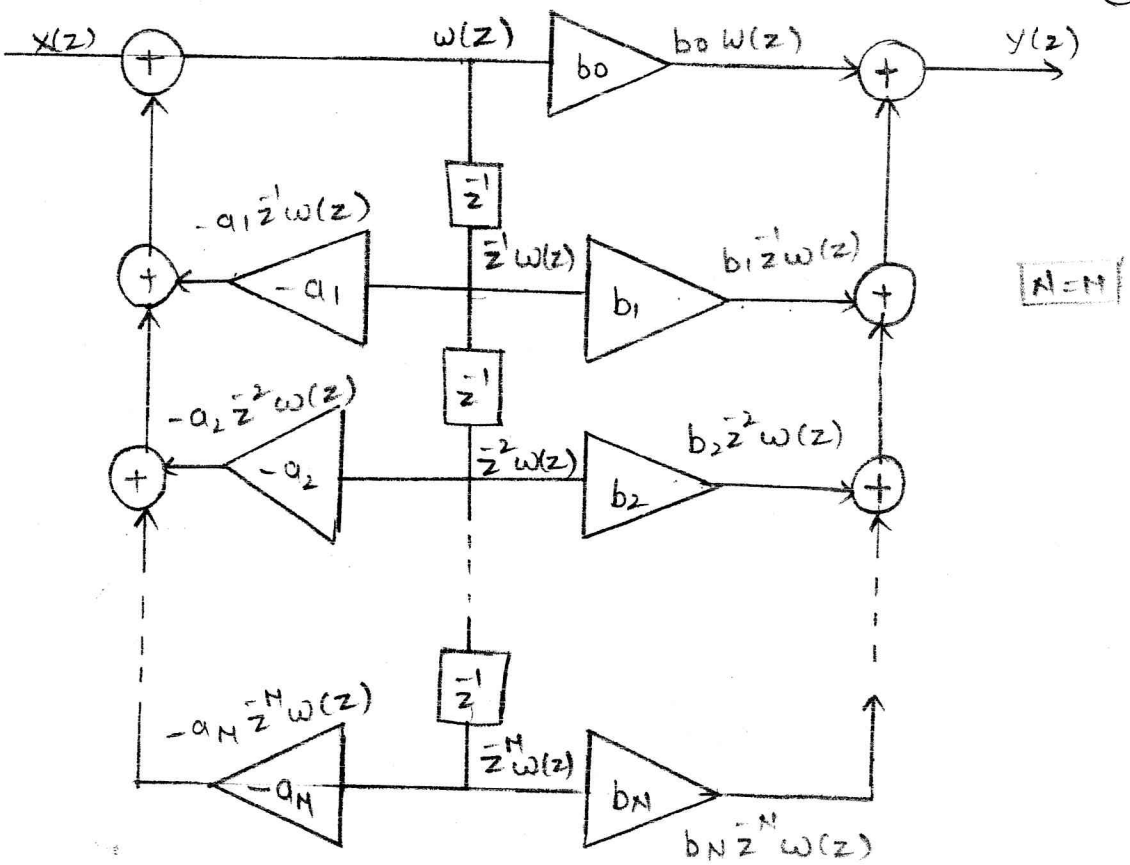
$$\frac{W(z)}{X(z)} = \frac{1}{1 + a_1 z^{-1} + \cdots + a_N z^{-N}}$$

$$W(z) + a_1 z^{-1} W(z) + \cdots + a_N z^{-N} W(z) = X(z)$$

$$W(z) = X(z) - a_1 z^{-1} W(z) - \cdots - a_N z^{-N} W(z) \longrightarrow (1)$$

& $$\frac{Y(z)}{W(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_N z^{-N}$$

$$Y(z) = b_0 W(z) + b_1 z^{-1} W(z) + \cdots + b_N z^{-N} W(z) \longrightarrow (2)$$

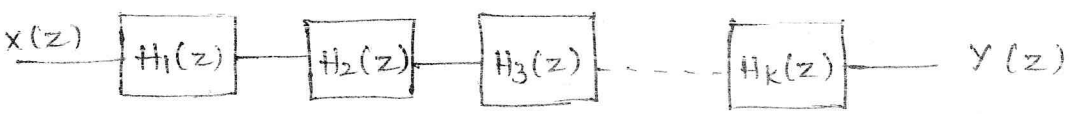


Cascade form Realization: For the cascade form realization $H(z)$ is realized as

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z) \cdots H_k(z)$$

Now realize

$H(z) = H_1(z), H_2(z), H_3(z), \cdots H_k(z)$ in direct form-II in the cascade



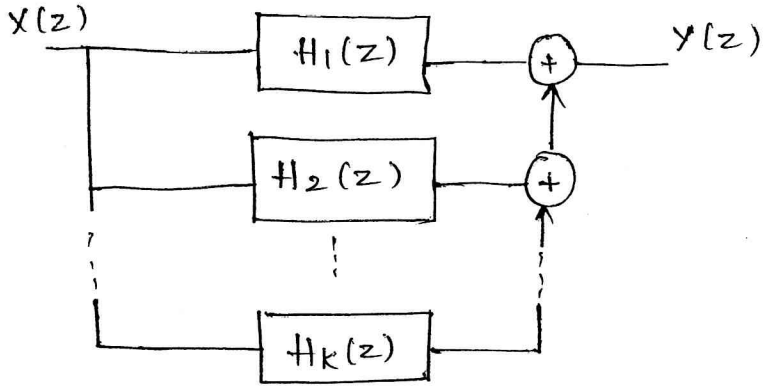
Parallel form Realization: Under the parallel form Realization $H(z)$ is converted in

$$H(z) = H_1(z) + H_2(z) + H_3(z) + \cdots + H_k(z)$$

using partial fraction

Now realize

$H(z) = H_1(z), H_2(z), H_3(z), \cdots H_k(z)$ in direct form-II and all the system in parallel



1. Obtain the Directive form-I & form II, cascade and parallel Realization form for the following system

$$y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + x(n) + 3x(n-1) + 2x(n-2).$$

Sol Given

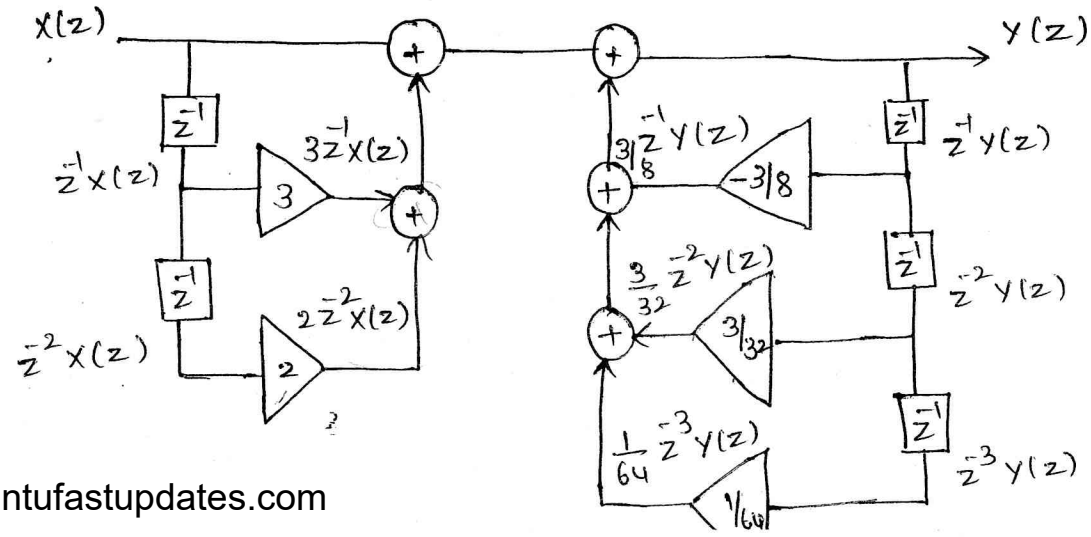
$$y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$$

Apply z-transform on both sides

$$Y(z) = -\frac{3}{8}z^{-1}Y(z) + \frac{3}{32}z^{-2}Y(z) + \frac{1}{64}z^{-3}Y(z) + X(z) + 3z^{-1}X(z) + 2z^{-2}X(z) \longrightarrow (1)$$

Direct form-I realization

Now, realize eq (1) in DFI



Direct form - II Realization:-

From eq ①

$$Y(z)\left[1+\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}\right]=X(z)\left[1+3z^{-1}+2z^{-2}\right]$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+3z^{-1}+2z^{-2}}{1+\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}}$$

Assume $\quad \frac{Y(z)}{X(z)}=\frac{W(z)}{X(z)}\cdot\frac{Y(z)}{W(z)}$

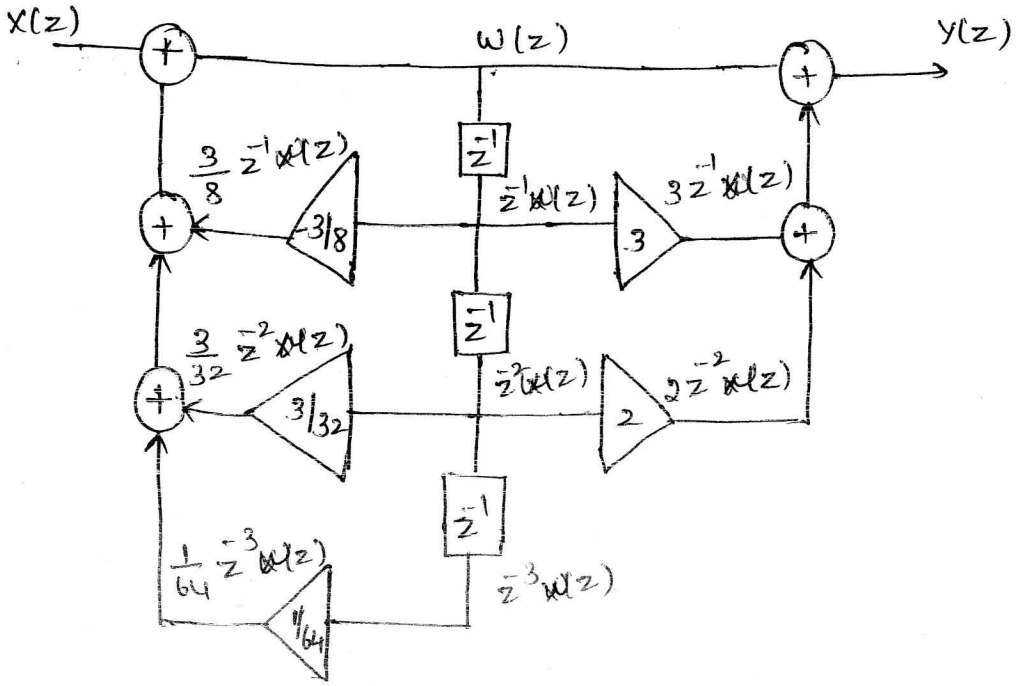
$$\frac{W(z)}{X(z)}=\frac{1}{1+\frac{3}{8}z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}}$$

$$W(z)=-\frac{3}{8}z^{-1}W(z)+\frac{3}{32}z^{-2}W(z)+\frac{1}{64}z^{-3}W(z)+X(z) \longrightarrow ②$$

$$\frac{Y(z)}{W(z)}=1+3z^{-1}+2z^{-2}$$

$$Y(z)=W(z)+3z^{-1}W(z)+2z^{-2}W(z) \longrightarrow ③$$

Now realize ② & ③ in DF - II



cascade form

we have

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3}}$$

$$= \frac{z^3(z^2+3z+2)}{z^2(z^3 + \frac{3}{8}z^2 - \frac{3}{32}z - \frac{1}{64})} = \frac{z(z^2+3z+2)}{(\frac{8}{8}z^3 + \frac{3}{8}z^2 - \frac{3}{32}z - \frac{1}{64})}$$

$$H(z) = \frac{z(z+1)(z+2)}{(z+0.125)(z+0.5)(z-0.25)}$$

$$= \frac{(1+z^{-1})(1+2z^{-1})}{(1+0.125z^{-1})(1+0.5z^{-1})(1-0.25z^{-1})}$$

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z)$$

consider

$$H_1(z) = \frac{1+z^{-1}}{1+0.125z^{-1}} = \frac{Y_1(z)}{X(z)} \quad ; \quad H_2(z) = \frac{1+2z^{-1}}{1+0.5z^{-1}} = \frac{Y_2(z)}{Y_1(z)}$$

$$H_3(z) = \frac{1}{1-0.25z^{-1}} = \frac{Y(z)}{Y_2(z)}$$

Now $H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_1(z)}{W(z)}$

consider

$$\frac{W(z)}{X(z)} = \frac{1}{1+0.125z^{-1}}$$

$$W(z) = X(z) - 0.125z^{-1}W(z) \quad \rightarrow (4)$$

$$\frac{Y_1(z)}{W(z)} = 1 + z^{-1}$$

$$Y_1(z) = W(z) + z^{-1}W(z) \quad \rightarrow (5)$$

Now $H_2(z) = \frac{Y_2(z)}{Y_1(z)} = \frac{W(z)}{Y_1(z)} \cdot \frac{Y_2(z)}{W(z)}$

consider

$$\frac{W(z)}{Y_1(z)} = \frac{1}{1+0.5z^{-1}}$$

$$W(z) = Y_1(z) - 0.5z^{-1}W(z) \quad \rightarrow (6)$$

$$\frac{Y_2(z)}{W(z)} = 1 + 2z^{-1}$$

$$Y_2(z) = W(z) + 2W(z)z^{-1} \quad \rightarrow (7)$$

$$H_3(z) = \frac{Y(z)}{Y_2(z)} = \frac{W(z)}{Y_2(z)} \cdot \frac{Y(z)}{W(z)}$$

consider

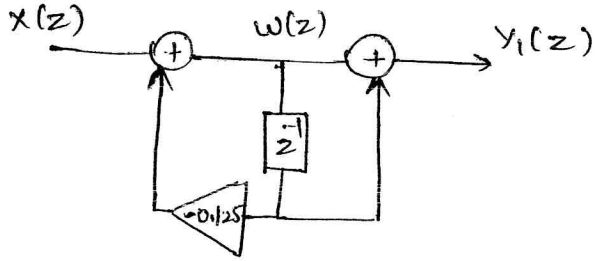
$$\frac{W(z)}{Y_2(z)} = \frac{1}{1 - 0.25 z^{-1}}$$

$$W(z) = Y_2(z) + 0.25 z^{-1} W(z) \longrightarrow (8)$$

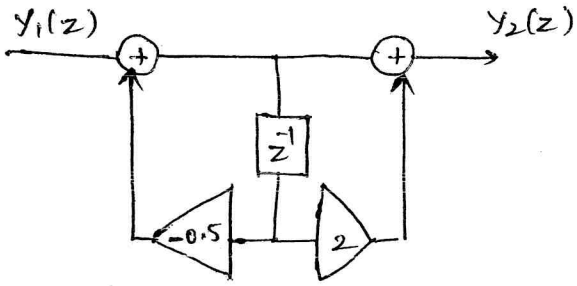
$$\frac{Y(z)}{W(z)} = 1$$

$$Y(z) = W(z) \longrightarrow (9)$$

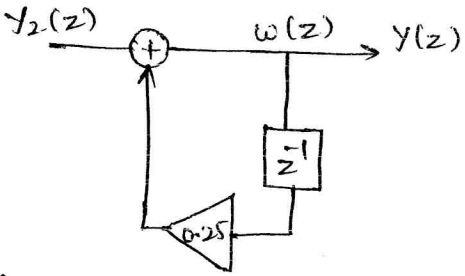
Now realize (4) & (5)



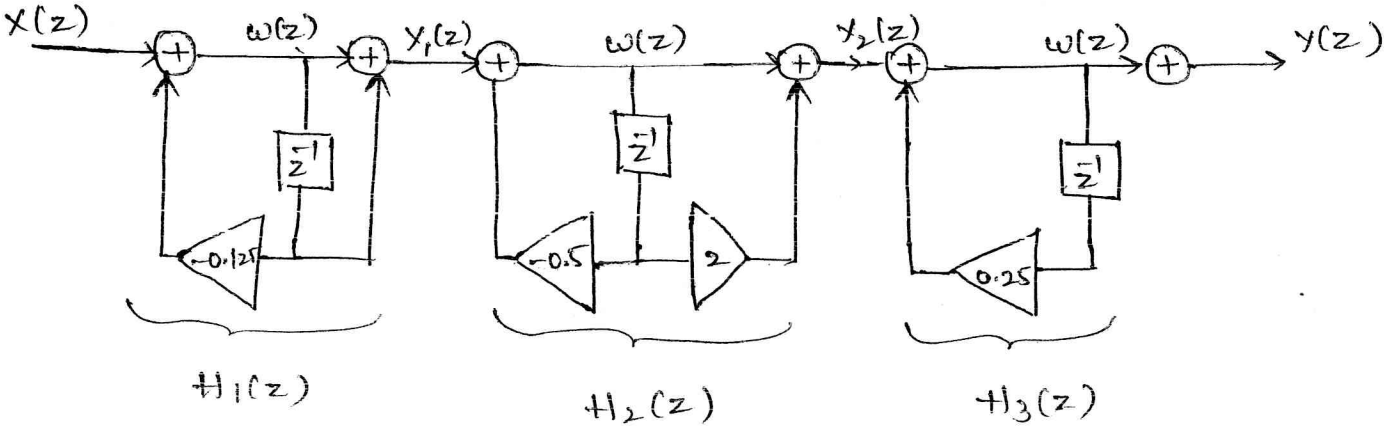
Now realize (6) & (7)



Now realize (8) & (9)



Now cascade $H_1(z)$, $H_2(z)$ and $H_3(z)$



Parallel form realization:

$$H(z) = \frac{z(z+1)(z+2)}{(z+0.125)(z+0.5)(z-0.25)}$$

$$\frac{H(z)}{z} = \frac{(z+1)(z+2)}{(z+0.125)(z+0.5)(z-0.25)}$$

Apply partial fractions

$$\frac{(z+1)(z+2)}{(z+0.125)(z+0.5)(z-0.25)} = \frac{A}{z+0.125} + \frac{B}{z+0.5} + \frac{C}{z-0.25}$$

$$(z+1)(z+2) = A(z+0.5)(z-0.25) + B(z+0.125)(z-0.25) + C(z+0.125)(z+0.5)$$

let $z = 0.25$

$$(1.25)(2.25) = C(0.375)(0.75)$$

$$2.8125 = C(0.28125)$$

$$\boxed{C = 10}$$

let $z = -0.5$

$$(0.5)(1.5) = B(-0.375)(-0.75)$$

$$0.75 = B(0.75)(0.375)$$

$$B = \frac{1}{0.375} = 2.666$$

$$\boxed{B = 2.66}$$

let $z = -0.125$

$$(0.875)(1.875) = A(0.375)(-0.375)$$

$$1.6406 = A(-0.140625)$$

$$\boxed{A = -11.666}$$

$$\frac{H(z)}{z} = \frac{-11.666}{z+0.125} + \frac{2.666}{z+0.5} + \frac{10}{z-0.25}$$

$$H(z) = \frac{(z)(-11.666)}{z+0.125} + \frac{(z)(2.666)}{z+0.5} + \frac{10z}{z-0.25}$$

$$H(z) = \frac{-11.666}{1+0.125z^{-1}} + \frac{2.666}{1+0.5z^{-1}} + \frac{10}{1-0.25z^{-1}}$$

Consider

$$H(z) = H_1(z) + H_2(z) + H_3(z)$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_1(z)}{W(z)}$$

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$$\frac{W(z)}{X(z)} = \frac{1}{1+0.125z^{-1}}$$

$$W(z) = X(z) - 0.125z^{-1}W(z) \longrightarrow (10)$$

$$\frac{Y_1(z)}{W(z)} = -11.666$$

$$Y_1(z) = -11.666\,W(z) \longrightarrow (11)$$

$$H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_2(z)}{W(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+0.5z^{-1}}$$

$$W(z) = X(z) - 0.5z^{-1}W(z) \longrightarrow (12)$$

$$\frac{Y_2(z)}{W(z)} = 2.666$$

$$Y_2(z) = 2.666\,W(z) \longrightarrow (13)$$

$$H_3(z) = \frac{Y_3(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_3(z)}{W(z)}$$

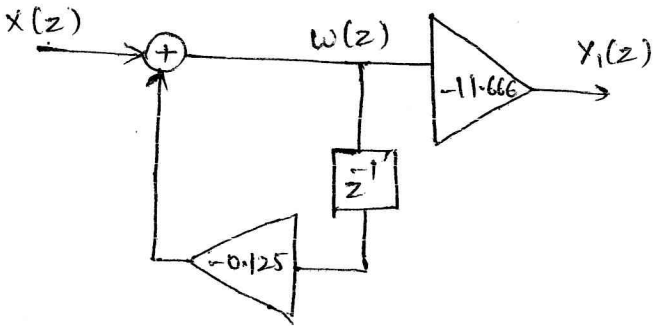
$$\frac{W(z)}{X(z)} = \frac{1}{1-0.25z^{-1}}$$

$$W(z) = X(z) + 0.25z^{-1}W(z) \longrightarrow (14)$$

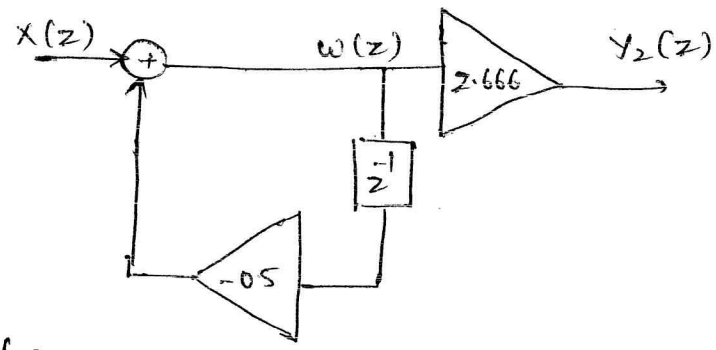
$$\frac{Y_3(z)}{W(z)} = 10$$

$$Y_3(z) = 10\,W(z) \longrightarrow (15)$$

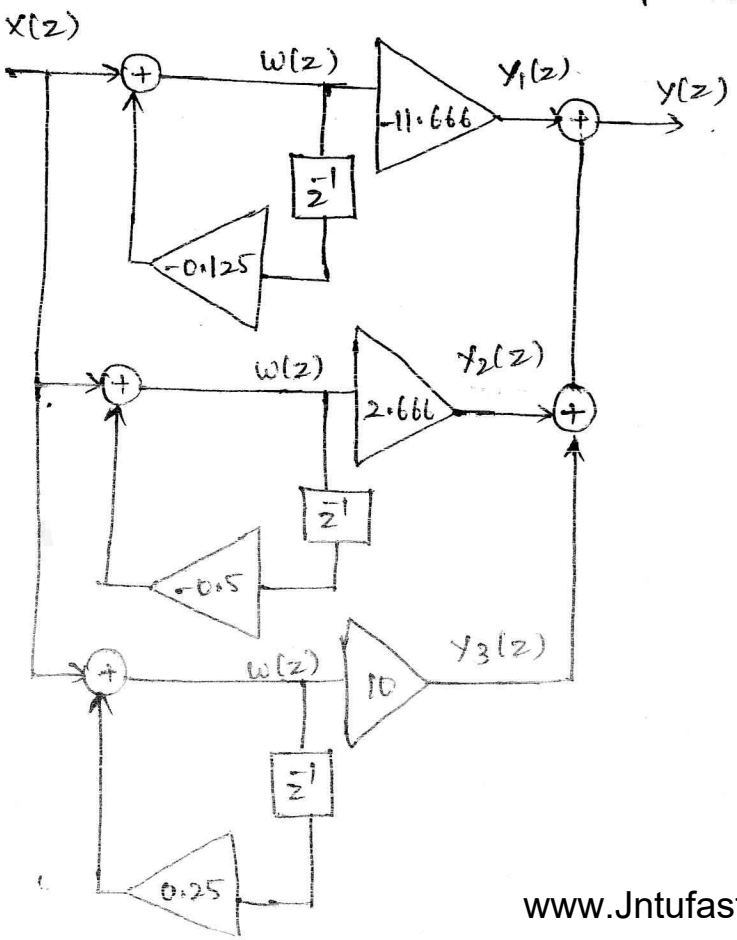
Realize (10) & (11) in DF II



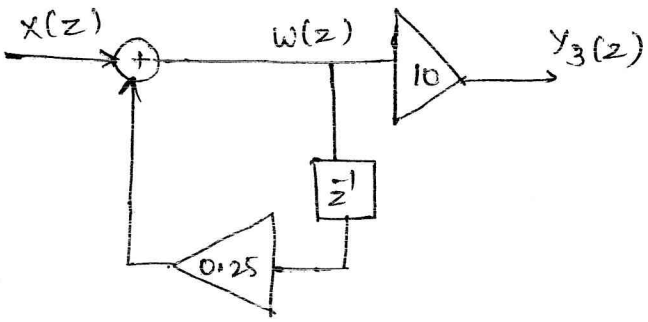
Realize (12) & (13) in DF-II



Realize $H_1(z)$, $H_2(z)$ & $H_3(z)$ in parallel form



Realize (14) & (15) in DF-II



2. Realize the given system in cascade and parallel forms

$$H(z) = \frac{1+1/2\,z^{-1}}{(1-z^{-1}+\frac{1}{4}z^{-2})(1-z^{-1}+\frac{1}{2}z^{-2})}$$

Sol

Given

$$H(z) = \frac{1+\frac{1}{2}z^{-1}}{(1-z^{-1}+\frac{1}{4}z^{-2})(1-z^{-1}+\frac{1}{2}z^{-2})}$$

Cascade Realization:

let $H(z) = H_1(z) \cdot H_2(z)$

consider

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{1+1/2\,z^{-1}}{1-z^{-1}+\frac{1}{2}z^{-2}}$$

$$\frac{Y_1(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_1(z)}{W(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1-z^{-1}+\frac{1}{2}z^{-2}}$$

$$W(z) = X(z) + W(z)z^{-1} - \frac{1}{2}z^{-2}W(z) \longrightarrow (1)$$

$$\frac{Y_1(z)}{W(z)} = 1+1/2\,z^{-1}$$

$$Y_1(z) = W(z) + \frac{1}{2}W(z)z^{-1} \longrightarrow (2)$$

consider

$$H_2(z) = \frac{Y(z)}{Y_1(z)} = \frac{1}{1-z^{-1}+\frac{1}{4}z^{-2}}$$

$$\frac{Y(z)}{Y_1(z)} = \frac{W(z)}{Y_1(z)} \cdot \frac{Y(z)}{W(z)}$$

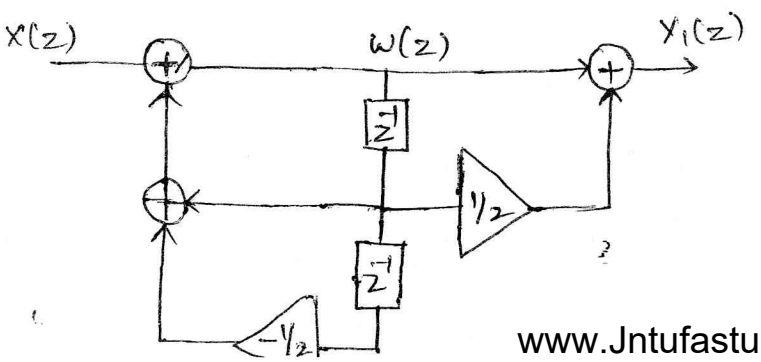
$$\frac{W(z)}{Y_1(z)} = \frac{1}{1-z^{-1}+\frac{1}{4}z^{-2}}$$

$$W(z) = Y_1(z) + z^{-1}W(z) - \frac{1}{4}z^{-2}W(z) \longrightarrow (3)$$

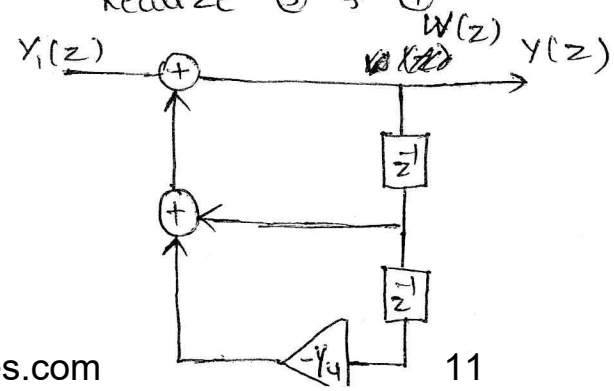
$$\frac{Y(z)}{W(z)} = 1$$

$$Y(z) = W(z) \longrightarrow (4)$$

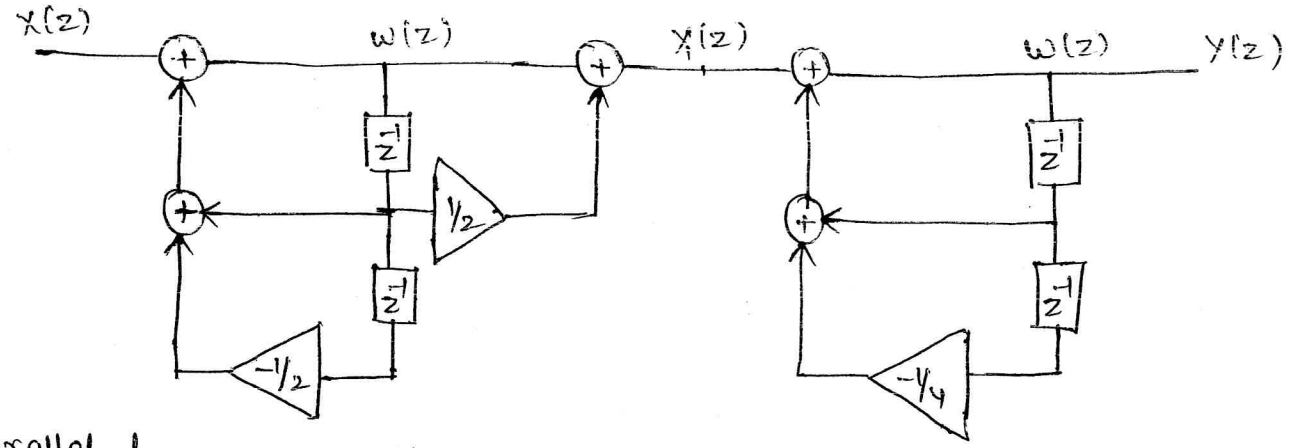
Realize (1) & (2)



Realize (3) & (4)



Now cascade $H_1(z)$, $H_2(z)$



Parallel form realization:

we have $$H(z) = \frac{1+\frac{1}{2}z^{-1}}{(1-z^{-1}+\frac{1}{4}z^{-2})(1-z^{-1}+\frac{1}{2}z^{-2})}$$

$$= \frac{(z+1/2)/z}{(z^2-z+\frac{1}{4})(z^2-z+1/2)/z^4}$$

$$H(z) = \frac{z^3(z+1/2)}{(z^2-z+1/4)(z^2-z+1/2)}$$

$$\frac{H(z)}{z} = \frac{z^2(z+1/2)}{(z^2-z+1/4)(z^2-z+1/2)}$$

Apply partial fractions

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$$\frac{z^2(z+1/2)}{(z^2-z+1/4)(z^2-z+1/2)} = \frac{Az+B}{z^2-z+1/4} + \frac{Cz+D}{z^2-z+1/2}$$

$$z^2(z+1/2) = (Az+B)(z^2-z+1/2)+(Cz+D)(z^2-z+1/4) \longrightarrow (5)$$

compare z^3 terms on both sides

$A+C=1 \longrightarrow (6)$

z^2 terms on both sides

$\frac{1}{2} = -A+B+D-C \longrightarrow (7)$

z terms on both sides

$0 = \frac{1}{2}A - B + \frac{1}{4}C - D \longrightarrow (8)$

constants on both sides

$0 = \frac{1}{2}B + \frac{1}{4}D \longrightarrow (9)$

Substitute (6) in (7)

$B+D-(A+C) = 1/2$

$B+D = \frac{1}{2}+1$

$B+D = 3/2 \longrightarrow (10)$

Solve (9) & (10)

$\frac{1}{2}B + \frac{1}{4}D = 0$

Eq (10) $\times \frac{1}{2}$ $= \frac{1}{2}B + \frac{1}{2}D = 3/4$

$-1/4\,D = -3/4$

Substitute D value in (10)

$B + 3 = 3/2$

$B = \frac{3}{2} - 3$

$\boxed{B = -3/2}$

From eq (6)

~~$A = 1 - C \Rightarrow A + C = 1$~~

substitute ~~A~~, B, D values in (8)

$\frac{1}{2}A + \frac{3}{2} + \frac{1}{4}C - 3 = 0$

$2A + 6 + C - 12 = 0$

$2A + C = 6 \rightarrow (11)$

solve (6) & (11)

$A + C = 1$

$2A + C = 6$

$-A = -5$

$\boxed{A = 5}$

substitute $A = 5$ in (6)

$5 + C = 1$

$\boxed{C = -4}$

$$\frac{H(z)}{z} = \frac{5z - 3/2}{z^2 - z + 1/4} + \frac{-4z + 3}{z^2 - z + 1/2}$$

$$H(z) = \frac{z(5z - 3/2)}{z^2 - z + 1/4} + \frac{z(-4z + 3)}{z^2 - z + 1/2}$$

$$H(z) = \frac{5 - \frac{3}{2}z^{-1}}{1 - z^{-1} + \frac{1}{4}z^{-2}} + \frac{-4 + 3z^{-1}}{1 - z^{-1} + \frac{1}{2}z^{-2}}$$

Consider

$H(z) = H_1(z) + H_2(z)$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_1(z)}{W(z)}$$

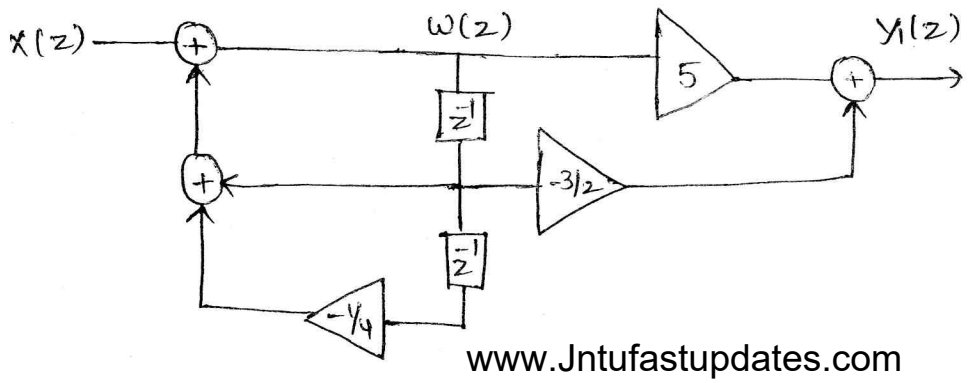
$$\frac{W(z)}{X(z)} = \frac{1}{1 - z^{-1} + \frac{1}{4}z^{-2}}$$

$$W(z) = X(z) + z^{-1}W(z) - \frac{1}{4}W(z)z^{-2} \rightarrow (12)$$

$$\frac{Y_1(z)}{W(z)} = 5 - \frac{3}{2}z^{-1}$$

$$Y_1(z) = 5W(z) - \frac{3}{2}z^{-1}W(z) \rightarrow (13)$$

Realize (12) & (13) in DF-II



Now $H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_2(z)}{W(z)}$

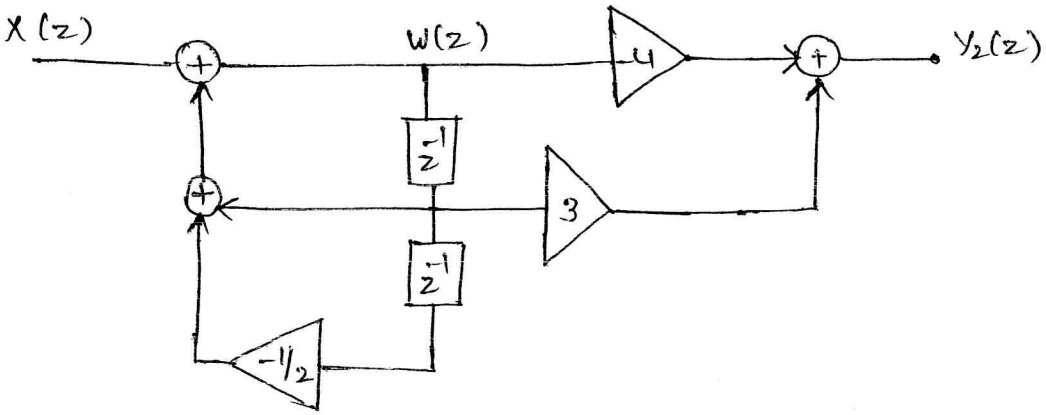
$$\frac{W(z)}{X(z)} = \frac{1}{1 - z^{-1} + \frac{1}{2} z^{-2}}$$

$$W(z) = X(z) + z^{-1} W(z) - \frac{1}{2} z^{-2} W(z) \longrightarrow (14)$$

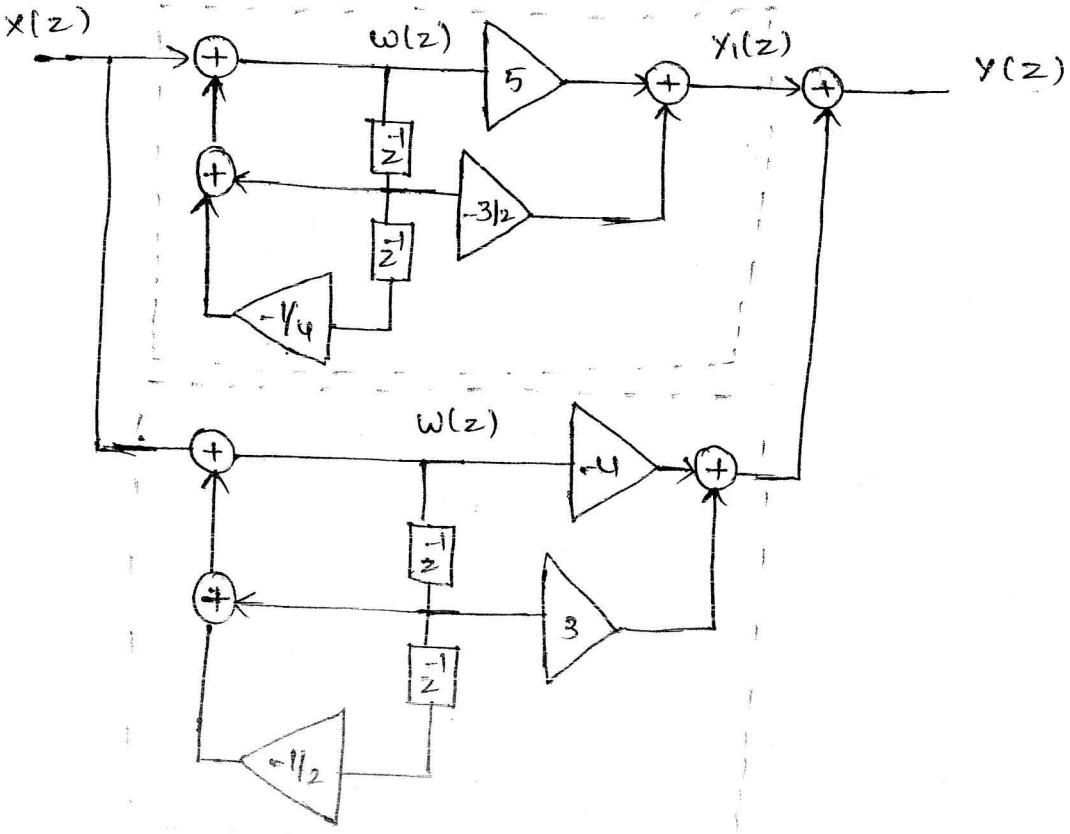
$$\frac{Y_2(z)}{W(z)} = -4 + 3z^{-1}$$

$$Y_2(z) = -4W(z) + 3z^{-1} W(z) \longrightarrow (15)$$

Realize (14) & (15) in DF-II



Realize $H_1(z)$ and $H_2(z)$ in parallel form



3. Obtain the cascade form and parallel form realizations for the following system

$$H(z) = \frac{2 + z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1}\right)}$$

Sol

Given $H(z) = \dfrac{2 + z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1}\right)}$

$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z)$

$H_1(z) = \dfrac{Y_1(z)}{X(z)} = \dfrac{2 + z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)}$

$\dfrac{Y_1(z)}{X(z)} = \dfrac{W(z)}{X(z)} \cdot \dfrac{Y_1(z)}{W(z)}$

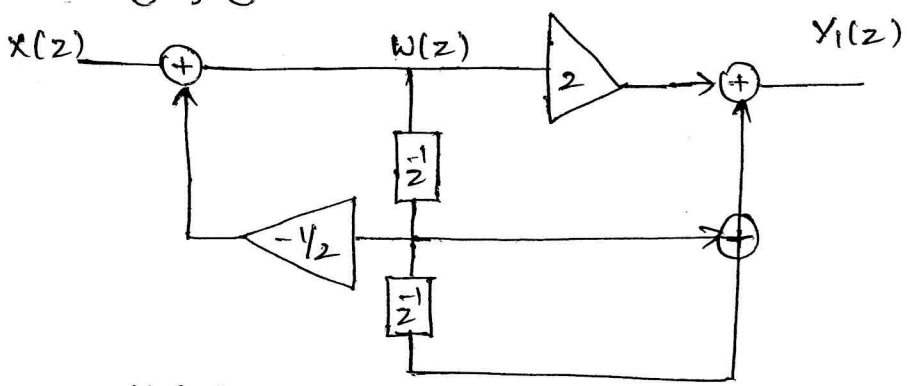
$\dfrac{W(z)}{X(z)} = \dfrac{1}{1 + \frac{1}{2}z^{-1}}$

$W(z) = X(z) - \frac{1}{2}z^{-1}W(z) \longrightarrow (1)$

$\dfrac{Y_1(z)}{W(z)} = 2 + z^{-1} + z^{-2}$

$Y_1(z) = 2W(z) + z^{-1}W(z) + z^{-2}W(z) \longrightarrow (2)$

Realize (1) & (2) in DF II



$H_2(z) = \dfrac{Y_2(z)}{Y_1(z)} = \dfrac{1}{1 - 1/2\, z^{-1}}$

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$\dfrac{Y_2(z)}{Y_1(z)} = \dfrac{W(z)}{Y_1(z)} \cdot \dfrac{Y_2(z)}{W(z)}$

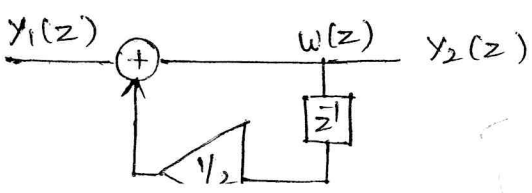
$\dfrac{W(z)}{Y_1(z)} = \dfrac{1}{1 - 1/2\, z^{-1}}$

$W(z) = Y_1(z) + \frac{1}{2}z^{-1}W(z) \longrightarrow (3)$

$\dfrac{Y_2(z)}{W(z)} = 1$

$Y_2(z) = W(z) \longrightarrow (4)$

Realize (3) & (4)



$$H_3(z) = \frac{1}{1+\frac{1}{2}z^{-1}} = \frac{Y(z)}{Y_2(z)}$$

$$\frac{Y(z)}{Y_1(z)} = \frac{W(z)}{Y_2(z)} \cdot \frac{Y(z)}{W(z)}$$

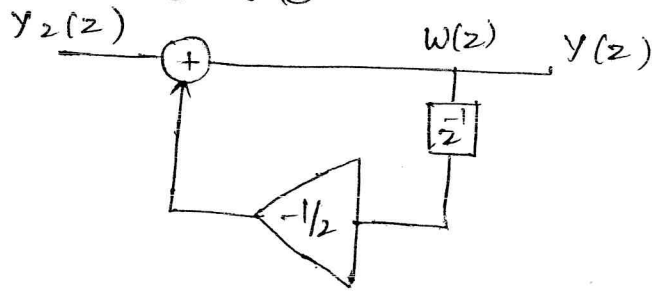
$$\frac{W(z)}{Y_2(z)} = \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$W(z) = Y_2(z) - \frac{1}{2}z^{-1}W(z) \longrightarrow (5)$$

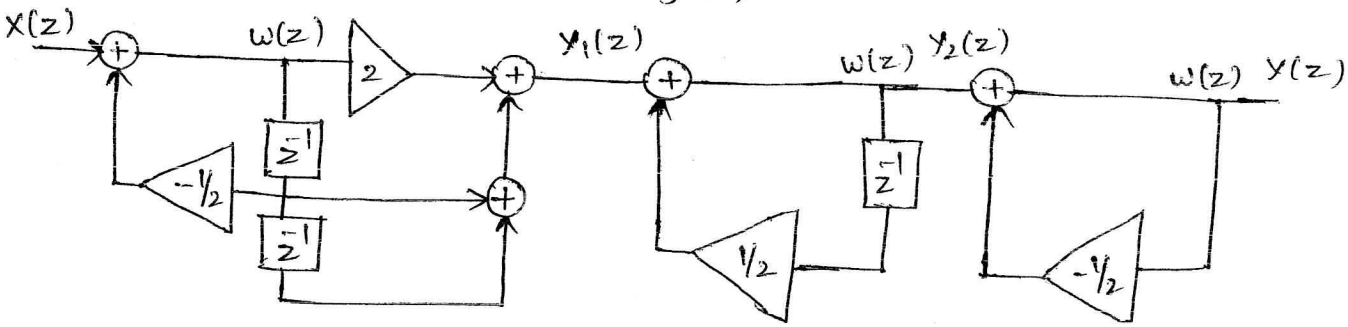
$$\frac{Y(z)}{W(z)} = 1$$

$$Y(z) = W(z) \longrightarrow (6)$$

Realize (5) & (6)



Cascade $H_1(z)$, $H_2(z)$ and $H_3(z)$



Parallel form Realization

$$H(z) = \frac{2+z^{-1}+z^{-2}}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})}$$

$$H(z) = \frac{(2z^2+z+1)z}{(z+\frac{1}{2})(z-\frac{1}{2})(z+1/2)}$$

$$\frac{H(z)}{z} = \frac{2z^2+z+1}{(z+\frac{1}{2})^2(z-1/2)}$$

Apply partial fractions

$$\frac{2z^2+z+1}{(z+1/2)^2(z-1/2)} = \frac{A}{z+1/2} + \frac{B}{(z+1/2)^2} + \frac{C}{z-1/2}$$

$$2z^2+z+1 = A(z+1/2)(z-1/2) + B(z-1/2) + C(z+1/2)^2$$

let $z = 1/2$

$$2\left(\frac{1}{4}\right)+\frac{1}{2}+1 = C\left(\frac{1}{2}+\frac{1}{2}\right)^2$$

$$\boxed{2 = C}$$

let $z = -1/2$

$$2\left(\frac{-1}{4}\right)-\frac{1}{2}+1 = B\left(-\frac{1}{2}-\frac{1}{2}\right)$$

$$1 = -B$$

$$\boxed{B = -1}$$

compare z^2 coef on both sides

$$2 = A + C$$

$$\boxed{A = 0}$$

$$\frac{H(z)}{z} = \frac{-1}{(z+1/2)^2} + \frac{2}{z-1/2}$$

$$H(z) = \frac{-z}{(z+\frac{1}{2})^2} + \frac{2z}{z-1/2}$$

$$H(z) = \frac{z^{-1}}{(1+\frac{1}{2}z^{-1})^2} + \frac{2}{1-\frac{1}{2}z^{-1}} = \frac{z^{-1}}{1+z^{-1}+\frac{1}{4}z^{-2}} + \frac{2}{1-\frac{1}{2}z^{-1}}$$

consider

$$H(z) = H_1(z) + H_2(z)$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_1(z)}{W(z)}$$

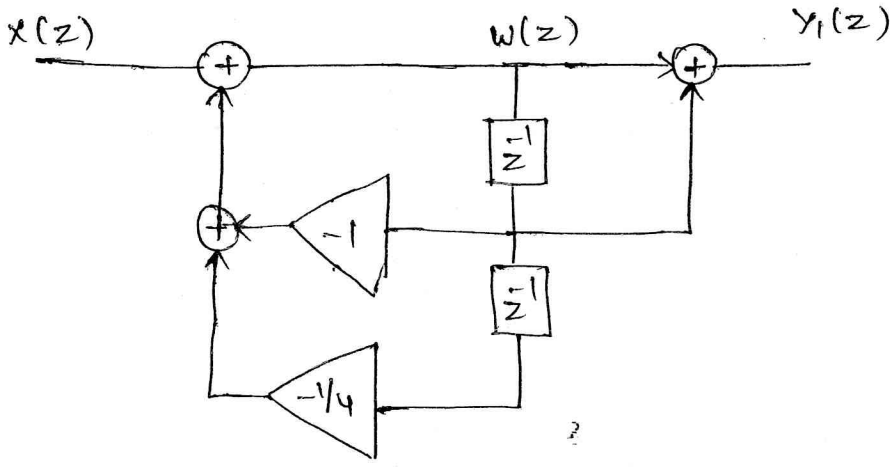
$$\frac{W(z)}{X(z)} = \frac{1}{1+z^{-1}+\frac{1}{4}z^{-2}}$$

$$W(z) = X(z) - z^{-1}W(z) - \frac{1}{4}z^{-2}W(z) \rightarrow (7)$$

$$\frac{Y_1(z)}{W(z)} = z^{-1}$$

$$Y_1(z) = z^{-1}W(z) \rightarrow (8)$$

Realize eq (7) & (8)



$$H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_2(z)}{W(z)}$$

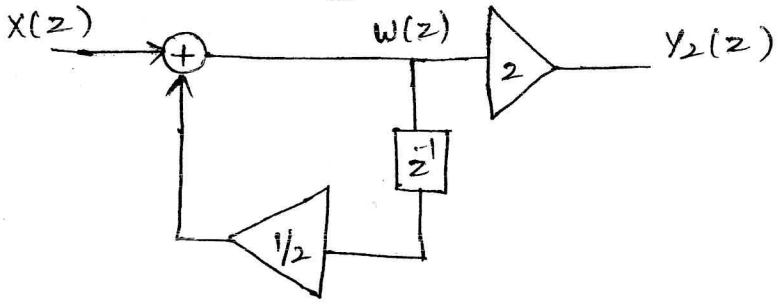
$$\frac{W(z)}{X(z)} = \frac{1}{1 - 1/2 z^{-1}}$$

$$W(z) = X(z) + \frac{1}{2} z^{-1} W(z) \longrightarrow (9)$$

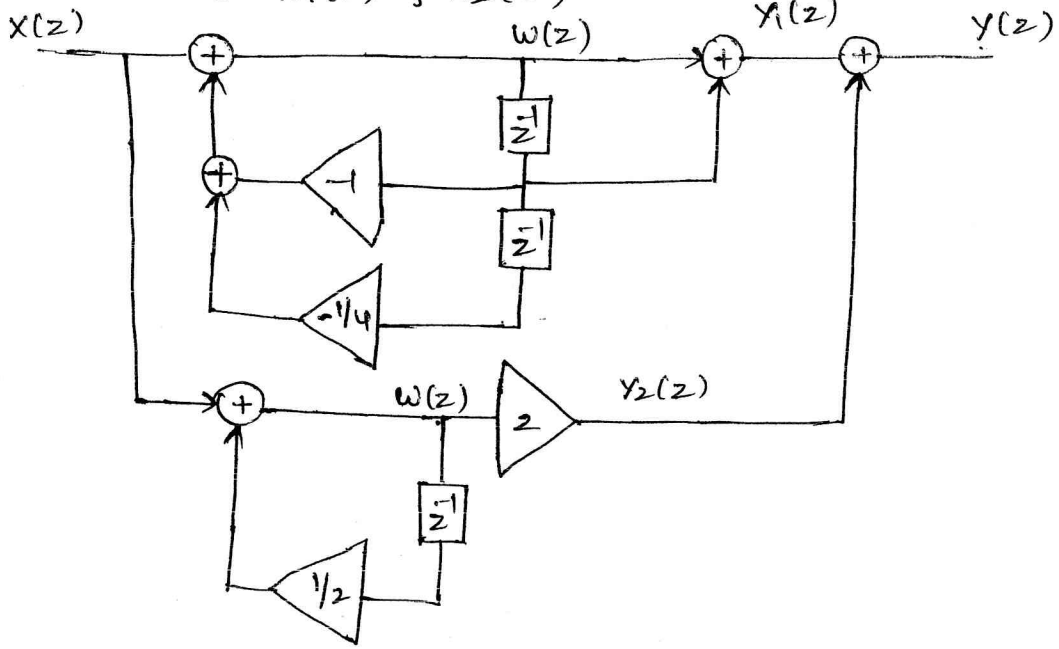
$$\frac{Y_2(z)}{W(z)} = 2$$

$$Y_2(z) = 2 W(z) \longrightarrow (10)$$

Realise (9) & (10)



Now realize $H_1(z)$ & $H_2(z)$



4. Obtain parallel form realization of the following system

$$H(z) = \frac{1 + 1/4 z^{-1}}{(1 + \frac{1}{2} z^{-1})(1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2})}$$

Sol

Given $$H(z) = \frac{1 + 1/4 z^{-1}}{(1 + \frac{1}{2} z^{-1})(1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2})}$$

$$H(z) = \frac{z^2 (z + 1/4)}{(z + 1/2)(z^2 + \frac{1}{2} z + 1/4)}$$

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$$\frac{H(z)}{z} = \frac{z (z + 1/4)}{(z + 1/2)(z^2 + \frac{1}{2} z + 1/4)}$$

Apply partial fractions

$$\frac{z(z+1/4)}{(z+1/2)(z^2+1/2z+1/4)} = \frac{A}{z+1/2} + \frac{Bz+C}{z^2+1/2z+1/4}$$

$$z(z+1/4) = A(z^2+\tfrac{1}{2}z+\tfrac{1}{4}) + (Bz+C)(z+1/2)$$

Let $z=-1/2$

$$-\frac{1}{2}(-\frac{1}{2}+\frac{1}{4}) = A(\frac{1}{4}-\frac{1}{4}+\frac{1}{4})$$

$$-\frac{1}{2}(\frac{-1}{4}) = A(\frac{1}{4})$$

$$\boxed{A=+1/2}$$

Compare z^2 terms

$$1 = A + B$$

$$B = 1 - 1/2$$

$$\boxed{B = 1/2}$$

Compare constan

$$0 = \frac{1}{4}A + \frac{1}{2}C$$

$$\frac{1}{2}C = -1/8$$

$$\boxed{C = -1/4}$$

$$\frac{H(z)}{z} = \frac{1/2}{z+1/2} + \frac{1/2\,z - 1/4}{z^2+\frac{1}{2}z+1/4}$$

$$H(z) = \frac{1/2\,z}{z+1/2} + \frac{z(\frac{1}{2}z-1/4)}{z^2+\frac{1}{2}z+1/4} = \frac{1/2}{1+1/2\,z^{-1}} + \frac{1/2 - 1/4\,z^{-1}}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

Now $H(z) = H_1(z) + H_2(z)$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_1(z)}{W(z)} = \frac{1/2}{1+1/2\,z^{-1}}$$

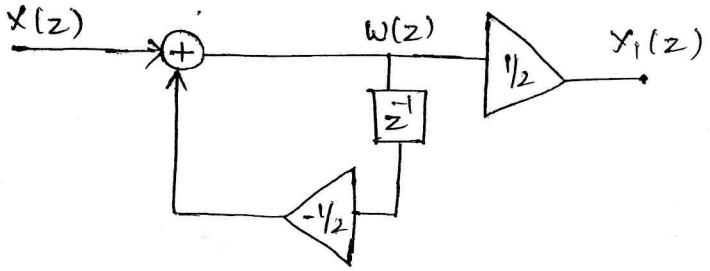
$$\frac{W(z)}{X(z)} = \frac{1}{1+1/2\,z^{-1}}$$

$$W(z) = X(z) - \frac{1}{2}z^{-1}W(z) \rightarrow (1)$$

$$\frac{Y_1(z)}{W(z)} = 1/2$$

$$Y_1(z) = 1/2\,W(z) \rightarrow (2)$$

Now realise (1) & (2)



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$$H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{1/2 - 1/4\,z^{-1}}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}} = \frac{W(z)}{X(z)} \cdot \frac{Y_2(z)}{W(z)}$$

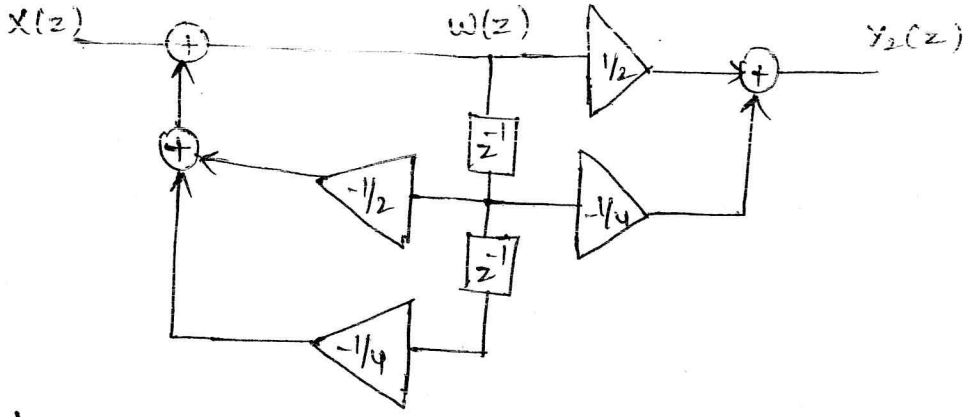
$$\frac{W(z)}{X(z)} = \frac{1}{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$$W(z) = X(z) - 1/2\,z^{-1}W(z) - \frac{1}{4}z^{-2}W(z) \rightarrow (3)$$

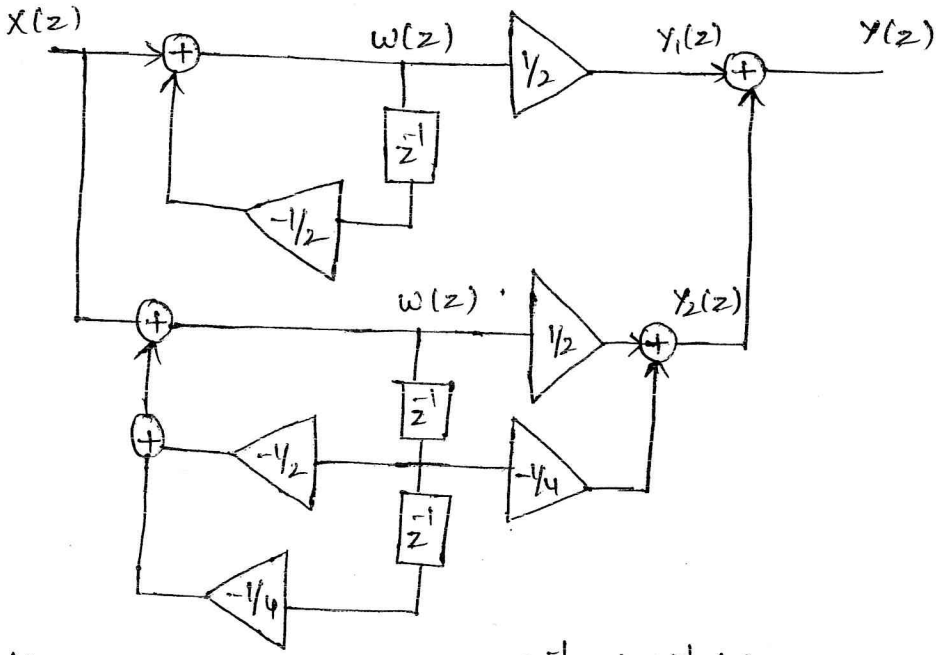
$$\frac{Y_2(z)}{W(z)} = \frac{1}{2} - \frac{1}{4}z^{-1}$$

$$Y_2(z) = \frac{1}{2}W(z) - \frac{1}{4}z^{-1}W(z) \rightarrow (4)$$

Now realise ③ & ④



Now realize $H_1(z)$ and $H_2(z)$



⑤ Realize the filter $H(z) = \dfrac{(z^{-1}-a)(z^{-1}-b)}{(1-az^{-1})(1-bz^{-1})}$ in cascade and parallel form

Sol Given $H(z) = \dfrac{(z^{-1}-a)(z^{-1}-b)}{(1-az^{-1})(1-bz^{-1})}$

Cascade Realization:-

Assume $H(z) = H_1(z) \cdot H_2(z)$

Now realize $H_1(z)$, $H_2(z)$ in DF-II

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_1(z)}{W(z)} = \frac{z^{-1}-a}{1-az^{-1}}$$

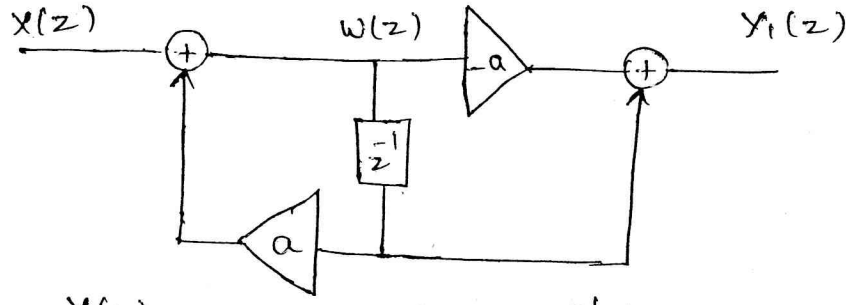
$$\frac{W(z)}{X(z)} = \frac{1}{1-az^{-1}}$$

$$W(z) = X(z) + az^{-1}W(z) \longrightarrow (1)$$

$$\frac{Y_1(z)}{W(z)} = z^{-1} - a$$

$$Y_1(z) = -aW(z) + z^{-1}W(z) \longrightarrow (2)$$

Realise eq ① & ②



$$H_2(z) = \frac{Y(z)}{Y_1(z)} = \frac{W(z)}{Y_1(z)} \cdot \frac{Y(z)}{W(z)} = \frac{z^{-1}-b}{1-bz^{-1}}$$

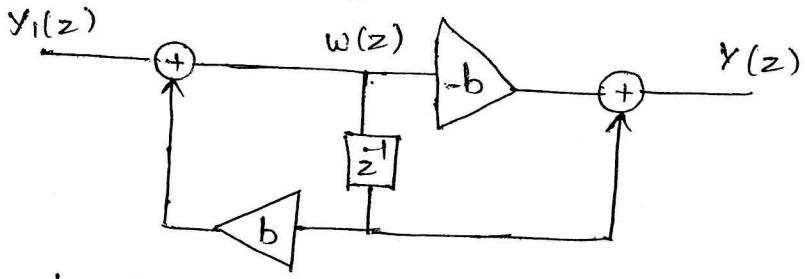
$$\frac{W(z)}{Y_1(z)} = \frac{1}{1-bz^{-1}}$$

$$W(z) = Y_1(z) + bz^{-1}W(z) \longrightarrow ③$$

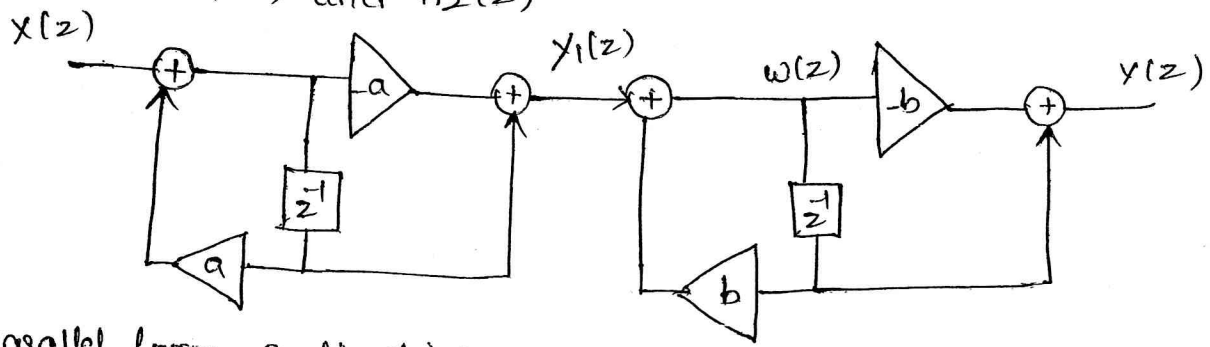
$$\frac{Y(z)}{W(z)} = z^{-1} - b$$

$$W(z)z^{-1} - bW(z) = Y(z) \longrightarrow ④$$

Realise eq ③ & ④



cascade $H_1(z)$ and $H_2(z)$



Parallel form Realization

$$H(z) = \frac{(z^{-1}-a)(z^{-1}-b)}{(1-az^{-1})(1-bz^{-1})}$$

$$H(z) = \frac{(1-az)(1-bz)}{(z-a)(z-b)}$$

Divide with z on both sides.

$$\frac{H(z)}{z} = \frac{(1-az)(1-bz)}{z(z-a)(z-a)}$$

Apply partial fractions

$$\frac{(1-az)(1-bz)}{z(z-a)(z-b)} = \frac{A}{z} + \frac{B}{z-a} + \frac{C}{z-b}$$

$$(1-az)(1-bz) = A(z-a)(z-b) + B(z-b)z + C(z-a)z$$

let $z = 0$

$1 = A(-a)(-b)$

$\boxed{A = 1/ab}$

let $z = a$

$(1-a^2)(1-ba) = B(a-b)a$

$B = \frac{(1-a^2)(1-ba)}{(a-b)a}$

let $z = b$

$(1-ba)(1-b^2) = C(b-a)b$

$C = \frac{(1-ba)(1-b^2)}{b(b-a)}$

$$H(z) = \frac{Az}{z} + \frac{Bz}{z-a} + \frac{Cz}{z-b}$$

$$H(z) = A + \frac{B}{1-az^{-1}} + \frac{C}{1-bz^{-1}} = H_1(z) + H_2(z) + H_3(z)$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = A$$

$$Y_1(z) = A\,X(z) \longrightarrow (5)$$

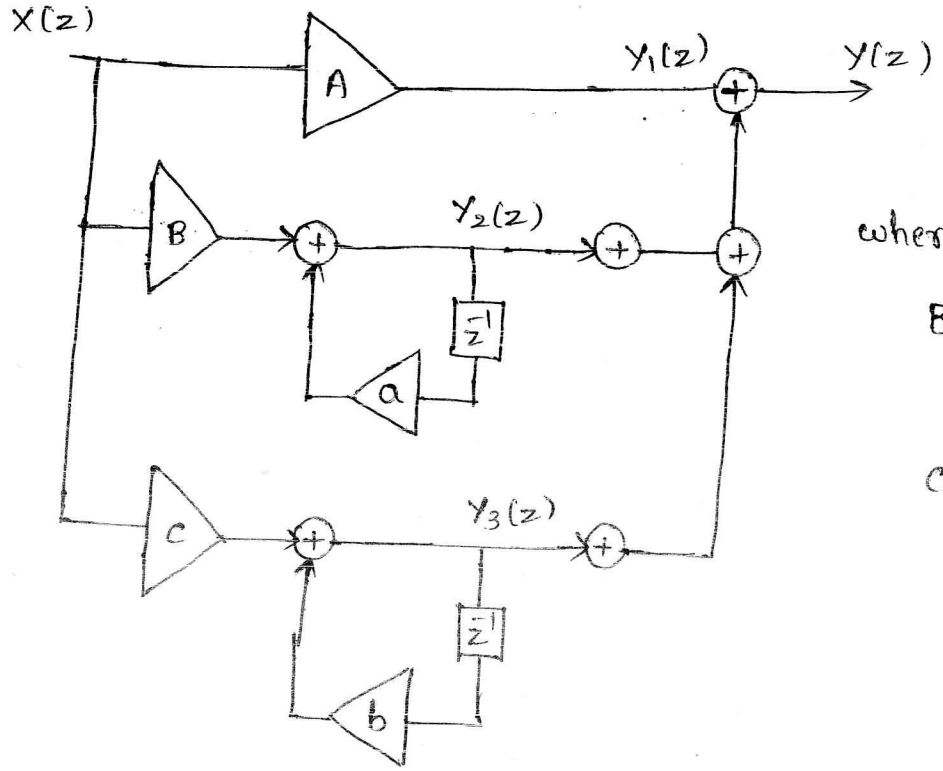
$$H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{B}{1-az^{-1}}$$

$$Y_2(z) = B\,X(z) + az^{-1}Y_2(z) \longrightarrow (6)$$

$$H_3(z) = \frac{Y_3(z)}{X(z)} = \frac{C}{1-bz^{-1}}$$

$$Y_3(z) = C\,X(z) + bz^{-1}Y_3(z)$$

Now realize $H_1(z)$, $H_2(z)$ & $H_3(z)$



where $A = 1/ab$

$B = \frac{(1-a^2)(1-ba)}{(a-b)a}$

$C = \frac{(1-b^2)(1-ba)}{(b-a)b}$

6. Obtain the direct form I & II & cascade form realization of the following system

$y(n) = 0.1\,y(n-1) + 0.2\,y(n-2) + 3\,x(n) + 3.6\,x(n-1) + 0.6\,x(n-2)$

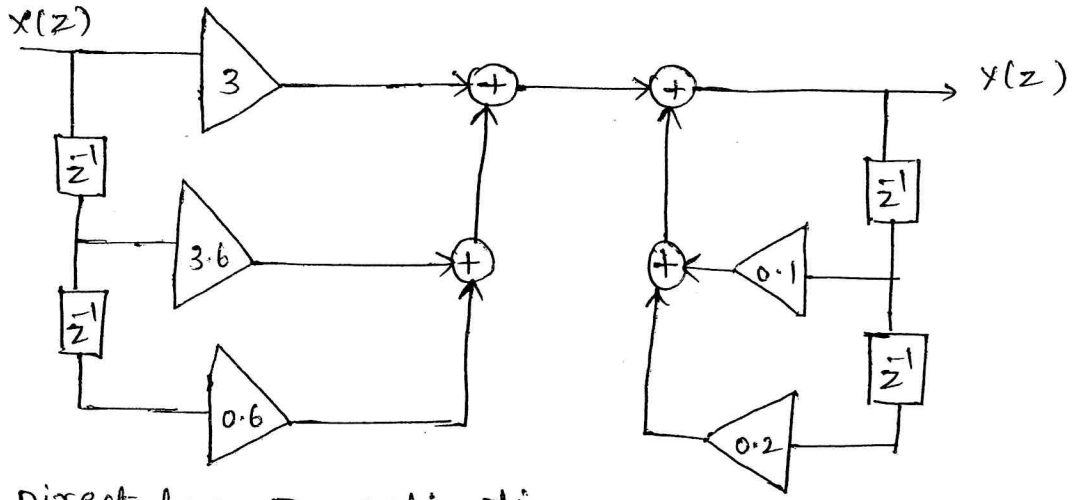
Sol Given

$y(n) = 0.1\,y(n-1) + 0.2\,y(n-2) + 3\,x(n) + 3.6\,x(n-1) + 0.6\,x(n-2)$

Apply z-transform on both sides

$$Y(z) = 0.1\,z^{-1}Y(z) + 0.2\,Y(z)\,z^{-2} + 3X(z) + 3.6X(z)\,z^{-1} + 0.6\,X(z)\,z^{-2} \longrightarrow (1)$$

Direct form I realization



Direct form II realization

From eq (1)

$$Y(z)\left[1 - 0.1z^{-1} - 0.2z^{-2}\right] = X(z)\left[3 + 3.6z^{-1} + 0.6z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 - 0.1z^{-1} - 0.2z^{-2}}$$

Assume $\dfrac{Y(z)}{X(z)} = \dfrac{W(z)}{X(z)} \cdot \dfrac{Y(z)}{W(z)}$

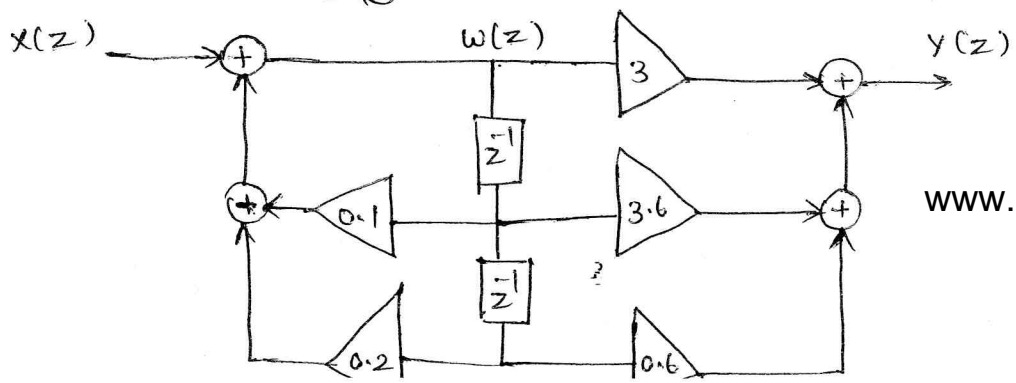
$$\frac{W(z)}{X(z)} = \frac{1}{1 - 0.1z^{-1} - 0.2z^{-2}}$$

$$W(z) = X(z) + 0.1z^{-1}W(z) + 0.2z^{-2}W(z) \longrightarrow (2)$$

$$\frac{Y(z)}{W(z)} = 3 + 3.6z^{-1} + 0.6z^{-2}$$

$$Y(z) = 3W(z) + 3.6z^{-1}W(z) + 0.6z^{-2}W(z) \longrightarrow (3)$$

Now realize (2) and (3)



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Cascade form :-

$$H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 - 0.1z^{-1} - 0.2z^{-2}}$$

$$H(z) = \frac{3z^2 + 3.6z + 0.6}{z^2 - 0.1z - 0.2} = \frac{3(z+0.2)(z+1)}{(z-0.5)(z+0.4)}$$

$$H(z) = \frac{(3z+0.6)(z+1)}{(z-0.5)(z+0.4)} = \frac{(3+0.6z^{-1})(1+z^{-1})}{(1-0.5z^{-1})(1+0.4z^{-1})}$$

Here $H(z) = H_1(z) \cdot H_2(z)$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_1(z)}{W(z)} = \frac{3z+0.6}{z-0.5}$$

$$H_1(z) = \frac{3 + 0.6z^{-1}}{1 - 0.5z^{-1}}$$

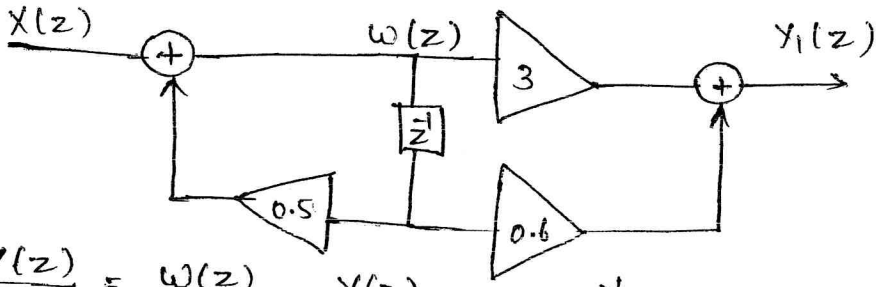
$$\frac{W(z)}{X(z)} = \frac{1}{1 - 0.5z^{-1}}$$

$$W(z) = X(z) + 0.5z^{-1}W(z) \rightarrow (4)$$

$$\frac{Y_1(z)}{W(z)} = 3 + 0.6z^{-1}$$

$$Y_1(z) = 3W(z) + 0.6z^{-1}W(z) \rightarrow (5)$$

Realise (4) & (5)



$$H_2(z) = \frac{Y(z)}{Y_1(z)} = \frac{W(z)}{Y_1(z)} \cdot \frac{Y(z)}{W(z)} = \frac{1 + z^{-1}}{1 + 0.4z^{-1}}$$

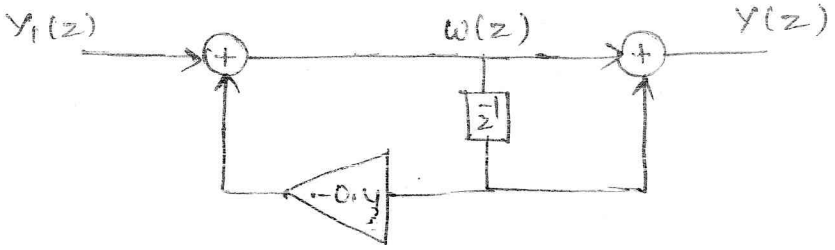
$$\frac{W(z)}{Y_1(z)} = \frac{1}{1 + 0.4z^{-1}}$$

$$W(z) = Y_1(z) - 0.4z^{-1}W(z) \rightarrow (6)$$

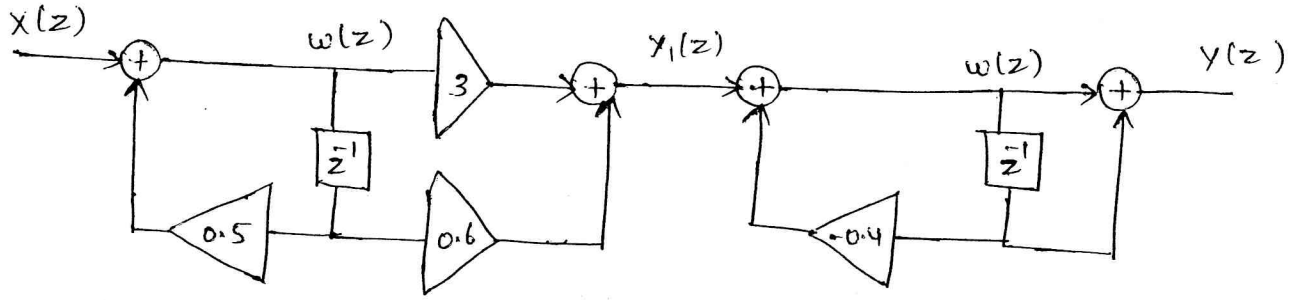
$$\frac{Y(z)}{W(z)} = 1 + z^{-1}$$

$$Y(z) = W(z) + W(z)z^{-1} \rightarrow (7)$$

Realise (6) & (7)



Now cascade $H_1(z)$ and $H_2(z)$



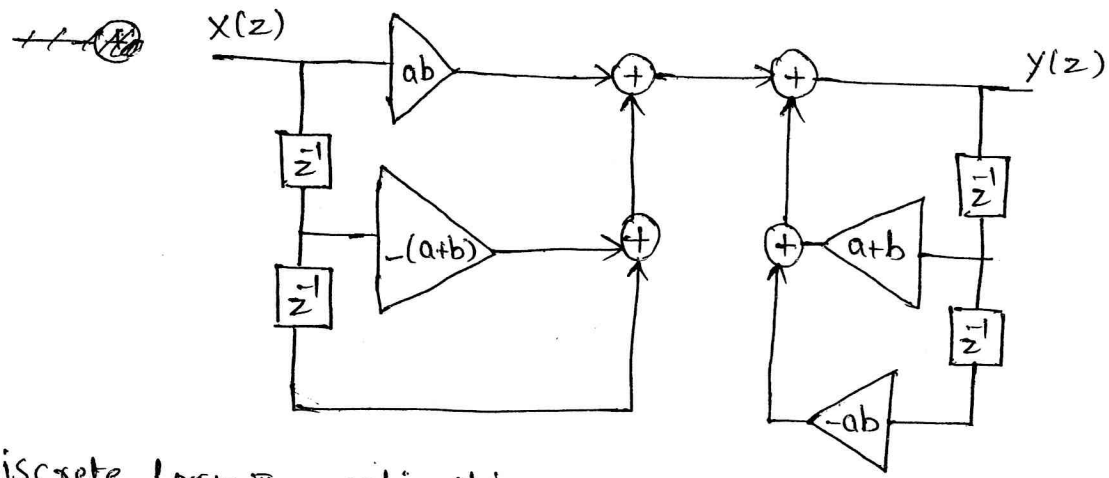
7. $H(z) = \dfrac{(z^{-1}-a)(z^{-1}-b)}{(1-az^{-1})(1-bz^{-1})}$ in DF-I & DF-II

Discrete form I realization

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(z^{-1}-a)(z^{-1}-b)}{(1-az^{-1})(1-bz^{-1})}$$

$$\frac{Y(z)}{X(z)} = \frac{z^{-2}-z^{-1}b-z^{-1}a+ab}{1-bz^{-1}-az^{-1}+abz^{-2}}$$

$$Y(z) = ab\,X(z) + z^{-2}X(z) - (a+b)z^{-1}X(z) + (a+b)z^{-1}Y(z) - abz^{-2}Y(z) \longrightarrow (1)$$



Discrete form II realization

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-2}-(a+b)z^{-1}+ab}{abz^{-2}-(a+b)z^{-1}+1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{w(z)}{X(z)} \cdot \frac{Y(z)}{w(z)}$$

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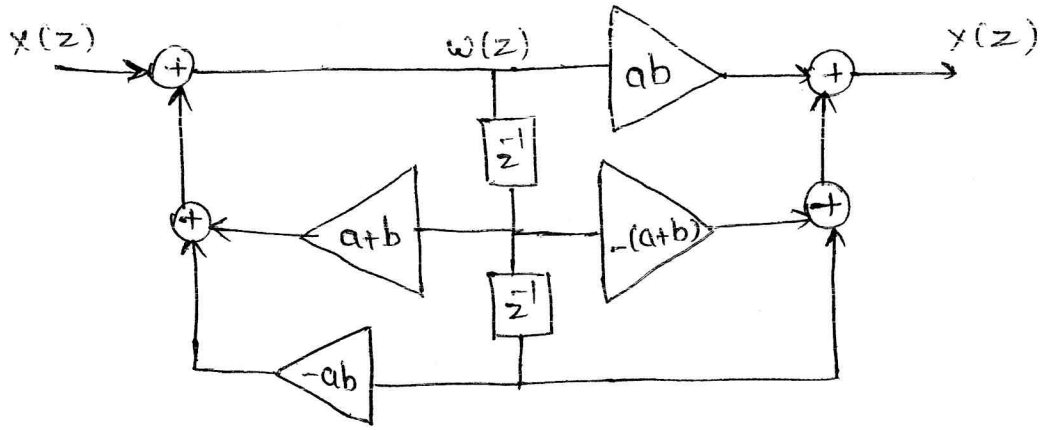
$$\frac{w(z)}{X(z)} = \frac{1}{abz^{-2}-(a+b)z^{-1}+1}$$

$$w(z) = X(z) + (a+b)z^{-1}w(z) - abz^{-2}w(z) \longrightarrow (2)$$

$$\frac{Y(z)}{w(z)} = z^{-2}-(a+b)z^{-1}+ab$$

$$Y(z) = ab\,w(z) - (a+b)z^{-1}w(z) + z^{-2}w(z) \longrightarrow (3)$$

Realize eq ② & ③



8. Realize the following system $y(n) = -0.1\,y(n-1) + 0.72\,y(n-2) + 0.7x(n) - 0.252\,x(n-2)$ in parallel form

Sol Given

$$y(n) = -0.1\,y(n-1) + 0.72\,y(n-2) + 0.7x(n) - 0.252x(n-2)$$

Apply z transform on both sides

$$Y(z) = -0.1\,Y(z)z^{-1} + 0.72\,Y(z)z^{-2} + 0.7X(z) - 0.252X(z)z^{-2}$$

$$Y(z)\left[1 + 0.1z^{-1} - 0.72z^{-2}\right] = X(z)\left[0.7 - 0.252z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

$$\frac{Y(z)}{X(z)} = \frac{0.7z^2 - 0.252}{z^2 + 0.1z - 0.72}$$

$$H(z) = \frac{0.7z^2 - 0.252}{(z+0.9)(z-0.8)}$$

Divide with z on both sides

$$\frac{H(z)}{z} = \frac{0.7z^2 - 0.252}{z(z+0.9)(z-0.8)}$$

Apply partial fractions on both sides

$$\frac{0.7z^2 - 0.252}{z(z+0.9)(z-0.8)} = \frac{A}{z} + \frac{B}{z+0.9} + \frac{C}{z-0.8}$$

$$0.7z^2 - 0.252 = A(z-0.8)(z+0.9) + B(z-0.8)(z) + C(z+0.9)(z)$$

Let $z=0$ | let $z=0.8$ | let $z=-0.9$

$-0.252 = A(-0.72)$ | $0.196 = C(1.36)$ | $0.315 = B(1.53)$

$\boxed{A = 0.35}$ | $\boxed{C = 0.1441}$ | $\boxed{B = 0.2058}$

$$\frac{H(z)}{z} = \frac{0.35}{z} + \frac{0.2058}{z+0.9} + \frac{0.1441}{z-0.8}$$

$$H(z) = 0.35 + \frac{0.2058z}{z+0.9} + \frac{0.1441z}{z-0.8}$$

$$H(z) = 0.35 + \frac{0.2058}{1+0.9z^{-1}} + \frac{0.1441}{1-0.8z^{-1}}$$

$$H(z) = H_1(z) + H_2(z) + H_3(z) = \frac{Y_1(z)}{X(z)} + \frac{Y_2(z)}{X(z)} + \frac{Y_3(z)}{X(z)}$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = 0.35 \Rightarrow Y_1(z) = 0.35\,X(z)$$

$$H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{0.2058}{1+0.9z^{-1}}$$

$$\frac{Y_2(z)}{X(z)} = \frac{W(z)}{X(z)} \cdot \frac{Y_2(z)}{W(z)}$$

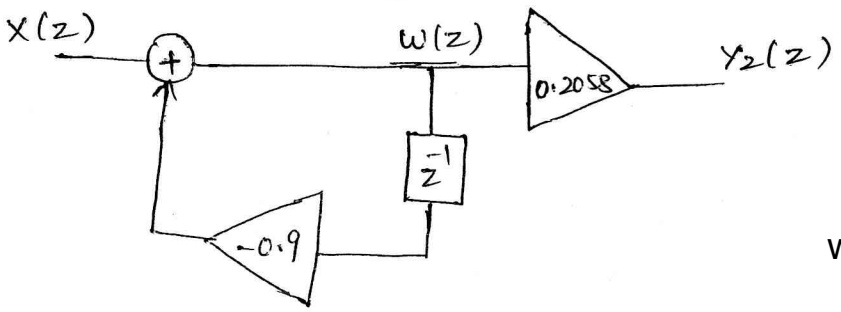
$$\frac{W(z)}{X(z)} = \frac{1}{1+0.9z^{-1}}$$

$$W(z) = X(z) - 0.9z^{-1}W(z) \longrightarrow (1)$$

$$\frac{Y_2(z)}{W(z)} = 0.2058$$

$$Y_2(z) = 0.2058\,W(z) \longrightarrow (2)$$

Now realize (1) & (2)



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$$H_3(z) = \frac{W(z)}{X(z)} \cdot \frac{Y_3(z)}{W(z)} = \frac{Y_3(z)}{X(z)} = \frac{0.1441}{1-0.8z^{-1}}$$

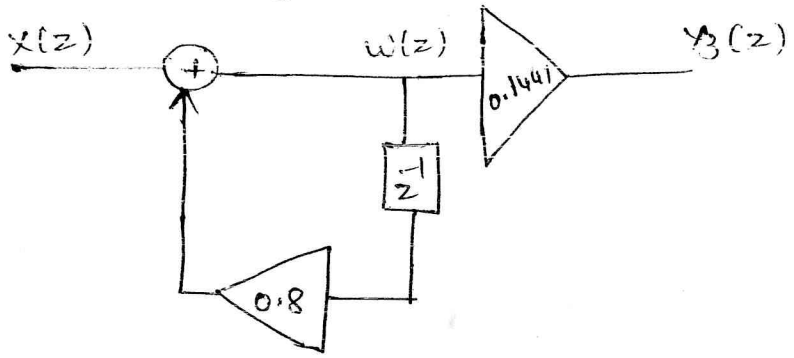
$$\frac{W(z)}{X(z)} = \frac{1}{1-0.8z^{-1}}$$

$$W(z) = X(z) + 0.8z^{-1}W(z) \longrightarrow (3)$$

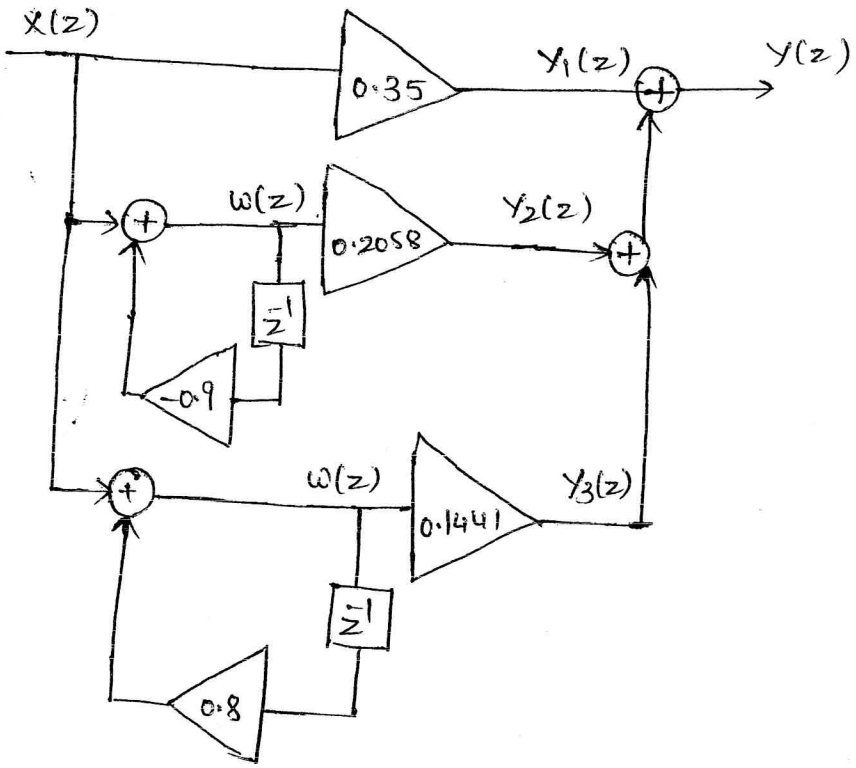
$$\frac{Y_3(z)}{W(z)} = 0.1441$$

$$Y_3(z) = 0.1441\,W(z) \longrightarrow (4)$$

Realize ③ & ④



Now $H_1(z)$, $H_2(z)$ and $H_3(z)$



1. Obtain parallel form realization for the IIR system described by the transfer function

$$H(z) = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.7z^{-2}}$$

2. Realize $H(z) = \frac{1 + 0.6z^{-2} + 0.2z^{-1}}{3 + 5z^{-1} + 4z^{-2}}$ using D.F I & DF-II structure

3. Obtain the DF-I, DF-II, cascade and parallel form realization of the system described by the equation

$$y(n) = y(n-1) - \frac{1}{2}y(n-2) + x(n) - x(n-1) + x(n-2)$$

4. Obtain the cascade form realization for the following system equation

$$H(z) = \frac{1 + 1/4z^{-1}}{(1 + 1/2z^{-1})(1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2})}$$

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Transposed forms (or) Transposed structure:-

The transposed of a structure is defined by the following operations

1. changing the directions of all branches
2. Interchange the input and output
3. Changing the summing point to branching point and viceversa

1. Determine the DF-II and transposed form for the following system $y(n)=\frac{1}{2}y(n-1)-\frac{1}{4}y(n-2)+x(n)+x(n-1)$

Sol Given $y(n)=\frac{1}{2}y(n-1)-\frac{1}{4}y(n-2)+x(n)+x(n-1)$

Apply z-transform on both sides

$$Y(z)=\frac{1}{2}Y(z)z^{-1}-\frac{1}{4}Y(z)z^{-2}+X(z)+X(z)z^{-1}$$

$$Y(z)\left[1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right]=X(z)\left[1+z^{-1}\right]$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{W(z)}{X(z)}\cdot\frac{Y(z)}{W(z)}$$

consider

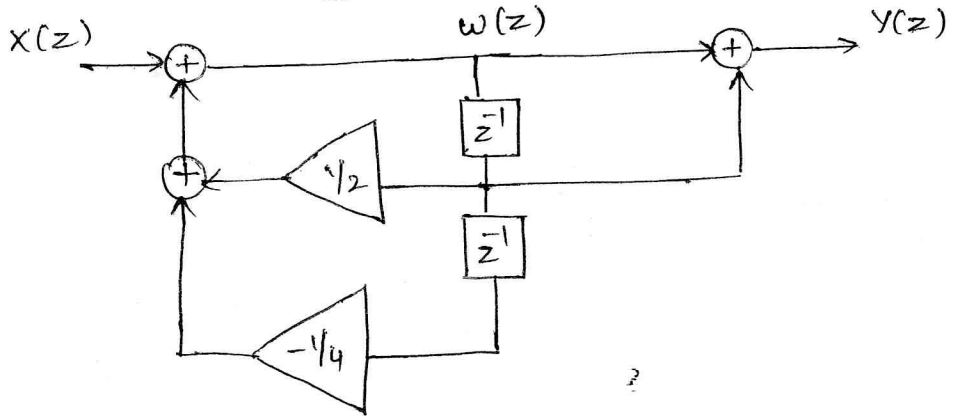
$$\frac{W(z)}{X(z)}=\frac{1}{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}$$

$$W(z)=X(z)+\frac{1}{2}z^{-1}W(z)-\frac{1}{4}z^{-2}W(z) \longrightarrow (1)$$

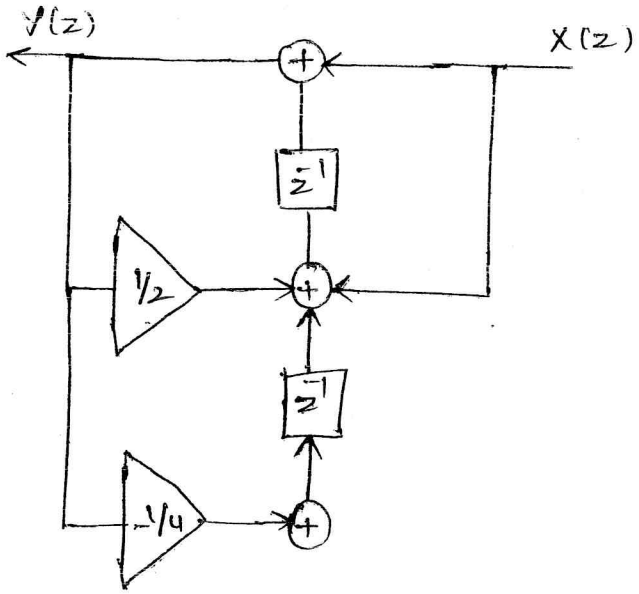
$$\frac{Y(z)}{W(z)}=1+z^{-1}$$

$$Y(z)=W(z)+z^{-1}W(z) \longrightarrow (2)$$

Realize eq (1) & (2)



Transposed structure:-



$$Y(z) = X(z) + z^{-1}x(z) + \frac{1}{2}z^{-1}y(z) - \frac{1}{4}z^{-2}y(z)$$

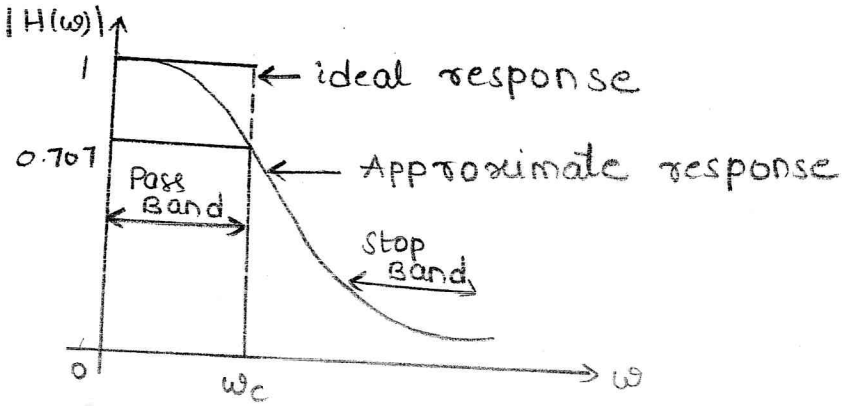
Design of Digital filters:-

Frequency selective filters:-

Filter: A filter is one which allows the desired frequency and reject the unwanted frequencies.

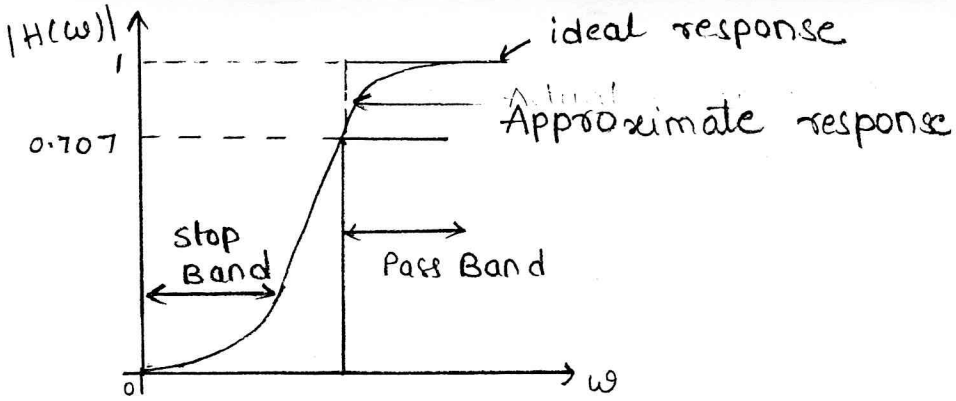
There are 4 types

1. Low pass filter:- It is one which allows the frequencies upto ω_c and reject the frequencies above the ω_c.

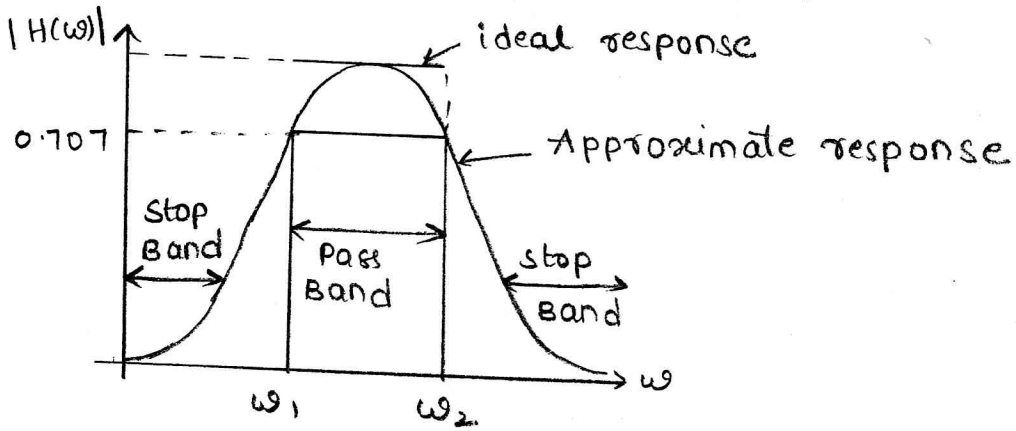


2. High pass filter:-

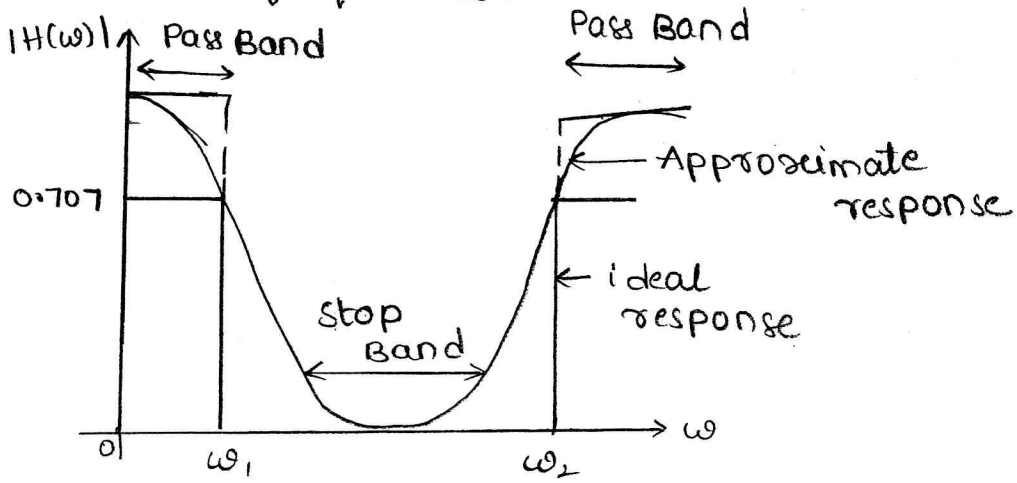
It is the one which allows the frequencies above the ω_c and reject the frequencies below the ω_c.



3. Band pass filter:- BPF is one which allows the frequencies between ω_1 and ω_2 and rejects all other frequencies.



4. Band reject filter (or) Elimination filter (or) Band stop filter: BRF is one which rejects the frequencies between ω_1 and ω_2 and allows all other frequencies.



Methods of designing IIR systems:-

The most common method for designing the IIR filter is indirect method. For indirect method we have to follow the following steps

1. Convert the digital filter specification into corresponding analog filter specification.

2. The analog filter specifications design the analog prototype H(s).

3. Convert the H(s) into H(z) using

i) Impulse invarient transformation (IIT)

ii) Bilinear Transformation (BLT)

1. Impulse Invariant Transformation:

By using impulse invariant transformation the s-domain is converted into the z-domain using the following forms.

i) $\frac{A_i \rightarrow \text{constant}}{S - P_i} \longrightarrow \frac{A_i}{1 - e^{+P_i T} z^{-1}}$

$S - P_i$: S → S-domain, P_i → Poles

ii) $\frac{S+a}{(S+a)^2 + b^2} \longrightarrow \frac{1 - e^{-aT}(\cos bT) z^{-1}}{1 - 2e^{-aT}(\cos bT) z^{-1} + e^{-2aT} z^{-2}}$

iii) $\frac{b}{(S+a)^2 + b^2} \longrightarrow \frac{e^{-aT}(\sin bT) z^{-1}}{1 - 2e^{-aT}(\cos bT) z^{-1} + e^{-2aT} z^{-2}}$

1. Design H(z) for the analog transfer function $H_a(S) = \frac{2}{(S+1)(S+2)}$ using impulse invariant transformation method and assume the sampling time period (i) T = 1 sec (ii) T = 0.1 sec

Sol Given that $H_a(S) = \frac{2}{(S+1)(S+2)}$

Apply partial fractions

$$H_a(S) = \frac{2}{(S+1)(S+2)} = \frac{A}{S+1} + \frac{B}{S+2}$$

$$2 = A(S+2) + B(S+1)$$

let $S = -2$

$2 = B(-1)$

$\boxed{B = -2}$

Let $S = -1$

$2 = A(1)$

$\boxed{A = 2}$

$$H_a(s) = \frac{2}{s+1} + \frac{-2}{s+2}$$

$$H_a(s) = \frac{2}{s-(-1)} + \frac{-2}{s-(-2)}$$

using impulse invarient transformation

$$\frac{A_i}{s-P_i} \longrightarrow \frac{A_i}{1-e^{P_i T} z^{-1}}$$

$$H_a(z) = \frac{2}{1-e^{P_1 T} z^{-1}} + \frac{-2}{1-e^{P_2 T} z^{-1}}, \text{ where } P_1 = -1 \text{ and } P_2 = -2$$

$$H_a(z) = \frac{2}{1-e^{-T} z^{-1}} + \frac{-2}{1-e^{-2T} z^{-1}}$$

If T = 1sec

$$H_a(z) = \frac{2}{1-e^{-1} z^{-1}} - \frac{2}{1-e^{-2} z^{-1}}$$

$$= \frac{2}{1-0.3678 z^{-1}} - \frac{2}{1-0.1353 z^{-1}}$$

$$= \frac{2(1-0.1353 z^{-1}) - 2(1-0.3678 z^{-1})}{(1-0.3678 z^{-1})(1-0.1353 z^{-1})}$$

$$= \frac{2 - 0.2706 z^{-1} - 2 + 0.7356 z^{-1}}{1 - 0.5031 z^{-1} + 0.04976 z^{-2}}$$

$$H_a(z) = \frac{0.465 z^{-1}}{1 - 0.5031 z^{-1} + 0.04976 z^{-2}}$$

If T = 0.1sec

$$H_a(z) = \frac{2}{1-e^{-0.1} z^{-1}} - \frac{2}{1-e^{-0.2} z^{-1}} = \frac{2}{1-0.9048 z^{-1}} - \frac{2}{1-0.8187 z^{-1}}$$

$$= \frac{2(1-0.8187 z^{-1}) - 2(1-0.9048 z^{-1})}{(1-0.9048 z^{-1})(1-0.8187 z^{-1})} = \frac{-1.6374 z^{-1} + 1.8096 z^{-1}}{1 - 1.7235 z^{-1} + 0.7407 z^{-2}}$$

$$H_a(z) = \frac{0.1729 z^{-1}}{1 - 1.7235 z^{-1} + 0.7407 z^{-2}}$$

2. Design H(z) for the following $H_a(s) = \frac{10}{s^2+7s+10}$ assume T = 0.2sec

Sol Given $H_a(s) = \frac{10}{s^2+7s+10}$

$$H_a(s) = \frac{10}{(s+2)(s+5)}$$

Apply partial fractions

$$\frac{10}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$

$$10 = A(s+5) + B(s+2)$$

Put $s = -5$

$10 = B(-3)$

$\boxed{B = -3.33}$

Put $s = -2$

$10 = A(3)$

$\boxed{A = 3.33}$

$$H_a(s) = \frac{3.33}{s+2} - \frac{3.33}{s+5} = \frac{3.33}{s-(-2)} + \frac{-3.33}{s-(-5)}$$

we know that $\frac{A_i}{s_i - P_i} = \frac{A_i}{1-e^{P_iT}z^{-1}}$

$$H_a(z) = \frac{3.33}{1-e^{P_1T}z^{-1}} - \frac{3.33}{1-e^{P_2T}z^{-1}} \quad \text{where } P_1 = -2 \text{ \& } P_2 = -5$$

$$H_a(z) = \frac{3.33}{1-e^{-2T}z^{-1}} - \frac{3.33}{1-e^{-5T}z^{-1}}$$

If T = 0.2sec

$$H_a(z) = \frac{3.33}{1-e^{-0.4}z^{-1}} - \frac{3.33}{1-e^{-1}z^{-1}}$$

$$= \frac{3.33}{1-0.6703z^{-1}} - \frac{3.33}{1-0.3678z^{-1}}$$

$$H_a(z) = \frac{3.33(1-0.3678z^{-1}) - 3.33(1-0.6703z^{-1})}{(1-0.6703z^{-1})(1-0.3678z^{-1})}$$

$$H_a(z) = \frac{1.0073z^{-1}}{1-1.0381z^{-1}+0.2465z^{-2}}$$

3. Design the H(z) for the analog transfer function

$$Ha(s) = \frac{5}{s^3+6s^2+11s+6}$$ using impulse invariant transformation?

Sol Given that

$$Ha(s) = \frac{5}{s^3+6s^2+11s+6}$$

$$Ha(s) = \frac{5}{(s+1)(s+2)(s+3)}$$

Apply partial fraction

$$\frac{5}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$5 = A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$

Let $s = -1$

$5 = A(2)$

$A = 5/2$

$\boxed{A = 2.5}$

Let $s = -2$

$5 = B(-1)$

$\boxed{B = -5}$

Let $s = -3$

$5 = C(2)$

$\boxed{C = 2.5}$

$$Ha(s) = \frac{2.5}{s+1} - \frac{5}{s+2} + \frac{2.5}{s+3}$$

$$Ha(s) = \frac{2.5}{s-(-1)} - \frac{5}{s-(-2)} + \frac{2.5}{s-(-3)}$$

we know that

$$\frac{Ai}{Si-Pi} = \frac{Ai}{1-e^{PiT}z^{-1}}$$

$$Ha(z) = \frac{2.5}{1-e^{-1T}z^{-1}} - \frac{5}{1-e^{-2T}z^{-1}} + \frac{2.5}{1-e^{-3T}z^{-1}}$$

Assume $T = 1$ sec

$$Ha(z) = \frac{2.5}{1-e^{-1}z^{-1}} - \frac{5}{1-e^{-2}z^{-1}} + \frac{2.5}{1-e^{-3}z^{-1}}$$

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$$Ha(z) = \frac{2.5}{1-0.3678z^{-1}} - \frac{5}{1-0.1353z^{-1}} + \frac{2.5}{1-0.0497z^{-1}}$$

$$Ha(z) = \frac{2.5 - 0.3382z^{-1} - 5 + 1.839z^{-1}}{1-0.5031z^{-1}+0.04976z^{-2}} + \frac{2.5}{1-0.0497z^{-1}}$$

$$H_a(z) = \frac{-2.5 + 1.5008z^{-1}}{1 - 0.5031z^{-1} + 0.04976z^{-2}} + \frac{2.5}{1 - 0.0497z^{-1}}$$

$$= \frac{-2.5 + 0.12425z^{-1} + 1.5008z^{-1} - 0.0745z^{-2} + 2.5 - 1.2575z^{-1} + 0.1244z^{-2}}{1 - 0.0497z^{-1} - 0.5031z^{-1} + 0.025z^{-2} + 0.04976z^{-2} - 0.00247z^{-3}}$$

$$H_a(z) = \frac{0.36735z^{-1} + 0.0499z^{-2}}{1 - 0.5528z^{-1} + 0.07476z^{-2} - 0.00247z^{-3}}$$

4. Design the $H(z)$ for analog TF $H_a(s) = \frac{s+3}{s^2+6s+25}$

Given $H_a(s) = \frac{s+3}{s^2+6s+25}$

$$H_a(s) = \frac{s+3}{s^2+6s+3^2-3^2+25} = \frac{s+3}{(s+3)^2+16}$$

we know that

$$\frac{s+a}{(s+a)^2+b^2} \rightarrow \frac{1 - e^{aT}\cos bT\, z^{-1}}{1 - 2e^{-aT}\cos bT\, z^{-1} + e^{-2aT}z^{-2}}$$

$$H_a(z) = \frac{1 - e^{-3T}\cos 4T\, z^{-1}}{1 - 2e^{-3T}\cos 4T\, z^{-1} + e^{-2(3)T}z^{-2}}$$

If $T = 1$ sec

$$H_a(z) = \frac{1 - e^{-3}\cos 4(1)z^{-1}}{1 - 2e^{-3}\cos 4(1)z^{-1} + e^{-6(1)}z^{-2}}$$

$$H_a(z) = \frac{1 - 0.049\,(-0.6536)z^{-1}}{1 - 2(0.049)(-0.6536)z^{-1} + (0.00247z^{-2})}$$

$$H_a(z) = \frac{1 + 0.032z^{-1}}{1 + 0.064z^{-1} + 0.00247z^{-2}}$$

Bilinear Transformation:

The Bilinear transformation the s-domain is converted into the z-domain by replacing the s by $\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$, where T is sampling time period.

1. Design the H(z) for the analog transfer function $H_a(s) = \frac{2s}{s^2+0.2s+1}$ using Bilinear transformation. Assume T = 1sec

Sol Given $H_a(s) = \frac{2s}{s^2+0.2s+1}$

using Bilinear transformation

s is replaced by $\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

$$H_a(z) = \frac{2\left[\frac{2}{1}\frac{1-z^{-1}}{1+z^{-1}}\right]}{\left[2\frac{1-z^{-1}}{1+z^{-1}}\right]^2 + 0.2\left[2\frac{1-z^{-1}}{1+z^{-1}}\right]+1}$$

$$H_a(z) = \frac{4(1-z^{-1})(1+z^{-1})}{4(1-z^{-1})^2 + 0.4(1-z^{-1})(1+z^{-1}) + (1+z^{-1})^2}$$

$$= \frac{4(1-z^{-2})}{4(1+z^{-2}-2z^{-1}) + 0.4(1-z^{-2}) + (1+z^{-2}+2z^{-1})}$$

$$H_a(z) = \frac{4(1-z^{-2})}{5.4 + 4.6z^{-2} - 6z^{-1}}$$

$$H_a(z) = \frac{4(1-z^{-2})/4.6}{z^{-2} - 1.3z^{-1} + 1.17}$$

$$H_a(z) = \frac{0.869(1-z^{-2})}{z^{-2} - 1.3z^{-1} + 1.17}$$

2. Design the H(z) for the analog transfer function $H_a(s) = \frac{s+0.3}{(s+0.3)^2+16}$ using bitinear transformation.

Sol Given $H_a(s) = \frac{s+0.3}{(s+0.3)^2+16}$

$$Ha(s) = \frac{s+0.3}{s^2+0.6s+0.09+16} = \frac{s+0.3}{s^2+0.6s+16.09}$$

using Bilinear transformation

s replaced by $\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

Assume T = 1 sec

$$Ha(z) = \frac{2\frac{1-z^{-1}}{1+z^{-1}} + 0.3}{\left[2\frac{1-z^{-1}}{1+z^{-1}}\right]^2 + \left[2\frac{1-z^{-1}}{1+z^{-1}}\right]0.6 + 16.09}$$

$$Ha(z) = \frac{[2(1-z^{-1}) + 0.3(1+z^{-1})](1+z^{-1})}{4(1-z^{-1})^2 + 2(1-z^{-1})(1+z^{-1})0.6 + 16.09(1+z^{-1})^2}$$

$$Ha(z) = \frac{(2-2z^{-1}+0.3+0.3z^{-1})(1+z^{-1})}{4(1+z^{-2}-2z^{-1}) + 1.2(1-z^{-2}) + 16.09(1+z^{-2}+2z^{-1})}$$

$$Ha(z) = \frac{(2.3-1.7z^{-1})(1+z^{-1})}{4+4z^{-2}-8z^{-1}+1.2-1.2z^{-2}+16.09+16.09z^{-2}+32.18z^{-1}}$$

$$Ha(z) = \frac{2.3+2.3z^{-1}-1.7z^{-1}-1.7z^{-2}}{18.89z^{-2}+24.18z^{-1}+21.29}$$

$$Ha(z) = \frac{2.3+0.6z^{-1}-1.7z^{-2}}{18.89z^{-2}+24.18z^{-1}+21.29}$$

3. Design the H(z) for analog transfor function $Ha(s) = \frac{s^3}{(s+1)(s^2+s+1)}$ using Bilinear transfermation.

Given that $Ha(s) = \frac{s^3}{(s+1)(s^2+s+1)}$

using Bilinear transformation

s replaced by $\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

Assume T = 1 sec

$$H_a(z) = \frac{\left[2\ \frac{1-z^{-1}}{1+z^{-1}}\right]^3}{\left[2\ \frac{1-z^{-1}}{1+z^{-1}}+1\right]\left[\left[2\ \frac{1-z^{-1}}{1+z^{-1}}\right]^2+2\ \frac{1-z^{-1}}{1+z^{-1}}+1\right]}$$

$$= \frac{8(1-z^{-1})^3}{2(1-z^{-1})+(1+z^{-1})\left[4(1-z^{-1})^2+2(1-z^{-1})(1+z^{-1})+(1+z^{-1})^2\right]}$$

$$= \frac{8(1-z^{-1})^3}{(2-2z^{-1}+1+z^{-1})\left[4(1+z^{-2}-2z^{-1})+2(1-z^{-2})+(1+z^{-2}+2z^{-1})\right]}$$

$$= \frac{8(1-z^{-1})^3}{(3-z^{-1})(7+3z^{-2}-6z^{-1})}$$

$$= \frac{8(1-z^{-1})^3}{(21+9z^{-2}-18z^{-1}-7z^{-1}-3z^{-3}+6z^{-2})}$$

$$H_a(z) = \frac{8(1-z^{-1})^3}{21+15z^{-2}-25z^{-1}-3z^{-3}}$$

Design procedure of Digital lowpass Butterworth filter:-

let A_1 = Gain at a passband frequency ω_1

A_2 = Gain at a stop band frequency ω_2

Ω_1 = Analog frequency corresponding to ω_1.

Ω_2 = Analog frequency corresponding to ω_2

1. choose either bilinear or impulse invariant transformation.
2. Calculate the ratio of Ω_2/Ω_1

For bilinear transformation Ω_2/Ω_1

$$\frac{\Omega_2}{\Omega_1} = \frac{\tan\omega_2/2}{\tan\omega_1/2} \longrightarrow (1)$$

For impulse invariant transformation $\frac{\Omega_2}{\Omega_1} = \frac{\omega_2}{\omega_1} \longrightarrow (2)$

3. Decide the order N of the filter. The order N should be greater than or equal to N_1, where N_1 is given by.

$$N_1 = \frac{1}{2} \frac{\log\left\{\left[\frac{1}{A_2^2}-1\right] / \left[\frac{1}{A_1^2}-1\right]\right\}}{\log \frac{\Omega_2}{\Omega_1}} \longrightarrow (3)$$

(choose N such that $N \geq N_1$)

4. Calculate the analog cut-off frequency Ω_c.

For bilinear transformation $\Omega_c = \frac{2}{T} \frac{\tan \omega_1/2}{\left[\frac{1}{A_1^2}-1\right]^{1/2N}} \longrightarrow (4)$

For impulse invariant transformation,

$$\Omega_c = \frac{\omega_1/T}{\left[\frac{1}{A_1^2}-1\right]^{1/2N}} \longrightarrow (5)$$

5. Determine the analog transfer function of the filter

Let, $H_a(s)$ = Analog filter transfer function.

when the order N is even for unity dc gain filter, $H_a(s)$ is given by the equation

$$H_a(s) = \prod_{k=1}^{N/2} \frac{\Omega_c^2}{s^2 + b_k \Omega_c s + \Omega_c^2} \longrightarrow (6)$$

when the order N is odd for unity gain dc filter, $H_a(s)$ is given by the equation

$$H_a(s) = \frac{\Omega_c}{s+\Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{\Omega_c^2}{s^2 + b_k \Omega_c s + \Omega_c^2} \longrightarrow (7)$$

The coefficient b_k is given by

$$b_k = 2 \sin\left[\frac{(2k-1)\pi}{2N}\right] \longrightarrow (8)$$

Note: For normalized case take Ω_c = 1rad/sec

6. using the choosen transformation, transform $H_a(s)$ to $H(z)$, where $H(z)$ is the transfer function of the digital filter.
7. Realize the digital filter transfer function $H(z)$ by a suitable structure.

1. The specification of desired low pass filter is

$$\frac{1}{\sqrt{2}} \le |H(\omega)| \le 1.0 \quad ; \quad 0 \le \omega \le 0.2\pi$$

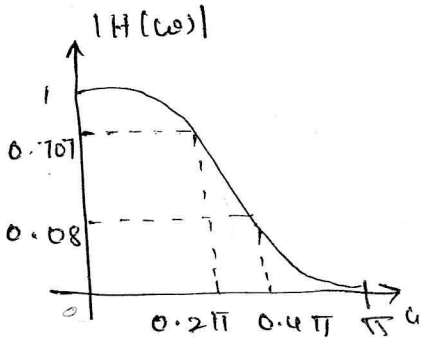
$$|H(\omega)| \le 0.08 \quad ; \quad 0.4\pi \le \omega \le \pi$$

Design a Butter worth digital filter using Bilinear transformation

Sol

Given that, $A_1 = \frac{1}{\sqrt{2}} = 0.707$; $\omega_1 = 0.2\pi$

$A_2 = 0.08$; $\omega_2 = 0.4\pi$



using Bilinear transformation

For Bilinear transformation, $\frac{\Omega_2}{\Omega_1}$ is

$$\frac{\Omega_2}{\Omega_1} = \frac{\tan(\omega_2/2)}{\tan(\omega_1/2)} = \frac{\tan(0.4\pi/2)}{\tan(0.2\pi/2)} = \frac{\tan(0.2\pi)}{\tan(0.1\pi)} = \frac{0.7265}{0.3249}$$

$$\frac{\Omega_2}{\Omega_1} = 2.236$$

$$N_1 = \frac{1}{2} \frac{\log\left\{\left[\frac{1}{A_2^2}-1\right]/\left[\frac{1}{A_1^2}-1\right]\right\}}{\log\frac{\Omega_2}{\Omega_1}} = \frac{1}{2} \frac{\log\left\{\left[\frac{1}{0.08^2}-1\right]/\left[\frac{1}{0.707^2}-1\right]\right\}}{\log 2.236}$$

$$= \frac{1}{2} \frac{\log\{155.25/1.0006\}}{0.3494} = \frac{1}{2} \frac{2.1907}{0.3494} = 3.1349$$

$N_1 = 3.1349$

Choose N such that $N \ge N_1$

$\boxed{N = 4}$, Assume $T = 1$ sec

Analog cut off frequency

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$$\Omega_c = \frac{2}{T} \times \frac{\tan(\omega_1/2)}{\left[\frac{1}{A_1^2}-1\right]^{\frac{1}{2N}}} = 2 \frac{\tan(0.2\pi/2)}{\left[\frac{1}{0.707^2}-1\right]^{1/8}}$$

$$\Omega_c = \frac{2(0.3249)}{(1.0006)^{1/8}} = 0.6497$$

$\boxed{\Omega_c = 0.6497}$

If N is even, the analog transfer function $H_a(s)$ is given by

$$H_a(s) = \prod_{k=1}^{N/2} \frac{\Omega_c^2}{s^2 + b_k \Omega_c s + \Omega_c^2} \quad ; \text{ where } b_k = 2\sin\left[\frac{(2k-1)\pi}{2N}\right]$$

Here $N=4$, $k=1,2$

If $k=1$ $\quad b_1 = 2\sin\left[\frac{(2-1)\pi}{2\times 4}\right] = 2\sin\left(\frac{\pi}{8}\right)$

$$\boxed{b_1 = 0.7653}$$

If $k=2$ $\quad b_2 = 2\sin\left[\frac{(4-1)\pi}{2\times 4}\right] = 2\sin\left(\frac{3\pi}{8}\right)$

$$\boxed{b_2 = 1.8477}$$

$$H_a(s) = \prod_{k=1}^{2} \frac{\Omega_c^2}{s^2 + b_k \Omega_c s + \Omega_c^2}$$

$$H_a(s) = \frac{(0.6497)^2}{s^2 + (0.7653)(0.6497)s + (0.6497)^2} * \frac{(0.6497)^2}{s^2 + (0.6497)(1.8477)s + (0.6497)^2}$$

$$H_a(s) = \frac{0.1781}{(s^2 + 0.4972s + 0.4221)(s^2 + 1.2004s + 0.4221)}$$

using Bilinear transformation s is Replaced by $\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

Assume here $T = 1$ sec

$$H(z) = \frac{0.1781}{\left[\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 0.4972\times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.4221\right]\left[\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 1.2004\times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.4221\right]}$$

$$H(z) = \frac{0.1781}{\left[4\frac{(1-z^{-1})^2}{(1+z^{-1})^2} + 0.9944\frac{(1-z^{-1})}{1+z^{-1}} + 0.4221\right]\left[4\frac{(1-z^{-1})^2}{(1+z^{-1})^2} + 2.4008\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.4221\right]}$$

$$H(z) = \frac{0.1781\,(1+z^{-1})^4}{\left[4(1+z^{-2}-2z^{-1}) + 0.9944 - 0.9944z^{-2} + 0.4221(1+z^{-2}+2z^{-1})\right]\left[4(1+z^{-2}-2z^{-1}) + 2.4008(1-z^{-1})(1+z^{-1}) + 0.4221(1+z^{-2}+2z^{-1})\right]}$$

$$H(z) = \frac{0.1781\,(1+z^{-1})^4}{(4+4z^{-2}-8z^{-1}+0.9944-0.9944z^{-2}+0.4221+0.4221z^{-2}+0.8442z^{-1})}$$

$$(4+4z^{-2}-8z^{-1}+2.4008-2.4008z^{-2}+0.4221+0.4221z^{-2}+0.8442z^{-1})$$

$$= \frac{0.1781\,(1+z^{-1})^4}{(3.4277z^{-2}-7.1558z^{-1}+5.4165)(2.0213z^{-2}-7.1558z^{-1}+6.8229)}$$

$$= \frac{0.1781\,(1+z^{-1})^4 / 5.4165 \times 6.8229}{(1-1.3211z^{-1}+0.6328z^{-2})(1-1.0487z^{-1}+0.2962z^{-2})}$$

$$H(z) = \frac{0.004819\,(1+z^{-1})^4}{(1-1.3211z^{-1}+0.6328z^{-2})(1-1.0487z^{-1}+0.2962z^{-2})}$$

2. The specification of the desired lowpass filter is

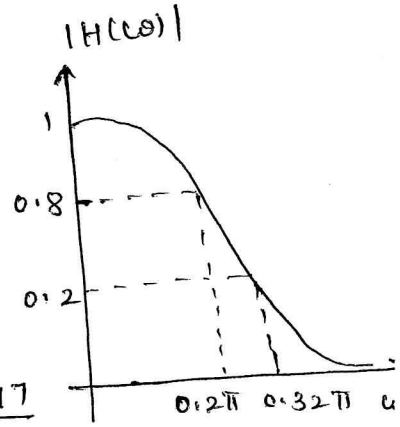
$0.8 \le |H(\omega)| \le 1.0$; $0 \le \omega \le 0.2\pi$

$|H(\omega)| \le 0.2$; $0.32\pi \le \omega \le \pi$

Sol

Given that, $A_1 = 0.8$; $\omega_1 = 0.2\pi$ rad/sec

$A_2 = 0.2$; $\omega_2 = 0.32\pi$ rad/sec



using Bilinear transformation

For Bilinear transformation, $\frac{\Omega_2}{\Omega_1}$ is

$$\frac{\Omega_2}{\Omega_1} = \frac{\tan(\omega_2/2)}{\tan(\omega_1/2)} = \frac{\tan(0.16\pi)}{\tan(0.1\pi)} = \frac{0.3249}{0.5497} = \frac{0.5497}{0.3249}$$

$$\frac{\Omega_2}{\Omega_1} = 1.692$$

$$N_1 = \frac{1}{2}\,\frac{\log\left\{\left[\frac{1}{A_2^2}-1\right]\Big/\left[\frac{1}{A_1^2}-1\right]\right\}}{\log\left(\frac{\Omega_2}{\Omega_1}\right)} = \frac{1}{2}\,\frac{\log\left\{\left[\frac{1}{0.2^2}-1\right]\Big/\left[\frac{1}{0.8^2}-1\right]\right\}}{\log 1.692}$$

$$= \frac{1}{2}\,\frac{\log(24/0.5625)}{0.2284} = \frac{1.63 \times 0.5}{0.2284}$$

$$\boxed{N_1 = 3.5683}$$

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Choose N such that $N \ge N_1$

$\boxed{N=4}$, Assume $T = 1$ sec

Analog cut off frequency

$$\Omega_c = \frac{2}{T}\frac{\tan(\omega_1/2)}{\left[\frac{1}{A_1^2}-1\right]^{\frac{1}{2N}}} = 2\frac{\tan(0.1\pi)}{\left[\frac{1}{0.8^2}-1\right]^{1/8}} = 2\frac{(0.3249)}{0.9306}$$

$$\Omega_c = 0.6982 \text{ rad/sec}$$

For N is even, analog transfer function $H_a(s)$ is given by

$$H_a(s) = \prod_{k=1}^{N/2} \frac{\Omega_c^2}{s^2 + b_k\Omega_c s + \Omega_c^2} \quad \text{where } b_k = 2\sin\left(\frac{(2k-1)\pi}{2N}\right)$$

Here $N=4$, $K=1,2$

If $K=1$

$$b_1 = 2\sin\left(\frac{(2-1)\pi}{2\times4}\right) = 2\sin\left(\frac{\pi}{8}\right)$$

$$\boxed{b_1 = 0.7653}$$

If $K=2$

$$b_2 = 2\sin\left(\frac{(4-1)\pi}{2\times4}\right) = 2\sin\left(\frac{3\pi}{8}\right)$$

$$\boxed{b_2 = 1.8477}$$

$$H_a(s) = \prod_{k=1}^{2} \frac{\Omega_c^2}{s^2 + b_k\Omega_c s + \Omega_c^2}$$

$$H_a(s) = \frac{(0.6982)^2}{s^2 + (0.7653)(0.6982)s + (0.6982)^2} \times \frac{(0.6982)^2}{s^2 + (1.8477)(0.6982)s + (0.6982)^2}$$

$$H_a(s) = \frac{0.2376}{(s^2 + 0.5343s + 0.4874)(s^2 + 1.295s + 0.4874)}$$

using Bilinear transformation s is Replaced by $\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

Assume here T = 1 sec

$$H(z) = \frac{0.2376}{\left[\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2 + 0.5343\times2\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 0.4874\right]\left[\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2 + 1.29\times2\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + 0.4874\right]}$$

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$$H(z) = \frac{0.2376}{\left[4\frac{(1-z^{-1})^2}{(1+z^{-1})^2} + 1.0686\frac{(1-z^{-1})}{(1+z^{-1})} + 0.4874\right]\left[4\frac{(1-z^{-1})^2}{(1+z^{-1})^2} + 2.58\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.4874\right]}$$

$$H(z) = \frac{0.2376(1+z^{-1})^4}{\left[4(1+z^{-2}-2z^{-1}) + 1.0686 - 1.0686z^{-2} + 0.4874(1+z^{-2}+2z^{-1})\right]\left[4(1+z^{-2}-2z^{-1}) + 2.58 - 2.58z^{-2} + 0.4874(1+z^{-2}+2z^{-1})\right]}$$

$$H(z) = \frac{0.2376(1+z^{-1})^4}{\left[4+4z^{-2}-8z^{-1}+1.0686-1.0686z^{-2}+0.4874+0.4874z^{-2}+0.9748z^{-1}\right]\left[4+4z^{-2}-8z^{-1}+2.58-2.58z^{-2}+0.4874+0.4874z^{-2}+0.9748z^{-1}\right]}$$

$$H(z) = \frac{0.2376(1+z^{-1})^4}{(3.4188z^{-2}-7.0252z^{-1}+5.556)(1.9074z^{-2}-7.0252z^{-1}+7.067}$$

$$H(z) = \frac{0.2376(1+z^{-1})^4/5.556\times7.0674}{(0.6153z^{-2}-1.2644z^{-1}+1)(0.2698z^{-2}-0.994z^{-1}+1)}$$

$$H(z) = \frac{0.006(1+z^{-1})^4}{(1-1.2644z^{-1}+0.6153z^{-2})(1+0.2698z^{-2}-0.994z^{-1})}$$

Using impulse invariant transformation.

$$H_a(s) = \frac{0.2376}{(s^2+0.5343s+0.4874)(s^2+1.29s+0.4874)}$$

By partial fraction expansion

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$$H_a(s) = \frac{0.2376}{(s^2+0.5343s+0.4874)(s^2+1.29s+0.4874)}$$

$$= \frac{As+B}{s^2+0.5343s+0.4874} + \frac{Cs+D}{s^2+1.29s+0.4874} \rightarrow (1$$

$$0.2376 = (As+B)(s^2+1.29s+0.4874) + (Cs+D)(0.4874+0.5343s+s^2$$

$$0.2376 = As^3+1.29As^2+0.4874As+Bs^2+1.29Bs+0.4874B + Cs^3+0.5343s^2C+0.4874Cs+Ds^2+0.5343$$

Equating constants on both sides of above equation

$$0.4874B + 0.4874D = 0.2376$$

$$B + D = 0.2376 / 0.4874$$

$$B + D = 0.4874 \longrightarrow (2)$$

Equating s coefficients on Both sides

$$0.4874A + 1.29B + 0.4874C + 0.5343D = 0$$

$$0.4874(A+C) + 1.29B + 0.5343D = 0 \longrightarrow (3)$$

Equating s^r coefficients on Both sides

$$1.29A + B + 0.5343C + D = 0$$

$$1.29A + 0.5343C = -(B+D)$$

$$1.29A + 0.5343C = -0.4874 \longrightarrow (4)$$

Equating s^3 coefficients on Both sides

$$A + C = 0 \longrightarrow (5)$$

Sub (5) in (3)

$$1.29B + 0.5343D = 0 \longrightarrow (6)$$

Solve eq (2) and (6) to get B and D values

Eq (2) $\times$ 1.29 $\Rightarrow$ $1.29B + 1.29D = 0.6287$

(6) $\times$ 1 $\Rightarrow$ $1.29B + 0.5343D = 0$

$$0.7557D = 0.6287$$

$$D = \frac{0.6287}{0.7557}$$

$$\boxed{D = 0.8319}$$

Substitute D value in eq (2)

$$B + D = 0.4874$$

$$B = 0.4874 - 0.8319$$

$$\boxed{B = -0.3445}$$

solve eq ④ and ⑤ to get A and C values

Eq ⑤ × 1.29 ⇒ $1.29A + 1.29C = 0$

Eq ④ × 1 ⇒ $1.29A + 0.5343C = -0.4874$

$$0.7557C = 0.4874$$

$$C = \frac{0.4874}{0.7557}$$

$$\boxed{C = 0.6449}$$

Substitute C value in ⑤

$$A = -C$$

$$\boxed{A = -0.6449}$$

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Now Ha(s) can be written as

$$Ha(s) = \frac{-0.6449s - 0.3445}{s^2 + 0.5343s + 0.4874} + \frac{0.6449s + 0.8319}{s^2 + 1.29s + 0.4874}$$

$$= \frac{-0.6449(s + 0.5341)}{(s^2 + 2 \times 0.26715s + 0.26715^2) + (\sqrt{0.4874 - 0.26715^2})^2} +$$

$$\frac{0.6449(s + 1.2899)}{(s^2 + 2 \times 0.645s + 0.645^2) + (\sqrt{0.4874 - 0.645^2})^2}$$

$$Ha(s) = \frac{-0.6449(s + 0.5341)}{(s + 0.26715)^2 + (0.645)^2} + \frac{0.6449(s + 1.2899)}{(s + 0.645)^2 + (0.2671)^2}$$

$$Ha(s) = \frac{-0.6449(s + 0.267 + 0.267)}{(s + 0.267)^2 + (0.645)^2} + \frac{0.6449(s + 0.645 + 0.645)}{(s + 0.645)^2 + (0.2671)^2}$$

$$= \frac{-0.6449(s + 0.267)}{(s + 0.267)^2 + (0.645)^2} - \frac{0.6449 \times 0.267}{0.645} \frac{0.645}{(s + 0.267)^2 + (0.645)^2}$$

$$+ \frac{0.6449(s + 0.645)}{(s + 0.645)^2 + (0.267)^2} + \frac{0.6449 \times 0.645}{0.267} \frac{0.267}{(s + 0.645)^2 + (0.267)^2}$$

$$H_a(s) = -0.6449\,\frac{(s+0.267)}{(s+0.267)^2+(0.645)^2} - 0.2669\,\frac{0.645}{(s+0.267)^2+(0.645)^2}$$

$$+ 0.6449\,\frac{(s+0.645)}{(s+0.645)^2+(0.267)^2} + 1.5579\,\frac{0.267}{(s+0.645)^2+(0.267)^2}$$

The analog transfer function can be transformed to digital transfer function using impulse invariant transformations

$$\frac{s+a}{(s+a)^2+b^2} \rightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2} \rightarrow \frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

Assume $T = 1$ sec $H_a(s)$ to get $H(z)$

$$H(z) = -0.6449\,\frac{1-e^{-0.267}(\cos 0.645)z^{-1}}{1-2e^{-0.267}(\cos 0.645)z^{-1}+e^{-0.534}z^{-2}}$$

$$- 0.2669\,\frac{e^{-0.267}\sin(0.645)z^{-1}}{1-2e^{-0.267}(\cos 0.645)z^{-1}+e^{-0.534}z^{-2}}$$

$$+ 0.6449\,\frac{1-e^{-0.645}(\cos 0.267)z^{-1}}{1-2e^{-0.645}(\cos 0.267)z^{-1}+e^{-1.29}z^{-2}}$$

$$+ 1.5579\,\frac{e^{-0.645}\sin(0.267)z^{-1}}{1-2e^{-0.645}(\cos 0.267)z^{-1}+e^{-1.29}z^{-2}}$$

$$H(z) = \frac{-0.6449+0.3945z^{-1}}{1-1.2236z^{-1}+0.5862z^{-2}} + \frac{-0.1228z^{-1}}{1-1.2236z^{-1}+0.5862z^{-2}}$$

$$= \frac{0.6449-0.3263z^{-1}}{1-1.012z^{-1}+0.2752z^{-2}} + \frac{0.2156z^{-1}}{1-1.012z^{-1}+0.2752z^{-2}}$$

$$H(z) = \frac{-0.6449 + 0.2717z^{-1}}{1 - 1.2236z^{-1} + 0.5862z^{-2}} + \frac{0.6449 - 0.1107z^{-1}}{1 - 1.012z^{-1} + 0.2752z^{-2}}$$

Design procedure for lowpass digital Chebyshev I IR filter:

A_1 = Gain at a passband frequency ω_1.

A_2 = Gain at a stopband frequency ω_2.

Ω_1 = Analog frequency corresponding to ω_1.

Ω_2 = Analog frequency corresponding to ω_2.

1. Choose either bilinear or impulse invariant transformation.
2. Calculate the attenuation constant ϵ

$$\epsilon = \left[\frac{1}{A_1^2} - 1\right]^{1/2}$$

3. Calculate the ratio $\frac{\Omega_2}{\Omega_1}$

For bilinear transformation, $\frac{\Omega_2}{\Omega_1} = \frac{\tan \omega_2/2}{\tan \omega_1/2}$

For impulse invariant transformation, $\frac{\Omega_2}{\Omega_1} = \frac{\omega_2}{\omega_1}$

4. Decide the order N of the filter. Choose N such that $N \geq N_1$, where N_1 is given by

$$N_1 = \frac{\cosh^{-1}\left\{\frac{1}{\epsilon}\left[\frac{1}{A_2^2} - 1\right]^{1/2}\right\}}{\cosh^{-1}\left(\frac{\Omega_2}{\Omega_1}\right)}$$

5. Calculate the analog cut off frequency Ω_c

For bilinear transformation, Ω_c

$$\Omega_c = \frac{2}{T} \frac{\tan \omega_1/2}{\left[\frac{1}{A_2^2} - 1\right]^{1/2N}}$$

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For impulse invariant transformation

$$\Omega_c = \frac{\omega_1 / T}{\left[\frac{1}{A_1^2} - 1\right]^{1/2N}}$$

6. Determine the analog transfer function $H_a(s)$ of the filter.

When the order N is even, $H_a(s)$ is given by equation

$$H_a(s) = \prod_{k=1}^{N/2} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

when the order N is odd, $H_a(s)$ is given by equation

$$H_a(s) = \frac{B_0 \Omega_c}{s + c_0 \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + c_k \Omega_c^2}$$

where, $b_k = 2 Y_N \sin\left[\frac{(2k-1)\pi}{2N}\right]$

$$c_k = Y_N^2 + \cos^2 \frac{(2k-1)\pi}{2N}$$

$$c_0 = Y_N$$

$$= \frac{1}{2}\left\{\left[\left(\frac{1}{\epsilon^2} + 1\right)^{1/2} + \frac{1}{\epsilon}\right]^{\frac{1}{N}} - \left[\left(\frac{1}{\epsilon^2} + 1\right)^{1/2} + \frac{1}{\epsilon}\right]^{-1/N}\right\}.$$

For even values of N and unity dc gain filter, find B_k's such that

$$H_a(0) = \frac{1}{(1+\epsilon^2)^{1/2}}$$

For odd values of N and unity dc gain filter, find B_k's such that

$$H_a(0) = 1$$

(It is normal practice to take $B_0 = B_1 = B_2 ---- = B_k$)

7. Using the chosen transformation, transform $H_a(s)$ to $H(z)$, where $H(z)$ is the transfer function of the digital filter.

8. Realize the digital filter transfer function $H(z)$ by a suitable structure.

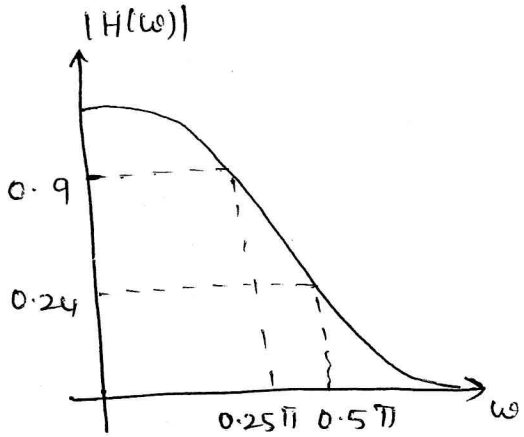
1. The specification of the desired lowpass digital filter is

$$0.9 \le |H(\omega)| \le 1.0 \; ; \; 0 \le \omega \le 0.25\pi$$

$$|H(\omega)| \le 0.24 \; ; \; 0.5\pi \le \omega \le \pi$$

Design a chebyschev digital filter using impulse invariant transformation.

Sol



Given that,

$A_1 = 0.9$; $\omega_1 = 0.25\pi$

$A_2 = 0.24$; $\omega_2 = 0.5\pi$

By using impulse invariant transformation

For impulse invariant transformation,

$$\frac{\Omega_2}{\Omega_1} = \frac{\omega_2}{\omega_1} = \frac{0.5\pi}{0.25\pi} = 2$$

The attenuation constant,

$$\epsilon = \left[\frac{1}{A_1^2} - 1\right]^{1/2} = \left[\frac{1}{0.9^2} - 1\right] = 0.484$$

$$N_1 = \frac{\cosh^{-1}\left\{\frac{1}{\epsilon}\left(\frac{1}{A_2^2} - 1\right)^{1/2}\right\}}{\cosh^{-1}\frac{\Omega_2}{\Omega_1}} = \frac{\cosh^{-1}\left\{\frac{1}{0.484}\left(\frac{1}{0.24^2} - 1\right)^{1/2}\right\}}{\cosh^{-1} 2}$$

$$= \frac{\cosh^{-1} 8.357}{\cosh^{-1} 2} = \frac{2.813}{1.317} = 2.136$$

Choose N such that $N \ge N_1$

let the order of filter $N = 3$

Assume $T = 1$ sec

The analog cut off frequency

$$\Omega_c = \frac{\omega_1/T}{\left(\frac{1}{A_1^2}-1\right)^{\frac{1}{2N}}} = \frac{0.25\pi}{\left[\frac{1}{0.9^2}-1\right]^{1/6}} = \frac{0.25\pi}{0.7855} = 0.99 \approx 1 \text{ rad/sec}$$

The transfer function of the analog filter for odd values

$$H_a(s) = \frac{B_0\Omega_c}{s+c_0\Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k\Omega_c^2}{s^2+b_k\Omega_c s+\Omega_c^2}$$

Here $N=3$, $k=1$

$$H_a(s) = \frac{B_0\Omega_c}{s+c_0\Omega_c} \times \frac{B_1\Omega_c^2}{s^2+b_1\Omega_c s+\Omega_c^2}$$

where $b_k = 2Y_N \sin\frac{(2k-1)\pi}{2N}$

$$c_k = Y_N^2 + \cos^2\frac{(2k-1)\pi}{2N}$$

$$c_0 = Y_N$$

where $Y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\epsilon^2}+1\right)^{1/2}+\frac{1}{\epsilon}\right]^{1/N} - \left[\left(\frac{1}{\epsilon^2}+1\right)^{1/2}+\frac{1}{\epsilon}\right]^{-1/N}\right\}$

$$= \frac{1}{2}\left\{\left[\left(\frac{1}{0.484^2}+1\right)^{1/2}+\frac{1}{0.484}\right]^{1/3} - \left[\left(\frac{1}{0.484^2}+1\right)^{1/2}+\frac{1}{0.484}\right]^{-1/3}\right\}$$

$$= \frac{1}{2}\left\{(2.2953+2.0661)^{1/3} - (2.2953+2.0661)^{-1/3}\right\}$$

$$= \frac{1}{2}\{1.634 - 0.612\}$$

$$\boxed{Y_N = 0.511}$$

$\Rightarrow c_0 = Y_N$

$$\boxed{c_0 = 0.511}$$

$$c_1 = Y_N^2 + \cos^2\frac{(2-1)\pi}{2\times 3}$$

$$c_1 = (0.511)^2 + \cos^2\pi/6$$

$$= 0.2611 + 0.75$$

$$\boxed{c_1 = 1.011}$$

$$b_1 = 2 y_N \sin\frac{\pi}{6}$$

$$= 2 \times 0.511 \times \frac{1}{2} = 0.511$$

$$\boxed{b_1 = 0.511}$$

$$\therefore H_a(s) = \frac{B_0(1)}{s + 0.511} \times \frac{B_1(1)^2}{s^2 + 0.511s + 1.011}$$

when $s = 0$, $H_a(0) = \dfrac{B_0 B_1}{0.511 \times 1.011} = 1.936 B_0 B_1$

let $H_a(0) = 1$ $\therefore$ $B_0 B_1 1.936 = 1$

$$\boxed{B_0 B_1 = 0.5165}$$

let $B_0 = B_1$

$$B_0^2 = 0.5165 \Rightarrow B_0 = 0.7186$$

$$\therefore B_0 = B_1 = 0.7186$$

$$\therefore H_a(s) = \frac{(0.7186)^2}{(s + 0.511)(s^2 + 0.511s + 1.011)} = \frac{0.5165}{(s + 0.511)(s^2 + 0.511s + 1.011)} \rightarrow (1)$$

Apply partial fractions.

$$H_a(s) = \frac{0.5165}{(s + 0.511)(s^2 + 0.511s + 1.011)} = \frac{A}{s + 0.511} + \frac{Bs + C}{s^2 + 0.511s + 1.011} \rightarrow (2)$$

$$0.5165 = A(s^2 + 0.511s + 1.011) + (Bs + C)(s + 0.511)$$

$$0.5165 = As^2 + 0.511As + 1.011A + Bs^2 + 0.511Bs + Cs + 0.511C$$

compare s^2 coefficients on both sides

$$A + B = 0 \rightarrow (3)$$

compare s coefficients on both sides

$$0.511A + 0.511B + C = 0 \rightarrow (4)$$

$$0.511(A + B) + C = 0$$

$$\boxed{C = 0} \rightarrow (5)$$

Comparing constants on both sides

$$0.5165 = 1.011A + 0.511C \rightarrow (6)$$

Substitute C value in above equation

$$1.011A = 0.5165$$

$$\boxed{A = 0.511} \rightarrow (7)$$

Substitute A value in eq (3)

$$A + B = 0$$

$$B + 0.51 = 0$$

$$\boxed{B = -0.511}$$

Now

$$H_a(s) = \frac{A}{s+0.511} + \frac{Bs+C}{s^2+0.511s+1.011}$$

$$= \frac{0.511}{s+0.511} - \frac{0.511s}{s^2+0.511s+1.011}$$

$$= \frac{0.511}{s+0.511} - \frac{0.511s}{(s^2+2\times0.256s+0.256^2)+(\sqrt{1.011-0.256^2})^2}$$

$$= \frac{0.511}{s+0.511} - 0.511\,\frac{s+(0.256-0.256)}{(s+0.256)^2+0.972^2}$$

$$= \frac{0.511}{s+0.511} - 0.511\,\frac{s+0.256}{(s+0.256)^2+0.972^2} + \frac{0.511\times0.256}{0.972}\times\frac{0.972}{(s+0.256)^2+(0.972)^2}$$

$$= \frac{0.511}{s+0.511} - 0.511\,\frac{s+0.256}{(s+0.256)^2+0.972^2} + 0.135\,\frac{0.972}{(s+0.256)^2+0.972^2}$$

The analog transfer function can be transformed into digital transfer function

$$\frac{Ai}{s+Pi} \rightarrow \frac{Ai}{1-e^{PiT}z^{-1}}$$

$$\frac{(s+a)}{(s+a)^2+b^2} \longrightarrow \frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-aT2}z^{-2}}$$

$$\frac{b}{(s+a)^2+b^2} \longrightarrow \frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$$

Assume $T = 1$ sec

$$H(z) = \frac{0.511}{1-e^{-0.511}z^{-1}} - 0.511\ \frac{1-e^{-0.256}(\cos 0.972)z^{-1}}{1-2e^{-0.256}(\cos 0.972)z^{-1}+e^{-0.512}z^{-2}}$$

$$+0.135\ \frac{e^{-0.256}(\sin 0.972)z^{-1}}{1-2e^{-0.256}(\cos 0.972)z^{-1}+e^{-0.512}z^{-2}}$$

$$= \frac{0.511}{1-0.6z^{-1}} + \frac{-0.511+0.223z^{-1}}{1-0.873z^{-1}+0.6z^{-2}} + \frac{0.086z^{-1}}{1-0.873z^{-1}+0.6z^{-2}}$$

$$= \frac{0.511}{1-0.6z^{-1}} + \frac{-0.511+0.223z^{-1}+0.086z^{-1}}{1-0.873z^{-1}+0.6z^{-2}}$$

$$H(z) = \frac{0.511}{1-0.6z^{-1}} + \frac{-0.511+0.309z^{-1}}{1-0.873z^{-1}+0.6z^{-2}}$$

2. The specification of the desired lowpass filter is

$$0.8 \le |H(\omega)| \le 1.0 \quad ; \quad 0 \le \omega \le 0.2\pi$$

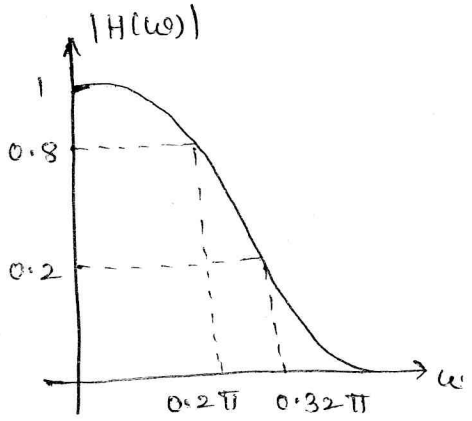
$$|H(\omega)| \le 0.2 \quad ; \quad 0.32\pi \le \omega \le \pi$$

Design chebyschev digital filter using bilinear transformation.

Sol)

Given that $A_1 = 0.8$; $\omega_1 = 0.2\pi$

$A_2 = 0.2$; $\omega_2 = 0.32\pi$



using Bilinear transformation

$$\frac{\Omega_2}{\Omega_1} = \frac{\tan \omega_2/2}{\tan \omega_1/2} = \frac{\tan 0.16\pi}{\tan 0.1\pi} = 1.692$$

The attenuation constant

$$\epsilon = \left[\frac{1}{A_1^2} - 1\right]^{1/2} = \left[\frac{1}{0.8^2} - 1\right]^{1/2} = 0.75$$

$$N_1 = \frac{\cosh^{-1}\left\{\frac{1}{\epsilon}\left[\frac{1}{A_2^2} - 1\right]^{1/2}\right\}}{\cosh^{-1}\frac{\Omega_2}{\Omega_1}} = \frac{\cosh^{-1}\left\{\frac{1}{0.75}\left[\frac{1}{0.2^2} - 1\right]^{1/2}\right\}}{\cosh^{-1} 1.692}$$

$$= \frac{\cosh^{-1}(6.532)}{1.1174} = \frac{2.5639}{1.1174} = 2.295$$

Choose the order N of the filter, $N \geq N_1$

let the order of filter, $\boxed{N = 3}$

The analog cut off frequency

$$\Omega_c = \frac{2}{T} \frac{\tan \omega_1/2}{\left[\frac{1}{A_1^2} - 1\right]^{\frac{1}{2N}}}$$

Assume $T = 1$ sec

$$\Omega_c = 2 \frac{\tan 0.1\pi}{\left[\frac{1}{0.8^2} - 1\right]^{1/6}} = \frac{0.6498}{0.9086} = 0.715$$

For the odd values of N, the transfer function of analog filter is

$$Ha(s) = \frac{B_0 \Omega_c}{s + C_0 \Omega_c} \prod_{k=1}^{\frac{N-1}{2}} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + C_k \Omega_c^2}$$

Here $N = 3$, $k = 1$

$$\therefore Ha(s) = \frac{B_0 \Omega_c}{s + C_0 \Omega_c} \times \frac{B_1 \Omega_c^2}{s^2 + b_1 \Omega_c s + C_1 \Omega_c^2}$$

$$y_N = \frac{1}{2}\left\{\left[\left(\frac{1}{\epsilon^2} + 1\right)^{1/2} + \frac{1}{\epsilon}\right]^{1/N} - \left[\left(\frac{1}{\epsilon^2} + 1\right)^{1/2} + \frac{1}{\epsilon}\right]^{-1/N}\right\}$$

$$= \frac{1}{2}\left\{\left[\left(\frac{1}{0.75^2} + 1\right)^{1/2} + \frac{1}{0.75}\right]^{1/3} - \left[\left(\frac{1}{0.75^2} + 1\right)^{1/2} + \frac{1}{0.75}\right]^{-1/3}\right\}$$

$= 0.7211 - 0.3467 = 0.3744$

$\boxed{Y_N = 0.3744}$

$C_0 = Y_N = 0.3744$

$\boxed{C_0 = 0.3744}$

$C_k = Y_N^2 + \cos^2 \frac{(2k-1)\pi}{2N}$

when $k=1$,

$C_1 = Y_N^2 + \cos^2 \left(\frac{\pi}{6}\right)$

$= 0.3744^2 + \cos^2(\pi/6) = 0.8902$

$\boxed{C_1 = 0.8902}$

$b_k = 2Y_N \sin \frac{(2k-1)\pi}{2N}$

$b_1 = 2 \times 0.3744 \sin\left(\frac{\pi}{6}\right) = 2 \times 0.3744 \times 1/2 = 0.3744$

$\boxed{b_1 = 0.3744}$

$$\therefore H_a(s) = \frac{0.715 B_0}{S + 0.3744 \times 0.715} \times \frac{(0.715)^2 B_1}{s^2 + 0.3744 \times 0.715 S + 0.8902 \times 0.715^2}$$

$$= \frac{0.3655 B_0 B_1}{(s+0.2677)(s^2 + 0.2677s + 0.4551)}$$

when $s=0$, $H_a(0) = \frac{0.3655 B_0 B_1}{0.2677 \times 0.4551} = 3 B_0 B_1$

let $H_a(0) = 1$

$3 B_0 B_1 = 1 \Rightarrow B_0 B_1 = 1/3$

let $B_0 = B_1$

$B_0^2 = 1/3 = 0.333$

$B_0 = 0.577$

$B_0 = B_1 = 0.577$

$$H_a(s) = \frac{0.3655 \times 0.577^2}{(s+0.2677)(s^2+0.2677s+0.4551)}$$

$$H_a(s) = \frac{0.1217}{(s+0.2677)(s^2+0.2677s+0.4551)}$$

For bilinear transformation put $s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

Assume $T = 1$ sec

$$H(z) = \frac{0.1217}{\left(2\frac{1-z^{-1}}{1+z^{-1}}+0.2677\right)\left(4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+0.2677\times 2\frac{1-z^{-1}}{1+z^{-1}}+0.4551\right)}$$

$$H(z) = \frac{0.1217(1+z^{-1})^3}{2(1-z^{-1})+0.2677(1+z^{-1})\left[4(1-z^{-1})^2+0.2677\times 2(1-z^{-2})+0.4551(1+z^{-1})^2\right]}$$

$$H(z) = \frac{0.1217(1+z^{-1})^3}{\left[2-2z^{-1}+0.2677+0.2677z^{-1}\right]\left[4+4z^{-2}-8z^{-1}+0.5354-0.5354z^{-2}+0.4551+0.4551z^{-2}+0.9102z^{-1}\right]}$$

$$H(z) = \frac{0.1217(1+z^{-1})^3}{(2.2677-1.7323z^{-1})(4.9905+3.9197z^{-2}-7.0898z^{-1})}$$

$$H(z) = \frac{0.1217(1+z^{-1})^3 / 2.2677\times 4.9905}{(1-0.7639z^{-1})(1+0.7854z^{-2}-1.4206z^{-1})}$$

$$H(z) = \frac{0.01075(1+z^{-1})^3}{(1-0.7639z^{-1})(1+0.7854z^{-2}-1.4206z^{-1})}$$