UNIT- III — ch. Murali Krishna

* DESIGN OF IIR DIGITAL FILTERS & REALIZATIONS *

Review of analog filters:-

1) Few parameters related to analog filters :-

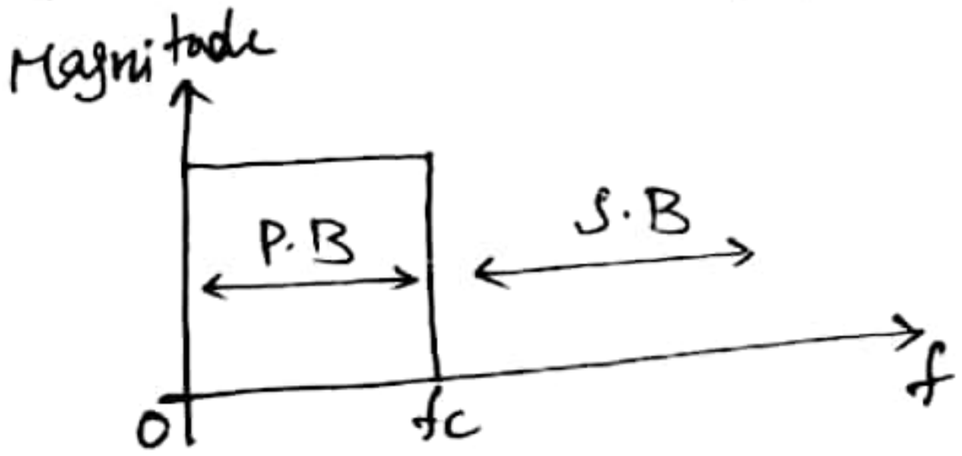
a). pass band : It passes certain range of frequencies.
In pass band, attenuation is zero

b). stop band : It suppresses certain range of frequencies.
In stop band, attenuation is infinity.

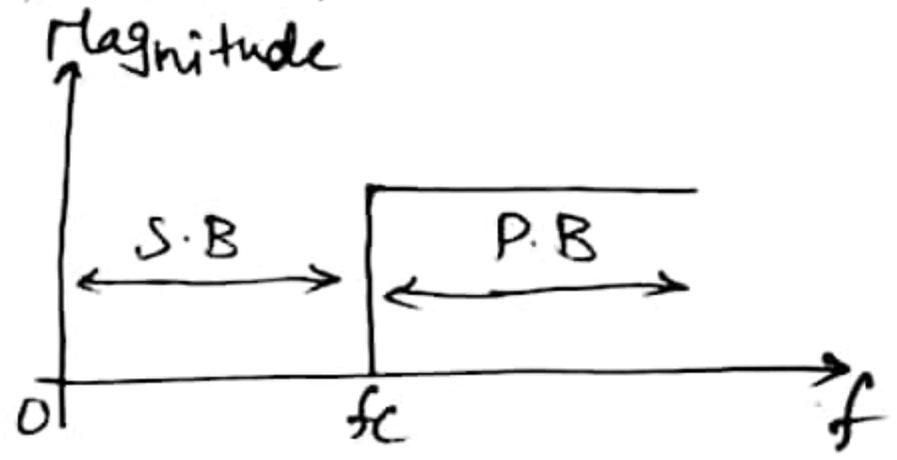
c) Cut off frequency: This is the frequency which seperates Pass band and stopband.

2). Classification of Analog filters:-

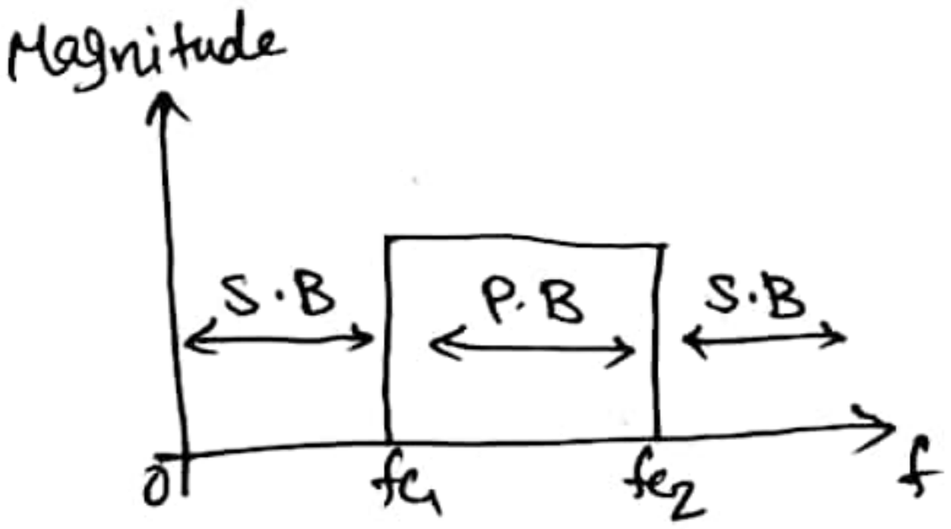
i). Low Pass filter (LPF)



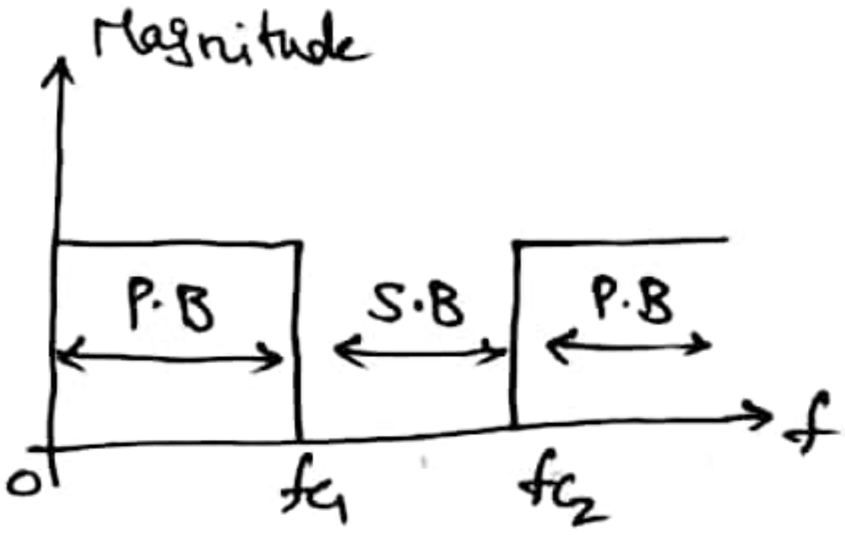
ii). High pass filter (HPF)



iii) Band pass filter (BPF)



iv). Band Reject filter.



v). All pass filter (APF)

It passes all frequencies. By using this filter, the phase of input signal can be modified.



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Filtering Concept :-

Analog fillers designed using analog components like resistors (R), Capacitors (C) and inductors (L). on otherhand, digital fillers are implemented using difference equations.

Advantages of Digital filter :-

1). Many input signals can be filtered by one digital filter without replacing hardware.

2). Unlike analog filters, digital filters are portable.

3). The digital filters are highly flexible.

4). The performance of digital filters doesnot vary with environmental parameters.

5). Digital fillers can be used at low frequencies, like Biomedical applications.

6). Multiple filtering is possible only in digital filter.

7). using VLSI technology, the hardware ~~and~~ of digital fillers can be reduced similarly power consumption can be reduced.

Disadvantages of Digital fillers :-

① Speed limitation :- In case of digital fillers, ADC & DAC are used. SO speed of digital filter depends on conversion time of ADC & Settling time of DAC.

② Finite wordlength effect :- The accuracy of digital filter depends on the wordlength used to encode them in binary form. Wordlength should be long enough to obtain the required accuracy.

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Introduction:-

The IIR filters design involves design of digital filters in the analog domain and Transforming the design into the digital domain.

The system function describing an analog filter may be written as

$$H_a(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} \longrightarrow (1)$$

Where a_k & b_k are filter coefficients.

The system function $H(s)$ can be described by linear constant coefficient differential equation.

i.e.. $$\sum_{k=0}^{N} a_k \cdot \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \longrightarrow (2)$$

where $x(t)$ = input signal
$y(t)$ = output signal

Design of IIR digital filters from analog filters:-

There are several techniques described for converting an analog filter into digital filter. If the conversion techniques are to be effective, it should possesses the following properties:

1). The $j\Omega$-axis in the S-plane should map into the unit circle in the Z-plane. This gives a direct relationship between the two frequency variables in the two domains.

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ii). The left half of the S-plane should map into the inside of unit circle in the Z-plane. Thus, a stable analog filter will be converted into a stable digital filter.

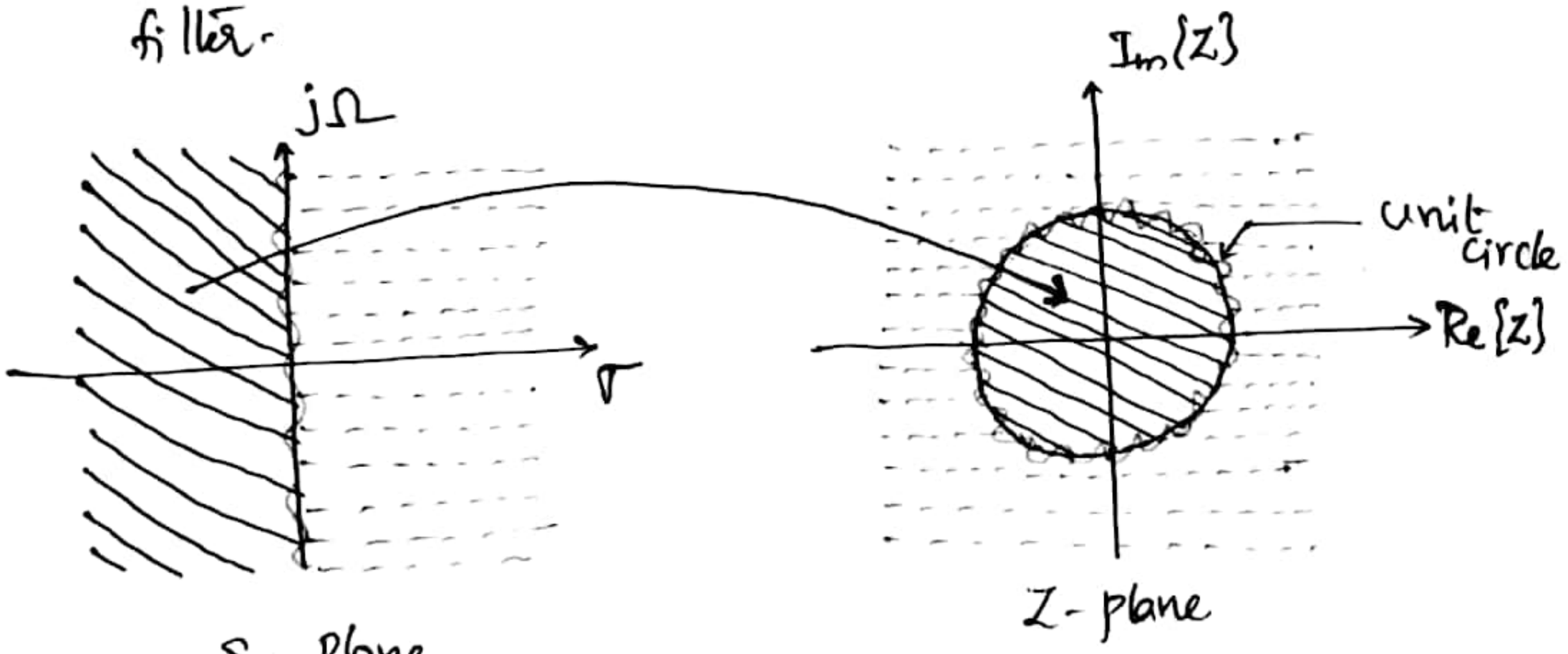


Fig:- Mapping from S-plane to Z-plane.

There are three methods for transforming the filter into digital domain. They are

i). Approximation of derivatives
ii). Impulse Invariance Method [IIM]
iii) Bilinear Transformation technique.

i). Approximation of derivatives:-

The system function of digital filter can be

as $$H(Z) = H_a(S)\Big|_{S=\frac{1-Z^{-1}}{T}}$$

where T = sampling interval.

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Eg:- Make use of the backwald differential for. the'.. derivative to convert the analog LPF having system function given below:

a) $H_a(S) = \frac{1}{S+2}$

Sol:- The backwald difference formula is

$$H(Z) = H_a(S)\Big|_{S=\frac{1-\bar{Z}^1}{T}}$$

$$\therefore H(Z) = \frac{1}{S+2}\Big|_{S=\frac{1-\bar{Z}^1}{T}}$$

$$= \frac{1}{\left[\frac{1-\bar{Z}^1}{T}\right]+2}$$

$$H(Z) = \frac{T}{1-\bar{Z}^1+2T}$$

If $T = 1$ sec then

$$H(Z) = \frac{1}{1-\bar{Z}^1+2} = \frac{1}{3-\bar{Z}^1}$$

$= 0 =$

b). $H(S) = \frac{1}{(S+0.1)^2+9}$

Sol:-

$$H(Z) = \frac{1}{(S+0.1)^2+9}\Big|_{S=\frac{1-\bar{Z}^1}{T}}$$

$$= \frac{1}{\left[\frac{1-\bar{Z}^1}{T}+0.1\right]^2+9}$$

$$H(Z) = \frac{T^2}{\left[1-\bar{Z}^1+0.1T\right]^2+9T^2}$$

If $T = 1$ sec then

$$H(Z) = \frac{1}{\left[1-\bar{Z}^1+0.1\right]^2+9}$$

$$= \frac{1}{1.21-2.2\bar{Z}^1+\bar{Z}^2+9}$$

$$\Rightarrow H(Z) = \frac{1}{\bar{Z}^2-2.2\bar{Z}^1+10.21}$$

$= 0 =$

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ii). IIR filter design by "Impulse Invariance Method":

In this method, we following different notations:

$h_a(t)$ = Impulse response in time domain

$H_a(S)$ = Transfer function of analog filter in S-domain

$h_a(nT)$ = Sampled version of $h_a(t)$, i.e. $t = nT$

Where T = Sampling interval

$H(Z)$ = Z-transform of $h(nT)$, response of digital filter

Ω = analog frequency

ω = digital frequency.

The system function of analog filter can be expressed as

$$H_a(S) = \frac{A_1}{S-P_1} + \frac{A_2}{S-P_2} + \cdots + \frac{A_N}{S-P_N}$$

$$\Rightarrow H_a(S) = \sum_{i=1}^{N} \frac{A_i}{S-P_i} \longrightarrow (1)$$

Here A_i = filter coefficients & P_i = Poles

Take Inverse Laplace transform on both sides,

$$h_a(t) = \sum_{i=1}^{N} A_i e^{P_i t} \longrightarrow (2)$$

$h(n)$ can be obtained by replacing 't' with nT

$$h(n) = \sum_{i=1}^{N} A_i e^{P_i nT} \longrightarrow (3)$$

The system transfer function of digital filter is

$$H(Z) = \sum_{n=0}^{\alpha} h(n) Z^{-n}$$

$$H(Z) = \sum_{n=0}^{\alpha} \left[\sum_{i=1}^{N} A_i e^{P_i nT} \right] Z^{-n} \quad [\because \text{from } (3)]$$

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Interchanging the order of summation,

$$H(Z) = \sum_{i=1}^{N} A_i \left[\sum_{n=0}^{\alpha} e^{P_i n T} \cdot Z^{-n} \right] = \sum_{i=1}^{N} A_i \left[\sum_{n=0}^{\alpha} \left(e^{P_i T} Z^{-1} \right)^n \right]$$

$$\Rightarrow H(Z) = \sum_{i=1}^{N} A_i \cdot \frac{1}{1 - e^{P_i T} Z^{-1}} \qquad \left[\because \sum_{n=0}^{\alpha} a^n = \frac{1}{1-a} \right]$$

$\longrightarrow$ (4)

This is the required transfer function of digital filter.

Comparing eqns (1) & (4),

$$\frac{1}{S - P_i} \longrightarrow \frac{1}{1 - e^{P_i T} Z^{-1}} \longrightarrow (5)$$

Equation (5) describes how poles from analog domain are transfered to digital domain. This transformation of poles is called "Mapping of poles".

Some standard expressions:-

a). $\frac{1}{S - P_i} \longrightarrow \frac{1}{1 - e^{P_i T} Z^{-1}} \longrightarrow (6)$

b). $\frac{S+a}{(S+a)^2 + b^2} \longrightarrow \frac{1 - e^{-aT} \cdot \cos bT \cdot Z^{-1}}{1 - 2e^{-aT} \cdot \cos bT \cdot Z^{-1} + e^{-2aT} \cdot Z^{-2}} \longrightarrow (7)$

c). $\frac{b}{(S+a)^2 + b^2} \longrightarrow \frac{e^{-aT} \cdot \sin bT \cdot Z^{-1}}{1 - 2e^{-aT} \cdot \cos bT \cdot Z^{-1} + e^{-2aT} \cdot Z^{-2}} \longrightarrow (8)$

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Designing steps for IIM:-

Step-1:- $H_a(s)$ will be given. If not given, obtain $H_a(s)$ from given specifications.

Step-2:- Expand $H_a(s)$ by using partial fraction method

Step-3:- obtain Z-Transform for each partial fraction expansion term using IIM [eqns ⑥, ⑦, ⑧]

Step-4:- obtain $H(z)$, this is the required digital filter.

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Eg:- obtain $H(z)$ using IIM. Take $T=1s$ for the given transfer function.

$$H_a(s) = \frac{1}{(s+1)(s+2)}$$

Sol:- Given that,

$$H_a(s) = \frac{1}{(s+1)\cdot(s+2)}$$

using partial fraction method,

$$H_a(s) = \frac{1}{s+1} - \frac{1}{s+2} = \frac{1}{s-(-1)} - \frac{1}{s-(-2)}$$

using eqn ⑥,

$$H(z) = \frac{1}{1-e^{-T}z^{-1}} - \frac{1}{1-e^{-2T}z^{-1}}$$

Substitute $T=1$,

$$H(z) = \frac{1}{1-e^{-1}z^{-1}} - \frac{1}{1-e^{-2}z^{-1}} = \frac{1}{1-0.368z^{-1}} - \frac{1}{1-0.135z^{-1}}$$

$$\therefore H(z) = \frac{0.233z^{-1}}{1-0.503z^{-1}+0.0497z^{-2}}$$

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Limitations of IIM:-

The IIM is Many-to-one mapping i.e., many Points in S-plane are mapped to a single point in the Z-plane. Therefore, infinite number of S-plane Poles must have the same real and imaginary parts that are diffed by multiple of $\frac{2\pi}{T}$, which is the draw back of IIM.

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iii). IIR filter design by "Bilinear transformation method":-

The IIR filter design methods i) Approximation of derivatives and ii) IIM are appropriate for LPF & BPF whose resonant frequencies are low. These techniques are not suitable for HPF & BRF. This limitation is overcome in the mapping technique is called "Bilinear transformation". [BLT].

→ This is a one-to-one mapping from S-domain to Z-domain

→ It is a conformal mapping that transforms the jΩ axis into the unit circle in the Z-plane, thus avoiding aliasing of frequency components.

→ The transformation of a stable analog filter results a stable digital filter.

→ The transformation is obtained by using the trapezoidal formula for numerical integration.

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Let system function of analog filter is

$$H_a(s) = \frac{b}{s+a} \longrightarrow (1)$$

The differential equation can be obtained as

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b}{s+a}$$

$$\Rightarrow s.Y(s) + a.Y(s) = b\,X(s)$$

taking inverse laplace transform,

$$\frac{d}{dt}y(t) + a.y(t) = b.x(t)$$

Integrate the above equation between $(nT-T)$ & nT.

$$\int_{nT-T}^{nT} \frac{d}{dt}y(t)dt + a.\int_{nT-T}^{nT} y(t)\,dt = b\int_{nT-T}^{nT} x(t)\,dt \longrightarrow (2)$$

Trapezoidal rule for numeric integration:

$$\int_{nT-T}^{nT} a(t)\,dt = \frac{T}{2}\left[a(nT) + a(nT-T)\right] \longrightarrow (3)$$

using (2) & (3),

$$y(nT) - y(nT-T) + a.\frac{T}{2}\left[y(nT)+y(nT-T)\right] = b.\frac{T}{2}\left[x(nT)+x(nT-T)\right]$$

Taking Z-transform, after simplifications, the system function expressed as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+a} \longrightarrow (4)$$

Comparing eqns (1) & (4), we get

$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] = \frac{2}{T}\left[\frac{z-1}{z+1}\right] \longrightarrow (5)$$

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We know that, $Z = re^{j\omega}$ substitute in eqn ⑤,

$$S = \frac{2}{T}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right]\left[\frac{re^{-j\omega}+1}{re^{-j\omega}+1}\right]$$

$$= \frac{2}{T}\left[\frac{r^2+re^{j\omega}-re^{-j\omega}-1}{r^2+re^{j\omega}+re^{-j\omega}+1}\right]$$

$$= \frac{2}{T}\left[\frac{r^2+r\cdot 2j\sin\omega-1}{r^2+r\cdot 2\cos\omega+1}\right] \qquad \left[\because \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j},\ \cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}\right]$$

$$\Rightarrow S = \frac{2}{T}\left[\frac{r^2-1}{r^2+2r\cdot\cos\omega+1} + j\cdot\frac{2r\sin\omega}{r^2+2r\cos\omega+1}\right] \longrightarrow ⑥$$

W.K.T, $S = \sigma + j\Omega$

Here $\sigma = \frac{2}{T}\left[\frac{r^2-1}{r^2+2r\cdot\cos\omega+1}\right] \rightarrow ⑦$ & $\Omega = \frac{2}{T}\left[\frac{2r\sin\omega}{r^2+2r\cos\omega+1}\right] \rightarrow ⑧$

From eqn ⑦, If $r<1$ then $\sigma<0$ & If $r>1$ then $\sigma>0$. Hence the left half of the S-plane maps onto points inside the unit circle in the Z-plane and it results in a stable digital system.

For unit magnitude ($r=1$), $\sigma=0$. In this case from eqn ⑧,

$$\Omega = \frac{2}{T}\cdot\left[\frac{2\sin\omega}{1+2\cos\omega+1}\right] = \frac{2}{T}\left[\frac{\sin\omega}{1+\cos\omega}\right] = \frac{2}{T}\left[\frac{2\sin\omega/2\cos\omega/2}{2\cos^2\omega/2}\right]$$

$$\Rightarrow \boxed{\Omega = \frac{2}{T}\cdot\tan\omega/2} \quad (or) \quad \boxed{\omega = 2\cdot\tan^{-1}\frac{\Omega T}{2}} \longrightarrow ⑨$$

Eqn ⑨ represents the relationship between frequencies in two domains as shown in fig. ⑥

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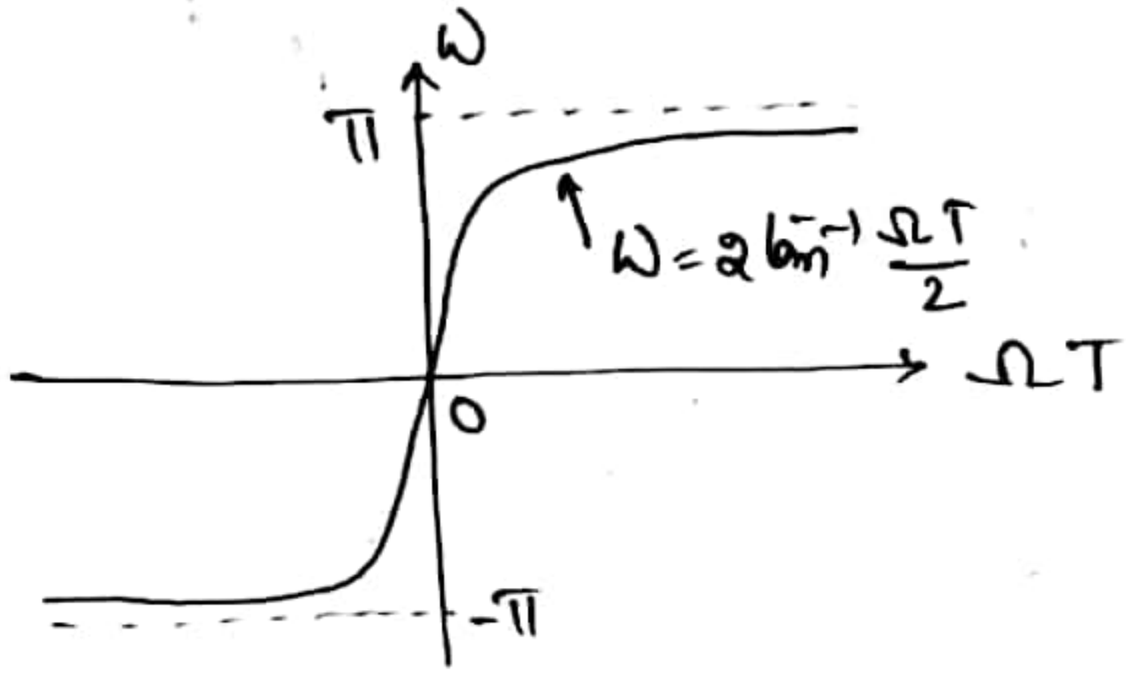


Fig :- Mapping between 'Ω' & 'ω' in BLT

From, fig, the entire range in 'Ω' is mapped only once into range; $-\pi \le \omega \le \pi$. But the mapping is non-linear and the lower frequencies in analog domain are expanded in the digital domain, whereas the higher frequencies are compressed. This is due to non-linearity of arc tangent function and is called "Frequency Warping".

Impulse Invariance Method	Bilinear Transformation method
①. Poles are transformed by using the expression: $\frac{1}{S-P_i} \rightarrow \frac{1}{1-e^{P_i T} z^{-1}}$	①. Poles are transformed by using the expression: $S = \frac{2}{T}\left[\frac{z-1}{z+1}\right]$
②. Mapping is Many-to-one.	②. Mapping is one-to-one.
③. Aliasing effect is present.	③. Aliasing effect is not present.
④. Suitable for LPF & BPF	④. Suitable for HPF & BRF
⑤. only poles of the system can be mapped.	⑤. Poles as well as Zeroes can be mapped.
⑥. No frequency warping effect	⑥. Frequency warping effect is present.

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Eg:- Convert the analog filter with system function $H(s) = \frac{s+0.1}{(s+0.1)^2+9}$ into a digital IIR filter by using BLT. The digital filter should have a resonant frequency if $\omega_r = \pi/4$.

Sol:- Given that, $H(s) = \frac{s+0.1}{(s+0.1)^2+9}$

Using BLT, $H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} = 7.246\left[\frac{z-1}{z+1}\right]$

We know that, $\Omega = \frac{2}{T}\tan\left(\frac{\omega_r}{2}\right) \Rightarrow T = \frac{2}{\Omega}\tan(\omega/2)$

Here $\Omega = 3$, $\omega = \pi/4$

$$T = \frac{2}{3}\tan\left(\pi/8\right) = 0.276 \text{ sec}.$$

$$\therefore H(z) = \frac{7.246\left(\frac{z-1}{z+1}\right)+0.1}{\left[\left[7.246\left(\frac{z-1}{z+1}\right)+0.1\right]^2+9\right]}$$

After simplification,

$$H(z) = \frac{1+0.027z^{-1}-0.973z^{-2}}{8.572-11.84z^{-1}+8.177z^{-2}}$$

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Eg:- An Analog filter has the following transfer function $H_a(s) = \frac{1}{s+1}$. Using BLT Technique, Determine the transfer function of digital filter $H(z)$ and also write the differential equation of digital filter.

Sol:- Given that, $H_a(s) = \frac{1}{s+1}$

Using BLT, $H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} = \frac{1}{\frac{2}{T}\left(\frac{z-1}{z+1}\right)+1}$

Let $T = 1$ sec,

$$H(z) = \frac{1}{2\left(\frac{z-1}{z+1}\right)+1} = \frac{z+1}{2(z-1)+z+1} = \frac{z+1}{3z-1}$$

This is the required transfer function of a digital filter.

Now..

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z+1}{3z-1}$$

$$\Rightarrow (3z-1).Y(z) = (z+1).X(z)$$

$$\Rightarrow 3z.Y(z) - Y(z) = z.X(z) + X(z)$$

$$\Rightarrow 3.Y(z) - z^{-1}.Y(z) = X(z) + z^{-1}.X(z)$$

Taking Inverse Z-transforms on both sides,

$$3.y(n) - y(n-1) = x(n) + x(n-1)$$

$$\Rightarrow y(n) = \frac{1}{3}\left[x(n) + x(n-1) + y(n-1)\right]$$

This is the differential equation of a digital filter.

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Eg:- Design a single pole LPF with a 3dB bandwidth of 0.2π by use of BLT applied to the analog filter.

$H_a(s) = \frac{\Omega_c}{\Omega_c + s}$; where Ω_c is the 3dB BW of analog filter.

Sol:- Given that, $H_a(s) = \frac{\Omega_c}{s+\Omega_c}$ and $\omega_c = 0.2\pi$

W.K.T, $\Omega_c = \frac{2}{T}\cdot\tan\left(\frac{\omega_c}{2}\right) = \frac{2}{T}\cdot\tan\left(\frac{0.2\pi}{2}\right) = \frac{0.65}{T}$

$\therefore H_a(s) = \frac{0.65/T}{s + \frac{0.65}{T}}$

using BLT method,

$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} = \frac{\frac{0.65}{T}}{\frac{2}{T}\left(\frac{z-1}{z+1}\right) + \frac{0.65}{T}}$

$\Rightarrow H(z) = \frac{0.65}{2\left(\frac{z-1}{z+1}\right) + 0.65} = \frac{0.65(z+1)}{2.65z - 1.35}$

$\Rightarrow H(z) = \frac{0.245(z+1)}{z - 0.509}$

This is the required transfer function of a digital filter and there is a single pole at 0.509.

In given problem, frequency of digital filter is $\omega_c = 0.2\pi$ and bandwidth is ~~30~~ 3dB, so need to check the frequency response.

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The frequency response can be obtained by substituting $Z = e^{j\omega}$ in transfer function

$$H(\omega) = H(Z)\Big|_{Z=e^{j\omega}} = \frac{0.245(e^{j\omega}+1)}{e^{j\omega}-0.509}$$

If $\omega = 0$:-

$$H(0) = \frac{0.245(e^{0}+1)}{e^{0}-0.509} = \frac{0.245(1+1)}{1-0.509} = 0.9979 \approx 1$$

If $\omega = 0.2\pi$:-

$$H(0.2\pi) = \frac{0.245(e^{j0.2\pi}+1)}{e^{j0.2\pi}-0.509} = \frac{0.245\left[\cos 0.2\pi + j\sin 0.2\pi + 1\right]}{\cos 0.2\pi + j\sin 0.2\pi - 0.509}$$

$$\Rightarrow H(0.2\pi) = \frac{0.245(1.809 + j0.589)}{0.3 + j0.587} = 0.5 - 0.5j$$

$$\Rightarrow |H(0.2\pi)| = 0.707 = 3\,dB$$

Hence, the desired response is obtained

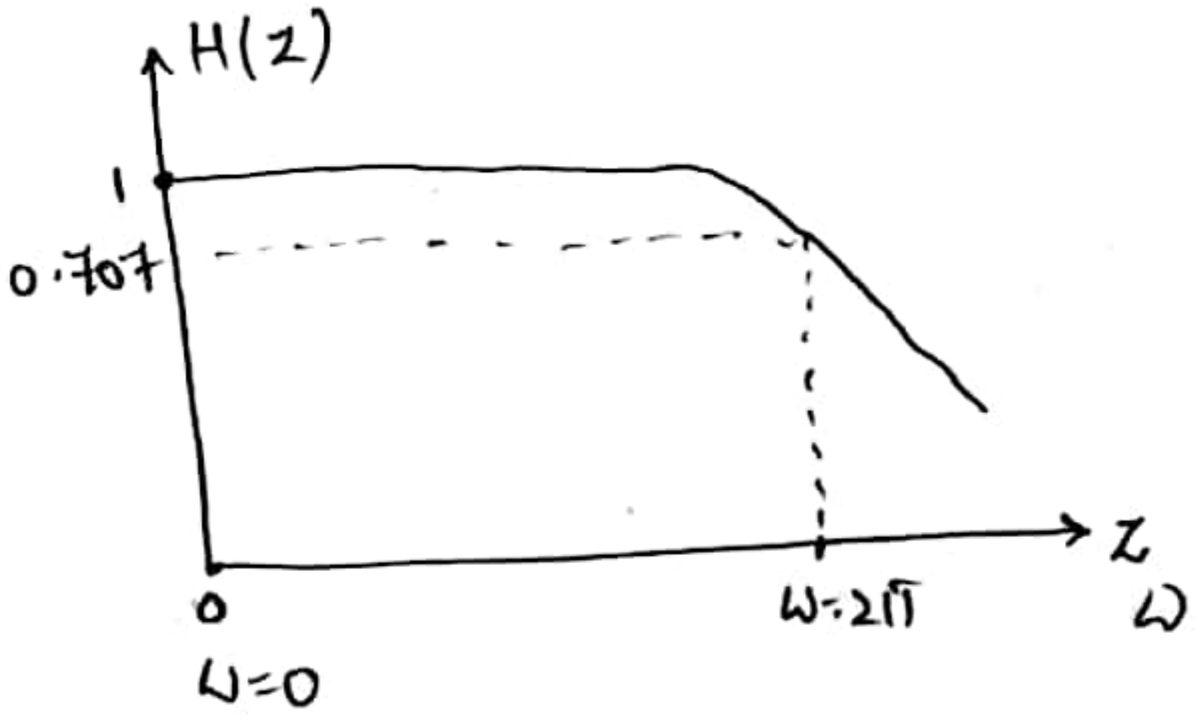


Fig:- Frequency response of given filter.

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Eg:- The analog transfer function of a LPF is $H_a(s) = \frac{1}{s+2}$ and its bandwidth is 1 rad/sec. Design the digital filter using BLT method where cutoff frequency is 20π and sampling time is 0.0167sec by Considering the Pre-Warping effect.

Sol:- Given that, $H_a(s) = \frac{1}{s+2}$ & $\omega_c = 20\pi$, $T = 0.0167s$

For the pre warping technique,

$$\Omega_c^* = \frac{2}{T}\tan\left(\frac{\omega_c T}{2}\right) = \frac{2}{0.0167}\tan\left[\frac{20\pi \times 0.0167}{2}\right] = 69.31$$

The value of $H_a^*(s)$ can be obtained by substituting $s = \frac{s}{\Omega_c^*}$, we get

$$H_a^*(s) = \frac{1}{\frac{s}{\Omega_c^*}+2} = \frac{1}{\frac{s}{69.31}+2} = \frac{69.31}{s+138.62}$$

By using BLT,

$$H(z) = H_a^*(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} = \frac{69.31}{\frac{2}{T}\left[\frac{z-1}{z+1}\right]+138.62}$$

Put $T = 0.0167s$,

$$H(z) = \frac{69.31(z+1)}{119.76(z-1)+138.62(z+1)}$$

$$\Rightarrow H(z) = \frac{z+1}{3.13z+0.272}$$

This is the required transfer function of a digital filter.

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Analog filter Approximations:-

In general IIR digital filters are designed from the analog filter. Many times, it is necessary to approximate the characteristics of analog filter. So this approximation is required because the practical characteristics of a filter is identical to the ideal characteristics.

There are three different approximation techniques:

1). Butterworth filter approximation
2). Chebyshev filter approximation
3). Elliptic filter approximation.

1). Butterworth filter Approximation:-

The characteristic of a butterworth LPF is as shown in fig. This type of response is called as "Butterworth response" because its main characteristic is that the passband is maximally flat. This means that there are no variations (ripples) in the passband.

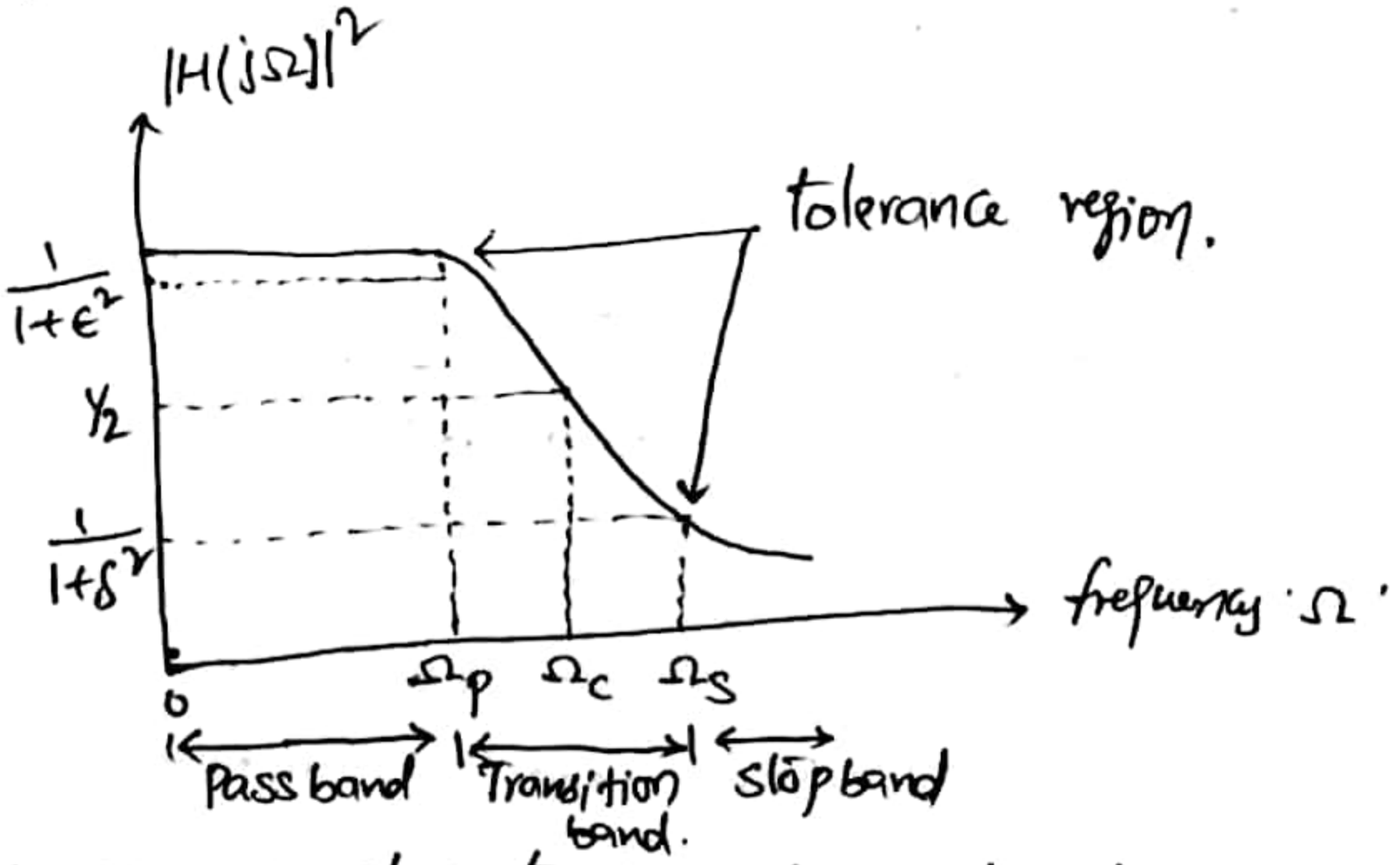


Fig:- Typical characteristic of analog LPF.

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Now, the magnitude squared response of low pass ~~filter~~ butterworth filter is given by

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}} \longrightarrow (1)$$

It can also expressed as

$$|H(j\Omega)|^2 = \frac{1}{1+\epsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}} \longrightarrow (2)$$

where, $|H(j\Omega)|$ = Magnitude of analog LPF

Ω_c = Cutoff frequency (3-dB frequency)

Ω_p = Passband edge frequency

Ω_s = Stopband edge frequency

$1+\epsilon^2$ = passband edge value

$1+\delta^2$ = Stopband edge value

ϵ = Parameter related to ripples in the Passband

δ = parameter related to ripples in the stopband

N = Order of the filter.

Ideal characteristic of LPF is shown by dotted line in figure. Ideally, at the value of Ω_c, the frequencies should be stopped. However, in practical cases this is not happening.

Now, 'N' represents order of filter it means no. of stages used in analog filter design. As 'N' increases, the response of filter is more close to ideal response as shown in fig (10)

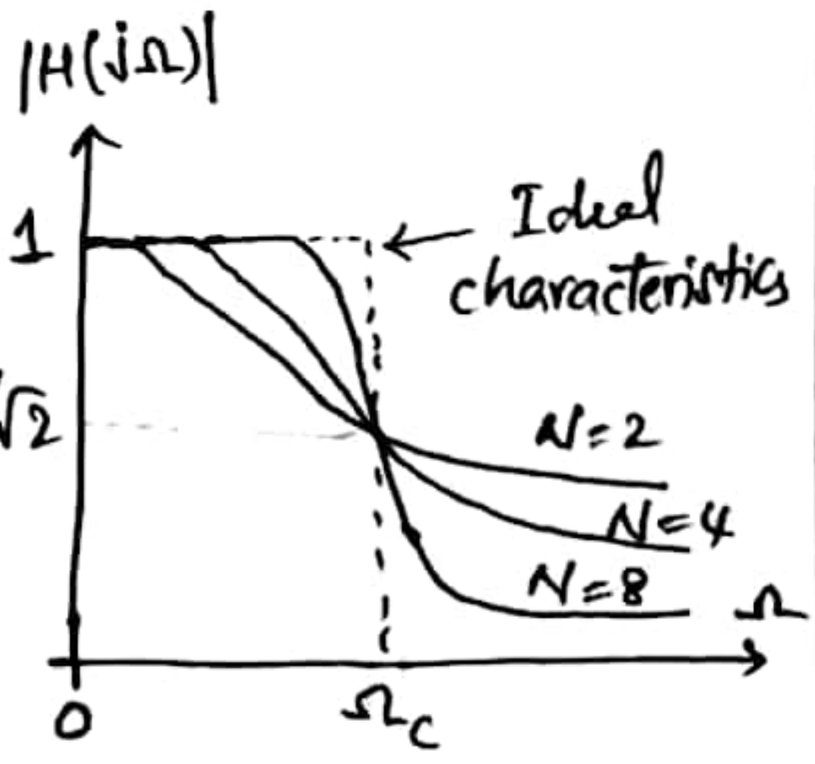


Fig:- Effect of 'N' on frequency response characteristics.

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Salient features of Low pass butterworth filter:-

→ The Magnitude response is nearly constant at lower frequencies. This means passband is maximally flat.

→ There are no ripples in the passband & stopband.

→ The Maximum gain occurs at $\Omega = 0$ i.e. $|H(0)| = 1$

→ The Magnitude response is monotonically decreasing.

Designing equations & Designing steps:-

Let A_p = Attenuation in passband ; Ω_p = Passband edge frequency

A_s = Attenuation in stopband ; Ω_s = Stopband edge frequency

Ω_c = Cutoff frequency.

From the given specifications of a digital filter, we obtain analog filter as under:

1). For Impulse Invariance method, $\Omega = \frac{\omega}{T}$

For BLT method, $\Omega = \frac{2}{T} \tan\frac{\omega}{2}$

Here Ω = frequency of analog filter

ω = frequency of digital filter

T = Sampling time.

2). Evaluate the order of filter 'N',

$$N = \frac{1}{2} \cdot \frac{\log\left[\frac{\left(\frac{1}{A_s^2} - 1\right)}{\left(\frac{1}{A_p^2} - 1\right)}\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

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If specifications are given in dB then

$$N = \frac{1}{2} \cdot \frac{\log\left[\frac{10^{0.1A_s(dB)}-1}{10^{0.1A_p(dB)}-1}\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

3). Now, determine the cutoff frequency, Ω_c.

For IIM, $\Omega_c = \frac{\omega_c}{T}$

For BLT, $\Omega_c = \frac{2}{T}\tan\frac{\omega_c}{2}$

If 'ω_c' is not given, $\Omega_c = \frac{\Omega_p}{\left[\frac{1}{A_p^2}-1\right]^{1/2N}}$

If Specifications are in dB then $\Omega_c = \frac{\Omega_p}{\left[10^{0.1A_p}-1\right]^{1/2N}}$

4). Next, we determine the poles using following expressions:

$$P_i = \pm \Omega_c \cdot e^{j(N+2i+1)\pi/2N}, \quad i = 0, 1, 2, \ldots N-1$$

If the poles are complex conjugate then poles as complex conjugate pairs that means S_1 & S_1^*, S_2 & S_2^* etc.,

[$\because a+ib, a-ib$]

5). Now, Calculate the system transfer function of analog filter using following expressions:

$$H_a(S) = \frac{\Omega_c^N}{(S-P_1)(S-P_2)\ldots(S-P_N)}$$

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If Poles are complex conjugate then

$$H_a(s) = \frac{\Omega_c^N}{(s-s_1)(s-s_1^*)(s-s_2)(s-s_2^*)}$$

c). Finally, we design the digital filter using IIM (or) BLT method.

Eg:- using BLT, Design a butterworth filter which satisfies following conditions :

$$0.8 \le |H(e^{j\omega})| \le 1 \quad ; \quad 0 \le \omega \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2 \quad ; \quad 0.6\pi \le \omega \le \pi$$

Sol:- Given that,

$$0.8 \le |H(e^{j\omega})| \le 1 \quad ; \quad 0 \le \omega \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2 \quad ; \quad 0.6 \le \omega \le \pi$$

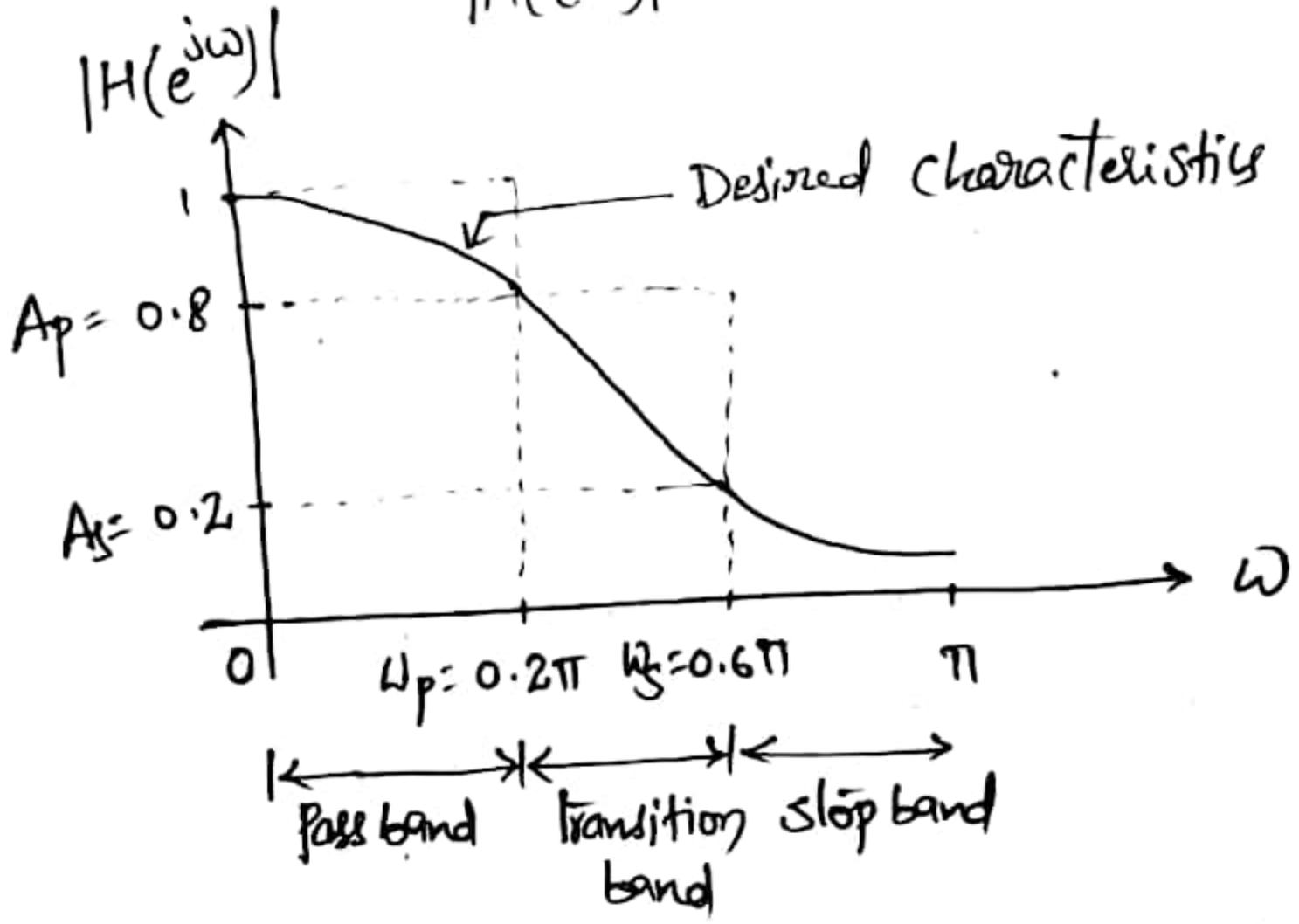


Fig:- Characteristic plot for given filter.

Given that, $A_p = 0.8$ $\omega_p = 0.2\pi$

$A_s = 0.2$ $\omega_s = 0.6\pi$

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Step-1:- Determine analog filter specifications, Ω_p & Ω_s:

We have to use BLT, Hence

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2} = 2\tan\left(\frac{0.2\pi}{2}\right) = 0.65 \qquad [\because \text{Assume } T = 1\text{sec}]$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2} = 2\tan\left(\frac{0.6\pi}{2}\right) = 2.753$$

Step-2:- Determine the order of filter, N.

$$N = \frac{1}{2}\cdot\frac{\log\left[\left(\frac{1}{A_s^2}-1\right)\Big/\left(\frac{1}{A_p^2}-1\right)\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{1}{2}\cdot\frac{\log\left[\frac{\left(\frac{1}{0.2^2}-1\right)}{\left(\frac{1}{0.8^2}-1\right)}\right]}{\log\left(\frac{2.753}{0.65}\right)} = 1.30$$

Thus, approximately like $N = 2$.

Step-3:- Determine the cut off frequency, Ω_c

$$\Omega_c = \frac{\Omega_p}{\left(\frac{1}{A_p^2}-1\right)^{1/2N}} = \frac{0.65}{\left[\frac{1}{0.8^2}-1\right]^{1/2\times2}} = 0.75$$

Step-4:- Determine the poles of $H_a(s)$

$$P_i = \pm\Omega_c\cdot e^{j(N+2i+1)\pi/2N}, \quad i = 0,1,\ldots N-1$$

$$= \pm 0.75\, e^{j(2+2i+1)\cdot\pi/2\times2}, \quad i = 0,1$$

$$P_i = \pm 0.75\, e^{j(3+2i)\pi/4}$$

If $i = 0$,

$$P_0 = \pm 0.75 e^{j3\pi/4} = \pm 0.75\cdot\left[\cos\frac{3\pi}{4} + j\sin 3\pi/4\right]$$

$$= \pm 0.75\left[-0.707 + j0.707\right]$$

$$\therefore P_0 = -0.53 + j0.53 \text{ & } 0.53 - j0.53$$

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If $i=1$,

$$P_1 = \pm 0.75 \cdot e^{j 5\pi/4} = \pm 0.75\left[\cos\frac{5\pi}{4} + j\sin\frac{5\pi}{4}\right]$$

$$= \pm 0.75\left[-0.707 - j0.707\right]$$

$$\Rightarrow P_1 = -0.53 - j0.53 \quad \& \quad 0.53 + j0.53.$$

The location of poles are shown in fig.

W.K.T, the stability of system is poles are located in LHS of S-plane. Thus, we select $-0.53 + j0.53$ and $-0.53 - j0.53$. Both these are Complex Conjugate of each other.

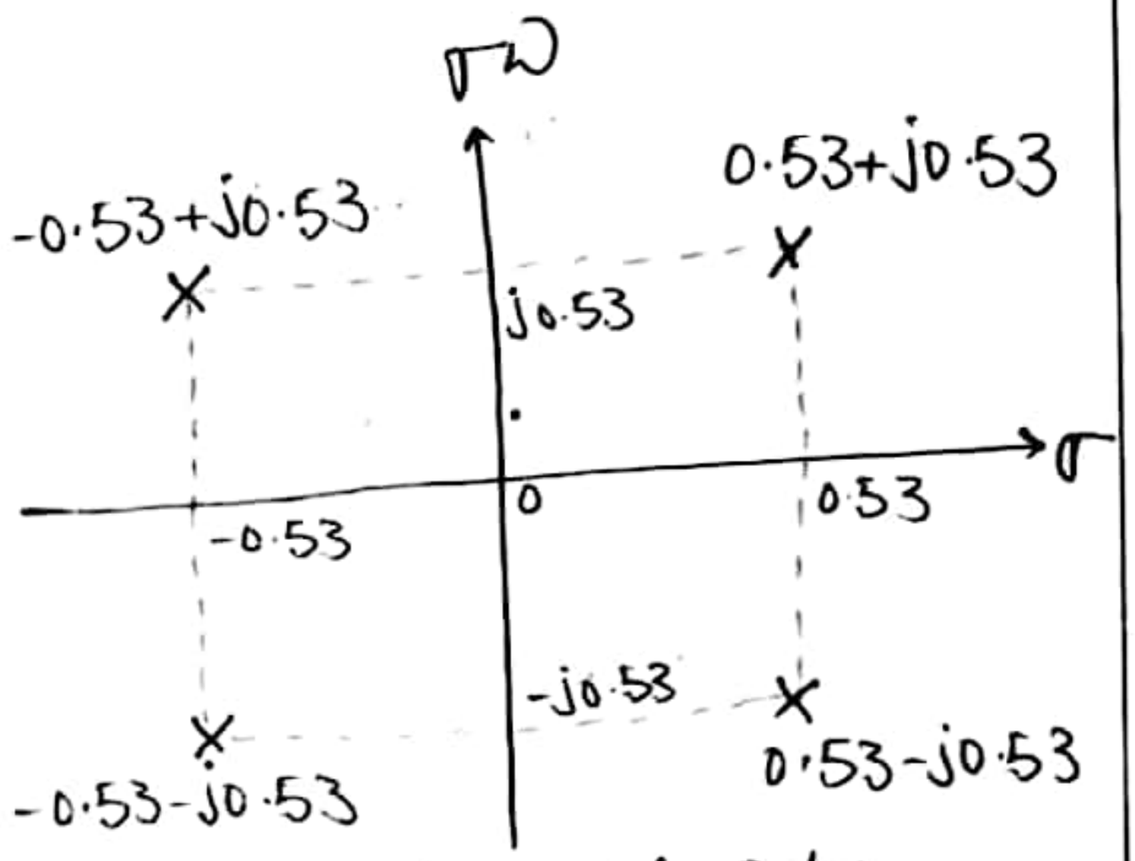


Fig:- Location of poles

Here $S_1 = -0.53 + j0.53$ & $S_1^* = -0.53 - j0.53$

Step-5:- Determine analog filter transfer function, $H_a(s)$

$$H_a(s) = \frac{\Omega_c^N}{(S-S_1)(S-S_1^*)} = \frac{0.75^2}{\left(S-[-0.53+j0.53]\right)\cdot\left(S-(-0.53-j0.53)\right]}$$

$$= \frac{0.75^2}{(S+0.53-j0.53)\cdot(S+0.53+j0.53)}$$

$$\Rightarrow H_a(s) = \frac{0.5625}{S^2 + 1.06S + 0.5618}$$

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Step-6:- Determination of Analog digital filter

using BLT, $S = \frac{2}{T}\left[\frac{Z-1}{Z+1}\right] = 2\left[\frac{Z-1}{Z+1}\right]$

Therefore,

$$H(Z) = H_a(S)\Big|_{S=2\left(\frac{Z-1}{Z+1}\right)} = \frac{0.56}{\left[2\left(\frac{Z-1}{Z+1}\right)\right]^2 + 1.06\left[2\left(\frac{Z-1}{Z+1}\right)\right] + 0.56}$$

$$\Rightarrow H(Z) = \frac{0.56\,(Z+1)^2}{6.68Z^2 - 6.68Z + 2.44}$$

∴ This is the required transfer function to design the digital filter.

= * =

Eg:- a). Find the order and cut off frequency of a digital filter with the following specifications:

$0.89 \le |H(e^{j\omega})| \le 1 \quad ; \quad 0 \le \omega \le 0.4\pi$

$|H(e^{j\omega})| \le 0.18 \quad ; \quad 0.6\pi \le \omega \le \pi$

use Impulse Invariance method.

Sol:- Given specifications are $A_p = 0.89$, $\omega_p = 0.4\pi$

$A_s = 0.18$, $\omega_s = 0.6\pi$

Step-1:- Determine Ω_p & Ω_s using IIM.

$\Omega_p = \frac{\omega_p}{T} = 0.4\pi$ & $\Omega_s = \frac{\omega_s}{T} = 0.6\pi$ $\quad [\because T = 1\,sec]$

Step-2:- Determine the filter order, N

$$N = \frac{1}{2}\cdot\frac{\log\left[\frac{1}{A_s^2}-1 \Big/ \frac{1}{A_p^2}-1\right]}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{1}{2}\cdot\frac{\log\left[\left(\frac{1}{0.18^2}-1\right)\Big/\left(\frac{1}{0.89^2}-1\right)\right]}{\log\left(\frac{0.6\pi}{0.4\pi}\right)} = 5.837 \simeq 6$$

∴ order of filter, N = 6

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2). Chebyshev filter approximation :-

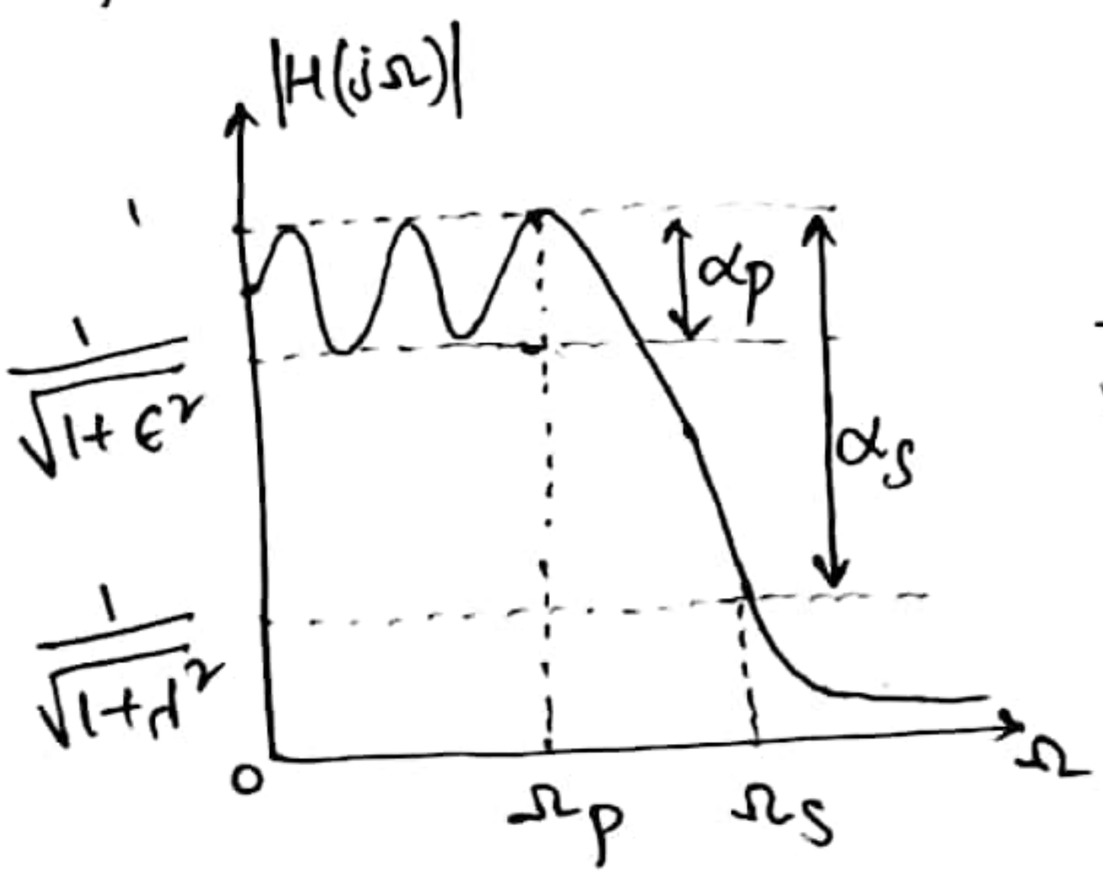
There are two types of chebyshev filters.

a). Chebyshev filter type - 1 :-

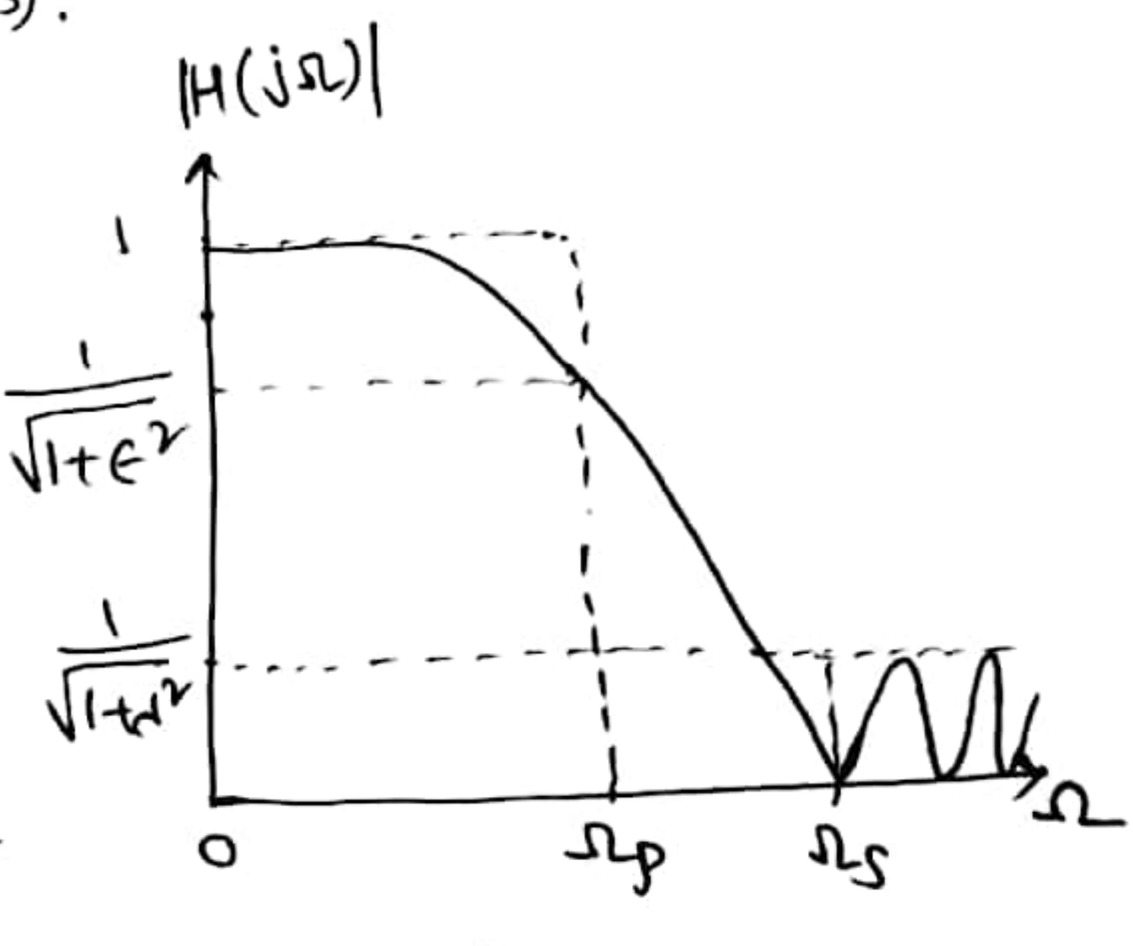
These filters are all pole filters. In the passband, these filters show equiripple behaviour and monotonic characteristics in the stop band. The filter characteristics are shown in fig. (a).

b). Chebyshev filter type - 2 :-

This filter consists of poles as well as zeroes. In pass band, zeroes and poles exhibits a monotonic behaviour in the pass band and an equipple behaviour in the stop band as shown in fig. (b).



a): Type -1



b). Type -2

Fig:- Characteristics of chebyshev filter.

Where α_p = Attenuation at the pass band frequency, dB

α_s = Attenuation at the stop band frequency. dB

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The Magnitude response of type-1 filter is given by.

$$|H(j\Omega)|^2 = \frac{1}{1+\epsilon^2 C_N^2\left(\frac{\Omega}{\Omega_p}\right)} \quad \cdot N = 1, 2, \ldots$$

The Magnitude squared response of type-2 filter is

$$|H(j\Omega)|^2 = \frac{1}{1+\epsilon^2\left[C_N^2\left(\frac{\Omega_s}{\Omega_p}\right)\Big/C_N^2\left(\frac{\Omega_s}{\Omega}\right)\right]}$$

Where ϵ = Ripple parameter in the pass band

$$\Rightarrow \epsilon = \left[\frac{1}{A_p^2} - 1\right]^{1/2} \quad (or) \quad \left[10^{0.1\,A_p(dB)} - 1\right]^{1/2}$$

Ω_p = pass band frequency

$C_N(x)$ = Chebyshev polynomial of order 'N'

$$\Rightarrow C_N(x) = \begin{cases} \cos(N\cdot\cos^{-1}x) & : |x| \le 1 \quad \text{(pass band)} \\ \cosh(N\cdot\cosh^{-1}x) & : |x| \ge 1 \quad \text{(stop band)} \end{cases}$$

The chebyshev polynomials are determined by using

$$C_{N+1}(x) = 2x\cdot C_N(x) - C_{N-1}(x)$$

with $C_0(x) = 1$ and $C_1(x) = x$.

S.NO	N	$C_N(x)$
1.	0	1
2.	1	x
3.	2	$2x^2-1$
4.	3	$4x^3-3x$
5.	4	$8x^4-8x^2+1$
6.	5	$16x^5-20x^3+5x$

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Chebyshev filter design steps:-

Step-1:- Determine, ϵ

If Ap in dB, $\epsilon = \left[10^{0.1 A_{pdB}} - 1\right]^{1/2}$

If Ap is not in dB, $\epsilon = \left[\frac{1}{A_p^2} - 1\right]^{1/2}$

Step-2:- Determine the filter order, N

When magnitude expressed in dB, Then

$$|H(j\Omega)| \text{ in dB} = -20 \cdot \log_{10}^{\epsilon} - 6(N-1) - 20N \log_{10}^{\Omega_S}$$

Step-3:- Determine the poles of chebyshev filter.

The poles are denoted by $S_i = \gamma \cdot \cos\theta_i + jR \cdot \sin\theta_i$

Where γ = minor axis of ellipse $= \Omega_p \cdot \frac{\beta^2 - 1}{2\beta}$

R = major axis of ellipse $= \Omega_p \cdot \frac{\beta^2 + 1}{2\beta}$

$$\beta = \left[\frac{\sqrt{1+\epsilon^2}+1}{\epsilon}\right]^{1/N}$$

$$\theta_i = \frac{\pi}{2} + \frac{(2i+1)\pi}{2N}, \quad i = 0, 1, 2, \ldots, N-1$$

Step-4:- Determine system transfer function

$$H_a(s) = \frac{i}{(s-s_0)(s-s_1)(s-s_2)\ldots}$$

After simplification,

$$H_a(s) = \frac{i}{s^N + b_{N-1}s^{N-1} + \ldots + b_0}$$

Where b_0 = Constant term in the denominator

$$i = \begin{cases} b_0 & : \text{for 'N' odd} \\ \frac{b_0}{\sqrt{1+\epsilon^2}} & : \text{for 'N' even.} \end{cases}$$

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Eg:- Design a digital LPF using chebyshev filter seeing Procedure that meets the following specifications: Pass band magnitude characteristics that is constant to within 1dB for recurrences below $\omega = 0.2\pi$ and stopband attenuation of atleast 15dB for frequencies between $\omega = 0.3\pi$ and π. use bilinear transformation.

Sol:- Given that, $A_p = 1\,dB$, $\omega_p = 0.2\pi$

$A_s = 15\,dB$, $\omega_s = 0.3\pi$

For the BLT,

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = 2\tan\left(\frac{0.2\pi}{2}\right) = 0.6498 \quad [\because T = 1\,sec]$$

$$\Omega_s = \frac{2}{T}\tan\left(\omega_s/2\right) = 2\tan\left(\frac{0.3\pi}{2}\right) = 1.019$$

step-1:-

$$\epsilon = \left[10^{0.1 A_p\,dB} - 1\right]^{1/2} = \left[10^{0.1\times 1} - 1\right]^{1/2} = 0.5088$$

step-2:-

$$|H(j\Omega)| \text{ in } dB = -20\cdot\log_{10}\epsilon - 6(N-1) - 20N\cdot\log_{10}\Omega_s$$

Given stop band attenuation = 15 dB for $\Omega_s = 1.019$ rad/sec.

$$-15 = -20\cdot\log_{10}0.5088 - 6(N-1) - 20N\cdot\log_{10}1.019$$

$$\Rightarrow -15 = 5.87 - 6N + 6 - 0.163N$$

$$\Rightarrow 6.163N = 26.87$$

$$\Rightarrow N = 4.36 \approx 5$$

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step-3:- $S_i = r.\cos\theta_i + jR.\sin\theta_i$

$$\beta = \left[\frac{\sqrt{1+\epsilon^2}+1}{\epsilon}\right]^{1/N} = \left[\frac{\sqrt{1+0.5088^2}+1}{0.5088}\right]^{1/5} = 1.33$$

$$r = \Omega_p \cdot \frac{\beta^2-1}{2\beta} = 0.6498\ \frac{1.33^2-1}{2\times1.33} = 0.187$$

$$R = \Omega_p \cdot \frac{\beta^2+1}{2\beta} = 0.6498\ \frac{1.33^2+1}{2\times1.33} = 0.67$$

Now, $\theta_i = \frac{\pi}{2} + \frac{(2i+1)\pi}{2N}$, $i = 0, 1, 2, 3, 4$

$i=0,\ \theta_0 = \frac{\pi}{2} + \frac{\pi}{10} = \frac{6\pi}{10} = \frac{3\pi}{5}$

$i=1,\ \theta_1 = \frac{\pi}{2} + \frac{3\pi}{10} = \frac{8\pi}{10} = \frac{4\pi}{5}$

$i=2,\ \theta_2 = \frac{\pi}{2} + \frac{5\pi}{10} = \pi$

$i=3,\ \theta_3 = \frac{\pi}{2} + \frac{7\pi}{10} = \frac{12\pi}{10} = \frac{6\pi}{5}$

$i=4,\ \theta_4 = \frac{\pi}{2} + \frac{9\pi}{10} = \frac{14\pi}{10} = \frac{7\pi}{5}$

i=0:- $S_0 = 0.187\cos\left(\frac{3\pi}{5}\right) + j0.67\sin\left(\frac{3\pi}{5}\right) = -0.057 + j0.637$

i=1:- $S_1 = 0.187\cos\left(\frac{4\pi}{5}\right) + j0.67\sin\left(\frac{4\pi}{5}\right) = -0.15 + j0.39$

i=2:- $S_2 = 0.187\cos\pi + j0.67\sin\pi = -0.187$

i=3:- $S_3 = 0.187\cos\left(\frac{6\pi}{5}\right) + j0.67\sin\left(\frac{6\pi}{5}\right) = -0.15 - j0.39$

i=4:- $S_4 = 0.187\cos\left(\frac{7\pi}{5}\right) + j0.67\sin\left(\frac{7\pi}{5}\right) = -0.057 - j0.637$

step-4:-

System transfer function, $H_a(s) = \dfrac{i}{(s-s_0)(s-s_1)(s-s_2)(s-s_3)(s-s_4)}$

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$$H_a(s) = \frac{i}{(s+0.057-j0.637).(s+0.15-j0.39).(s+0.187).(s+0.15+j0.39).(s+0.057+j0.637)}$$

$$\Rightarrow H_a(s) = \frac{i}{(s+0.187).(s^2+0.35+0.1746).(s^2+0.114s+0.409)}$$

Here $N = 5$ (odd),

thus $i = b_0$ = Constant term in denominator $= 0.187 \times 0.1746 \times 0.409$

$= 0.0133$

$$\therefore H_a(s) = \frac{0.0133}{(s+0.187)(s^2+0.35+0.1746)(s^2+0.114s+0.409)}$$

This is the required transfer function of analog filter.

<u>step-5:</u> using BLT,

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right) = 2.\left(\frac{z-1}{z+1}\right) \qquad \left[\because T = 1sec\right]$$

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left[\frac{z-1}{z+1}\right]} = \frac{0.0133}{\left[2\left(\frac{z-1}{z+1}\right)+0.187\right].\left[4\left(\frac{z-1}{z+1}\right)+0.6\left(\frac{z-1}{z+1}\right)+0.1746\right].\left[4\left(\frac{z-1}{z+1}\right)^2+0.228\left(\frac{z-1}{z+1}\right)+0.409\right]}$$

$\therefore$ This is the required transfer function for a given digital filter.

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Chebyshev filter properties:-

① $C_N(x) = C_N(-x)$ for 'N' even
$= -C_N(-x)$ for 'N' odd

② $C_N(0) = (-1)^{N/2}$ for 'N' even
$= 0$ for 'N' odd

③ $C_N(1) = 1$ for all N

④ $C_N(-1) = 1$ for 'N' even
$= -1$ for 'N' odd

⑤ $C_N(x)$ oscillates with equi ripple between ± 1 for $|x| \leq 1$

⑥ For all N, $0 \leq |C_N(x)| \leq 1$ for $0 \leq |x| \leq 1$
$|C_N(x)| > 1$ for $|x| \geq 1$

⑦ $C_N(x)$ is monotonically increasing for $|x| > 1$ for all N.

⑧ The poles of chebychev filter lie on the ellipse.

Butterworth filter	Chebyshev filter
① The magnitude response of filter depends on 'Ω', as 'Ω' increases its response decrease monotonically	① Magnitude response has ripple in passband ~~and~~ (or) stopband depending on type of filter used.
② It has more transition band than chebycher filter.	② The transition band is less Compared to Butterworth filter.
③ The Poles lying on circle in S-plane.	③ The poles lying on ellipse in S-plane
④ Design Procedure of filter require less number of parameters to be calculated.	④ Design procedure of the filter require more number of Parameters to be calculated.
⑤ Order of the filter $N \geq \frac{\log\left[\sqrt{\frac{10^{0.1A_{3dB}}-1}{10^{0.1A_{pdB}}-1}}\right]}{\log\left(\frac{\Omega_S}{\Omega_p}\right)}$	⑤ Order of filter $N \geq \frac{\cosh^{-1}\left[\sqrt{\frac{10^{0.1A_S}-1}{10^{0.1A_p}-1}}\right]}{\cosh^{-1}\left(\frac{\Omega_S}{\Omega_p}\right)}$

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Analog and Digital Frequency Transformations:-

upto now, we have studied the design of LPF only. Now, if it is asked to design like high pass, bandpass (or) band reject filter then we have to use frequency transformation.

If the cut off frequency of LPF is equal to 1 that means if $\Omega_c = 1$ then it is called as "Normalised filter". To design other filters, first the system function of normalized LPF is obtained.

Now, we have a normalized LPF having cut-off frequency Ω_c.

a). Low pass to Low pass:-

To design another LPF with new passband edge frequency Ω_{Lp}. Then use the frequency transformation.

$$S \longrightarrow \frac{\Omega_c}{\Omega_{Lp}} S$$

b). Low pass to high pass:-

To design HPF with cut off frequency, Ω_{Hp} then

$$S \longrightarrow \frac{\Omega_c \Omega_{Hp}}{S}$$

c). Low pass to bandpass:-

To design BPF with higher cutoff frequency, Ω_u and lower cut off frequency, Ω_l then

$$S \longrightarrow \Omega_c \cdot \frac{S^2 + \Omega_l \Omega_u}{S \cdot (\Omega_u - \Omega_l)}$$

d). Low pass to band stop (or) band reject (or) Notch :-

$$S \longrightarrow \Omega_c \cdot \frac{S(\Omega_u - \Omega_l)}{S^2 + \Omega_u \Omega_l}$$

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Basic structures of IIR filters :-

The Causal IIR systems are characterized by the constant difference equation i.e.,

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \longrightarrow (1)$$

The real transfer function is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \longrightarrow (2)$$

* Recursive system :-

A Causal recursive system is described by an input-output equation of the form:

$$y(n) = F[y(n-1), \ldots . y(n-N), x(n), x(n-1), \ldots x(n-M)] \rightarrow (3)$$

* Non-Recursive system :-

A Causal non-recursive system do not depend on past values of the output and hence are described by an input-output equation of the form:

$$y(n) = F[x(n), x(n-1), \ldots . x(n-M)] \longrightarrow (4)$$

From eqns (1) & (2), it may be observed that the realization of IIR systems involves a recursive Computational algorithm.

IIR filters Can be realized in many forms. They are

1). Direct form-I realization
2). Direct form-II realization
3). Cascade form realization
4). Parallel form realization
5). Lattice-Ladder structure.

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Direct form realization of IIR systems:-

$$\text{W.K.T.,} \quad H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

The multipliers in the feed forward paths are the numerator coefficients and the multipliers in the feed negative paths are the denominator coefficients.

$$\text{Let} \quad H_1(z) = \sum_{k=0}^{M} b_k z^{-k} \longrightarrow (3) \quad \text{and}$$

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \longrightarrow (4)$$

Substituting (3) & (4) in eqn (2), we get

$$H(z) = H_1(z) \cdot H_2(z) \longrightarrow (5)$$

This equation can be represented as the cascade connection of $H_1(z)$ & $H_2(z)$ as shown in fig.1.

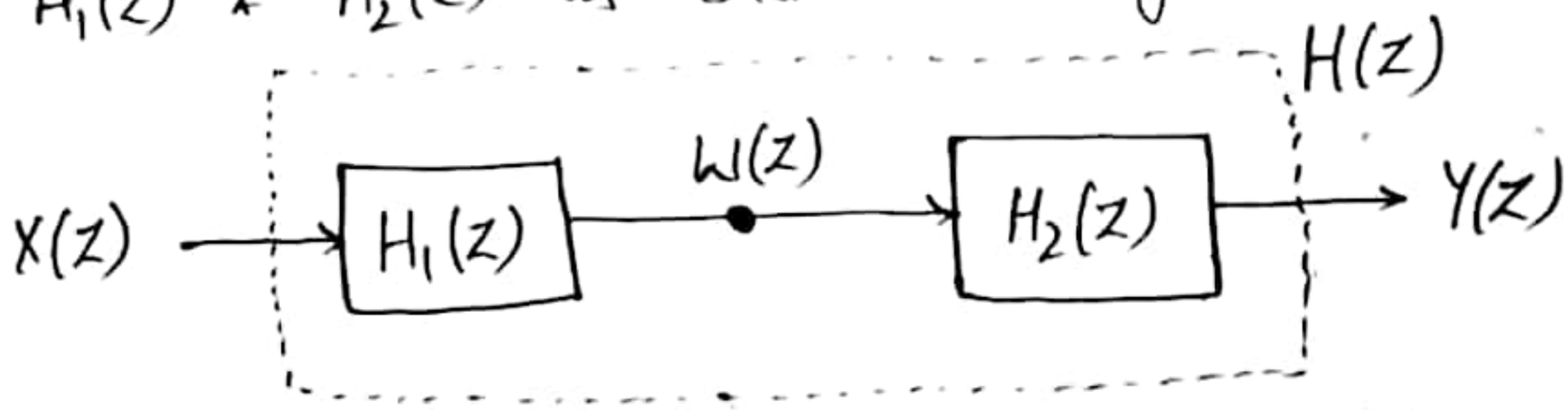


Fig 1:- Cascade connection of $H_1(z)$ & $H_2(z)$

W.K.T, $H_1(z)$ is the transfer function of numerator and it always contain zeroes of the system. So, $H_1(z)$ is called as "All zero system". Now, $H_2(z)$ is the transfer function of the denominator and it always contain poles of the system. So, $H_2(z)$ is called as "All pole system."

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i). Direct form-I structure:-

From fig.①.,

$$H_1(z) = \frac{W(z)}{X(z)} \quad \text{and} \quad H_2(z) = \frac{Y(z)}{W(z)}$$

a). Direct form structure for $H_1(z)$:-

from ③., $H_1(z) = \frac{W(z)}{X(z)} = \sum_{k=0}^{M} b_k z^{-k} \longrightarrow ⑥$

$$\Rightarrow W(z) = \sum_{k=0}^{M} b_k \cdot z^{-k} \cdot X(z)$$

taking inverse Z-transform on both sides,

$$w(n) = \sum_{k=0}^{M} b_k \cdot x(n-k)$$

$$\Rightarrow w(n) = b_0 \cdot x(n) + b_1 x(n-1) + b_2 \cdot x(n-2) + \ldots + b_M \cdot x(n-M) \longrightarrow ⑦$$

Here $b_0, b_1, \ldots b_M$ are the coefficients. The direct form realization of eqn.⑦ shown in fig.2.

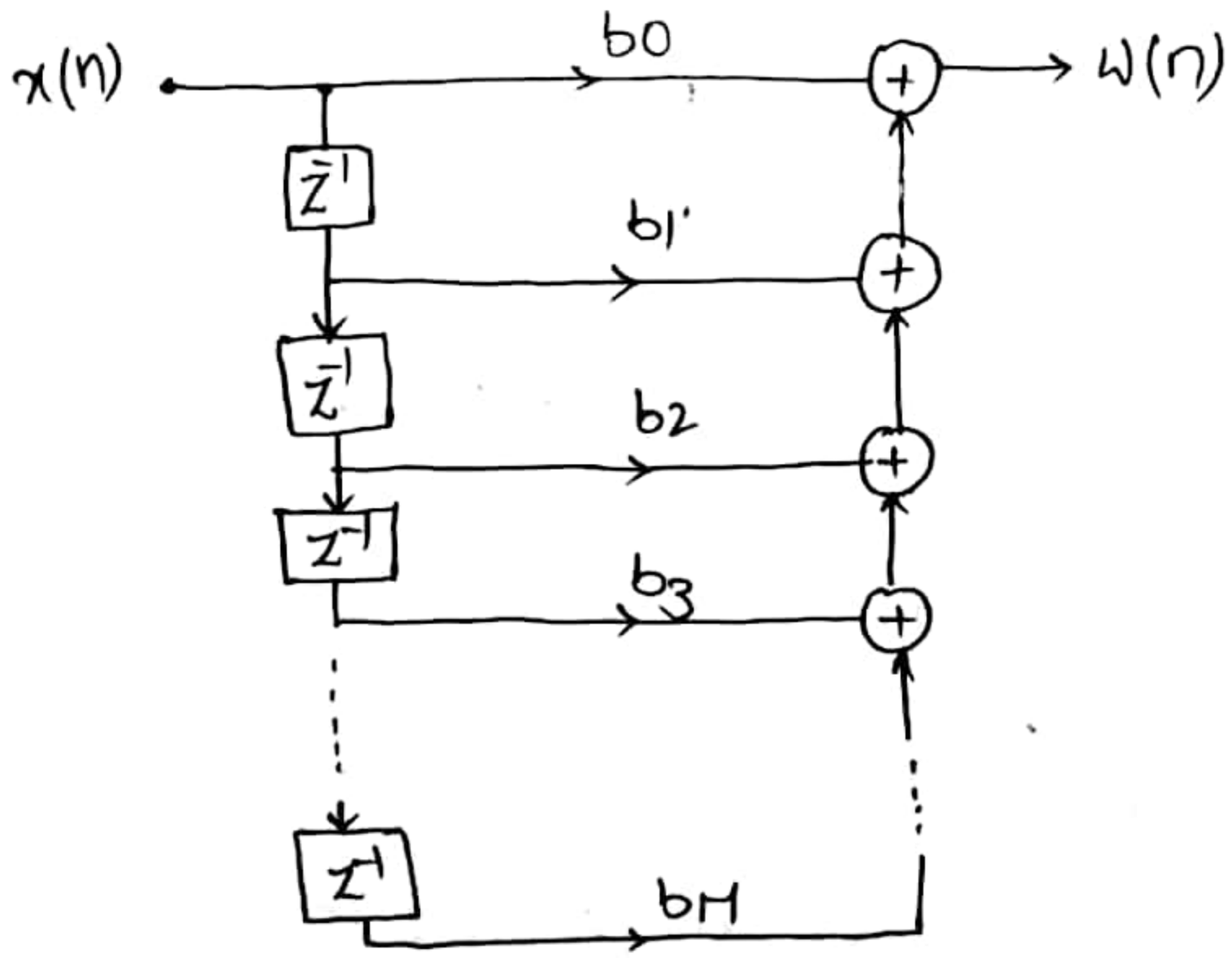


Fig.2:- Direct form realization for of $H_1(z)$
[All zero system]

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b). Direct form structure for $H_2(z)$:-

W.K.T, $H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$ ⟶ (8)

$\Rightarrow Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = W(z)$

$\Rightarrow Y(z) + \sum_{k=1}^{N} a_k z^{-k} \cdot Y(z) = W(z)$

$\Rightarrow Y(z) = W(z) - \sum_{k=1}^{N} a_k \cdot z^{-k} \cdot Y(z)$

taking inverse Z-transform on both sides

$y(n) = w(n) - \sum_{k=1}^{N} a_k \cdot y(n-k)$ ⟶ (9)

$\Rightarrow y(n) = w(n) - a_1 \cdot y(n-1) - a_2 \cdot y(n-2) - \cdots - a_N \cdot y(n-N)$

Where $-a_1, -a_2, \ldots, -a_N$ are the coefficients. of ~~filter~~)

The direct form of $y(n)$ is shown in fig.3

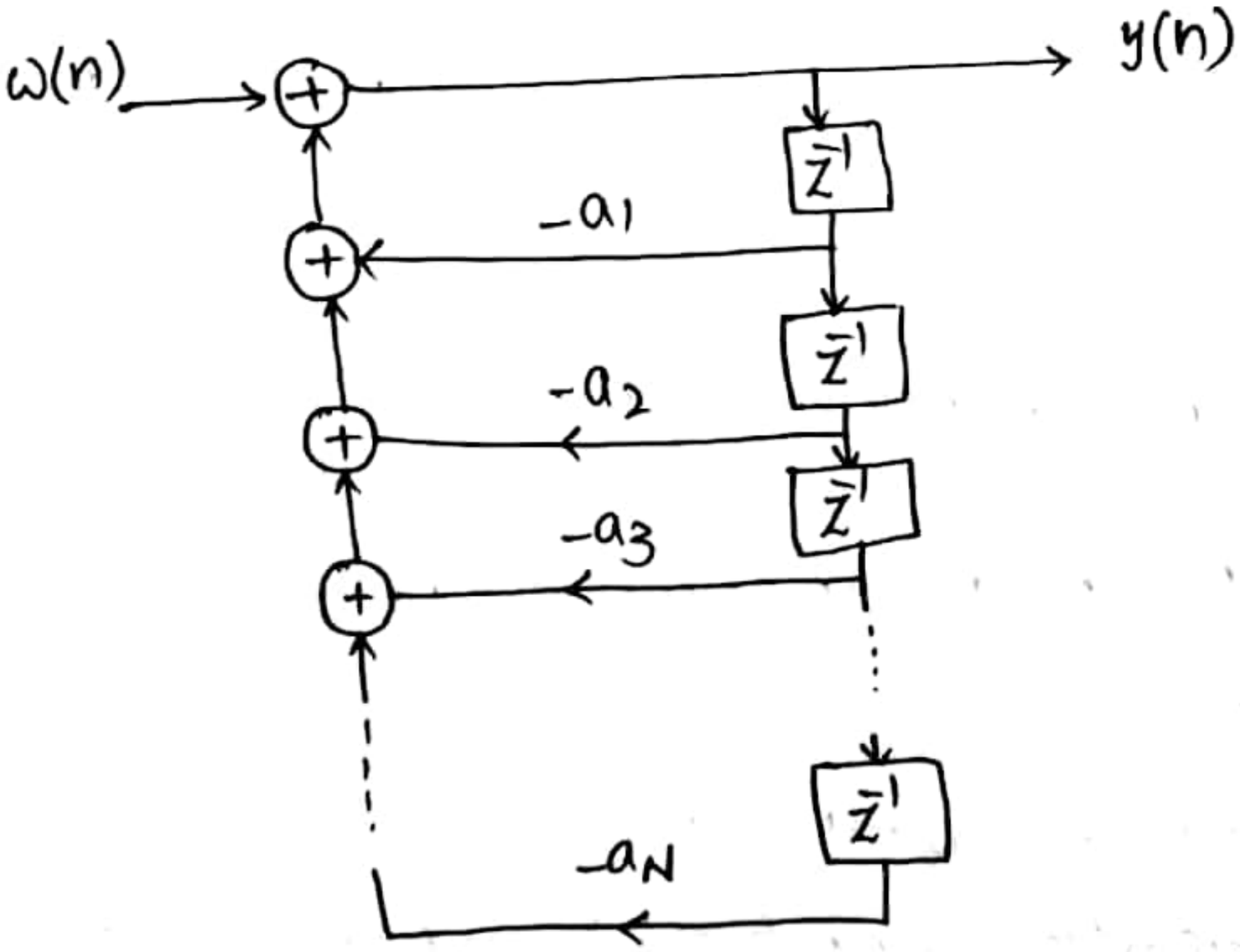


Fig.3:- Direct form realization of $H_2(z)$
[All Pole System]

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Now, direct form-I structure can be obtained by cascading $H_1(z)$ & $H_2(z)$. It is shown in fig (4).

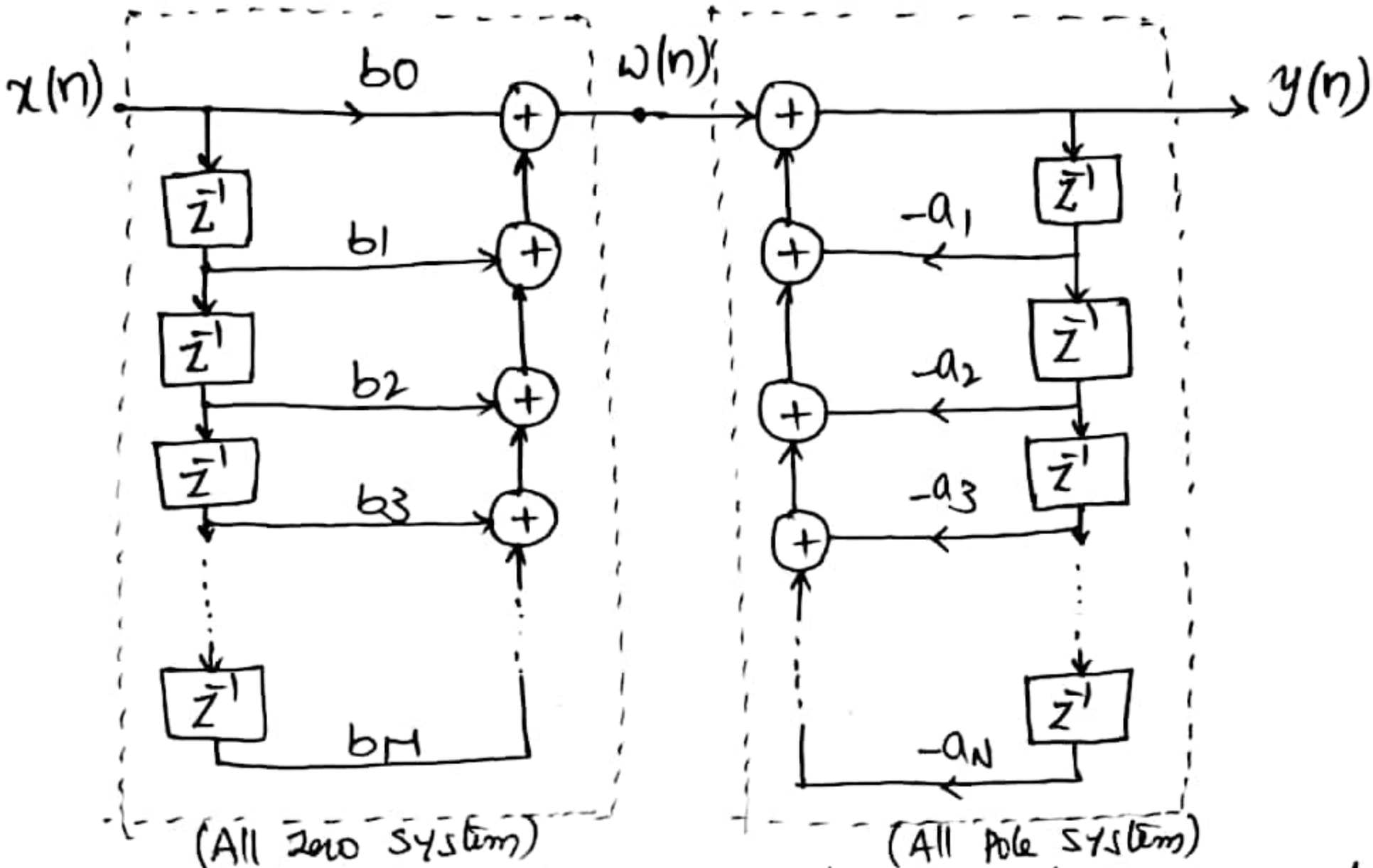


Fig.4 :- Direct form-I realization of IIR system.

Eg:- Determine a direct form-I structure realization of the following difference equation:

$$y(n) = b_0 \cdot x(n) + b_1 \cdot x(n-1) + b_2 \cdot x(n-2) + b_3 \cdot x(n-3) - a_1 \cdot y(n-1) - a_2 \cdot y(n-2) - a_3 \cdot y(n-3).$$

Sol:- Given that,

$$y(n) = b_0 \cdot x(n) + b_1 \cdot x(n-1) + b_2 \cdot x(n-2) + b_3 \cdot x(n-3) - a_1 \cdot y(n-1) - a_2 \cdot y(n-2) - a_3 \cdot y(n-3) \longrightarrow (1)$$

Let $w(n) = b_0 \cdot x(n) + b_1 \cdot x(n-1) + b_2 \cdot x(n-2) + b_3 \cdot x(n-3) \longrightarrow (2)$

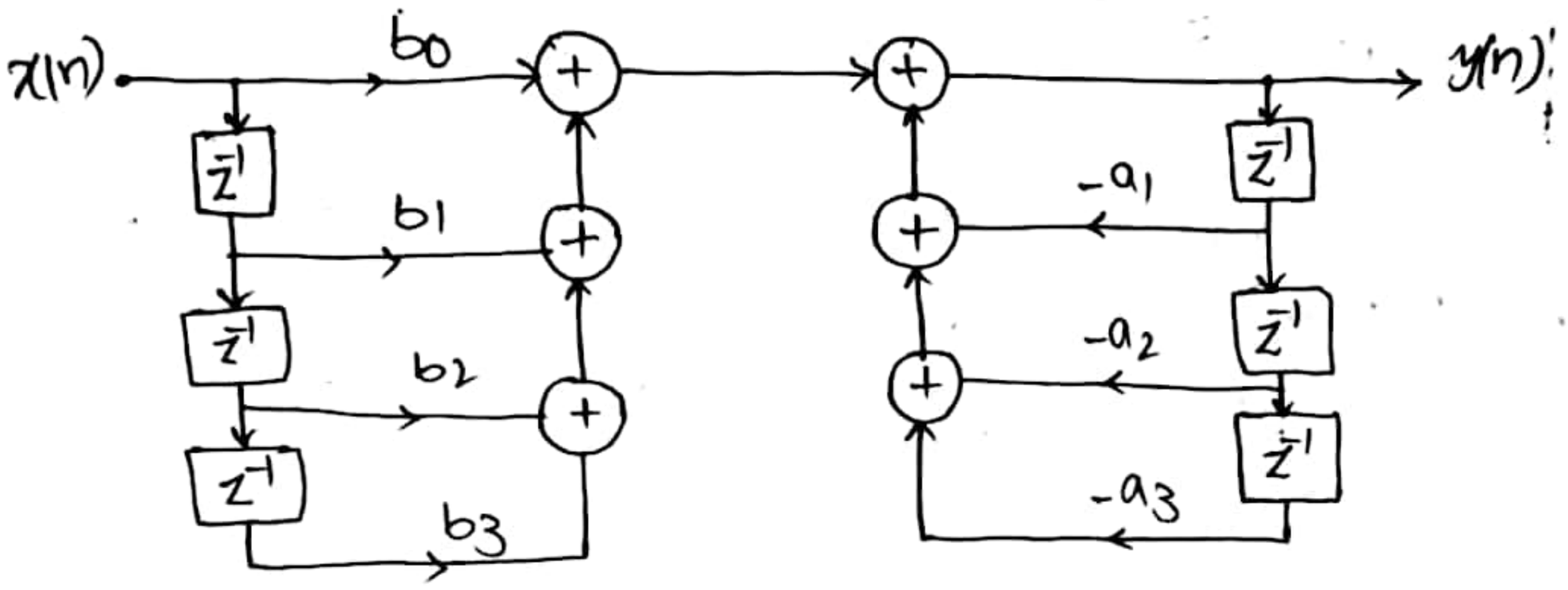
Thus, eqn (1) becomes

$$y(n) = w(n) - a_1 y(n-1) - a_2 \cdot y(n-2) - a_3 \cdot y(n-3) \longrightarrow (3)$$

The corresponding direct form-I realization structure has been illustrated in fig.5.

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Fig·5:

Eg:- Obtain the direct form-I realization for the system described by difference equation $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$.

Sol:- Given that,

$$y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1) \longrightarrow (1)$$

Let $w(n) = x(n) + 0.4x(n-1) \longrightarrow (2)$

Thus, $y(n) = w(n) + 0.5y(n-1) - 0.25y(n-2) \longrightarrow (3)$.

Realizing eqns (2) & (3) and then combining, we get

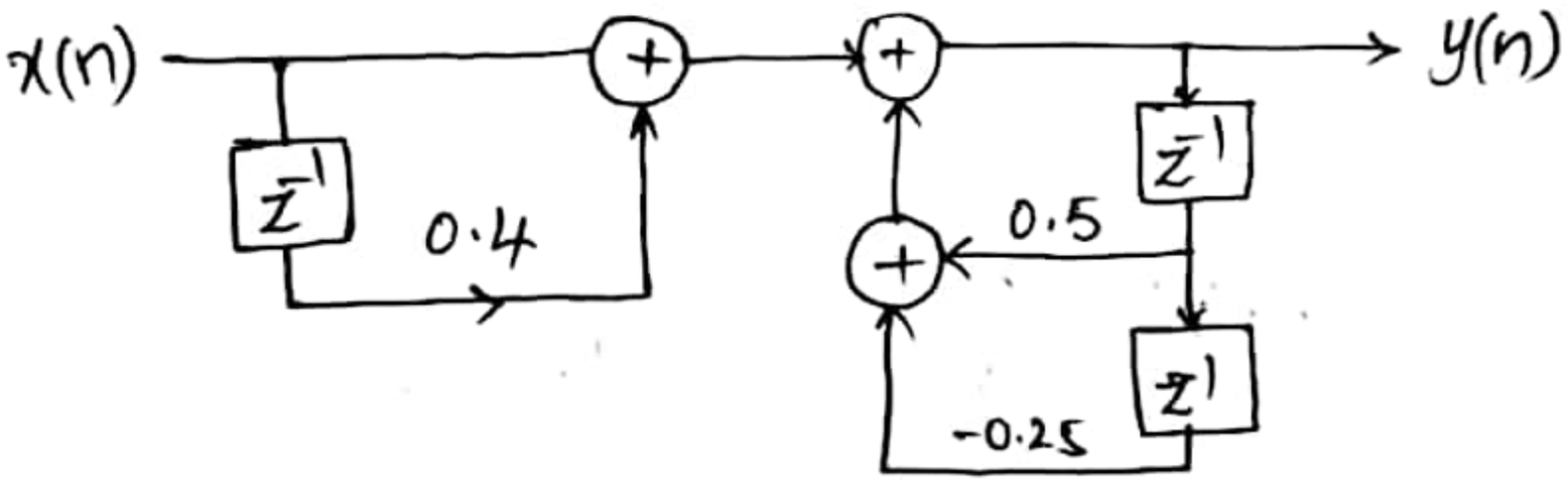


fig. 6 :-

Computational Complexity :-

1). Number of multiplications = M+N+1

2). Number of Additions = M-N

3). Number of memory locations = M+N+1

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2). Direct form-II structure :-

Now, we are studying for LTI [Linear Time Invariant] systems. since the system is linear. We can interchange the positions of $H_1(z)$ and $H_2(z)$. This property gives direct form-II structure. Hence, Poles of $H(z)$ are realised first and Zeroes are realised second.

So, $$H_1(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \longrightarrow (1)$$

$$H_2(z) = \sum_{k=0}^{M} b_k z^{-k} \longrightarrow (2)$$

The Cascade Connection of $H(z)$ is shown in fig.1

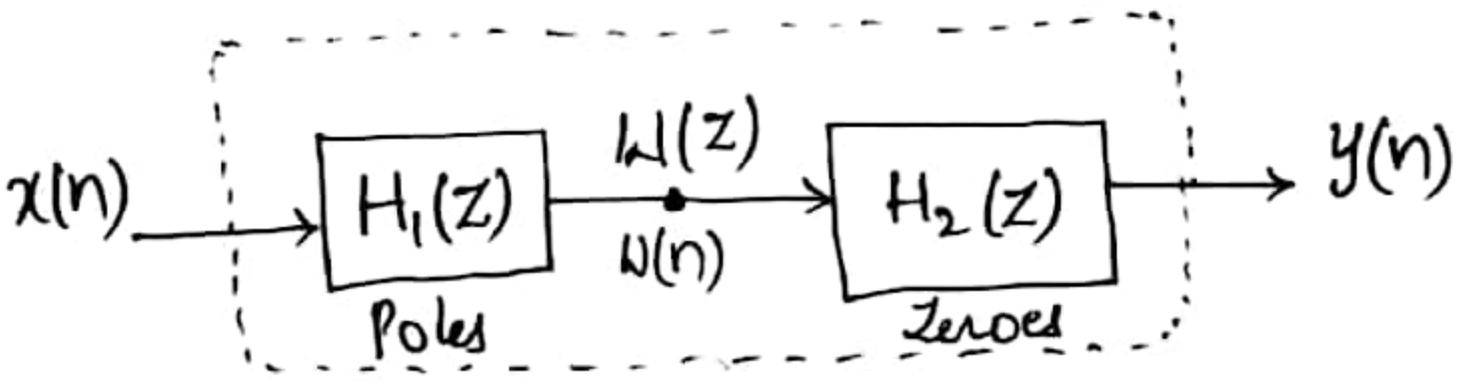


Fig.1 :- Decomposition of direct form-II structure

a). All Pole system :-

From fig.(1)., $$H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \longrightarrow (3)$$

$$\Rightarrow W(z) + \sum_{k=1}^{N} a_k \cdot z^{-k} \cdot W(z) = X(z)$$

Applying Inverse Z-transform on both sides

$$w(n) + \sum_{k=1}^{N} a_k \cdot w(n-1) = x(n)$$

$$\Rightarrow w(n) = x(n) - \sum_{k=1}^{N} a_k \cdot w(n-1) \longrightarrow (4)$$

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The direct form implementation of eqn.④ as shown in fig.②

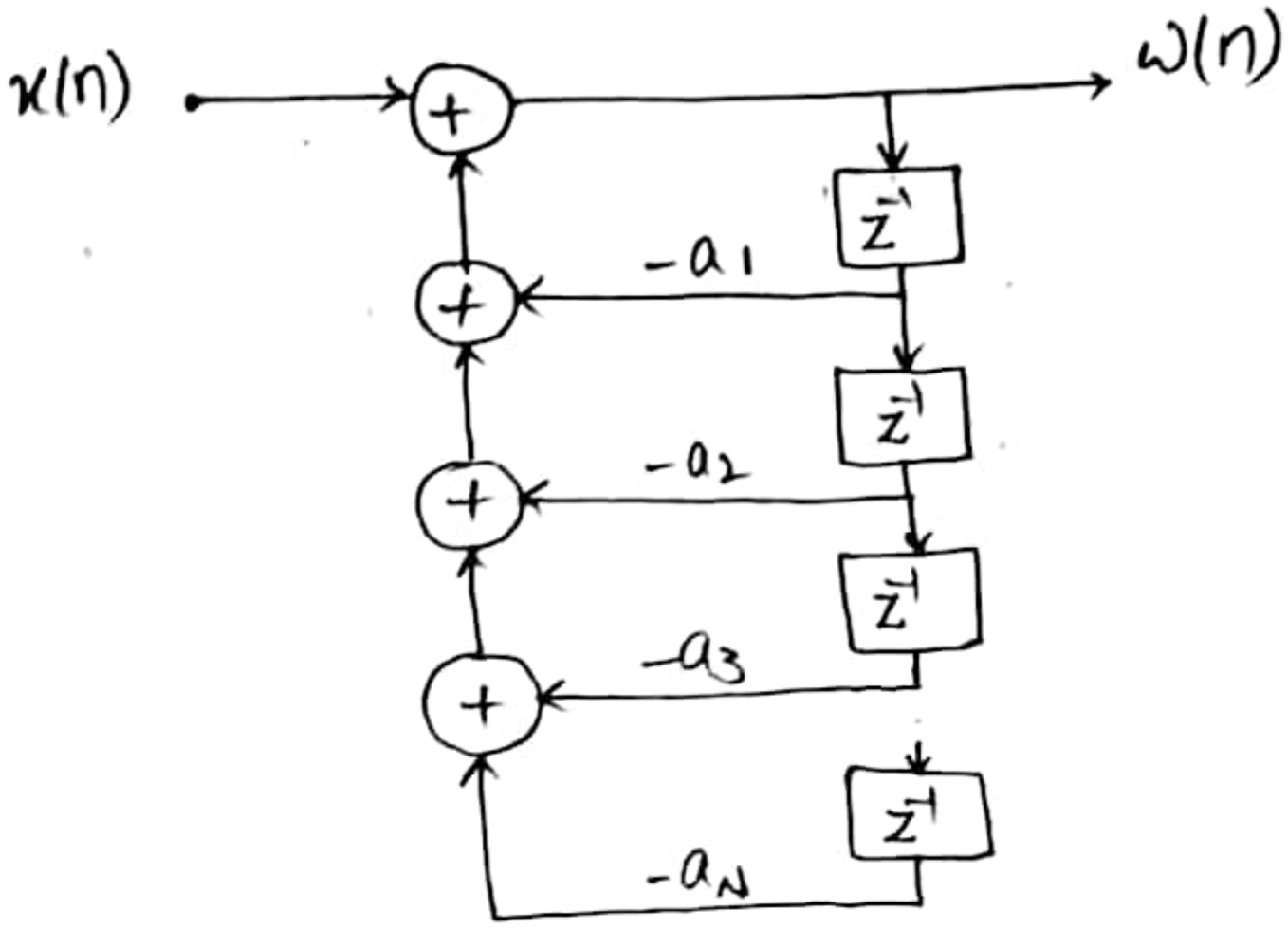


Fig.2:- Direct form realization of $H_1(z)$ [All pole system]

b). All zero system:-

From fig ①, $H_2(z) = \frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k \cdot z^{-k}$ ⟶ ⑤

$$\Rightarrow Y(z) = \sum_{k=0}^{M} b_k z^{-k} \cdot W(z)$$

Applying Inverse Z-transform on both sides

$$y(n) = \sum_{k=0}^{M} b_k \cdot w(n-k) \longrightarrow ⑥$$

The Direct form implementation of eqn ⑥ as shown in fig ③.

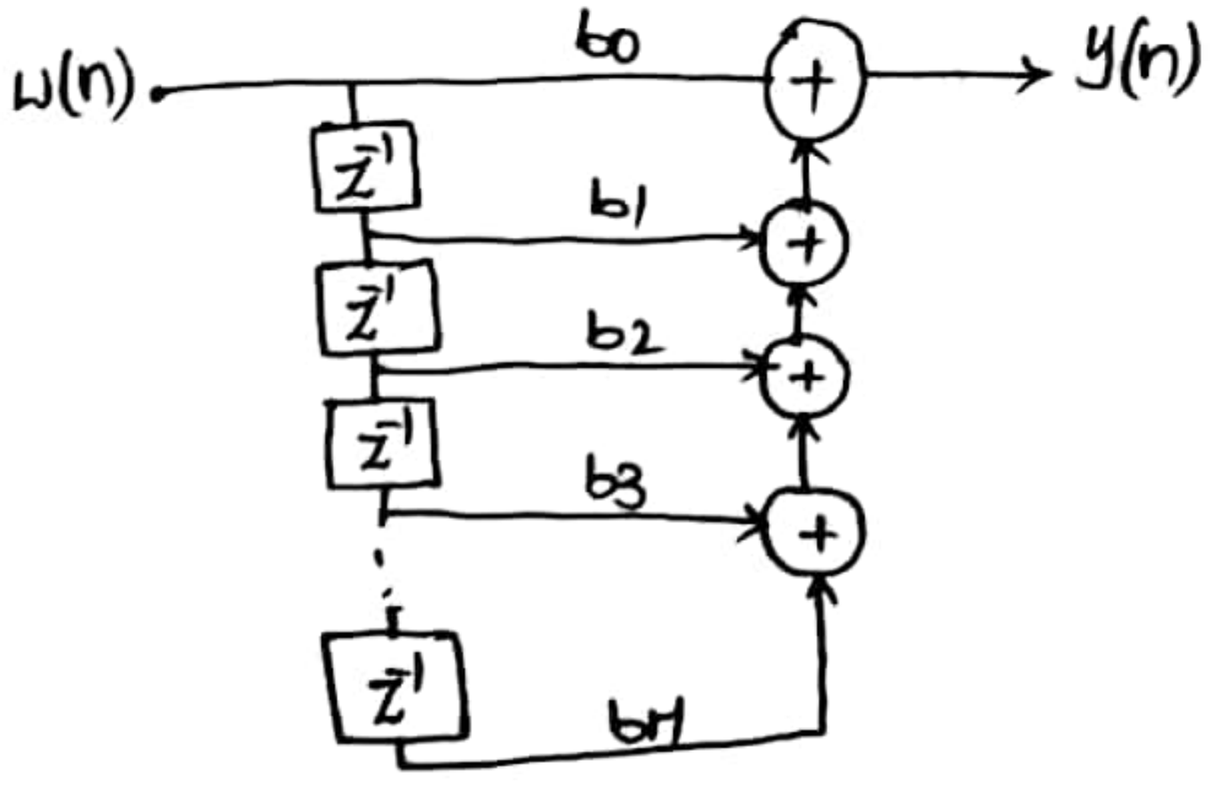


Fig.3:- Direct form realization of $H_2(z)$ [All zero system] ⑦

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Now, direct form-II structure can be obtained by cascading $H_1(z)$ and $H_2(z)$. The implementation of $H(z)$ as shown in fig. 4

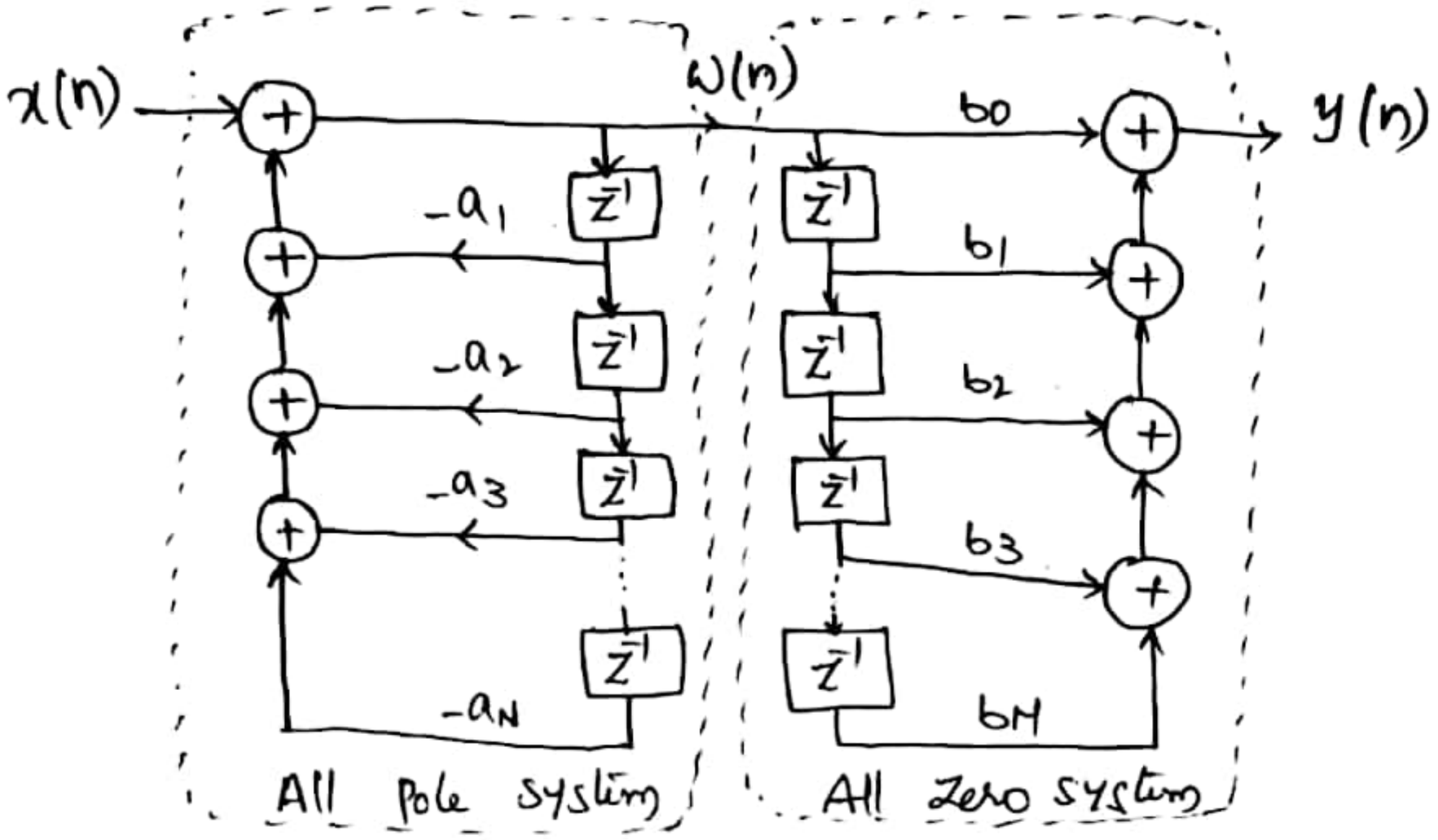


Fig: 4 Direct form-II realisation of IIR systems.

In fig 4, observe that ~~x(n)~~ w(n) is delayed and used in all pole system. Similarly, it is delayed ~~by~~ and used in all zero system. But, we have used two seperate delay elements. In fact, a single delay element can be used to get w(n-1). This can be used by all pole system as well as all zero system. This means that the two delay elements of all pole & all zero systems can be merged into single delay element. The resulting realization has been shown in fig. 5.

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Let us Consider N=M.

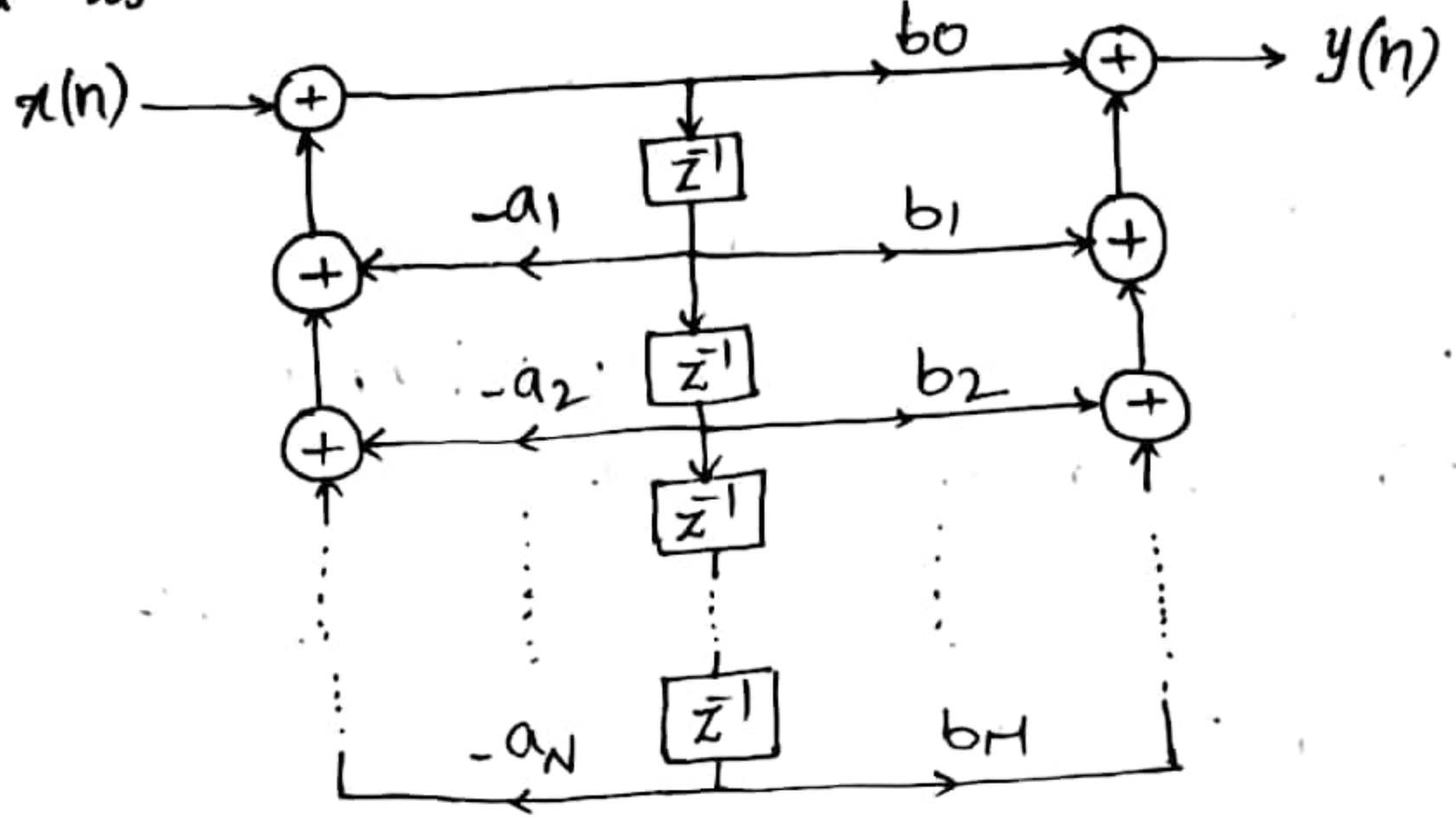


Fig.5 :- Direct form-II realization using common delay elements, Here N=M.

Computations/ Complexity :-

i). Number of Multiplications = M+N+1

ii). Number of Additions = M+N

iii). Number of delay elements are reduced in direct form-II structure. Compared to direct form-I structure. This means that the memory locations are reduced. The memory location required for direct form-II structure are Max [M,N].

Advantages :-

→ The advantage of direct form realization is its implementation is easy.

→ Less memory locations are required for direct form-II compared to direct form-I.

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Eg:- Obtain direct form - I & - II realization of a system described by

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}\cdot x(n-1)$$

Sol:- Given Ket,

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-1)$$

Applying Z- transform on both sides

$$Y(z) - \frac{3}{4}z^{-1}\cdot Y(z) + \frac{1}{8}z^{-2}\cdot Y(z) = X(z) + \frac{1}{2}\cdot z^{-1}\cdot X(z)$$

$$\Rightarrow Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = X(z)\left[1 + \frac{1}{2}z^{-1}\right]$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Direct form - I :-

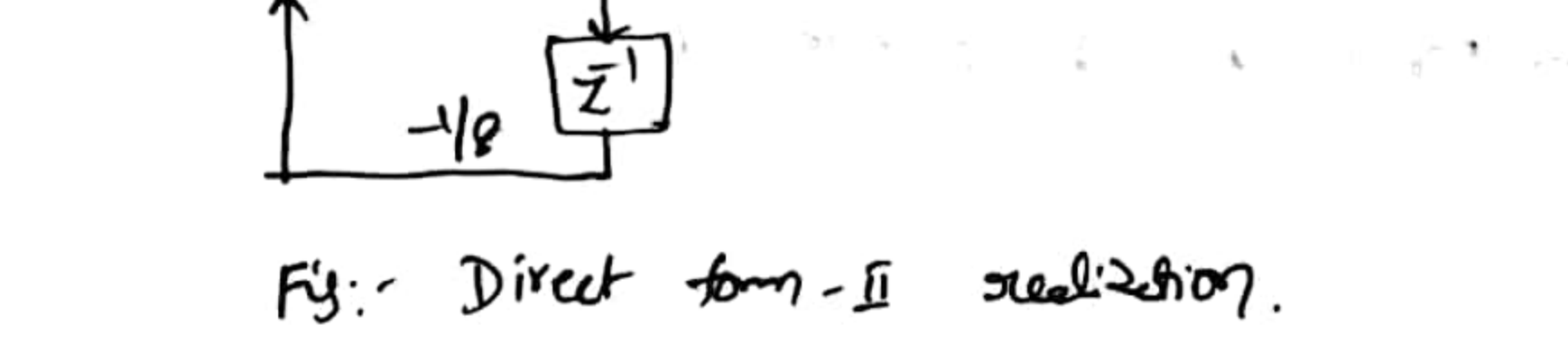


Fig:- Direct form - I realization

Direct form - II :-

Fig:- Direct form - II realization.

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Eg:- Find direct form - I & - II realizations for the second order filter is given by

$$y(n) = 2b.\cos\omega_0.y(n-1) - b^2.y(n-2) + x(n) - b.\cos\omega_0.x(n-1)$$

Sol:- Given that,

$$y(n) = 2b.\cos\omega_0.y(n-1) - b^2.y(n-2) + x(n) - b.\cos\omega_0.x(n-1)$$

taking Z-transform on both sides,

$$Y(z) = 2b.\cos\omega_0.z^{-1}.Y(z) - b^2.z^{-2}.Y(z) + X(z) - b.\cos\omega_0.z^{-1}.X(z)$$

$$\Rightarrow Y(z).\left[1 - 2b.\cos\omega_0 z^{-1} + b^2 z^{-2}\right] = \left[1 - b.\cos\omega_0.z^{-1}\right].X(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 - b.\cos\omega_0 z^{-1}}{1 - 2b\cos\omega_0.z^{-1} + b^2.z^{-2}}$$

Direct form - I :-

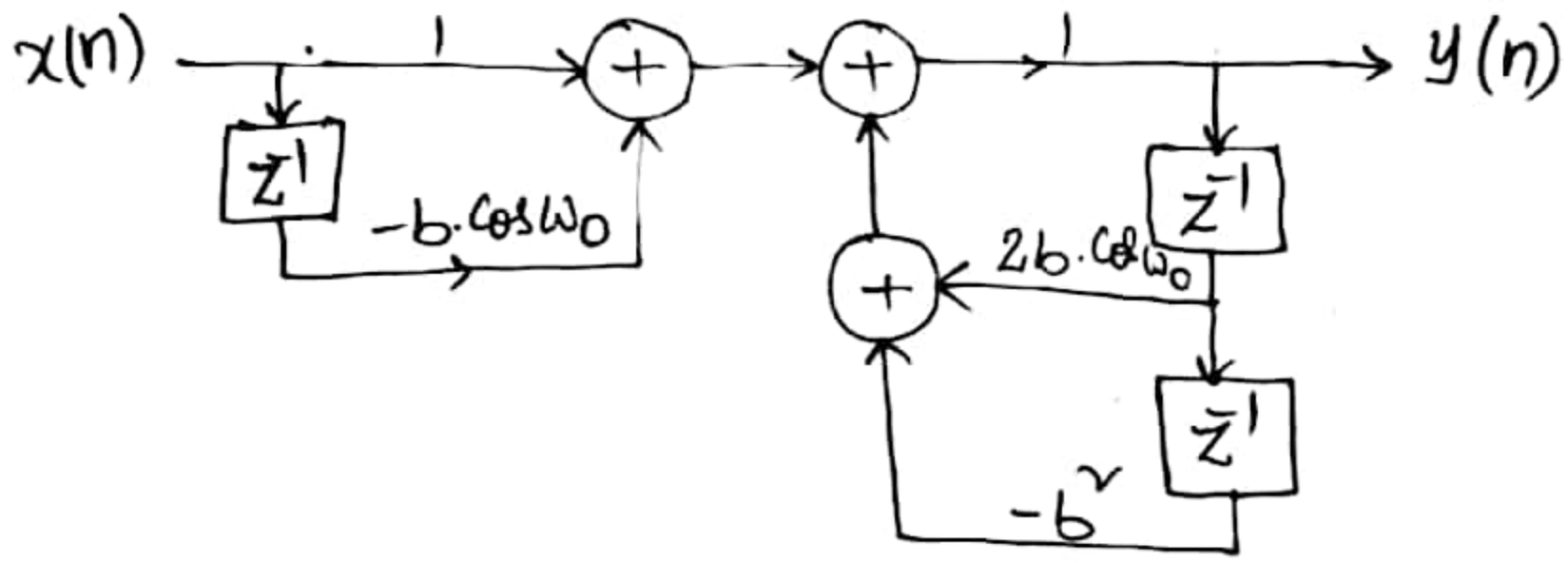


Fig:- Direct form - I realization

Direct form - II :-

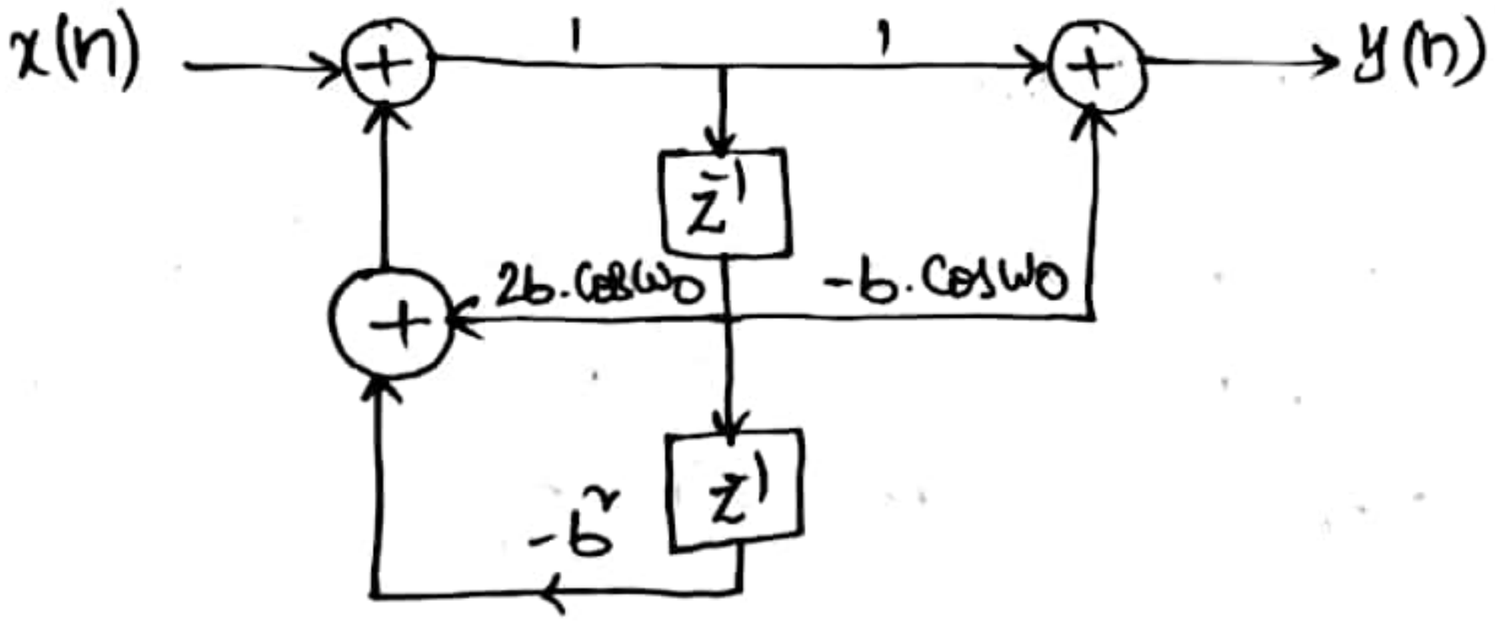


Fig:- Direct form - II realization

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Cascade form realization of IIR filters:-

To obtain the cascade form realization, the numerator & denominator of given transfer function $H(z)$ is factored into the product of second order terms.

The total transfer function is expressed as

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z) \cdot \ldots \cdot H_K(z)$$

Here $H_1(z), H_2(z), \ldots H_K(z)$ are second order polynomials. The total transfer function is obtained by connecting all second order subsystems in series as shown in fig.

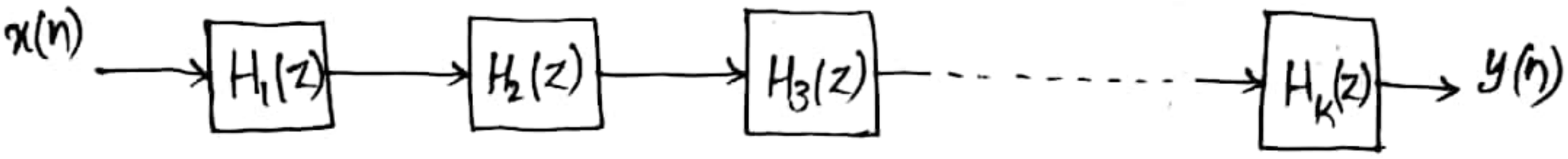


Fig:- Cascade form realization

The general difference equation for D.T-LTI systems is given by

$$H(z) = \frac{\sum_{K=0}^{M} b_K z^{-K}}{1 + \sum_{K=1}^{N} a_K \cdot z^{-K}}$$

Since $H_1(z)$ & $H_2(z)$ are second order polynomials, so substitute $M=N=2$.

$$H(z) = \frac{\sum_{K=0}^{2} b_K z^{-K}}{1 + \sum_{K=1}^{2} a_K z^{-K}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

In this case, we have second order equation. Hence, we can draw the direct form-II structure for above eqn. by substituting $M=N=2$ as shown in fig.

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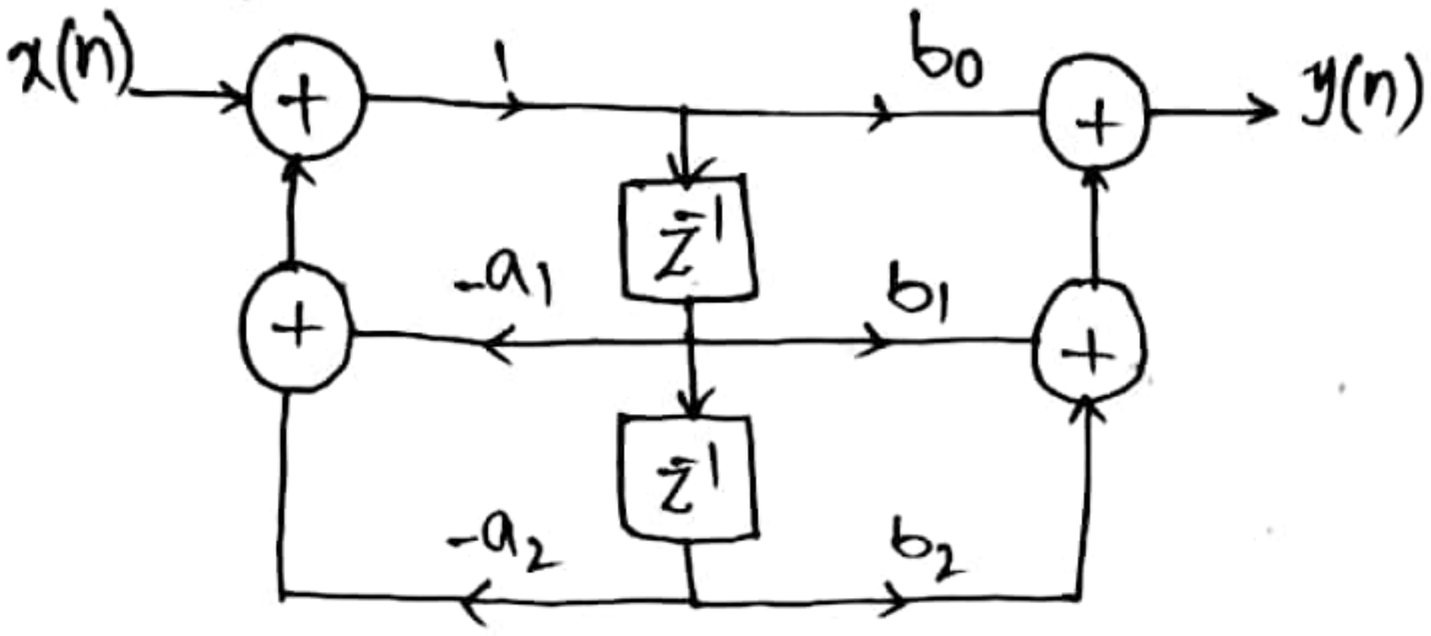


Fig:- Direct form - II realization of second order subsystem.

Therefore, all such subsystems must be connected in series to obtain the cascade form realization of IIR systems.

Eg:- Realize the system with difference equation $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$ in cascade form.

Sol:- Given ket,

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$$

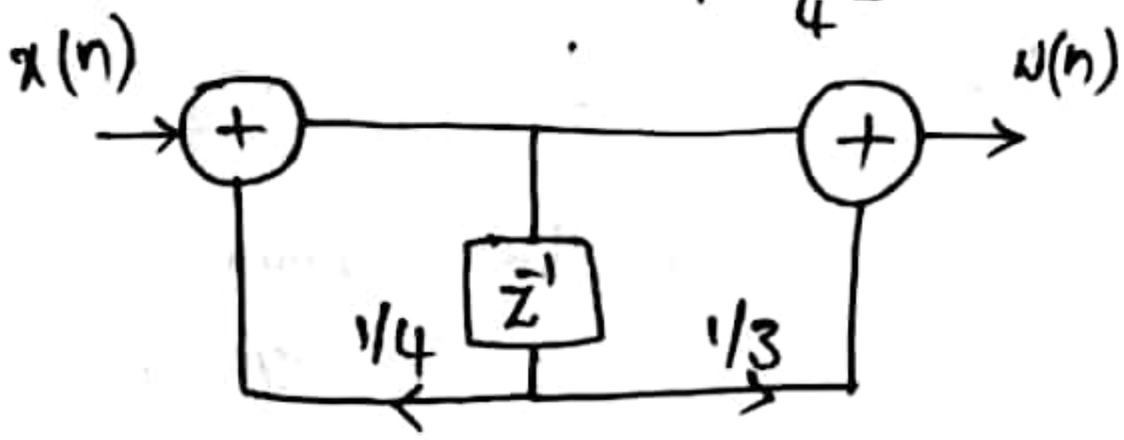
Applying Z-transform on both sides

$$Y(z) = \frac{3}{4}z^{-1}\cdot Y(z) - \frac{1}{8}z^{-2}\cdot Y(z) + X(z) + \frac{1}{3}z^{-1}\cdot X(z)$$

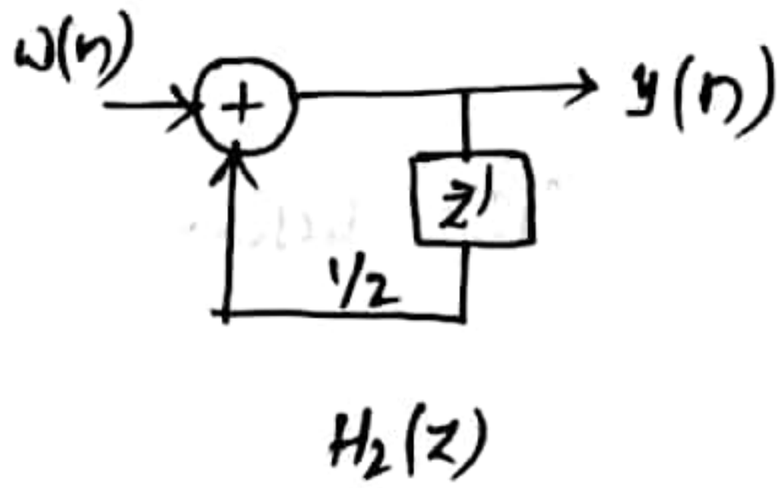
$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$\Rightarrow H(z) = \frac{1+\frac{1}{3}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)} = H_1(z)\cdot H_2(z)$$

Where $H_1(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{4}z^{-1}}$ & $H_2(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$



$H_1(z)$



$H_2(z)$

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Now cascading the realization of $H_1(z)$ & $H_2(z)$ as shown in fig.

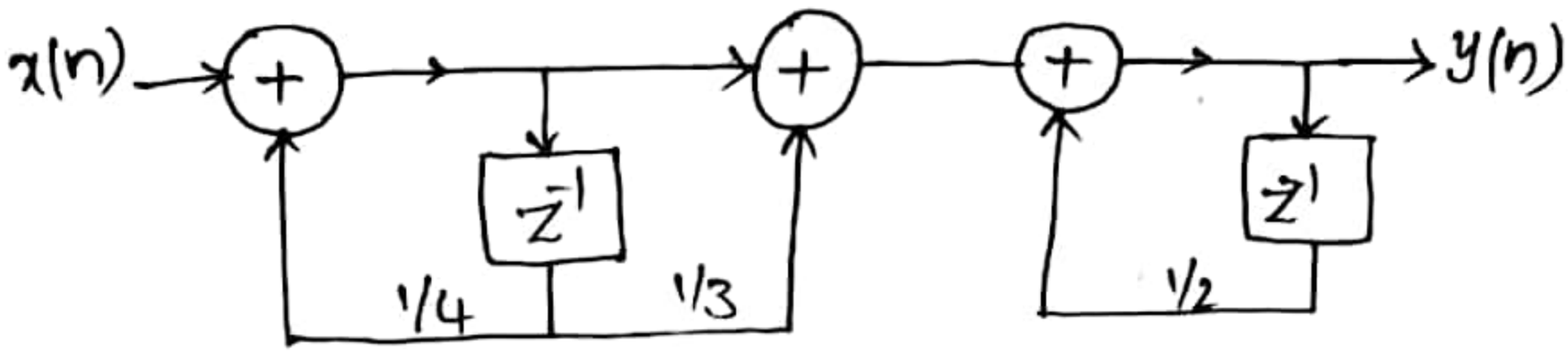


Fig:- Cascade realization

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Eg:- Determine a cascade realization of the system characterised by the transfer function which is expressed as under :

$$H(Z) = \frac{2(Z+2)}{Z(Z-0.1)(Z+0.5)(Z+0.4)}$$

Sol:- Given ket,

$$H(Z) = \frac{2\cdot(Z+2)}{Z\cdot(Z-0.1)\cdot(Z+0.5)\cdot(Z+0.4)}$$

Multiplying numerator and denominator with z^{-4}, we get

$$H(z) = \frac{2\cdot z^{-3}\cdot(1+2z^{-1})}{(1-0.1z^{-1})\cdot(1+0.5z^{-1})\cdot(1+0.4z^{-1})}$$

$$\Rightarrow H(Z) = 2\cdot z^{-1}\cdot z^{-1}\cdot z^{-1}\cdot \frac{1+2z^{-1}}{1-0.1z^{-1}}\cdot\frac{1}{1+0.5z^{-1}}\cdot\frac{1}{1+0.4z^{-1}}$$

Fig. shows the cascade realization of the given transfer function.

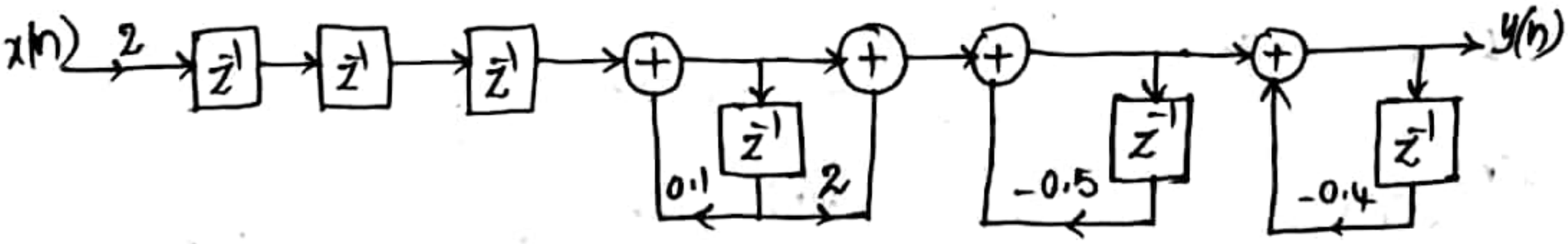


Fig:- Cascade form realization of given transfer function.

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Parallel form realization of IIR Systems:-

The general difference equation for IIR system which is given by

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

By using partial fraction expansion, we can express overall transfer function $H(z)$ as

$$H(z) = C + H_1(z) + H_2(z) + \cdots + H_K(z).$$

Where 'C' is constant and $H_1(z), H_2(z), \ldots H_K(z)$ are second order subsystems.

The general block schematic of parallel form realization or structure for IIR system shown in fig.

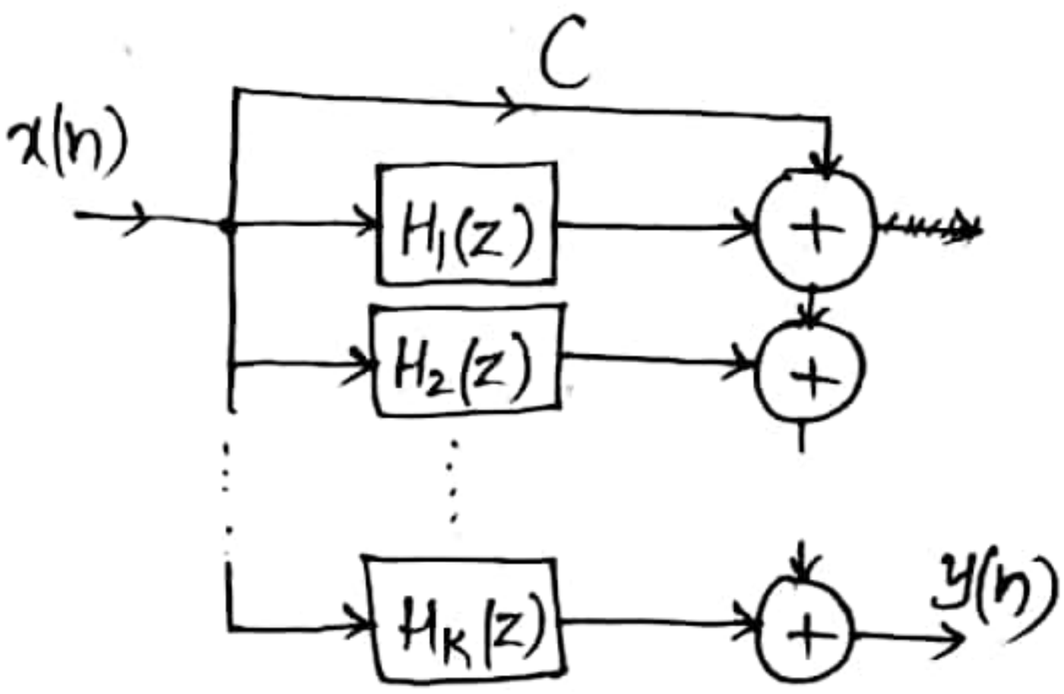


Fig:- Parallel form realization for IIR system.

Applications:-

The parallel form realization is generally used for high speed filtering applications. Because this is a parallel connection. The processing of filtering operation is performed Parallely.

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Eg:- Obtain the parallel form realization of the IIR digital filter transfer function:

$$H(z) = \frac{3(2z^2+5z+4)}{(2z+1)\cdot(z+2)}$$

Sol:- Given that,

$$H(z) = \frac{3\cdot(2z^2+5z+4)}{(2z+1)(z+2)}$$

Divide with 'z' on both sides.

$$\frac{H(z)}{z} = \frac{3\cdot(2z^2+5z+4)}{2z(z+\frac{1}{2})\cdot(z+2)} = \frac{3}{2}\cdot\left[\frac{A}{z}+\frac{B}{z+\frac{1}{2}}+\frac{C}{z+2}\right]$$

After simplification,

$$\frac{H(z)}{z} = \frac{3}{2}\cdot\left[\frac{4}{z} - \frac{8}{3(z+1/2)} + \frac{2}{3(z+2)}\right]$$

$$\Rightarrow \frac{H(z)}{z} = \frac{6}{z} - \frac{4}{z+1/2} + \frac{1}{z+2}$$

$$\Rightarrow H(z) = 6 - 4\cdot\frac{z}{z+1/2} + \frac{z}{z+2}$$

$$\Rightarrow H(z) = 6 - 4\cdot\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{1}{1+2z^{-1}}$$

Fig. shows the parallel form realization of the transfer function.

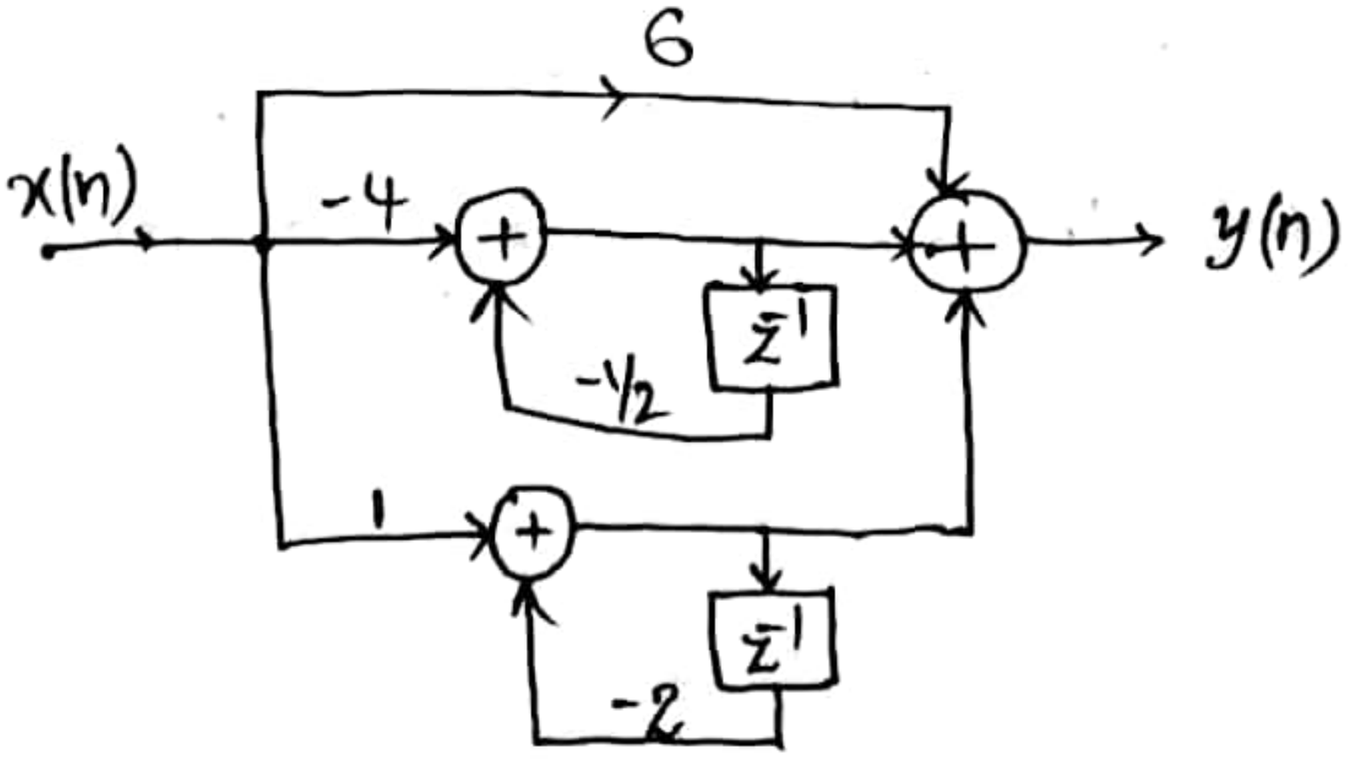


Fig :- Parallel form realization of given transfer function.

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Eg:- Realize the system given by difference equation $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$ in Parallel form.

Sol:- Given that,

$$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$$

Applying z-transform on both sides,

$$Y(z) = -0.1z^{-1}.Y(z) + 0.72z^{-2}Y(z) + 0.7X(z) - 0.252z^{-2}.X(z)$$

$$\Rightarrow Y(z).\left[1+0.1z^{-1}-0.72z^{-2}\right] = X(z).\left[0.7-0.252z^{-2}\right]$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{0.7-0.252z^{-2}}{1+0.1z^{-1}-0.72z^{-2}} = \frac{0.7z^2-0.252}{z^2+0.1z-0.72}$$

both sides divide with 'z'.

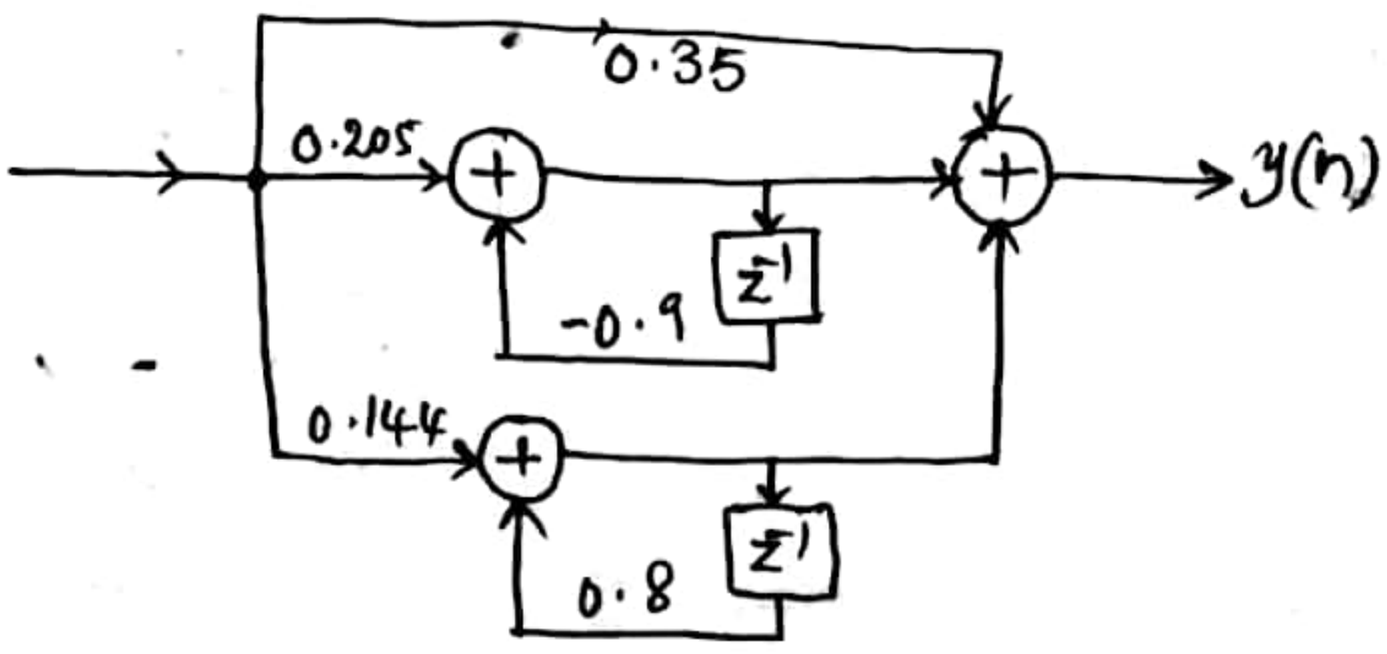
$$\Rightarrow \frac{H(z)}{z} = \frac{0.7z^2-0.252}{z.(z+0.9)(z-0.8)} = \frac{A}{z} + \frac{B}{z+0.9} + \frac{C}{z-0.8}$$

After simplification,

$$\frac{H(z)}{z} = \frac{0.35}{z} + \frac{0.205}{z+0.9} + \frac{0.144}{z-0.8}$$

$$\Rightarrow H(z) = 0.35 + \frac{0.205z}{z+0.9} + \frac{0.144z}{z-0.8}$$

$$\Rightarrow H(z) = 0.35 + \frac{0.205}{1+0.9z^{-1}} + \frac{0.144}{1-0.8z^{-1}}$$



(26) Fig:-

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Transposed forms:-

When two digital filters having same transfer function then they are called as equivalent structures. A simple way to produce an equivalent structure from a given realization structure is by the "Transpose operation". The transpose of structure is defined by the following operations:

1). Reverse the direction of all branches in the signal flow graph.
2). Interchange the inputs & outputs
3). Reverse the roles of all nodes in the flow graph
4). Summing points become branch points.
5). Branching points become summing points.

According to transposition theorem, the system transfer function remain unchanged by transposition.

W.K.T. the transfer function of second order subsystem is

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

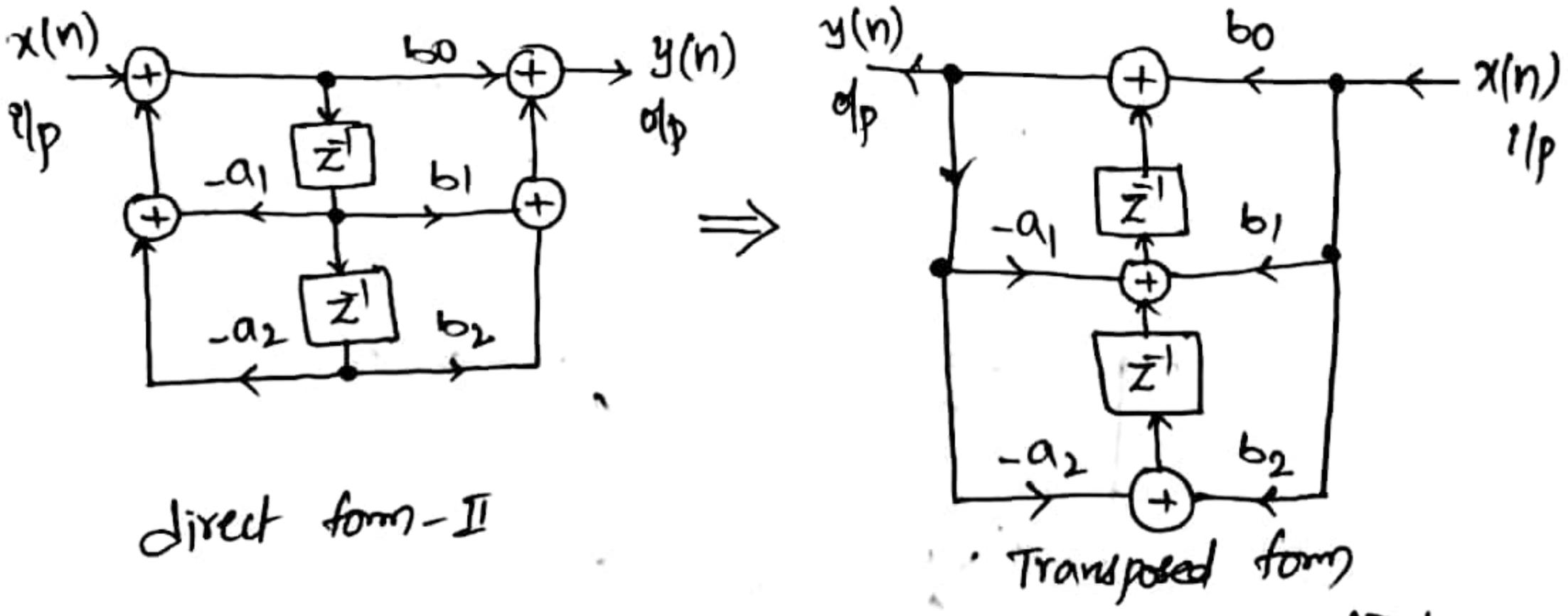


Fig:- Realization of transposed form structure from direct form-II structure.

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Eg:- Determine the direct form - II and transposed direct form - II for the given system

$$y(n) = \frac{1}{2}y(n-1) - \frac{1}{4}y(n-2) + x(n) + x(n-1)$$

Sol:- Given that

$$y(n) = \frac{1}{2}\cdot y(n-1) - \frac{1}{4}y(n-2) + x(n) + x(n-1)$$

Applying Z- transform on both sides

$$Y(z) = \frac{1}{2}z^{-1}\cdot Y(z) - \frac{1}{4}z^{-2}Y(z) + X(z) + z^{-1}X(z)$$

$$\Rightarrow Y(z)\left[1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}\right] = X(z)\cdot\left[1 + z^{-1}\right]$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}$$

The direct form - II realization for this transfer function is shown in fig.

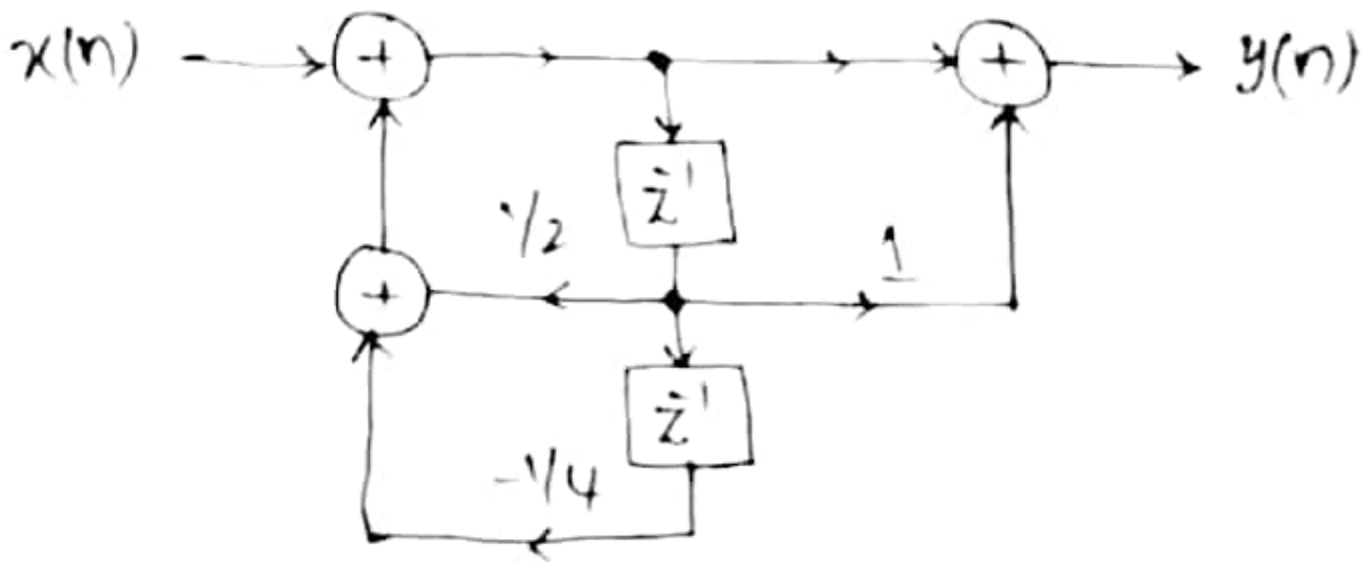


Fig:- Direct form - II realization

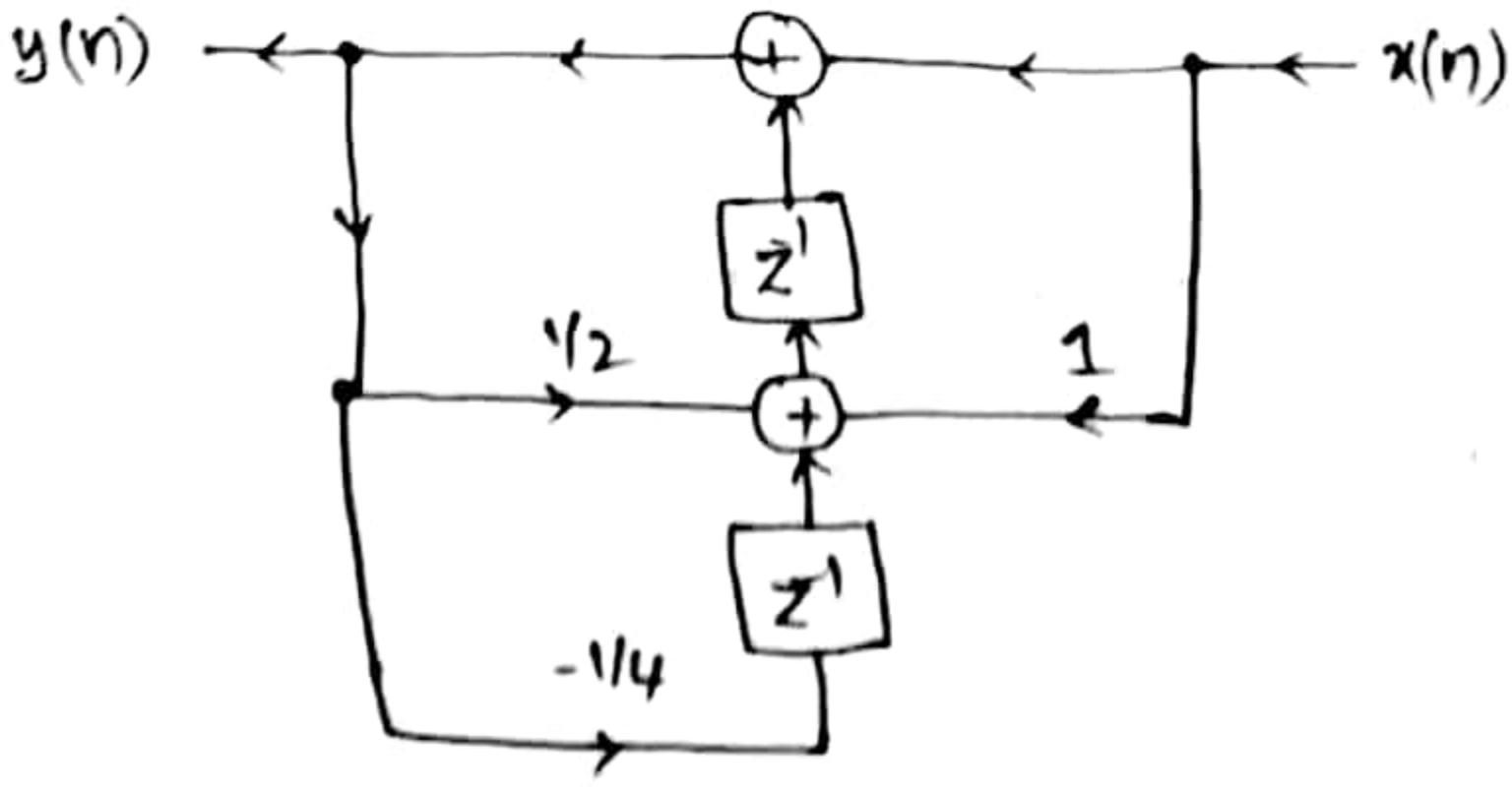


Fig:- Transposed structure

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Canonic and Non-Canonic structures:-

If the number of delays in the basic realisation block diagram is equal to the order of the difference equation (or) the order of the transfer function of a digital filter, then the realisation structure is known as "Canonic structure".

On otherhand, if the no. of delays in the structure are not same as order then it is called "Non-Canonic structure".

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Why IIR filters do not have linear phase?

W.K.T. $$H(e^{j\omega}) = |H(e^{j\omega})| \cdot e^{j\theta(\omega)}$$

we define phase delay, $\tau_p = \frac{-\theta(\omega)}{\omega}$ & group delay, $\tau_g = \frac{-d\theta(\omega)}{d\omega}$

For a linear filter, $\theta(\omega) \propto \omega$. The linear phase filter does not alter the shape of the signal. If the phase response of the filter is non-linear then the output signal can be distorted one. In several cases, a linear phase characteristics is needed throughout the passband of the filter to preserve the shape of a given signal within the passband. IIR filters cannot produce a linear phase.

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UNIT – IV

DESIGN OF FIR DIGITAL FILTERS & REALIZATIONS

Short Answer Questions:

1. Draw the indirect form realizations of FIR systems? (Apr 2018:S2)
2. Why FIR filters are always stable? (Apr 2018:S4)
3. Draw the direct form realization of FIR system. (Apr 2017:S2)
4. What are the advantages of Kaiser Window? (Apr 2017:S3)
5. What is the necessary and sufficient condition for linear phase Characteristics in FIR filter? (Apr 2017:S4)
6. Explain Gibb's phenomenon. (Dec 2016:S1)
7. What conditions are to be satisfied by the impulse response of an FIR system in order to have a linear phase? (Apr 2016:S1)

Long Answer Questions:

1. Explain the need for the use of window sequence in the design of FIR filter. Describe the window sequence generally used and compare the properties. (Apr 2018:S1)
2. Design a HPF of length 7 with cut off frequency of 2 rad/sec using Hamming window. (Apr 2018:S2)
3. Distinguish between "maximally flat magnitude response" and "equiripple magnitude response" filters. (Apr 2018:S3)
4. A low pass filter is to be designed with the following desired frequency response.

$$H_d(e^{j\omega}) = e^{-3j\omega},\ -\pi/4 \le \omega \le \pi/4$$
$$= 0,\ \pi/4 \le |\omega| \le \pi$$

Determine the filter coefficients $h_d(n)$ if the window function is defined as $\omega(n) = 1,\ 0 \le n \le 4$; $=0$, otherwise. Also determine the frequency response $H(e^{j\omega})$ of the designed filter. (Apr 2018:S4 & Apr 2016:S1)

5. Design an ideal high pass filter with a frequency response $H_d(e^{jw}) = 1$ for $\pi/4 \le |w| \le \pi$; $= 0$ for $|w| \le \pi/4$ Find the values of h(n) for N = 11 using Hamming window. Find H (z) and determine the magnitude response. (Apr 2017:S2)
6. Comparison of FIR and IIR filters. (Apr 2017:S3)
7. What is Hamming Window function? Obtain its frequency domain characteristics. (Apr 2017:S3)
8. Prove that FIR filter has linear phase if the unit impulse response satisfies the condition h(n)=h(N-1-n), n=0,1,.....M-1. Also discuss symmetric and antisymmetric cases of FIR filter. (Apr 2017:S4)
9. Realize an FIR filter with impulse response is given by
   $h(n)=(1/2)^n[u(n)-u(n-5)]$ (Dec 2016:S1)
10. The desired frequency response of a low-pass filter is

$$H_d(\omega) = \begin{cases} e^{-3j\omega}, \dfrac{-3\pi}{4} \le \omega \le \dfrac{3\pi}{4} \\ 0, \dfrac{3\pi}{4} < \omega \le \pi \end{cases}$$

Determine $H(e^{jw})$ for M=7 using a rectangular window. (Dec 2016:S1)

11. Discuss the computationally efficient implementation of interpolator in an FIR filter. (Dec 2016:S1 & Apr 2016:S3)

12. What are the effects of windowing? (Apr 2016:S1)

13. What is a Kaiser window? In what way is it superior to other window functions? (Apr 2016:S2)

14. What is a Hamming window function? Obtain its frequency domain characteristics. (Apr 2016:S3)

15. The desired frequency response of a low-pass filter is

$$H_d(\omega) = \begin{cases} 1, \frac{-\pi}{2} \le \omega \le \frac{\pi}{2} \\ 0, \frac{\pi}{2} < \omega \le \pi \end{cases}$$

Determine $h_d(n)$ for M=7 using a rectangular window. (Apr 2016:S4)

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