UNIT- II

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Introduction:-

Fourier analysis is a family of mathematical techniques, all based on decomposing signals into sinusoids.

A signal can be either continuous or discrete, and it can be either periodic or aperiodic. The combination of these two features generates the four categories.

	Continuous	Discrete
Periodic	Fourier Series (FS)	Discrete fourier transform (DFT)
Aperiodic	Fouriey transform (FT)	Discrete time fourier transform (DTFT)

1). Fourier Series:

Any waveform that repeats itself in a regular ~~form~~ pattern from negative to positive infinity. This version of fourier transform is called the "fourier series". Examples are sine waves and square waves.

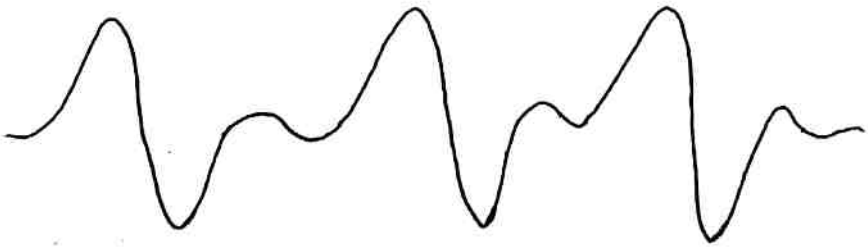


fig.1 :- fourier series - example signal

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2). Fourier Transform :-

These signals extend to both positive and negative infinity without repeating in a periodic pattern. The fourier transform of this type of signals is simply called "fourier transform".

Example : Decaying exponentials & Gaussian curve.

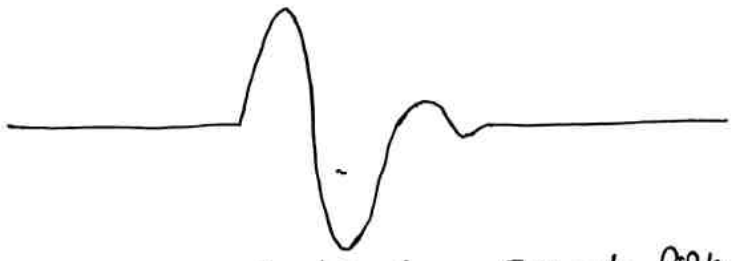


Fig:- Fourier Transform - Example signal.

3). Discrete fourier Transform :

These are discrete signals that repeat themselves in a periodic manner from negative to positive infinity. This class of fourier transforms some times called "Discrete fourier series", but is most called "Discrete fourier transforms".



Fig:- DFT - Example signal.

4). Discrete time fourier Transform :

These signals are only defined at discrete points between positive and negative infinity and do not repeat themselves in a periodic manner. This type of fourier transform is called the "Discrete time fourier Transform".



Fig:- DTFT - Example signal.

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Discrete fourier series:

* Fourier series representation of a continuous-time [C.T] periodic signal can consist of an infinite number of frequency components, whereas the frequency spacing between two successively harmonically related frequencies is $1/T_p$, where 'T_p' is fundamental time period.
* The frequency range for Discrete-time signals [D.T] is unique over $[-\pi, \pi]$ (or) $(0, 2\pi)$. A D.T. signal of fundamental period 'N' can consists of frequency components seperated by $2\pi/N$ radians (or) $f = \frac{1}{N}$ cycles. The fourier series representation of D.T. periodic signal will contain 'N' frequency components.

This is the basic difference b/w the fourier series representation for C.T & D.T. periodic signals.

Fourier Series	C.T - Periodic signal	D.T - Periodic signal
Number of frequency components	Infinite	'N'

Let $x(n)$ is a periodic sequence with period 'N', i.e., $x(n) = x(n+N)$.

The fourier series representation for $x(n)$ consists of 'N' harmonically related exponential functions,

$$e^{j2\pi Kn/N}, \quad K = 0, 1, 2, 3, \ldots N-1$$

and is expressed as

$$x(n) = \sum_{K=0}^{N-1} C_K \cdot e^{j2\pi Kn/N} \longrightarrow (1)$$

Where $\{C_K\}$ are fourier series coefficients.

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The fourier coefficients $\{C_k\}$ can be obtained by multiplying eqn.① with $e^{-j2\pi ln/N}$ and summing the product from $n=0$ to $N-1$.

Thus,

$$\sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi ln}{N}} = \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} C_k \cdot e^{j\frac{2\pi kn}{N}} \cdot e^{-j\frac{2\pi ln}{N}}$$

$$= \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} C_k \cdot e^{j2\pi (k-l)n/N}$$

Change the order of summation on right side,

$$\sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi ln}{N}} = \sum_{k=0}^{N-1} C_k \left[\sum_{n=0}^{N-1} e^{j2\pi (k-l)n/N} \right] \rightarrow ②$$

Formula:

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi kn}{N}} = \begin{cases} N & ; k = 0, \pm N, \pm 2N, \ldots \\ 0 & ; \text{otherwise} \end{cases} \rightarrow ③$$

By using ③, eqn ② written as

$$\sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi lk}{N}} = C_k \cdot N \qquad [\because k-l=0]$$

$$\Rightarrow \quad C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi kn}{N}} \rightarrow ④$$

This is the desired expression for the fourier coefficients interms of the signal $x(n)$.

Frequency analysis of Discrete-time Periodic signals	
Synthesis equation: $x(n) = \sum_{k=0}^{N-1} C_k \cdot e^{j\frac{2\pi kn}{N}}$	Analysis equation: $C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi kn/N}$

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Power density Spectrum of Periodic Signals:

W.K.T., the average power of a D.T. periodic signal with period 'N' defined as

$$P_{avg} = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 \longrightarrow (5)$$

Now, derive this expression interms of $\{c_k\}$,

$$P_{avg} = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cdot x^*(n)$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x(n) \left[\sum_{k=0}^{N-1} c_k \cdot e^{jk2\pi kn/N} \right]^*$$

$[\because$ using eqn (1)$]$

$$\Rightarrow P_{avg} = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cdot \sum_{k=0}^{N-1} c_k^* \cdot e^{-j2\pi kn/N}$$

Changing the order of summation,

$$P_{avg} = \frac{1}{N} \sum_{k=0}^{N-1} c_k^* \left[\sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi kn/N} \right]$$

Substitute eqn (4), in above expression,

$$P_{avg} = \sum_{k=0}^{N-1} c_k^* \cdot c_k = \sum_{k=0}^{N-1} |c_k|^2 \longrightarrow (6)$$

~~It can also be~~

Similarly,

$$E = \sum_{n=0}^{N-1} |x(n)|^2 = N \cdot \sum_{k=0}^{N-1} |c_k|^2 \longrightarrow (7)$$

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Properties of Discrete fourier Series:-

① . Linearity :-

Consider two periodic sequences $x_{1p}(n)$, $x_{2p}(n)$ with Period 'N', such that:

$DFS[x_{1p}(n)] = X_{1p}(K)$ and $DFS[x_{2p}(n)] = X_{2p}(K)$ then

$$DFS[a_1 \cdot x_{1p}(n) + a_2 \cdot x_{2p}(n)] = a_1 \cdot X_{1p}(K) + a_2 \cdot X_{2p}(K).$$

② . Time Shifting :-

If $x_p(n)$ is a periodic with 'N' samples and $DFS[x_p(n)] = X_p(K)$ then

$$DFS[x_p(n-m)] = e^{-j2\pi mK/N} \cdot X_p(K)$$

Where $x_p(n-m)$ is shifted version of $x_p(n)$.

③ . Symmetry property :-

If $DFS[x_p(n)] = X_p(K)$ then

$$DFS[x_p^*(n)] = X_p^*(-K) \quad ; \quad DFS[x_p^*(-n)] = X_p^*(K)$$

then and

$$DFS[Re[x_p(n)]] = DFS\left[\frac{x_p(n) + x_p^*(n)}{2}\right] = \frac{1}{2}[X_p(K) + X_p^*(-K)] = X_{pe}(K)$$

$$DFS[jim[x_p(n)]] = DFS\left[\frac{x_p(n) - x_p^*(n)}{2}\right] = \frac{1}{2}[X_p(K) - X_p^*(-K)] = X_{po}(K)$$

we can write,

$$x_p(n) = x_{pe}(n) + x_{po}(n)$$

Where $x_{pe}(n) = \frac{1}{2}[x_p(n) + x_p^*(-n)]$ and

$$x_{po}(n) = \frac{1}{2}[x_p(n) - x_p^*(-n)]$$

then

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$$DFS[x_{pe}(n)] = DFS\left[\frac{1}{2}\left(x_p(n)+x_p^*(-n)\right)\right] = \frac{1}{2}\left[X_p(k)+X_p^*(k)\right] = Re\{X_p(k)\}$$

$$DFS[x_{po}(n)] = DFS\left[\frac{1}{2}\left(x_p(n)-x_p^*(-n)\right)\right] = \frac{1}{2}\left[X_p(k)-X_p^*(k)\right] = j\,Im\{X_p(k)\}$$

④. Periodic Convolution :-

Let $x_{1p}(n)$ & $x_{2p}(n)$ be two periodic sequences with period 'N' with $DFS[x_{1p}(n)] = X_{1p}(k)$ & $DFS[x_{2p}(n)] = X_{2p}(k)$.

$$\text{If} \quad x_{3p}(n) = x_{1p}(n) * x_{2p}(n)$$

$$\Rightarrow x_{3p}(n) = \sum_{m=0}^{N-1} x_{1p}(m) \cdot x_{2p}(n-m)$$

$$\Rightarrow DFS[x_{3p}(n)] = DFS[x_{1p}(n) * x_{2p}(n)] = X_{1p}(k) \cdot X_{2p}(k)$$

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Dirchlet Conditions :-

①. The signal $x(t)$ has a finite number of discontinuities in any period

②. The signal $x(t)$ contains a finite number of maxima and minima during any period.

③. The signal $x(t)$ is absolutely integrable in any period, that is,

$$\int_{T_p} |x(t)|\,dt < \infty$$

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Fourier Transform:-

If $x(n)$ is a discrete sequence then

$$X(\omega) = F[x(n)] = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n} \longrightarrow (1)$$

For causal systems,

$$\Rightarrow x(n) = \frac{1}{N} \sum_{k=0} \quad X(\omega) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}$$

$$\Rightarrow x(n) = \frac{1}{N} \sum_{\omega=0}^{2\pi} X(\omega) e^{j\omega n} \longrightarrow (2)$$

Discrete fourier transform:-

When the input signal is sampled at discrete intervals of time $x(n)$ and discrete frequency components $X(k)$. This formulation is known as "Discrete fourier transform [DFT]".

It is expressed as

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi kn}{N}} \longrightarrow (3)$$

Eqn (3). is also known as "N-Point DFT".

Note:-

In fourier transform, 'ω' has many values. If we use a digital computer to compute 'N' equally spaced points over the interval $0 \le \omega \le 2\pi$, then 'N' points should be located at

$$\omega_k = \frac{2\pi k}{N}, \quad k = 0, 1, 2, \ldots N-1 \longrightarrow (4)$$

The N-equally spaced frequency samples of DTFT are known as DFT denoted by $X(k)$.

$$X(k) = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}}, \quad 0 \le k \le N-1 \longrightarrow (5)$$

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From eqn ③, we obtain discrete sequence $x(n)$ as

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j2\pi Kn/N} \longrightarrow ⑥$$

This is known as "Inverse discrete fourier transform [IDFT]" (or) "N-Point IDFT".

Twiddle factor [W_N]:

Let us define $W_N = e^{\frac{-j2\pi}{N}}$ $\longrightarrow ⑦$

This is called as "Twiddle factor". Twiddle factor makes the computation of DFT a bit easy and fast.

using twiddle factor, eqn ③ & ⑥ can be written as

DFT : $$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK} \longrightarrow ⑧$$

IDFT : $$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) W_N^{-nK} \longrightarrow ⑨$$

- The integer 'n' is known as time index since it denotes the time instant.
- The integer 'K' denotes discrete frequency and is called frequency index.

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Eg-1:- Find the DFT of a sequence $x(n)=\{1,1,0,0\}$.

Sol:- We know that $X(K)=\sum_{n=0}^{N-1} x(n) W_N^{-nK} = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi Kn}{N}}$ → (1)

Given that, length of the sequence is 4, i.e., $N=4$

and $x(0)=1,\ x(1)=1,\ x(2)=0,\ x(3)=0$

Substitute $K=0$ in eqn (1).

$$X(0)=\sum_{n=0}^{3} x(n)\cdot e^{0} = x(0)+x(1)+x(2)+x(3)$$
$$= 1+1+0+0$$
$$= 2$$

If $K=1$, $X(1)=\sum_{n=0}^{3} x(n)\cdot e^{\frac{-j2\pi n}{4}} = x(0)+x(1)\cdot e^{\frac{-j2\pi}{4}}+x(2)\cdot e^{\frac{-j4\pi}{4}}+x(3)\cdot e^{\frac{-j6\pi}{4}}$

$$= 1+1\cdot\left[\cos\tfrac{\pi}{2}-j\cdot\sin\pi/2\right]$$
$$= 1+1[0-j]$$
$$= 1-j$$

If $K=2$, $X(2)=\sum_{n=0}^{3} x(n)\cdot e^{-j2\pi n/4} = \sum_{n=0}^{3} x(n)\cdot e^{-j\pi n}$

$$= x(0)+x(1)e^{-j\pi}+x(2)e^{-j2\pi}+x(3)e^{-j3\pi}$$
$$= 1+1[\cos\pi-j\sin\pi]$$
$$= 1-1$$
$$= 0$$

If $K=3$, $X(3)=\sum_{n=0}^{3} x(n)\cdot e^{-j6\pi n/4} = \sum_{n=0}^{3} x(n)\cdot e^{-j3\pi n/2}$

$$= x(0)+x(1)\cdot e^{-j3\pi/2}+x(2)\cdot e^{-j3\pi}+x(3)\cdot e^{-j9\pi/2}$$
$$= 1+1\left[\cos 3\pi/2-j\sin 3\pi/2\right]$$
$$= 1+j$$

$$\therefore\ X(K)=\{2,\ 1-j,\ 0,\ 1+j\}$$

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Eg-2:- Find the DFT of the sequence $x[n]=\{1,2,3,4,5,6,7,8\}$.

Sol:- Given that, $x[n]=\{1,2,3,4,5,6,7,8\}$

Length of the sequence, $N=8$

W.K.T, $X(K)=\sum_{n=0}^{N-1} x(n)\cdot e^{-j2\pi Kn/N} = \sum_{n=0}^{7} x(n)\cdot e^{-j\frac{\pi Kn}{4}}$

If $K=0$,

$$X(0)=\sum_{n=0}^{7} x(n)\cdot e^{0} = x(0)+x(1)+x(2)+x(3)+x(4)+x(5)+x(6)+x(7)$$
$$= 1+2+3+4+5+6+7+8$$
$$= 36$$

If $K=1$,

$$X(1)=\sum_{n=0}^{7} x(n)\, e^{-j\pi n/4} = x(0)+x(1)\cdot e^{-j\pi/4}+x(2)\cdot e^{-j\pi/2}+x(3)\cdot e^{-j3\pi/4}+x(4)\cdot e^{-j\pi}$$
$$+x(5)\cdot e^{-j5\pi/4}+x(6)\cdot e^{-j3\pi/2}+x(7)\cdot e^{-j7\pi/4}$$

$$= 1+2\cdot[0.707-j0.707]+3\cdot[-j]+4\cdot[-0.707-j0.707]+5(-1)+$$
$$6[-0.707+j0.707]+7[j]+8[0.707+j0.707]$$

$$= -4+9.656j$$

If $K=2$, $X(2)=\sum_{n=0}^{7} x(n)\cdot e^{-j\pi n/2} = x(0)+x(1)\cdot e^{-j\pi/2}+x(2)e^{-j\pi}+x(3)\cdot e^{-j3\pi/2}+x(4)\cdot e^{-j2\pi}+$
$$x(5)\cdot e^{-j5\pi/2}+x(6)e^{-j3\pi}+x(7)\cdot e^{-j7\pi/2}$$

$$= 1+2[-j]+3[-1]+4[j]+5[j]+6[-j]+7[-1]+8[j]$$

$$= -4+4j$$

If $K=3$, $X(3)=\sum_{n=0}^{7} x(n)\cdot e^{-j3\pi n/4} = x(0)+x(1)\cdot e^{-j3\pi/4}+x(2)\cdot e^{-j6\pi/4}+x(3)e^{-j9\pi/4}+$
$$x(4)\, e^{-j12\pi/4}+x(5)\cdot e^{-j15\pi/4}+x(6)\cdot e^{-j18\pi/4}+x(7)\cdot e^{-j21\pi/4}$$

$$= 1+2\cdot[-0.707-0.707j]+3[j]+4[0.707-0.707j]+$$
$$5[-1]+6[0.707+0.707j]+7[-j]+8[-0.707+0.707j]$$

$$= -4+1.656j$$

If $K=4$, $X(4)=\sum_{n=0}^{7} x(n)\cdot e^{-j\pi n} = x(0)+x(1)e^{-j\pi}+x(2)e^{-j2\pi}+x(3)e^{-j3\pi}+x(4)e^{-j4\pi}$
$$+x(5)e^{-j5\pi}+x(6)e^{-j6\pi}+x(7)e^{-j7\pi}$$

$$= 1+2[-1]+3[1]+4[-1]+5[1]+6[-1]+7[1]+8[-1]$$

$$= -4$$

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If $K=5$,

$$X(5)=\sum_{n=0}^{7} x(n)\cdot e^{-j5\pi n/4} = x(0)+x(1)e^{-j5\pi/4}+x(2)e^{-j10\pi/4}+x(3)e^{-j15\pi/4}+x(4)e^{-j20\pi/4}+$$
$$x(5)e^{-j25\pi/4}+x(6)e^{-j30\pi/4}+x(7)\cdot e^{-j35\pi/4}$$

$$= 1+2[-0.707+0.707j]+3[-j]+4[0.707+0.707j]+$$
$$5[-1]+6[0.707-0.707j]+7[j]+8[-0.707-0.707j]$$

$$= -4-1.656j$$

If $K=6$,

$$X(6)=\sum_{n=0}^{7} x(n)\cdot e^{-j6\pi n/4} = \sum_{n=0}^{7} x(n)\cdot e^{-j3\pi n/2} = x(0)+x(1)e^{-j3\pi/2}+x(2)e^{-j3\pi}+x(3)e^{-j9\pi/2}$$
$$+x(4)e^{-j6\pi}+x(5)\cdot e^{-j15\pi/2}+x(6)\cdot e^{-j9\pi}+x(7)\cdot e^{-21\pi/2}$$

$$= 1+2[j]+3[-1]+4[-j]+5[1]+6[j]+7[-1]+8[-j]$$

$$= -4-4j$$

If $K=7$,

$$X(7)=\sum_{n=0}^{7} x(n)\cdot e^{-j7\pi n/4} = x(0)+x(1)\cdot e^{-j7\pi/4}+x(2)\cdot e^{-j14\pi/4}+x(3)\cdot e^{-j21\pi/4}+x(4)e^{-j28\pi/4}$$
$$+x(5)\cdot e^{-j35\pi/4}+x(6)e^{-j42\pi/4}+x(7)\cdot e^{-j49\pi/4}$$

$$= 1+2[0.707+0.707j]+3[j]+4[-0.707+0.707j]+5[-1]+$$
$$6[-0.707-0.707j]+7[-j]+8[0.707-0.707j]$$

$$= -4-9.656j$$

$$\therefore X(K)=\{-36,\ -4+9.656j,\ -4+4j,\ -4+1.656j,\ -4,$$
$$-4-1.656j,\ -4-4j,\ -4-9.656j\}$$

$$=0=$$

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Eg-3:- Compute N-Point DFT of following sequences.

i) $x(n) = \delta(n-n_0)$, where $0 \le n_0 \le N$

Sol:- W.K.T., $X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi Kn/N}$

$x(n) = \delta(n-n_0) = 1 \; ; n = n_0$

$= 0 \; ; n \neq n_0$

$\therefore X(K) = 1 \cdot e^{-j2\pi Kn_0/N}$

ii). $x(n) = a^n$, $0 \le n \le N$

Sol:- $X(K) = \sum_{n=0}^{N-1} a^n \cdot e^{-j2\pi Kn/N} = \sum_{n=0}^{N-1} \left[a \cdot e^{-j2\pi K/N}\right]^n$

$= \dfrac{1 - \left[a \cdot e^{-j2\pi K/N}\right]^N}{1 - a \cdot e^{-j2\pi K/N}}$ $\left[\because \sum_{n=0}^{N} a^n = \dfrac{1-a^n}{1-a}\right]$

$= \dfrac{1 - a^N \cdot e^{-j2\pi K}}{1 - a \cdot e^{-j2\pi K/N}}$

W.K.T. $e^{-j2\pi K} = \cos 2\pi K - j \sin 2\pi K = 1 - 0 = 1$ [$\because$ 'K' is an integer]

$\Rightarrow X(K) = \dfrac{1 - a^N}{1 - a \cdot e^{-j2\pi K/N}}$

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Eg-4:- Find the DFT of a sequence, $x(n) = 1 \; ; 0 \le n \le 2$

$= 0$; otherwise

for $N = 4$ · plot $|X(K)|$ & $\angle X(K)$. Comment on result.

Sol:- Given that, $x(n) = 1 \; ; 0 \le n \le 2$

$= 0$; otherwise

i.e., $x(n) = \{1, 1, 1\}$

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Length of the given sequence, $L=3$.

For $N=4$, the periodic extension of $x(n)$ can be obtained by adding zero (i.e., zero padding).

$$\therefore x(n) = \{1, 1, 1, 0\}$$

$$\therefore X(K) = \sum_{n=0}^{3} x(n) \cdot e^{-j\frac{\pi nk}{2}}$$

If $K=0$, $X(0) = \sum_{n=0}^{3} x(n) = 1+1+1+0 = 3$

If $K=1$, $X(1) = \sum_{n=0}^{3} x(n) \cdot e^{-j\pi n/2} = -j$

If $K=2$, $X(2) = \sum_{n=0}^{3} x(n) \cdot e^{-j\pi n} = 1$

If $K=3$, $X(3) = \sum_{n=0}^{3} x(n) \cdot e^{-j3\pi n/2} = j$

$$\therefore X(K) = \{3, -j, 1, j\}$$

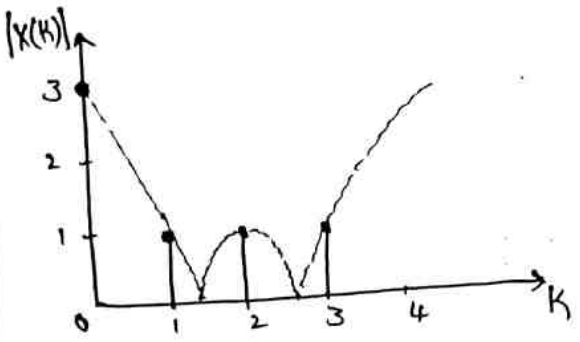
$\left[\begin{array}{l} a+jb \\ M = \sqrt{a^2+b^2} \\ \theta = \tan^{-1}(b/a) \end{array}\right]$

Magnitude:

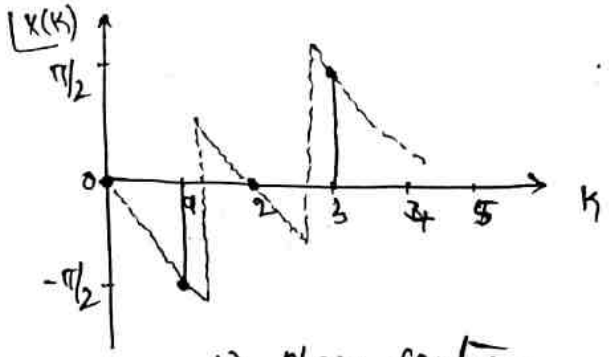
$$|X(K)| = \{3, 1, 1, 1\}$$

Phase:-

$$\angle X(K) = \{0, -\pi/2, 0, \pi/2\}$$



a). Magnitude spectrum



b) phase spectrum

Observe that, it is extrapolate the entire frequency spectrum with $N=4$. That is the resolution of the spectrum is very poor. In order to increase the resolution, we must increase 'N'.

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Eg-4: Find the IDFT of $Y(k)=\{1,0,1,0\}$.

Sol: Given that, $Y(k)=\{1,0,1,0\}$

Here $N=L=4$.

W.K.T. $y(n)=\frac{1}{N}\sum_{k=0}^{N-1} Y(k)\cdot e^{j2\pi kn/N}=\frac{1}{4}\sum_{k=0}^{3} Y(k)\cdot e^{j\frac{\pi kn}{2}}$

If $n=0$,

$$y(0)=\frac{1}{4}\sum_{k=0}^{3} y(k)=\frac{1}{4}[Y(0)+Y(1)+Y(2)+Y(3)]=\frac{1}{4}[1+0+1+0]=1/2$$

If $n=1$,

$$y(1)=\frac{1}{4}\sum_{k=0}^{3} Y(k)e^{j\pi k/2}=\frac{1}{4}\left[Y(0)+Y(1)e^{j\pi/2}+Y(2)e^{j\pi}+Y(3)e^{j3\pi/2}\right]$$

$$=\frac{1}{4}[1+0+1[-1]+0]=0$$

If $n=2$,

$$y(2)=\frac{1}{4}\sum_{k=0}^{3} Y(k)e^{j\pi k}=\frac{1}{4}\left[Y(0)+Y(1)e^{j\pi}+Y(2)e^{j2\pi}+Y(3)e^{j3\pi}\right]$$

$$=\frac{1}{4}[1+0+1[1]+0]=1/2$$

If $n=3$,

$$y(3)=\frac{1}{4}\sum_{k=0}^{3} Y(k)e^{j\frac{3\pi k}{2}}=\frac{1}{4}\left[Y(0)+Y(1)e^{j3\pi/2}+Y(2)e^{j3\pi}+Y(3)e^{j9\pi/2}\right]$$

$$=\frac{1}{4}[1+0+1[-1]+0]=0$$

$$\therefore y(n)=\left\{\frac{1}{2},0,\frac{1}{2},0\right\}$$

$$=0=$$

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Properties of discrete fourier transform :-

① Linearity :-

If $x_1(n) \leftrightarrow X_1(K)$, $x_2(n) \leftrightarrow X_2(K)$ then

$$a \cdot x_1(n) + b \cdot x_2(n) \xleftrightarrow{DFT} a \cdot X_1(K) + b \cdot X_2(K)$$

Proof :- Let $x(n) = a \cdot x_1(n) + b \cdot x_2(n)$

From def. $$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi Kn/N} \longrightarrow (1)$$

$$= \sum_{n=0}^{N-1} \left[a \cdot x_1(n) + b \cdot x_2(n)\right] \cdot e^{-j2\pi Kn/N}$$

$$= a \cdot \sum_{n=0}^{N-1} x_1(n) \cdot e^{-j2\pi Kn/N} + b \cdot \sum_{n=0}^{N-1} x_2(n) \cdot e^{-j2\pi Kn/N}$$

$$= a \cdot X_1(K) + b \cdot X_2(K)$$

$\therefore$ $DFT\left[a \cdot x_1(n) + b \cdot x_2(n)\right] = a \cdot X_1(K) + b \cdot X_2(K)$.

② Periodicity :-

If $x(n) \xleftrightarrow[N]{DFT} X(K)$ then $x(n+N) = x(n)$ for all 'n'

$X(K+N) = X(K)$ for all 'K'

Proof :-

From def. eqn ①, replacing 'K' by 'K+N'.

$$X(K+N) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi (K+N)n/N}$$

$$= \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi Kn/N} \cdot e^{-j2\pi Nn/N}$$

Here 'n' is an integer, so $e^{-j2\pi n} = \cos 2\pi n - j \sin 2\pi n = 1$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi Kn/N}$$

$$= X(K)$$ [$\because$ from eqn ①]

$$\Rightarrow X(K+N) = X(K)$$

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③. Shifting property (or) Circular time shift :-

If $x(n) \xleftrightarrow[N]{DFT} X(K)$ then $x(n-l) \xleftrightarrow[N]{DFT} e^{\frac{-j2\pi Kl}{N}} \cdot X(K)$

Proof :-

W.K.T, $x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j2\pi Kn/N}$

~~multiplying both sides with~~ $e^{-j2\pi lK/N}$,

$x(n) \cdot e^{-j2\pi lK/N} = \frac{1}{N} \sum_{K=0}^{N-1}$

replace 'n' by 'n-l',

$$x(n-l) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j2\pi K(n-l)/N}$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{\frac{j2\pi Kn}{N}} \cdot e^{-j2\pi Kl/N}$$

$$= e^{\frac{-j2\pi Kl}{N}} \left[\frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{j2\pi Kn/N} \right]$$

$$= e^{-j2\pi Kl/N} \cdot X(K)$$

Therefore, $DFT[x(n-l)] = e^{-j2\pi Kl/N} \cdot X(K)$

④. Circular frequency shift :-

If $x(n) \xleftrightarrow[N]{DFT} X(K)$ then $DFT[x(n) e^{j2\pi ln/N}] = X(K-l)$

Proof :- using eqn ①,

$$DFT\left[x(n) e^{j2\pi ln/N}\right] = \sum_{n=0}^{N-1} x(n) \cdot e^{j2\pi ln/N} \cdot e^{-j2\pi Kn/N}$$

$$= \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi (K-l)n/N}$$

$$= X(K-l)$$

Therefore, $x(n) \cdot e^{j2\pi ln/N} \xleftrightarrow[N]{DFT} X(K-l)$

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⑤ Multiplication of two sequences :-

If $x_1(n) \xleftrightarrow{DFT} X_1(K)$, $x_2(n) \xleftrightarrow{DFT} X_2(K)$ then

$$x_1(n) \cdot x_2(n) \xleftrightarrow{DFT} \frac{1}{N}\left[X_1(K) \textcircled{N} X_2(K)\right].$$

Where $\textcircled{N}$ means convolution.

⑥ Circular Convolution :-

If $x_1(n) \xleftrightarrow{DFT} X_1(K)$, $x_2(n) \xleftrightarrow{DFT} X_2(K)$ then

$$DFT\left[x_1(n) \textcircled{N} x_2(n)\right] = X_1(K) \cdot X_2(K).$$

⑦ Parseval's theorem :-

If $x_1(n) \xleftrightarrow{DFT} X_1(K)$, $x_2(n) \xleftrightarrow{DFT} X_2(K)$ then

$$\sum_{n=0}^{N-1} x_1(n)\, x_2^*(n) = \frac{1}{N} \sum_{K=0}^{N-1} X_1(K) X_2^*(K).$$

=0=

Eg-5 :- Show that with $x(n)$ as an N-point sequence and $X(K)$ as its N-point DFT.

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{K=0}^{N-1} |X(K)|^2$$

Sol :-

$$\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n)\, x^*(n)$$

$$= \sum_{n=0}^{N-1} x(n) \cdot \left[\frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j2\pi Kn/N}\right]^*$$

$$= \sum_{n=0}^{N-1} x(n) \cdot \left[\frac{1}{N} \sum_{K=0}^{N-1} X^*(K)\, e^{-j2\pi Kn/N}\right]$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X^*(K) \cdot \left[\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi Kn/N}\right]$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} X^*(K) \cdot X(K)$$

$$= \frac{1}{N} \sum_{K=0}^{N-1} |X(K)|^2$$

=0=

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Linear Convolution :-

The output $y(n)$ is obtained by taking convolution of $x(n)$ and $h(n)$. This is called "Linear Convolution".

Therefore, $y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

Computational methods of linear convolution :-

1). Using Mathematical equation
2). Graphical method
3). Tabulation method
4). Diagonal method.

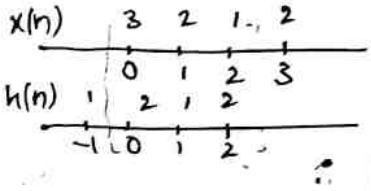
Eg-6 :- Determine the convolution sum of two sequences

$x(n) = \{3, 2, 1, 2\}$ and $h(n) = \{1, 2, 1, 2\}$ (↑ at 2)

Sol :- Given that, $x(n) = \{3, 2, 1, 2\}$ & $h(n) = \{1, 2, 1, 2\}$ (↑ at 1)

Here $x(n)$ starts at $n_1 = 0$ & $h(n)$ starts at $n_2 = -1$.

Therefore, the starting time for evaluating output sequence is $n = n_1 + n_2 = -1$.



i). Using Mathematical equation :-

$$y(n) = \sum_{k=-\infty}^{\infty} x(k).h(n-k)$$

If $n=-1$, $y(-1) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k) = x(0).h(-1) + x(1).h(-2) + \cdots$

$= 3(1) + 0$

$= 3$

If $n=0$, $y(0) = \sum_{k=-\infty}^{\infty} x(k).h(-k) = x(-1)h(1) + x(0)h(0) + x(1)h(-1) + x(2)h(-2)$

$= 0(1) + 3(2) + 2(1) + 1(0)$

$= 8$

If $n=1$, $y(1) = \sum_{k=-\infty}^{\infty} x(k).h(1-k) = \cdots + x(-1)h(2) + x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2) + \cdots$

$= 0(2) + 3(1) + 2(2) + 1(1) + 2(0)$

$= 8$

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If $n=2$, $y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k) = x(-1)h(3)+x(0)h(2)+x(1)h(1)+x(2)h(0)+$
$x(3)h(-1)$

$= 0+3(2)+2(1)+2(2)+0$

$= 12$

If $n=3$, $y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k) = x(-1)h(4)+x(0)h(3)+x(1)h(2)+x(2)h(1)+$
$x(3)h(0)$

$= 0+3(0)+2(2)+1(1)+0+2(2)$

$= 9$

If $n=4$, $y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k) = x(-1)h(5)+x(0)h(4)+x(1)h(3)+x(2)h(2)+$
$x(3)h(1)+x(4)h(0)$

$= 0+3(0)+2(0)+1(2)+2(1)+0$

$= 4$

If $n=5$, $y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k) = x(-1)h(6)+x(0)h(5)+x(1)h(4)+x(2)h(3)+$
$x(3)h(2)+x(4)h(1)+x(5)h(0)$

$= 0+0+0+0+2(2)+0+0$

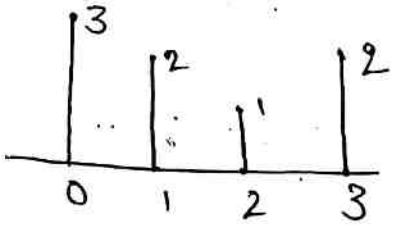
$= 4$

$\therefore y(n) = \{3, 8, 8, 12, 9, 4, 4\}$
$\uparrow$

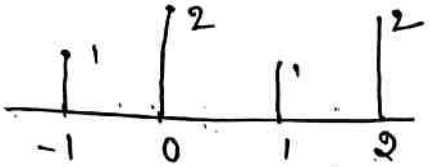
ii) <u>using graphical method:-</u>

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$x(n) = \{3, 2, 1, 2\}$
$\uparrow$

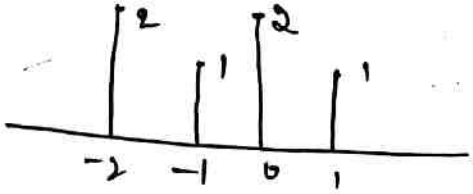


$h(n) = \{1, 2, 1, 2\}$
$\uparrow$



folding

$h(-n) = \{2, 1, 2, 1\}$

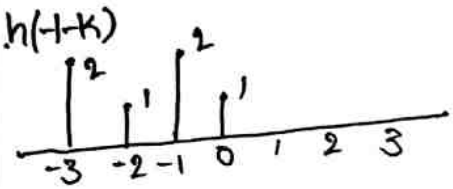


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If $n=-1$,

$$y(-1)=\sum_{k=-\infty}^{\infty} x(k)\,h(-1-k)$$

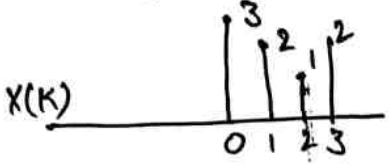


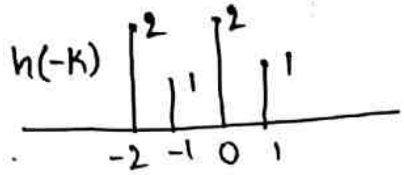


$\therefore y(-1)=3\times1=3$

If $n=0$,

$$y(0)=\sum_{k=-\infty}^{\infty} x(k)\,h(-k)$$

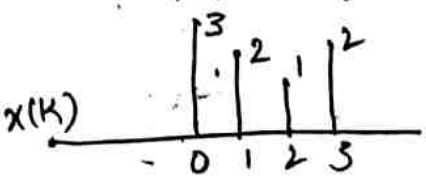


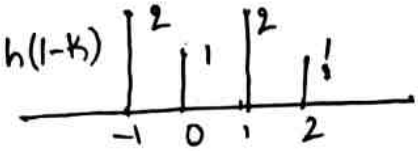


$y(0)=(3\times2)+(2\times1)=8$

If $n=1$

$$y(1)=\sum_{k=-\infty}^{\infty} x(k)\,h(1-k)$$



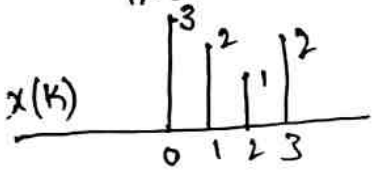


$y(1)=(3\times1)+(2\times2)+(1\times1)$

$=8$

If $n=2$

$$y(2)=\sum_{k=-\infty}^{\infty} x(k)\,h(2-k)$$



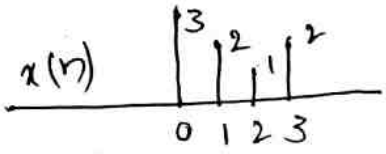


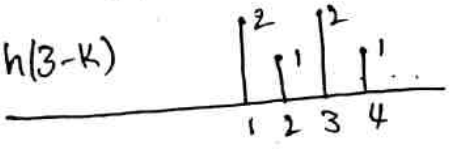
$y(2)=(3\times2)+(2\times1)+(1\times2)+(2\times1)$

$=12$

If $n=3$

$$y(3)=\sum_{k=-\infty}^{\infty} x(k)\,h(3-k)$$



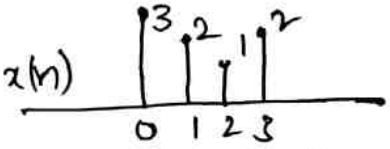


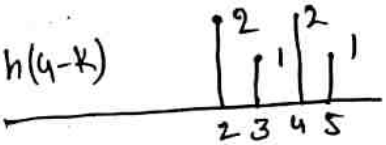
$y(3)=(2\times2)+(1\times1)+(2\times2)$

$=9$

If $n=4$

$$y(4)=\sum_{k=-\infty}^{\infty} x(k)\,h(4-k)$$



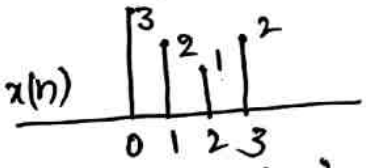


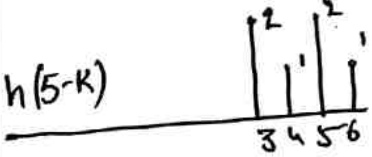
$y(4)=(1\times2)+(2\times1)$

$=4$

If $n=5$,

$$y(5)=\sum_{k=-\infty}^{\infty} x(k)\,h(5-k)$$





$y(5)=2\times2$

$=4$

$\therefore y(n)=\{3, 8, 8, 12, 9, 4, 4\}$

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iii) using tabulation method :-

K		-3	-2	-1	0	1	2	3	4	5	6
x(K)					3	2	1	2			
n=-1	h(-1-K)	2	1	2	1						
n=0	h(0-K)		2	1	2	1					
n=1	h(1-K)			2	1	2	1				
n=2	h(2-K)				2	1	2	1			
n=3	h(3-K)					2	1	2	1		
n=4	h(4-K)						2	1	2	1	
n=5	h(5-K)							2	1	2	1

$y(-1) = 3\times1 = 3$; $y(0) = (3\times2)+(2\times1) = 8$; $y(1) = (3\times1)+(2\times2)+(1\times1) = 8$

$y(2) = (3\times2)+(2\times1)+(1\times2)+(2\times1) = 12$; $y(3) = (2\times2)+(1\times1)+(2\times2) = 9$;

$y(4) = (1\times2)+(2\times1) = 4$; $y(5) = 2\times2 = 4$

$\therefore y(n) = \{3, 8, 8, 12, 9, 4, 4\}$

iv). using Diagonal method :-

- Divide the elements in diagonal
- starting from left, sum all the elements in each strip

x(n) → / h(n) ↓	3	2	1	2
1	3	2	1	2
2	6	4	2	4
1	3	2	1	2
2	6	4	2	4

$y(n) = \{3, 8, 8, 12, 9, 4, 4\}$

=0=

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Circular Convolution:

If $x(n)$ contains 'L' no. of samples and $h(n)$ contains 'M' no. of samples then we perform circular convolution between the two using $N = \max(L, M)$,

by adding $(L-M)$ no. of zero samples to $h(n)$ if $L > M$.

So both sequences are periodic with 'N'.

There are two methods used to find circular convolution

i). Concentric circle method

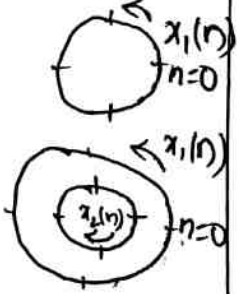
ii). Matrix Multiplication method.

i). Concentric circle method:

Given $x_1(n)$ & $x_2(n)$ are two sequences, the circular convolution of these two sequences $x_3(n) = x_1(n) \textcircled{N} x_2(n)$ can be found by using following steps:

→ Graph 'N' samples of $x_1(n)$ as equally spaced points around an outer circle in counterclockwise direction

→ start at samp point as $x_1(n)$ graph 'N' samples of $x_2(n)$ as equally spaced points around an inner circle in clockwise direction.



→ Multiply corresponding samples on two circles & sum the products to produce output.

→ Rotate the inner circle one sample at a time in counterclockwise direction and go to step 3: to obtain next value of output.

→ Repeat step 4, until the inner circle comes into initial state.

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ii). Matrix Multiplication method:-

$$\begin{bmatrix} x_3(0) \\ x_3(1) \\ \vdots \\ x_3(N-1) \end{bmatrix} = \begin{bmatrix} x_2(0) & x_2(N-1) & x_2(N-2) & \cdots & x_2(1) \\ x_2(1) & x_2(0) & x_2(N-1) & \cdots & x_2(2) \\ \vdots & & & & \\ x_2(N-1) & x_2(N-2) & x_2(N-3) & \cdots & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ \vdots \\ x_1(N-1) \end{bmatrix}$$

The sequence $x_2(n)$ is repeated via circular shift of samples and represented in $N \times N$ matrix.

Eg-7:- Find the circular convolution of two finite duration sequences $x_1(n) = \{1, -1, -2, 3, -1\}$, $x_2(n) = \{1, 2, 3\}$

Sol:- Given that, $x_1(n) = \{1, -1, -2, 3, -1\}$, $L = 5$

$x_2(n) = \{1, 2, 3\}$ $M = 3$

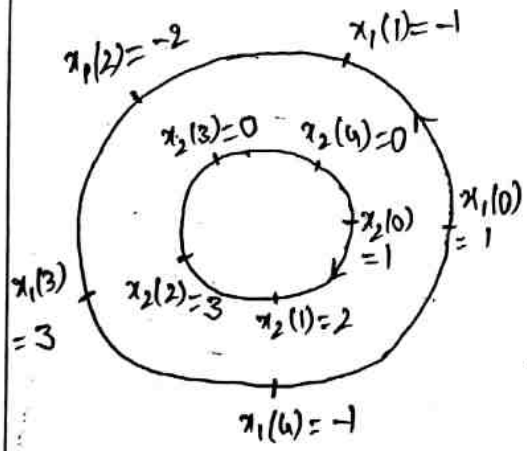
$\therefore L > M$, $N = \max(L, M) = 5$ so $(L-M) = 2$ zeros

samples adding to $x_2(n)$,

$x_2(n) = \{1, 2, 3, 0, 0\}$

a). using circular convolution method:-

If $n=0$

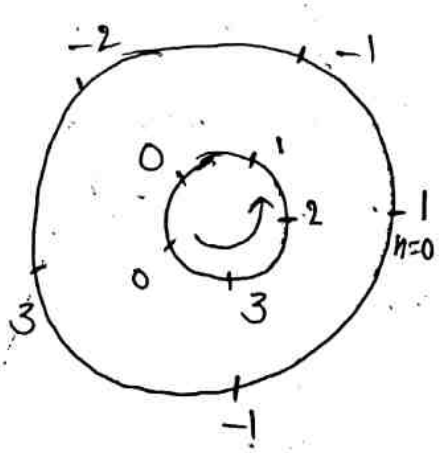


$y(0) = (1\times1) + (-1\times0) + (-2\times0) + (3\times3) + (-1\times2)$

$= 1 + 9 - 2$

$= 8$

If $n=1$

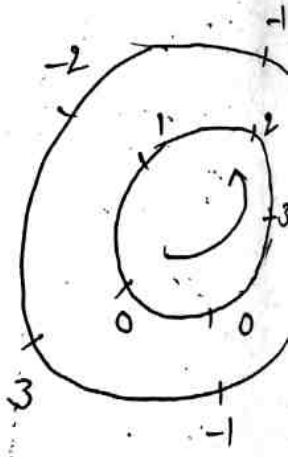


$y(1) = (1\times2) + (-1\times1) + (-2\times0) + (3\times0) + (-1\times3)$

$= 2 - 1 - 3$

$= -2$

If $n=2$



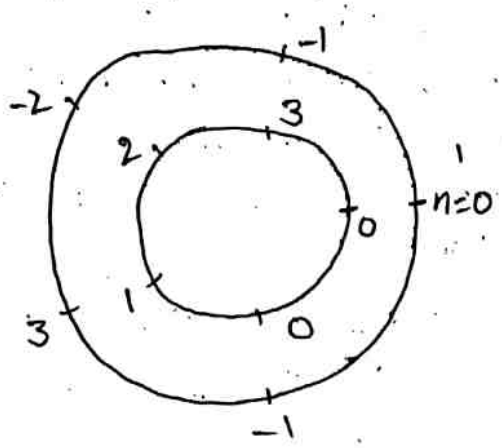
$y(2) = (1\times3) + (-1\times2) + (-2\times1) + (3\times0) +$

$= 3 - 2 - 2$

$= -1$

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If $n=4$

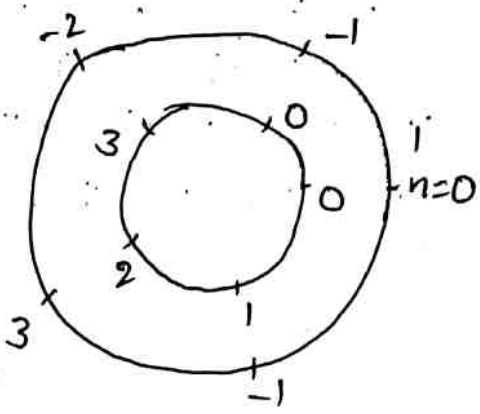


$y(4) = (0\times0)+(-1\times3)+(-2\times2)+(3\times1)+(-1\times0)$

$= -3-4+3$

$= -4$

If $n=5$



$y(5) = (1\times0)+(-1\times0)+(-2\times3)+(3\times2)+(-1\times1)$

$= -6+6-1$

$= -1$

$\therefore y(n) = \{8, -2, -1, -4, -1\}$

b). Using Matrix Multiplication method :-

$$\begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \\ x_3(4) \end{bmatrix} = \begin{bmatrix} x_2(0) & x_2(4) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(4) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(4) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) & x_2(4) \\ x_2(4) & x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \\ x_1(4) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ -2 \\ -1 \\ -4 \\ -1 \end{bmatrix}$$

$\therefore x_3(n) = \{8, -2, -1, -4, -1\}$

$=0=$

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Linear Convolution	Circular Convolution
①. If $x(n)$ is a sequence of 'L' no. of samples & $h(n)$ with 'M' no. of samples, after Convolution $y(n)$ will contain $N = L+M-1$ samples	①. If $x(n)$ is a sequence of 'L' no. of samples & $h(n)$ with 'M' no. of samples, after convolution $y(n)$ will contain $N = \max(L,M)$ samples.
②. Linear convolution can be used to find the response of a linear filter	②. Circular convolution can't be used to find the response of a linear filter.
③. Zero padding is not necessary to find the response of a linear filter.	③. Zero padding is necessary to find the response of a filter.

=o=

Eg-8:- Use the four point DFT & IDFT to determine the circular convolution of following sequences:

$$x_1(n) = \{1, 2, 3, 1\}, \quad x_2(n) = \{4, 3, 2, 2\}$$

Sol:- W.K.T, $x_1(n) \xleftrightarrow[N]{DFT} X_1(K)$

It can also be expressed as

$$[X_1(K)] = [W_4][x_1(n)]$$

Where $W_4 = W_4^{nK} =$

$$\begin{array}{c} \\ K=0 \\ K=1 \\ K=2 \\ K=3 \end{array} \begin{array}{c} \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array} \\ \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$X_1(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

$$\therefore X_1(K) = \{7, -2-j, 1, -2+j\}$$

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Similarly,

$$X_2(K) = [W_4][x_2(n)] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$$\therefore X_2(K) = \{11, 2-j, 1, 2+j\}$$

According to circular convolution property,

$$X_3(K) = DFT\left[x_1(n) \textcircled{N} x_2(n)\right] = X_1(K) \cdot X_2(K)$$

$$= \{77, -5, 1, -5\}$$

Now, $x_3(n)$ can be obtained by computing IDFT of $X_3(K)$.

$$x_3(n) = \frac{1}{N}\left[W_N^*\right]\left[X_3(K)\right]$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 77 \\ -5 \\ 1 \\ -5 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 68 \\ 76 \\ 88 \\ 76 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$$

$$\therefore x_3(n) = \{17, 19, 22, 19\}.$$

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Linear filtering of long sequences:-

In real time applications, the input data sequence is very long. It is not possible to do the calculations for such long data sequences. This is because, the digital computer has limited memory.

Hence, long data input sequences is broken into small length sequences. The computation of each block is done seperately. Then all processed blocks are fitted one after other to get the final output.

There are two methods to obtain such computation

i). Overlap save method

ii). Overlap add method.

i). Overlap save method:-

In this method, the filtering of each block is done seperately. Let 'M' be the impulse response length of a filter. Let 'N' be the length of each block. This long data sequence is divided into blocks.

→ Let us form the blocks of length 'N'

→ For first block, $x_1(n)$, there are no previous segments. So this block is padded with 'M-1' zeros and last 'L' samples of are taken from input sequence

$$x_1(n) = \{\underbrace{0, 0, \ldots 0}_{(M-1)\text{ samples}}, \underbrace{x(0), x(1), \ldots x(L-1)}_{\text{'L' samples from i/p sequence}}\}$$

So, total length, $N = L + M - 1$

→ For second block $x_2(n)$, taking last 'M-1' samples from $x_1(n)$ and next 'L' samples from input sequence $x(n)$

$$x_2(n) = \{\underbrace{x(L-M+1) \ldots x(L-1)}_{(M-1)\text{ samples from Previous segment}}, \underbrace{x(L), x(L+1), \ldots x(2L-1)}_{\text{next 'L' samples from i/p sequence}}\}$$

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→ Likewise the different blocks are formed with length 'N'.

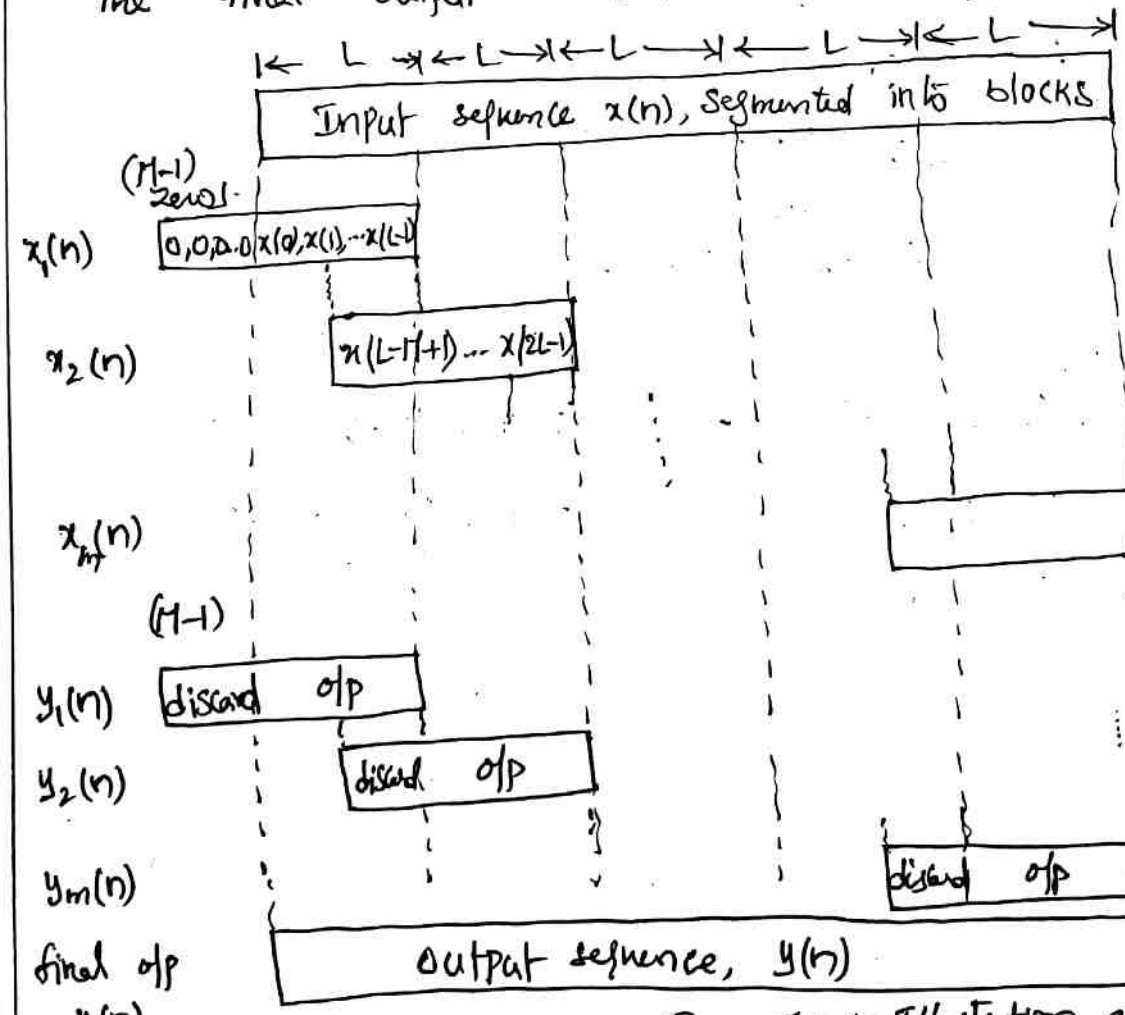
To avoid loss of data due to aliasing :-

Observe that each input block $x_1(n)$, $x_2(n)$, ... ~~Each~~ consists of initial 'M-1' samples taken from the previous block. This means that there is overlap of sequences. While the 'last' 'L' samples of each block are the actual input samples.

To avoid aliasing,

→ first (M-1) samples of $y_m(n)$ are discarded. This means that after computing time domain sequences, initial 'M-1' samples are discarded.

→ Then each block is fitted one after other to obtain the final output.



(15) Fig :- Illustration of overlap save method.

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Eg-9:- Find linear convolution using overlap save method of the following sequences:

$$x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$$

$$h(n) = \{1, 2, 3\}$$

Sol:- Given that, $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$

$h(n) = \{1, 2, 3\}$

Here M = length of impulse response = 3

Let $L = 2$ = length of previous segment.

So, $N = L + M - 1 = 4$ = length of each block.

→ For first block, $(M-1) = 2$ zeros add & $L = 2$ bits from input sequence

$$x_1(n) = \{\underbrace{0, 0}_{(M-1)\text{ zeroes}}, \underbrace{1, 2}_{L\text{ samples from input}}\}$$

→ for second block, $(M-1) = 2$ samples from previous block & 'L' samples from input sequence

$$x_2(n) = \{\underbrace{1, 2}_{(M-1)\text{ from previous segment}}, -1, 2\}$$

→ similarly.

$x_3(n) = \{-1, 2, 3, -2\}$

$x_4(n) = \{3, -2, -3, -1\}$

$x_5(n) = \{-3, -1, 1, 1\}$

$x_6(n) = \{1, 1, 2, -1\}$

$x_7(n) = \{2, -1, 0, 0\}$

using zero padding, $h(n) = \{1, 2, 3, 0\}$

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W.K.T, $y(n) = x(n) \textcircled{N} h(n)$

$$y_1(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times1+2\times2 \\ 3\times2 \\ 1\times1 \\ 2\times1+1\times2 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ 1 \\ 4 \end{bmatrix}$$

$\therefore y_1(n) = \{\textcircled{7}, 6, 1, 4\}$

$$y_2(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 10 \\ 6 \\ 6 \end{bmatrix}$$

$\therefore y_2(n) = \{\textcircled{2}, 10, 6, 6\}$

$$y_3(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -6 \\ 4 \\ +10 \end{bmatrix}$$

$\therefore y_3(n) = \{\textcircled{4}, -6, 4, +10\}$

$$y_4(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ -3 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 \\ 1 \\ 2 \\ -13 \end{bmatrix}$$

$\therefore y_4(n) = \{\textcircled{-8}, 1, 2, -13\}$

$$y_5(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -4 \\ -10 \\ 0 \end{bmatrix}$$

$\therefore y_5(n) = \{\textcircled{2}, -4, -10, 0\}$

$$y_6(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 0 \\ 7 \\ 6 \end{bmatrix}$$

$\therefore y_6(n) = \{\textcircled{5}, 0, 7, 6\}$

$$y_7(n) = \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 3 \\ 4 \\ -3 \end{bmatrix}$$

$\therefore y_7(n) = \{\textcircled{2}, 3, 4, -3\}$

The final output $y(n)$ can be obtained by discarding initial 2 samples from each output block and connecting all output blocks one after other.

$\therefore y(n) = \{1, 4, 6, 6, 4, +10, 2, -13, -10, 0, 7, 6, 4, -3\}$

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ii). Overlap add method:-

In this method, the size of each input block is 'N'. Each input data block is formed by taking 'L' samples from input sequence and adding 'M-1' zeroes at the end of each input sequence.

Therefore, $x_1(n) = \{\underbrace{x(0), x(1), \ldots x(L-1)}_{\text{'L' samples from i/p sequence}}, \underbrace{0, 0, \ldots 0}_{(M-1)\text{ zeroes}}\}$

Also, $x_2(n)$ is formed by taking next 'L' samples from input sequence and padding 'M-1' zeroes.

$$x_2(n) = \{\underbrace{x(L), x(L+1), \ldots x(2L-1)}_{\text{Next 'L' samples from i/p sequence}}, \underbrace{0, 0, 0 \ldots 0}_{(M-1)\text{ zeroes.}}\}$$

Impulse response of the filter $h(n)$ & its length is 'M'. Its length must be equal to 'N'. Therefore, 'L-1' zeroes are added to form sequence $h(n)$.

$$\therefore h(n) = \{\underbrace{h(0), h(1), \ldots h(M-1)}_{\text{'M' samples of impulse response}}, \underbrace{0, 0, \ldots 0}_{(L-1)\text{ zeroes padding}}\}$$

Now, the output of m^{th} block is obtained by multiplying DFT of m^{th} input by $H(K)$

i.e., $Y_m(K) = H(K) \cdot X_m(K)$

In this method, (M-1) samples are not discarded because there is no aliasing effect.

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Eg-10: Solve Eg.⑨ using overlap add method.

Sol: Given that, $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$

$h(n) = \{1, 2, 3\}$

In this method, each block is obtained by adding (M-1) = 2 zeroes at the end.

$x_1(n) = \{1, 2, 0, 0\}$ $x_2(n) = \{-1, 2, 0, 0\}$ $x_3(n) = \{3, -2, 0, 0\}$

$x_4(n) = \{-3, -1, 0, 0\}$ $x_5(n) = \{1, 1, 0, 0\}$ $x_6(n) = \{2, -1, 0, 0\}$

We have $y_1(n) = x_1(n) \textcircled{N} h(n)$

$y_1(n) = x_1(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 4 \\ 7 \\ 6 \end{bmatrix}$$

$\therefore y_1(n) = \{1, 4, 7, 6\}$

$y_2(n) = x_2(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \\ 1 \\ 6 \end{bmatrix}$$

$\therefore y_2(n) = \{-1, 0, 1, 6\}$

$y_3(n) = x_3(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 3 & 2 \\ 2 & 1 & 0 & 3 \\ 3 & 2 & 1 & 0 \\ 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

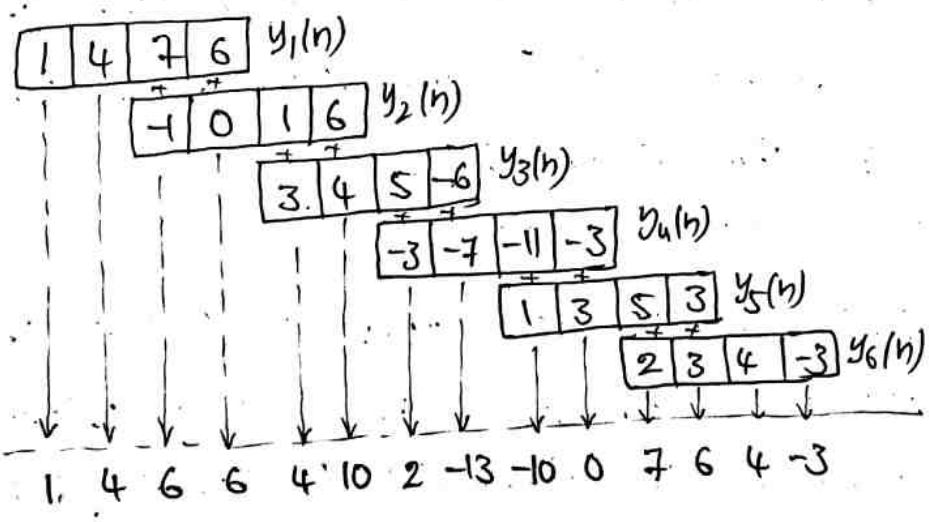
$$= \begin{bmatrix} 3 \\ 4 \\ 5 \\ -6 \end{bmatrix}$$

$\therefore y_3(n) = \{3, 4, 5, -6\}$

Similarly,

$y_4(n) = \{-3, -7, -11, -3\}$ $y_5(n) = \{1, 3, 5, 3\}$ $y_6(n) = \{2, 3, 4, -3\}$

The final output can be obtained by overlapping and adding individual outputs:



$\therefore y(n) = \{1, 4, 6, 6, 4, 10, 2, -13, -10, 0, 7, 6, 4, -3\}$

=o= (17)

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Fast Fourier Transform [FFT]:

FFT is not a transform; it is an algorithm used to compute DFT efficiently.

Computational problems of DFT:

Basically, the computational problem for the DFT is to compute the sequence $X(k)$ of N-complex valued numbers from given sequence $x(n)$ of length 'N', according to following expression.

DFT: $$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \quad , 0 \le k \le N-1$$

Similarly,

IDFT: $$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-nk} \quad , 0 \le n \le N-1$$

As a result to compute all 'N' values of the DFT requires N^2 complex multiplications & $N(N-1)$ complex additions.

Let $N = 1024$, No. of Multiplications $= N^2 = 1048576$

No. of additions $= N(N-1) = 1047552$

Direct computation of DFT is basically inefficient because it doesn't exploit any symmetry & periodicity properties of phase factor W_N.

Advantages of FFT over DFT:

FFT exploits the symmetry & periodic properties of twiddle factor W_N to reduce DFT computation time. It is based on the fundamental principle of decomposing the computation of DFT of $x(n)$ 'N' into smaller DFTs.

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Properties of twiddle factor:-

We know that twiddle factor $W_N = e^{-j2\pi/N}$ ⟶ (1)

i). Periodicity property:-

$$W_N^K = W_N^{K+N}$$

Proof:-

$$W_N^{K+N} = \left[e^{-j\frac{2\pi}{N}}\right]^{K+N} = e^{-j\frac{2\pi K}{N}} \cdot e^{-j\frac{2\pi N}{N}} = e^{-j\frac{2\pi K}{N}} = \left[e^{-j\frac{2\pi}{N}}\right]^K$$

$$= W_N^K$$

$$\left[\because e^{-j2\pi} = \cos 2\pi - j\sin 2\pi = 1\right]$$

$$\therefore \boxed{W_N^{K+N} = W_K^N}$$ ⟶ (2)

ii). Symmetric property:-

$$W_N^{K+\frac{N}{2}} = -W_N^K$$

Proof:-

$$W_N^{K+\frac{N}{2}} = \left[e^{-j\frac{2\pi}{N}}\right]^{K+\frac{N}{2}} = e^{-j\frac{2\pi K}{N}} \cdot e^{-j\frac{2\pi}{N}\cdot\frac{N}{2}} = -e^{-j\frac{2\pi K}{N}} = -\left[e^{-j\frac{2\pi}{N}}\right]^K$$

$$= -W_N^K$$

$$\therefore \boxed{W_N^{K+\frac{N}{2}} = -W_N^K}$$

$$\left[\because e^{-j\pi} = \cos\pi - j\sin\pi = -1\right]$$

⟶ (3)

iii) $W_{N/2} = W_N^2$

Proof:-

$$W_{N/2} = \left[e^{-j\frac{2\pi}{N/2}}\right] = e^{-j\frac{4\pi}{N}} = \left[e^{-j\frac{2\pi}{N}}\right]^2 = W_N^2$$

$$\therefore \boxed{W_{N/2} = W_N^2}$$ ⟶ (4)

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In FFT,

No. of Multiplications, $= \frac{N}{2} \cdot \log_2 N$

No. of additions $= N \cdot \log_2 N$

Let $N = 1024$, No. of Multiplications $= \frac{1024}{2} \cdot \log_2 1024 = 5120$

No. of additions $= 1024 \cdot \log_2 1024 = 10240$

$\therefore$ Improvement in Speed $= \frac{N^2}{\frac{N}{2}\log_2 N} = \frac{1048576}{5120} = 204.8$ times

Applications of FFT :-

→ Linear filtering

→ Correlation

→ Spectrum analysis

=o=

FFT algorithms:-

FFT algorithms are based on the fundamental Principle of decomposing the computation of DFT of a sequence of length 'N' into successively smaller DFTs.

Basically, there are two algorithms

i). Decimate - in - time algorithm [DIT-FFT]

ii). Decimate - in - frequency algorithm [DIF-FFT].

The word 'decimate' means to break into parts.

Therefore,

- DIT indicates dividing the sequence in time domain
- DIF indicates dividing the sequence in frequency domain.

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i). Decimate-in-Time algorithm [DIT-FFT] :- (or) Radix-2 DIT FFT algorithm

DIT-FFT algorithm is also known as Radix-2 DIT FFT algorithm which means the number of output points 'N' can be expressed as power of 2, i.e., $N=2^M$ where 'M' is an integer.

Let $x(n)$ is an N-point sequence, where 'N' is powers of 2. Decimate this sequence into two sequences of length $N/2$, one consisting even & other consisting odd values of $x(n)$.

i.e., $x(n) = x_e(n) + x_o(n)$ ——→ (1)

Here $x_e(n)$ = even sequence $= x(2n)$, $n=0,1,2,\ldots,\frac{N}{2}-1$

$x_o(n)$ = odd sequence $= x(2n+1)$, $n=0,1,2,\ldots,\frac{N}{2}-1$

W.K.T, N-Point DFT can be written as

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK}, \quad K=0,1,2,\ldots N-1 \longrightarrow (2)$$

$$= \underbrace{\sum_{n=0}^{N-1} x(n) W_N^{nK}}_{\text{(even)}} + \underbrace{\sum_{n=0}^{N-1} x(n) W_N^{nK}}_{\text{(odd)}}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) \cdot W_{N/2}^{2nK} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) \cdot W_N^{(2n+1)K}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_e(n) \cdot W_N^{2nK} + W_N^K \cdot \sum_{n=0}^{\frac{N}{2}-1} x_o(n) \cdot W_N^{2nK}$$

——→ (3)

W.K.T., $W_N^2 = \left(e^{-j\frac{2\pi}{N}}\right)^2 = e^{-j\frac{2\pi}{N/2}} = W_{N/2}$ ——→ (4)

using (3) & (4),

$$X(K) = \sum_{n=0}^{\frac{N}{2}-1} x_e(n) \cdot W_{N/2}^{nK} + W_N^K \cdot \sum_{n=0}^{\frac{N}{2}-1} x_o(n) \cdot W_{N/2}^{nK} \longrightarrow (5)$$

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From def eqn (5) written as

$$X(K) = X_e(K) + W_N^K X_o(K) \longrightarrow (6)$$

The value 'K' ranges from $K = 0, 1, 2, \cdots N-1$, each sum is only computed for $n = 0, 1, 2, \cdots \frac{N}{2}-1$. Since $X_e(K)$ & $X_o(K)$ are periodic in 'K' with period $\frac{N}{2}$.

For $K \geq N/2$, $W_N^{K+N/2} = -W_N^K$

eqn (6) for $K \geq N/2$,

$$X(K) = X_e\left(K-\frac{N}{2}\right) - W_N^{K-\frac{N}{2}} X_o\left(K-\frac{N}{2}\right) \longrightarrow (7)$$

for $K = \frac{N}{2}, \frac{N}{2}+1, \cdots, N-1$

Similarly, if we again decompose the sequence $X_e(K)$ & $X_o(K)$ into two subsequences then the amount of computation again be cut in half.

Note:

Let $N = 64$,

For direct computation DFT,

No. of Multiplications $= N^2 = 64^2 = 4096$

No. of additions $= N(N-1) = 64 \times 63 = 4032$

For DIT algorithm,

No. of Multiplications $= N + \frac{N^2}{2} = 64 + \frac{64^2}{2} = 2112$

No. of additions $= \frac{N^2}{2} = \frac{64^2}{2} = 2048$

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Let $N=8$,

from eqn ⑥ : $X(K) = X_e(K) + W_8^K X_o(K)$; $0 \le K \le 3$

from eqn ⑦ : $X(K) = X_e(K-4) - W_8^{K-4} X_o(K-4)$, $4 \le K \le 7$ → ⑧

for different values of 'K',

$K=0$, $X(0) = X_e(0) + W_8^0 X_o(0)$; $K=4$, $X(4) = X_e(0) - W_8^0 X_o(0)$

$K=1$, $X(1) = X_e(1) + W_8^1 X_o(1)$; $K=5$, $X(5) = X_e(1) - W_8^1 X_o(1)$

$K=2$, $X(2) = X_e(2) + W_8^2 X_o(2)$; $K=6$, $X(6) = X_e(2) - W_8^2 X_e(2)$

$K=3$, $X(3) = X_e(3) + W_8^3 X_o(3)$; $K=7$, $X(7) = X_e(3) - W_8^3 X_o(3)$ → ⑨

From the above set of equations, $X(0)$ & $X(4)$, $X(1)$ & $X(5)$, $X(2)$ & $X(6)$, $X(3)$ & $X(7)$ have same inputs.

The operation of $X(0)$ & $X(4)$ can be represented by butterfly diagram as shown in fig.

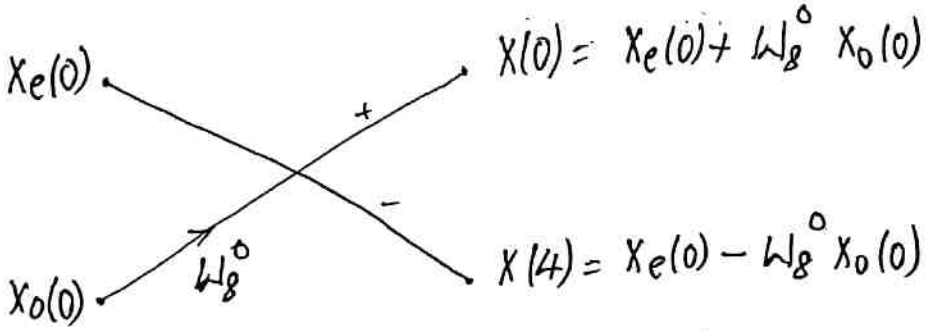


Fig:- Flow graph of butterfly diagram

Now, we can find that initially the sequence $x(n)$ is shuffled into $x_e(n)$ & $x_o(n)$ and then transformed to $X_e(K)$ & $X_o(K)$. For $K=0, 1, 2, 3 \ldots 7$ the values of $X_e(K)$ & $X_o(K)$ are combined acording to ⑨ and using butterfly structure shown in fig:1. The 8-Point DFT constructed from two 4-Point DFT samples as shown in fig:2.

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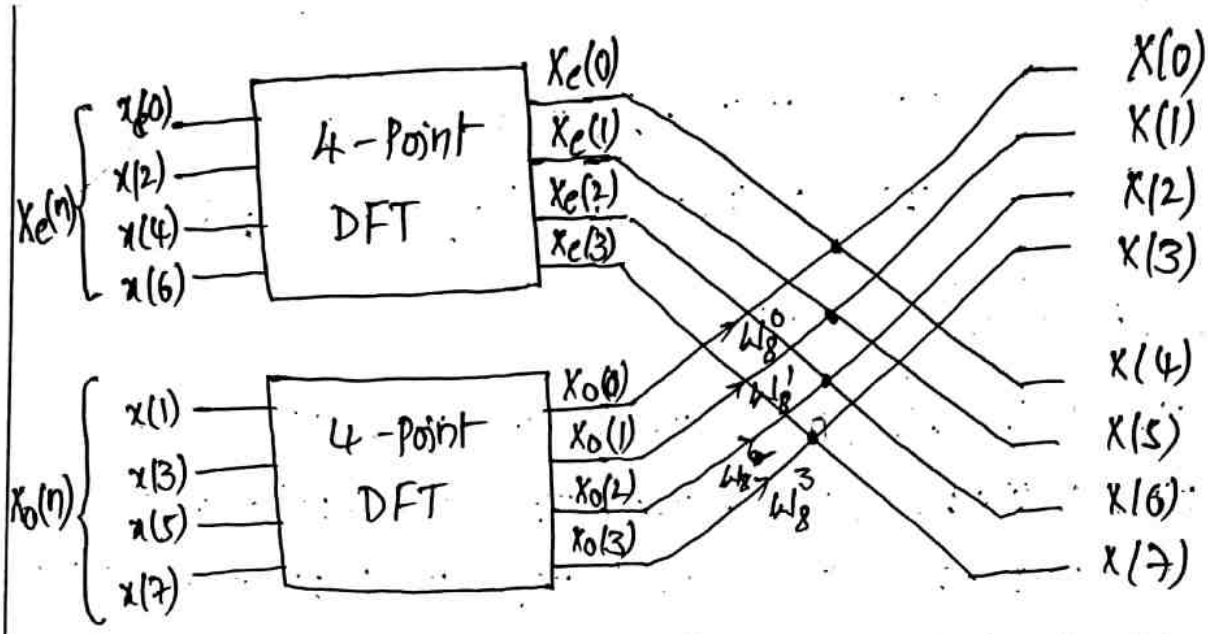


Fig.2: Construction of an 8-point DFT from two 4-point DFTs.

Similarly, we can decompose each $N/2$ sample of DFT. This can be done by dividing the $x_e(n)$ & $x_o(n)$ into two sequences consisting of even & odd members of sequences. The $\frac{N}{2}$ point DFT can be expressed as a ~~decomposition~~ combination of $\frac{N}{4}$ point DFTs.

$$X_e(n) = X_{ee}(n) + X_{eo}(n)$$
$$X_o(n) = X_{oe}(n) + X_{oo}(n)$$

the construction of 8-point DFT from 4 2-point DFTs as shown in fig. (3).

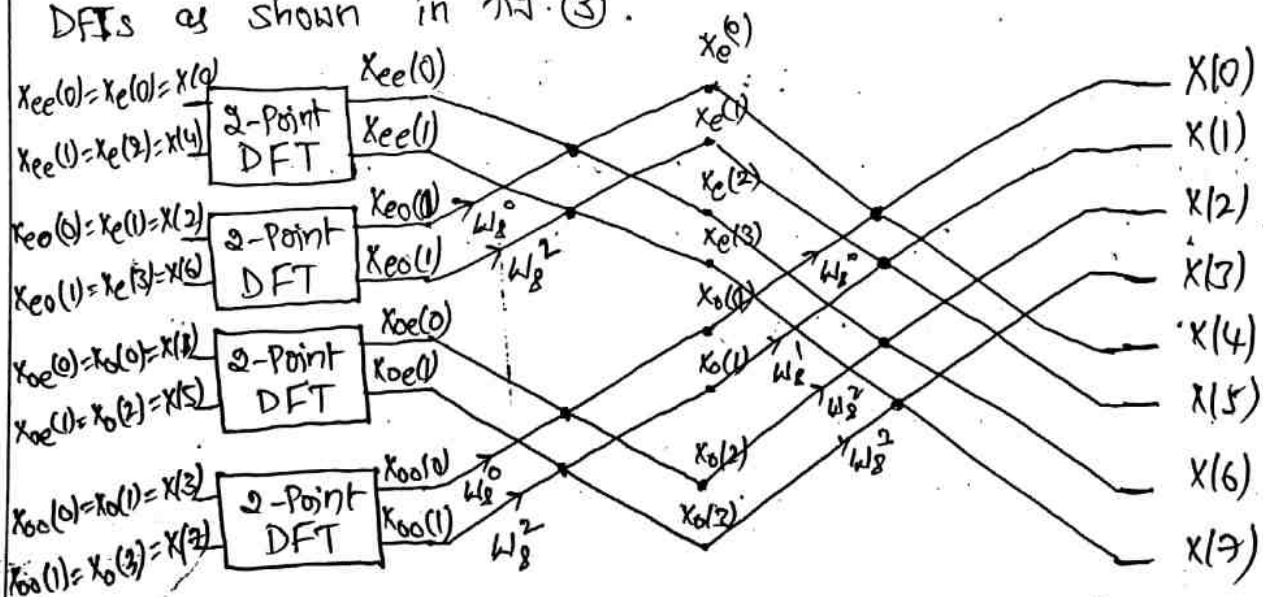


Fig.3:- Construction of 8-point DFT from four 2-point DFTs

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The two point DFT can be easily found by adding and subtracting the input sequences as twiddle factor associated with first stage is W_8^0. That is the first stage involves no multiplication but addition and subtraction.

$$X_{ee}(0) = x_{ee}(0) + x_{ee}(1) = x_e(0) + x_e(2) = x(0) + x(4)$$

$$X_{ee}(1) = x_{ee}(0) - x_{ee}(1) = x_e(0) - x_e(2) = x(0) - x(4)$$

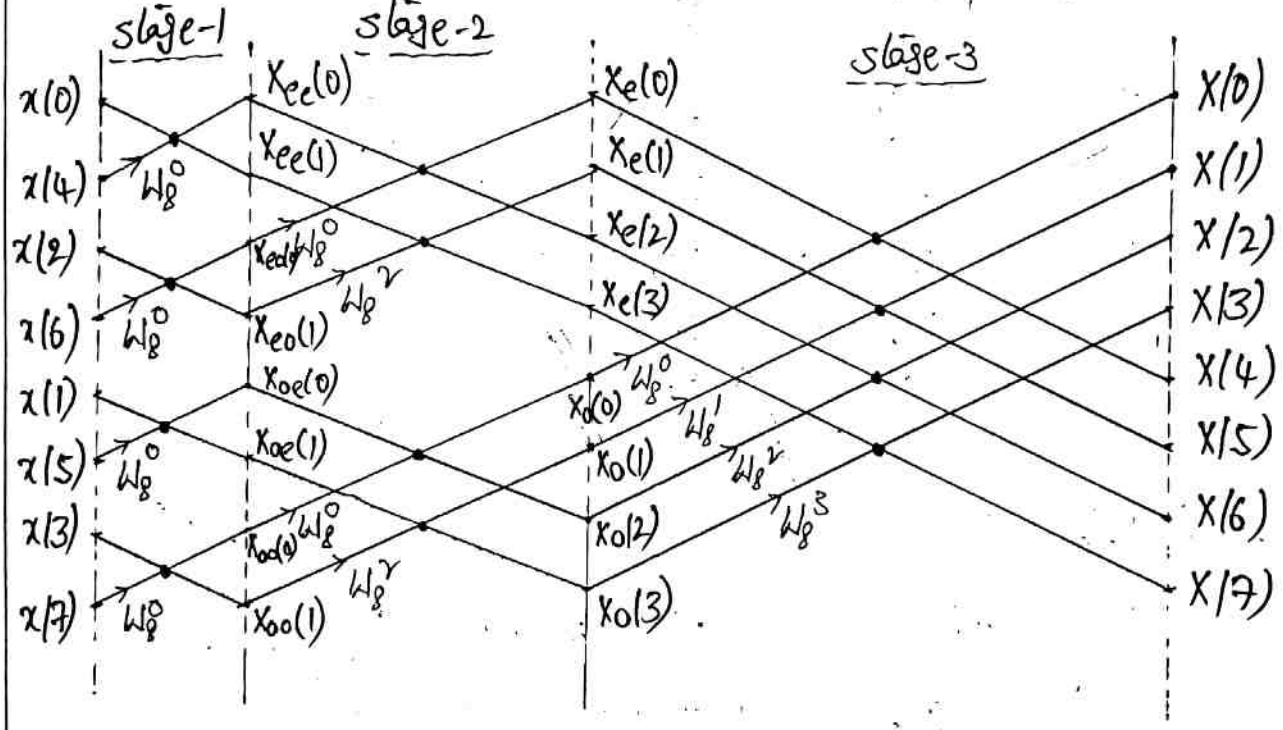


Fig. 4:- Flow graph of 8-point DIT algorithm

The algorithm has been called "Decimation in time" since at each stage, the input sequence is divided into smaller sequences i.e., the input sequences are decimated at each stage.

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Observations:-

1). Bit reversal:-

From fig ⑤, in DIT algorithm output sequence is in natural order (i.e. X(K), K=0, 1, ... N-1) and the input has to be stored in shuffled order.

For N=8, the bit reversal process is shown in table.

Table:- Bit reversal process for N=8

Input Samplex Index	Binary representation	Bit reversed binary	Bit reversed Sample index
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

2). Computations:-

Total number of complex multiplications required for calculating DIT-FFT = $\frac{N}{2}\log_2 N$

Total number of complex additions = $N.\log_2 N$

for N=64	DFT	DIT-FFT
No. of complex multiplications	$N^2 = 64^2 = 4096$	$\frac{N}{2}.\log_2^N = \frac{64}{2}.\log_2^{64} = 192$
No. of Complex additions	$N(N-1) = 4032$	$N.\log_2^N = 384$

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Steps of Radix-2 DIT-FFT Algorithm:-

1). The number of input samples, $N=2^M$, where 'M' is an integer.

2). The input sequence is shuffled through bit reversal

3). No. of stages in the flow graph is $M=\log_2 N$

4). Each stage consists of $\frac{N}{2}$ butterflies.

5). Inputs/outputs for each butterfly are depeated by 2^{m-1} samples, where 'm' is stage index. i.e. for first stage, $m=1$ & second stage, $m=2$.

6). No. of complex multiplications is given by $\frac{N}{2}\log_2 N$.

7). No. of complex additions is given by $N.\log_2 N$.

8). The twiddle factor exponents are a function of stage index 'm' i.e.,

$$K=\frac{Nt}{2^m} \quad ; t=0,1,2,\ldots,2^{m-1}-1.$$

for first stage, $m=1$, $t=0$, $K=0$

for second stage, $m=2$, $t=0,1$, $K=0, \frac{N}{4}$

for third stage, $m=3$, $t=0,1,2,3$, $K=0, \frac{N}{8}, \frac{N}{4}, \frac{3N}{8}$

9). No. of sets of butterflies in each stage, 2^{M-m}

10). The Exponent repeat factor [ERF], is the number of times the exponent sequence associated with 'm' is repeated and is given by 2^{M-m}.

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Eg-11:- Draw the flow graph of 16-point DIFFT.

Sol:-

1). The number of input samples, $N = 16 = 2^4$

2). The input sequence is shuffled through bit reversal as shown in table

Index	Binary representation	Bit-reversal order	Bit-reversal index
0	0000	0000	0
1	0001	1000	8
2	0010	0100	4
3	0011	1100	12
4	0100	0010	2
5	0101	1010	6
6	0110	0110	10
7	0111	1110	14
8	1000	0001	1
9	1001	1001	9
10	1010	0101	5
11	1011	1101	13
12	1100	0011	3
13	1101	1011	11
14	1110	0111	7
15	1111	1111	15

3). No. of stages, $M = \log_2^N = \log_2^{16} = 4$

4). No. of butterflies per each stage, $\frac{N}{2} = 8$

5). The inputs/outputs for each butterfly in stage 'm' is seperated by 2^{m-1} samples.

for stage 1, inputs/outputs for each butterfly are seperated by 1 sample

for stage 2, seperated by 2 samples

for stage 3, seperated by 4 samples

for stage 4, seperated by 8 samples.

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6). No. of complex multiplications, $\frac{N}{2}\log_2 N = 8.\log_2 16 = 32$.

7). No. of complex additions, $N.\log_2 N = 16.\log_2 16 = 64$.

8). No. of sets of butterflies in each stage, $2^{M-m} = 2^{4-m}$

for stage 1, $m=1$, 8 sets of butterflies

for stage 2, $m=2$, 4 sets of butterflies

for stage 3, $m=3$, 2 sets of butterflies

for stage 4, $m=4$, 1 set of butterfly.

9). Twiddle factor exponents for each stage are given by

$$K = \frac{Nt}{2^m} = \frac{16t}{2^m}, \quad t = 0, 1, \ldots, 2^{m-1} - 1.$$

for $m=1$, $t=0$, $K=0$ i.e., exponent is zero.

for $m=2$, $t=0,1$, $K=0,4$ i.e., exponents are 0, 4

for $m=3$, $t=0,1,2,3$, $K=0,2,4,6$

for $m=4$, $t=0,1,2,3,4,5,6,7$, $K=0,1,2,3,4,5,6,7$

10). The Exponent repeat factor (ERF), which is no. of times the exponent sequence is associated with 'm' repeat is given by $2^{M-m} = 2^{4-m}$

for stage 1, $m=1$, $ERF=8$

for stage 2, $m=2$, $ERF=4$

for stage 3, $m=3$, $ERF=2$

for stage 4, $m=4$, $ERF=1$

using the above steps, the complete flow graph of 16-point DFT using DIT algorithm is shown in fig. 5.

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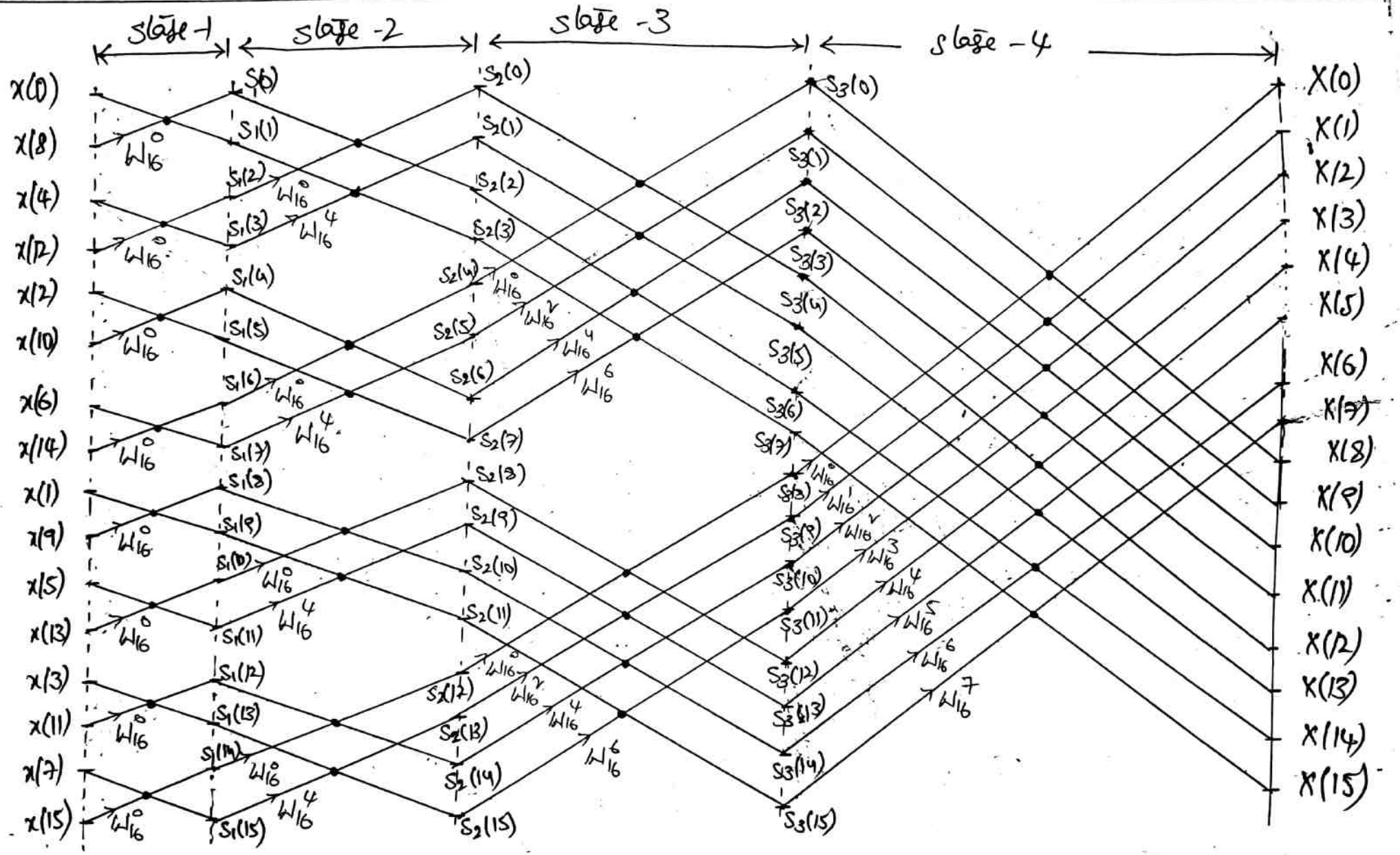


Fig 5: Flow graph of 16-Point DIT-FFT

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Eg-12:- Determine the 8-point DFT of the following sequence using Radix-2 DIT FFT algorithm.

$$x(n) = \left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0\right\}.$$

Sol:- Given that, $x(n) = \left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0\right\}$

Here $N=8$, no. of stages $= 3$.

Twiddle factors are:

$W_8^0 = 1$; $W_8^1 = e^{-j\frac{2\pi}{8}} = 0.707 - j0.707$

$W_8^2 = e^{-j4\pi/8} = -j$; $W_8^3 = e^{-j\frac{6\pi}{8}} = -0.707 - j0.707$

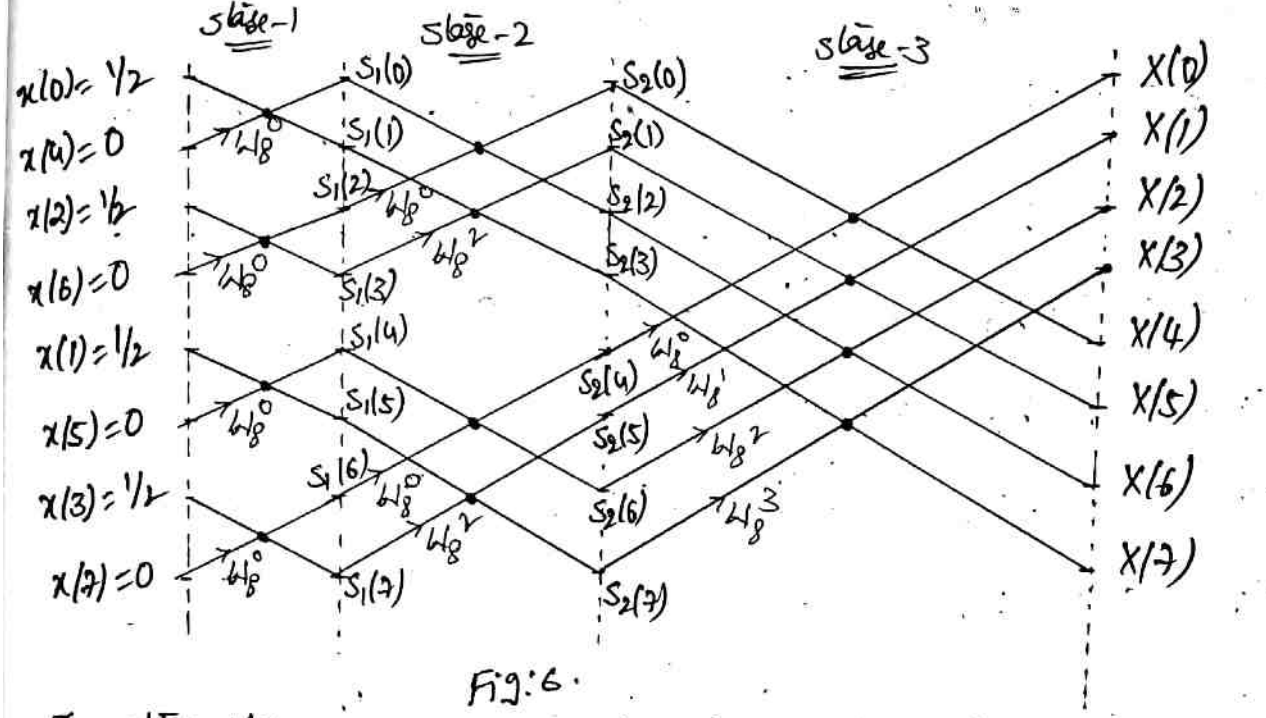


Fig:6.

For stage-1:-

$S_1(0) = x(0) + W_8^0 x(4) = \frac{1}{2}$; $S_1(4) = x(1) + W_8^0 x(5) = 1/2$

$S_1(1) = x(0) - W_8^0 x(4) = 1/2$; $S_1(5) = x(1) - W_8^0 x(5) = 1/2$

$S_1(2) = x(2) + W_8^0 x(6) = 1/2$; $S_1(6) = x(3) + W_8^0 x(7) = 1/2$

$S_1(3) = x(2) - W_8^0 x(6) = 1/2$; $S_1(7) = x(3) - W_8^0 x(7) = 1/2$

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For stage-2:-

$S_2(0) = S_1(0) + W_8^0 S_1(2) = 1$

$S_2(1) = S_1(1) + W_8^2 S_1(3) = \frac{1}{2} - \frac{j}{2}$

$S_2(2) = S_1(0) - W_8^0 S_1(2) = 0$

$S_2(3) = S_1(1) - W_8^2 S_1(2) = \frac{1}{2} + \frac{j}{2}$

$S_2(4) = S_1(4) + W_8^0 S_1(6) = 1$

$S_2(5) = S_1(5) + W_8^2 S_1(7) = \frac{1}{2} - \frac{j}{2}$

$S_2(6) = S_1(4) - W_8^0 S_1(6) = 0$

$S_2(7) = S_1(5) - W_8^2 S_1(7) = \frac{1}{2} + \frac{j}{2}$

For stage-3:-

$X(0) = S_2(0) + W_8^0 \cdot S_2(4) = 1 + 1 = 2$

$X(1) = S_2(1) + W_8^1 S_2(5) = \left(\frac{1}{2} - \frac{j}{2}\right) + (0.707 - j0.707)\cdot\left(\frac{1}{2} - \frac{j}{2}\right) = 0.5 - 1.207j$

$X(2) = S_2(2) + W_8^2 S_2(6) = 0$

$X(3) = S_2(3) + W_8^3 S_2(7) = \left(\frac{1}{2} + \frac{j}{2}\right) + (-0.707 - j0.707)\left(\frac{1}{2} + \frac{j}{2}\right) = 0.5 - 0.207j$

$X(4) = S_2(0) - W_8^0 S_2(4) = 1 - 1 = 0$

$X(5) = S_2(1) - W_8^1 S_2(5) = \left(\frac{1}{2} - \frac{j}{2}\right) - (0.707 - j0.707)\left(\frac{1}{2} - \frac{j}{2}\right) = 0.5 + 0.207j$

$X(6) = S_2(2) - W_8^2 S_2(6) = 0$

$X(7) = S_2(3) - W_8^3 S_2(7) = \left(\frac{1}{2} + \frac{j}{2}\right) - (-0.707 - j0.707)\left(\frac{1}{2} + \frac{j}{2}\right) = 0.5 + 1.207j$

$\therefore X(k) = \{2;\ 0.5 - 1.207j,\ 0,\ 0.5 - 0.207j,\ 0,\ 0.5 + 0.207j,\ 0,\ 0.5 + 1.207j\}$

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Eg-13:- Compute DFT of the sequence $x(n) = \sin(n\pi/2)$. using 4-Point DIT algorithm.

Sol:- Given that, $x(n) = \sin\left(\frac{n\pi}{2}\right)$

Here $N=4$, no. of stages, $m=2$

The range of 'n' is 0 to N-1 i.e., 0, 1, 2, 3

If $n=0$, $x(n)=0$; If $n=1$, $x(n) = \sin\frac{\pi}{2} = 1$

If $n=2$, $x(n) = \sin\pi = 0$; If $n=3$, $x(n) = \sin\frac{3\pi}{2} = -1$

$\therefore x(n) = \{0, 1, 0, -1\}$

Twiddle factors are

$W_4^0 = 1$; $W_4^1 = e^{-j\frac{2\pi}{4}} = e^{-j\pi/2} = -j$

The flow graph of 4-Point DIT FFT as shown in fig.

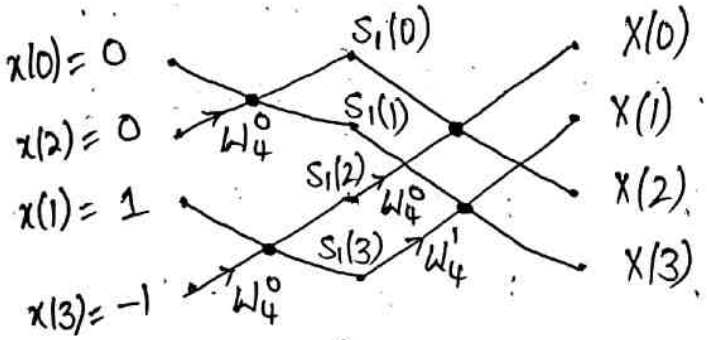


fig: 7.

First stage:-

$S_1(0) = x(0) + W_4^0 x(2) = 0$

$S_1(1) = x(0) - W_4^0 x(2) = 0$

$S_1(2) = x(1) + W_4^0 x(3) = 0$

$S_1(3) = x(1) - W_4^0 x(3) = 2$

Second stage:-

$X(0) = S_1(0) + W_4^0 S_1(2) = 0$

$X(1) = S_1(1) + W_4^1 S_1(3) = -2j$

$X(2) = S_1(0) - W_4^0 S_1(2) = 0$

$X(3) = S_1(1) - W_4^1 S_1(3) = 2j$

$\therefore X(K) = \{0, -2j, 0, 2j\}$

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ii) Decimate-in-frequency algorithm [DIF-FFT]:-

Radix-2 DIF FFT algorithm:-

- DIT algorithm is based on the decomposition of DFT computation by forming smaller and smaller subsequences of the sequence $x(n)$.
- In DIF algorithm, the output $X(K)$ is divided into smaller and smaller subsequences.

W.K.T., $$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{nK} \longrightarrow (1)$$

In this algorithm, input sequence $x(n)$ is partitioned into two sequences each of length $\frac{N}{2}$ samples. Let $x_1(n)$ be first sequence of first $\frac{N}{2}$ samples of $x(n)$ and $x_2(n)$ be second sequence of last $\frac{N}{2}$ samples of $x(n)$.

i.e., $$x_1(n) = x(n), \quad n = 0, 1, 2, \ldots \frac{N}{2}-1 \longrightarrow (2)$$
$$x_2(n) = x(n), \quad n = \frac{N}{2}, \frac{N}{2}+1, \ldots N-1.$$

Let $n = m + \frac{N}{2}$ then $m + \frac{N}{2} = \frac{N}{2}, \frac{N}{2}+1, \ldots N-1$

$$\Rightarrow m = 0, 1, 2, \ldots \frac{N}{2}-1$$

Therefore, $x_2(n) = x\left(n+\frac{N}{2}\right)$, $n = 0, 1, 2, \ldots \frac{N}{2}-1 \longrightarrow (3)$

Eqn (1) can be written as

$$X(K) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n) W_N^{nK} + \sum_{n=\frac{N}{2}}^{N-1} x_2(n) W_N^{nK}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_1(n) W_N^{nK} + \sum_{n=0}^{\frac{N}{2}-1} x_2(n) \cdot W_N^{\left(n+\frac{N}{2}\right)K}$$

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$$\Rightarrow X(k) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n) W_N^{nk} + W_N^{\frac{Nk}{2}} \sum_{n=0}^{\frac{N}{2}-1} x_2(n) W_N^{nk} \longrightarrow (4)$$

Here. $W_N^{Nk/2} = \left(e^{\frac{-j2\pi}{N}}\right)^{\frac{Nk}{2}} = e^{-j\pi k}$ $\longrightarrow (5)$

If 'k' is even in eqn (5), eqn (4) written as

$$X(2k) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n) W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x_2(n) W_N^{2nk} \qquad \left[\because e^{-j\pi \cdot (2k)} = 1\right]$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x_1(n) + x_2(n)\right] W_N^{2nk}$$

$$\Rightarrow X(2k) = \sum_{n=0}^{\frac{N}{2}-1} \left[x_1(n) + x_2(n)\right] W_{N/2}^{nk} \qquad \left[\because W_N^2 = W_{N/2}\right]$$

$\longrightarrow (6)$

$$\Rightarrow X(2k) = \sum_{n=0}^{\frac{N}{2}-1} f(n) W_{N/2}^{nk} \longrightarrow (7)$$

Where $f(n) = x_1(n) + x_2(n)$ $\longrightarrow (8)$

Eqn (7) is the $\frac{N}{2}$-point DFT of the $\frac{N}{2}$-point sequence $f(n)$ obtained by adding the first half and last half of sequence.

If 'k' is odd in eqn (5), eqn (4) written as

$$X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} x_1(n) W_N^{n\cdot(2k+1)} - \sum_{n=0}^{\frac{N}{2}-1} x_2(n) \cdot W_N^{n(2k+1)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x_1(n) - x_2(n)\right] \cdot W_N^{n(2k+1)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} [x_1(n) - x_2(n)] W_N^n \cdot W_N^{2nk}$$

$$\Rightarrow X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} g(n) \cdot W_{N/2}^{nk} \longrightarrow (9)$$

where $g(n) = [x_1(n) - x_2(n)] W_N^n \longrightarrow (10)$

Eqn (9) is the $\frac{N}{2}$-point DFT of sequence $g(n)$ obtained by subtracting second half of sequence from first half and then multiplying the resulting sequence with W_N^n.

From eqn (7) & (9), Even & odd samples of DFT can be obtained from the $\frac{N}{2}$-point DFTs of $f(n)$ & $g(n)$ respectively. Equation (7) & (9) can be represented by a butterfly as shown in fig.8. This is the basic operation of DIF algorithm.

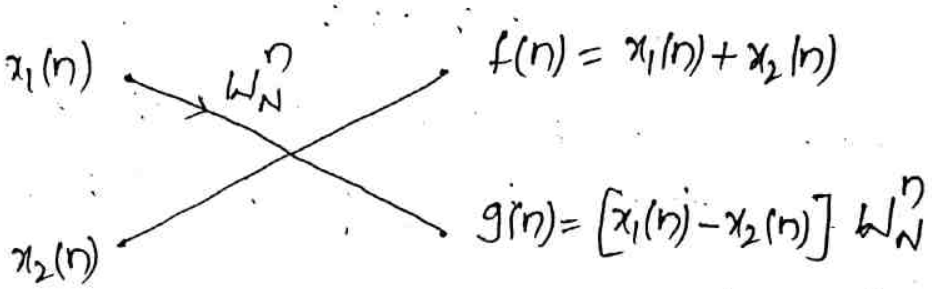


Fig.8: Flow graph of basic butterfly diagram for DIF algorithm.

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8-Point DFT using DIF-FFT algorithm:-

Let $x(n)$ be 8-point input sequence. Consider first four half sequences as $x_1(n)$ and second four half sequences as $x_2(n)$.

using eqn (8), $f(n) = x_1(n) + x_2(n)$

If $n=0$, $f(0) = x_1(0) + x_2(0)$

If $n=1$, $f(1) = x_1(1) + x_2(1)$

If $n=2$, $f(2) = x_1(2) + x_2(2)$

If $n=3$, $f(3) = x_1(3) + x_2(3)$

using eqn (10), $g(n) = [x_1(n) - x_2(n)] \cdot W_8^n$

If $n=0$, $g(0) = [x_1(0) - x_2(0)] W_8^0$

If $n=1$, $g(1) = [x_1(1) - x_2(1)] W_8^1$

If $n=2$, $g(2) = [x_1(2) - x_2(2)] W_8^2$

If $n=3$, $g(3) = [x_1(3) - x_2(3)] W_8^3$

using the above information, butterfly structure is shown in fig. 9.

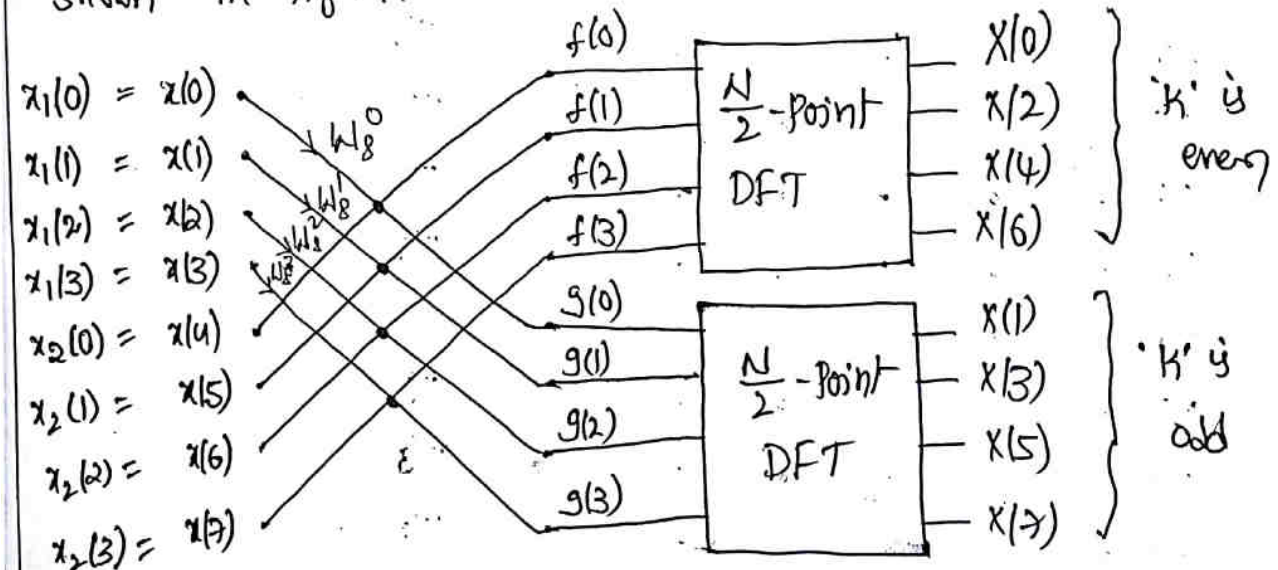


Fig. 9:- Flow graph of 8-point DIF decomposition into 2 4-point DFT's.

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Similarly $\frac{N}{2}$-point DFT can be divided into two $\frac{N}{4}$ point DFTs as shown in fig. 10

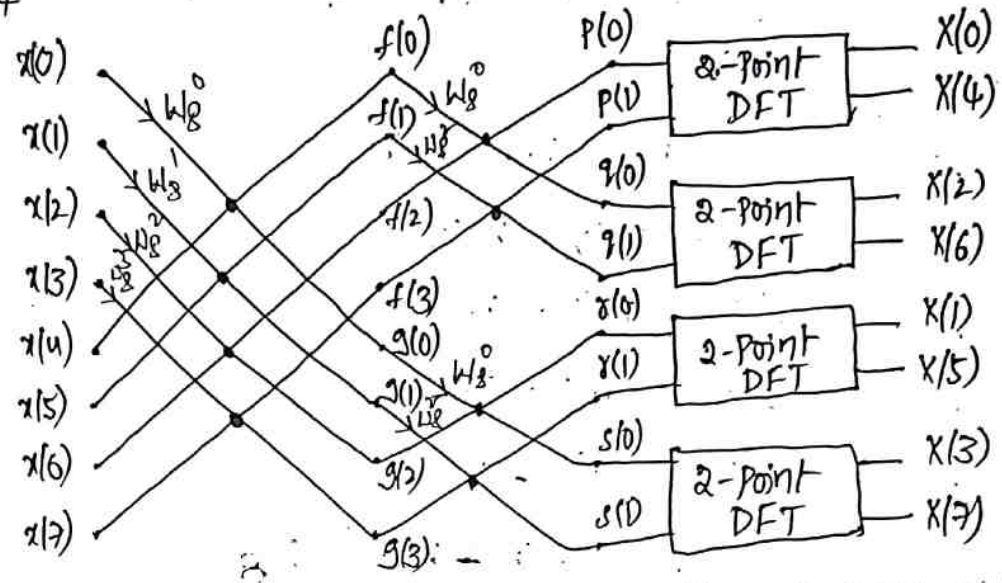


Fig. 10:- Decomposition of 8-point DFT into four 2-point DFTs using DIF FFT algorithm

The 2-point DFT can be found by adding and subtracting the inputs. Fig. 10 can be further reduced as fig. 11, which is the flow graph of complete DIF 8-point DFT computation.

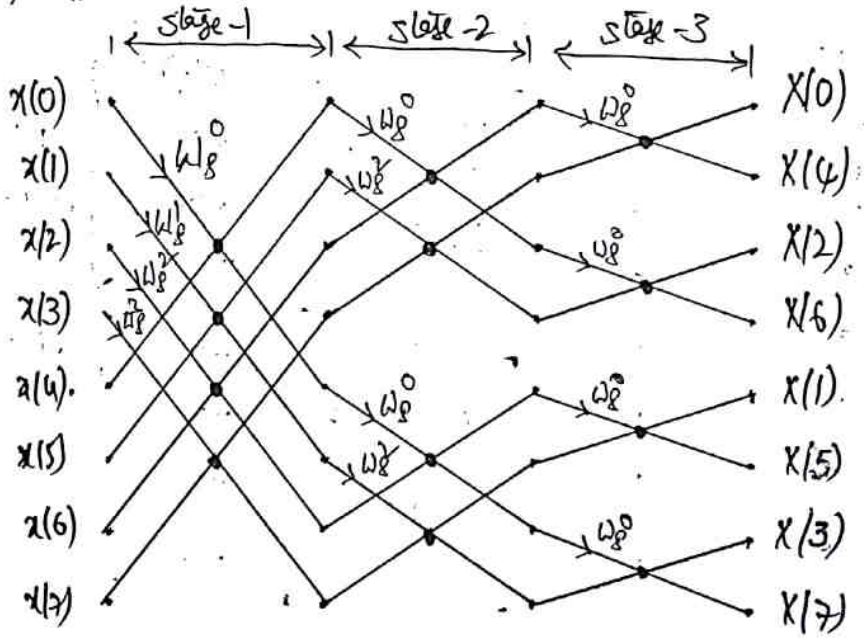


Fig. 11 :- Flow graph of complete DIF decomposition of an 8-point DFT computation.

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Steps for Radix-2 DIF FFT Algorithm:-

1). The number of input samples $N = 2^M$, where 'M' is an integer

2). The input sequence is in natural order.

3). The number of stages in the flow graph is $M = \log_2^N$

4). Each stage consists of $\frac{N}{2}$ butterflies.

5). Inputs/outputs of each butterfly are seperated by 2^{M-m} samples.

6). The Number of complex multiplications is $\frac{N}{2}\log_2^N$

7). The number of complex additions is $N.\log_2^N$

8). The twiddle factor exponents are a function of stage index 'm' and is given by

$$K = \frac{Nt}{2^{M-m+1}}, \quad t = 0, 1, 2, \cdots, 2^{M-m} - 1.$$

9). The number of sections of ~~each~~ butterflies in each stage is given by 2^{m-1}.

10). The exponent repeat factor [ERF], which is number of times the exponent associated with 'm' repeated is given by 2^{m-1}.

Differences between DIT & DIF Algorithms:-

DIT	DIF
①. Input is bit reversed while output is natural order	①. Input is natural order and output is bit reversal.
②. DFTs are combined at every stage	②. Sequences is split at every stage.

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Eg-14:- Given $x(n) = \{1,2,3,4,4,3,2,1\}$. Find $X(k)$ using DIF FFT algorithm.

Sol:- Given that $x(n) = \{1,2,3,4,4,3,2,1\}$

Here $N = 8$, no. of stages are 3.

Twiddle factors are $W_8^0 = 1$, $W_8^1 = 0.707 - j0.707$, $W_8^2 = -j$,

$W_8^3 = -0.707 - j0.707$.

The complete flow graph of 8-point DFT using DIF FFT as shown in fig. 12.

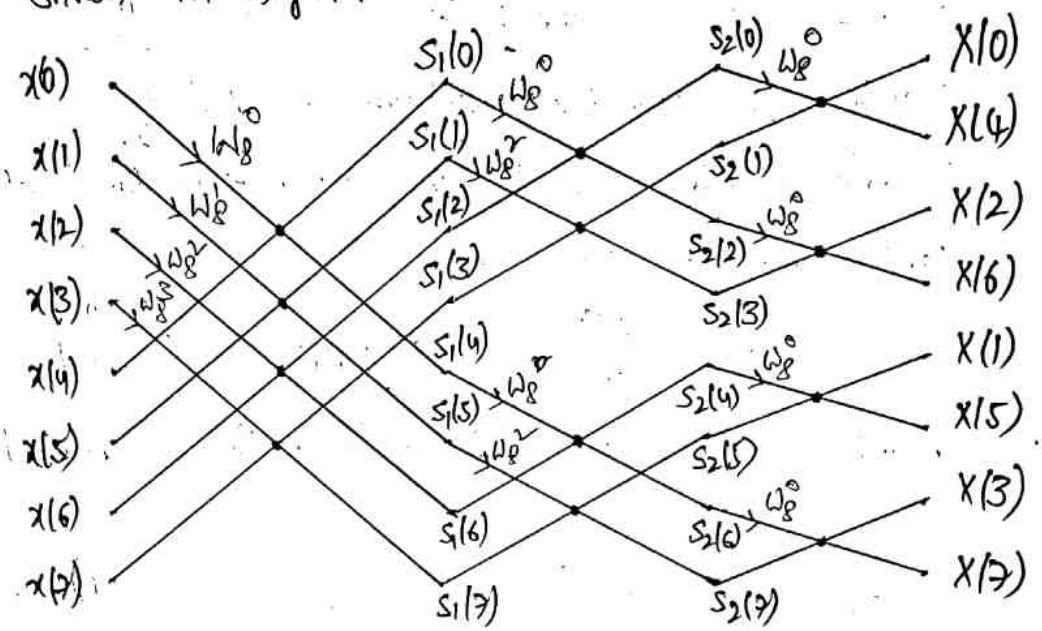


Fig. 12:- 8-point DFT flow graph using DIF FFT

Stage-1:-

$S_1(0) = [x(0) + x(4)] = 1+4 = 5$

$S_1(1) = x(1) + x(5) = 2+3 = 5$

$S_1(2) = x(2) + x(6) = 3+2 = 5$

$S_1(3) = x(3) + x(7) = 4+1 = 5$

$S_1(4) = [x(0) - x(4)] W_8^0 = (1-4) \cdot 1 = -3$

$S_1(5) = [x(1) - x(5)] W_8^1 = (2-3)(0.707 - j0.707) = -0.707 + j0.707$

$S_1(6) = [x(2) - x(6)] W_8^2 = (3-2)(-j) = -j$

$S_1(7) = [x(3) - x(7)] W_8^3 = (4-1)(-0.707 - j0.707) = -2.121 - 2.121j$

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Stage-2:-

$S_2(0) = S_1(0) + S_1(2) = 5+5 = 10$

$S_2(1) = S_1(1) + S_1(3) = 5+5 = 10$

$S_2(2) = [S_1(0) - S_1(2)]\, W_8^0 = 0$

$S_2(3) = [S_1(1) - S_1(3)]\, W_8^2 = 0$

$S_2(4) = S_1(4) + S_1(6) = -3 - j$

$S_2(5) = S_1(5) + S_1(7) = -0.707 + j0.707 - 2.121 - j2.121 = -2.828 - j1.414$

$S_2(6) = [S_1(4) - S_1(6)]\, W_8^0 = [-3+j]\cdot 1 = -3+j$

$S_2(7) = [S_1(5) - S_1(7)]\, W_8^2 = [-0.707 + j0.707 + 2.121 + j2.121](-j)$

$= 2.828 - j1.414$

Stage-3:-

$X(0) = S_2(0) + S_2(1) = 10+10 = 20$

$X(4) = S_2(0) - S_2(1) = 10-10 = 0$

$X(2) = S_2(2) + S_2(3) = 0$

$X(6) = S_2(2) - S_2(3) = 0$

$X(1) = S_2(4) + S_2(5) = -3-j-2.828-j1.414 = -5.828 - j2.414$

$X(5) = S_2(4) - S_2(5) = -3-j+2.828+j1.414 = -0.172 + j0.414$

$X(3) = S_2(6) + S_2(7) = -3+j+2.828-j1.414 = -0.172 - j0.414$

$X(7) = S_2(6) - S_2(7) = -3+j-2.828+j1.414 = -5.828 + j2.414$

$\therefore X(k) = \{20,\ -5.828-j2.414,\ 0,\ -0.172-j0.414,\ 0,\ -0.172+j0.414,\ 0,\ -5.828+j2.414\}$

$= 0 =$

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IDFT using FFT algorithm:-

FFT algorithms can be used to compute an IDFT without any change in the algorithm.

The IDFT of an N-point sequence $X(K)$, $K=0,1,\ldots N-1$ is defined as

$$x(n)=\frac{1}{N}\sum_{K=0}^{N-1} X(K)\cdot W_N^{-nK} \longrightarrow (1)$$

Take complex conjugate and multiply by 'N', we obtain

$$N\cdot x^*(n)=\sum_{K=0}^{N-1} X^*(K)\,W_N^{nK}$$

The right hand side of above equation is DFT of sequence $X^*(K)$. The desired output sequence can be found by

$$x(n)=\frac{1}{N}\left[\sum_{K=0}^{N-1} X^*(K)\,W_N^{nK}\right]^* \longrightarrow (2)$$

Eg-15: Compute IDFT of the sequence $X(K)=\{7,-0.707-j0.707, -j, 0.707-j0.707, 1, 0.707+j0.707, j, -0.707+j0.707\}$ using i) DIT ii) DIF algorithm.

Sol: Given that $X(K)=\{7,-0.707-j0.707,-j,0.707-j0.707,1,0.707+j0.707,j,-0.707+j0.707\}$

Here $N=8$, no. of stages $=3$

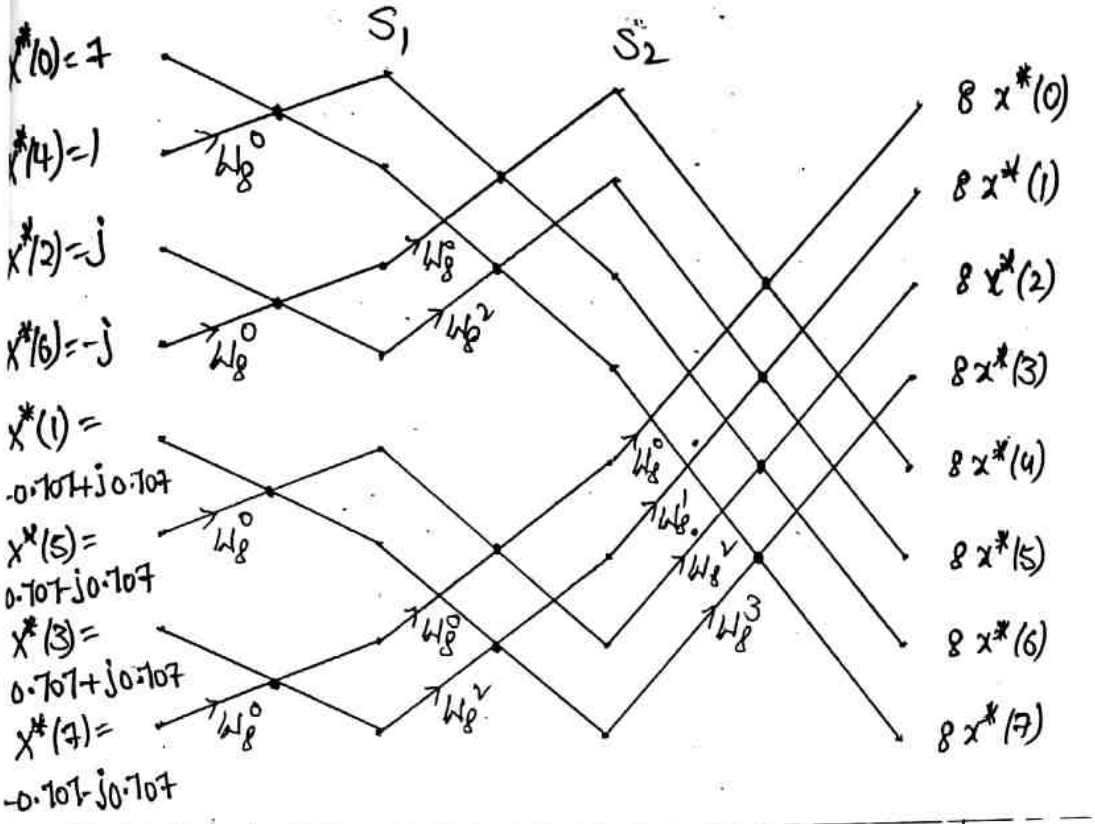
Take complex conjugate,

$$X^*(K)=\{7,-0.707+j0.707, j, 0.707+j0.707, 1, 0.707-j0.707, -j, -0.707-j0.707\}$$

Twiddle factors are $W_8^0=1$, $W_8^1=0.707-j0.707$, $W_8^2=-j$, $W_8^3=-0.707-j0.707$

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i) DIT FFT algorithm :-



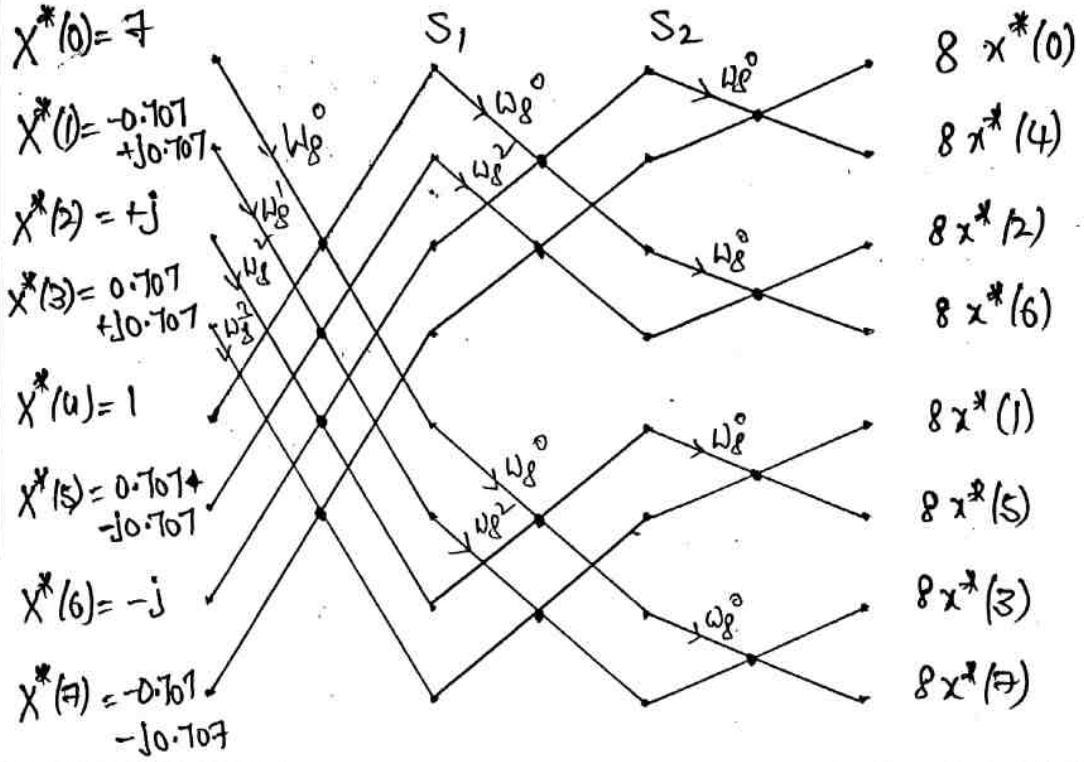
input	stage - 1	stage - 2	output
7	7+1 = 8	8+0 = 8	8+0 = 8
1	7-1 = 6	6+(-j)2j = 8	8+0 = 8
j	j-j = 0	8-0 = 8	8+0 = 8
-j	j+j = 2j	6-(-j)2j = 4	4+(-0.707-j0.707)·(-2.828+j2.828) = 8
-0.707+j0.707	0	0	8-0 = 8
0.707-j0.707	-1.414+j1.414	(-1.414+j1.414)+(-j)·(1.414+j1.414)	8-0 = 8
0.707+j0.707	0	= 0 0	8-0 = 8
-0.707-j0.707	1.414+j1.414	(-1.414+j1.414)-(-j)·(1.414+j1.414) = -2.828+j2.828	4-(-0.707-j0.707)·(-2.828+j2.828) = 0

∴ output, $N\,x^*(n) = \{8,8,8,8,8,8,8,0\}$

⇒ $x(n) = \{1,1,1,1,1,1,1,0\}$

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ii) DIF FFT Algorithm:-



	Input	stage -1	stage -2	output
0	7	7+1 = 8	8	8
1	-0.707 +j0.707	0	0	8
2	j	0	8	8
3	0.707 +j0.707	0	0	8
4	1	6	8	8
5	0.707 -j0.707	(-1.414+j1.414)·(0.707-j0.707) = 2j	0	8
6	-j	(j+j)(-j) = 2	4	8
7	-0.707 -j 0.707	(1.414+j1.414)(-0.707-j0.707) = -2j	(2j+2j)(-j) = 4	0

Therefore, output $N\,x^*(n) = \{8,8,8,8,8,8,8,0\}$

$x(n) = \{1,1,1,1,1,1,1,0\}$

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Use of FFT algorithms in linear filtering and correlation:-

We know that linear filtering is same as linear convolution. ~~Now~~, ~~here~~ Let us discuss how to obtain linear convolution using FFT algorithms.

~~1. Let the input sequence x(n) is segmented into the block of L points.~~

1) Consider x(n) is input sequence and h(n) is the impulse response of a filter.

2) using DIF FFT algorithm, the DFT of input blocks is performed. W.K.T., DIF-FFT algorithm doesnot require bit reversal of input sequences. But output sequence is in the bit reversed order.

3). DFT of impulse response h(n) is H(K) obtained by using DIF FFT algorithm. Therefore, H(K) is also in bit reversed order.

4). Now, output of m^{th} block is obtained by multiplying H(K) with $X_m(K)$. Here $X_m(K)$ is DFT of m^{th} input block.

$$Y_m(K) = H(K) \cdot X_m(K)$$

Here $Y_m(K)$ is also in bit reversed order.

5). Now, IDFT is obtained using DIF FFT algorithm. This requires bit reversed input $Y_m(K)$ which produces the output y(n) in the natural order.

Note:- W.K.T., Correlation is nothing but the convolution with one of the sequences folded. Thus, by folding the sequence h(n), we can compute linear filtering using FFT. This means we can compute the correlation.

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DIGITAL SIGNAL PROCESSING

UNIT-I
INTRODUCTION

Short Answer Questions:

1. What are the elementary discrete time signals? (Apr 2018:S1)
2. Define discrete time signal and give examples. (Apr 2018:S2)
3. Determine whether the following system given by $y(n)=\log_{10}\{x(n)\}$ is casual or not. (Apr 2018:S3)
4. Determine whether system defined by $y(n)=x(-n^2-2)$ is time i· ariant or not. (Apr 2018:S4)
5. Test whether the following signal is periodic or not,if periodic find the fundamentalperiod sin $\sqrt{2}\pi t$. (Apr 2017:S1)
6. Test whether the following signal is periodic or not,if periodic find the fundamentalperiod $\sin 20\pi t + \sin 5\pi t$. (Apr 2017:S2)
7. Test the following systems for time invariance $y(n)=n\,x^2(n)$. (Apr 2017:S3)
8. Test the following systems for time invariance $a^{x(n)}$. (Apr 2017:S4)
9. What is BIBO stability? What are the conditions for BIBO st ility? (Dec 2016/Apr 2016:S4)
10. Find the power of the given signal below? (Apr 2016:S1)

$$x(n) = \begin{cases} 3(-1)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

11. Show that the following systems are nonlinear and time invariant. (Apr 2016:S2)

$$y(n) - x(n)y(n-1) = x(n)$$

12. Show that the following system is nonlinear and time invariant. (Apr 2016:S3)

$$y(n+2) + 2y(n) = x(n+1) + 2$$

Long Answer Questions:

1. a) Find the periodicity of the signal $x(n)=\sin(2\pi n/3)+\cos(\pi n/$
   b)Explain the frequency response of discrete time system.
   c)What is the causality condition for an LTI system?(Apr 2018:S1)
2. a) Discuss the frequency domain representation of linear time-invariant systems.
   b) Determine the frequency response for the system given by(Apr 2018:S2)

$$y(n)-3/4y(n-1)+1/8\,y(n-2) = x(n)-x(n-1)$$

3. a) Consider a signal $x[n] = (-a)^n u[n]$. Determine the spectrum X(w). (APR 2018:S3)
   b) Determine the response of Second order Discrete Time system governed by thedifference equation $y(n)-2y(n-1)-3y(n-2)= x(n)+4x(n-1)$, $n\geq 0$,when the input signalis $x(n)= 2nu(n)$, with initial conditions $y(-2)=0, y(-1)=5$.
4. a) For each case determine the system is stable or causal(Apr 2018:S4)

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i) $h(n)= \sin(\pi n/2)$ ii) $h(n) = \delta(n) + \sin \pi n$ iii) $h(n) = 2nu(-n)$

b) Show that an LTI system can be described by its unit sample response.

5. a) Determine whether each of the following systems defined below is (i) casual (ii) linear(iii) dynamic (iv) time invariant

(i) $y(n) = \log10[\{x(n)\}]$(ii) $y(n) = x(-n-2)$(iii) $y(n) = \cosh[nx, + x(n-1)]$

b) Give the frequency domain representation of discrete time signals.(Apr 2017:S1)

6. a) Determine the impulse response of the filter defined by $y(n)=x(n)+by(n-1)$.

b) A system has unit sample response h(n) given by$h(n)=-1/\delta(n+1)+1/2\delta(n)-1-1/4\ \delta(n-1)$. Is the system BIBO stable? Is the filtercausal? Justify your answer.(Apr 2017:S2)

7. a) Determine and sketch the magnitude and phase response of the following systems

(i) $y(n) = 1/3[x(n)+ x(n-1) + x(n-2)]$(ii) $y(n) = ½[x(n) – x(n-1)]$ (iii) $y(n)-1/2y(n-1)=x(n)$

b) Determine the impulse response of the filter defined by $y(n)=x(n)+by(n-1)$.

(Apr 2017:S3)

8. a) Explain the method of obtaining the frequency response of linear shift-invariant systems.

b) For the following discrete time signals, determine whethe or not the system is linear, shiftinvariant, causal and stable.(i) $y(n)=x(n+7)$ (ii) $y(n)= x3(n)$

c) Determine the magnitude and phase response of the following system: $y(n)=[x(n) + x(n-1)]/2$. (Dec 2016)

9. a) Find the solution to the following linear constant coefficient difference equation$y(n)-3/2\ y(n-1) +1/2\ y(n-2) = (1/2)^n$ for $n\geq0$ with initial conditions $y(-1) = 4$ and $y(-2) = 10$.

b) Derive the relationship between impulse response and frequency response of adiscrete time system. (Apr 2016:S1)

10. a) Determine the frequency response, magnitude and phase responses and timedelay of the systems given by $y(n)-1/2\ y(n-1) = x(n)$.

b) Explain causality and stability of a linear time invariant sy m. (Apr 2016:S2)

11. a) Determine the frequency response, magnitude and phase responses and time delayof the systems given by$y(n) = x(n) – x(n-1) + x(n - 2)$

b) State and explain the transfer function of an LTI system. (Apr 2016:S3)

12. a) Determine frequency, magnitude and phase responses and time delay for thesystem. $y(n) + ¼\ y(n-1) = x(n) –x(n-1)$

b) Define the terms: linearity, time invariance and causality for a discrete timesystem. (Apr 2016:S4)

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UNIT-II
DISCRETE FOURIER SERIES & FOURIER TRANSFORMS

Short Answer Questions:

1. Find the IDFT of Y (k) = (1, 1, 1, 0) (Apr 2018:S1)
2. What are the advantages FFT over DFT. (Apr2018:S2)
3. What are the properties of convolution sum? (Apr 2018:S3)
4. What is FFT? How many multiplications and additions are required to compute N pointDFT using redix-2 FFT? (Apr 2018:S4)
5. Find the DFT of a sequence x(n) ={1 ,1, 2, 2}. (Apr 2017:S1)
6. Find the values of W^N_k, When N=8, k=2 and also for k=3. (Apr 2017:S2)
7. Define DFT and IDFT. (Apr 2017:S3)
8. What are the advantages of FFT over DFT. (Apr 2017:S4)
9. Distinguish between linear and circular convolutions of two sequences. (Dec 2016)
10. Compare overlap-add method and overlap-save method. (Apr 2016:S1)
11. Write computation efficiency of FFT over DFT. (Apr 2016:S2)
12. State all properties of DFT. (Apr 2016:S3)
13. How FFT is more efficient to determine DFT of sequence? (Apr 2016:S4)

Long Answer Questions:

1. a) Find the DFT of $x[n] = a^n$ for $0 \le n \le 3$ = 0 otherwise. (Apr 2018:S1)
   b) Find the linear convolution of the sequences x[n]={ 1,4,0.9,-1} and h[n]= {-3,-4,0,7}.
2. a) Find the DFT of the sequence x[n]={1,2,1,2,1,2,1,2} using decimation in timealgorithm.
   b) State and prove any four Properties of discrete Fourier series. (Apr 2018:S2)
3. a) Explain the significance of FFT algorithms. Draw the basic butterfly diagram forradix - 2 DIT-FFT.
   b) Find the DFT of x[n]={0.5,0.5,0.5,0.5,-1,-1,-1,-1} using decimation in timealgorithm. (Apr 2018:S3)
4. a) State and prove convolution Properties of DFT. (Apr 2018:S4)
   b) Compute the DFT for the sequence (0.5, 0.5, 0.5, 0.5,1,1,1,1) using DIF-FFT.
5. a) Compute the DFT for the sequence {1, 2, 0, 0, 0, 2, 1, 1}. Using radix -2 DIF FFT and radix-2 DIT- FFT algorithm. (Apr 2017:S1)
   b) Derive the equation to implement a butterfly structure In DITFFT algorithm.
6. a) Find the DFT of the sequence x[n]={1,2,3,4,5,6,7,8}. (Apr 2017:S2)
   b) Explain the use of FFT algorithms in linear filtering and correlation. 4 4 8
7. a) Determine IDFT of the following(i) X(k)={1,1-j2,-1,1+j2} (ii)X(k)={1,0,1,0}
   b) Find the DFT of the sequence x[n]={1,2,3,4,5,6,7,8} using DITFFT. (Apr 2017:S3)
8. a) How is the FFT algorithm applied to determine inverse discrete Fourier transform?
   b) Derive the equation to implement a butterfly structure In DIFFFT algorithm (Apr 2017:S4)
9. a) State shifting property of the DFT. (Dec 2016)

b) Compute the FFT for the sequence x(n)= n2+1 where N=8 using DIT algorithm.

c) What is DIT FFT algorithm?

10. a) Compute the DFT of the sequence $x(n) = \sin[n\pi/4]$, where N=8 using DIT FFTalgorithm.

b) Determine the IDFT of the sequenceX(K) = (6,-√2-j4.8284, -2+j2, √2-j0.8284, -2, √2+j0.8284, -2-j2, -√2-j4.8284). (Apr 2016:S1)

11. a) Find the DFT of the following sequence using FFT DIF?X(n) = {1,2,3,5,5,3,2,1}

b) Compute the DFTs of the sequence $x(n) = 2^{-n}$, where N = 8 using DITalgorithm. (Apr 2016:S2)

(Apr 2016:S3)

12. a) Find the N-point DFT for $x(n) = a^n$ for $0 < a < 1$?

b) Given x(n) = {1,2,3,4,4,3,2,1}, find X(k) using DIF FFT algorithm.

13. a) Compute the FFT for the sequence x(n) = n+1 where N =8 using DIT algorithm. (Apr 2016:S4)

b) State and prove the periodicity property in DFT.

DFS - prop
DFT - prop
DFT Prob
IDFT Pb
DIT & Pb - 3
DIF & - 3.
IDFT - 1
2 - 1 (12)

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