UNIT-I

* INTRODUCTION - DSP *

Signal :-

"A function of one or more independent variables which contains some information is called a signal".

(or)

"A signal is also defined as any physical quantity that varies with time, space or any other independent variable".

Mathematically, a signal as a function of one or more independent variables.

Eg :- $x(t) = at$ [one independent variable 't']

$S(x,y) = 3x + 2xy + 10y^2$ [two independent variables x & y]

The signals can be one-dimensional (or) multi-dimensional.

1). One dimensional signal :-

When the function depends on a single variable, the signal is said to be one dimensional.

Eg :- Speech signal - amplitude varies with time

2). Multi dimensional signal :-

When the function depends on two or more independent variables, the signal is said to be multi-dimensional.

Eg :- Image signal - two dimensional signal with horizontal & vertical coordinates.

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Classification of Signals :-

Based upon their nature and characteristics in the time domain, the signals may be classified as :

i). Continuous time signals :-

The signals that are defined for every instant of time are known as "Continuous time signals". This signal is also known as "analog signal". A Continuous time signal is represented by $x(t)$.

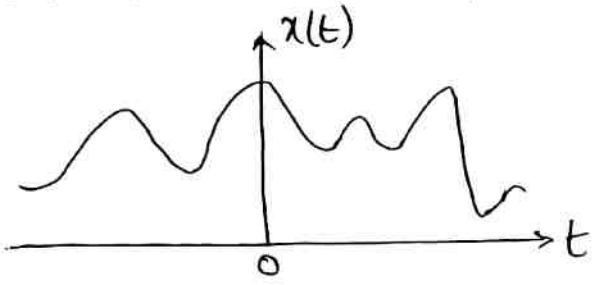


Fig :- Continuous time signal

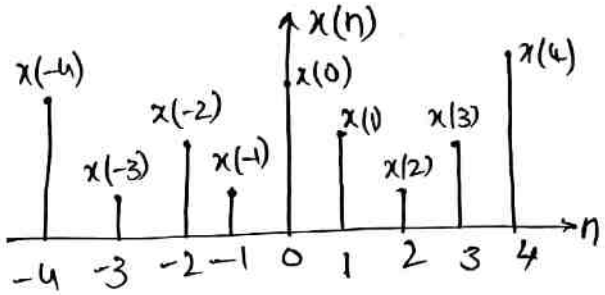


Fig :- Discrete time signal.

ii). Discrete time signals :-

The signals that are defined at discrete instants of time are known as "discrete time signals". They are denoted by $x(n)$.

iii) Digital signals :-

The signals that are discrete in time and quantized in amplitude are digital signals.

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Representation of Signals :-

Sketch the continuous time (C.T) signal $x(t) = 2.e^{-2t}$ for an interval. $0 \le t \le 2$. Sample the C.T signal with a sampling Period $T = 0.2$ sec and Sketch the Discrete time (D.T). Signal.

Sol :- Given that; $x(t) = 2e^{-2t}$

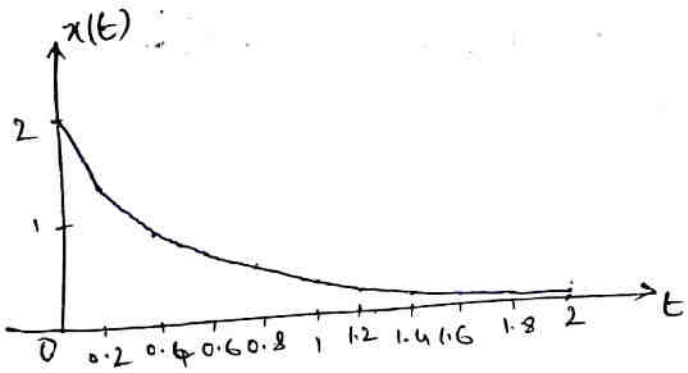
t	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
x(t)	2	1.3406	0.8987	0.6024	0.4038	0.2707	0.1814	0.1216	0.0815	0.0546	0.0366

Given Sampling period, $T = 0.2$

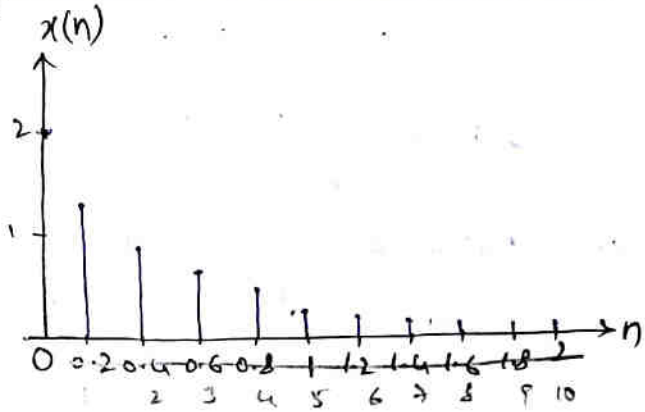
$$x(nT) = x(t)\big|_{t=nT} = x(0.2n)$$

$$\therefore x(n) = 2e^{-2n} \Rightarrow x(0.2n) = 2.e^{-0.4n}$$

n	0	1	2	3	4	5	6	7	8	9	10
x(n)	2	1.3406	0.8987	0.6024	0.4038	0.2707	0.1814	0.1216	0.0815	0.0546	0.0366



a). Continuous time signal



b). Discrete time signal

Representation of Discrete time signals :-

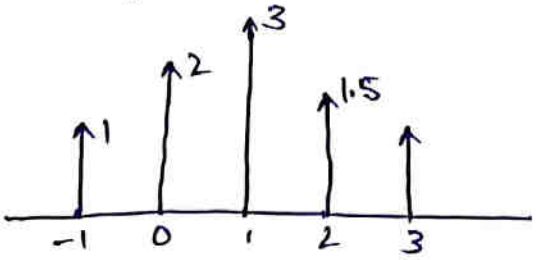
① . Sequence representation

$$x(n) = \{1, \underset{\uparrow}{2}, 3, 1.5, 1\}$$

② . Tabular representation

n	-1	0	1	2	3
x(n)	1	2	3	1.5	1

③ . Graphical representation



④ . Functional representation

$$x(n) = \begin{cases} 1 & ; n = -1 \\ 2 & ; n = 0 \\ 3 & ; n = 1 \\ 1.5 & ; n = 2 \\ 1 & ; n = 3 \end{cases}$$

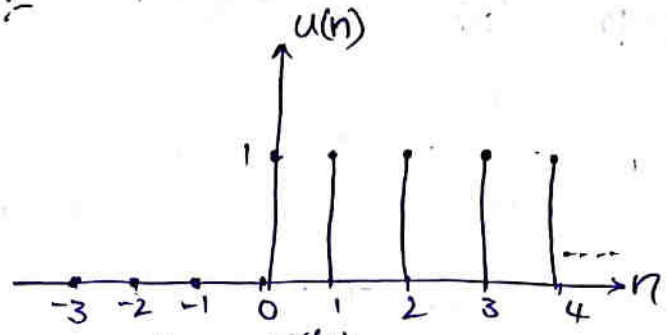
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Elementary Discrete time signals :-

① . unit step sequence :

$u(n) = 1$ for $n \geq 0$

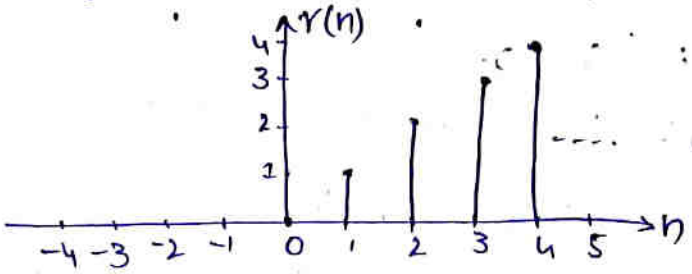
$= 0$ for $n < 0$



② . unit ramp sequence :

$r(n) = n$ for $n \geq 0$

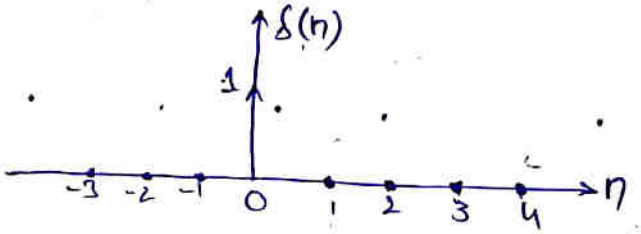
$= 0$ for $n < 0$



③ . unit sample sequence (or) unit impulse sequence

$\delta(n) = 1$ for $n = 0$

$= 0$ for $n \neq 0$



Properties :-

$\delta(n) = u(n) - u(n-1)$

$u(n) = \sum_{k=-\infty}^{n} \delta(k)$

$\sum_{n=-\infty}^{\infty} x(n) \cdot \delta(n-n_0) = x(n_0)$

④ . Exponential sequence :

$x(n) = a^n$ for all 'n'

⑤ . Sinusoidal signal :

$x(n) = A \cdot \cos(\omega_0 n + \phi)$

⑥ . Complex exponential signal :

$x(n) = a^n \cdot e^{j(\omega_0 n + \phi)}$

$= a^n \cdot \cos(\omega_0 n + \phi) + j \cdot a^n \sin(\omega_0 n + \phi)$

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Classification of Discrete-time Signals:-

①. Energy and Power Signals:-

For a discrete time signal $x(n)$,

The energy 'E' is defined as $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

The power 'P' is defined as $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

* A signal is said to be energy signal, which has finite energy and zero average power.

$$\boxed{0 < E < \infty \quad \text{and} \quad P = 0}$$

* A signal is said to be power signal, which has finite average power and infinite energy

$$\boxed{0 < P < \infty \quad \text{and} \quad E = \infty}$$

Eg:- Determine whether the signals is a power signal or energy signal or neither.

i). $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n\right|^2 \cdot u(n) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$\therefore$ Since energy is finite and non-zero, therefore it is a energy signal with $E = \frac{4}{3}$.

ii). $x(n) = u(n)$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2 = \lim_{N\to\infty} \frac{1}{2N+1} [N+1] = \frac{1}{2}$$

$\left[\because \sum_{n=0}^{N} (1)^2 \text{ means } 1+1+1+\ldots \text{ for } n=0 \text{ to } N \text{ (or) } 1+1+1+\ldots (N+1) \text{ times} = N+1\right]$

$\therefore$ Since power is finite and energy has infinite, therefore given signal is power signal.

iii) $x(n) = e^{2n} \cdot u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} e^{4n} = 1 + e^4 + e^8 + \cdots + \infty = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} e^{4n}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\frac{e^{4(n+1)} - 1}{e^4 - 1} \right]$$

$$= \infty$$

$\therefore$ The signal is neither energy nor power signals.

$= 0 =$

② Periodic and Aperiodic Signals :-

A signal which represents itself after a fixed time period is called as a periodic signal.

Mathematically, $x(t) = x(t + T_0)$

Where 'T_0' is period of signal.

For a non-periodic (or) Aperiodic signal, $x(t) \neq x(t + T_0)$.

For a discrete time signal $x(n)$, the condition of periodicity is

$$x(n) = x(n+N)$$

Where 'N' is the period of the signal

For Non-periodic D.T. signal, $x(n) \neq x(n+N)$.

Condition for periodicity of a D.T. Signal :

Let $x(n)$ be a Cosine wave, $x(n) = A \cdot \cos(2\pi f_0 n + \theta)$

$x(n+N) = A \cdot \cos(2\pi f_0 n + 2\pi f_0 N + \theta)$

To satisfy periodicity, $A \cdot \cos(2\pi f_0 n + \theta) = A \cdot \cos(2\pi f_0 n + 2\pi f_0 N + \theta)$

$\therefore 2\pi f_0 N = 2\pi K$.

$\Rightarrow f_0 = \frac{K}{N}$ Where 'K' is an integer

Thus, D.T. signal is periodic if its frequency 'f_0' is rational.

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Periodicity Condition for $x(n) = x_1(n) + x_2(n)$:

According to Condition of Periodicity,

$$f_1 = \frac{K_1}{N_1} \quad \& \quad f_2 = \frac{K_2}{N_2}$$

The resultant signal $x(n)$ is periodic if $\frac{N_1}{N_2}$ is ratio of two integers. The period of $x(n)$ will be least common multiple of N_1 & N_2.

Eg:- Determine whether the following D.T. Signals are periodic or not ? If Periodic, determine the fundamental Period.

i). $x(n) = \cos 0.01\pi n$

Sol:- $x(n) = \cos 0.01\pi n$

Condition is $2\pi f_0 = 0.01\pi \Rightarrow f_0 = \frac{0.01}{2} = \frac{1}{200} = \frac{K}{N}$

Here 'f' is expressed as ratio of two integers. Hence, the Signal is periodic with Period $N = 200$.

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ii). $x(n) = \cos\frac{2\pi n}{5} + \cos\frac{2\pi n}{7}$

Sol:- $x(n) = \cos\frac{2\pi n}{5} + \cos\frac{2\pi n}{7}$

We have, $2\pi f_1 = \frac{2\pi}{5}$; $2\pi f_2 = \frac{2\pi}{7}$

$\Rightarrow f_1 = \frac{1}{5} = \frac{K_1}{N_1}$; $\Rightarrow f_2 = \frac{1}{7} = \frac{K_2}{N_2}$

$\Rightarrow N_1 = 5$ $\Rightarrow N_2 = 7$

Since $\frac{N_1}{N_2} = \frac{5}{7}$ is the ratio of two integers. Hence, the Signal is Periodic.

The Period of $x(n) = LCM(N_1, N_2) = LCM(5, 7) = 35$

$\therefore$ The sequence is periodic with Period = 35.

=o=

iii). $x(n) = \cos\left(\frac{n}{8}\right) \cdot \cos\left(\frac{n\pi}{8}\right)$

Sol:- We have, $2\pi f_1 = \frac{1}{8}$

$\Rightarrow f_1 = \frac{1}{16\pi}$

which is irrational

$\Rightarrow \cos\left(\frac{n}{8}\right)$ is non-Periodic

$2\pi f_2 = \frac{\pi}{8}$

$\Rightarrow f_2 = \frac{1}{16}$

which is rational

$\Rightarrow \cos\left(\frac{n\pi}{8}\right)$ is Periodic

∴ Hence, $x(n)$ is non-Periodic signal.

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③. Causal and Non Causal Signals:-

A signal is said to be causal if its value is zero for $n<0$. otherwise, the signal is non-causal.

Eg:-

for causal signals:

$x_1(n) = a^n \cdot u(n)$

$x_2(n) = \{1, -2, 1, 4, 3\}$ (↑ at 1, ‡ at 4)

for non-causal signals:

$x_1(n) = a^n \cdot u(-n+1)$

$x_2(n) = \{1, -2, 1, 4, 3\}$ (↑ at 3)

* A signal that is zero for all $n \geq 0$ is called an anticausal signal.

④. Symmetric (even) and Antisymmetric (odd) signals:-

A D.T. signal $x(n)$ is said to be a symmetric (even) signal if it satisfies the condition:

$x(-n) = x(n)$ for all 'n'.

Eg:- $x(n) = A.\cos\omega_0 n$

The signal is said to be an odd signal if it satisfies the condition:

$x(-n) = -x(n)$ for all 'n'.

Eg:- $x(n) = A \cdot \sin\omega_0 n$

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System :-

"A system is defined as a physical device that performs an operation on a signal".

(or)

"A system is an interconnection of components. It is a physical device that performs an operation on an input signal and produces another signal as output".

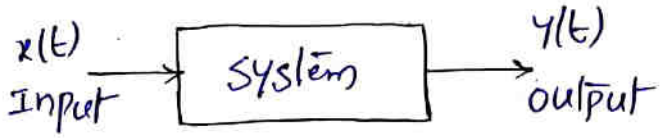


Fig:- A system

Mathematically, the relationship between $x(t)$ & $y(t)$ of a system is

$$y(t) = T[x(t)]$$

Classification of systems :-

1). Continuous time system :-

A continuous time system is one which operates on a continuous-time signal and produces a continuous-time output signal.

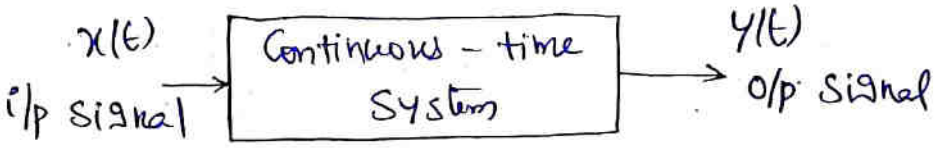


Fig:- A continuous-time system

Eg:- Amplifiers, filters, Communication systems etc.

2). Discrete time system :-

A discrete time system is one which operates on a discrete time signal and produces a discrete time o/p signal.

$$y(n) = T[x(n)]$$

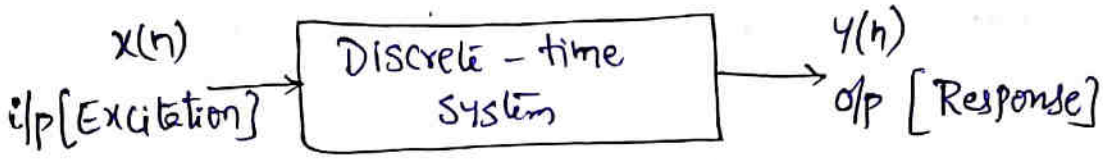
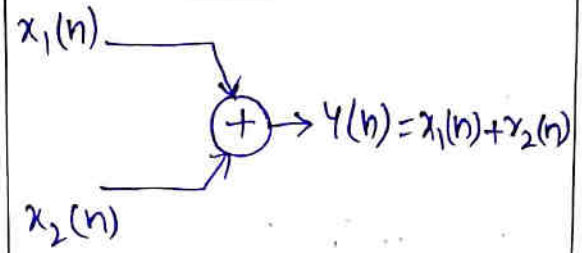
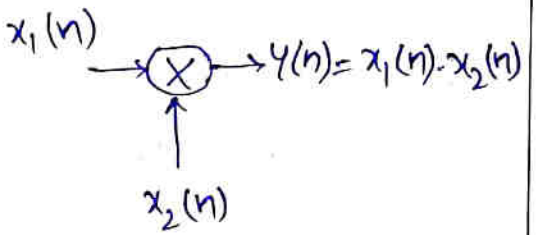
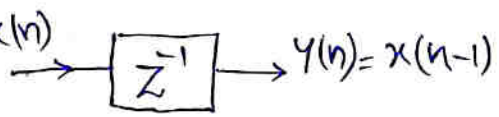
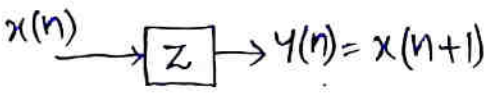


Fig :- A Discrete - time system

Eg:- Population model, savings account etc..

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Representation of Discrete - time systems :-

S.No	Name	Symbol	Significance
1.	Adder		It is memoryless operation
2.	Constant multiplier		It is memory less operation
3.	signal multiplier		It is memoryless operation
4.	unit delay		It requires memory to store the previous input
5.	unit advance		This operation is not realizable in practice.

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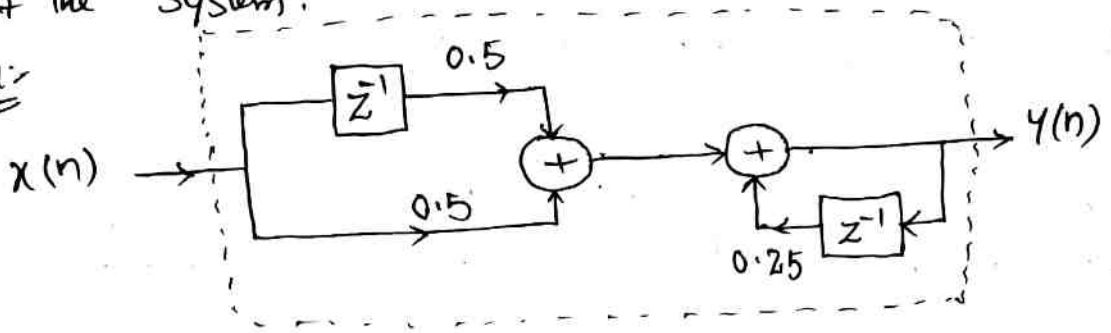
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Eg:- Using basic building blocks, sketch the block diagram representation of the discrete-time system described by the i/p - o/p relation

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

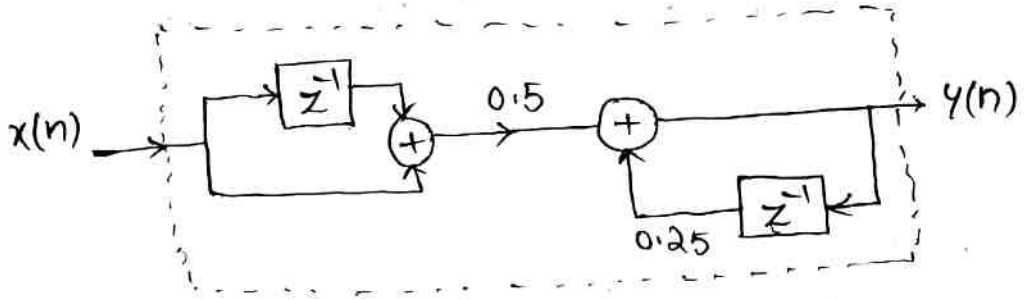
where $x(n)$ is the input and $y(n)$ is the output of the system.

Sol:-



(a)

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}\left[x(n) + x(n-1)\right]$$



(b)

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Classification of Discrete-time systems:-

According to their general properties & characteristics, D.T. systems are classified into:

1). static and Dynamic system
2). Time-invariant and Time-variant systems
3). Linear and Non-Linear systems
4). Causal and Non-Causal systems
5). stable and unstable systems
6). FIR & IIR systems

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①. Static and Dynamic systems:-

A D.T. system is called "static or memoryless" if its output at any instant of time depends on the input samples at the same time but not on past (or) future samples of the input.

Eg:- a). $y(n) = a \cdot x(n)$ b). $y(n) = x^2(n) + x(n)$

The output $y(n)$ depends on the input at that instant only.

A D.T. system is called "dynamic or memory" if its output at any instant of time depends on input sample at the same time as well as other times.

Eg:- a). $y(n) = x(n) + 5 \cdot x(n-1)$ b). $y(n) = 3 \cdot x(n+2) + x(n)$

The output $y(n)$ depends on past & future inputs.

Eg-1:- check whether the following systems are static or Dynamic.

i). $y(n) = 3 \cdot x(n) + 5$

Sol:- The o/p depends on present input only hence it is static system.

ii). $y(n) = n \cdot x(n) + 6 \cdot x^2(n)$

Sol:- Note that $x^2(n)$ means, ~~the~~ square the amplitude of input signal. So it is not related to time shifting. Since the equation doesnot contain any advanced or delay input terms. It is static system.

iii). $y(n) = x(-n)$

Sol:- W.K.T., $x(-n)$ is folded input signal.

From the observations, o/p's depends on past & future samples. Hence the system is dynamic.

At $n=-1$, $y(-1) = x(1)$
At $n=0$, $y(0) = x(0)$
At $n=1$, $y(1) = x(-1)$
At $n=2$, $y(2) = x(-2)$

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②. Causal and Non-Causal Systems:-

A system is said to be Causal system if output at any instant of time depends only on present and past inputs. But, the output does not depend on future inputs.

Eg:- $y(n) = x(n) + x(n-1)$

A system is said to be Non-Causal (or) Anti Causal system if its output depends not only on present and past inputs but also on future inputs.

Eg:- $y(n) = x(2n)$

Eg-2:- Determine whether the following systems are Causal or non-Causal.

i). $y(n) = x(n^2)$

Sol:- For all values of 'n', the system depends on future inputs. So, the system is non-Causal.

ii). $y(n) = x(n) + n \cdot x(n+1)$

Sol:- Here the system output depends on present and future inputs. So, it is non-Causal.

iii). $y(n) = x(n) + \frac{1}{x(n-1)}$

Sol:- For all values of 'n', the output depends on present and past inputs. Therefore, the system is Causal.

③. Time-variant & Time Invariant systems:-

A system is said to be time-invariant (or) shift invariant system if the characteristic of the system do not change with time.

A system is time-invariant if and only if

$x(n) \xrightarrow{T} y(n)$ implies that $x(n-k) \xrightarrow{T} y(n-k)$

Now delay the input sequence by 'k' samples and find output sequence, denote it as

$$y(n,k) = T[x(n-k)]$$

Delay the o/p sequence by 'k' samples denote it as $y(n-k)$. if $y(n,k) = y(n-k)$ for all possible values of 'k', the system is time-invariant.

if output $y(n,k) \neq y(n-k)$, then the system is time-variant.

Eg-3 :- Determine whether the following system is LIV (or) not.

i). $y(n) = x(n) - x(n-1)$

Sol:- → First delay the i/p by 'k' samples and denote output by $y(n,k)$.

Thus, $y(n,k) = x(n-k) - x(n-k-1)$

→ replace 'n' by 'n-k' throught the equation.

Therefore, $y(n-k) = x(n-k) - x(n-k-1)$

Here $y(n,k) = y(n-k)$. Thus, the system is time invariant.

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ii). $y(n) = n.x^2(n)$

Sol:- → first delay input by 'k' units and denoted as $y(n,k)$

Thus, $y(n,k) = n.x^2(n-k)$

→ replace 'n' by 'n-k' in given equation

$$y(n-k) = (n-k).x^2(n-k)$$

Therefore, $y(n,k) \neq y(n-k)$, the system is time variant.

= o =

④. Linear and Non-Linear systems:-

A system that satisfies the superposition principle is said to be a linear system.

* Superposition principle states that the response of the system to a weighted sum of signals should be equal to the corresponding weighted sum of the outputs of the system to each of the individual input signal.

A system is linear if and only if

$$T[a_1x_1(n) + a_2.x_2(n)] = a_1.T[x_1(n)] + a_2.T[x_2(n)]$$

A system that does not satisfy the superposition Principle is called "Non-linear".

Eg-4:- Determine whether the following systems are linear or not.

i). $y(n) = e^{x(n)}$

Sol:- $y_1(n) = e^{x_1(n)}$; $y_2(n) = e^{x_2(n)}$

$$y_3(n) = e^{x_1(n)+x_2(n)} = e^{x_1(n)}.e^{x_2(n)}$$

$\therefore y_3(n) \neq y_1(n) + y_2(n) \Rightarrow e^{x_1(n)}.e^{x_2(n)} \neq e^{x_1(n)} + e^{x_2(n)}$

Therefore, the system is non-linear.

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ii). $y(n) = n.x(n)$

Sol:- $y_1(n) = n.x_1(n)$: $y_2(n) = n.x_2(n)$

$y_3(n) = n.[x_1(n) + x_2(n)] = n.x_1(n) + n.x_2(n)$

Therefore, $y_3(n) = y_1(n) + y_2(n)$.

Hence, the system is linear.

=o=

⑤. Stable and unstable systems:-

To define stability of the system, we will use the term 'BIBO'. It stands for Bounded Input Bounded Output. The meaning of "bounded" is some finite value.

An LTI system is stable if it produces a bounded output sequence for every bounded input sequence.

If for bounded input sequence, the output is unbounded, the system is unstable.

Conditions for BIBO stability:-

Let us consider a finite number 'M_x', whose value is less than infinity. Then, if input is bounded. we write

$$|x(n)| < M_x < \infty$$

Similarly, Consider a finite number 'M_y', whose value is less than infinity. Then, if output is bounded. we write

$$|y(n)| < M_y < \infty$$

Let $x(n)$ be a bounded i/p sequence, $h(n)$ be the impulse response of the system and $y(n)$ be the output sequence..

We have $|y(n)| = \left| \sum_{k=-\infty}^{\infty} h(k).x(n-k) \right|$

$$\Rightarrow |y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)| \cdot |x(n-k)|$$

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Let the bounded value of input is equal to 'M_x', then

$$|y(n)| \leq M_x \cdot \sum_{k=-\infty}^{\infty} |h(k)|$$

The above condition will be satisfied when

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

So, the necessary and sufficient condition for stability is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

=o=

⑥. FIR and IIR Systems:-

If the impulse response of the system is of finite duration, then the system is called a "finite impulse response" system.

Eg:-

$$h(n) = \begin{cases} 1 & : \text{for } n = -1, 2 \\ 2 & : \text{for } n = 1 \\ 3 & : \text{for } n = 0, 3 \\ 0 & : \text{otherwise} \end{cases}$$

If the impulse response of the system has infinite duration then the system is called "Infinite Impulse response" system.

Eg:- $h(n) = a^n \cdot u(n)$

=o=

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Invertibility:-

A system 'T' is invertible if every output signal corresponds to a unique input signal. If so, then there exists an inverse system 'T^{-1}' that can recover the input signal.

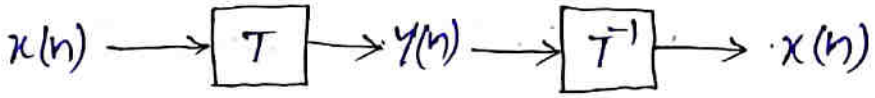
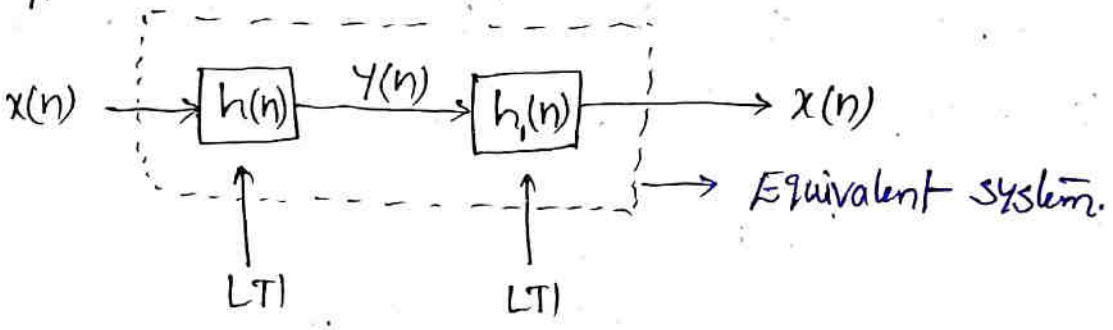
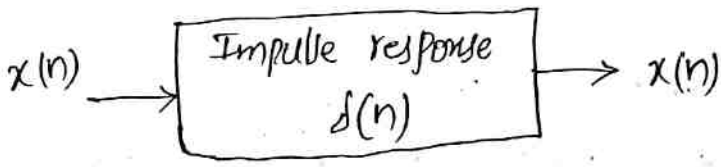


Fig:- Invertable System

If an LTI system is invertible then its corresponding inverse system is also LTI.

Suppose h(n) is the impulse response.

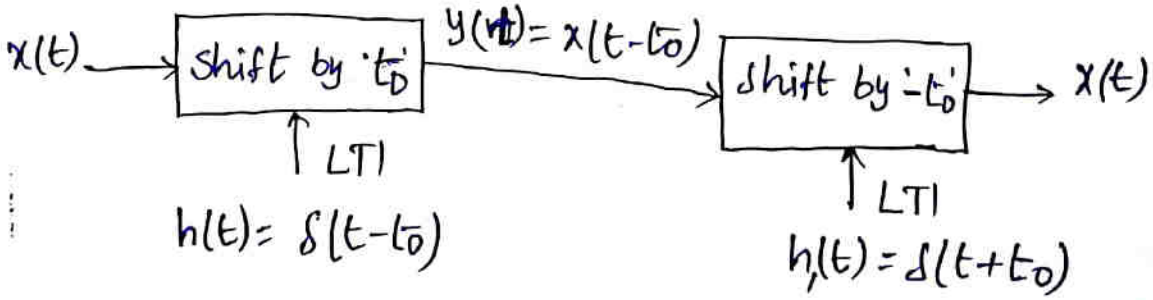




The LTI system with h(n) is invertible with $h_1(n)$ such that

$$h(n) * h_1(n) = \delta(n)$$

Eg:-



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Impulse response and Convolution Sum:-

Let the i/p signal $x(n)$ is the system excitation and $y(n)$ is the system response.

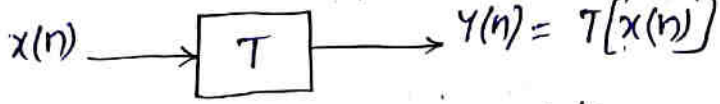


Fig:- A D.T. system representation

If the input to the system is a unit impulse i.e., $x(n) = \delta(n)$ then the output of the system is known as "Impulse response" denoted by $h(n)$.

Where $h(n) = T[\delta(n)]$ ⟶ (1)

W.K.T., arbitrary sequence $x(n)$ can be represented as a weighted sum of discrete impulses.

i.e., $$x(n) = \sum_{k=-\infty}^{\infty} x(k).\delta(n-k)$$ ⟶ (2)

Now, the system response is given by,

$$y(n) = T[x(n)] = T\left[\sum_{k=-\infty}^{\infty} x(k).\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k).\, T[\delta(n-k)]$$ ⟶ (3)

The response to the shifted impulse sequence can be denoted by $h(n,k)$ defined as

$$h(n,k) = T[\delta(n-k)]$$ ⟶ (4)

For a time invariant systems, $h(n,k) = h(n-k)$ ⟶ (5)

using (3), (4) & (5):-

$$y(n) = \sum_{k=-\infty}^{\infty} x(k).h(n-k)$$ ⟶ (6)

Which is known as Convolution sum.

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Eqn ⑥ can be represented as

$$y(n) = x(n) * h(n) \longrightarrow ⑦$$

Where '*' denotes the convolution sequence.

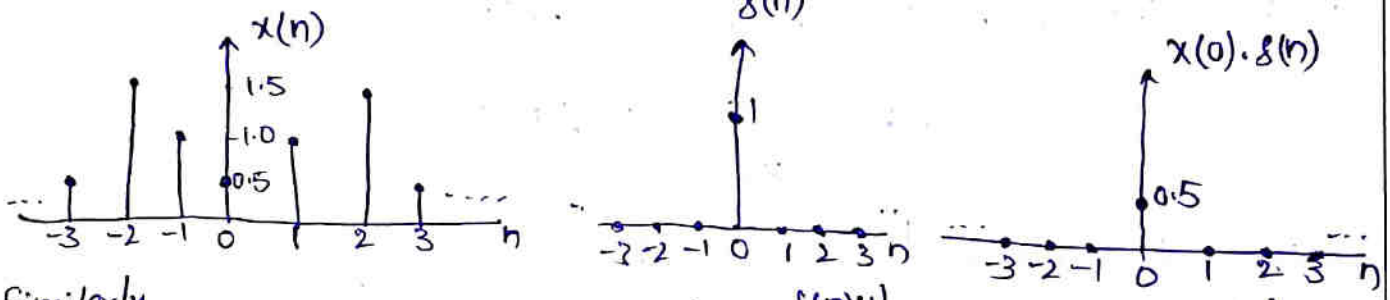
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Representation of an arbitrary sequence:

Any arbitrary sequence $x(n)$ can be represented interms of delayed and scaled impulse sequence $\delta(n)$.

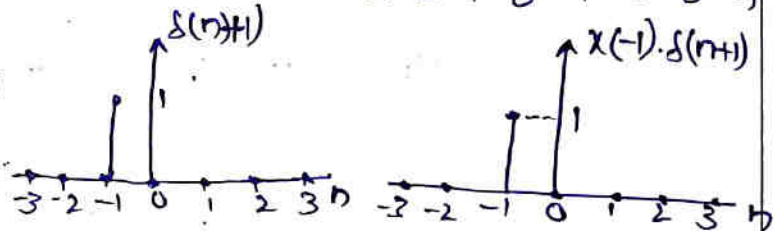
The sample $x(0)$ can be obtained by multiplying $x(0)$ with unit impulse $\delta(n)$ as shown in fig.

i.e., $$x(0).\delta(n) = \begin{cases} x(0) & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

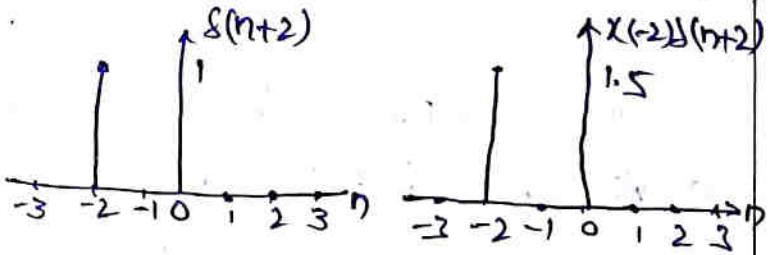


Similarly,

$$x(-1).\delta(n+1) = \begin{cases} x(-1) & \text{: for } n=-1 \\ 0 & \text{: for } n \neq -1 \end{cases}$$



$$x(-2).\delta(n+2) = \begin{cases} x(-2) & \text{: for } n=-2 \\ 0 & \text{: for } n \neq -2 \end{cases}$$



Similarly, the sum of all sequences given by

$$x(n) = \ldots + x(-3)\delta(n+3) + x(-2)\delta(n+2) + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n-2) + x(3).\delta(n-3) + \ldots$$

$$\Rightarrow x(n) = \sum_{k=-\infty}^{\infty} x(k).\delta(n-k)$$

Where $\delta(n-k)$ is unity for $n=k$ and zero for all other terms

=0=

Linear Constant Coefficient Difference Equation:-

W.K.T, the output of D.T. System can be calculated using linear convolution i.e.,

$$y(n) = \sum_{k=-\infty}^{\infty} x(k).h(n-k)$$

→ These difference equations are used to implement the D.T. Systems.

→ These difference equations are especially used for designing the digital filter.

If the system is causal then the impulse raponse $h(n)$ is zero for all negative values of 'n'.

Hence for a causal LTI system.

$$y(n) = \sum_{k=0}^{\infty} x(k).h(n-k)$$

$$\Rightarrow y(n) = \sum_{k=0}^{\infty} h(k).x(n-k)$$

Since the o/p depends on impulse response $h(n)$. The no. of samples present in $h(n)$ plays an important role to determine o/p $y(n)$. The number of samples present in $h(n)$ is called the length of impulse response.

Depending upon the length of impulse response, D.T. Systems are classified into two types:

i). Finite Impulse Response [FIR] systems.

ii). Infinite Impulse Response [IIR] systems.

Time Response Analysis of D.T. Systems:- (or)

Solution of Linear Constant Coefficient Difference Equation:-

Depending upon Initial Conditions, two types of responses are obtained.

i). **Zero state response (or) Forced response:-**

If initial condition $y(-1)$ of the ~~Zero~~ system is Zero then the response of the input signal is called as "Zero state response".

(or)

The Zero state response of the system is the response of the system due to input alone when the initial state of the system is Zero. That is, the system is initially relaxed at time $n=0$.

ii). **Zero input response (or) Natural response:-**

The input is not applied to the system and initial condition $y(-n) \neq 0$. This kind of o/p without applying i/p $x(n)$ is called as "Zero input response" (or) "Natural response".

(or)

The Zero input response depends only on the initial state of the system. That is, the input is Zero.

The response of any D.T. System can be expressed as

Total response = Zero state response + Zero input response.

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Basically, there are two methods to obtain solution of a given linear constant coefficient difference equations:

1). Direct Method 2). Indirect Method

→ Indirect Method is based on Z-transformation.

→ In direct method, the solution consists of two parts:

a). Homogeneous (or) Complementary solution $[y_h(n)]$

b). Particular solution $[y_p(n)]$

Thus, the total solution is

$$y(n) = y_h(n) + y_p(n).$$

a). Homogeneous solution of difference equation:-

The general form of difference equation of an N^{th} order LTI-DT system is

$$y(n) = -\sum_{k=1}^{N} a_k \cdot y(n-k) + \sum_{k=0}^{M} b_k \cdot x(n-k) \longrightarrow (1)$$

Where 'a_k' & 'b_k' are constants.

It can also be written as

$$\sum_{k=0}^{N} a_k \cdot y(n-k) = \sum_{k=0}^{M} b_k \cdot x(n-k) \longrightarrow (2)$$

$[\because a_0 = 1]$

The natural response (or) homogeneous solution will obtain by substituting $x(n) = 0$.

Thus, eqn (2) becomes

$$\sum_{k=0}^{N} a_k \cdot y(n-k) = 0 \longrightarrow (3)$$

Let the homogeneous solution for above equation is

$$y_h(n) = \lambda^n \longrightarrow (4)$$

replace 'n' by 'n-k', $y_h(n-k) = \lambda^{n-k} \longrightarrow (5)$

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Put eqn ⑤ in eqn ③, we get

$$\sum_{K=0}^{N} a_K \cdot \alpha^{n-K} = 0 \qquad [\because a_0 = 1]$$

$$\Rightarrow \alpha^n + a_1 \cdot \alpha^{n-1} + a_2 \cdot \alpha^{n-2} + \dots + a_N \cdot \alpha^{n-N} = 0$$

$$\Rightarrow \alpha^{n-N} \cdot [\alpha^N + a_1 \cdot \alpha^{N-1} + a_2 \cdot \alpha^{N-2} + \dots + a_N] = 0$$

$$\Rightarrow \alpha^N + a_1 \cdot \alpha^{N-1} + a_2 \cdot \alpha^{N-2} + \dots + a_N = 0 \longrightarrow ⑥$$

This polynomial is called as "characteristic polynomial of the system". This polynomial has 'N' roots denoted by $\alpha_1, \alpha_2, \dots \alpha_N$. These roots can be real valued (or) complex valued.

The nature of natural response depends upon the types of roots:

- real roots - real exponential
- Imaginary roots - sinusoids
- Complex roots - Exponentially damped sinusoids.

Distinct roots :- If the roots $\alpha_1, \alpha_2, \dots \alpha_n$ are distinct then it has 'N' solutions $C_1\alpha_1^n, C_2\alpha_2^n, \dots C_N\alpha_N^n$.

The general solution of the form is

$$y_h(n) = C_1\alpha_1^n + C_2\alpha_2^n + \dots + C_N\alpha_N^n \longrightarrow ⑦$$

where 'C_1', 'C_2' ... 'C_N' are arbitrary constants. These constants are determined by applying initial conditions.

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Repeated roots:- If the roots d_1 is repeated 'm' times and the remaining $(N-m)$ roots are distinct then the characteristic equation of the system is

$$(d-d_1)^m.(d-d_{m+1}).(d-d_{m+2}).\cdots\cdots(d-d_N)=0$$

and its general solution is

$$y_h(n)=(C_1+C_2n+C_3n^2+\cdots+C_mn^{m-1}).d_1^n+C_{m+1}d_{m+1}^n+C_{m+2}d_{m+2}^n$$
$$+\cdots+C_Nd^N \longrightarrow (8)$$

Complex roots:- If roots are complex then

$$d_1=d=a+jb$$
$$d_2=d^*=a-jb$$

Then the homogeneous solution is of the form

$$y_h(n)=r^n[A_1.\cos n\theta+A_2.\sin n\theta] \longrightarrow (9)$$

where $r=\sqrt{a^2+b^2}$, $\theta=\tan^{-1}\frac{b}{a}$, A_1 & A_2 are constants.

Eg:- Find the natural response of the system described by difference equation

$$y(n)+2.y(n-1)+y(n-2)=x(n)+x(n-1).$$

with initial condition $y(-1)=y(-2)=1$

Sol:- Given that, $y(n)+2.y(n-1)+y(n-2)=x(n)+x(n-1) \longrightarrow (1)$

The homogeneous solution can be obtained by equating the input being to zero. i.e.,

$$y(n)+2.y(n-1)+y(n-2)=0 \longrightarrow (2)$$

The homogeneous solution of this form is

$$y_h(n)=d^n \longrightarrow (3)$$

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from ② & ③..

$$\alpha^n + 2\cdot\alpha^{n-1} + \alpha^{n-2} = 0$$

$$\Rightarrow \alpha^{n-2}\cdot[\alpha^2 + 2\alpha + 1] = 0$$

$$\Rightarrow (\alpha+1)^2 = 0$$

$$\Rightarrow \alpha_1 = -1,\ \alpha_2 = -1$$

The roots are repeated, therefore the general solution of homogeneous equation is

$$y_h(n) = (C_1 + n\cdot C_2)(-1)^n \longrightarrow ④$$

Let $n=0$, $y(0) = C_1 \rightarrow (a)$ & $n=1$, $y(1) = -(C_1+C_2) \rightarrow (b)$

Put $n=0$ in eqn ①, $y(0) + 2\cdot y(-1) + y(-2) = 0$

Given that, $y(-1) = y(-2) = 1$

$\therefore y(0) + 2 + 1 = 0 \Rightarrow y(0) = -3 \rightarrow (c)$

Put $n=1$ in eqn ②, $y(1) + 2\cdot y(0) + y(-1) = 0$

$\therefore y(1) + 2(-3) + 1 = 0 \Rightarrow y(1) = 5 \rightarrow (d)$

from (a) & (c), $C_1 = -3$

from (a), (b), (c) & (d).. $5 = -(-3 + C_2) \Rightarrow C_2 = -2$

Substitute 'C_1' & 'C_2' values in eqn. ④, we get

$$y_h(n) = (-3 - 2n)(-1)^n \quad , \text{ for } n \geq 0$$

$$\Rightarrow y_h(n) = [-3u(n) - 2n\,u(n)](-1)^n$$

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Eg:- What do you mean by impulse response of a system? Find it for a system described as $y(n) = x(n) + y(n-1)$, assuming that initially system is relaxed.

Sol:- Given that, $y(n) = x(n) + y(n-1)$ ⟶ ①

$\Rightarrow y(n) - y(n-1) = x(n)$

for homogeneous solution, $x(n) = 0$

Thus, $y(n) - y(n-1) = 0$ ⟶ ②

Impulse response is the response of system when input is $x(n) = \delta(n)$.

The homogeneous solution for eqn ② is

$$y_h(n) = d^n \; ; \; y_h(n-1) = d^{n-1}$$

Substitute in eqn ②, $d^n - d^{n-1} = 0$

$\Rightarrow d^{n-1}[d-1] = 0$

$\Rightarrow d = 1$

The homogeneous solution is given by $y_h(n) = c_1(1)^n$ ⟶ ③

Given input is $x(n) = \delta(n)$ and $y(n) = h(n)$

Putting these values in eqn ①, we get

$$h(n) = \delta(n) + h(n-1)$$

Put $n=0$, $h(0) = \delta(0) + h(-1)$

$\delta(0) = 1$ since initial conditions are zero, $h(-1) = 0$ $\left[\because \delta(n) = 1 \; ; n=0 \;\; = 0 \; ; n \neq 0\right]$

$\therefore h(0) = 1$ ⟶ ④

from ③, $h(n) = c_1(1)^n$

Put $n=0$, $h(0) = c_1 = 1$

Therefore, $h(n) = 1$

=0=

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b). Particular solution, $y_p(n)$:-

The particular solution is calculated by assuming that the output has same form at that of input.

For example, if input $x(n) = u(n)$ then the output is assumed to be $y_p(n) = K.u(n)$, where 'K' is arbitrary constant.

Some commonly used input signals and the particular solution that we have to assume is shown in following table.

S.NO	Input $x(n)$	Particular solution
1.	Constant term (A)	K
2.	$u(n)$	$K.u(n)$
3.	AC^n	KC^n
4.	$A^n.n^c$	$K_0n^c + K_1n^{M-1} + \cdots + K_c$
5.	$A.\cos\omega_0 n$	$K_1\cos\omega_0 n + K_2.\sin\omega_0 n$
6.	An^c	

Eg:- Find out the particular solution for the following difference equation $y(n) + 3y(n-1) = x(n)$. Assume $x(n) = u(n)$.

Sol:- Given that, input $x(n) = u(n)$ ⟶ (1)

Now corresponding particular solution is $y_p(n) = K.u(n)$ ⟶ (2)

Given eqn. is $y(n) + 3.y(n-1) = x(n)$ ⟶ (3)

Substitute (1) & (2) in (3), we get

$$K.u(n) + 3K.u(n-1) = u(n) \longrightarrow (4)$$

Put $n=1$, $\therefore K.u(1) + 3.K.u(0) = u(1)$

$$\Rightarrow K + 3K = 1$$

$$\Rightarrow 4K = 1$$

$$\Rightarrow K = \frac{1}{4}$$

$\therefore$ The particular solution is $y_p(n) = \frac{1}{4}u(n)$

=o=

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Eg:- Determine the particular solution of the difference equation $y(n) = 3 \cdot y(n-1) - 4 \cdot y(n-2) + x(n)$. When the forcing function $x(n) = 2^n$, $n \geq 0$ and zero everywhere.

Sol:- Given that, $y(n) = 3 \cdot y(n-1) - 4 \cdot y(n-2) + x(n) \longrightarrow (1)$

input is $x(n) = 2^n$, $n \geq 0$ Then $y_p(n) = K \cdot 2^n \cdot u(n) \longrightarrow (2)$

$\Rightarrow x(n) = 2^n \cdot u(n)$

Substitute eqn (2) in (1), we get

$$K \cdot 2^n = 3K \cdot 2^{n-1} - 4 \cdot 2^{n-2} + 2^n$$

Substitute eqn (2) in (1), we get

$$K 2^n u(n) = 3K 2^{n-1} u(n-1) - 4K 2^{n-2} u(n-2) + 2^n \cdot u(n)$$

Put $n = 2$,

$$K \cdot 4 \cdot u(2) = 3K \cdot 2 \cdot u(1) - 4K\, u(0) + 4 \cdot u(2)$$

$$\Rightarrow 4K = 6K - 4K + 4$$

$$\Rightarrow 2K = 4 \Rightarrow K = 2$$

$\therefore$ The particular solution of this difference equation is

$$y_p(n) = 2 \cdot 2^n u(n)$$

$= 0 =$

Note:- If total solution is asked to calculate then first obtain $y_h(n)$ and $y_p(n)$ seperately. The total solution is

$$y(n) = y_h(n) + y_p(n)$$

$= 0 =$

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Eg:- Find the forced response of the system (or) total solution described by the difference equation

$$y(n) + 2\cdot y(n-1) + y(n-2) = x(n) + x(n-1)$$

for input $x(n) = \left(\frac{1}{2}\right)^n u(n)$.

Sol:- Given equation is $y(n) + 2\cdot y(n-1) + y(n-2) = x(n) + x(n-1)$ $\longrightarrow$ (1)

and input $x(n) = \left(\frac{1}{2}\right)^n u(n)$ $\longrightarrow$ (2)

The total solution of the system is

$$y(n) = y_h(n) + y_p(n) \longrightarrow (3)$$

i) $y_h(n)$:-

Put input, $x(n) = 0$ for all values of 'n'

$$y(n) + 2\cdot y(n-1) + y(n-2) = 0 \longrightarrow (3a)$$

The General solution for the system is $y_h(n) = \alpha^n$

So, $\alpha^n + 2\cdot\alpha^{n-1} + \alpha^{n-2} = 0$

$\Rightarrow \alpha^{n-2}[\alpha^2 + 2\alpha + 1] = 0$

$\Rightarrow (\alpha+1)^2 = 0$

$\Rightarrow \alpha = -1, -1$

$\therefore$ The homogeneous solution of the given system is

$$y_h(n) = (C_1 + nC_2)(-1)^n \longrightarrow (4)$$

Put $n=0$ in eqn (4), $y(0) = C_1$ $\longrightarrow$ (5)

Put $n=1$ in eqn (4), $y(1) = -C_1 - C_2$ $\longrightarrow$ (6)

Put $n=0$ in eqn (3a), $y(0) + 2\cdot y(-1) + y(-2) = 0$

Assume initial conditions, $y(-1) = y(-2) = 0$

then $y(0) = 0$ $\longrightarrow$ (7)

from (5) & (7), $C_1 = 0$

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Put $n=1$ in eqn (3a), $y(1)+2\cdot y(0)+y(-1)=0$

$\Rightarrow y(1)=0$

---

ii). $y_p(n)$:-

For input $x(n)=\left(\frac{1}{2}\right)^n u(n)$, the particular solution of the form is

$$y_p(n)=K\cdot\left(\frac{1}{2}\right)^n u(n) \longrightarrow (5)$$

Substitute eqn (2) & (5) in (1), we get

$$K\cdot\left(\frac{1}{2}\right)^n u(n)+2\cdot K\cdot\left(\frac{1}{2}\right)^{n-1}u(n-1)+K\cdot\left(\frac{1}{2}\right)^{n-2}\cdot u(n-2)=\left(\frac{1}{2}\right)^n u(n)+\left(\frac{1}{2}\right)^{n-1}\cdot u(n-1)$$

Put $n=2$,

$$K\cdot\frac{1}{4}u(2)+2K\left(\frac{1}{2}\right)\cdot u(1)+K\cdot u(0)=\frac{1}{4}\cdot u(2)+\frac{1}{2}\cdot u(1)$$

$$\Rightarrow \frac{K}{4}+K+K=\frac{1}{4}+\frac{1}{2}$$

$$\Rightarrow \frac{9K}{4}=\frac{3}{4}$$

$$\Rightarrow K=1/3$$

Therefore, $y_p(n)=\frac{1}{3}\cdot\left(\frac{1}{2}\right)^n u(n) \longrightarrow (6)$

Substitute (4) & (6) in eqn (3), we get

$$y(n)=(C_1+nC_2)(-1)^n+\frac{1}{3}\left(\frac{1}{2}\right)^n \longrightarrow (7)$$

Put $n=0$ in eqn (7), $y(0)=C_1+\frac{1}{3} \longrightarrow (8)$

Put $n=1$ in eqn (7), $y(1)=-(C_1+C_2)+\frac{1}{6} \longrightarrow (9)$

Put $n=0$ in eqn (1) $\cdot$ $y(0)+2\cdot y(-1)+y(-2)=x(0)+x(-1)$

Assume initial conditions, $y(-1)=y(-2)=0$

$$y(0)=\left(\frac{1}{2}\right)^0 u(0)+\left(\frac{1}{2}\right)^{-1}u(-1) \qquad \left[\because u(-1)=0\right]$$

$$\Rightarrow y(0)=1 \longrightarrow (10)$$

from (8) & (10), $C_1+\frac{1}{3}=1 \Rightarrow C_1=2/3$

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Put $n=1$ in eqn (1), $y(1)+2\cdot y(0)+y(-1)=x(1)+x(0)$

$\Rightarrow y(1)+2=\frac{1}{2}+1$

$\Rightarrow y(1)=-1/2 \longrightarrow (11)$

from (9), $-\frac{1}{2}=-\frac{2}{3}-c_2+\frac{1}{6}$

$\Rightarrow c_2=0$

Substitute c_1 & c_2 values in eqn. (7), we get

$$y(n)=\frac{2}{3}(-1)^n+\frac{1}{3}\left(\frac{1}{2}\right)^n$$

$=0=$

Eg:- Determine the impulse response of the filter defined by $y(n)=x(n)+b\cdot y(n-1)$

Sol:- Given that, $y(n)=x(n)+b\cdot y(n-1)$

We know that, the total response is $y(n)=y_h(n)+y_p(n)$

For impulse $x(n)=\delta(n)$, $y_p(n)=0$

Therefore, $y(n)=y_h(n)$

The homogeneous solution can be obtained by $x(n)=0$,

i.e., $y(n)=b\cdot y(n-1)$

$\Rightarrow y(n)-b\cdot y(n-1)=0$

Its general solution is $y_h(n)=\alpha^n$

Thus, $\alpha^n-b\cdot\alpha^{n-1}=0$

$\Rightarrow \alpha^{n-1}[\alpha-b]=0$

$\Rightarrow \alpha-b=0 \Rightarrow \alpha=b$

The general form of solution of homogeneous eqn. is

$$y_h(n)=c_1\alpha_1^n=c_1\cdot b^n$$

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Put $n=0$ in above equation, $y(0) = C_1$

Put $n=0$ in given equation, $y(0) = x(0) + b \cdot y(-1)$

Assume initial conditions, $y(-1)=0$ & $x(0)=\delta(0)=1$

$\therefore y(0) = 1$

So, $y(0) = C_1 = 1$.

$\therefore$ The impulse response of the given system is

$$y(n) = b^n \cdot u(n)$$

=o=

Frequency domain representation of Discrete time Signals:-

W.K.T., the fourier series representation for C.T. Periodic signal can consist of an infinite no.of frequency components, with frequency range extends from $-\infty$ to $+\infty$. It means, signals contain an infinite no.of frequency components.

The frequency range for D.T. signals is unique over the interval $(-\pi, \pi)$ (or) $(0, 2\pi)$. A D.T. signal of fundamental period 'N' can consist of frequency components seperated by $\frac{2\pi}{N}$ radians (or) $f = \frac{1}{N}$ cycles.

This is the basic diffelence between the fourier series representation for C.T & D.T. periodic signals.

Let $x(n)$ be a periodic sequence with period 'N' i.e., $x(n) = x(n+N)$ for all 'n'. The fourier series representation for $x(n)$ consists 'N' harmonically related exponential functions. $e^{\frac{j2\pi kn}{N}}$, $k = 0, 1, 2, \ldots N-1$

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Thus, $x(n)$ can be expressed as

$$x(n) = \sum_{k=0}^{N-1} C_k \cdot e^{\frac{j2\pi kn}{N}} \longrightarrow (1)$$

where 'C_k' are the coefficients in the series expansion.

To derive the expression for fourier coefficients, we use

$$\sum_{n=0}^{N-1} e^{\frac{j2\pi kn}{N}} = \begin{cases} N & ; k = 0, \pm N, \pm 2N, \ldots \\ 0 & ; \text{otherwise} \end{cases} \longrightarrow (2)$$

Multiply eqn (1) on both sides with $e^{-j2\pi ln/N}$ and summing the product from $n=0$ to $N-1$. Thus,

$$\sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi ln/N} = \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} C_k \cdot e^{j2\pi (k-l)n/N} \longrightarrow (3)$$

To obtain fourier coefficients, we have from (2).,

$$\sum_{n=0}^{N-1} e^{j2\pi (k-l)n/N} = \begin{cases} N & ; k-l = 0, \pm N, \pm 2N, \ldots \\ 0 & ; \text{otherwise} \end{cases} \longrightarrow (4)$$

Eqn (3). can be reduced as

$$\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi ln/N} = N \cdot C_l$$

$$\Rightarrow C_l = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi ln/N} \longrightarrow (5)$$

This is the desired expression for fourier coefficients interms of the signal $x(n)$.

Frequency Analysis of D.T Periodic Signals	
Synthesis equation	$x(n) = \sum_{k=0}^{N-1} C_k \cdot e^{\frac{j2\pi kn}{N}}$
Analysis equation	$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cdot e^{\frac{-j2\pi kn}{N}}$

=0=

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Frequency domain representation of LTI-D.T Systems:-

W.K.T, the output $y(n)$ of any LTI system is an input signal $x(n)$ can be obtained by using convolution sum

$$y(n) = \sum_{k=-\infty}^{\infty} h(k).x(n-k) \longrightarrow (1)$$

where $h(n)$ is the impulse response of the system.

Let us consider a complex exponential signal, $x(n) = e^{j\omega n}$ as input to the system. The o/p is given by

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, e^{j\omega(n-k)}$$

$$= e^{j\omega n}\left[\sum_{k=-\infty}^{\infty} h(k)\, e^{-j\omega k}\right] \longrightarrow (2)$$

$$\Rightarrow y(n) = e^{j\omega n}.\ H(e^{j\omega})$$

$H(e^{j\omega})$ → frequency response

where $$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h(k)\, e^{-j\omega k} \longrightarrow (3)$$

The quantity $H(e^{j\omega})$ is also called as frequency response of the system. Since the frequency response $H(e^{j\omega})$ is a complex valued function. It can be expressed in polar form as

$$H(e^{j\omega}) = |H(e^{j\omega})|.\ e^{j\theta(\omega)} \longrightarrow (4)$$

where $|H(e^{j\omega})|$ = Magnitude response of the system

$\theta(\omega)$ = phase response of the system.

Transfer function:-

Eqn ①. Can be written as

$$y(n) = h(n) * x(n)$$

Applying fourier transform on both sides,

$$Y(e^{j\omega}) = H(e^{j\omega}) \cdot X(e^{j\omega})$$

$$\Rightarrow H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \longrightarrow (5)$$

where $H(e^{j\omega})$ is known as transfer function of the system and $h(n) = F^{-1}[H(e^{j\omega})]$ is known as impulse response of the system.

Eg:- Determine and sketch the magnitude and phase response of $y(n) = \frac{1}{2}[x(n) + x(n-2)]$.

Sol:- Given that $y(n) = \frac{1}{2}[x(n) + x(n-2)]$

W.K.T., $Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y(n) \cdot e^{-j\omega n}$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} [x(n) + x(n-2)] \cdot e^{-j\omega n}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n} + \frac{1}{2} \sum_{n=-\infty}^{\infty} x(n-2) \cdot e^{-j\omega n}$$

$$= \frac{1}{2} X(e^{j\omega}) + \frac{1}{2} e^{-2j\omega} \cdot X(e^{j\omega})$$

$$\Rightarrow Y(e^{j\omega}) = \frac{1}{2}[1 + e^{-2j\omega}] \cdot X(e^{j\omega})$$

$$\Rightarrow H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 + e^{-2j\omega}}{2}$$

This is the transfer function of given system

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$$H(e^{j\omega}) = \frac{1+\cos 2\omega - j\sin 2\omega}{2}$$

Magnitude response is given by

$$|H(e^{j\omega})| = \frac{1}{2}\sqrt{(1+\cos 2\omega)^2 + \sin^2 2\omega} = \frac{1}{2}\sqrt{(2\cos^2\omega)^2 + (2\sin\omega\cos\omega)^2}$$

$$= \frac{1}{2}\sqrt{4\cos^4\omega + 4\sin^2\omega\cos^2\omega}$$

$$\Rightarrow |H(e^{j\omega})| = \cos\omega$$

Phase response is

$$\angle H(e^{j\omega}) = \tan^{-1}\left[\frac{-\sin 2\omega}{1+\cos 2\omega}\right] = \tan^{-1}\left[\frac{-2\sin\omega\cos\omega}{2\cos^2\omega}\right] = \tan^{-1}[-\tan\omega]$$

$$\Rightarrow \angle H(e^{j\omega}) = -\omega \text{ for } |H(e^{j\omega})| > 0$$
$$= -\omega + \pi \text{ for } |H(e^{j\omega})| < 0$$

ω	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$\lvert H(e^{j\omega})\rvert$	1	0.812	0.707	0.5	0	0.5	0.707	1
$\angle H(e^{j\omega})$	0	$-\pi/6$	$-\pi/4$	$-\pi/3$	$-\pi/2$	$\pi/3$	$\pi/4$	0°

W.K.T. $|H(e^{j\omega})|$ is symmetrical & $|H(e^{j\omega})|$ is antisymmetrical with period of 2π in ω. We also need to know over $0 \le \omega \le \pi$.

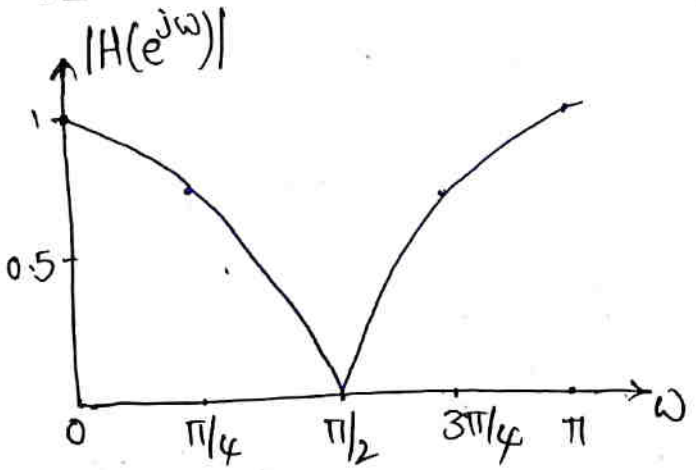


fig: Magnitude response

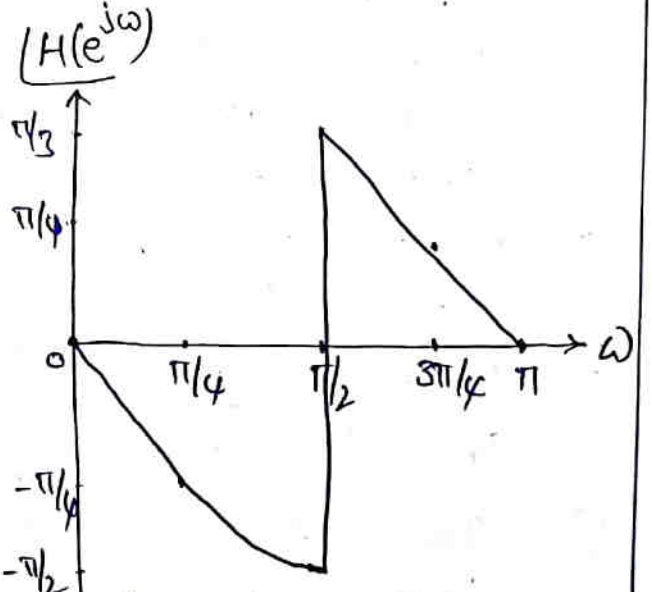


fig: Phase response

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Z-Transforms:-

Z-Transform plays in the analysis of D-T signals and LTI systems as well as Laplace Transform plays in the analysis of C.T signals and LTI systems.

For example, in the Z-domain the convolution of two time domain signals is equivalent to the multiplication of their corresponding Z-Transforms.

The Z-Transform of a D.T. signal $x(n)$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

Where 'z' is a complex variable. It is denoted by

$$X(z) = Z[x(n)]$$

$$\Rightarrow \quad x(n) \xleftrightarrow{Z} X(z)$$

Advantages of Z-Transform:-

1. D.T. signals and LTI systems can be completely characterized using Z-Transform.
2. The stability of LTI systems can be determined using Z-Transform
3. Mathematical calculations are reduced using Z-Transform.
   For example, convolution operation is transformed into simple multiplication operation.
4. By calculating Z-Transform of given signal, DFT & FT can be calculated.
5. Entire family of digital filters can be obtained from one Proto-type design using Z-Transform.
6. The solution of differential equations can be simplified using Z-Transform.

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Some Important Z-Transform Pairs and Corresponding ROCs:-

S.No	D.T. Signal $x(n)$	Z-Transform $X(z)$	ROC
1.	$\delta(n)$	1	All z-plane
2	$\delta(n-k)$	z^{-k}	$\lvert z\rvert>0$, $k>0$ $\lvert z\rvert<\alpha$, $k<0$
3	$u(n)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$	$\lvert z\rvert>1$
4	$-u(-n-1)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$	$\lvert z\rvert<1$
5	$n.u(n)$	$\frac{z^{-1}}{(1-z^{-1})^2} = \frac{z}{(z-1)^2}$	$\lvert z\rvert>1$
6.	$a^n.u(n)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$	$\lvert z\rvert>\lvert a\rvert$
7.	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$	$\lvert z\rvert<\lvert a\rvert$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert>\lvert a\rvert$
9	$-na^n u(-n-1)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert<\lvert a\rvert$
10	$n^2 u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert>1$
11.	$\sin\omega_0 n$	$\frac{z.\sin\omega_0}{z^2-2z.\cos\omega_0+1}$	$\lvert z\rvert>1$
12.	$\cos\omega_0 n$	$\frac{z.(z-\cos\omega_0)}{z^2-2z.\cos\omega_0+1}$	$\lvert z\rvert>1$
13.	$a^n.\sin\omega_0 n$	$\frac{z.a\sin\omega_0}{z^2-2za\cos\omega_0+a^2}$	$\lvert z\rvert>\lvert a\rvert$
14.	$a^n.\cos\omega_0 n$	$\frac{z.(z-a\cos\omega_0)}{z^2-2za.\cos\omega_0+a^2}$	$\lvert z\rvert>\lvert a\rvert$
15.	$u(-n)$	$\frac{1}{1-z}$	$\lvert z\rvert>1$
16.	$a^n u(n)+b^n u(-n-1)$	$\frac{z}{z-a}-\frac{z}{z-b}$	$\lvert a\rvert<\lvert z\rvert<\lvert b\rvert$

(20)

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Properties of the Z-transform:-

S.No	Property	Time domain $\xleftrightarrow{Z}$ Z-domain	ROC
1.	Linearity	$a_1x_1(n)+a_2x_2(n) \xleftrightarrow{Z} a_1X_1(z)+a_2X_2(z)$	Intersection of ROC of $X_1(z)$ & $X_2(z)$
2.	Time shifting	$x(n-k) \xleftrightarrow{Z} z^{-k}\cdot X(z)$	Same as ROC of $X(z)$
3.	Scaling in 'z'	$a^n x(n) \xleftrightarrow{Z} X\left(\frac{z}{a}\right)$	$\lvert a\rvert r_1 < \lvert z\rvert < \lvert a\rvert r_2$
4.	Time reversal	$x(-n) \xleftrightarrow{Z} X(z^{-1})$	$\frac{1}{r_1} < \lvert z\rvert < \frac{1}{r_2}$
5.	Differentiation in Z-domain	$n\cdot x(n) \xleftrightarrow{Z} -z\cdot\frac{d}{dz}X(z)$	Same as ROC of $X(z)$
6.	Convolution	$x_1(n) * x_2(n) \xleftrightarrow{Z} X_1(z)\cdot X_2(z)$	ROC is atleast intersection
7.	Multiplication	$x_1(n)\cdot x_2(n) \xleftrightarrow{Z} \frac{1}{2\pi j}\oint_C X_1(v)\cdot X_2\left(\frac{z}{v}\right)v^{-1}dv$	ROC is atleast $r_{1l}r_{2l} < \lvert z\rvert < r_{1u}r_{2u}$
8.	Initial value theorem	Causal $x(n)$: $x(0) = \lim_{z\to\infty} X(z)$	-
9.	Final value theorem	Causal $x(n)$: $x(\infty) = \lim_{z\to 1} X(z)$	-

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Solution of Difference equations using Z-transform:-

We solved Linear Constant Coefficient difference equations which involved finding the particular and homogeneous solution.

Now, we use the time shift property of one-sided (or) unilateral Z-transform to the difference equation so that we can solve the transform of the output. Then applying inverse transform, we can find the o/p sequence.

Z-transform of D.T. signal $x(n)$ is defined as

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} \longrightarrow (1)$$

Time shifting property: $x(n-k) \xleftrightarrow{Z} z^{-k} X(z) \longrightarrow (2)$

Eg:- Find the impulse response of the system described by difference equation

$y(n) - 3 \cdot y(n-1) - 4 \cdot y(n-2) = x(n) + 2 \cdot x(n-1)$ using Z-transform.

Sol:- Given that,

$$y(n) - 3 \cdot y(n-1) - 4 \cdot y(n-2) = x(n) + 2 \cdot x(n-1)$$

Applying Z-transform on both sides using eqn (2).

$$Y(z) - 3z^{-1} \cdot Y(z) - 4 \cdot z^{-2} Y(z) = X(z) + 2 \cdot z^{-1} \cdot X(z)$$

$$\Rightarrow (1 - 3z^{-1} - 4z^{-2}) \cdot Y(z) = (1 + 2z^{-1}) \cdot X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1}}{1 - 3z^{-1} - 4z^{-2}} = \frac{z \cdot (z+2)}{z^2 - 3z - 4}$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{z \cdot (z+2)}{(z+1) \cdot (z-4)}$$

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$$\frac{H(z)}{z} = \frac{z+2}{(z+1)(z-4)} = \frac{-\frac{1}{5}}{5(z+1)} + \frac{6}{5(z-4)}$$

$$\Rightarrow H(z) = -\frac{1}{5}\cdot\frac{z}{z+1} + \frac{6}{5}\cdot\frac{z}{z-4}$$

Here $H(z)$ is called as transfer function (or) system function of a given D.T. system.

To find the impulse response of the system, applying inverse transform,

$$h(n) = \frac{6}{5}\cdot 4^n u(n) - \frac{1}{5}(-1)^n u(n)$$

=o=

The system function:-

We know that, A linear constant coefficient difference equation of a system is described by

$$y(n) = -\sum_{k=1}^{N} a_k \cdot y(n-k) + \sum_{k=0}^{M} b_k \cdot x(n-k)$$

Taking Z-transform on both sides,

$$Y(z) = -\sum_{k=1}^{N} a_k \cdot z^{-k} \cdot Y(z) + \sum_{k=0}^{M} b_k \cdot z^{-k} \cdot X(z)$$

$$\Rightarrow \left[1 + \sum_{k=1}^{N} a_k \cdot z^{-k}\right] \cdot Y(z) = \sum_{k=0}^{M} b_k z^{-k} \cdot X(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k \cdot z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \longrightarrow (3)$$

Where $H(z)$ is known as transfer function (or) system function. $H(z)$ is the Z-transform of the impulse response $h(n)$.

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Eg:- Find the system function and impulse response of the system described by the difference equation

$$y(n) = \frac{1}{5} y(n-1) + x(n).$$

Sol:- Given that, $y(n) = \frac{1}{5} y(n-1) + x(n)$

Applying 'Z'-Transform on both sides

$$Y(z) = \frac{1}{5} z^{-1} \cdot Y(z) + X(z)$$

$$\Rightarrow \left(1 - \frac{1}{5} z^{-1}\right) \cdot Y(z) = X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{5} z^{-1}}$$

∴ Now, the system function $H(z) = \frac{1}{1 - \frac{1}{5} z^{-1}} = \frac{z}{z - 1/5}$

using inverse Z-Transform,

$$h(n) = \left(\frac{1}{5}\right)^n \cdot u(n)$$

=o=

Eg:- Determine the unit step response of the system whose difference equation is

$$y(n) - 0.7y(n-1) + 0.12\, y(n-2) = x(n-1) + x(n-2) \text{ if } y(-1) = y(-2) = 1.$$

Sol:- Given that input $x(n) = u(n)$ and

$$y(n) - 0.7\, y(n-1) + 0.12\, y(n-2) = x(n-1) + x(n-2)$$

Applying Z-Transform on both sides,

$$Y(z) - 0.7\left[z^{-1} Y(z) + y(-1)\right] + 0.12\left[z^{-2} Y(z) + z^{-1} y(-1) + y(-2)\right] =$$

$$\left[z^{-1} X(z) + x(-1)\right] + \left[z^{-2} X(z) + z^{-1} x(-1) + x(-2)\right]$$

Given that, $y(-1) = y(-2) = 1$ and $x(-1) = x(-2) = 0$

$$\left[\because x(n) = u(n) = 1 ; n \geq 0 \right.$$
$$\left. = 0 ; n < 0 \right]$$

$$\Rightarrow Y(z) - 0.7\left[z^{-1}Y(z) + 1\right] + 0.12\left[z^{-2}Y(z) + z^{-1} + 1\right] = z^{-1}X(z) + z^{-2}X(z)$$

$$\Rightarrow Y(z)\left[1 - 0.7z^{-1} + 0.12z^{-2}\right] - 0.7 + 0.12z^{-1} + 0.12 = X(z)\left[z^{-1} + z^{-2}\right]$$

$$x(n) = u(n) \xleftrightarrow{Z} X(z) = \frac{1}{1 - z^{-1}}$$

$$\Rightarrow Y(z)\left[1 - 0.7z^{-1} + 0.12z^{-2}\right] - 0.58 + 0.12z^{-1} = \frac{z^{-1}}{1 - z^{-1}} + \frac{z^{-2}}{1 - z^{-1}}$$

$$\Rightarrow Y(z) = \frac{0.58 - 0.12z^{-1}}{1 - 0.7z^{-1} + 0.12z^{-2}} + \frac{z^{-1} + z^{-2}}{(1 - 0.7z^{-1} + 0.12z^{-2}) \cdot (1 - z^{-1})}$$

$$\Rightarrow Y(z) = \frac{+(0.58z^2 - 0.12z)}{z^2 - 0.7z + 0.12} + \frac{z^2 + z}{(z^2 - 0.7z + 0.12) \cdot (z - 1)}$$

$$\Rightarrow Y(z) = \frac{z(0.58z - 0.12)}{(z - 0.3)(z - 0.4)} + \frac{z(z + 1)}{(z - 1)(z - 0.3)(z - 0.4)}$$

$$\Rightarrow \frac{Y(z)}{z} = \frac{1.12}{z - 0.4} - \frac{0.54}{z - 0.3} + \frac{4.762}{z - 1} + \frac{18.57}{z - 0.3} - \frac{23.33}{z - 0.4}$$

$$\Rightarrow Y(z) = \frac{1.12z}{z - 0.4} - \frac{0.54z}{z - 0.3} + \frac{4.762z}{z - 1} + \frac{18.57z}{z - 0.3} - \frac{23.33z}{z - 0.4}$$

Taking inverse Z-transform on both sides,

$$y(n) = 1.12(0.4)^n u(n) - 0.54(0.3)^n u(n) + 4.762\,u(n) + 18.57(0.3)^n u(n) - 23.33(0.4)^n u(n)$$

$$= 0 =$$

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Eg:- Using Z-transform, solve the difference equation

$$y(n+2) - \frac{3}{2}y(n+1) + \frac{1}{2}y(n) = \left(\frac{1}{4}\right)^n : \text{for } n \geq 0$$

with initial conditions $y(0) = 10$ and $y(1) = 4$.

Sol:- Given that,

$$y(n+2) - \frac{3}{2}y(n+1) + \frac{1}{2}y(n) = \left(\frac{1}{4}\right)^n$$

applying 'Z'-transform

$$\left[z^2 Y(z) - z^2 y(0) - z\,y(1)\right] - \frac{3}{2}\left[z^1 Y(z) - z\,y(0)\right] + \frac{1}{2}Y(z) = \frac{z}{z-1/4}$$

Note:-

Time-delay property : $x(n-k) \leftrightarrow z^{-k}\left[X(z) + \sum_{n=1}^{k} x(-n)z^n\right]$

Time-advance property : $x(n+k) \leftrightarrow z^k\left[X(z) - \sum_{n=0}^{k-1} x(n)z^{-n}\right]$, $k \geq 0$

Substitute $y(0) = 10$ & $y(1) = 4$

$$z^2 Y(z) - 10z^2 - 4z - \frac{3}{2}z\,Y(z) + 15z + \frac{1}{2}Y(z) = \frac{z}{z-1/4}$$

$$\Rightarrow \left[z^2 - \frac{3}{2}z + \frac{1}{2}\right]Y(z) - \left[10z^2 - 11z\right] = \frac{z}{z-1/4}$$

$$\Rightarrow Y(z) = \frac{10z^2 - 11z}{z^2 - \frac{3}{2}z + \frac{1}{2}} + \frac{z}{\left(z^2 - \frac{3}{2}z + \frac{1}{2}\right)\cdot\left(z - 1/4\right)}$$

$$= \frac{z\cdot(10z-11)}{(z-1)\cdot(z-1/2)} + \frac{z}{(z-1)\cdot(z-1/2)\cdot(z-1/4)}$$

$$\Rightarrow \frac{Y(z)}{z} = \frac{10z-11}{(z-1)(z-1/2)} + \frac{1}{(z-1)\cdot(z-1/2)\cdot(z-1/4)}$$

$$\Rightarrow \frac{Y(z)}{z} = \frac{-2}{z-1} + \frac{12}{z-1/2} + \frac{8}{3(z-1)} - \frac{8}{(z-1/2)} + \frac{16}{3\cdot(z-1/4)}$$

$$\Rightarrow Y(z) = \frac{-2\cdot z}{z-1} + \frac{12\cdot z}{z-1/2} + \frac{8}{3}\cdot\frac{z}{z-1} - \frac{8\cdot z}{z-1/2} + \frac{16}{3}\cdot\frac{z}{z-1/4}$$

applying inverse Z-Transform on both sides,

$$y(n) = -2\,u(n) + 12\left(\tfrac{1}{2}\right)^n u(n) + \frac{8}{3}u(n) - 8\cdot\left(\tfrac{1}{2}\right)^n u(n) + \frac{16}{3}\left(\tfrac{1}{4}\right)^n u(n)$$

$$\Rightarrow y(n) = \frac{2}{3}u(n) + 4\cdot\left(\tfrac{1}{2}\right)^n u(n) + \frac{16}{3}\left(\tfrac{1}{4}\right)^n u(n)$$

=o=

Eg:- Find the impulse response & frequency response of a second order D-T system described by the difference equation

$$y(n) - y(n-1) + \frac{3}{16}y(n-2) = x(n) - \frac{1}{2}x(n-1).$$

Sol: Given that, $y(n) - y(n-1) + \frac{3}{16}y(n-2) = x(n) - \frac{1}{2}x(n-1)$

taking Z-Transform on both sides,

$$Y(z) - z^{-1}Y(z) + \frac{3}{16}z^{-2} = X(z) - \frac{1}{2}z^{-1}X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{2}z^{-1}}{1 - z^{-1} + \frac{3}{16}z^{-2}} = \frac{z(z-1/2)}{z^2 - z + \frac{3}{16}}$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{z(z-1/2)}{(z-3/4)\cdot(z-1/4)}$$ $\Rightarrow$ System function

The frequency response $H(e^{j\omega}) = H(z)\big|_{z=e^{j\omega}} = \dfrac{e^{j\omega}(e^{j\omega} - 1/2)}{(e^{j\omega} - 3/4)\cdot(e^{j\omega} - 1/4)}$

$$\therefore \frac{H(z)}{z} = \frac{1}{2}\cdot\frac{z}{z-\frac{3}{4}} + \frac{1}{2}\cdot\frac{z}{z-1/4}$$

take inverse Z-T.

impulse response, $h(n) = \frac{1}{2}\cdot\left(\frac{3}{4}\right)^n u(n) + \frac{1}{2}\left(\frac{1}{4}\right)^n u(n)$

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Signal processing:-

Signal processing is any operation performed by the system that can changes the characteristic of a signal.

Depending upon the type of signal, signal processing can be classified into two types:

1). Analog signal processing [ASP]

2). Digital signal Processing [DSP]

1). Analog signal processing (ASP):-

The continuous time signals (or) Analog signals has been processed by a system is known as "Analog signal Processing".

Most of the signals encountered in science & engineering are analog in nature. These signals are functions of a ~~time~~, continuous variable such as time (or) space. Such signals may be processed directly by appropriate analog systems such as filters, freq. analyzers and multipliers for the purpose of changing their characteristic.

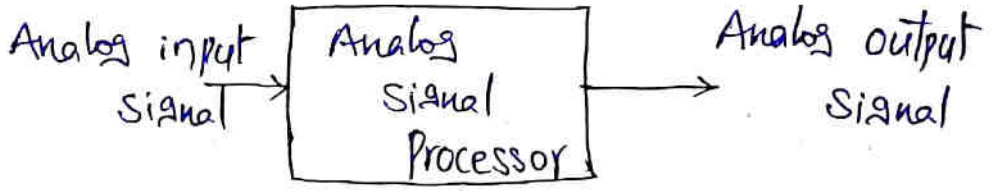


Fig:- Analog Signal Processing

2). Digital Signal processing [DSP]:-

DSP is a numerical processing of signals on a digital computer (or) some other data processing machine.

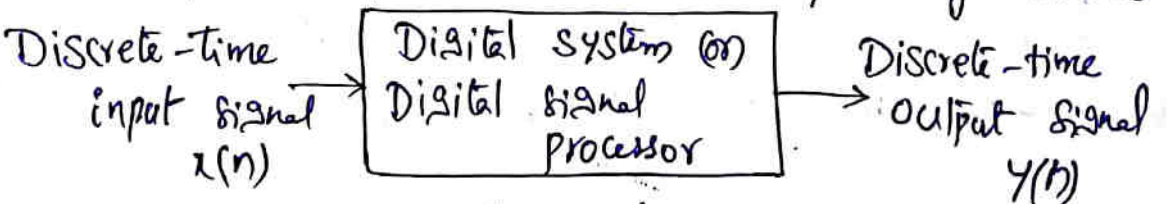


Fig:- Digital Signal Processing

(a)

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Advantages of Digital Signal processing :-

① **Flexibility :-**
The system can be reconfigured for some other operation by simply changing the software program.
Eg :- HPF can be changed to low pass digital filter.

② **Greater accuracy :-** The tolerance of the circuit components used to design the analog filters affects the accuracy, whereas the DSP provides superior control of accuracy.

③ **Easy data storage :-** The digital signals can be easily stored on the storage media such as magnetic tapes and disks.

④ **Implementation of Sophisticated Algorithms :-** Mathematical operations can be accurately performed on digital signals compared to analog signals.

⑤ **Adaptability :-** The DSP systems are easily upgradable since they are software controlled. But such easy upgradation is not possible in analog systems.

⑥ **Size and Reliability :-** The DSP systems are small in size, more reliable and less expensive compared to the analog systems.

Limitations of DSP :-

① **System Complexity :-** When the analog systems have wide bandwidth then high speed A/D converters are required. Such high speed A/D converters are difficult to achieve for same signals. For such applications, analog systems must be used.

② **Power Consumption :-** Analog processing algorithms can be implemented using passive elements like resistors, inductors & capacitors that do not need much power, whereas a DSP chip containing over 4 lakh transistors dissipates more power.

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Basic elements of DSP :-

Fig. shows the block diagram of a typical digital signal processing.

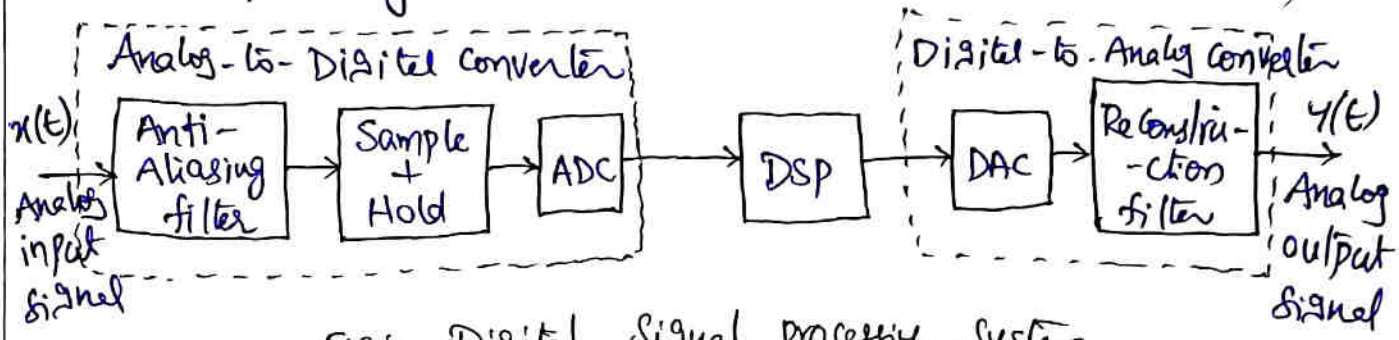


Fig:- Digital signal processing system

The source of the analog input signal is from a transducer (or) a communication channel.

a). Anti Aliasing filter :- This is a LPF used to remove the high frequency signal and to limit the signal.

b). Sample & hold :- The sample and hold device provides the input to the ADC and will be required if the input signal must remain relatively constant during the conversion of the analog to digital format.

c). ADC :- The o/p of the ADC is an N-bit binary number depending on the value of the analog signal at its input.

d). DSP :- Once signal converted to digital format, the signal can be processed using digital techniques.

e). DAC :- The ~~signal~~ digital signal from the processor is applied to the i/p of DAC. The o/p of DAC is continuous but not smooth. The signal contains unwanted high frequency components.

f). Reconstruction filter : To eliminate high frequency components, the o/p of DAC is applied to reconstruction filter. The o/p of this filter is a smooth continuous signal.

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