UNIT IV

WAVEGUIDE COMPONENTS AND APPLICATIONS -II

Ferrites - Composition and characteristics :-

→ A ferrite is a non-metallic material and it has the magnetic property like that of ferrous metals.

→ A ferrite is made of material having useful magnetic properties as well as high resistance to current flow.

→ The resistivity (ρ) of the ferrite materials is nearly 10^{14} times greater than metals

→ The dielectric constant of ferrites is around 10-15.

→ The relative permiability is of the order of 1000.

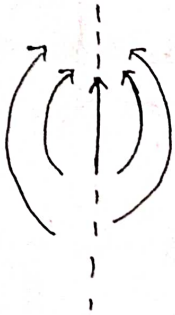
→ These are oxide based compounds having general composition of the form "$MeO.Fe_2O_3$" - ie, mixture of metallic oxide & ferric oxide. where MeO represents any divalent metallic oxide such as MnO, ZnO, CdO, Ni or a mixture of these. They are obtained by firing powered oxides of materials at 1100°C or more and processing them into different shapes. This processing gives them the added characteristics of ceramic insulators so that they can be used at microwave frequencies.

→ Because of high resistivity they can be used upto 100GHz.

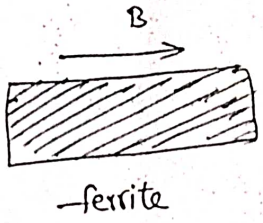
→ Ferrites exhibits non-reciprocal property. when two circularly polarized waves one rotating clockwise and other anticlockwise are made to propogate through ferrite, the material reacts differently to the two rotating fields, there by presenting different effective permiabilities to both the waves. ie, $\epsilon_{r1}, \mu_{r1}, \rho_1$ for left circularly polarised wave and $\epsilon_{r2}, \mu_{r2}, \rho_2$ for the right circularly polarised wave.

Faraday Rotation :-

If a circularly polarized wave is made to pass through a ferrite rod which has been influenced by an axial magnetic field B, then the axis of polarization gets tilted in clockwise direction. The amount of tilt depends upon the strength of magnetic field &



Circularly polarised wave





Tilted polarised wave

Ferrite Components :- (Microwave Engineering — T. Jayanthy, K.S. Shaji)

The ferrite components make use of faraday rotation. The three ferrite components are

(1) Isolator
(2) Gyrator
(3) Circulator

(1) Isolator :-

[An isolator is a 2 port device which provides very small amount of attenuation from port ① to port ② but provides maximum attenuation for transmission from port ② to port ①. This requirement is very much desirable when we want to match a source with a variable load]

An isolator is a non reciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line. An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Hence it is called as uniline.

Application :-

used to improve frequency stability of microwave generator such as klystron and magnetrons

The output amplitude and frequency of microwave generators [illegible] this is due to

waves.

If an isolator is placed between the microwave microwave generator and load, it offers zero attenuation for the generator output which is coupled to the load if any reflections from the load is absorbed completely by isolator. Thus the generator appears to be matched to loads.

Construction :-

An isolator consists of piece of circular waveguide with transition to rectangular waveguide at both ends. The rectangular to circular transition is being twisted by 45°. The rectangular waveguid supports TE_{10} mode & the circular waveguide supports TE_{11} mode.

A thin ferrite rod is located inside the circular waveguide & it is surrounded by permanent magnet which generates d.c magnetic field in ferrite core. Ferrite rod is tapered at both the ends to reduce attenuation & also for smooth rotation of polarised wave.

Also resistive card is placed inside the rectangular waveguide so as to absorb any wave whose plane of polarisation is parallel to the plane of the resistive card. The resistive card do not absorb any wave whose plane of polarisation is perpendicular to its own plane

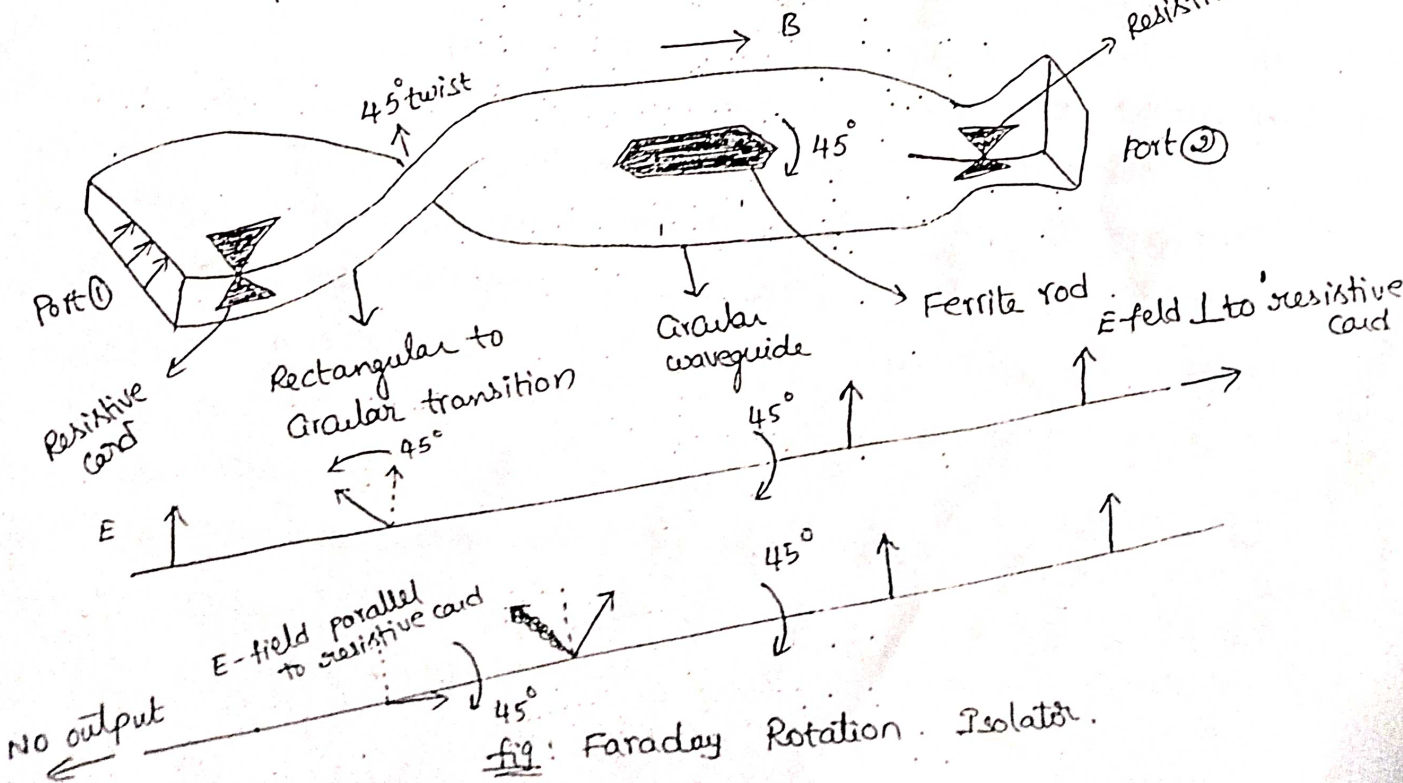


fig: Faraday Rotation Isolator.

A wave passing from port 1, gets shifted by 45° because of twist in rectangular to circular transition in anticlockwise direction. Because of the applied dc magnetic field, the wave passing through the ferrite in the forward direction will have its plane of polarisation shifted in 45° clockwise direction & come out of port 2 with zero attenuation as the resistive card is placed perpendicular to the plane of polarisation.

But a wave entering at port 2 passes ferrite rod and gets rotated by an angle 45° in the clockwise direction & further it will be rotate by an angle of 45° because of the twist in the waveguide. Now the plane of polarisation is parallel to the resistive card & the wave will be completely absorbed. The power is dissipated in the resistive card as heat.

In practice 20 to 30 dB isolation is obtained for transmission from port ② to port ①.

2. Gyrator :-

Gyrator is a two port device that has a relative phase difference of 180° for transmission from port ① to port ② & no phase shift or 0° phase shift for transmission from port ② to port ①.

Construction :-

It consists of a circular waveguide with transition to rectangular waveguide at both ends. Rectangular to circular waveguide transition is twisted by 90°. A thin ferrite rod is placed inside the circular waveguide & it is surrounded by a permanent magnet which generates dc magnetic field in the ferrite core. The ferrite is tapered at both ends to reduce the attenuation and also for smooth rotation of the polarised wave.

Operation :-

When a wave enters at port 1 the plane of polarisation is rotated by 90° in anticlockwise direction because of twist in the rectangular to circular transition & it again

undergoes faraday rotation through 90 anticlockwise & ... comes out of port 2 with a phase shift of 180° when compared to the wave entering at port 1.

But when the wave enters port 2 it will be rotated 90° anti clockwise because of passing through ferrite rod & again it will be rotated 90° in clockwise direction because of the 90° twist present at circular to rectangular transition. Hence the comes out of port ① with 0° phase shift.

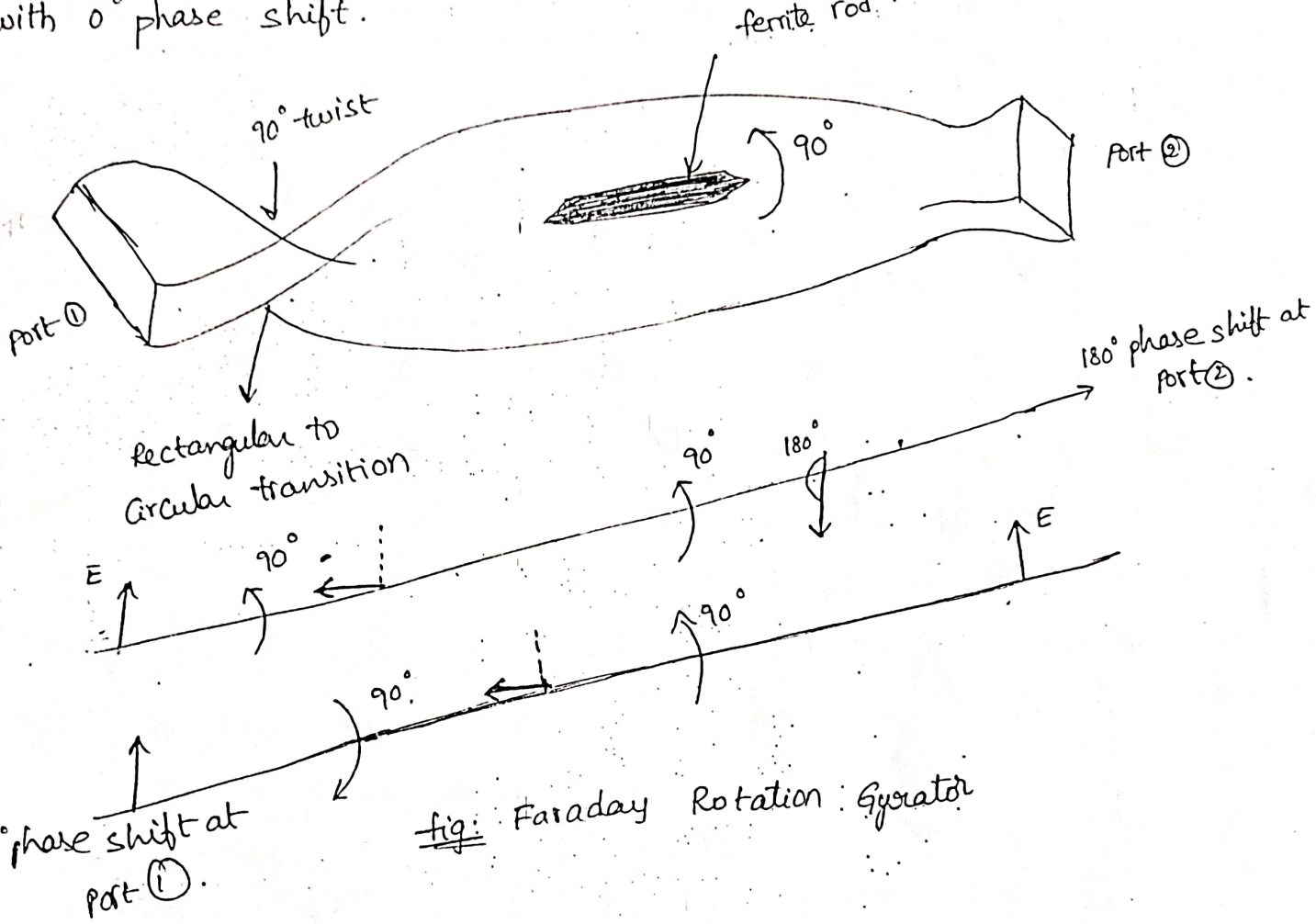
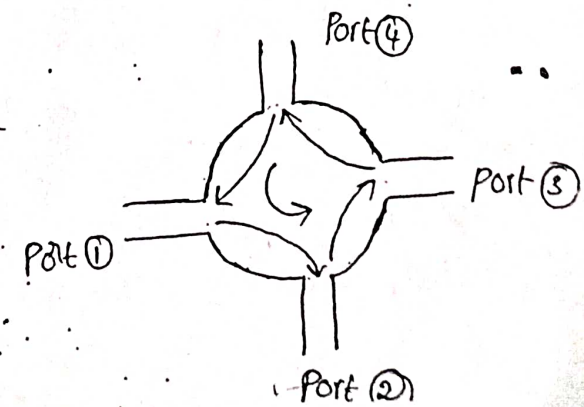


fig: Faraday Rotation Gyrator

Circulator :-

A Circulator is a multiport waveguide junction in which the wave can flow only from the n^{th} port to $(n+1)^{th}$ port in one direction. Although there is no restriction on the number of ports, the 4 port Circulator is the most common.



Here port ① is connected to port ② but not to port ③ & ④. port ② is connected to port ③ but not to port ① & ④ and so on.

a gyrator as shown below

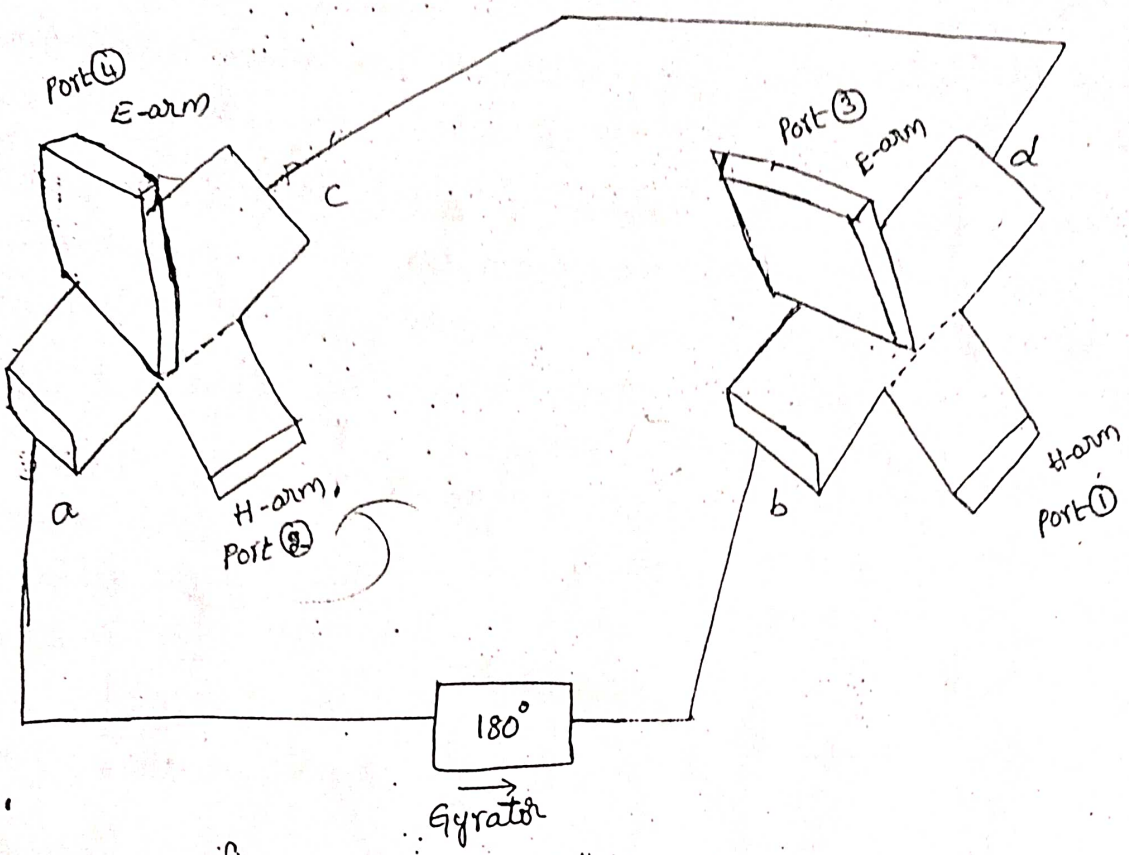


fig : Four Port Circulator

Here the gyrator is used to provide the 180° phase shift from 'a' to 'b'. For propagation from a to b, b to a, c to d, d to c, the electrical paths are equal.

→ Consider a wave incident in port ①, Then it will split into two waves of equal amplitude & phase and will arrive at 'a' & 'd'. Hence will emerge from port ②.

→ The wave incident at port ② will split into two waves equal in amplitud & phase & will appear at 'd' as it is from 'c' and at 'b' with 180° phase shift from 'a' because of the gyrator placed inbetween 'a' & 'b'. Hence the output emerge from port ③.

→ The wave incident at port ③ will split into two waves equal in magnitude & opposite ie; 180° outof phase and will appear across 'a' & 'c'. Hence the output emerge from port ④.

→ The wave incident at port ④ will split into two waves equal in amplitude & opposite in phase, but now the gyrator restore the phase equality, so the two waves will be in phase & output

merge from port ①.

Scattering (or) S parameters :-

At low frequencies circuits can be described by 2port networks. which can be described by a number of parameter sets, such as z, h, y, ABCD parameters. All these parameters relate the total voltages and total currents at each of the two ports.

But at microwave frequencies, we talk of travelling waves with associated powers instead of voltages and currents. In a microwave circuit, the incoming wave is "scattered" by the circuit & its energy is partitioned between all the possible outgoing waves on all the other transmission lines connected to the circuit. The scattering parameters are the fixed properties of the (linear) circuit which describe how the energy couples between each pair of ports & transmission lines connected to the circuit.

At microwave frequencies h, y, z and ABCD parameters cannot be measured for the following reasons.

① Equipment is not readily available to measure total voltage and total current at the ports of the network.

② short and open circuits are difficult to achieve over a broad band of frequencies

③ Active devices, such as power transistors and tunnel diodes, frequently will not have stability for a short & open circuit.

The S-parameters are expressed as

$$[b] = [S][a]$$

where S is a $n \times n$ square matrix

for a two port network $n = 2$.

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

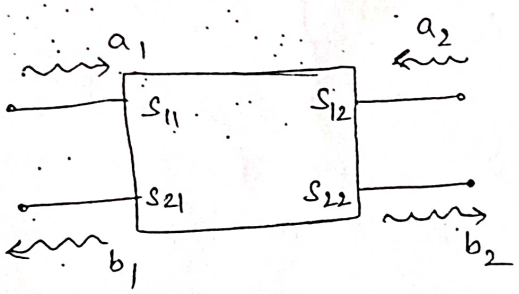


fig: Two port network

for a '3' port network

$$b_1 = S_{11} a_1 + S_{12} a_2 + S_{13} a_3$$

$$b_2 = S_{21} a_1 + S_{22} a_2 + S_{23} a_3$$

$$b_3 = S_{31} a_1 + S_{32} a_2 + S_{33} a_3$$

$$b_1 = S_{11}a_1 + S_{12}a_2 + \cdots + S_{1n}a_n$$
$$b_2 = S_{21}a_1 + S_{22}a_2 + \cdots + S_{2n}a_n$$
$$\vdots$$
$$b_n = S_{n1}a_1 + S_{n2}a_2 + \cdots + S_{nn}a_n$$

In matrix form

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix}_{n\times n} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}_{n\times 1}$$

Column matrix corresponding to reflected (or) output waves

Scattering matrix of order $n \times n$

Column matrix corresponding to Incident (or) input waves.

Properties of S-matrix :-

① [S] is always a square matrix of the order $n \times n$.

② [S] is a symmetric matrix

ie, $S_{ij} = S_{ji}$

③ [S] is a unitary matrix

ie, $[S][S]^* = I$.

where $[S]^*$ = Complex conjugate of [S]

[I] = unit (or) Identity matrix of same order as that of [S].

④ The sum of products of each term of any row (or column) multiplied by the complex conjugate of the corresponding terms of any other row (or column) is zero.

ie, $\sum_{i=1}^{n} S_{ij} S_{ik}^* = 0$ $\quad k = 1, 2, 3 \ldots n$, $j = 1, 2, 3 \ldots n$

⑤ If any of the terminal or reference planes are moved away from the junction by an electric distance βl, each of the

⑥ Zero diagonal element for perfect matched network

ie, $S_{ii} = 0$ for perfectly matched networks

⑦ $\sum_{i=1}^{n} S_{ij} S_{ij}^* = 1$ $\quad j = 1, 2, 3 \ldots n$

S-matrix for E-plane Tee :-

① As E-plane Tee is a 3-port device the S-matrix is given by

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

② From symmetry property of S-matrix $S_{ij} = S_{ji}$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$S_{12} = S_{21}$

$S_{13} = S_{31}$

$S_{23} = S_{32}$

③ Scattering coefficients

Since output from port ① & ② are 180° out of phase with an input at port ③

$$S_{23} = -S_{13}$$

④ If the port ③ is perfectly matched to the junction

$$S_{33} = 0.$$

with the above properties

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

⑤ From unitary property of S-matrix $[S][S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$ ——— ①

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$ ——— ②

$R_3 R_3$: $|S_{13}|^2 + |S_{13}|^2 = 1$ ——— ③

$R_3 C_1$: $S_{13} S_{11}^* - S_{13} S_{12}^* = 0$ ——— ④

from ④ $S_{11}^* = S_{12}^* \implies S_{11} = S_{12}$

from ① & ② $S_{11} = S_{22}$

$\therefore S_{11} = S_{22} = S_{12}$

from ③

$$2|S_{13}|^2 = 1$$

$$S_{13} = \frac{1}{\sqrt{2}}$$

Substituting in ①

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

$$2|S_{11}|^2 = 1/2$$

$$S_{11} = 1/2$$

$$S_{11} = S_{22} = S_{12} = 1/2$$

$$S_{13} = 1/\sqrt{2}$$

Substituting the values in the S-matrix

$$S = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

we know that $[b] = [s][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{a_1}{2} + \frac{a_2}{2} + \frac{a_3}{\sqrt{2}}$$

$$b_2 = \frac{a_1}{2} + \frac{a_2}{2} - \frac{a_3}{\sqrt{2}}$$

$$b_3 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}}$$

(case i) $a_1 = a_2 = 0$, $a_3 \neq 0$

$$b_1 = \frac{a_3}{\sqrt{2}}, \quad b_2 = \frac{-a_3}{\sqrt{2}}, \quad b_3 = 0.$$

ie, when the input is fed at port ③ divides between port ① & ② equal but 180° out of phase.

case (ii)

$$a_1 = a_2 = a, \quad a_3 = 0$$

$$b_1 = \frac{a}{2} + \frac{a}{2} = a, \quad b_2 = \frac{a}{2} + \frac{a}{2} = a, \quad b_3 = \frac{a}{\sqrt{2}} - \frac{a}{\sqrt{2}} = 0.$$

ie, equal inputs at port ① & ② no output at port ③

case (iii)

$$a_1 = a, \quad a_2 = -a, \quad a_3 = 0$$

$$b_1 = \frac{a}{2} - \frac{a}{2} = 0, \quad b_2 = \frac{a}{2} - \frac{a}{2} = 0, \quad b_3 = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} = \sqrt{2}a$$

ie, Different phase inputs at port ① & ② output at port ③ only.

case (iv)

$$a_1 \neq 0, \quad a_2 = a_3 = 0$$

$$b_1 = \frac{a_1}{2}, \quad b_2 = \frac{a_1}{2}, \quad b_3 = \frac{+a_1}{\sqrt{2}}$$

case (v)

$$a_2 \neq 0, \quad a_1 = a_3 = 0$$

$$b_1 = \frac{a_2}{2}, \quad b_2 = \frac{a_2}{2}, \quad b_3 = \frac{-a_2}{\sqrt{2}}$$

S-matrix for H-plane Tee :-

① As H-plane Tee is a 3-port device, the s-matrix is given by

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

② from symmetry property of s-matrix

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$S_{ij} = S_{ji}$

$\therefore S_{12} = S_{21}$

$S_{13} = S_{31}$

$S_{23} = S_{32}$

③ Scattering coefficients

In H-plane Tee when Input is fed at port ③ then it will divided

$S_{13} = S_{23}$

(14) If the port ③ is perfectly matched to the junction $S_{33} = 0$.

with the above properties

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

(15) From unitary property of s-matrix $[S][S]^* = I$.

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_1C_1: $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$ ——— (1)

R_2C_2: $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$ ——— (2)

R_3C_3: $|S_{13}|^2 + |S_{13}|^2 = 1$ ——— (3)

R_3C_1: $S_{13}S_{11}^* + S_{13}S_{12}^* = 0$ ——— (4)

from (1) & (2) $|S_{11}|^2 = |S_{22}|^2 \Rightarrow S_{11} = S_{22}$

from (3) $2|S_{13}|^2 = 1$

$S_{13} = 1/\sqrt{2}$

from (4) $S_{11}^* = -S_{12}^* \Rightarrow S_{11} = -S_{12}$

$\therefore S_{11} = S_{22} = -S_{12}$

from (1) $|S_{11}|^2 + |S_{11}|^2 + 1/2 = 1$

$2|S_{11}|^2 = 1/2$

$S_{11} = 1/2$

$\therefore S_{11} = S_{22} = 1/2$, $S_{12} = -1/2$, $S_{13} = 1/\sqrt{2}$

$$\therefore S = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

we know that

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{a_1}{2} - \frac{a_2}{2} + \frac{a_3}{\sqrt{2}}$$

$$b_2 = \frac{-a_1}{2} + \frac{a_2}{2} + \frac{a_3}{\sqrt{2}}$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}}$$

Case(i)

$$a_1 = a_2 = 0, \quad a_3 \neq 0$$

$$b_1 = a_3/\sqrt{2}, \quad b_2 = a_3/\sqrt{2}, \quad b_3 = 0.$$

ie, when the input is incident at port3 it will divide equally to Port ② and port ①.

Case(ii)

$$a_1 = a_2 = a, \quad a_3 = 0.$$

$$b_1 = \frac{a}{2} - \frac{a}{2} = 0, \quad b_2 = -\frac{a}{2} + \frac{a}{2} = 0, \quad b_3 = \sqrt{2}a$$

ie, when the two inputs of same phase are fed at ① & ② output appear across port ③

Case(iii)

$$a_1 = a, \quad a_2 = -a, \quad a_3 = 0$$

$$b_1 = \frac{a}{2} + \frac{a}{2} = a, \quad b_2 = \frac{-a}{2} - \frac{a}{2} = -a, \quad b_3 = 0.$$

ie, when the two inputs of 180° outof phase are fed at ① & ② then output appear across port ③.

① As S-matrix is a 4-port device the s-matrix is given by

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

② From the symmetry property of S-matrix

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}$$

$S_{ij} = S_{ji}$

$S_{12} = S_{21}$

$S_{31} = S_{13}$

$S_{23} = S_{32}$

$S_{34} = S_{43}$

$S_{14} = S_{41}$

$S_{24} = S_{42}$

③ Scattering coefficients because of E-arm ie, port ④

$$S_{14} = -S_{24}$$

④ Scattering coefficients because of H-arm ie, port ③

$$S_{13} = +S_{23}$$

⑤ If the input is fed at port ③ there will be no output at ④ & viceversa. ie, port ③ & ④ are isolated ports

$$S_{34} = S_{43} = 0$$

⑥ port ③ & port ④ are perfectly matched ie,

$$S_{33} = S_{44} = 0$$

Substituting the above conditions then S-matrix is given by

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & +S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$

From the unitary property of ...

$$[S][S]^* = I$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ —— ①

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ —— ②

R_3C_3 : $|S_{13}|^2 + |S_{13}|^2 = 1$ —— ③

R_4C_4 : $|S_{14}|^2 + |S_{14}|^2 = 1$ —— ④

from ① & ② $S_{11} = S_{22}$

from ③ $2|S_{13}|^2 = 1$

$S_{13} = 1/\sqrt{2}$

from ④ $2|S_{14}|^2 = 1$

$S_{14} = 1/\sqrt{2}$

from ① $|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$

$|S_{11}|^2 + |S_{12}|^2 = 0$

ie, $S_{11} = S_{12} = 0$

$\therefore S_{11} = S_{12} = S_{22} = 0$, $S_{13} = S_{14} = 1/\sqrt{2}$

∴ The S-matrix is given by

$$S = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

$[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$b_1 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}}, \quad b_2 = \frac{a_3}{\sqrt{2}} - \frac{a_4}{\sqrt{2}}$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}}, \quad b_4 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}}$$

Case(i)

$a_3 \neq 0, \quad a_1 = a_2 = a_4 = 0$

$b_1 = \frac{a_3}{\sqrt{2}}, \quad b_2 = \frac{a_3}{\sqrt{2}}; \quad b_3 = 0, \quad b_4 = 0.$

ie, when input is fed at port ③ ie, H-arm. It will split equally into port ① & ②.

Case(ii):

$a_4 \neq 0, \quad a_1 = a_2 = a_3 = 0.$

$b_1 = \frac{a_4}{\sqrt{2}}, \quad b_2 = \frac{-a_4}{\sqrt{2}}, \quad b_3 = 0, \quad b_4 = 0.$

ie, when the input is fed at port ④ ie, E-arm. It will split equally into port ① & ② but 180° outof phase.

Case(iii)

$a_1 = a_2 = a, \quad a_3 = a_4 = 0.$

$b_1 = 0, \quad b_2 = 0, \quad b_3 = \sqrt{2}a, \quad b_4 = 0.$

ie, when the same inputs of same phase are given at port ① & ② output across H-arm ie, port ③ only.

Case(iv)

$a_1 = a, \quad a_2 = -a; \quad a_3 = a_4 = 0.$

$b_1 = 0, \quad b_2 = 0, \quad b_3 = 0, \quad b_4 = \sqrt{2}a$

ie, when the same inputs of 180° outof phase are given at port ① & port ② then output across E-arm. ie port ④ only.

sol(5)

$a_1 \neq 0, a_2 = a_3 = a_4 = 0$

$b_1 = 0, b_2 = 0, b_3 = \frac{a_1}{\sqrt{2}}, b_4 = \frac{a_1}{\sqrt{2}}$

ie, when input is fed at port① nothing comes out of port-② eventhough they are collinear arms (magic !!). Hence port-① & ③ are isolated ports.

S-matrix for hybrid ring :-

① As hybrid ring is a 4 port device S-matrix is given by -

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

② From symmetry property of S-matrix $S_{ij} = S_{ji}$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}$$

$S_{12} = S_{21}$
$S_{13} = S_{31}$
$S_{14} = S_{41}$
$S_{23} = S_{32}$
$S_{24} = S_{42}$

③ All ports are perfectly matched

$$S_{11} = S_{22} = S_{33} = S_{44} = 0.$$

④ scattering coefficients

when input fed at port① $S_{31} = S_{13} = 0$

when input fed at port② $S_{42} = S_{24} = 0$

when input fed at port③ $S_{13} = S_{31} = 0$

when input fed at port④ $S_{24} = S_{42} = 0$.

Substituting the above conditions

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{24} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

⑤ from unitary property

$$[S][S]^* = I$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $|S_{12}|^2 + |S_{14}|^2 = 1$ ——— (1)

R_2C_2 : $|S_{12}|^2 + |S_{23}|^2 = 1$ ——— (2)

R_3C_3 : $|S_{23}|^2 + |S_{34}|^2 = 1$ ——— (3)

R_4C_4 : $|S_{14}|^2 + |S_{34}|^2 = 1$ ——— (4)

R_1C_3 : $S_{12}S_{23}^* + S_{14}S_{34}^* = 0$ ——— (5)

from (1) & (2) $S_{23} = S_{14}$

from (3) & (4) $S_{12} = S_{34}$

let us assume $S_{12} = S_{34} = P$, a positive & real value

$\therefore S_{34}^* = S_{34}$ [$\because S_{34}$ real]

$\Rightarrow S_{34}^* = P$.

from (5) $PS_{23}^* + PS_{14}^* = 0$.

$P(S_{23}^* + S_{14}) = 0$.

let us assume

$S_{23} = S_{14} = jq$ where q is positive & real

from (1)

$|P|^2 + |q|^2 = 1$.

$P^2 + q^2 = 1$.

$\therefore$ The S-matrix of a hybrid ring is given by

$$S = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$

S-matrix for 4 port directional coupler:

① As directional coupler is a 4 port device

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

② From symmetry property of S-matrix $S_{ij} = S_{ji}$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}$$

$S_{12} = S_{21}$
$S_{13} = S_{31}$
$S_{14} = S_{41}$
$S_{23} = S_{32}$
$S_{24} = S_{42}$

③ All ports are perfectly matched

$$S_{11} = S_{22} = S_{33} = S_{44} = 0.$$

④ Scattering coefficients

Port ① & ③ are isolated ports $S_{13} = S_{31} = 0.$

Port ② & ④ are isolated ports $S_{24} = S_{42} = 0.$

Substituting the above conditions

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{24} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \quad S_{34}$$

⑤ From unitary property

$$[S][S]^* = I$$

$$\therefore \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{24} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

$R_1C_1 : |S_{12}|^2 + |S_{14}|^2 = 1$ ——— ①

$R_2C_2 : |S_{12}|^2 + |S_{23}|^2 = 1$ ——— ②

$R_3C_3 : |S_{23}|^2 + |S_{34}|^2 = 1$ ——— ③

$R_4C_4 : |S_{14}|^2 + |S_{24}|^2 = 1$ ——— ④

$R(S_{31}) = S_{12} S_{13}^* + S_{14} S_{24}^* = 0$

from ① & ③ $S_{23} = S_{14}$

from ② & ④ $S_{12} = S_{34}$

let us assume $S_{12} = S_{34} = P$, where P = a real & positive value

$\therefore S_{34}^* = P$

from ③ $P S_{23}^* + P S_{14} = 0 \Rightarrow P(S_{23}^* + S_{14}) = 0$

$P(S_{23}^* + S_{14}) = 0$ $\quad P(S_{33}^* + S_{11}) = 0$ $\quad S_{23} = S_{14}$

$S_{23}^* = -S_{14}$

let its assume $S_{23} = S_{14} = jq$, where q = positive & real

from ① $P^2 + q^2 = 1$.

$\therefore$ The S-matrix for directional coupler is given by

$$S = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$

Smatrix for Isolator :-

① As Isolator is a ② port device the S-matrix is given by

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

S_{ij}
j = input port
i = output port

② Isolator is a non-reciprocal device. $S_{12} \neq S_{21}$

③ port ① & port ② are perfectly matched $S_{11} = S_{22} = 0$.

④ If input is fed at port ① output at port ② is NOT ZERO $S_{21} \neq 0$ & $S_{12} = 0$

$\therefore$ The S-matrix for an isolator is given by

$$S = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

we know that $[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \qquad b_1 = 0, \quad b_2 = a_1$$

case(i): $a_1 \neq 0$, $a_2 = 0$.

$\Rightarrow b_1 = 0$, $b_2 = a_1 \neq 0$.

case(ii) $a_2 \neq 0$, $a_1 = 0$.

$b_1 = 0$, $b_2 = 0$

S-matrix for Circulator :-

3 port circulator :-

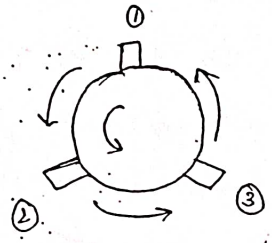
① As circulator is a 3 port circulator

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

② As it is a non-reciprocal device, $S_{ij} \neq S_{ji}$

③ from property of circulator

$$S_{31} = S_{23} = S_{12} = 0.$$



④ All ports are perfectly matched

$$S_{11} = S_{22} = S_{33} = 0.$$

$$\therefore S = \begin{bmatrix} 0 & 0 & S_{13} \\ S_{21} & 0 & 0 \\ 0 & S_{32} & 0 \end{bmatrix}$$

⑤ From property of S-matrix for non-reciprocal device

$$\sum_{i=1}^{n} S_{ij} S_{ij}^{*} = 1$$

$$\begin{bmatrix} 0 & 0 & S_{13} \\ S_{21} & 0 & 0 \\ 0 & S_{32} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & S_{13}^{*} \\ S_{21}^{*} & 0 & 0 \\ 0 & S_{32}^{*} & 0 \end{bmatrix}$$

$$\therefore S_{13} S_{13}^{*} = 1 \Rightarrow |S_{13}|^2 = 1 \Rightarrow S_{13} = 1$$

$$S_{21} S_{21}^{*} = 1 \Rightarrow |S_{21}|^2 = 1 \Rightarrow S_{21} = 1$$

$$S_{32} S_{32}^{*} = 1 \Rightarrow |S_{32}|^2 = 1 \Rightarrow S_{32} = 1$$

$$\therefore S_{13} = S_{21} = S_{32} = 1$$

$$\therefore S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[b] = [S][a]$$

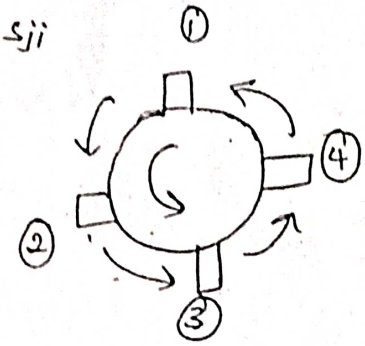
$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = a_3, \quad b_2 = a_1, \quad b_3 = a_2$$

① → As the circulator is a 4-port circuit

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

② → As circulator is a non-reciprocal device $S_{ij} \neq S_{ji}$



Port ② Port ③ Port ④ Port ①

$S_{12} = 0$, $S_{23} = 0$, $S_{34} = 0$, $S_{31} = 0$

$S_{42} = 0$, $S_{13} = 0$, $S_{24} = 0$, $S_{41} = 0$

③ → All ports are perfectly matched

$S_{11} = S_{22} = S_{33} = S_{44} = 0$.

$$\therefore S = \begin{bmatrix} 0 & 0 & 0 & S_{14} \\ S_{21} & 0 & 0 & 0 \\ 0 & S_{32} & 0 & 0 \\ 0 & 0 & S_{43} & 0 \end{bmatrix}$$

④ → from S-matrix properties for a non-reciprocal device

$$\sum_{i=1}^{n} S_{ij} S_{ij}^* = 1.$$

$$\begin{bmatrix} 0 & 0 & 0 & S_{14} \\ S_{21} & 0 & 0 & 0 \\ 0 & S_{32} & 0 & 0 \\ 0 & 0 & S_{43} & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & S_{14}^* \\ S_{21}^* & 0 & 0 & 0 \\ 0 & S_{32}^* & 0 & 0 \\ 0 & 0 & S_{43}^* & 0 \end{bmatrix}$$

$R_1 R_1^*$: $S_{14} \cdot S_{14}^* = 1 \Rightarrow S_{14} = 1$

$R_2 R_2^*$: $S_{21} \cdot S_{21}^* = 1 \Rightarrow S_{21} = 1$

$R_3 R_3^*$: $S_{32} \cdot S_{32}^* = 1 \Rightarrow S_{32} = 1$

$R_4 R_4^*$: $S_{43} \cdot S_{43}^* = 1 \Rightarrow S_{43} = 1$.

$\therefore S_{14} = S_{21} = S_{32} = S_{43} = 1$

$$\therefore S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

we know that $[b] = [s][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$b_1 = a_4$, $b_2 = a_1$, $b_3 = a_2$, $b_4 = a_3$.

Problems

1) A matched isolator has insertion loss of 0.5dB & isolation 25dB. Find the scattering coefficients

Insertion loss $= -10\log|S_{21}|^2 = -20\log|S_{21}| = 0.5$ dB

$S_{21} = 10^{-0.5/20} = 0.94$

Isolation $= -10\log|S_{12}|^2 = -20\log|S_{12}| = 25$

$S_{12} = 10^{-25/20} = 0.056$

Ports are perfectly matched $S_{11} = S_{22} = 0$

$\therefore S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

$\therefore S = \begin{bmatrix} 0 & 0.056 \\ 0.94 & 0 \end{bmatrix}$

2) A 3 port circulator has an insertion loss of 1dB, isolation 30dB & VSWR = 1.5. Find S-matrix

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Insertion loss $= -10\log|S_{21}|^2 = -10\log|S_{32}|^2$

$= -10\log|S_{13}|^2 = 1.$

$\therefore S_{21} = S_{32} = S_{13} = 10^{-1/20} = 0.89$

Isolation $= -10\log|S_{12}|^2 = -10\log|S_{23}|^2 = -10\log|S_{31}|^2 = 30.$

$\therefore S_{12} = S_{23} = S_{31} = 10^{-3/2} = 0.032$

Given VSWR = 1.5

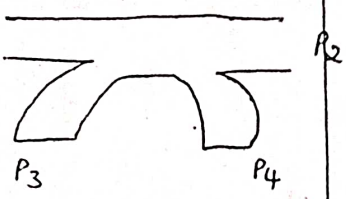
Reflection coefficients $S_{11} = S_{22} = S_{33} = \frac{VSWR-1}{VSWR+1} = \frac{1.5-1}{1.5+1} = 0.2$

$$\therefore S = \begin{bmatrix} 0.2 & 0.032 & 0.89 \\ 0.89 & 0.2 & 0.032 \\ 0.032 & 0.89 & 0.2 \end{bmatrix}$$

Determine the S-parameters for a 10dB directional coupler. The directivity is 30dB. Assume the directional coupler is lossless and the VSWR at each port is 1.0 under matched conditions.

Given $C = 10dB$ $\quad VSWR = 1.$

$D = 30dB$ $\quad \therefore S_{11} = S_{22} = S_{33} = \frac{VSWR-1}{VSWR+1} = 0.$



$I = $ Isolation $= C + D = 40$ dB.

$$C = -10\log|S_{41}|^2 = -10\log|S_{32}|^2 = 10$$

$$\therefore S_{41} = S_{32} = 10^{-1/2} = 0.3162$$

$$I = -10\log|S_{31}|^2 = -10\log|S_{42}|^2 = 40.$$

$$S_{31} = S_{42} = 10^{-2} = 0.01.$$

The directional coupler is a reciprocal device hence from symmetry property

$$S_{12} = S_{21},\ S_{13} = S_{31},\ S_{14} = S_{41},\ S_{23} = S_{32},\ S_{24} = S_{42},\ S_{34} = S_{43}$$

$$\therefore S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0.01 & 0.3162 \\ S_{12} & 0 & 0.3162 & 0.01 \\ 0.01 & 0.3162 & 0 & S_{34} \\ 0.3162 & 0.01 & S_{34} & 0 \end{bmatrix}$$

From unitary property of S-matrix $[S][S]^* = I$

$$\begin{bmatrix} 0 & S_{12} & 0.01 & 0.3162 \\ S_{12} & 0 & 0.3162 & 0.01 \\ 0.01 & 0.3162 & 0 & S_{34} \\ 0.3162 & 0.01 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0.01 & 0.3162 \\ S_{12}^* & 0 & 0.3162 & 0.01 \\ 0.01 & 0.3162 & 0 & S_{34}^* \\ 0.3162 & 0.01 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 C_1^*$: $|S_{12}|^2 + (0.01)^2 + (0.3162)^2 = 1. \Rightarrow S_{12} = 0.9486$

$R_4 C_4^*$: $(0.3162)^2 + (0.01)^2 + |S_{34}|^2 = 1 \Rightarrow S_{34} = 0.9486.$

$$\therefore S = \begin{bmatrix} 0 & 0.9486 & 0.01 & 0.3162 \\ 0.9486 & 0 & 0.3162 & 0.01 \\ 0.01 & 0.3162 & 0 & 0.9486 \\ 0.3162 & 0.01 & 0.9486 & 0 \end{bmatrix}$$

→ A 20mw signal is fed into the series arm of a lossless 'U' junction. Calculate the power delivered through each port when other ports are terminated in matched load.

S-matrix for magic tee is

$$S = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

Sol: Series arm = E-arm

Power applied at E-arm $a_4 = 20mw$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$b_1 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}}$$

$$b_2 = \frac{a_3}{\sqrt{2}} - \frac{a_4}{\sqrt{2}}$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}}$$

$$b_4 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}}$$

Given - $a_4 = 20mw$, $a_1 = a_2 = a_3 = 0$. then

$b_1 = \frac{a_4}{\sqrt{2}} = \frac{20m}{\sqrt{2}} = 14.142mw$, the power delivered to port ①

$b_2 = -\frac{a_4}{\sqrt{2}} = -\frac{20m}{\sqrt{2}} = -14.142mw$, Power delivered to port ②

$b_3 = 0$, power delivered to port ③.

UNIT - IV

WAVEGUIDE COMPONENTS AND APPLICATIONS - I

Coupling Mechanisms :- (Microwave Engineering - Arpana Singh, Namita Jain)

Special devices must be used to put energy into a waveguide at one end and remove it from other end. Coupling means to couple a co-axial line to a waveguide, To couple a signal to a waveguide & to couple waveguides to each other by means of slots in common walls.

The three devices used to inject & remove energy from waveguide are

(i) Probes (ii) loops (iii) slots (iv) Apertures (v) windows.

(i) Probe Coupling :-

when a small probe is inserted into a waveguide and supplied with microwave energy, it acts as a quarter wave antenna. current flows in the Probe and sets up an E field as shown in fig(a). The E lines detach themselves from the probe. when the probe is located at the point of highest efficiency, the E-lines set up a E-field of considerable intensity. The most efficient place to locate the probe is in the centre of the "x" wall, Parallel to the "y" wall & one-quarter wavelength from the shortened end of the waveguide as shown in fig(b). This is the point at which the E-field is maximum in the dominant mode. Therefore, energy transfer (coupling) is maximum at this point. The quarter wavelength spacing is at the frequency required to propagate the dominant mode.

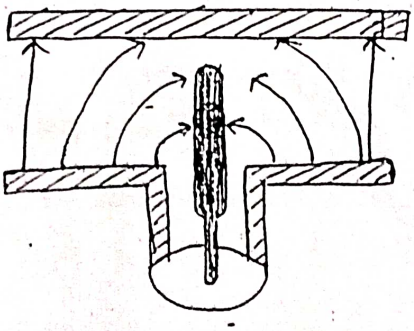


fig (a)

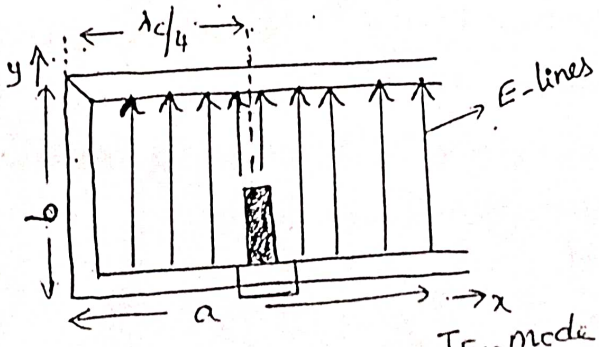


fig (b)

fig: Probe coupling in rectangular waveguide

called loose coupling is desirable. The amount of energy transfer can be reduced by decreasing the length of the probe, by moving it out of the centre of the "x" wall & by shielding it. where the degree of coupling must be varied frequently the probe is made retractable so the length can be easily changed.

The size and shape of the probe determines its frequency, bandwidth and power handling capability. As the diameter of the probe increases, the bandwidth increases. A probe similar in shape to a door knob is capable of handling much higher power and larger bandwidth than a conventional probe. Removal of energy from a waveguide is simply a reversal of the injection process using the same type of probe.

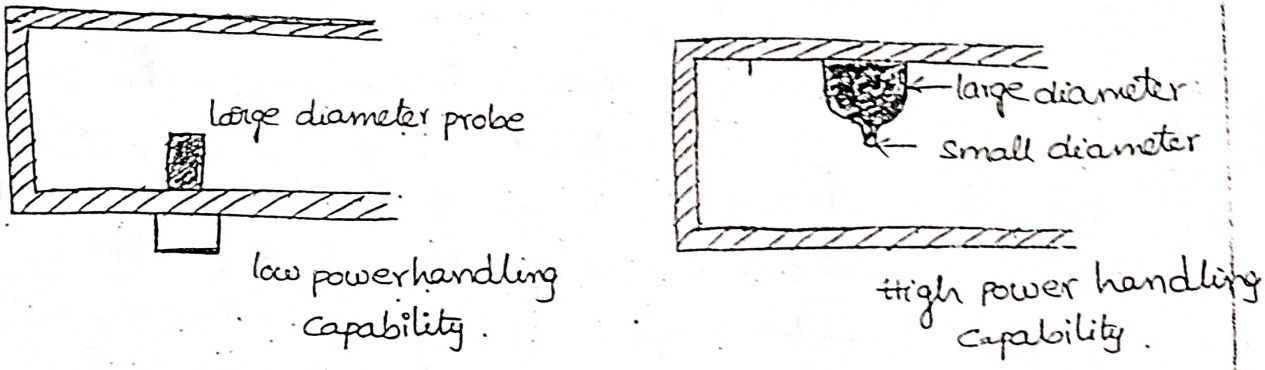
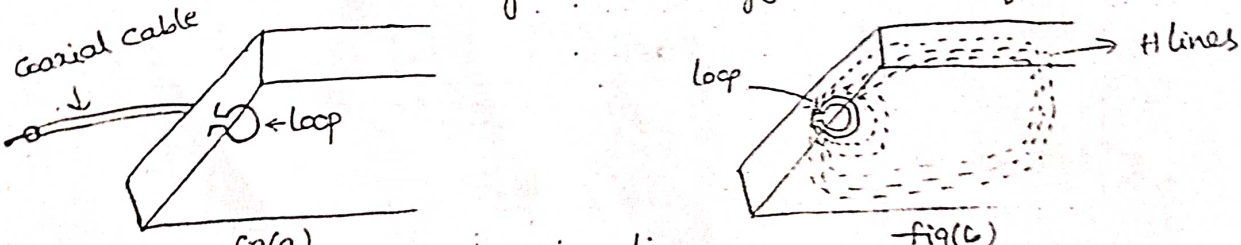


fig: Broad Bandwidth Probes.

Loop Coupling :-

Another way of injecting energy into a waveguide is by setting up an H-field in the waveguide. This can be accomplished by inserting a small loop which carries a high current into the waveguide, as shown in fig(a). A magnetic field builds up around the loop and expands to fit the waveguide as shown in fig(b). If the frequency of the current in the loop is within the bandwidth of the waveguide, energy will transferred to the waveguide



fig(a) fig(b)

fig: loop coupling in rectangular waveguide

for the most efficient coupling with the ... , ... , ... as one of several points where the magnetic field will be of greatest streangth. Four of these points are shown in below figure.

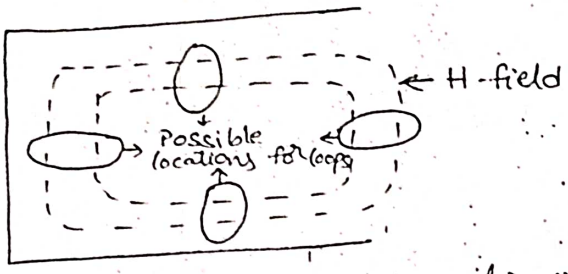


fig: loop coupling in rectangular waveguide.

when less efficient coupling is desired, the loops can be rotated or moved until it encircles a smaller number of H lines. when the diameter of the loop is increased its power handling capabilit increases. The bandwidth can be increased by increasing the size of the wire used to make the loop. when a loop is introduced into a waveguide in which an H-field is present, a current is induced ~~into a waveguide~~ in the loop. when this condition exists, energy is removed from the waveguide.

Aperture or slot coupling :-

Slots or apertures are sometimes used when very loose coupling is desired. In this method energy is coupled through a small slot in the waveguide and the E-field expands to the waveguide. Minimum reflections occur when energy is injected or removed if the size of the slot is properly proportional to the frequency of the energy.

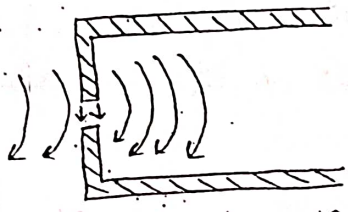


fig: Apesture coupling in rectangular waveguide

In this method when the energy leaves a waveguide, fields form around the end of the waveguide. These fields cause an impeda mismatch which inturn causes the development of standing wave and a drastic loss in efficiency.

be reflections, so it is necessary to terminate the waveguide with its characteristic impedance as in transmission line. In waveguides, some discontinuities are placed for matching purposes. Any susceptance appearing across the guide, causing mismatch (and production of standing waves) needs to be cancelled by introducing another susceptance of the same magnitude but of opposite nature. The discontinuities like irises (also called windows, apertures, diaphragms & obstacles) are used for this purpose. The iris is moved inside the waveguide until a proper match is obtained.

Inductive window (or) Iris :-

when the window is placed where the magnetic field is strong (or where electric field is relatively weak). As the energy is stored in the magnetic field, this type of iris is called inductive.

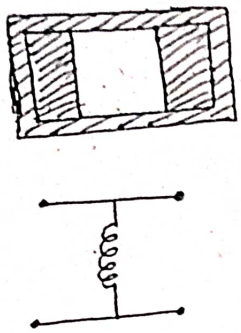
Capacitive window (or) Iris :-

when the window is placed where the electric field is strong (or where the magnetic field is relatively weak). As the energy is stored in the electric field, this type of iris is called capacitive.

Resonant windows :-

The inductive & capacitive windows if combined suitably (correctly shaped and positioned) the inductive & capacitive reactances introduced will be equal & the iris becomes a parallel resonant circuit. The parallel resonant window supports only dominant mode, and other modes are completely attenuated. & the resonant iris acts as a bandpass filter to suppress unwanted modes.

A series resonant iris supported by a non-metallic material is transparent to the flow of microwave energy.



fig(a): Inductive window

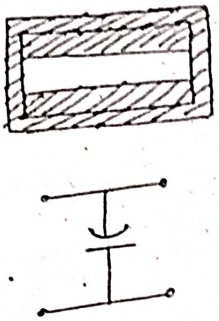


fig: capacitive window

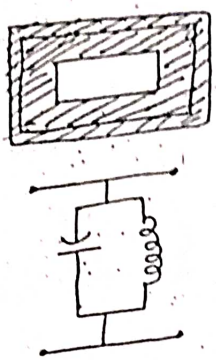


fig: parallel resonant window

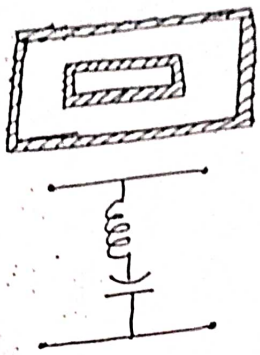
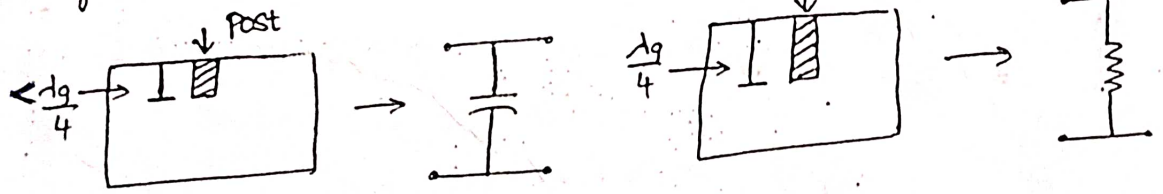


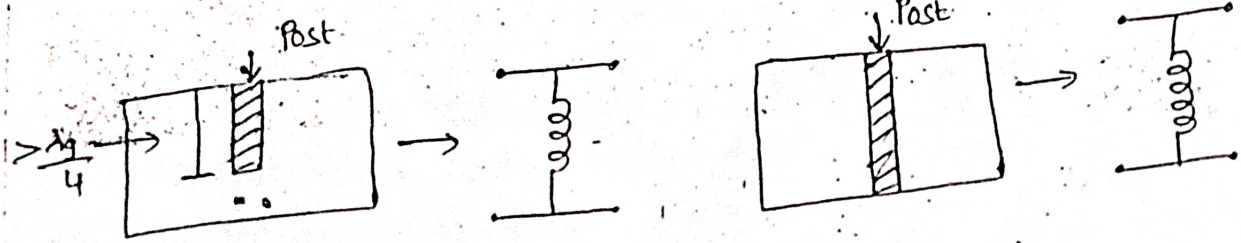
fig: Serier resonant window.

Posts :-

when a metallic cylindrical post is introduced into the broader side of waveguide, it produces the same effect as an iris in Providing lumped reactance at that point. If the post extends only a short distance ($<\lambda_g/4$) into the waveguide, it behaves capacitively, and this capacitive susceptance increases with depth of penetration. when the depth is equal to $\lambda_g/4$, the post behaves resistively, if it is $>\lambda_g/4$, the post behaves inductively and this inductive susceptance decreases when the post is moved further away from the centre of the waveguide. when the post is extended completely accross the waveguide, the post becomes inductive.



(a) capacitive post (b) Resistive post



(c) Inductive post (d) Inductive Post.

The amount of susceptance decreases as the diameter of the post is reduced. If the post is made thicker, effective Q will be

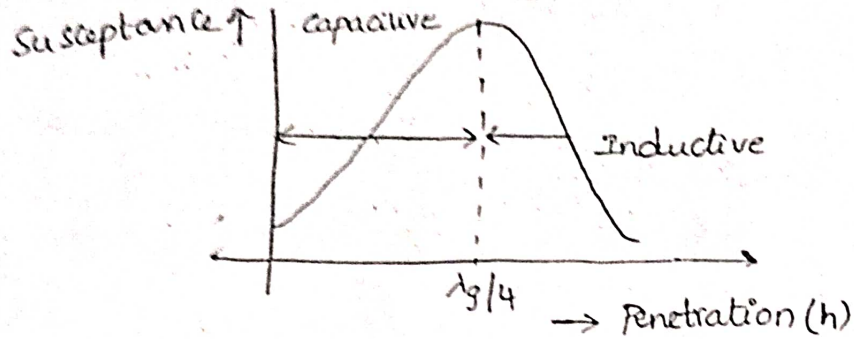
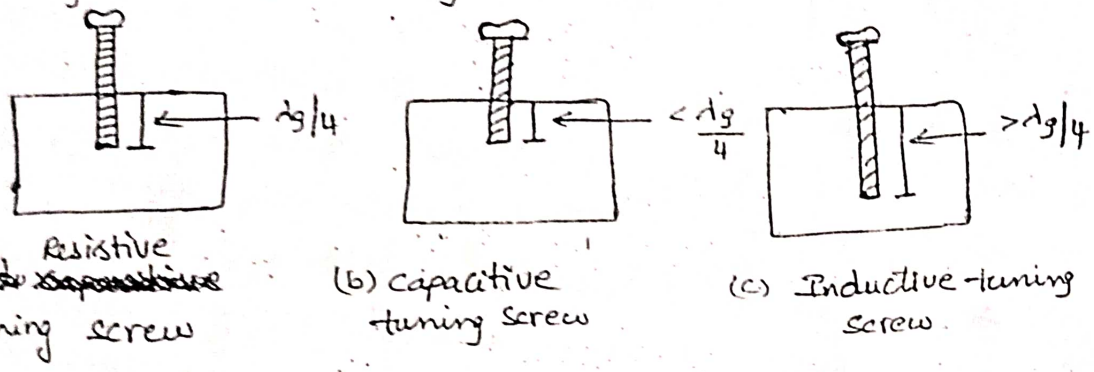


fig: Susceptance Vs penetration characteristics.

Tuning Screws :-

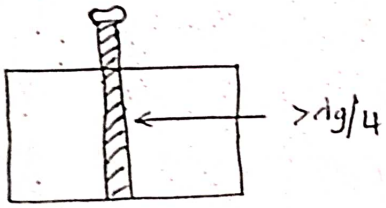
An adjustable post is known as a screw or stung. The adjustable or tuning screws are shown below.



(a) ~~Resistive~~ ~~Capacitive~~ tuning screw

(b) capacitive tuning screw

(c) Inductive tuning screw



(d) Inductive tuning screw

As incase of posts depending upon the depth of penetration, the tuning screw may be inductive, Resistive or capacitive.

Matched loads :-

The matched load absorbs all the incident power upon it and reflects nothing. This is equivalent to connecting the transmission line with its characteristic impedance or equivalent to an infinitely long transmission line. Matching of the load is a Prerequisite requirement in a waveguide system as mismatching in waveguide will cause reflection of energy. Hence the load should be capable of absorbing the energy without reflection,

on the power.

For low power the matched load is a lossy material inserted into the guide. since, the material is lossy, the incident power is absorbed For high powers water loads are used. The water loads consists of water circulating through a hollow glass tube mounted in the waveguide Increase in water-temperature is proportional microwave power absorbed

Waveguide Attenuators :- (Microwave Engineering - R.Gowri)

The function of attenuator is to reduce the power of the signal through it.

Attenuators are commonly used for measuring powergain & loss in dBs, for providing isolation between instruments for reducing the powerinput to a particular stage to prevent overloading and also for providing the signal generators with a means of calibrating their outputs accurately so that precise measurement could be made.

Attenuators are of 2 types.

1. Fixed type Attenuators 2. Variable Attenuators.

Fixed Attenuators :-

Fixed Attenuators are used for providing fixed amount of attenuation.

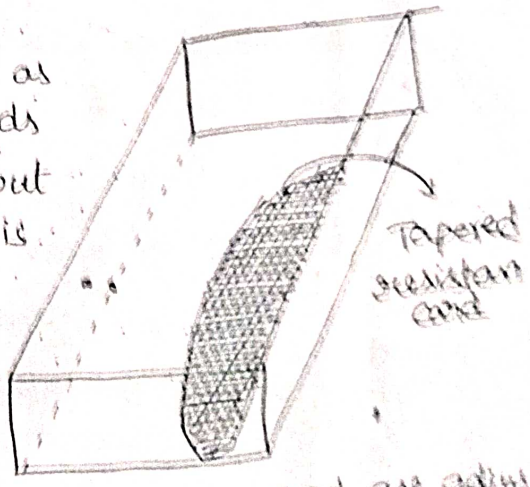
Variable Attenuators :-

Variable Attenuators provide continuous & stepwise variable attenuation.

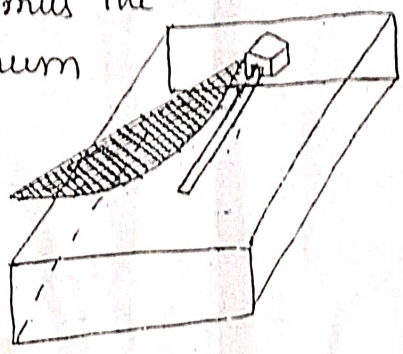
Resistive Card Attenuators :-

This may be either fixed & variable.

In fixed type, the resistive card is fixed as shown in figure. This is tapered at both ends in order to maintain both input and output SWR low. Maximum attenuator per length is achieved by having the card parallel to the electric field and the centre of the guide, where the electric field is maximum as in case of TE_{10} mode (dominant mode). The Conductivity & dimensions of the card are chosen for the desired value of attenuation. In high power applications ceramic



The variable type resistive card attenuator is called "flap." attenuator. In this the resistive card is inserted into a longitudinal slot cut along the centre of the waveguide wider dimension. The depth of insertion of the flap permits the attenuation to vary upto the maximum of 30dB. when the flap is removed, there will be '0' attenuation.



Rotatory Vane Attenuator :-

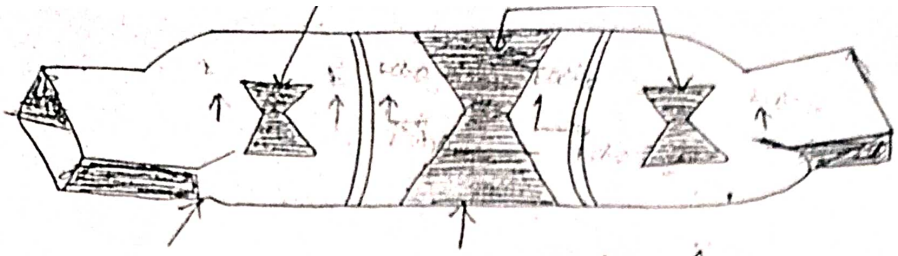
A rotatory attenuator is a precision waveguide attenuator in which attenuation follows a mathematical law. In this device attenuation is independent of frequency. They can be used in cascade to obtain large ranges of attenuation.

A rotatory vane attenuator has three sections of circular waveguides with a resistance card in the centre. These resistive cards are long enough to absorb completely any signal whose electric field is parallel to them. Both at input and output ends rectangular to circular waveguide transitions are there to Provide low SWR (standing Wave Ratio). The centre position is rotatable independently of the ends & the two end sections of the circular waveguide are fixed with their resistive cards perpendicular to the electric field of the input & output signal.

→ If the resistive card of the centre waveguide is parallel to other two, there will be minimum attenuation.

→ If the resistive card of the centre waveguide is perpendicular to other two, there will be maximum attenuation.

→ For any arbitrary angle θ, the field component parallel to the rotatable resistive card ($E\sin\theta$) will be absorbed and the field component perpendicular to rotatable resistive card ($E\cos\theta$) will be passed through it



Rectangular to Circular waveguide transition

Rotating section of Circular waveguide

when the electric field passes through the first section of resistive card then the signal will be passed through it as it is perperdicular to the electric field. As the rotating section is at an angle θ then the electric field will be split into two components. one component, $E\cos\theta$ perpendicular to rotating resistive card & it will be passed through this section another component, $E\sin\theta$ parallel to the rotating resistive card & it will be attenuated. The $E\cos\theta$ component will again split into 2 components when it passed through the third fixed section. one component $E\cos^2\theta$ perperdicular to the fixed resistive card & it will be passed to the output. another component $E\cos\theta\sin\theta$ parallel to the fixed resistive card which will be attenuated.

∴ The resulting attenuation is equal to

$$\alpha = 20\log_{10}\cos^2\theta$$

$$\boxed{\alpha = 40\log_{10}\cos\theta}$$

From the above equation, attenuation is independent of frequency making it precised attenuator.

waveguide phase shifters (or) phase changers :-

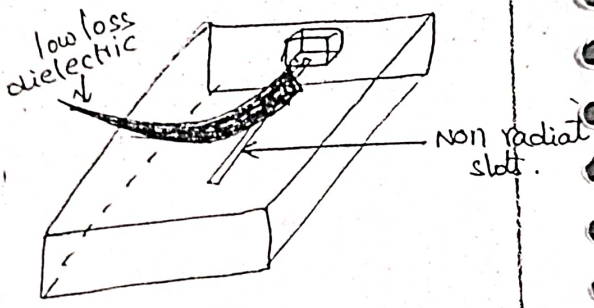
A phaseshifter is used to produce required phase shift in a waveguide system. It changes the phase angle of the wave passing through it. Ideally a phase shifter must change the phase only, it should not behave like an antennuator for the incident wave.

These are of 2 types

1. Dielectric phase shifter
2. Rotatory vane phase shifter

Dielectric phase shifter :-

In this low loss dielectric is inserted into a longitudinal slot cut along the centre of the waveguide wider dimension.



Insertion of dielectric with $\epsilon_r > 1$ will reduce the guide wavelength as

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} = \frac{c}{f\sqrt{\frac{f^2-f_c^2}{f}}} = \frac{1}{\sqrt{\mu\epsilon}\sqrt{f^2-f_c^2}}$$

$\lambda = c/f$

$c = \frac{1}{\sqrt{\mu\epsilon}}$

$$= \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}\sqrt{f^2-f_c^2}}$$

as ϵ_r increases with the insertion of dielectric λ_g decreases.

and

$$\beta_g = \frac{2\pi}{\lambda_g}$$

As λ_g decreases the phase constant β_g increases. Hence the phase of the output signal increases.

Rotatary Vane phase shifter :-

Its construction is similar to that of a rotatory attenuator. Here the centre resistive card is replaced by a halfwave plate and the two fixed resistive cards are replaced by quarter wave plates. These plates are constructed from a slab of dielectric material.

A quarterwave plate produces a circularly polarised wave when a linearly polarised wave is incident upon it. A circularly polarised field with x & y components of electric field that are equal in magnitude but 90° apart in time phase. When the polarization is perpendicular to the slab if propagation constant is β_1 & when the polarization is parallel to the slab if the propagation constant is β_2 then differential phase change is

$$(\beta_2 - \beta_1)\, l \qquad \text{where } \beta_2 > \beta_1.$$

where $l = \lambda/4$ for quarter wave plate

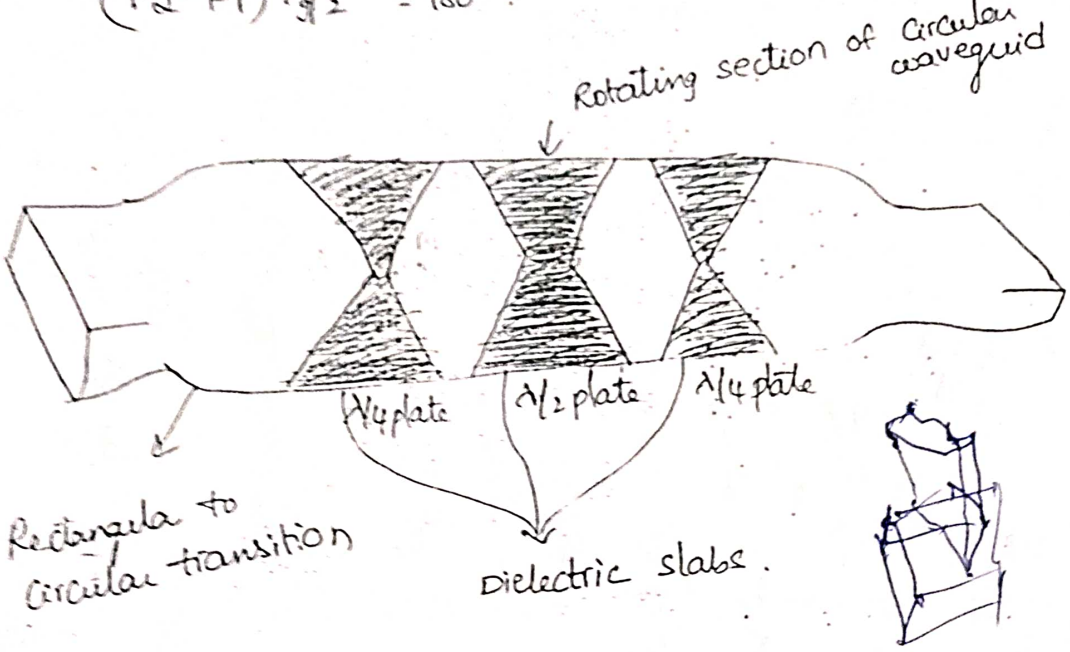
$= \lambda/2$ for Half wave plate.

The phase shift using quarter wave plate

$$(\beta_2 - \beta_1)\frac{\lambda}{4} = 90^\circ$$

The phase shift using Halfwave plate

$$(\beta_2 - \beta_1)\frac{\lambda}{2} = 180^\circ$$



Waveguide Multiport Junctions :-

In waveguide system, it is necessary to split all or part of microwave energy into particular directions. This is achieved by using microwave junctions. Alternately the same junction may be used to combine two or more signals. In general microwave junction is a interConnection of two or more microwave Components.

Microwave T-junctions :

Microwave Tee is a 3 port junction. These are of 2 types

1. E-plane Tee 2. H-plane Tee

1. E-plane Tee (or) Series Tee :-

An E-plane Tee is designed by cutting a rectangular slot along the broader wall of a waveguide section and attaching another waveguide called side arm / auxiliary arm

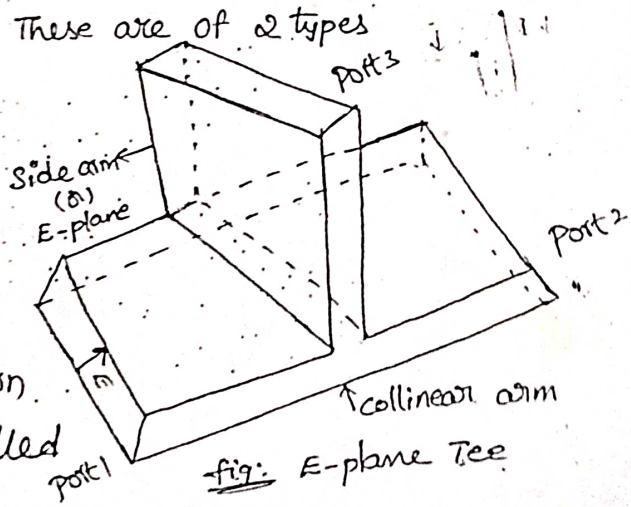


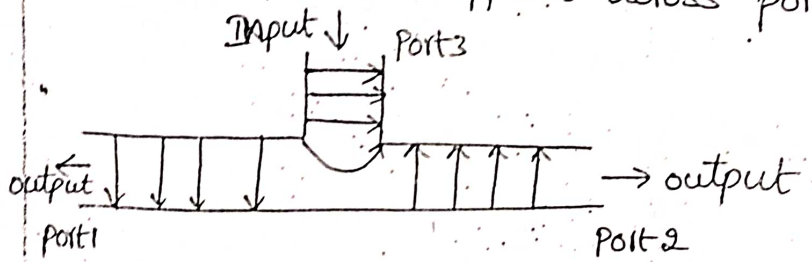
fig: E-plane Tee

→ when the input is fed at port3 then the outputs appearing at port1 & port2 will be opposite in phase and same in magnitude fig(a)

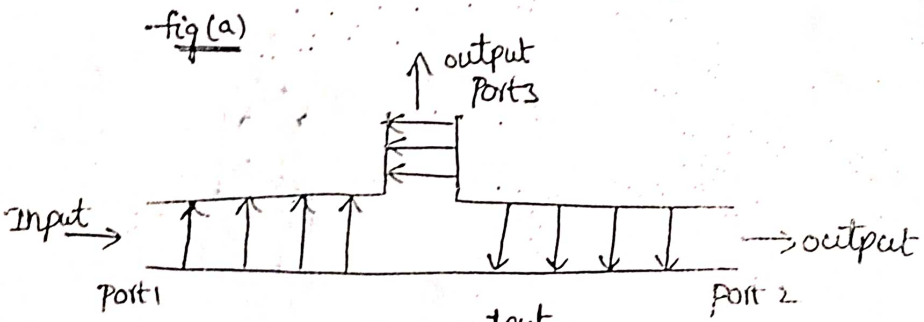
→ when the input is fed at port1 or port2 then the outputs appear at port2 & port3 (or) at port1 & port3 as shown in fig(b)

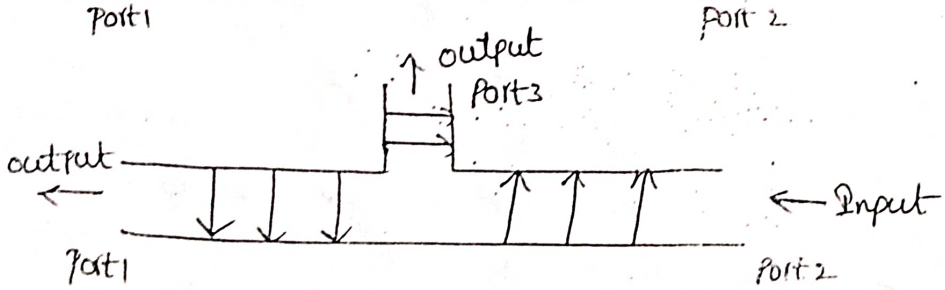
→ when two inputs of same phase are fed at port1 & port2 then no output will appear across port3 as in fig(c).

→ when two inputs of 180° out of phase are fed at port1 & port2 then the output will appear across port3 as in fig(d).

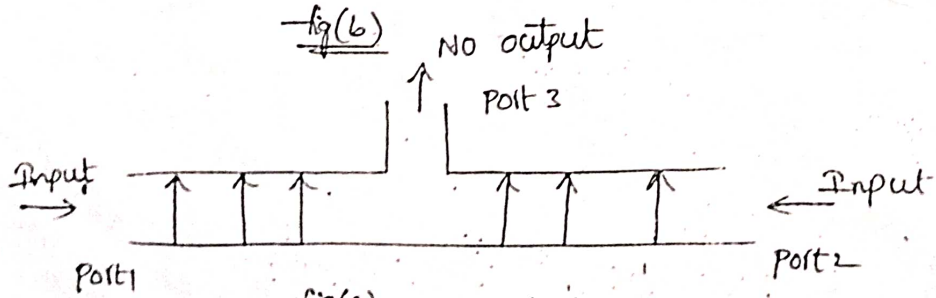


fig(a)

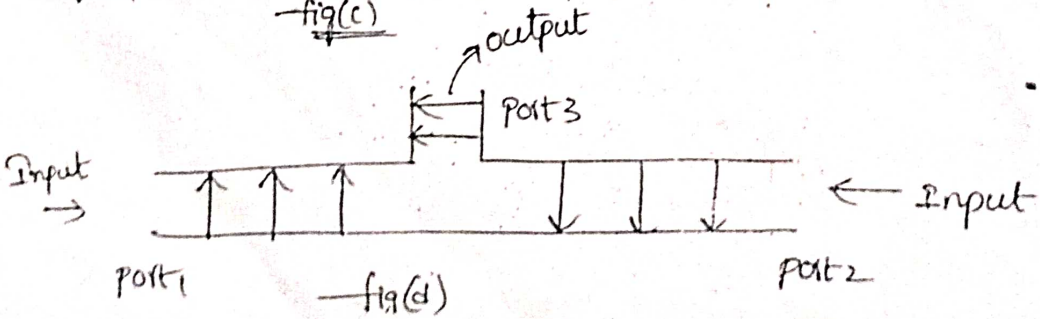




fig(b)



fig(c)



fig(d)

... connected in series with the main arm hence it is also called as series Tee.

2. H-plane Tee / Shunt Tee :-

H-plane Tee is designed by cutting a rectangular slot along the narrow arm of the main waveguide & attaching other waveguide called auxiliary waveguide as shown in figure

As the H-arm is attached in shunt with the main waveguide this is also called as shunt Tee.

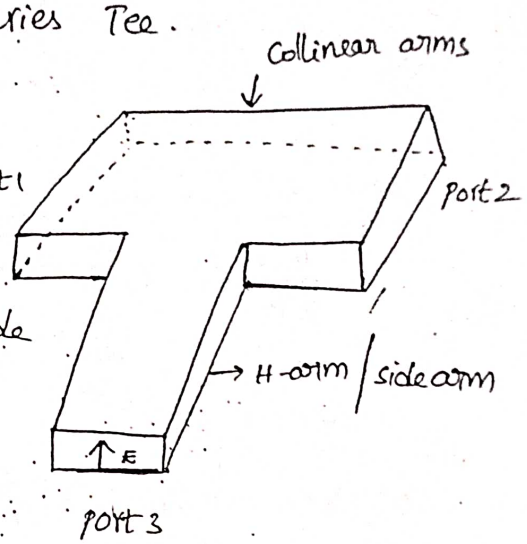


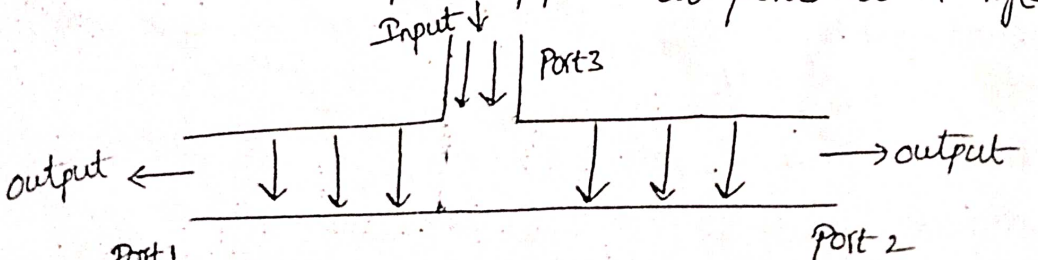
fig: H-plane Tee.

→ when the input is fed at port3 then the outputs appearing across port1 & port2 will be same in phase and magnitude as in fig(a).

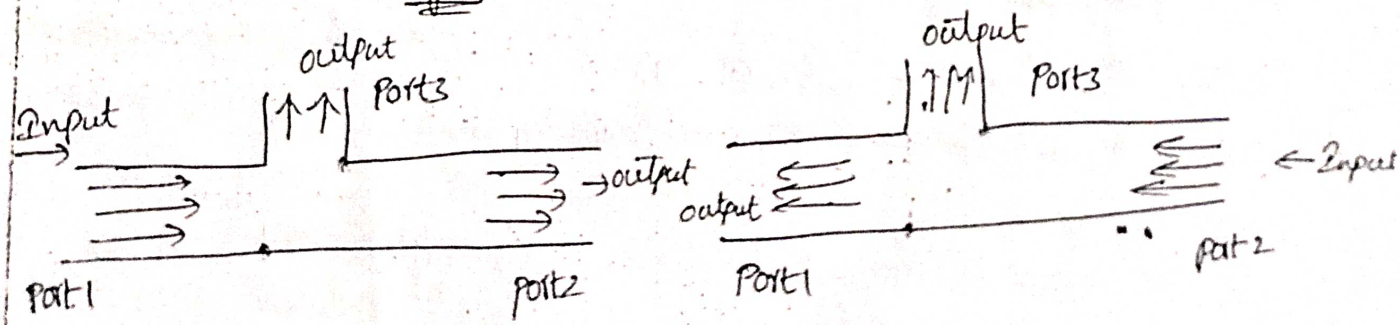
→ when the input is fed at port1 or port2 Then the outputs appear at port2 & port3 or port1 & port3 as shown in fig(b).

→ when the two inputs of same phase are applied at port1 & 2 then the output appear at port3 as in fig(c)

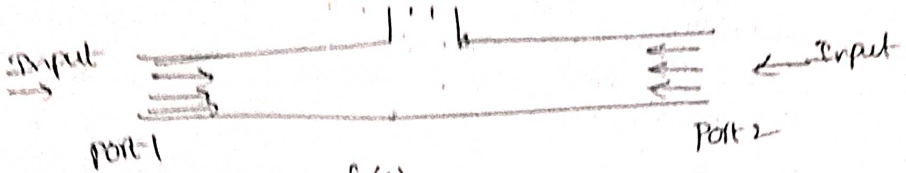
→ when the two inputs of 180° out of phase are applied at port1 & port2 then no output appear at port3 as in fig(d):



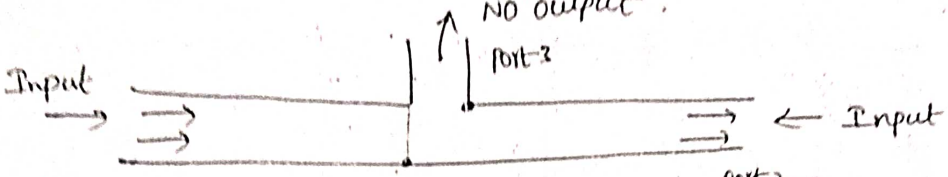
fig(a)



fig(b)



fig(c)



fig(d)

Hybrid Junctions:-

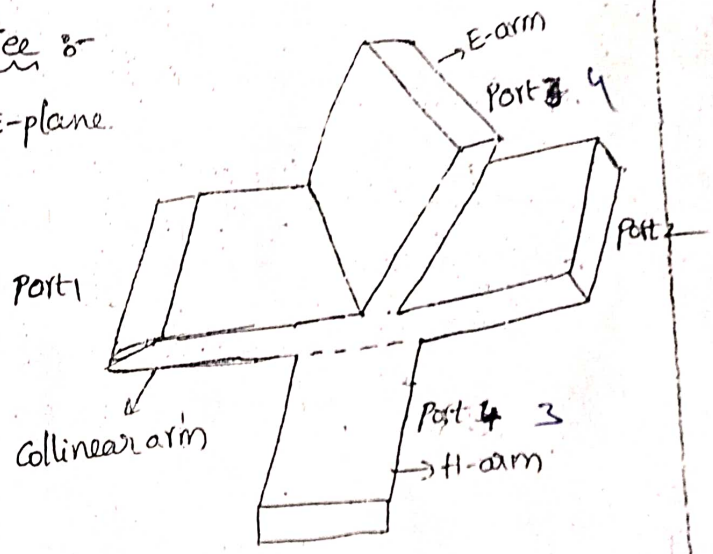
Hybrid Junctions are 4 port devices. These are of 2 types.

1. E-H plane Tee | Hybrid Tee | Magic Tee
2. Hybrid Ring | Rat race Junction

E-H plane Tee | Magic | Hybrid Tee or Magic Tee

Magic Tee is a Combination of E-plane and H-plane Tee.

→ If two waves of equal magnitude and same phase are fed into port 1 and port 2, the output at port 3 will be zero and additive at port 4.



→ If the input is fed at port 4 (H-arm), it will be divided equally between port 1 & port 2 of the collinear arms & will not appear at port 3 (E-arm).

→ when the input is fed at port 3 (E-arm), it will produce an output of equal magnitude and opposite phase at port 1 & port 2, output at port 4 is zero.

→ If the input is fed at any one of the collinear arms ie, at port 1 & port 2, it will not appear in the other collinear arm at port 2 & port 1 because E-arm causes a phase delay & Harm

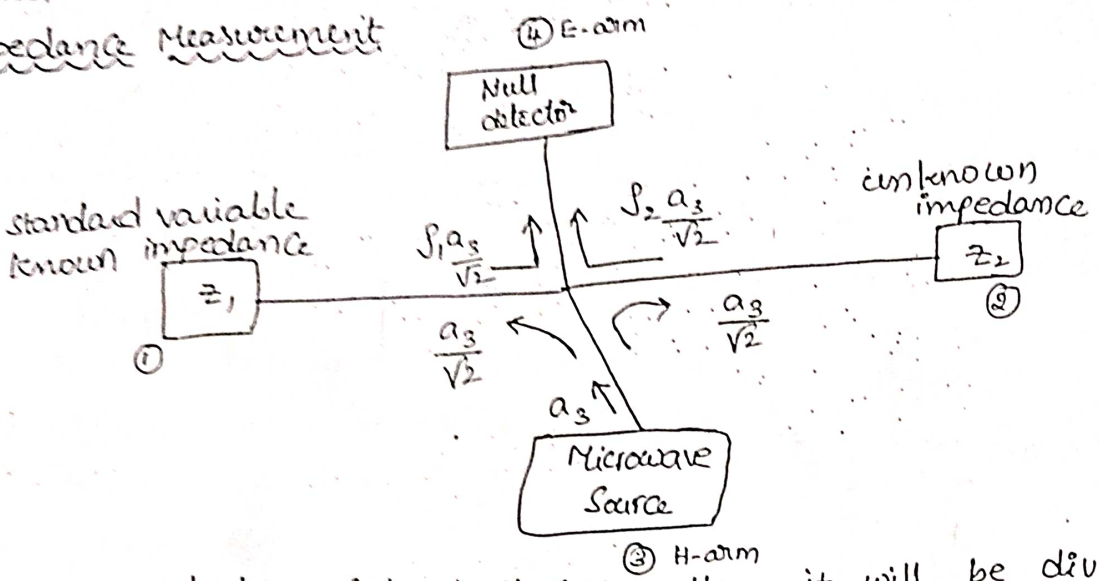
causes a phase advance.

Applications of Magic Tee:-

Magic Tee is commonly used for (or) used as

1. Impedance Measurement
2. Duplexer
3. Mixer

1- Impedance Measurement



when the input a_3 fed at H-arm ③ then it will be divided equally to port ① & ② as $\frac{a_3}{\sqrt{2}}$ ie, to standard variable impedance and unknown impedance. These impedances are not equal to characteristic impedance z_0 and hence there will be reflection from ports ① & ②. If s_1 & s_2 are reflection coefficients, powers $\frac{s_1 a_3}{\sqrt{2}}$ $\frac{s_2 a_3}{\sqrt{2}}$ enter the magic tee junction from ① & ②. The resultan wave into arm ④ ie, null detector can be calculated as

$$\frac{1}{\sqrt{2}}\left(s_1 \frac{a_3}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\left(s_2 \frac{a_3}{\sqrt{2}}\right) = \frac{a_3}{2}(s_1 - s_2).$$

For perfect balancing the bridge null detection must be equal to zero.

$$\frac{a_3}{2}(s_1 - s_2) = 0.$$

$$\Rightarrow s_1 = s_2 \Rightarrow z_1 = z_2$$

Thus the unknown impedance can be measured by adjusting the standard variable impedance till the bridge is balanced & both impedances equal.

Duplex is a device which connects the antenna to the transmitter during transmission & the antenna to the receiver during the reception.

Duplexers are used in Radar.

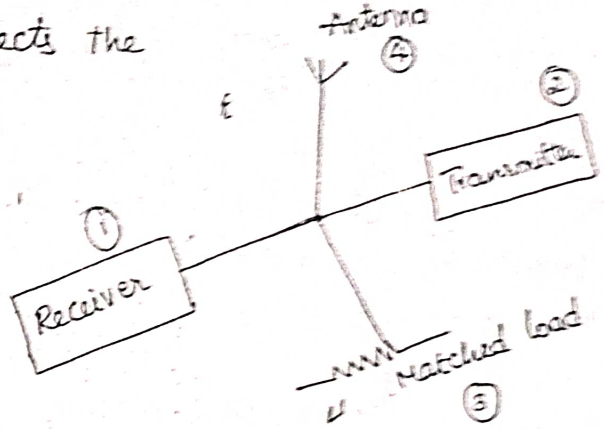


fig: Magic Tee as Duplexer

The transmitter and receiver are connected in port ② & port① respectively, antenna is connected to E-arm & Matched load is connected to H-arm.

→ During the transmission, Half of the transmitted power reaches the antenna & it will be radiated into space and the other half power reaches the matched load & it will be absorbed without reflections.

→ During the reception, Half of the power reaches the receiver and the other half goes to the transmitter.

3. Magic Tee as Mixer:-

A Magic Tee can be used in microwave receivers as a mixer where the signals from antenna and local oscillator are fed into E & H-planes as shown in figure.

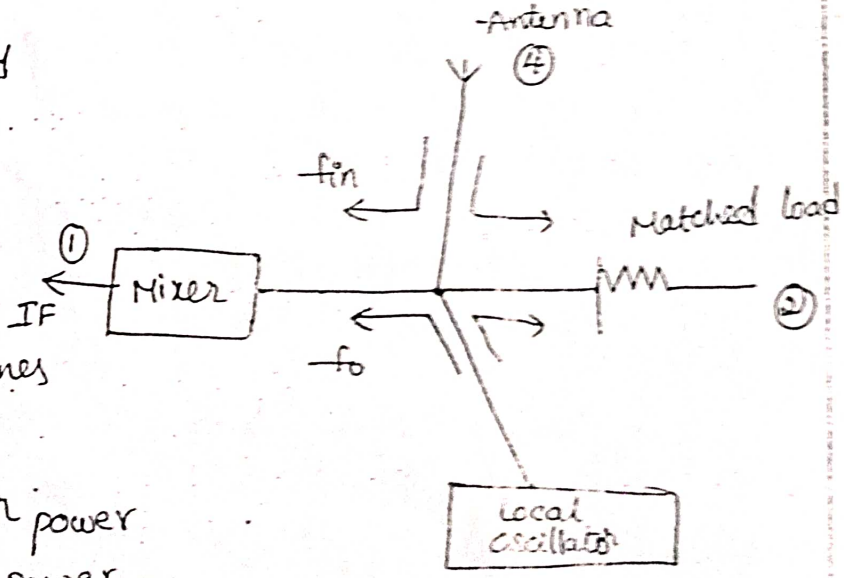


fig: Magic Tee as Mixer.

Half of the local oscillator power and half of the antenna power goes to the mixer where they are mixed to generate the IF frequency.

$$IF = f_0 \sim fin$$

Hybrid Ring

It is a fourport junction, The four arms are connected in a ring manner by means of series & parallel junctions. These ports are seperated by proper electrical lengths to sustain standing waves.

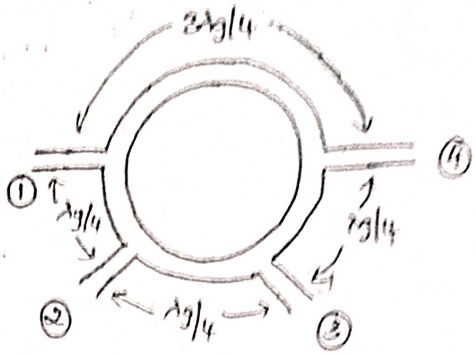


fig: Hybrid Ring

For proper operation, it is necessary that the mean circumference of the total race be $1.5\lambda_g$.

The hybrid ring has the characteristics similar to those of the hybrid Tee.

→ when a input is fed at port 1 it will split equally into port 2 and port 4 because the phase shifts for the waves travelling in clockwise direction (ie, $3\lambda_g/4$ distance) and in anticlockwise direction (ie, $\lambda_g/4$ distance) is 90°. There will be no output at port 3 because the difference of phase shifts travelling in clockwise direction (ie, λ_g distance) and in anticlockwise direction (ie, $\lambda_g/2$ distance is 180°.

Similarly

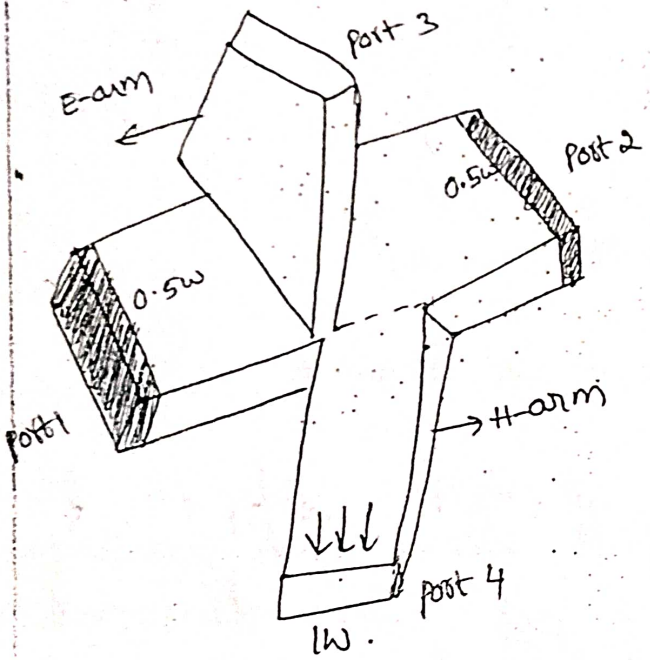
→ when the input is fed at port 2 it will split equally into port 1 & port 3 & there will be no output at port 4

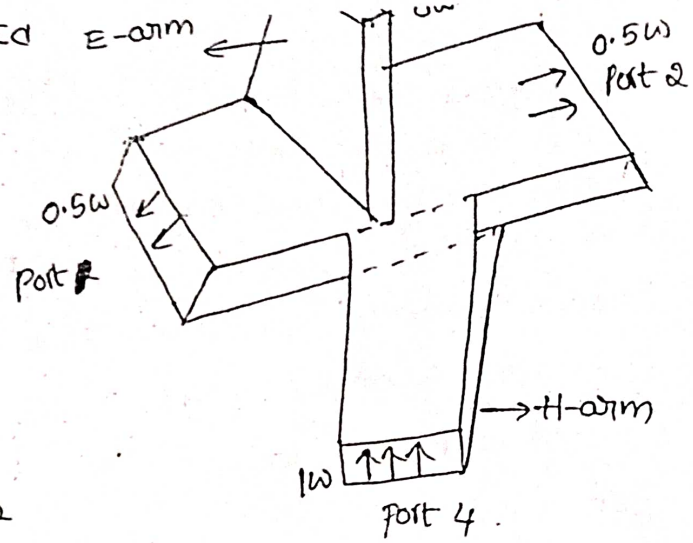
→ when the input is fed at port 3 it will split equally into port 2 & port 4 & there will be no output at port 1.

→ when the input is fed at port 4 it will split equally into port 1 & port 3 & there will be no output at port 2.

(Pb) → A matched generator with a power of one watt is connected to the H-arm of magic tee (Port 4). The E-arm (Port 3) is matched terminated and the lenth of the co-planar arm is the same. Compute the power delivered to the termination at port 1, 2 and 3 and the power reflected at port 4 when port 1 & port 2 are matched-terminated.

(i) When a power of 1W is connected to H-arm it will be divided equally to port 1 & port 2 ie, 0.5 W at port 1 & 0.5W at port 2. and the power at port 3 is 0W.





when port 1 & port 2 are matched terminated, the total power will reflect into port 4.

$0.5W + 0.5W = 1W.$

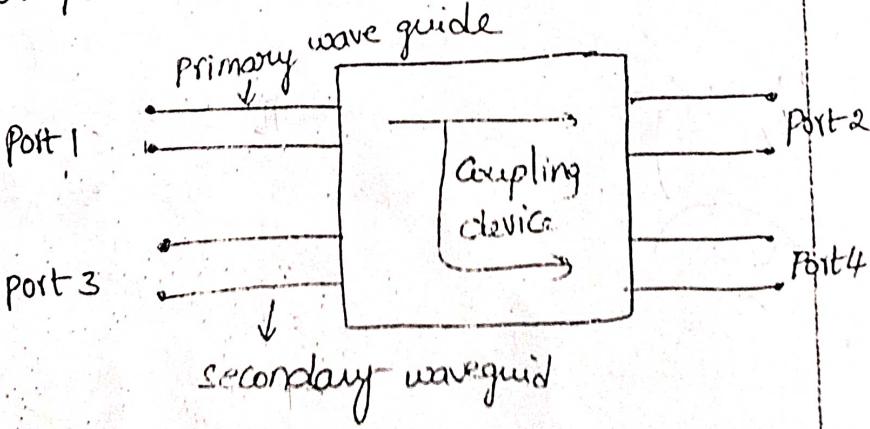
∴ Reflected power = 1W.

Directional Coupler :-

A directional coupler is a four-port junction. It consists of a primary waveguide 1-2 and a secondary waveguide 3-4. when all ports are terminated in their characteristic impedances, there is free transmission of power; without reflection, between port 1 & port 2 & there is no power at port 3

→ A portion of power travelling from 'port ① to port ②' is coupled to port ④ but not to port ③.

→ A portion of the power travelling from port ② to port ① is coupled to port ③ but not to port ④.



→ port ① & port ③, port ② & port ④ are isolated ports.

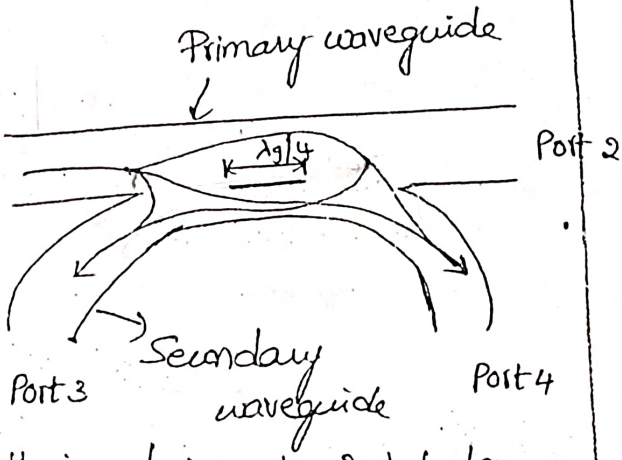
Types of directional couplers:

① 2-hole directional coupler

② Bethe hole (or) single directional coupler.

2-hole directional Coupler :-

It consists of two waveguides ie, mainwaveguide & secondary waveguide with two tiny holes common in between them. The two holes are at a distance $\lambda_g/4$.



→ The power out of holes ① & ② both in phase at 2nd hole hence they add up contributing to P_f.

→ The power out of holes ① & ② both in 180° out of phase at 1st hole hence they will be cancelled at port 3 making $P_b = 0$ (ideally).

→ The magnitude of the power coming out of 2 holes depends upon the dimensions of the two holes. Since the distance between holes is $\lambda_g/4$, P_b is made '0' [since the incident power will have to travel a distance of $(\lambda_g/4 + \lambda_g/4 = \lambda_g/2)$ when it comes back from hole ② resulting in 180° phase shift.]

Bethe (or) single-hole Coupler :-

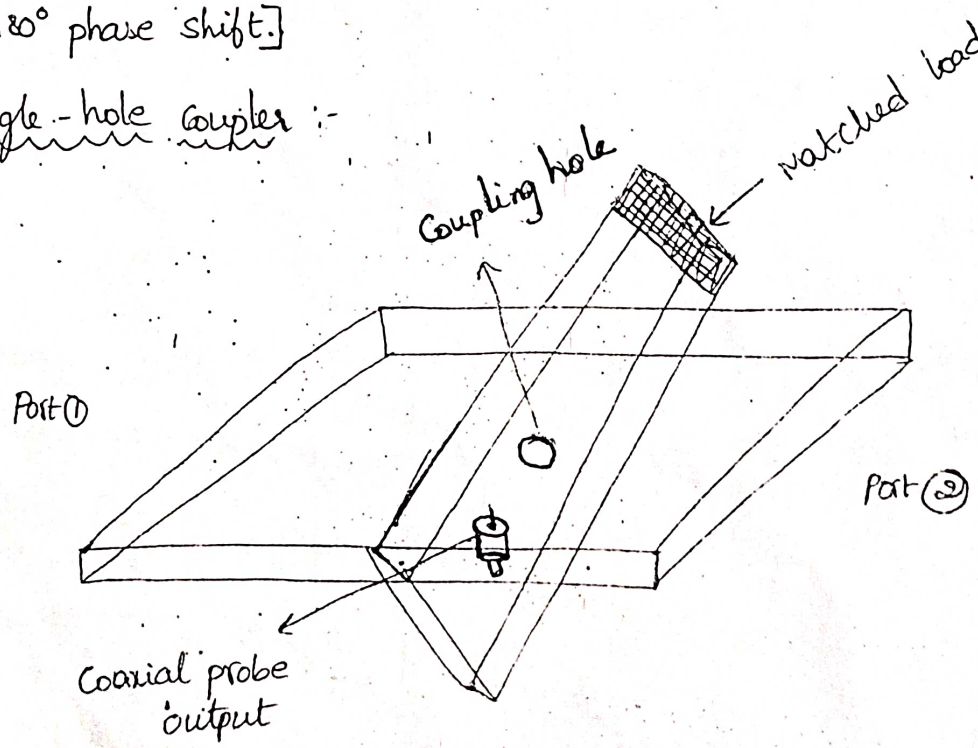


fig: Bethe (or) single hole coupler

A single-hole directional coupler has improved directivity when compared to 2-hole directional coupler as the coupling process involves a single hole.

→ The power entering port ① is coupled to the co-axial probe output and the power entering port ② is absorbed by the matched load.

→ The auxiliary guide is placed at such an angle where the magnitude of electric & magnetic fields are equal.

Pb → A 20dB coupler has a directivity of 30dB. Calculate the value of Isolation

Given data

$C = 20dB$

$D = 30dB$

$$D_{dB} = I - C \quad dB$$

$$\Rightarrow I_{dB} = (D + C) \quad dB$$

$$\boxed{I = 50\,dB}$$

Pb → There are two identical couplers connected back to back to sample incident & reflected powers. The outputs of the couplers are 12mw & 0.12mw respectively. what is the VSWR in the guide.

Given data

The incident power at port ④ = 12mw

The reflected power = 0.12mw

$$\text{Reflection coefficient} \quad \gamma = \sqrt{\frac{P_r}{P_i}} = \sqrt{\frac{0.12}{12}} = 0.1.$$

$$VSWR(\rho) = \frac{1+|\gamma|}{1-|\gamma|} = \frac{1+0.1}{1-0.1} = 1.22$$

$$\boxed{\rho = 1.22}$$

⇒ A 20dB directional coupler is found to give 3dBm in output power through coupled port. If the isolation is specified as 55dB, find the power available at the isolated port. Explain the relations used.

$C = 20dB$

$I = 55dB$

$P_f = 3dBm = 3\times10^{-3}\ dB \Rightarrow P_f = 1.00069\ W.$

$P_b = ?$

Directivity (dB) = Isolation (dB) − Coupling factor (dB)

$= 55 - 20$

$= 35dB.$

$$10\log\left(\frac{P_f}{P_b}\right) = 35dB$$

$$\log\left(P_f/P_b\right) = 3.5$$

$$P_f/P_b = 10^{3.5}$$

$$P_b = \frac{1}{10^{3.5}}$$

$$\boxed{P_b = 0.316\ mW}$$

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