UNIT-II

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## MICROWAVE TUBES -I

At microwave frequencies, the size of electronic devices required for generation of microwave energy becomes smaller & smaller. This results in lesser power handling capability & increased noise levels. Conventional triodes, tetrodes and pentodes are useful only at low frequencies. Electronic devices such as tubes & transistors will be required even at microwave frequencies. The tubes can provide higher output powers while transistors (semiconductor devices) are smaller have lesser noise, better reliability with reduced output power levels.

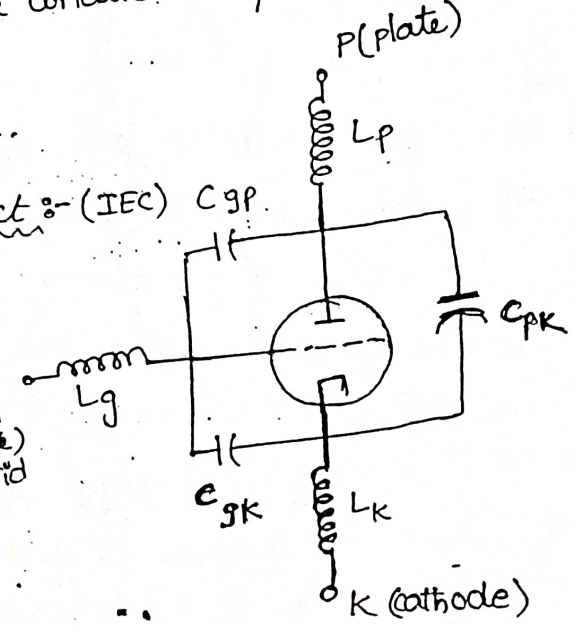
### High frequency (or) Microwave frequency limitations of conventional tubes

At microwave frequencies conventional tubes such as triode, tetrode pentodes cannot be used for generation of power, amplification & oscillation purpose because of following limitations.

1. Inter electrode capacitance effect } → Circuit Reactance
2. Lead inductance effect
3. Transit time effect
4. Gain bandwidth limitation
5. Effect due to RF losses (The conductance / skindepth or $I^2R$ losses & the dielectric losses)
6. Effect due to radiation losses.

1. Inter electrode capacitance effect :- (IEC) $C_{gp}$.

As frequency increases, the reactance $X_C = \frac{1}{2\pi fC}$ decreases and the output voltage decreases due to shunting effect. Because at higher frequency $X_C$ becomes almost short, $C_{gp}$, $C_{gk}$, $C_{pk}$ becomes short almost.




Remedies :-

This can be minimised by reducing $C_{gk}$, $C_{gp}$, $C_{pk}$. These can be reduced by decreasing the area of electrodes or by increasing distance

between electrodes

Since $C = \frac{A\epsilon}{d}$

2. Lead Inductance Effect :-

As frequency increases, the reactance $X_L = 2\pi fL$ increases and hence the voltages appearing at the active electrodes are less than the voltages at the base pins. This results in reduced gain for the tube amplifier. $L_K$, $L_P$ & $L_g$ are the lead inductances that limit the performance of the tubes.

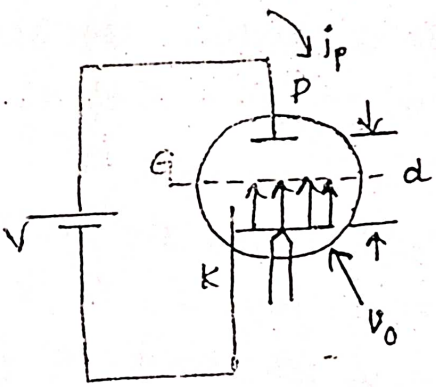
Remedy :-

It can be minimised by decreasing 'L' & L can be decreased by using large size leads or by decreasing 'A'. However this reduces the power handling capability

$$L = \frac{\mu A}{d}$$

3. Transit Time effect :-

Transit time is the time taken for the electron to travel from cathode to anode as shown in figure.




fig

$$\text{Transit time } \tau = \frac{d}{v_0}$$

where

d = distance between cathode & anode

$v_0$ = velocity of electrons

static energy of electrons = eV

kinetic energy = $\frac{1}{2} m v_0^2$

under equilibrium,

static energy = kinetic energy

$$eV = \frac{1}{2} m v_0^2$$

$$v_0 = \sqrt{\frac{2eV}{m}}$$

$$\tau = \frac{d}{\sqrt{2eV/m}}$$

$$A_{eff} \propto \frac{1}{\sqrt{\omega}}$$

$$A_{eff} \propto \frac{1}{\sqrt{f}}$$

& $$R = \frac{\rho l}{A_{eff}} = \frac{\rho l}{\frac{1}{\sqrt{f}}} = \rho l \sqrt{f}$$

As frequency increases resistance 'R' increases. Hence losses increas

Remedy:- By increasing the (size) (Area) of the conductor these losses can be reduced.

(b) Dielectric Losses :-

This occurs in various types of insulating materials used in the device ie, spacers, glass envelope, silicon & plastic encapsulations etc. This loss is generally expressed as

$$P = \pi f V_0^2 \epsilon_r \tan\sigma$$

$\epsilon_r$ = Relative permittivity of dielect
$\sigma$ = loss angle of the dielectric

As 'f' increases power loss increases

Remedy :- Eliminating tube base and reducing surface area of glass.

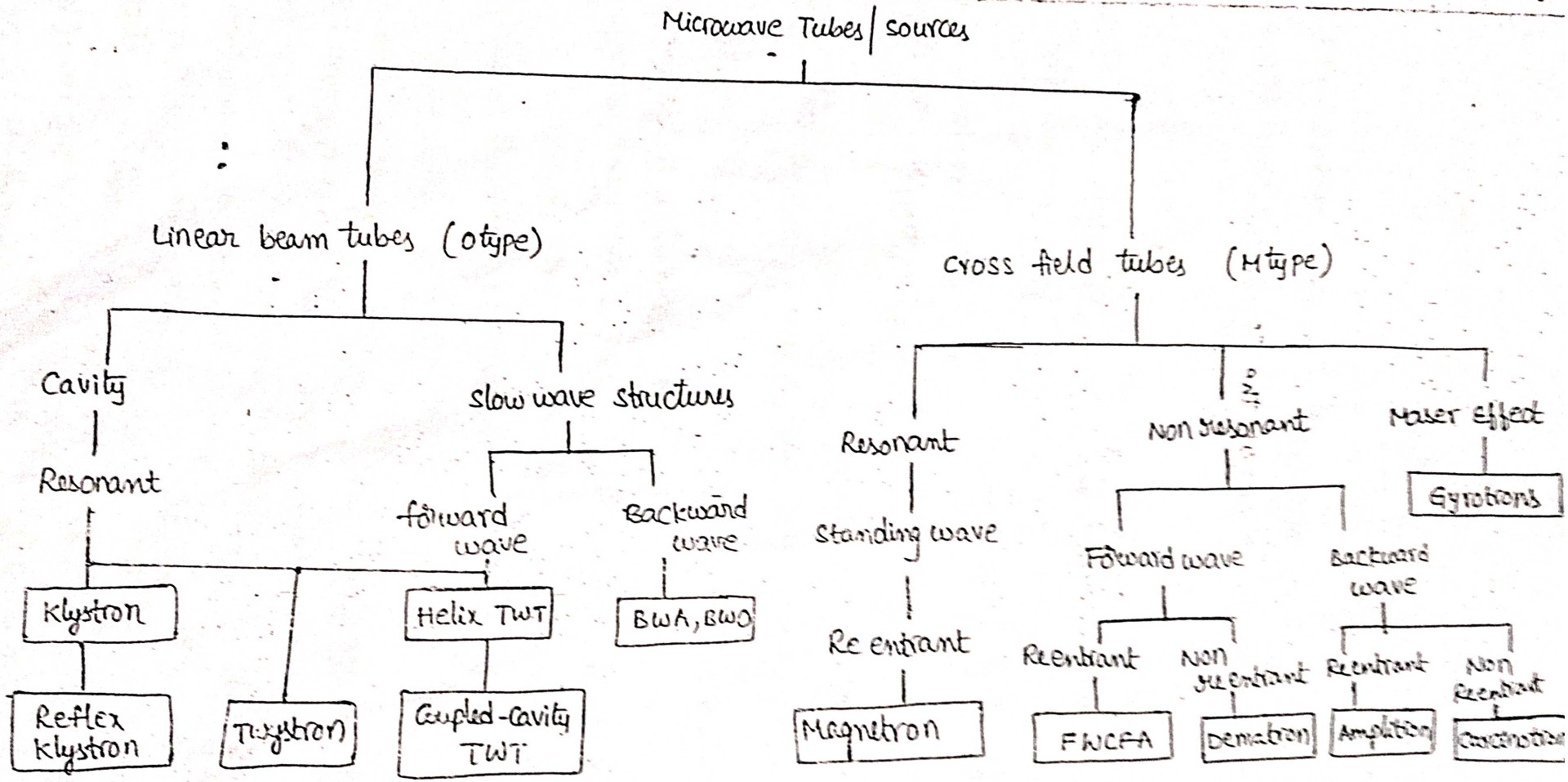
6. Radiation losses :-

whenever the dimensions of the wire approaches the wavelength ($\lambda = c/f$), it will emit radiation, ie, radiation losses increases with increase in frequency.

Remedy :- using proper shielding of the tubes and its circuitry.

Microwave tubes :-

Microwave tubes are constructed so as to overcome the limitations of conventional tubes. These tubes make use of transit time. Infact, large transit time is required for their operation. The basic principle of operation of microwave tubes involves "transfer of power from a source of dc voltage to a source of ac voltage by means of current density modulated electron beam." The density modulation of electron bea allows more electrons to be retarded by ac field than accelerated by dc fie which therefore makes possible a net energy to be delivered to the ac electric field.



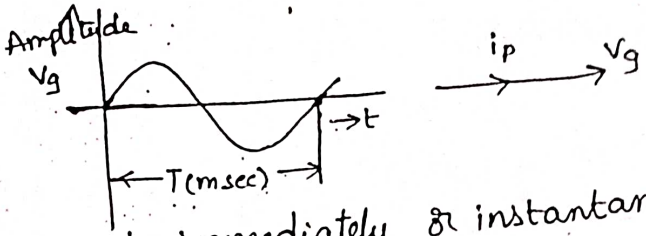

TWT – Travelling wave Tube
BWA – Back ward wave Amplifier
BWO – Back ward wave Oscillator

FWCFA – Forward wave Cross field Amplifier

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At low frequencies, of the signal as shown in figure.

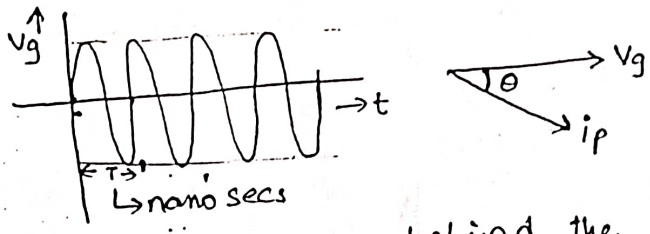
$$g_m = \frac{\Delta i_p}{\Delta V_g}$$




ie, both $V_g$ & $i_p$ are in phase.

Therefore the plate current '$i_p$' responds immediately or instantaneously to changes in control grid voltage '$V_g$' ie, $V_g$ & $i_p$ are in phase.

At high frequencies the transit time '$\tau$' is comparable with the period of the signal which is very small - nano seconds, as shown in figure.

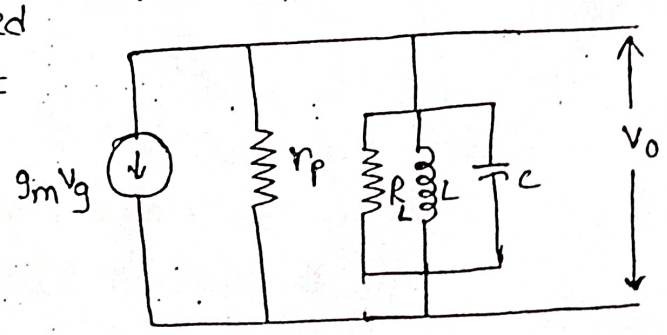



$i_p$ lags $V_g$ & $g_m$ becomes a complex quantity. Therefore change in plate current occurs after a finite delay with respect to change in control grid voltage $V_g$. Hence the plate current lags behind the control grid voltage & $g_m$ becomes a complex quantity as now it has a lagging phase angle. Gain becomes complex. ($\because A = -g_m r_p R_L / r_p + R_L$,

Remedy:- To minimise transit time ($\tau = d/\sqrt{2eV/m}$), 'd' the seperation between electrode can be decreased (but this increases Inter Electrode capacitance), and the plate to cathode potential 'V' can be increased. Therefore trade of between IEC & transit time is a must.

4. Gain Bandwidth limitation :-

Maximum gain is achieved when the tuned circuit is at resonance. $f_r = \frac{1}{2\pi\sqrt{LC}}$




Gain $A = \frac{V_0(s)}{V_i(s)}$

Replacing $r_p$ & $R_L$ with $R = \frac{1}{r_p} + \frac{1}{R_L}$

$$\frac{1}{Z_0(s)} = Y_0(s) = Cs + \frac{1}{Ls} + \frac{1}{R} = \frac{s^2LCR + Ls + R}{RLs}$$

$$Z_0(s) = \frac{s/C}{s^2 + \frac{s}{CR} + \frac{1}{LC}}$$

the roots of the characteristic equation

$$s^2 + \frac{s}{CR} + \frac{1}{LC} = 0. \qquad G = \frac{1}{R}$$

$$\omega_{1,\omega_2} = \frac{-\frac{1}{CR} \pm \sqrt{\left(\frac{1}{CR}\right)^2 - \frac{4}{LC}}}{2} = -\frac{G}{2C} \pm \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{-G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}} + \frac{G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

$$= 2\sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

for $\left(\frac{G}{2C}\right)^2 \gg 1/LC$

$$B.W = \frac{2G}{2C} = \frac{G}{C}.$$

The maximum gain at resonance is $A_{max} = g_m/G$

$\therefore$ Gain Bandwidth Product $= A_{max} \times B_\omega$

$$= \frac{G}{C} \times \frac{g_m}{G}$$

$$= g_m/C.$$

The gain bandwidth product is independent of frequency. For a given tube, a higher gain can be achieved only at the expense of a narrower bandwidth. This restriction is applicable to resonant circuit only. In microwave devices these restrictions can be overcome by use of

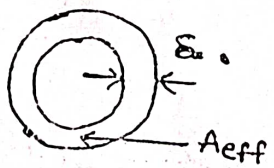
(1) Re-entrant cavities (2) Slow wave structures

to obtain a possible overall high gain over a broad bandwidth.

5. Effect due to RF losses :-

(a) Skin depth | Conductor ($\delta$) $I^2R$ losses :-

These losses come into play at higher frequencies at which the current has the tendency to confine itself to a smaller cross section of the conductor towards its outer surface.




$\delta$ = Skin depth $= \sqrt{\frac{2}{\omega\mu\sigma}}$

$\delta \propto \frac{1}{\sqrt{\omega\mu\sigma}}$ and $\delta \propto A_{eff}$

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In linear-beam tube a magnetic field whose axis coincides with that of the electron beam is used to hold the beam together as it travels the length of the tube. O-type is derived from the word "Original type of tube". In these tubes electrons receive potential energy from the dc beam voltage before they arrive in the microwave interaction region, and this energy is converted into their kinetic energy. In the microwave interaction region electrons are accelerated or deaccelerated by the microwave field & then bunched as they drift down the tube. The bunche electrons, inturn, induce current in the output structure. The electrons then give up their kinetic energy to the microwave fields and are collected by the collector.

O-type travelling type tubes are suitable for amplification. At present, klystron and TWT amplifiers can deliver a peak power output upto 30MW with a beam voltage on the order of 100KV at the frequency of 10GHz. The average power outputs are upto 700KW. The gain of these tubes is in the order of 30 to 70dB, & the efficiency is from 15 to 60%. The Bandwidth is from 1 to 8% for klystrons & 10 to 15% for TWTS.

## 2 Cavity Klystrons :-

A klystron is a vacuum tube that can be used either as a generator or as an amplifier of power at microwave frequencies. This was invented by Russel H. Varian at Stanford University in 1939 in association with his brother S.P. Varian.

A two-cavity klystron is a widely used microwave amplifier operated by the principles of velocity & current modulation.

## Construction and Operation:-

It consists of two cavities buncher cavity & catcher cavity & hence the name two cavity klystron amplifier. All electrons injected from the cathode arrive at the first cavity with uniform velocity. The RF signal is to be amplified is used for exciting the input buncher cavity there by developing an alternative voltage signal frequency across the gap A. The electrons passing through the positive half cycles of the gap voltage

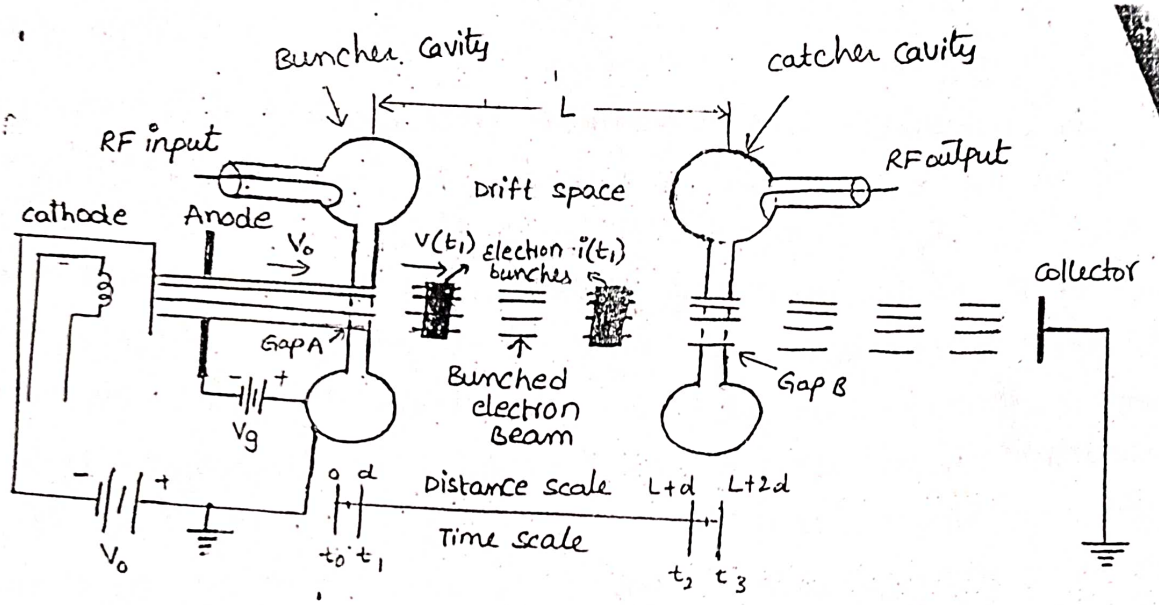



fig: 2-cavity klystron amplifier

undergo an increase in velocity; those passing through zero of the gap voltage will pass with unchanged velocity; those passing through the negative half cycles of the gap voltage undergo decrease in velocity. As a result of these actions, the electrons gradually bunch together as they travel down the drift space. "The variation in electron velocity in the drift space is known as velocity modulation." The density of the electrons in the gap B varies cyclically with time. The electron beam contains an ac component & is said to be current modulated. The maximum bunching occur approximately midway between the second cavity grid during its retarding phase. Thus the kinetic energy is transferred from the electrons to the field of the second cavity. The electrons then emerge from the second cavity with reduced velocity & finally terminate at the collector.

*Characteristics of 2-cavity klystron amplifier

1. Efficiency : about 40%.
2. power output : Average power upto 500kw
   pulsed power upto 30MW at 10GHz
3. power gain : about 30dB.

velocity modulation process :-

when electrons are first accelerated by the high dc voltage $V_0$ before entering the buncher grids, their velocity is uniform

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \quad m/s.$$

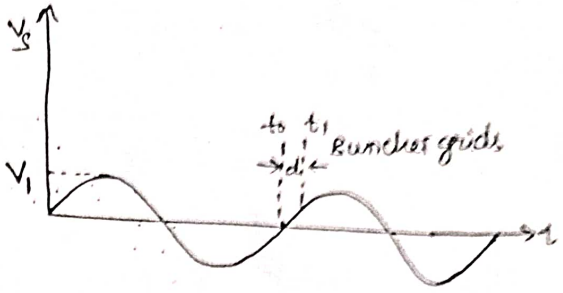
It is assumed that electrons leave the cathode with zero velocity. when a microwave signal is applied, to the input terminal the gap voltage between the buncher grids appears as

$$V_s = V_1 \sin(\omega t)$$

$V_1 << V_0$

The average transit time through the buncher gap distance d is

$$\tau = \frac{d}{v_0} = t_1 - t_0$$




The average gap transit angle can be expressed as

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0}$$

The average microwave voltage in the buncher gap can be found in the following way

$$\langle V_s \rangle = \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin(\omega t)\, dt$$

$\omega t_1 = \frac{\omega d}{v_0} + \omega t_0$

$$= \frac{-V_1}{\omega\tau} \left[\cos(\omega t_1) - \cos(\omega t_0)\right] = \frac{V_1}{\omega\tau}\left[\cos(\omega t_0) - \cos(\omega t_1\right.$$

$$= \frac{V_1}{\omega\tau}\left[\cos(\omega t_0) - \cos\left(\omega t_0 + \frac{\omega d}{v_0}\right)\right]$$

Let $\omega t_0 + \frac{\omega d}{2v_0} = \omega t_0 + \frac{\theta_g}{2} = A$

$\frac{\omega d}{2v_0} = \frac{\theta_g}{2} = B$

$A - B =$

$\cos(A-B) - \cos(A+B) = 2\sin A \sin B$

$$\langle V_s \rangle = \frac{V_1}{\omega\tau}\left[2\sin\left(\frac{2\omega t_0}{2} + \frac{\omega d}{2v_0}\right) \sin\left(\frac{\omega d}{2v_0}\right)\right]$$

$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

$$= \frac{2V_1}{\frac{\omega d}{v_0}}\left[\sin\left[\omega t_0 + \frac{\omega_d}{2v_0}\right] \sin\left[\frac{\omega d}{2v_0}\right]\right]$$

$$\langle V_s \rangle = \frac{V_1}{\theta_g/2} \sin(\theta_g/2) \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

$$= V_1 \beta_i \sin(\omega t_0 + \theta_g/2)$$

Let us defined $\beta_i = \frac{\sin(\theta_g/2)}{\theta_g/2}$ = Beam Coupling Coefficient of input cavity gap

As $\theta_g$ decreases the coupling between the (Coupling Coefficients) electron beam & buncher cavity increases.

The exit velocity from buncher gap is given by

$$v(t_1) = \sqrt{\frac{2e}{m}\left[V_0 + V_1 \beta_i \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$= \sqrt{\frac{2e}{m} V_0 \left[1 + \beta_i \frac{V_1}{V_0} \sin(\omega t_0 + \theta_g/2)\right]}$$

where $\beta_i \frac{V_1}{V_0}$ = depth of velocity modulation

$$\beta_i V_1 << V_0$$

$$v(t_1) = v_0 \sqrt{1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)}$$

$(1+x)^{1/2} \approx 1 + \frac{x}{2} + \frac{x^2}{3} + \ldots$ neglecting higher order terms

(or)

$$v(t_1) = v_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \quad \text{——(1)}$$

(or)

$$v(t_1) = v_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \quad \text{——(2)}$$

The above 2 equations are equations of velocity modulation.

Applegate diagram :- (Arpana Singh, Namitha Jain)

The below figure shows the applegate diagram of klystron amplifier. At point y on the input RF cycle, the alternating voltage is zero and going positive. At this instant, the electric field across gap A is zero and an electron which passes through gap A is unaffected by the RF signal. let this electron be called the reference electron $e_R$ which travels with an unchanged velocity $v_0 = \sqrt{\frac{2eV}{m}}$ where V is the anode to cathode voltage.

At point 'z' of the RF cycle an electron which leaves gap A later

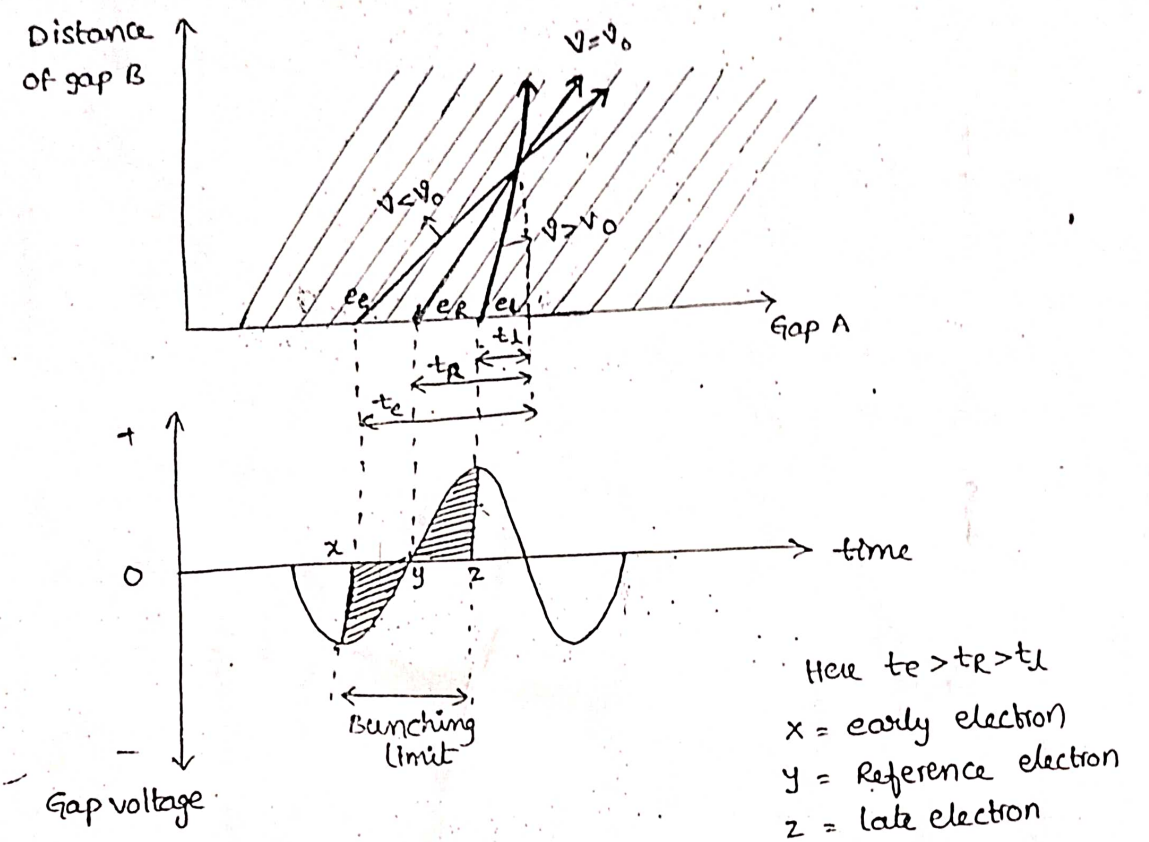



fig: Applegate diagram for 2-cavity klystron.

than reference electron '$e_R$', called the late electron. $e_l$ is subjected to maximum positive RF voltage and hence travels towards gap B with an increase velocity ($v > v_0$) and this electron tries to overtake the reference electron '$e_R$'.

Similarly an early electron $e_e$ that passes the gap 'A' slightly before the reference electron '$e_R$' is subjected to maximum negative field. Hence this early electron is deaccelerated & travels with a velocity ($v < v_0$). This electron $e_e$ falls back & reference electron $e_R$ catches up with the early electron.

Therefore the velocity of electron varies in accordance with RF input voltage, resulting in velocity modulation of the electron beam

Bunching Process:-

once the electrons leave the buncher cavity, they drift with a velocity given by equations ① & ② along in the field-free space between the two cavities. The effect of velocity modulation produces bunching of the electron beam - or current modulation. The electrons that pass the bunch

the bunching centre. Those electrons that pass the buncher cavity during the positive half cycles of the microwave input voltage $V_s$ travels faster than the electrons that passed the gap when $V_s = 0$. Those electrons that pass the buncher cavity during negative half cycles of the voltage $V_s$ travels slower than the electrons that passed the gap when $V_s = 0$. At a distance of $\Delta L$ along the beam from the buncher cavity, the beam electrons have drifted into dense clusters.

The trajectories of minimum, zero and maximum electron accelerations are shown in figure.

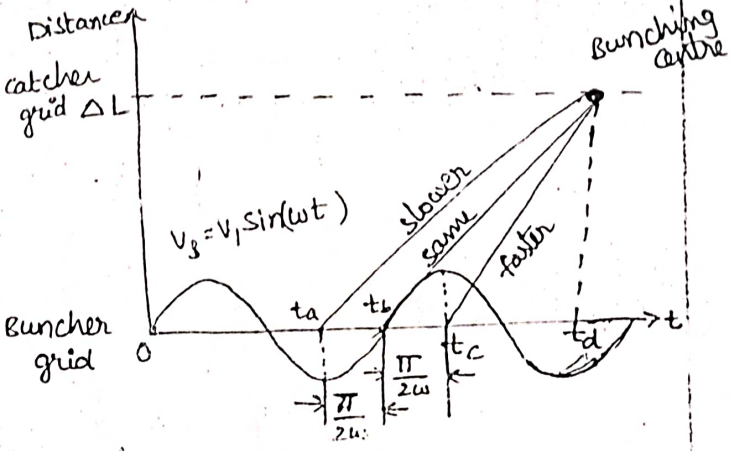



fig: Bunching distance

The distance from the buncher grid to the location of dense electron bunching for the electron at $t_b$ is

$$\Delta L = v_0 (t_d - t_b)$$

Similarly, the distances for the electrons at $t_a$ & $t_c$ are

$$\Delta L = v_{min}(t_d - t_a) = v_{min}\left(t_d - t_b + \frac{\pi}{2\omega}\right) \quad \text{---} ③$$

$$\Delta L = v_{max}(t_d - t_c) = v_{max}\left(t_d - t_b - \frac{\pi}{2\omega}\right) \quad \text{---} ④$$

from equations of velocity modulation (eqns ① & ②)

$$v_{min} = v_0\left[1 - \frac{\beta_i V_1}{2V_0}\right]$$

$$v_{max} = v_0\left[1 + \frac{\beta_i V_1}{V_0}\right]$$

Substituting in ③ & ④

$$\Delta L = v_0(t_d - t_b) + \left[\frac{v_0 \pi}{2\omega} - \frac{v_0 \beta_i V_1}{2V_0}(t_d - t_b) - v_0 \frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega}\right]$$

$$\Delta L = v_0(t_d - t_b) + \left[-\frac{v_0 \pi}{2\omega} + \frac{v_0 \beta_i V_1}{2V_0}(t_d - t_b) + \frac{v_0 \beta_i V_1}{2V_0}\frac{\pi}{2\omega}\right]$$

The necessary condition for those electrons at $t_a$, $t_b$ & $t_c$ to meet at the same distance $\Delta L$ is (61)

$$v_0\frac{\pi}{2\omega} - v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) - v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega} = 0.$$

and $$-v_0\frac{\pi}{2\omega} + v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) + v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega} = 0.$$

Consequently

$$t_d - t_b = \frac{v_0\frac{\pi}{2\omega} - v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega}}{v_0\frac{\beta_i V_1}{2V_0}}$$

$$= \frac{v_0\frac{\pi}{2\omega}\left[1 - \frac{\beta_i V_1}{2V_0}\right]}{v_0\frac{\beta_i V_1}{2V_0}}$$

$$= \frac{\pi V_0}{\omega\beta_i V_1} - \frac{\pi}{\omega}\cdot\frac{\frac{\beta_i V_1}{2V_0}}{\frac{\beta_i V_1}{V_0}}$$

$$= \frac{\pi V_0}{\omega\beta_i V_1} - \frac{\pi}{2\omega}$$

$$t_d - t_b \simeq \frac{\pi V_0}{\omega\beta_i V_1}$$

$$\therefore \Delta L = v_0(t_d - t_b) = v_0\frac{\pi V_0}{\omega\beta_i V_1}$$

This distance is not for a maximum degree of bunching

If the distance between the buncher & catcher cavities in order to achieve a maximum degree of bunching, be 'L'. Since the drift region is field free the transit time for an electron to travel a distance of 'L' is given by

$$T = t_2 - t_1 = \frac{L}{v(t_1)} = T_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

where $T_0 = \frac{L}{v_0}$ is the dc transit time.

$$T = \frac{L}{v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]} = \frac{T_0}{1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)}$$

$(1+x)^{-1}$ for $|x| \ll 1$ is $1 - x$

where $\theta_0 = \omega T_0 = \frac{\omega L}{v_0} = 2\pi N$ = dc transit angle between cavities

$$X = \omega T_0 \frac{\beta_i V_1}{2V_0} = \frac{\beta_i V_1}{2V_0} \theta_0$$

Here 'X' is the bunching parameter of a klystron.

At the buncher gap a charge $dQ_0$ passing through at a time interval $dt_0$ is given by

$$dQ_0 = I_0 dt_0$$

where $I_0$ = dc current. From the principle of conservation of charges this same amount of charge $dQ_0$ also passes the catcher at a later time interval $dt_2$. Hence

$$I_0 |dt_0| = i_2 |dt_2| \quad \text{---------} (5)$$

where the absolute value signs are necessary because a negative value of the time ratio would indicate a negative current. Current $i_2$ is the current at the catcher gap.

$$t_2 = t_0 + T + T_0\left[1 - \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \quad \text{---------} (6)$$

Alternatively $\omega t_2 = \omega t_0 + \theta_g + \theta_0 - X \sin\left(\omega t_0 + \theta_g/2\right)$

$$\omega t_2 - \left(\theta_0 + \frac{\theta_g}{2}\right) = \left(\omega t_0 + \frac{\theta_g}{2}\right) - X \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)$$

where $\omega t_0 + \theta_g/2$ is the buncher cavity departure angle

$\omega t_2 - (\theta_0 + \theta_g/2)$ is the catcher cavity arrival.

Differentiating (6) with respect to $t_0$

$$dt_2 = dt_0\left[1 - T_0 \frac{\beta_i V_1}{2V_0} \omega \cos\left(\omega t_0 + \theta_g/2\right)\right]$$

$$dt_2 = dt_0\left[1 - X \cos\left(\omega t_0 + \theta_g/2\right)\right]$$

The current arriving at the catcher cavity is then given by

$$i_2(t_0) = I_0 \frac{dt_0}{dt_2} = \frac{I_0}{1 - X\cos\left(\omega t_0 + \theta_g/2\right)}$$

interms of $t_2$ the current is

$$i_2(t_2) = \frac{I_0}{1 - x\cos(\omega t_2 - \theta_0 - \theta_{g/2})} \quad \text{---} (4)$$

$\because \omega t_2 = \omega t_0 + \frac{\omega t_g}{2} + \omega T_0$
$= \omega t_0 + \theta_g + \theta_0$ (6)

Here $i_2(t_2)$ is called the beam current

The beam current at the catcher cavity is a periodic waveform of period $\frac{2\pi}{\omega}$ about dc current. Therefore the current $i_2$ can be express in fourier series form as

$$i_2 = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega t_2) + b_n \sin(n\omega t_2)] \quad \text{---} (5)$$

where $n$ = an integer

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} i_2 \, d(\omega t_2)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos(n\omega t_2) \, d(\omega t_2)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \sin(n\omega t_2) \, d(\omega t_2)$$

Substituting $i_2 |dt_2| = I_0 |d t_0|$ from (5) in above (3) equations

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} I_0 \, (d\omega t_0) = I_0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} I_0 \cos\left[(n\omega t_0 + n\theta_g + n\theta_0) + n x \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] d(\omega t_0)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} I_0 \sin\left[(n\omega t_0 + n\theta_g + n\theta_0) + n x \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] d(\omega t_0)$$

The definite integrals in the expressions '$a_n$' & '$b_n$' is given by Bessels functions

$$a_n = 2 I_0 J_n(nx) \cos(n\theta_g + n\theta_0)$$

$$b_n = 2 I_0 J_n(nx) \sin(n\theta_g + n\theta_0)$$

where $J_n(nx)$ is the $n^{th}$ order Bessel's function of first kind.

since $i_2$ is an even function sinterm $b_n = 0$.

Substituting $a_n$ in (8)

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nX) \cos[n\omega(t_2 - T - T_0)]$$

The fundamental component of the beam current at the catcher cavity has a magnitude

$$\boxed{I_{f2} = 2I_0 J_1(X)} \quad [\because n=1]$$

The fundamental component $I_f$ has a maximum amplitude at

$$X = 1.841 = \frac{\beta_i V_1}{2V_0}\theta_0 = \frac{\beta_i V_1}{2V_0}\frac{\omega L}{v_0}$$

∴ The optimum distance 'L' at which the maximum fundamental component of current occurs is computed as

$$\boxed{L_{optimum} = \frac{3.682\, V_0 v_0}{\omega \beta_i V_1}}$$

It is interesting to note that the distance $\Delta L$ is approximately 15% less than '$L_{optimum}$'.

Expression for o/p power :-

The current induced by the electron beam in the walls of the catcher cavity

$$i_{2ind} = \beta_0 i_2 = \beta_0 2I_0 J_1(X) \cos[\omega(t_2 - T - T_0)]$$

$\beta_0$ = Beam coupling coefficient of the catcher gap.

If the buncher & catcher cavities are identical then $\beta_i = \beta_0$. The fundamental component of current induced in the catcher cavity

$$\boxed{I_{2ind} = \beta_0 I_2 = \beta_0 2I_0 J_1(X)}$$

The below figure shows an o/p equivalent circuit in which $R_{sh_0}$ represents the wall resistance of catcher cavity, $R_B$ the beam loading resistance, $R_L$ the external load resistance, and $R_{sh}$ the effective shunt resistance

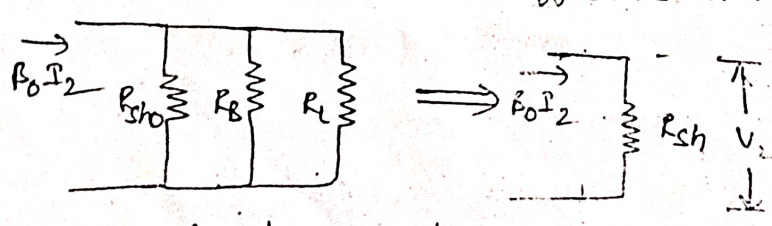



fig: o/p equivalent circuit.

$$P_{out} = \frac{(\beta_0 I_2)^2}{2} R_{sh} = \frac{\beta_0 I_2 V_2}{2} \Rightarrow \boxed{P_{out} = \frac{\beta_0 I_2 V_2}{2}}$$

where $R_{sh}$ is the total equivalent shunt resistance of catcher cavity, including the load, & $V_2$ is the fundamental component of the catcher gap voltage.

Expression for Efficiency :-

The electronic efficiency of the klystron amplifier is defined as the ratio of the output power to input power.

$$\text{Efficiency} = \frac{P_{out}}{P_{in}}$$

$$\boxed{P_{in} = V_0 I_0}$$

$$\boxed{\text{Efficiency} = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}}$$

If the coupling is perfect $\beta_0 = 1$.

Max. current $I_{2max} = 2I_0 \times 0.582$ $[\because \beta_0 = 1, J_1(x) = 0.582]$

Max. voltage $V_2 = V_0$

$$\therefore \text{Max. efficiency} = \frac{2I_0 \times 0.582 V_0}{2 I_0 V_0} = 0.582$$

$\therefore$ % Efficiency = 58.2%.

In practice, the efficiency is in the range 15 to 30%.

Two Cavity Klystron Oscillator :-

A klystron amplifier can be converted into an oscillator by feeding back a part of the catcher output into the buncher in proper phase so as to satisfy Barkhausen Criterion. The schematic is same as klystron amplifier except that a feedback loop needs to be added. The feedback must be adjusted to give correct polarity & amplitude which basically depends on the cavity tuning & the various dc voltages. If $\theta$ is the total phase shift in the resonator & the feedback cable, the criterion for oscillations is $\theta + \alpha + \frac{\pi}{2} = 2\pi n$ radians

& catcher voltages. If the two ~~oscillators~~ resonators oscillate in same phase, then $\theta = 0$, the maximum output power is obtained by substituting $\theta = 0$ in the above expression

$$\alpha + \pi/2 = 2\pi n$$

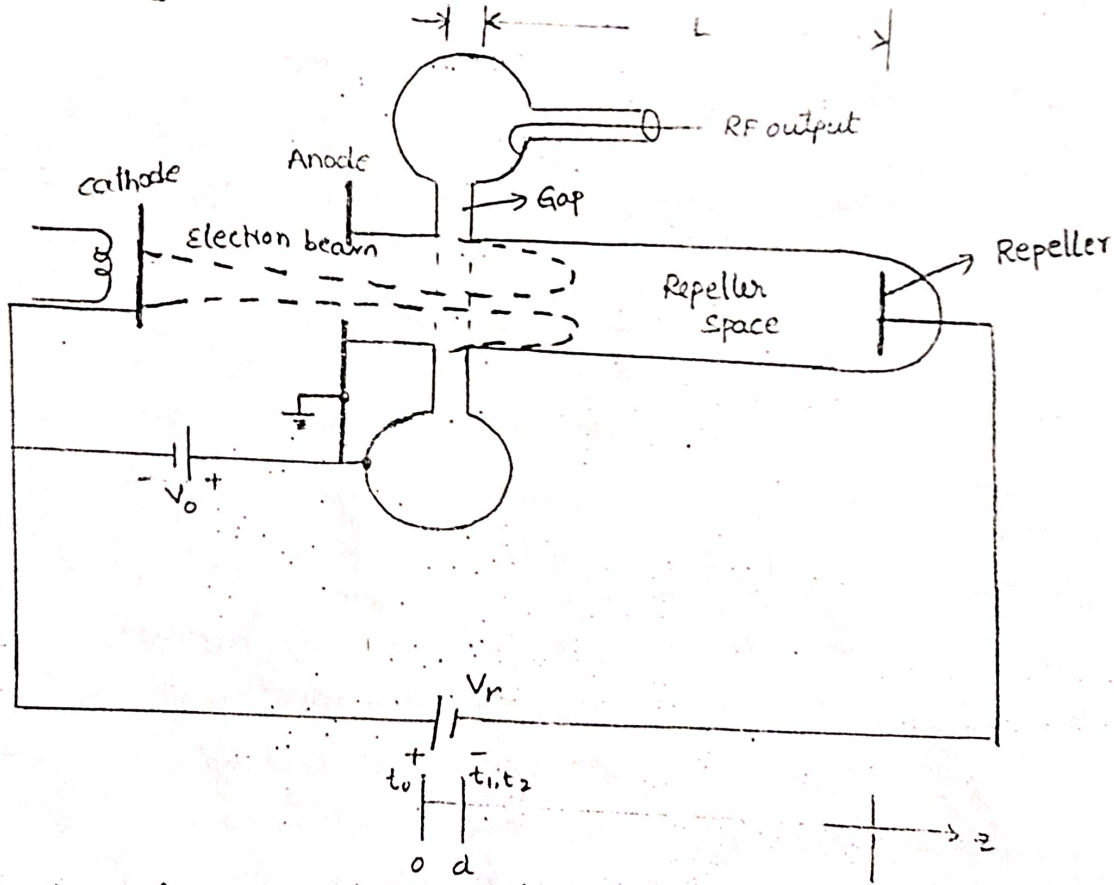
$$\boxed{\alpha = 2\pi n - \pi/2}$$

This is the condition for obtaining sustained oscillations.

Reflex klystrons:-

A two-cavity klystron oscillator is usually not constructed because, when oscillation frequency is varied, the resonant frequency of each cavity & the feedback path phase shift must be readjusted for a positive feedback.

The reflex klystron is a single cavity klystron that overcomes the disadvantage of two cavity klystron oscillator. It is a low power low efficiency microwave oscillator.




$t_2$ = time for same electron returned by retarding field $z = d$ & collected on walls of cavity

fig: Schematic diagram of Reflex klystron

$t_0$ = time for electron entering cavity gap at $z = 0$

$t_1$ = time for same electron having cavity gap at $z = d$

The electron beam injected from the cathode is first passed through the cavity. After passing through the gap in the cavity, electrons travel towards repeller, which is at high negative potential. The electrons will never reach the repeller because of negative field. hence the electrons turned around and oscillations are sustained. Because of these oscillations RF voltage is produced across the gap. ~~by the cavity~~ This will cause velocity modulation. oscillator output energy is taken from the cavity.

The following are the characteristics of Reflex klystron oscillator.

Output power : 10 - 500 mW at a frequency range of 1 to 25GHz.

Efficiency : 20 to 30%.

Applications :-

→ used in laboratory for microwave measurements

→ used as local oscillator in microwave receivers, in commercial, military and airborne doppler radars, in missiles.

Applegate diagram :-

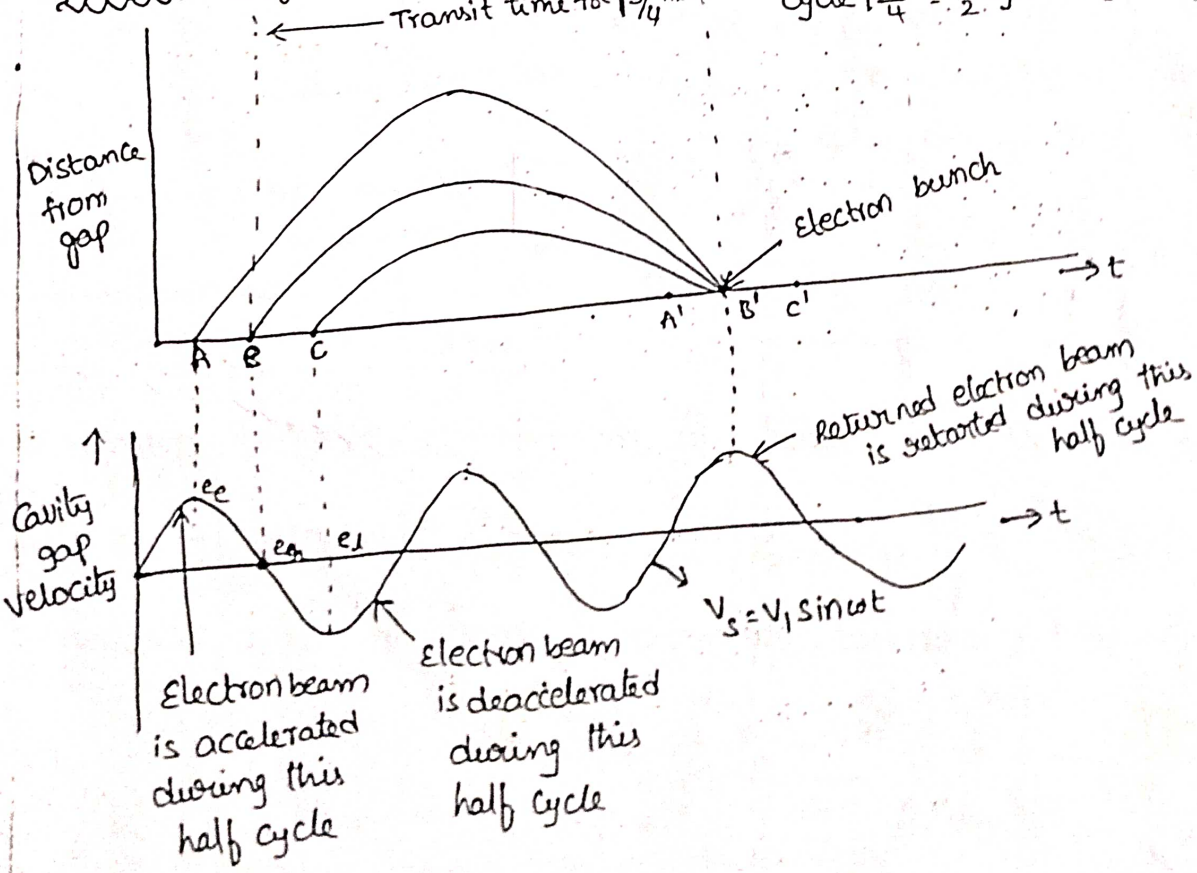



fig: Applegate diagram of reflex klystron

The early electron $e_e$ passes through the gap & is accelerated. The late electron $e_l$ passes through the gap & is deaccelerated. For sustained oscillations the transit time (time taken by the electrons to travel into the repeller space & back to the gap) must have an optimum value

$$T = n + \frac{3}{4} \text{ & } n - 1/4$$

where $n$ = integer.

Mathematical theory of bunching :-

velocity modulation

The analysis of a reflex klystron is similar to that of two cavity klystron. The electrons entering the cavity gap has the uniform velocity of

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0}$$

The velocity of electrons at the time of leaving the gap

$$v(t_1) = v_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin(\omega t_1 - \theta_g/2)\right]$$

The electrons are forced back to cavity at time $t_2$ by the retarding electric field E, which is given by

$$E = \frac{\text{Total voltage}}{\text{Drift space}}$$

$$E = \frac{V_r + V_0 + V_1 \sin(\omega t)}{L}$$

The force on the electron in the repeller region is given by

$$F = -eE = -e\frac{V_r + V_0}{L} = m\frac{d^2z}{dt^2} \quad |V_1 \sin(\omega t)| << V_r + V_0$$

here E is assumed to be a constant in the z direction

$$\frac{d^2z}{dt^2} = \frac{-e(V_r + V_0)}{mL}$$

Integrating the above eqn

$$\frac{dz}{dt} = \frac{-e(V_r + V_0)}{mL} \int_{t_1}^{t} dt = \frac{-e(V_r + V_0)}{mL}(t - t_1) + K_1$$

at $t=t_1 \Rightarrow$

$$\frac{dz}{dt} = v(t_1) = K_1 \text{ then } \frac{dz}{dt} = \frac{-e(V_r+V_0)}{mL}(t-t_1) + v_g(t_1).$$

$$\Rightarrow z = \frac{-e(V_r+V_0)}{mL}\int_{t_1}^{t}(t-t_1)dt + v_1(t)\int_{t_1}^{t}dt$$

$$z = \frac{-e(V_r+V_0)}{2mL}(t-t_1)^2 + v(t_1)(t-t_1) + K_2$$

at $t=t_1 \Rightarrow z=d=K_2$ Then

$$z = \frac{-e(V_r+V_0)}{2mL}(t-t_1)^2 + v(t_1)(t-t_1) + d.$$

The electron leaves the cavity at $z=d$ & time $t_1$ with a velocity $v(t_1)$ and returns the gap at $z=d$ & time $t_2$

then at $t=t_2$ $z=d$.

$$d = \frac{-e(V_r+V_0)}{2mL}(t_2-t_1)^2 + v(t_1)(t_2-t_1) + d$$

$$\frac{-e(V_r+V_0)}{2mL}(t_2-t_1)^2 + v(t_1)(t_2-t_1) = 0.$$

$$\Rightarrow [t_2-t_1]\left[\frac{-e(V_r+V_0)}{2mL}(t_2-t_1) + v(t_1)\right] = 0.$$

The roundtrip transit time in the repeller region is given by

$$T' = t_2-t_1 = \frac{2mL}{e(V_r+V_0)}v(t_1) = \frac{2mL}{e(V_r+V_0)}v_0\left[1+\frac{\beta_i V_1}{2V_0}\sin(\omega t_1 - \theta_g/2)\right]$$

$$t_2-t_1 = T' = T_0'\left[1+\frac{\beta_i V_1}{2V_0}\sin(\omega t_1 - \theta_g/2)\right]$$

where $T_0' = \frac{2mLv_0}{e(V_r+V_0)}$ $\longrightarrow$ Round trip dc transit time

$$\omega(t_2-t_1) = \omega T_0' + \omega T_0'\frac{\beta_i V_1}{2V_0}\sin(\omega t_1 - \theta_g/2)$$

$$= \theta_0' + X'\sin(\omega t_1 - \theta_g/2)$$

where $\theta_0' = \omega T_0'$ $\longrightarrow$ Round trip transit angle

$X' = \frac{\beta_i V_1}{2V_0}\theta_0'$ $\longrightarrow$ Bunching parameter of reflex klystron oscillator.

For maximum energy transfer, the round trip transit angle can be given by

$$\theta_0' = (n - 1/4)2\pi = 2\pi n - \frac{\pi}{2} \quad \text{---(1)} \quad n = \text{integer}$$

$n - 1/4$ = no. of modes.

we know that $v_0 = \sqrt{\frac{2eV_0}{m}}$ ---(2)

$\Rightarrow V_0 = \frac{v_0^2 m}{2e}$

Substituting eq(1) in eq(2)

$$\theta_0' = \frac{2mLv_0\omega}{e(V_r + V_0)}$$

$$v_0 = \frac{e(V_r+V_0)}{2mL\omega}\theta_0'$$

$$v_0^2 = \frac{e^2(V_r+V_0)^2}{4L^2m^2\omega^2}(\theta_0')^2$$

$$\therefore V_0 = \frac{m}{2e}\frac{e^2(V_r+V_0)^2}{4L^2m^2\omega^2}\cdot(\theta_0')^2$$

$$V_0 = \frac{m}{2e}\cdot\frac{e^2(V_r+V_0)^2}{4L^2m^2\omega^2}\left(2\pi n - \frac{\pi}{2}\right)^2$$

$$\therefore \frac{V_0}{(V_r+V_0)^2} = \frac{1}{8}\frac{e}{\omega^2L^2m}\left[2\pi n - \frac{\pi}{2}\right]^2$$

Power output of reflex klystron :-

The dc. power supplied by the beam voltage $V_0$ is

$$P_{dc} = V_0 I_0$$

The ac power delivered to the load

$$P_{ac} = \frac{\beta_i V_1 I_2}{2}$$

The fundamental component of current $I_2$ is given by

$$I_2 = 2I_0 J_1(X')$$

$$X' = \frac{\beta_i V_1}{2V_0}\theta_0' = \frac{\beta_i V_1}{2V_0}\left[2\pi n - \frac{\pi}{2}\right]$$

$$\frac{V_1}{V_0} = \frac{2X'}{\beta_i (2\pi n - \pi/2)}$$

$$V_1 = \frac{2X' V_0}{\beta_i (2\pi n - \pi/2)}$$

$$\therefore P_{ac} = \frac{V_1 I_2}{2} = \frac{2X' V_0}{2\beta_i (2\pi n - \pi/2)} \cdot 2 I_0 \beta_i J_1(X')$$

$$\therefore P_{out} = P_{ac} = \frac{2 V_0 I_0 X' J_1(X')}{2\pi n - \pi/2}$$

Efficiency of reflex klystron :-

$$\text{efficiency} = \frac{P_{ac}}{P_{dc}} = \frac{2 V_0 I_0 X' J_1(X')}{2\pi n - \pi/2} \Big/ V_0 I_0$$

$$\therefore \text{efficiency} = \frac{2X' J_1(X')}{2\pi n - \pi/2}$$

The factor $X' J_1(X')$ reaches maximum value of 1.25 at $X' = 2.408$ and $J_1(X') = 0.52$. In practice, the mode $n=2$ has the most power output.

If $n=2$ or $1\frac{3}{4}$ mode, the max. electronic efficiency becomes

$$\text{Efficiency}_{max} = \frac{2(2.408) J_1(2.408)}{2\pi(2) - \pi/2} = 22.7\%.$$

Electronic admittance:

The induced current can be written in phasor form as

$$I_2 = 2I_0 \beta_i J_1(x') e^{-j\theta_0'}$$

The voltage across the gap at time $t_2$ can be written in phasor form

$$V_2 = V_1 e^{-j\pi/2}$$

the ratio of $I_2$ & $V_2$ is defined as the electronic admittance of the reflex klystron, that is

$$Y_e = \frac{I_2}{V_2} = \frac{2I_0 \beta_i J_1(x') e^{-j\theta_0'}}{V_1 e^{-j\pi/2}}$$

putting $V_1 = \frac{2V_0 x'}{\beta_i \theta_0'}$

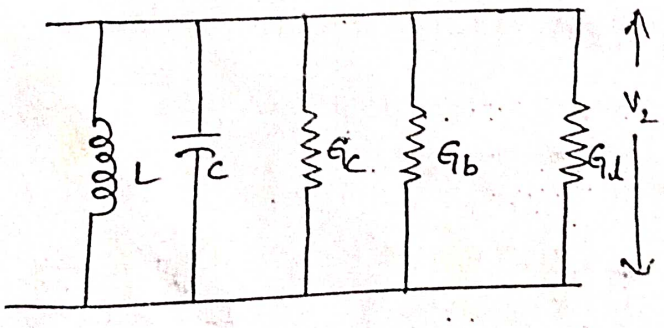
$$Y_e = \frac{2I_0 \beta_i J_1(x') e^{j(\pi/2 - \theta_0')}}{\frac{2V_0 x'}{\beta_i \theta_0'}}$$

$$Y_e = \frac{I_0}{V_0} \beta_i^2 \theta_0' \frac{2J_1(x')}{x'} e^{j(\pi/2 - \theta_0')} = G_e + jB_e$$

(Rectangular form)

This indicates that the electronic admittance is a function of the dc beam admittance, the dc transit angle & the second transit of the electron beam through the cavity gap. It is evident that the electronic admittance is non linear, since it is proportional to the factor $\frac{J_1(x')}{x'}$ & $x'$ is proportional to the signal voltage. when the signal voltages goes to zero, the factor $\frac{J_1(x')}{x'}$ approaches unity.

The equivalent circuit of a reflex klystron is given as




L, C = storage elements of cavity
$G_c$ = Copper losses of cavity
$G_b$ = Beam loading Conductance
$G_l$ = load conductance.

The necessary condition for oscillations is that magnitude of the negative real part of the electronic admittance must be greater than the total conductance of the cavity circuit.

$$|-G_e| \geq G$$

where $G = G_c + G_b + G_l = 1/R_{sh}$, $R_{sh}$ = effective shunt resistance.

The phase of the electronic admittance is $\pi/2$ when $\theta_0'$ is zero.

The plot of $Y_e$ is a spiral.

Any value of $\theta_0'$ for which the spiral lies in the area to the left of line $(-G-jB)$ will yield oscillation.

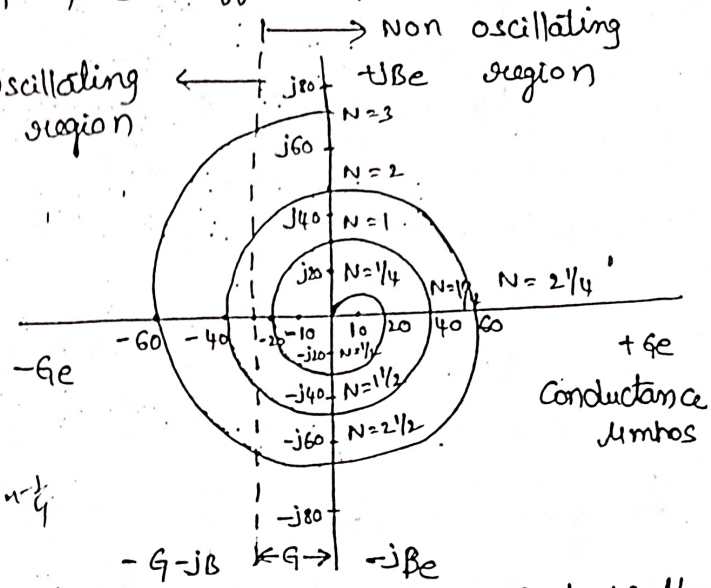



fig: electronic admittance spiral of reflex klystron

$$\theta_0' = (n - 1/4)2\pi = N2\pi$$

where $N = n - 1/4$ is the mode number as indicated in plot.

Relation between repeller voltage & frequency of operation of reflex klystron (Microwave Engineering – Manoj Mitra)

$$\frac{V_0}{(V_r + V_0)^2} = \frac{1}{8}\frac{1}{\omega^2 L^2}\frac{e}{m}\left[2\pi n - \pi/2\right]^2$$

$$(V_r + V_0)^2 = \frac{8\omega^2 L^2 m V_0}{\left(2\pi n - \frac{\pi}{2}\right)^2 \cdot e}$$

differentiating w.r.t $\omega$

$$2(V_r + V_0)\frac{dV_r}{d\omega} = \frac{8 \cdot 2\omega m L^2 V_0}{(2n\pi - \pi/2)^2 \cdot e} \Rightarrow \frac{dV_r}{d\omega} = \frac{8\omega m L^2 V_0}{(2n\pi - \pi/2)^2 \cdot e}\frac{1}{V_r + V_0}$$

$$\frac{dV_r}{d\omega} = \frac{8\omega m L^2 V_0}{(2n\pi - \pi/2)^2 \cdot e} \times \frac{(2n\pi - \pi/2)\cdot\sqrt{e}}{4\omega L\sqrt{8 m V_0}}$$

$$\frac{dV_r}{d\omega} = \sqrt{\frac{8 m L^2 V_0}{(2n\pi - \pi/2)^2 e}}$$

$$\frac{\omega}{df} = \frac{\omega_{n-}}{\left(2n\pi - \pi/2\right)\sqrt{\frac{8mV_0}{e}}} \qquad d\omega = 2\pi$$

The frequency depends on repeller voltage & dc anode voltage

Tuning :-

Tuning means the adjustment of the frequency of the resonance to get maximum output. Tuning is of 2 types

1. Electronic tuning 2. Mechanical tuning

1. Electronic tuning :-

Here the tuning can be achieved by varying repeller voltage. The Electronic Tunining Range (ETR) can be defined as the total change in frequency from the end of the mode to the other.

The Electronic tuning sensitivity (ETS) is measure of change in frequency with the change in repeller voltage & is usually measured in MHz/volt.

$$ETS = \frac{f_2 - f_1}{V_{r2} - V_{r1}} \quad MHz/volt.$$

where $f_1$ & $f_2$ are the frequencies in MHz at which the mode power falls to half of its value at the top.

2. Mechanical tuning :-

The frequency of oscillation of a klystron depends on its cavity dimensions. The dimensions of the cavity can be varied either by flexing a potion of the cavity wall or, by changing the space between cavity grids. In this type of mechanical tuning the frequency of the output power can be varied mechanically over a wide range:

Problems

① In a two cavity klystron the parameters are input power = 10mW, voltage gain = 20dB, $R_{sh}$ of input cavity 25kΩ, $R_{sh}$ of output cavity = 35kΩ, load resistance = 40kΩ. Find the input voltage, output voltage and power to the load.

$$P_{in} = \frac{V_{in}^2}{R_{sh\,in}}$$

$$V_{in} = \sqrt{P_{in}\, R_{sh\,in}}$$

$$= \sqrt{10m \times 25K}$$

$$V_{in} = 15.81\,V$$

$$20\log_{10}\left(\frac{V_o}{V_{in}}\right) = A_V = 20$$

$$\frac{V_o}{V_{in}} = 10$$

$$V_o = 10 \times 15.81 = 158.1\,V$$

$$P_{out} = \frac{V_o^2}{R_{sh\,out}} = \frac{158.1^2}{35K} = 0.714 \text{ Watts} = 714\,mW.$$

Given

$P_{in} = 10mV$

$A_V = 20dB$

$R_{sh\,in} = 25K\Omega$

$R_{sh\,out} = 35K\Omega$

$R_L = 40K\Omega$

$V_{in} = ?$

$V_o = ?$

$P_o = ?$

(2) A two cavity klystron amplifier has the following parameters $V_0 = 1200V$, $I_0 = 25mA$, $R_0 = 30K\Omega$, $f_0 = 10GHz$, $d = 1mm$, $L = 4cm$, $R_{sh} = 30K\Omega$. Calculate

(i) The input gap voltage for maximum output voltage

(ii) voltage gain in decibels

(iii) efficiency.

Given

$V_0 = 1200V$

$I_0 = 25mA$

$R_0 = 30K\Omega$

$f_0 = 10GHz$

$d = 1mm$

$L = 4cm$

$R_{sh} = 30K\Omega$

(i) For maximum output voltage $V_2$, $J_1(X)$ must be maximum. $J_1(X)$ is maximum 0.582 at $X = 1.841$

Gap voltage

$$\boxed{V_{1.max} = \frac{2XV_0}{\beta_i \theta_0}}$$

$X = 1.841$

$$\theta_0 = \omega T_0 = \frac{\omega L}{v_0} = \frac{2\pi f L}{v_0}$$

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} = 0.593 \times 10^6 \sqrt{1200} = 20.5 \times 10^6\ m$$

$$\theta_0 = \frac{2\pi \times 10 \times 10^3 \times 4 \times 10^{-2}}{20.5 \times 10^6} = 122.34 \text{ radians}$$

$$\beta_i = \frac{\sin \theta_g/2}{\theta_g/2}$$

$$\theta_g = \omega T = \frac{\omega d}{v_0} = 2\pi f \times \frac{d}{v_0} = \frac{2\pi \times 10 \times 10^9 \times 1 \times 10^{-3}}{20.5 \times 10^6} = 3.06 \text{ radians}$$

$$\beta_i = \frac{\sin(3.06/2)}{\frac{3.06}{2}} = \frac{2 \times 0.999}{3.06} = 0.653$$

$$\therefore V_{1max} = \frac{2 \times 1.841 \times 1200}{0.653 \times 122.34} = 55.303 \text{ V}$$

(ii) voltage gain

$$A_V = \frac{V_2}{V_{1max}}$$

$$V_2 = (\beta_0 I_2) R_{sh}$$

$$I_2 = 2 I_0 J_1(X) \qquad J_1(X) = 0.582$$

$$= 2 \times 25m \times 0.582 \qquad \beta_0 = \beta_i$$

$$I_2 = 29.1mA$$

$$V_2 = 0.653 \times 29.1m \times 30k\Omega = 570.069V$$

$$A_V = \frac{V_2}{V_{1(max)}} = \frac{570.069}{55.303} = 10.314$$

$$A_V(dB) = 20\log(10.314) = 20.269 \text{ dB}$$

(iii) Efficiency $\eta = \frac{P_{out}}{P_{in}} = \frac{\beta_i I_2 V_2}{2 I_0 V_0} \times 100\%$

$$= \frac{0.582 \times 29.1m \times 570.069}{2 \times 25m \times 1200} \times 100\%$$

$$= 0.1609 \times 100\% = 16.09\%$$

(11)

A reflex klystron operates with, $V_0 = 400V$, $R_{sh} = 20k\Omega$, $f = 9GHz$, $L = 10^{-3}m$, $n = 2$. Find the repeller voltage & electronic efficiency.

Sol:

The relation between operating voltage & repeller voltage is given by

$$\frac{V_0}{(V_r + V_0)^2} = \frac{1}{8} \frac{e}{\omega^2 L^2 m} \left[2\pi n - \frac{\pi}{2}\right]$$

Given

$V_0 = 400V$

$R_{sh} = 20k\Omega$

$f = 9GHz$

$L = 10^{-3}m$

$n = 2$

$e = 1.6 \times 10^{-19}$

$m = 9.1 \times 10^{-31}$

$\omega = 2\pi f$

$$\frac{400}{(V_r + 400)^2} = \frac{1}{8} \times \frac{1.6 \times 10^{-19}}{(2\pi \times 9 \times 10^9)^2} \frac{\left(2\pi \times 2 - \frac{\pi}{2}\right)}{(10^{-3})^2 \times 9.1 \times 10^{-31}}$$

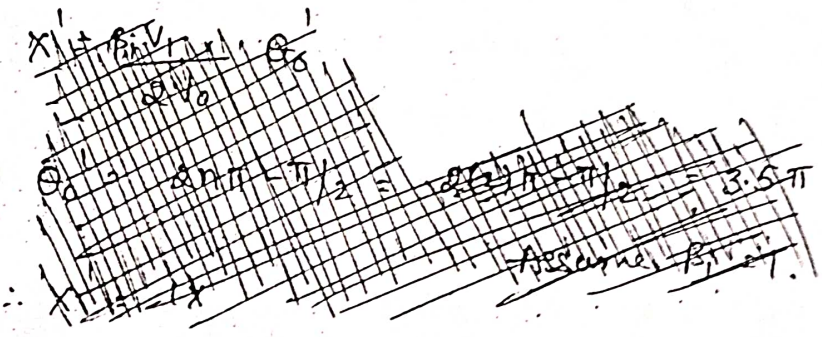
$$\frac{400}{(V_r + 400)^2} = 9.44 \times 10^{-6}$$

$$V_r + 400 = \sqrt{\frac{400}{9.44 \times 10^{-6}}} = 294V$$

$$V_r = 294V.$$

Electronic Efficiency

$$\eta = \frac{2X' J_1(X')}{2\pi n - \pi/2} \times 100\%.$$



$X' = 1.841$, $J_1(X') = 0.582$

$$\eta = \frac{2 \times 1.841 \times 0.582}{2(2)\pi - \pi/2} \times 100\% = 0.194 \times 100\%.$$

$$\eta = 19.4\%.$$

at a frequency of 10GHz with a retarding field of 500V. If the cavity is returned to 9GHz. what must be the new value of retarding field for oscillations in the same mode to take place?

Sol

Given

$V_0 = 300V$

$f_1 = 10GHz$

$f_2 = 9GHz$

$V_{r_1} = 500V$

$V_{r_2} = ?$

$$\frac{V_0}{(V_r+V_0)^2} = \frac{1}{8}\frac{e}{\omega^2 L^2 m}\left[2\pi n - \frac{\pi}{2}\right]^2$$

$$\frac{V_0}{(V_r+V_0)^2} \propto \frac{1}{\omega^2}$$

$$\frac{V_0}{(V_r+V_0)^2} \propto \frac{1}{f^2} \Rightarrow \frac{\frac{V_0}{(V_{r_1}+V_0)^2}}{\frac{V_0}{(V_{r_2}+V_0)^2}} = \frac{f_2^2}{f_1^2}$$

$$\frac{(V_{r_2}+V_0)^2}{(V_{r_1}+V_0)^2} = \frac{f_2^2}{f_1^2} \Rightarrow \frac{V_{r_2}+V_0}{V_{r_1}+V_0} = \frac{f_2}{f_1}$$

$$\frac{V_{r_2}+300}{500+300} = \frac{9\times10^9}{10\times10^9}$$

$$V_{r_2} = \frac{800\times9}{10} - 300$$

$$= 720 - 300$$

$$\therefore V_{r_2} = 420\ V$$

(5) A reflex klystron operates under the following conditions $V_0 = 600V$, $I_0 = 11.45mA$, $L = 1mm$, $R_{sh} = 15k\Omega$, $f_r = 9GHz$. The tube is oscillating at $f_r$ at the peak of $n = 1\frac{3}{4}$ mode. Assume $\beta_i = 1$.

(i) The microwave gap voltage

(ii) Repeller voltage for the mode $1\frac{3}{4}$.

Given

$V_0 = 600V$

$I_0 = 11.45mA$

$L = 1mm$

$R_{sh} = 15k\Omega$

$f_r = 9GHz$

$n = 1\frac{3}{4}(8V)$ $n = 2$.

Sol (i) Microwave gap voltage.

$$V_1 = \frac{2x' V_0}{\beta_i(2n\pi - \pi/2)}$$

$V_1 = \frac{}{2 \times 2\pi - \pi/2}$ 10.99

$V_1 = \beta_i I_2 R_{sh} = 2 I_0 J_1(x') \cdot R_{sh}$

$= 200 \cdot 26 V$

(ii) Repeller voltage $V_r$

$$\frac{V_0}{(V_r + V_0)^2} = \frac{1}{8} \frac{1}{\omega^2 L^2} \frac{e}{m} \left[2n\pi - \pi/2\right]^2 \qquad \omega = 2\pi f$$

$$\frac{600}{(V_r + 600)^2} = \frac{1}{8} \frac{1.6 \times 10^{-19} \times [2 \times 2\pi - \pi/2]^2}{(2\pi \times 9 \times 10^9)^2 \times (1 \times 10^{-3})^2 \times 9.1 \times 10^{-31}}$$

$$= \frac{1.934 \times 10^{-17}}{2.32 \times 10^{-14}} = 8.30 \times 10^{-4}$$

$$(V_r + 600)^2 = \frac{600}{8.30 \times 10^{-4}} = 72.222 \times 10^3$$

$V_r + 600 = 268.74$

$V_r = 250 V$

(6) A reflex klystron has following parameters. $V_0 = 800V$, $L = 1.5mm$, $R_{sh} = 15$

a $f = 9GHz$

Calculate

(i) The repeller voltage for which the tube can oscillate in 1 3/4 mod

(ii) The direct current necessary to give a microwave gap voltage. 200V

(iii) Electron efficiency.

Given $V_0 = 800V$

$L = 1.5mm$

$R_{sh} = 15k\Omega$

$f = 9GHz$

$n = 1 3/4$ mode (or) $n = 2$.

(ii) Repeller voltage

$$\left(\frac{V_0}{(V_0+V_R)^2}\right)^2 = \frac{1}{8}\frac{1}{\omega^2 L^2 m}\left(2n\pi - \pi/2\right)^2$$

$$= \frac{1}{8} \times \frac{1.6\times10^{-19} \times (4\pi - \pi/2)^2}{(2\pi\times 9\times10^9)^2 \times (1.5\times10^{-3})^2 \times 9.1\times10^{-31}}$$

$$\frac{800}{(800+V_R)^2} = 0.37\times10^{-3}$$

$$(V_R+800)^2 = \frac{800}{0.37\times10^{-3}}$$

$$V_R + 800 = 1470$$

$$V_R = 670V.$$

(iii) D.C. current $I_0$.

Given $V_1 = 200V$

$V_1 = \beta_0 I_2 R_{sh}$ Assume $\beta_0 = 1$.

$I_2 = 2I_0 J_1(x')$

$V_1 = 2I_0 J_1(x') R_{sh}$

$$I_0 = \frac{V_1}{2J_1(x')R_{sh}} = \frac{200}{2\times0.582\times15k} = 11.45mA$$

(iii) Efficiency $\eta = \dfrac{2x' J_1(x')}{2n\pi - \pi/2} = \dfrac{2\times1.841\times0.582}{2\times2\pi - \pi/2}\times100$

$x' = 1.841$, $J_1(x') = 0.582$

$\eta = 0.194 \times 100$

$\eta = 19.4\%$

(7) A reflex klystron operates at the peak of n=1 & 3/4 mode. The D.C power input is 40mw and ratio of $V_1$ to $V_0$ is 0.278

(i) Determine the efficiency of the reflex klystron oscillator

(ii) find the total power output in mw

(iii) If 20% of the power delivered by the electron beam is dissipated in the cavity walls find the power delivered to the load.

(i) $\eta = \frac{2x' J_1(x')}{2n\pi - \pi/2} = \frac{2 \times 1.582 \times 0.583}{2\pi - \pi/2}$

$= 0.455 \times 100$

$\eta = 45.5\%$

(ii) $P_{out} = P_{ac} = \eta \times P_{in}$

$= 0.455 \times 40m$

$P_{out} = 18.2\ mW$

(iii) 20% of the total power is dissipated in the cavity walls. Therefore 80% of power is delivered to load

ie, $P_L = 80\%\ (P_{out})$

$= \frac{80}{100} \times 18.2m$

$\therefore P_L = 14.56\ mA$

Given

$P_{dc} = P_{in} = 40mW$

$\frac{V_1}{V_0} = 0.278$

$n = 1.$

$x' = \frac{\beta_i V_0}{2V_1} \theta_0 = \frac{\beta_i V_0}{2V_1} (2n\pi)$

$= 1.528$

$J_1(x') = 0.582$