UNIT - II

CIRCULAR WAVEGUIDES

A circular waveguide is a tubular, circular conductor. "It is a hollow metallic tube with circular cross section".

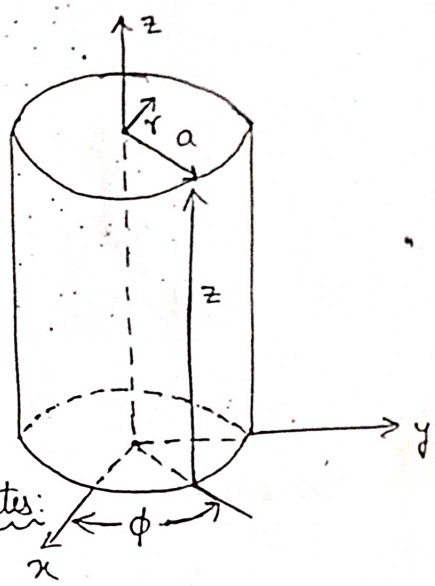


fig: Cylindrical Co-ordinates

Advantages:-

→ Manufacturing circular waveguide is easy compared to that of rectangular waveguide.

→ Circular waveguides are easily connected to each other.

→ It converts a linearly polarized wave into a circular polarised wave.

Solution of wave equation in cylindrical Co-ordinates:

The Helmholtz equation is given by

$$\nabla^2 \psi = \gamma^2 \psi$$

The Helmholtz equation in cylindrical co-ordinates is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2} = \gamma^2 \psi \quad \text{---} (1)$$

Using the method of seperation of variables, the solution is assumed in the form

$$\psi = R(r)\,\phi(\phi)\,Z(z) \quad \text{---} (2)$$

where $R(r)$ = Function of r co-ordinates only

$\phi(\phi)$ = " " ϕ " "

$Z(z)$ = " " z " "

Substituting (2) in (1) & dividing the resultant by (2) we get

$$\frac{1}{rR}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{r^2\phi}\frac{d^2\phi}{d\phi^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = \gamma^2 \quad \text{---} (3)$$

Since the sum of the three terms on the left hand side is a constant and each term is independently variable, it follows that each term must be equal to a constant.

Let

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = \gamma_g^2 \implies \frac{d^2Z}{dz^2} = Z\gamma_g^2 \quad \text{---} (4)$$

the solution to. This will be in the form of

$$Z = Ae^{-\gamma_g z} + Be^{\gamma_g z} \quad \text{——— (a)}$$

Substituting (4) in (3) & multiplying with r^2

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{\phi}\frac{d^2\phi}{d\phi^2} - (\gamma^2 - \gamma_g^2)r^2 = 0 \quad \text{—— (5)}$$

let

$$\frac{1}{\phi}\frac{d^2\phi}{d\phi^2} = -n^2 \implies \frac{d^2\phi}{d\phi^2} = -n^2\phi \quad \text{—— (6)}$$

The solution to this will be in the form of

$$\phi = A_n \sin(n\phi) + B_n \cos(n\phi) \quad \text{——— (b)}$$

Substituting (6) in (5) & multiplying with R

$$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) - n^2R + k_c^2r^2R = 0$$

$$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \left[(k_cr)^2 - n^2\right]R = 0 \quad \text{—— (7)}$$

$\gamma_g^2 = \gamma^2 + k_c^2$

$\therefore \gamma_g^2 - \gamma^2 = k_c^2$

$\gamma^2 - \gamma_g^2 = -k_c^2$

This is Bessel's equation of order n & the solution will be in the form

$$R = C_n J_n(k_c r) + D_n N_n(k_c r) \quad \text{——— (c)}$$

where $J_n(k_c r)$ = n^{th} order Bessel function of the first kind, representing a standing wave of $\cos(k_c r)$ for $r<a$.

$N_n(k_c r)$ = n^{th} order Bessel function of the second kind, representing a standing wave of $\sin(k_c r)$ for $r>a$.

$\therefore$ The total solution is given by

$$\psi = \left[C_n J_n(k_c r) + D_n N_n(k_c r)\right]\left[A_n \sin(n\phi) + B_n \cos(n\phi)\right] e^{-j\beta_g z}$$

At $r=0 \implies k_c r = 0 \implies D_n N_n(k_c r) = 0$

$$A_n \sin(n\phi) + B_n \cos(n\phi) = \sqrt{A_n^2 + B_n^2}\left[\frac{A_n}{\sqrt{A_n^2+B_n^2}}\sin(n\phi) + \frac{B_n}{\sqrt{A_n^2+B_n^2}}\cos(n\phi)\right]$$

let $\frac{A_n}{\sqrt{A_n^2+B_n^2}} = \sin\theta$

$$\cos\theta = \frac{B_n}{\sqrt{A_n^2+B_n^2}}, \quad \tan\theta = \frac{A_n}{B_n} \Rightarrow \theta = \tan^{-1}\left(\frac{A_n}{B_n}\right).$$

$$A_n\cos(n\phi) + B_n\sin(n\phi) = \sqrt{A_n^2+B_n^2}\left[\sin\theta\sin(n\phi) + \cos\theta\cos(n\phi)\right]$$

$$= \sqrt{A_n^2+B_n^2}\cos(n\phi-\theta)$$

$$= \sqrt{A_n^2+B_n^2}\cos\left[n\phi - \tan^{-1}\left(\frac{A_n}{B_n}\right)\right]$$

$$= F_n\cos(n\phi).$$

$$\therefore \psi = C_n J_n(k_c r)\, F_n\cos(n\phi)\, e^{-j\beta_g z}$$

finally the solution is reduced as

$$\psi = \psi_0 J_n(k_c r)\cos(n\phi)\, e^{-j\beta_g z}$$

TE mode in circular waveguides :-

Here the wave is propagating in the z-direction.

In TE mode $E_z = 0$.

The magnetic field component H_z must exist in the guide inorder to have electromagnetic energy transmission. Helmholtz equation for H_z, in a circular guide is given by

$$\nabla^2 H_z = \gamma^2 H_z$$

The solution to this is given by

$$H_z = H_{0z} J_n(k_c r)\cos(n\phi)\, e^{-j\beta_g z}$$

For a lossless dielectric, Maxwell's equation in frequency domain are given

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \times H = j\omega\epsilon E$$

In cylindrical Co-ordinates

$$\nabla \times E = \frac{1}{r}\begin{vmatrix} i & rj & k \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ E_r & rE_\phi & E_z \end{vmatrix} = -j\omega\mu\left[H_r i + H_\phi j + H_z k\right]$$

$$\frac{1}{r}\frac{\partial E_z}{\partial \phi} - \frac{\partial E\phi}{\partial z} = -j\omega\mu H_r$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu H\phi$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE\phi) - \frac{1}{r}\frac{\partial E_r}{\partial \phi} = -j\omega\mu H_z$$

$$\nabla \times H = \frac{1}{r}\begin{vmatrix} i & rj & k \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_r & rH_\phi & H_z \end{vmatrix} = j\omega\epsilon\left[E_r i + E_\phi j + E_z k\right]$$

Comparing similar terms

$$\frac{1}{r}\frac{\partial H_z}{\partial \phi} - \frac{\partial H\phi}{\partial z} = j\omega\epsilon E_r$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\omega\epsilon E\phi$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH\phi) - \frac{1}{r}\frac{\partial H_r}{\partial \phi} = j\omega\epsilon E_z$$

Substituting $\frac{\partial}{\partial z} = -j\beta_g$ & $E_z = 0$.

$$\beta_g E_\phi = -\omega\mu H_r \quad \text{..... (a)}$$

$$\beta_g E_r = \omega\mu H\phi \quad \text{..... (b)}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) - \frac{1}{r}\frac{\partial E_r}{\partial \phi} = -j\omega\mu H_z \quad \text{..... (c)}$$

$$\frac{1}{r}\frac{\partial H_z}{\partial \phi} + j\beta_g H\phi = j\omega\epsilon E_r \quad \text{..... (d)}$$

$$-j\beta_g H_r - \frac{\partial H_z}{\partial r} = j\omega\epsilon E\phi \quad \text{..... (e)}$$

$$\frac{\partial}{\partial r}(rH\phi) = \frac{\partial H_r}{\partial \phi} \quad \text{..... (f)}$$

Solving the above six equations for E_r, $E\phi$, H_r, $H\phi$ in terms of H_z will give the TE mode Equations.

from (a) & (c)

$$E\phi = \frac{-\omega\mu H_r}{\beta_g}, \quad H_r = \frac{-\beta_g E\phi}{\omega\mu}$$

Substituting in (e)

$$-j\beta_g H_r - \frac{\partial H_z}{\partial r} = j\omega\epsilon\left[\frac{-\omega\mu H_r}{\beta_g}\right]$$

$$H_r\left[-j\beta_g + \frac{j\omega^2\mu\epsilon}{\beta_g}\right] = \frac{\partial H_3}{\partial r}$$

$$H_r\left[\frac{j(\omega^2\mu\epsilon - \beta_g^2)}{\beta_g}\right] = \frac{\partial H_3}{\partial r}$$

$\gamma_g^2 - \gamma^2 = +k_c^2$

$\omega^2\mu\epsilon - \beta_g^2 = +k_c^2$

$$\boxed{H_r = \frac{-j\beta_g}{k_c^2}\frac{\partial H_3}{\partial r}} \quad \text{—— (1)}$$

$$-j\beta_g\left[\frac{-\beta_g E_\phi}{\omega\mu}\right] - \frac{\partial H_3}{\partial r} = j\omega\epsilon E_\phi$$

$$E_\phi\left[\frac{j\beta_g^2}{\omega\mu} - j\omega\epsilon\right] = \frac{\partial H_3}{\partial r}$$

$$E_\phi\left[\frac{j(\beta_g^2 - \omega^2\mu\epsilon)}{\omega\mu}\right] = \frac{\partial H_3}{\partial r}$$

$$\boxed{E_\phi = \frac{j\omega\mu}{k_c^2}\frac{\partial H_3}{\partial r}} \quad \text{—— (2)}$$

from (b) & (d)

$$E_r = \frac{\omega\mu H_\phi}{\beta_g}, \quad H_\phi = \frac{E_r\beta_g}{\omega\mu}$$

Substituting in (d)

$$\frac{1}{r}\frac{\partial H_z}{\partial\phi} + j\beta_g H_\phi = j\omega\epsilon\left[\frac{\omega\mu H_\phi}{\beta_g}\right]$$

$$H_\phi\left[\frac{j\omega^2\mu\epsilon}{\beta_g} - j\beta_g\right] = \frac{1}{r}\frac{\partial H_3}{\partial\phi}$$

$$H_\phi\left[\frac{j(\omega^2\mu\epsilon - \beta_g^2)}{\beta_g}\right] = \frac{1}{r}\frac{\partial H_3}{\partial\phi}$$

$$\boxed{H_\phi = \frac{-j\beta_g}{rk_c^2}\frac{\partial H_3}{\partial\phi}} \quad \text{—— (3)}$$

$$\frac{1}{r}\frac{\partial H_3}{\partial\phi} + j\beta_g\left[\frac{E_r\beta_g}{\omega\mu}\right] = j\omega\epsilon E_r$$

$$E_r\left[j\omega\epsilon - \frac{j\beta_g^2}{\omega\mu}\right] = \frac{1}{r}\frac{\partial H_3}{\partial\phi}$$

$$E_r\left[\frac{j(\omega^2\mu\epsilon - \beta_g^2)}{\omega\mu}\right] = \frac{1}{r}\frac{\partial H_3}{\partial\phi}$$

$$H_r = \frac{-j\omega}{r k_c^2} \frac{\partial H_z}{\partial \phi} \quad \text{---(11)}$$

$$\frac{\partial H_z}{\partial r} = H_{0z} J_n'(k_c r) \cos(n\phi) e^{-j\beta_g z}$$

$$\frac{\partial H_z}{\partial \phi} = -n H_{0z} J_n(k_c r) \sin(n\phi) e^{-j\beta_g z}$$

The boundary conditions are at $r = a$, the tangential component of electric field $E_\phi = 0 \Rightarrow \frac{\partial H_z}{\partial r} = 0$.

$$\Rightarrow \frac{\partial H_z}{\partial r} = H_{0z} J_n'(k_c a) \cos(n\phi) e^{-j\beta_g z} = 0.$$

$$\therefore J_n'(k_c a) = 0.$$

where J_n' indicates the derivative of J_n.

Since J_n are oscillatory functions, $J_n'(k_c a)$ are also oscillatory functions.

The permissible values of k_c can be written in TE_{np}

$$k_c = \frac{x_{np}'}{a}$$

∴ The TE_{np} modes in circular waveguides are given as

$$E_r = E_{0r} J_n\left(\frac{x_{np}' r}{a}\right) \sin(n\phi) e^{-j\beta_g z}$$

$$E_\phi = E_{0\phi} J_n'\left(\frac{x_{np}' r}{a}\right) \cos(n\phi) e^{-j\beta_g z}$$

$$E_z = 0$$

$$H_r = H_{0r} J_n'\left(\frac{x_{np}' r}{a}\right) \cos(n\phi) e^{-j\beta_g z}$$

$$H_\phi = H_{0\phi} J_n\left(\frac{x_{np}' r}{a}\right) \sin(n\phi) e^{-j\beta_g z}$$

$$H_z = H_{0z} J_n\left(\frac{x_{np}' r}{a}\right) \cos(n\phi) e^{-j\beta_g z}$$

where $n = 0, 1, 2, 3 \ldots$

$p = 1, 2, 3 \ldots$

where n = no. of full cycles of field variations in one revolution through 2π radians of ϕ.

P = No. of zeros of E_ϕ ie, $J_n'\left(\frac{x_{np}' r}{a}\right)$ along the radial of a guide but the zero on the x-axis is excluded if it exists.

Propagation constant

$$\beta_g = \sqrt{\omega^2\mu\epsilon - k_c^2}$$

$$= \sqrt{\omega^2\mu\epsilon - \left(\frac{x_{np}'}{a}\right)^2}$$

$\gamma_g^2 - \gamma^2 = k_c^2$

$\omega^2\mu\epsilon - \beta_g^2 = k_c^2$

$\beta_g = \sqrt{\omega^2\mu\epsilon -$

Cutoff wave number

$$k_c = \frac{x_{np}'}{a} = \omega_c\sqrt{\mu\epsilon}$$

cut off frequency

$$f_c = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{x_{np}'}{2\pi a\sqrt{\mu\epsilon}}$$

phase velocity

$$v_p = \frac{c}{\sqrt{1-(f_c/f)^2}}$$

Group velocity

$$v_g = c\sqrt{1-(f_c/f)^2}$$

Guide wavelength

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$$

wave impedance

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$x_{np}' = 3.832$ for TE

$x_{np}' = 1.841$ for TE_1

$x_{np}' = 3.054$ for TE_2

$x_{np}' = 4.201$ for TE_3

$x_{np}' = 5.317$ for TE_4

$x_{np}' = 6.416$ for TE_{51}

Here the wave is propagating in the z-direction.

In TM mode $H_z = 0$.

The electric field component E_z must exist in the guide inorder to have electromagnetic energy transmission. Helmholtz equation to E_z in a circular waveguide

$$\nabla^2 E_z = \gamma^2 E_z$$

The solution to this is given by

$$E_z = E_{0z} J_n(k_c r) \cos(n\phi)\, e^{-j\beta_g z}$$

For a lossless dielectric maxwell's equation in frequency domain are given by

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \times H = j\omega\epsilon E$$

In cylindrical co-ordinates

$$\nabla \times E = \frac{1}{r}\begin{vmatrix} i & rj & k \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ E_r & rE_\phi & E_z \end{vmatrix} = -j\omega\mu\left[H_r i + H_\phi j + H_z k\right]$$

Comparing similar term

$$\frac{1}{r}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega\mu H_r$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu H_\phi$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) - \frac{1}{r}\frac{\partial E_r}{\partial \phi} = -j\omega\mu H_z$$

$$\nabla \times H = \frac{1}{r}\begin{vmatrix} i & rj & k \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_r & H_\phi & H_z \end{vmatrix} = j\omega\epsilon\left[E_r i + E_\phi j + E_z k\right]$$

Comparing Similar terms

$$\frac{1}{r}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega\epsilon E_r$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\omega\epsilon E_\phi$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_\phi) - \frac{1}{r}\frac{\partial H_r}{\partial \phi} = j\omega\epsilon E_z$$

Substituting $\frac{\partial}{\partial z} = -j\beta_g$ & $H_z = 0$.

$$\frac{1}{r}\frac{\partial E_z}{\partial \phi} + j\beta_g E_\phi = -j\omega\mu H_r \quad \text{(a)}$$

$$-\frac{\partial E_z}{\partial r} - j\beta_g E_r = -j\omega\mu H_\phi \quad \text{(b)}$$

$$\frac{\partial}{\partial r}(rE_\phi) = \frac{\partial E_r}{\partial \phi} \quad \text{(c)}$$

$$\beta_g H_\phi = \omega\epsilon E_r \quad \text{(d)}$$

$$\beta_g H_r = -\omega\epsilon E_\phi \quad \text{(e)}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_\phi) - \frac{1}{r}\frac{\partial H_r}{\partial \phi} = j\omega\epsilon E_z \quad \text{(f)}$$

Solving the above six equations for E_r, E_ϕ, H_r, H_ϕ interms of E_z will give the TM mode equations.

from (a) & (e)

$$H_r = \frac{-\omega\epsilon}{\beta_g} E_\phi \quad , \quad E_\phi = \frac{-\beta_g}{\omega\epsilon} H_r$$

Substituting in (a)

$$\frac{1}{r}\frac{\partial E_z}{\partial \phi} + j\beta_g\left[\frac{-\beta_g}{\omega\epsilon} H_r\right] = -j\omega\mu H_r$$

$$H_r\left[-j\omega\mu + \frac{j\beta_g^2}{\omega\epsilon}\right] = \frac{1}{r}\frac{\partial E_z}{\partial \phi}$$

$$H_r\left[\frac{j(\beta_g^2 - \omega^2\mu\epsilon)}{\omega\epsilon}\right] = \frac{1}{r}\frac{\partial E_z}{\partial \phi}$$

$$\boxed{H_r = \frac{j\omega\epsilon}{rk_c^2}\frac{\partial E_z}{\partial \phi}} \quad \text{---} \quad (1)$$

$$\frac{1}{r}\frac{\partial E_z}{\partial \phi} + j\beta_g E_\phi = -j\omega\mu\left[\frac{-\omega\epsilon}{\beta_g} E_\phi\right]$$

$$\left[\frac{}{\beta g} \quad j\gamma g\right] = \frac{1}{r}\frac{\partial E_z}{\partial \phi}$$

$$E_\phi\left[\frac{j(\omega^2\mu\epsilon - \beta g^2)}{\beta g}\right] = \frac{1}{r}\frac{\partial E_z}{\partial \phi}$$

$$\boxed{E_\phi = \frac{-j\beta g}{r k_c^2}\frac{\partial E_z}{\partial \phi}} \quad \text{——} (2)$$

from (5) & (2)

$$H\phi = \frac{\omega\epsilon}{\beta g} E_r, \quad E_r = H\phi \frac{\beta g}{\omega\epsilon}$$

substituting in (5)

$$-\frac{\partial E_z}{\partial r} - j\beta g E_r = -j\omega\mu\left[\frac{\omega\epsilon}{\beta g} E_r\right]$$

$$E_r\left[\frac{+j\omega^2\mu\epsilon}{\beta g} - j\beta g\right] = \frac{\partial E_z}{\partial r}$$

$$E_r\left[\frac{j(\omega^2\mu\epsilon - \beta g^2)}{\beta g}\right] = \frac{\partial E_z}{\partial r}$$

$$\boxed{E_r = \frac{-j\beta g}{k_c^2}\frac{\partial E_z}{\partial r}} \quad \text{——} (3)$$

$$\frac{-\partial E_z}{\partial r} - j\beta g\left[H\phi \frac{\beta g}{\omega\epsilon}\right] = -j\omega\mu H\phi$$

$$H\phi\left[+j\omega\mu - \frac{j\beta g^2}{\omega\epsilon}\right] = \frac{\partial E_z}{\partial r}$$

$$H\phi\left[\frac{j(\omega^2\mu\epsilon - \beta g^2)}{\omega\epsilon}\right] = \frac{\partial E_z}{\partial r}$$

$$\boxed{H\phi = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial r}} \quad \text{——} (4)$$

$\frac{\partial E_z}{\partial r} = E_{0z} J_n(k_c r) \cos(n\phi) e^{-j\beta_g z}$

$\frac{\partial E_z}{\partial \phi} = -n E_{0z} J_n(k_c r) \sin(n\phi) e^{-j\beta_g z}$

The boundary conditions are, at $r=a$, the tangential component of electric field $E_\phi = 0 \Rightarrow \frac{\partial E_z}{\partial \phi} = 0$.

$\Rightarrow \frac{\partial E_z}{\partial \phi} = -n E_{0z} J_n(k_c a) \sin(n\phi) e^{-j\beta_g z} = 0.$

$\therefore J_n(k_c a) = 0.$

The permissible values of k_c can be written in TM_{np}

$$k_c = \frac{x_{np}}{a}$$

$\therefore$ The TM_{np} modes in circular waveguide are given as

$$E_r = E_{0r} J_n'\left(\frac{x_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g z}$$

$$E_\phi = E_{0\phi} J_n\left(\frac{x_{np} r}{a}\right) \sin(n\phi) e^{-j\beta_g z}$$

$$E_z = E_{0z} J_n\left(\frac{x_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g z}$$

$$H_r = H_{0r} J_n\left(\frac{x_{np} r}{a}\right) \sin(n\phi) e^{-j\beta_g z}$$

$$H_\phi = H_{0\phi} J_n'\left(\frac{x_{np} r}{a}\right) \cos(n\phi) e^{-j\beta_g z}$$

$$H_z = 0.$$

where $n = 0, 1, 2 \ldots$

$p = 1, 2, 3 \ldots$

Propagation Constant $\beta_g = \sqrt{\omega^2 \mu\epsilon - k_c^2} = \sqrt{\omega^2 \mu\epsilon - \left(\frac{x_{np}}{a}\right)^2}$

Cutoff wave number $k_c = \frac{x_{np}}{a} = \omega_c \sqrt{\mu\epsilon}$.

cut off frequency $f_c = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{x_{np}}{2\pi a \sqrt{\mu\epsilon}}$

phase velocity $V_p = \frac{c}{\sqrt{1-(fc/f)^2}}$

Group velocity $V_g = c\sqrt{1-(fc/f)^2}$

Guide wavelength $\lambda_g = \frac{\lambda}{\sqrt{1-(fc/f)^2}}$

wave impedance $Z_{TM} = \eta\sqrt{1-(fc/f)^2}$

$X_{np} = 2.405$ for TM_{01}

$X_{np} = 3.832$ for TM_{11}

$X_{np} = 5.136$ for TM_{21}

$X_{np} = 6.380$ for TM_{31}

$X_{np} = 7.588$ for TM_{41}

$X_{np} = 8.771$ for TM_{51}

Dominant mode :- In circular waveguides the dominant mode is the TE_{11}. This has the smallest value of the product

$$k_c a = 1.841$$

TEM modes in Circular waveguides :-

TEM mode is characterized by $E_z = H_z = 0$.

The Maxwell's equation in frequency domain are given by

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \times H = j\omega\epsilon E$$

$$\frac{1}{r}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega\mu H_r$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu H_\phi$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) - \frac{1}{r}\frac{\partial E_r}{\partial \phi} = -j\omega\mu H_z$$

$$\frac{1}{r}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega\epsilon E_r$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\omega\epsilon E_\phi$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH_\phi) - \frac{1}{r}\frac{\partial H_r}{\partial \phi} = j\omega\epsilon E_z$$

Substituting $E_z = H_z = 0$, $\frac{\partial}{\partial z} = -j\beta_g$

$$-\beta_g E_\phi = +\omega\mu H_r \quad \text{..... (a)}$$

$$\beta_g E_r = \omega\mu H_\phi \quad \text{..... (b)}$$

$$\frac{\partial}{\partial r}(rE_\phi) = \frac{\partial E_r}{\partial \phi} \quad \text{..... (c)}$$

$$\beta_g H_\phi = \omega \epsilon E_r \quad \text{---------- (d)}$$

$$\beta_g H_r = -\omega \epsilon E_\phi \quad \text{---------- (e)}$$

$$\frac{\partial}{\partial r}(rH_\phi) = \frac{\partial H_r}{\partial \phi} \quad \text{---------- (f)}$$

from (a) & (e)

$$\frac{H_r}{E_\phi} = \frac{-\beta_g}{\omega \mu} \quad , \quad \frac{H_r}{E_\phi} = \frac{-\omega \epsilon}{\beta_g}$$

$$\therefore \frac{-\beta_g}{\omega \mu} = \frac{-\omega \epsilon}{\beta_g}$$

Propagation constant $\beta_g = \omega \sqrt{\mu \epsilon}$ ——— (1)

which is the phase constant of the wave in transmission line. Comparing this phase constant with the phase constant in the waveguide i.e,

$$\beta_g = \sqrt{\omega^2 \mu \epsilon - k_c^2} \quad \text{——— (2)}$$

from (1) & (2) $k_c = 0$.

phase velocity $v_p = \frac{\omega}{\beta_g} = \frac{1}{\sqrt{\mu \epsilon}}$

which is the velocity of the light in an unbounded dielectric

wave impedance $\eta(TEM) = \frac{E_r}{H_\phi} = \frac{\beta_g}{\omega \epsilon} = \frac{\omega \sqrt{\mu \epsilon}}{\omega \epsilon} = \sqrt{\frac{\mu}{\epsilon}}$

which is the impedance of a lossless transmission line in a dielectric

Ampere's law states that the line integral of H about any closed path is exactly equal to the current enclosed by that path. This is

$$\oint H \cdot dl = I = I_0 e^{-j\beta_g z} = 2\pi r H_\phi .$$

where I is the complex current that must be supported by the centre conductor of a co-axial line. This clearly demonstrates that the TEM mode can only exist in the two conductor system — not in the hollow waveguide because the centre conductor does not exist.

∴ The properties of TEM modes in a lossless medium are

→ Its cutoff frequency is zero
→ Its transmission line is a two conductor system
→ Its waveimpedance is the impedance in unbounded dielectric
→ Its propagation constant is the constant in an unbounded dielectric
→ Its phase velocity is the velocity of light in an unbounded dielectric.

Power Transmission in Circular waveguides

The power transmitted through circular waveguides can be calculated by means of the Complex poynting theorem.

For a lossless dielectric, the time-average power transmitted through a circular guide can be given by

$$P_{tr} = \int P \cdot ds$$
$$= \int \frac{1}{2}(E \times H) \cdot da$$
$$= \frac{1}{2z_g} \int |E|^2 da$$
$$= z_g \int |H|^2 da$$

$$P_{tr} = \frac{1}{2z_g} \int_0^{2\pi} \int_0^a \left[|E_\phi|^2 + |E_r|^2 \right] r \, dr \, d\phi$$

$$P_{tr} = \frac{z_g}{2} \int_0^{2\pi} \int_0^a \left[|H_\phi|^2 + |H_r|^2 \right] r \, dr \, d\phi$$

$$|E|^2 = |E_r|^2 + |E_\phi|^2$$
$$|H|^2 = |H_r|^2 + |H_\phi|^2$$

where $z_g = \frac{E_r}{H_\phi} = \frac{-E_\phi}{H_r}$

a = Radius of circular guide.

For TE_{np} mode $\quad z_g = \eta \sqrt{1-(f_c/f)^2}$

$$P_{tr} = \frac{\sqrt{1-(f_c/f)^2}}{2\eta} \int_0^{2\pi} \int_0^a \left[|E_\phi|^2 + |E_r|^2 \right] r \, dr \, d\phi$$

for TM_{np} mode $\quad z_g = \eta \sqrt{1-(f_c/f)^2}$

$$P_{tr} = \frac{1}{2\eta\sqrt{1-(f_c/f)^2}} \int_0^{2\pi} \int_0^a \left[|E_\phi|^2 + |E_r|^2 \right] r \, dr \, d\phi$$

for TEM mode $\quad z_g = \eta$

$$P_{tr} = \frac{1}{2\eta} \int_0^{2\pi} \int_0^a \left[|E_\phi|^2 + |E_r|^2 \right] r \, dr \, d\phi$$

line is assumed to be

$$I_z = I_0 e^{-j\beta_g z}$$

The magnetic intensity introduced by the current around the centre conductor is given by Ampere's law as

$$H\phi = \frac{I_0}{2\pi r} e^{-j\beta_g z}$$

The potential rise from the outer conductor to the centre conductor is given by

$$V_r = -\int_b^a E_r dr = -\int_b^a \eta H\phi dr = \frac{I_0 \eta}{2\pi} \ln\left(\frac{b}{a}\right) e^{-j\beta_g z}$$

The characteristic impedance of a co-axial line is

$$Z_0 = \frac{V}{I} = \frac{\eta}{2\pi} \ln(b/a) \quad \text{---} \quad (1)$$

∴ The power transmitted by TEM modes in a co-axial line can be expressed

$$P_{tr} = \frac{1}{2\eta} \int_0^{2\pi} \int_a^b |\eta H\phi|^2 r\, dr\, d\phi = \frac{\eta I_0^2}{4\pi} \ln(b/a) \quad \text{---} \quad (2)$$

from (1) & (2)

$$\therefore P_{tr} = \frac{1}{2} V_0 I_0$$

This shows that the power transmission derived from the poynting theory is same as from the circuit theory for an ordinary transmission line.

Power losses in Circular waveguides :-

The power loss for the TE & TM modes in Circular waveguides are sameas that of rectangular waveguides.

Power loss for TEM mode in co-axial lines can be computed from transmission line theory by means of

$$P_L = 2\alpha P_{tr}$$

where P_L = power loss per unit length
P_{tr} = transmitted power
α = Attenuation Constant.

wireless medium, the attenuation constant is given by

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right).$$

Micro Strip lines :-

A stripline is a balanced line in which there are two large ground planes and flat conducting strip is placed in between these two ground planes. The space between the two ground planes is filled with a homogeneous dielectric.

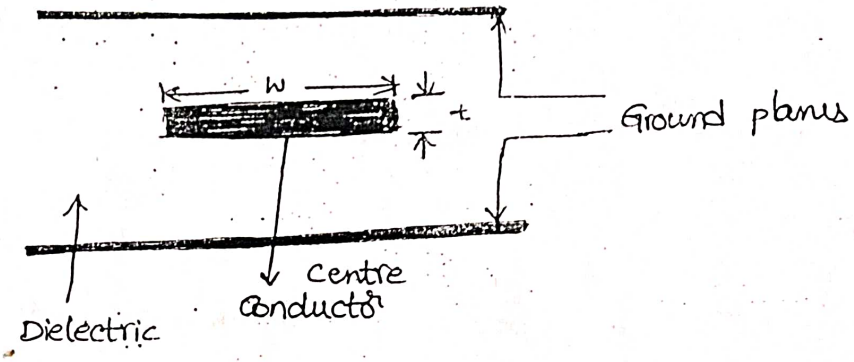


fig: stripline transmission lines.

The mode of propagation in a strip line is TEM. Ground planes will be at zero potential with field lines concentrated near the central conductor as shown in below figure.

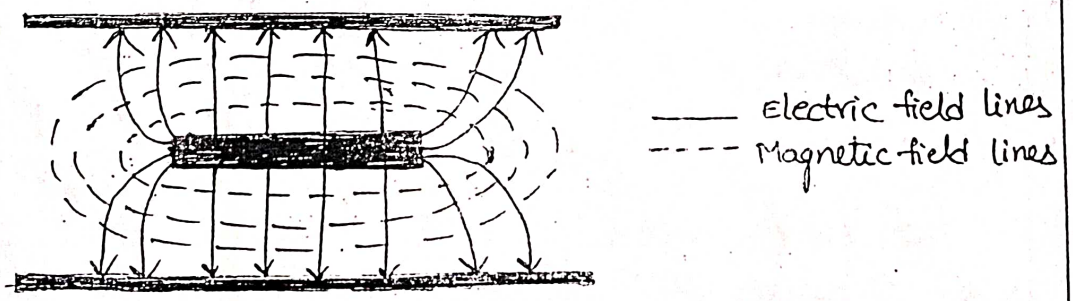


fig: TEM mode of stripline

There are certain advantages of microstriplines over striplines, co-axial lines and waveguides.

A microstrip line is an unsymmetrical stripline that is nothing but a parallel plate transmission line having dielectric substrate, the one face of which is metallised ground and the other (top) face has a thin conducting strip of certain width 'w' and thickness 't'. The top ground plane is not present in microstrip as compared to

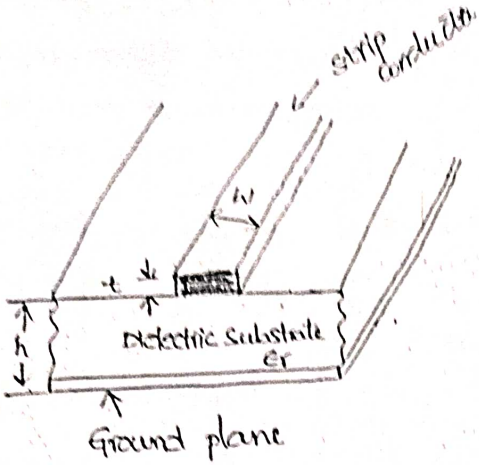


fig: schematic diagram of striplines

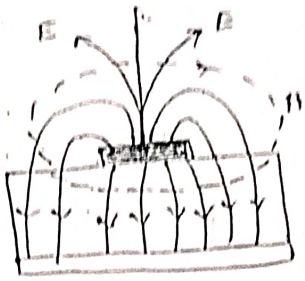


fig: Field lines.

Advantages of microstriplines:-

① Complete conductor pattern may be deposited and processed on a single dielectric substrate which is supported by a single metal ground plane. Thus fabrication costs would be substantially lower than stripline, co-axial & waveguide circuits.

② Due to the planar nature of the microstrip structure both packed and unpacked semiconductor chips can be conviniently attached to the microstrip element.

③ Also there is an easy access to the top surface making it easy to mount passive & active discrete devices and also for making minor adjustments after the circuit has been fabricated. This also allows access for probing and measurement purpose.

Limitations of microstriplines:-

① Due to openness of the microstrip structure, they have higher radiation losses & interference due to near by conductors. These can be reduced by choosing thin substrates with high dielectric constants

② Because of the proximity of the air-dielectric interface with the microstrip conductor at the interface, a discontinuity in the electric and magnetic fields is generated. This results in a microstrip configuration that becomes a mixed dielectric transmission structure with unpure TEM modes propagating. This makes the analysis complicated.

Characteristic Impedance of Microstrip lines :-

The characteristic impedance of a microstrip line is a function of the stripline width (w), thickness (t) and the distance between the line and the ground plane (h), and the homogeneous dielectric constant of the broad material.

The characteristic impedance of microstripline can be derived from the well known equation and making some changes. This method is called Comparative, or indirect method.

The well known equation of the characteristic impedance of a wire-over-ground transmission line is given by

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln \frac{4h}{d} \quad \text{for } h \gg d \quad \text{---} ①$$

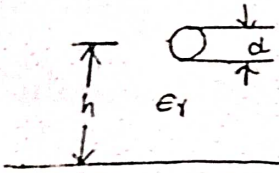


fig: a wire-over-ground

where ϵ_r = dielectric constant of the ambient medium
h = the height from the center of the wire to the ground plane.
d = diameter of the wire.

If the effective or equivalent values of the relative dielectric constant ϵ_r of the ambient medium and the diameter d of the wire can be determined for the microstripline, the characteristic impedance of the microstripline can be calculated.

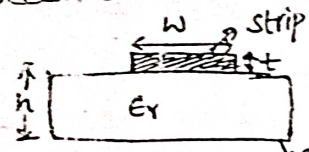


fig: stripline

Effective dielectric Constant :-

For a homogeneous dielectric medium, the propagation-delay time per unit length is

$$T_d = \sqrt{\mu\epsilon} = \sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}$$

$\mu_0 = 4\pi \times 10^{-7}$ H/m

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

In transmission lines used for interconnections, the relative permiability $\mu_r = 1$.

$$T_d = 3.3\sqrt{\epsilon_r} \text{ ns/m} \quad \text{---} ②$$

DiGiacomo & his coworkers discovered an empirical equation for the effective relative dielectric constant of a microstripline by measuring the propagation delay time and the relative dielectric constant of several board materials, such as fiberglass-epoxy and nylon phenolic.

The equation is expressed as

$$\epsilon_{re} = 0.475\epsilon_r + 0.67 \quad \text{---} ③$$

ϵ_r = Relative dielectric constant of the board material

of microstripline

Transformation of a rectangular conductor into an equivalent circular conductor :-

The cross-section of a microstripline is rectangular, so the rectangular conductor must be transformed into an equivalent circular conductor. Springfield dicovered an equation for the transformation.

$$d = 0.67w\left(0.8 + \frac{t}{w}\right) \quad \text{---- (4) (5)}$$

where d = diameter of the wire overground
w = width of the microstripline
t = Thickness of the microstripline.

here the limitation of the ratio of thickness to width is between 0.1 & 0.8.

Substituting (3) & (4) in (1)

$$z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8w + t}\right] \quad \text{for } (h < 0.8w)$$

The above equation is the characteristic impedance for a narrow microstripline.

The characteristic impedance for a wide microstripline was derived by Assadourian & is given by

$$z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\epsilon}} = \frac{377}{\sqrt{\epsilon_r}}\frac{h}{w} \quad \text{for } (w \gg h).$$

Losses in Microstriplines :-

Attenuation is a characteristic of the microstripline. The attenuation constant of the dominant microstrip mode depends on geometric factors, electrical properties of the substrate, conductors and on the frequency. For a non-magnetic dielectric substrate, two types of losses occur.

(1) Dielectric loss in the substrate (α_d)
(2) ohmic skin loss in the strip conductor & the ground plane (α_c).

Dielectric Losses (α_d) :-

When the conductivity of a dielectric cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase. In that case the dielectric attenuation constant

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} \text{ nplm} \quad \sigma = \text{conductivity of dielectric substrate board (s}$$

The dielectric constant can be expressed interms of dielectric loss tange

$$\tan\delta = \sigma/\omega\epsilon$$

$$\alpha_d = \frac{\omega}{2}\sqrt{\mu\epsilon}\,\tan\delta \quad \text{Np/cm}$$

$$\boxed{\alpha_d = 27.3\left(\frac{q\epsilon_r}{\epsilon_{re}}\right)\frac{\tan\delta}{\lambda_g} \quad \text{dB}/\lambda_g}$$

where $\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{re}}} = \frac{c}{f\sqrt{\epsilon_{re}}}$

$$q = \frac{\epsilon_{re}-1}{\epsilon_{re}+1} \qquad \therefore \text{Np} = 8.686\ \text{dB}.$$

Ohmic losses :-

The microstrip conductor contributes a major part of the ohmic loss because of the non-uniformity of current density in the strip conductor & ground conductor as shown in below figure.

The conducting attenuation constant of a wide microstripline is given by

$$\alpha_c = \frac{8.686\,R_s}{Z_0 w} \text{ dB/cm for } \frac{w}{v} > 1$$

here $R_s = \sqrt{\frac{\pi f\mu}{\sigma}}$

= surface skin resistance Ω/square

$= \frac{1}{\delta\sigma}$ Ω/square.

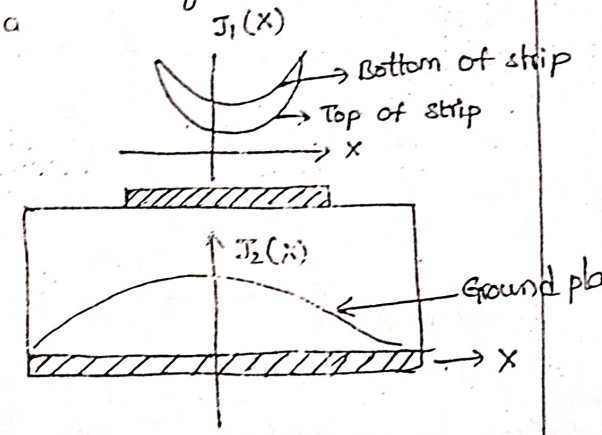


fig: Current density on microstrip conductors.

$\delta = \sqrt{\frac{1}{\pi f\mu\sigma}}$ = skin depth (cm).

∴ Total attenuation constant $\alpha = \alpha_d + \alpha_c$.

Radiation losses :-

The radiation loss depends on the substrate's thickness and dielectric constant, as well as its geometry.

The ratio of radiated power to the total dissipated power for an open circuited microstripline is

$$\frac{P_{rad}}{P_t} = \frac{R_r}{Z_0}$$

where R_r = Radiation resistance of an open-circuited microstrip

$$= 20\pi^2 \left[\frac{h}{\lambda_0}\right]^2 F(\epsilon_{re})$$

$F(\epsilon_{re})$ = Radiation factor.

$$= \frac{\epsilon_{re}+1}{\epsilon_{re}} - \frac{\epsilon_{re}-1}{2\epsilon_{re}\sqrt{\epsilon_{re}}} \ln \frac{\sqrt{\epsilon_{re}}+1}{\sqrt{\epsilon_{re}}-1}$$

Quality-factor of Microstriplines :-

Many microwave integrated circuits require very high quality resonant circuits. The quality-factor of a microstrip line is very high, but is limited by the radiation losses of the substrate and with low dielectric constant.

The ohmic attenuation constant $\alpha_c = \frac{8.686\, R_s}{Z_0 w}$ dB/cm.

The characteristic impedance of wide microstripline

$$Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\epsilon}} = \frac{377}{\sqrt{\epsilon_r}} \frac{h}{w} \quad \Omega.$$

The quality-factor Q_c related to conductor attenuation constant

$$Q_c = \frac{27.3}{\alpha_c}$$

$$= 39.5 \left(\frac{h}{R_s}\right) f_{GHz}$$

where h in cm & $R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = 2\pi \sqrt{\frac{f_{GHz}}{\sigma}}$ Ω/square

$$Q_c = 0.63 h \sqrt{\sigma f_{GHz}}$$

for a copper strip $\sigma = 5.8 \times 10^7$ ℧/m.

$$Q_{cu} = 4780 h \sqrt{f_{GHz}}$$

Similarly, the quality factor Q_d related to the dielectric attenuation constant

$$Q_d = \frac{27.3}{\alpha_d} \quad \therefore \quad \alpha_d = 27.3 \left(\frac{q\epsilon_r}{\epsilon_{re}}\right) \frac{\tan\theta}{\lambda_g} \text{ dB/d}$$

$$Q_d = \frac{\lambda_0}{\sqrt{\epsilon_{re}}\tan\theta} \approx \frac{1}{\tan\theta}$$

Cavity resonators

Cavity resonators are metallic enclosers that confine the electromagnetic energy within it. Infact a cavity resonator is a piece of waveguide closed off at both ends with metallic plates. The magnetic & electric energy of the wave constitutes the inductance, capacitance & resistance.

Cavity resonators are employed as tuned circuits at microwave frequencies. If waveguide is terminated by conducting end walls, then reflections of standing waves will take place, if a source is located between the walls. The standing waves are in the form of oscillations if the both walls are seperated by a distance of $n\lambda_g/2$, where n is an integer. In this case there will be resonance in the hallow space (cavity) of the waveguide and hence it is called cavity resonator.

If first wall is placed in a waveguide, there will be standing waves. Now placing the second wall such that, it does not disturb the pattern due to first wall, oscillations are achieved. so this behaves like an LC-tuned circuit. This oscillations may be sustained if energy is constantly supplied. Cavity resonators can resonate at only one particular frequency just as a parallel resonant circuit.

Rectangular Cavity Resonator :-

The wave equation in the rectangular resonator should satisfy the boundary conditions. $z=0, z=d$.

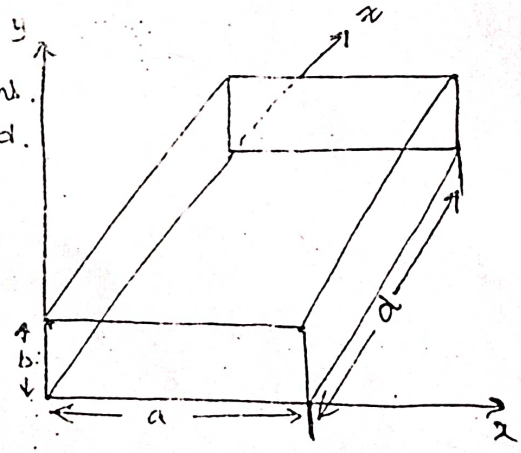


fig: Co-ordinates of rectangular cavity

for TE_{mnp} mode : $E_z = 0$

$$H_z = H_{0z} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \sin\left(\frac{p\pi z}{d}\right)$$

where

m = Number of halfwave periodicity in the x-direction. 0, 1, 2, 3...

n = 0, 1, 2, 3 .... Number of halfwave periodicity in the y-direction

p = 1, 2, 3, 4 - - - - - - Number of halfwave periodicity in the z-direction

for TM_{mnp} mode : $H_z = 0$

$$E_z = E_{0z} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \cos\left(\frac{p\pi}{d}z\right)$$

where m = 1, 2, 3 .... n = 1, 2, 3 .... , p = 0, 1, 2, 3 ....

The seperation equation for both TE & TM modes is given by

$$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

For a lossless dielectric $k^2 = \omega^2 \mu\epsilon$

∴ The resonant frequency is expressed by

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2} \qquad \omega = 2\pi f$$

For $a > b < d$, the dominant mode is the TE_{101} mode.

Circular cavity Resonator :-

A Circular cavity resonator is a circular waveguide with two ends enclosed by a metal wall. The wave function in the circular resonator should satisfy maxwell's equations, subject to the same boundary conditions.

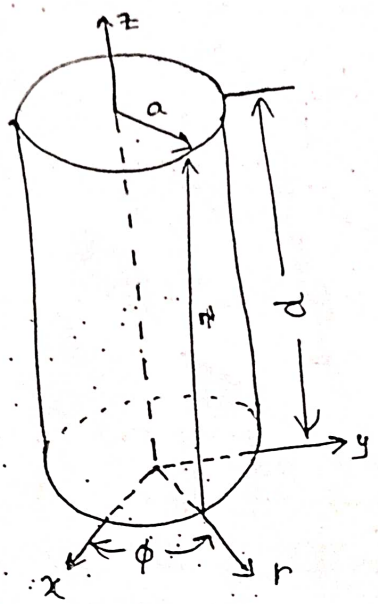


fig: Co-ordinates of a Circular resonator.

For TE_{npq} mode : $E_z = 0$

$$H_z = H_{0z} J_n\left(\frac{x_{np}' r}{a}\right) \cos(n\phi) \sin\left(\frac{q\pi}{d} z\right)$$

where

$n = 0, 1, 2, 3 \ldots$ Number of periodicity in the ϕ-direction

$P = 1, 2, 3, 4 \cdots$ Number of zeros of the field in the radial direction

$q = 1, 2, 3, 4 \cdots$ No. of halfwaves in the axial direction

J_n = Bessel's function of the first kind

for TM_{npq} mode : $H_z = 0$

$$E_z = E_{0z} J_n\left(\frac{x_{np} r}{a}\right) \cos(n\phi) \cos\left(\frac{q\pi}{d} z\right)$$

where

$n = 0, 1, 2, 3 \ldots$

$P = 1, 2, 3, 4 \ldots$

$q = 0, 1, 2, 3 \ldots$

for TE mode

$$k^2 = \left(\frac{x_{np}'}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2$$

for TM mode

$$k^2 = \left(\frac{x_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2$$

For lossless dielectric $k^2 = \omega^2 \mu\epsilon$

$$f_r = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{x_{np}'}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \quad \text{for TE mode}$$

$$f_r = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{x_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \quad \text{for TM mode}$$

Dominant modes :- TE_{111} when $a \geq 2a$

TM_{110} when $2a > d$

Q-factor of a cavity resonator :-

The quality factor Q is a measure of the frequency selectivity of a resonant or antiresonant circuit, and it is defined as

$$Q = 2\pi \times \frac{\text{Maximum Energy Stored}}{\text{Energy dissipated per Cycle}} = \frac{\omega W}{P} \quad \text{---(1)}$$

where W = Max. stored Energy

P = Avg. power loss

At resonant frequency, the electric & magnetic energies are equal and in time quadrature. when the electric energy is maximum, the magnetic energy is zero and vice versa. The total energy stored in the resonator is obtained by integrating the energy density over the volume of the resonator.

$$W_e = \int_V \frac{\epsilon}{2} |E|^2 dv = W_m = \int \frac{\mu}{2} |H|^2 dv = W \quad \text{---(2)}$$

where |H| & |E| are the peak values of the field intensities.

The avg. power loss in the resonator can be evaluated by integrating the power density over the inner surface of the resonator

$$P = \frac{R_s}{2} \int_S |H_t|^2 \, da \quad \text{——— (3)}$$

H_t is the peak value of the tangential magnetic intensity & R_s is the surface resistance of the resonator.

Substituting (2) & (3) in (1)

$$\therefore Q = \frac{\omega\mu \int_V |H|^2 \, dv}{R_s \int_S |H_t|^2 \, da}$$

we know that

$$|H|^2 = |H_t|^2 + |H_n|^2$$

where H_n is the peak value of the normal magnetic intensity, The average value of $|H_t|^2$ at the resonator walls is approximately twice the value of $|H|^2$ averaged over the volume.

$$\therefore Q = \frac{\omega\mu \, (\text{volume})}{2R_s \, (\text{surface area})}$$

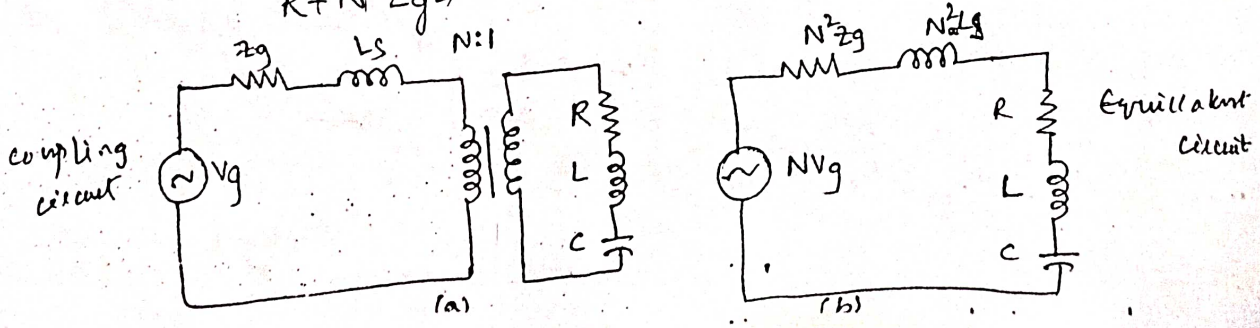
An unloaded resonator can be represented by either a series or a parallel resonant circuit. The resonant frequency and the unloaded Q_0 of cavity resonator are

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q_0 = \frac{\omega_0 L}{R}$$

If the cavity is coupled by means of an ideal N:1 transformer and series inductance L_s to a generator having internal impedance Z_g then the loaded Q_L of the system is given by

$$Q_L = \frac{\omega_0 L}{R + N^2 Z_g} \quad \text{for } |N^2 L_s| << |R + N^2 Z_g|.$$



$$K = \frac{N^2 z_g}{R}$$

$$\therefore Q_L = \frac{\omega_0 L}{R(1+K)} = \frac{Q_0}{1+K}$$

If $Q_{ext} = \frac{Q_0}{K} = \frac{\omega_0 L}{RK}$ is the external Q

Then

$$\boxed{\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}}$$

There are three types of Coupling coefficients.

1. Critical Coupling :-

If the resonator is matched to the generator, then $K=1$.

$$Q_L = \frac{1}{2} Q_{ext} = \frac{1}{2} Q_0$$

2. Over Coupling :-

If $K>1$, the cavity terminals are at a voltage maximum in the input line at resonance. The normalised impedance at the voltage maximum is the standing wave ratio S. That is $K = S$.

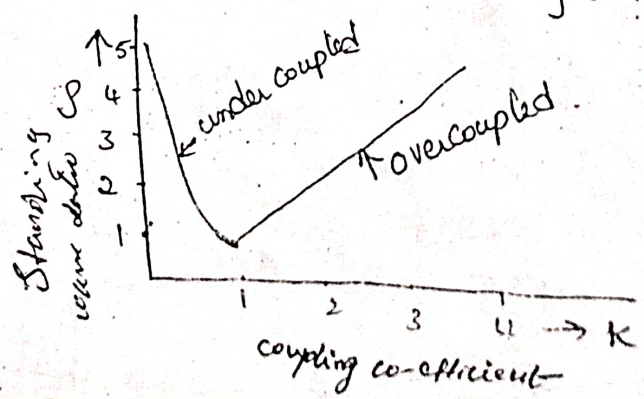
$$Q_L = \frac{Q_0}{1+S}$$

3. Under Coupling :-

If $K<1$, the cavity terminals are at a voltage minimum and the input terminal impedance equal to the reciprocal of the standing-wave ratio. That is, $K = 1/S$.

$$Q_L = \frac{S Q_0}{S+1}$$

The relationship of the Coupling coefficient & the Standing wave ratio is shown in below figure.



Problems

① A cylindrical waveguide has a inner radius of 2cm. Find the cutoff frequency for the guide operating in TE_{11} mode. Calculate λ_g and Z_{TE} at 10GHz ($\lambda_0 = 3cm$).

Given data

$a = 2cm$.

$\lambda_0 = 3cm$, $f = 10GHz$.

TE_{11} mode of propagation

$X_{np}' = k_c a = 1.841$ for TE_{11} mode.

$$k_c = \frac{1.841}{2\times10^{-2}} = 92.05$$

$$f_c = \frac{c \times k_c}{2\pi} = \frac{3\times10^8 \times 92.05}{2\pi} = 4.46GHz.$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-(f_c/f)^2}} = \frac{3\times10^{-2}}{\sqrt{1-\left(\frac{4.46}{10G}\right)^2}} = 0.033m = 3.3cm.$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} = \frac{120\pi}{\sqrt{1-\left(\frac{4.4}{10}\right)^2}} = 423.58\ \Omega.$$

② An air filled circular wave guide of 2cm inside radius is operated in the TE_{01} mode.

(i) Compute the cutoff frequency.

(ii) If the guide is to be filled with a dielectric material of $\epsilon_r = 2.25$, to what value must its radius be changed inorder to maintain the cutoff frequency at its original value.

Given data

$a = 2cm$

mode of propagation TE_{01},

for TE_{01} $X_{np}' = 3.832$.

$$f_c = \frac{c \cdot X_{np}'}{2\pi a} = \frac{3\times10^8 \times 3.832}{2\pi \times 2\times10^{-2}} = 9.148GHz.$$

$\frac{c}{a}$ = constant ie, $\frac{1}{\sqrt{\mu\epsilon}\,a}$ = constant

$$\frac{a_1}{a_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$a_1 = 2cm$

$\epsilon_1 = \epsilon_0$

$\epsilon_2 = \epsilon_0 \epsilon_r$

$$\frac{2\times10^{-2}}{a_2} = \sqrt{\frac{\epsilon_r \epsilon_0}{\epsilon_0}} = \sqrt{2.25} = 1.5$$

$$a_2 = \frac{2\times10^{-2}}{1.5} = 0.013 \text{ m}$$

$\therefore a_2 = 1.3 cm$

(3) A certain microstripline has the following parameters $\epsilon_r = 5.23$, $h = 7$ mills, $t = 2.8$ mills, $w = 10$ mills calculate

(i) Effective dielectric constant

(ii) Diameter of the wire over ground

(iii) characteristic impedance

(iv) Microstrip wavelength at a frequency of 2GHz.

Sol (i) Effective dielectric constant

$$\epsilon_{re} = 0.475 \epsilon_r + 0.67$$

$$\epsilon_{re} = 3.154$$

(ii) Diameter of the wire over ground

$$d = 0.67w \left(0.8 + \frac{t}{w}\right)$$

$$= 0.67 \times 10 \left(0.8 + \frac{2.8}{10}\right)$$

$$d = 7.236 \text{ mills}$$

(iii) characteristic impedance

$$Z_0 = \frac{87}{\sqrt{\epsilon_{re}+1.41}} \ln\left[\frac{5.98h}{0.8w+t}\right]$$

$$= \frac{87}{\sqrt{5.23+1.41}} \ln\left[\frac{5.98 \times 7}{0.8\times10 + 2.8}\right]$$

$$Z_0 = 45.74 \Omega.$$

(14) An air-filled resonant cavity with dimension $a = 5cm$, $b = 4cm$, $c = 10cm$ is made of copper ($\sigma_c = 5.8 \times 10^7$ mhos/m). It is filled with a lossless material ($\mu_r = 1$), ($\epsilon_r = 3$). Find the resonant frequency f_r, and the ~~quality factor~~ for TE_{101} mode. (32)

Sol: Given data

$a = 5cm$, $b = 4cm$, $c = 10cm$

$\sigma_c = 5.8 \times 10^7$ v/m.

$\mu_r = 1$, $\epsilon_r = 3$.

$f_r = ?$

$Q = ?$

TE_{101} mode

$m = 1, n = 0, p = 1.$

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$= \frac{1}{2\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} \sqrt{\left(\frac{1}{5\times10^{-2}}\right)^2 + 0 + \left(\frac{1}{10\times10^{-2}}\right)^2}$$

$$= \frac{3\times10^8}{2\sqrt{3}} \sqrt{\left(\frac{100}{5}\right)^2 + \left(\frac{100}{10}\right)^2}$$

$f_r = 1.936$ GHz