UNIT 5

# Inventory Models

**Syllabus**

Inventory Costs, Models with Deterministic Demand, Model (a) Demand Rate Uniform and Production Rate Infinite, Model (b) Demand Rate Non-uniform and Production Rate Infinite, Model (c) Demand Rate Uniform and Production Rate Finite.

## 5.1 Introduction

An inventory may be defined as any idle resource of an enterprise. It is a physical stock of items held in any business for the purpose of future affairs. In a factory the inventory may be in the form of raw materials, parts, semi-finished goods and finished goods. In a hospital it is used to indicate number of beds, specialized personnel, stocks of drugs etc. In airline company the inventories could be in the form of parts of engine repairs, stewardness, mechanics etc.

### 5.1.1 Reasons for Maintaining Inventories

Though inventory of materials is an idle resource (not to be used immediately) it is almost essential to maintain certain inventories for smooth functioning of an organization.

For example, in a service oriented organization such as transport corporation, absence of spare part when it is required will affect the maintenance of vehicles as a result of which the quality of service will deteriorate.

Consider an enterprise which has no inventory of materials. On receiving a sales order, the enterprise will have to order out, for purchase of raw materials required to complete the order, wait till they arrive and then start production. This would keep the customers to wait for a long time for the delivery of goods ordered.

Some of the important reasons for carrying inventories are listed below,

1) To ensure smooth and efficient running of business.

2) To ensure an adequate and prompt supply of items to the customer to avoid the reputation for constantly being out of stock and avoid the shortage at a minimum cost.


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3) To provide service to customers with varying demand at various locations by maintaining adequate supply to meet their immediate and seasonal needs.

4) To provide service to the customer at a short notice. Immediate deliveries can fetch more goodwill and orders.

5) To reduce the possibility of duplicating of orders.

6) To maintain economy by absorbing some of the fluctuations when the demand for an item fluctuates or is seasonal.

7) To minimize the losses due to deterioration, obsolescence, damage and pilferage etc.

Though the inventories are essential and provide an alternative to production/purchase in future, it also locks up capital of the enterprise. Frequently, money is borrowed from banks or financial institutions and interest is to be paid for the idle resources. The organization has to incur various types of costs to maintain inventory. It includes the expenses on stores, equipments, personnel etc. Also the goods stocked are insured for which premium has to be paid. Therefore excess inventories are undesirable. The larger inventories do not necessarily lead to high volume of output. Many items which are overstocked for a long period cannot be used due to technological change and other reasons. Lack of inventories might hamper production.

Out problem is to balance between the advantage of having inventories (or losses that may be expected from lack of inventories) and costs of carrying them to arrive an optimal level of inventories to minimize the total inventory cost. This calls for controlling the inventories in the most profitable way. The basic objective of inventory control is to release capital for more productive use.

## 5.2 Types of Inventories

1) Process (or transit or pipeline or transportation) inventories.

2) Lot size (or cycle) inventories.

3) Seasonal inventories.

4) Buffer (or safety) inventories.

5) Decoupling inventories.


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## 3 Inventory Costs

Four categories of inventory costs are associated with keeping inventories of items. They are,

1) Item (or production or purchase) cost.

2) Ordering (or set up) cost.

3) Carrying (or holding) cost.

4) Shortage (or stock out) cost.

1) **Item Cost :** The costs associated with item whether it is manufactured or purchased. If the item is manufactured, then the cost includes items such as direct materials and labour, indirect materials and labour and overhead expenses. If the item is purchased, then the item cost is the purchase price of 1 unit. The item cost is denoted by Rs. $C_1$ per item.

2) **Odering Cost :** The cost associated with ordering of raw materials for production purposes is known as ordering cost. This cost includes factors such as, advertisements, administrative (paper work, telephone calls, postage), rent for space used by the purchasing department, transportation of items ordered, receiving and inspection of goods, processing payments etc. This is denoted by Rs. $C_2$ per order.

3) **Carrying Cost :** The costs associated with carrying (or holding) inventory in stock are known as carrying costs. They include the costs such as rent for space used for storage, interest on the money invested in inventory, insurance of stored equipment (inflammable and explosive items), depreciation of equipment, taxes, deterioration of quality and furniture used etc. This cost is denoted by $C_3$/item/unit time.

4) **Shortage Cost :** When the demand or consumption of an item exceeds the available stock, there is a shortage. If there is a shortage in raw material or spareparts the production is going to suffer. Therefore an item cannot be supplied on the customers demand. The cost associated with either a delay in meeting demand or the inability to meet it all are called shortage costs. These cost include the following items,

   i) Overtime requirements due to shortage.

   ii) loss of goodwill of customers due to delay.

   iii) cost of expediting.


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The cost arising out of shortage can be classified into two different types,

If customer is unwilling to wait and switch to a rival product, the shortage cost is only proportional to the amount of shortage. Here the profit on the sales as well as the goodwill is lost.

On the otherhand, the unfilled demand can be filled at a later stage (when item demand becomes available) when a customers can wait. In this case the company does not lose customer permanently. Here the costs are proportional to quantity that is short as well as the delay time costs incurred. In this case are costs for extra paper work, possible expediting of orders etc.

Shortage cost is denoted by Rs. $C_4$ per item per unit time of shortage.

**Note :** Total inventory cost = Purchase cost of inventory item + Ordering cost + Carrying costs + Shortage costs.

## 5.4 Factors Involved in Inventory Analysis

**Demand :** Demand is the number of units required per period. Also known as size of demand. It may be either known exactly (deterministic) or known in terms of probabilities or be completely unknown.

If the demand is deterministic, the demand during each period is known with certainty. It may be either fixed or variable with time. Such demands are called static and dynamic demands respectively.

The demand is probability if the requirement for a period of time is not known with certainty but can be described by a known probability distribution.

The rate of demand is the size of demand over a particular unit of time.

The pattern of demand is the manner in which items are drawn from inventory. The demand over a given period may be satisfied instantaneously at the beginning of the period (or) uniformly during the period.

**Lead Time :** The time gap between placing an order of item and actually receiving an item in the inventory is known as Lead time. It may be deterministic (constant or variable) or probabilistic. If the lead time is zero, there is no need for placing an order in advance. If it is not zero, but known with certainty, then it is required to place an order in advance by an amount of time equal to the lead time, provided the demand is also known. If the demand is known probabilitically, then place an order only after considering the expected costs or holding and shortage over the lead time.


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**Order Cycle :** The time period between placement of two successive orders is known as order cycle. The order may be initiated in one of the two basic ways,

1) **Continuous Review :** The record of inventory level is updated continuously until a certain lower limit (known as reorder level) is reached where a new order of a fixed amount is placed. This is often known as the two-bin system.

2) **Periodic Review :** In this system the inventory level is examined at fixed periodic intervals and orders are placed at such intervals. This amount of order depends among other things, on the consumption between two review periods and may vary from one review date to another.

**Time Horizon :** The time period over which the inventory level will be controlled is known as time horizon.

## 5.5 Classification of Inventory Control Model

The inventory model can be classified into two categories,

1) Deterministic inventory models.

2) Probabilistic inventory models.

### 5.5.1 Deterministic Inventory Model

Here the parameters like demand, ordering quantity cost etc., are already known (i.e.,) there is no uncertainty in parameters. Here we shall consider two models,

**Model I :** No shortages permitted.

**Model II :** Shortages permitted.

In this section, we shall deal with inventory problems in which demand is assumed to be fixed and completely known. Such problems are usually called economic order quantity (or) lot size problems.

Economic Order Quantity (EOQ) is that size of order which minimizes the total annual costs of carrying inventory and cost of ordering.

The objective of the study of economic order quantity model is to determine an optimum order quantity such that the total inventory cost in minimized.

The concept of EOQ is applicable under the following conditions,

1) The item is replenished in lots or batches, either by purchasing or producing.

2) Consumption of items is uniform and continuous.




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The model is described under the following conditions,

1) Planning period is one year.

2) Demand is deterministic and D units per year.

3) Cost of purchase is C, per unit.

4) Ordering cost (or procurement cost or set up cost or replenishment cost) is $C_0$ or $C_3$.

5) Inventory carrying cost (or cost of holding stock) is $C_1$ or $C_n$ per unit per year.

6) Shortage cost is $C_2$ or $C_s$ per unit per year.

7) Order size is denoted by Q.

8) Lead time is L.

9) Cycle period in replenishment is t.

**Sensitivity of EOQ Formula :** In most of the cases, a slight variation from the EOQ will result in only a slight increase in costs (the total annual cost). The order quantity will not affect annual purchasing cost, The Fig. shows the affect of changes in the order quantity on annual holding cost HC and ordering cost OC.

Let HC(q) = Annual holding cost if the order quantity is q.

OC(q) = Annual ordering cost if the order quantity is q.

**Table 5.5.1** Cost Calculations

| q | HC(q) | OC(q) | HC(q) + OC(q) |
|---|---|---|---|
| 50 | 2.0 | 50.00 | 52.00 |
| 100 | 4.0 | 25.00 | 29.00 |
| 150 | 6.0 | 16.67 | 22.67 |
| 200 | 8.0 | 12.50 | 20.50 |
| 220 | 8.8 | 11.36 | 20.16 |
| 240 | 9.6 | 10.42 | 20.02 |
| 250 | 10.0 | 10.00 | 20.00 |
| 260 | 10.4 | 9.62 | 20.02 |
| 280 | 11.2 | 8.93 | 20.13 |
| 300 | 12.0 | 8.33 | 20.33 |
| 350 | 14.0 | 7.14 | 21.14 |
| 400 | 16.0 | 6.25 | 22.25 |


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We find that, $HC(q) = \frac{1}{2}(0.08q) = 0.04q$

$$OC(q) = 5\left(\frac{500}{q}\right) = \frac{2,500}{q}$$

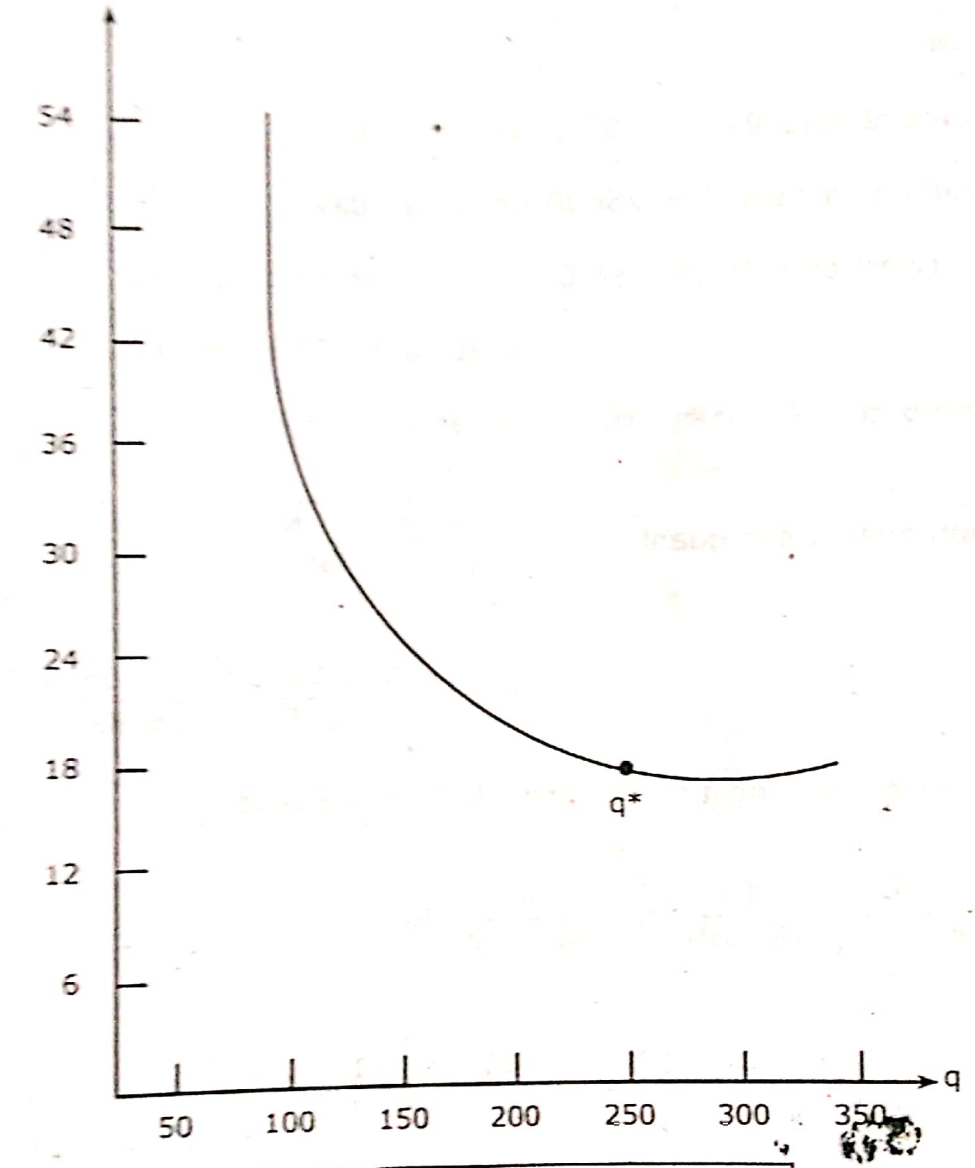



**Fig. 5.5 1** OC(q) + HC(q) for Example

We obtain the graph of holding cost and ordering cost (HC(q) + OC(q)) with respect to q by using the information in table, . The fig. shows that the cost curve is very flat near q*. The flatness of the HC(q) + OC(q) curve is very important, because it is often difficult to estimate holding cost ($C_1$) and setup cost ($C_s$). Inaccurate estimatimation of holding cost and setup cost may result in a value of q that differs slightly from the actual EOQ.

The flatness of the cost curve indicates that even a moderate error in the determination of the EOQ will only increase costs by a slight amount.


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**Example**

**Ex. 1) A Contractor has to supply 10,000 paper cones per day to a textile unit. He finds that when he starts a production run he can produce 25,000 paper cones per day. The cost of holding a paper cone in stock for one year is 2 paise and the setup cost of a production run is Rs. 18. How frequently should a production run be made?**

*Sol. :* Given,

Demand rate R = 10000 units per day,

Production rate K = 25000 units per day,

Carrying or holding cost $C_1$ = 0.02 per unit per year

= Rs. 0.000055 per unit per day

Setup cost $C_3$ = Rs. 18 per order

Optimum order quantity $q^* = \sqrt{\frac{2C_3R}{C_1}} \cdot \sqrt{\frac{K}{K-R}}$

$$= \sqrt{\frac{2 \times 18 \times 10,000}{0.000055}} \cdot \sqrt{\frac{25,000}{25,000 - 10,000}}$$

Average waiting time of arrivals in the queue

$$W_q = \frac{L_q}{\lambda} = \frac{1}{(S-1)!}\left(\frac{\lambda}{\mu}\right)^S \cdot \frac{\mu}{(S\mu - \lambda)^2} \cdot P_0$$

$$= \frac{1}{(2-1)!}\left(\frac{3}{2}\right)^2 \cdot \frac{20}{(2 \times 20 - 30)^2} \times \frac{1}{7}$$

$$= \frac{9}{4} \times \frac{20}{100} \times \frac{1}{7}$$

$$= \frac{9}{4} \times \frac{1}{5} \times \frac{1}{7}$$

$$= \frac{9}{140} \text{ hours}$$

= 3.86 min.



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Combined waiting time with increased service time when,

$\lambda' = 30$/hour and $\frac{1}{\mu'} = 3.5$min. (or) $\mu = \frac{60}{3.5} = \frac{120}{7}$ per hour.

$\frac{\lambda'}{S\mu'} = \frac{30}{2(120/7)} = \frac{7}{8}$ i.e., $< 1$ and $\frac{\lambda'}{\mu'} = \frac{30}{120/7} = \frac{7}{4}$

$$P_0 = \left[\sum_{n=0}^{1} \frac{1}{n!}\left(\frac{\lambda'}{\mu'}\right)^n + \frac{1}{S!}\left(\frac{\lambda'}{\mu'}\right)^S \cdot \frac{S\mu}{S\mu - \lambda'}\right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{1}{n!}\left(\frac{7}{4}\right)^n + \frac{1}{2!}\left(\frac{7}{4}\right)^2 \cdot \frac{2(120/7)}{2(120/7) - 30}\right]^{-1}$$

$$= \left[\frac{1}{0!}\left(\frac{7}{4}\right)^0 + \frac{1}{1!}\left(\frac{7}{4}\right)^1 + \frac{1}{2\times 1} \times \frac{49}{16} \cdot \frac{2(120)}{240 - 210}\right]^{-1}$$

$$= \left[1 + \frac{7}{4} + \frac{49}{4}\right]^{-1}$$

$$= [15]^{-1}$$

$$= \frac{1}{15}$$

= 104446.5936 paper cones.

Times between orders $= \frac{q^*}{R} = \frac{1,04,446.5936}{10,000} = 10.4$ days

Times of manufacture $= \frac{q^*}{R} = \frac{1,04,446.5936}{25,000} = 4.17 \Rightarrow 4$ days (approx).

∴ The production or supply or manufacture cycle starts at an interval of 10.4 days and production continues for 4 days. In each cycle a batch of 1,04447 paper cones is produced.



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## 5.5.1.1 Model I : Economic Order Quantity with No Shortages

### 5.5.1.1.1 Model (I a) Demand Rate Uniform and Production Rate Infinite

Here we have the following assumptions,

1) Demand rate is constant and known.

2) Shortage are not permitted i.e., as soon as the level of inventory reaches zero, the inventory is replenished. Thus the cost of shortage is infinite.

3) Production time is neglibile i.e., production rate is infinite so that replacement of items is instantaneous. (lead time is zero).

4) The cost coefficients $C_1$, $C_2$ and $C_3$ are constants i.e., purchase price (or) production price (or) cost per unit is constant. Therefore, quantity discounts are not allowed. And unit costs of carrying inventory and ordering are known and constant.

Q = Order quantity.

With a constant rate of demand,

$$\text{Average inventory} = \frac{\text{Maximum level inventory} + \text{Minimum level inventory}}{2}$$

$$= \frac{Q+0}{2} = \frac{Q}{2}$$

We order a quantity Q, time $t = 0$, at which is stored as maximum inventory.

We require the optimum level of Q so that the total inventory cost is minimized.

The inventory costs are determined as follows,

N = Number of orders per year

$$= \frac{\text{Total annual demand}}{\text{Quantity ordered each time}}$$

$$= \frac{D}{Q}$$

Ordering cost = Number of orders per year × Ordering cost per year

$$= NC_2 = \frac{D}{Q}C_2$$


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Carrying cost = Average inventory × carrying cost per unit

$$= \frac{Q}{2} \times C_3$$

$$\text{Total inventory cost} = \frac{D}{Q}C_2 + \frac{Q}{2}C_3$$

The total cost is minimum at a point where ordering costs equal carrying costs.

(i.e.,) economic order quantity occurs at a point where,

Ordering cost = Carrying cost

$$\therefore \frac{D}{Q}C_2 = \frac{Q}{2}C_3$$

$$\Rightarrow Q^2 = \frac{2DC_2}{C_3}$$

$$\Rightarrow Q = \sqrt{\frac{2DC_2}{C_3}}$$

∴ The economic order quantity, denoted by Q*, is given by

$$Q^* = \sqrt{\frac{2DC_2}{C_3}}$$

which is known as optimal lot size formula. It is also known as Wilson lot size formula, due to R.H. Wilson.

Optimum number of orders per year is,

$$N^* = \frac{D}{Q^*}$$

Where, D is the demand per year.

Optimum length of the time between order is,

$$T^* = \frac{T}{N^*}$$

where, T (total time horizon) is one year.

Minimum annual inventroy cost,

$$TC^* = \sqrt{2DC_2C_3}$$


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### 5.5.1.1.2 Solved Problems

**P 1)** **A manufacturer has to supply 12,000 units of a product per year to his customer. The ordering cost is Rs. 100 per order and the carrying cost is Rs. 0.80 per item per months. The shortage cost is not allowed and the replacement is instantaneous. Determine,**

**i) The economic order quantity.**

**ii) The time between orders.**

**iii) The number of orders per year.**

**iv) The optimum annual cost if the cost of item is Rs. 2 per item.**

*Sol. :* Given,

$C_1$ = Rs. 2/item

$C_2$ = Rs. 100/order

$C_3$ = 0.8/item/month

= Rs. 9.6/item/year

D = 12,000 items/year

i) EOQ : $Q^* = \sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2(12,000)\ (100)}{9.6}}$ = 500 units.

ii) Number of orders per year

$$N^* = \frac{D}{Q^*} = \frac{12,000}{500} = 24$$

iii) The line between orders $= \frac{1}{N^*} = \frac{1}{24}$ year $= \frac{12}{24}$ month = 0.5 month

iv) The optimum annual cost = Number of orders × cost of one cycle

$$= N \times \left( C_1 Q + C_2 + \frac{Q}{2} C_3 t \right)$$

$$= 24 \left[ 2 \times 500 + 100 + \frac{500}{2} \times 0.8 \times 0.5 \right]$$

= Rs. 28,800



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P 2) **The demand rate for a particular item is 12,000 units per year. The set up cost per run is Rs. 350 and the holding cost is Re. 0.20 per unit per month. If no shortages are allowed and the replacement is instantaneous. Determine,**

i) **The optimum run size.**

ii) **Optimum scheduling period.**

iii) **Minimum total expected annual cost.**

*Sol. :* Given,

Demand rate D = 12,000/year

$C_2$ = Rs. 350 per run

$C_3$ = Re. 0.20/unit/month

= Rs. 2.40/unit/year

i) Optimum run size $Q^* = \sqrt{\frac{2DC_2}{C_3}}$

$$= \sqrt{\frac{2 \times 12000 \times 350}{2.40}}$$

= 1870 units

ii) Optimum scheduling period $= \frac{Q^*}{D}$ $\left(\text{or } \frac{1}{N^*}\right)$

$$= \frac{1,870}{12,000} \text{ year}$$

$$= \frac{1,870 \times 12}{12,000} \text{ month}$$

= 1.87 months

iii) Minimum total expected annual cost $= \sqrt{2\ C_3 C_2\ D}$

$$= \sqrt{2 \times 2.4 \times 350 \times 12,000}$$

= Rs. 4,490/year



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**P 3)** **A dealer has to supply his customer with 400 units of a product per every week. The dealer gets the product from the manufacturer at a cost of Rs. 50 per unit. The cost of ordering from the manufacturers is Rs. 75 per order. The cost of holding inventory is 7.5% per year of the product cost.**

**Determine,**

**i) Economic order quantity.**

**ii) The optimum total cost.**

*Sol. :* D = 400 unit per week

$C_2$ = Rs. 75 per order

$C_3$ = 7.5% per year of the cost of the product

$$= \frac{7.5}{100} \times 50 \text{ / unit / year}$$

$$= \frac{7.5}{100} \times \frac{50}{52} \text{ / unit / week}$$

$$= \text{Rs. } \frac{3.75}{52} \text{ / unit / week}$$

i) Economic order quantity $Q^* = \sqrt{\frac{2DC_2}{C_3}}$

$$= \sqrt{\frac{2 \times 400 \times 75 \times 52}{3.75}}$$

$$= 912 \text{ units/order}$$

ii) The optimum total cost $= (400 \times 50) + \sqrt{2C_2C_3D}$

$$= 20,000 + \sqrt{2 \times 75 \times \frac{3.75}{52} \times 400}$$

$$= \text{Rs. } 20,065.80/\text{week}$$

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**P 4)** **A manufacturing company purchases 9,000 parts of a machine for its annual requirement, ordering one month's usage at a time. Cost of each part is Rs. 20. The order cost is Rs. 15/order and the carrying cost is 15% per year of the average inventory. You have been assigned to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?**

***Sol. :*** D = 9,000 parts/year

$C_1$ = Rs. 20/paret

$C_2$ = Rs. 15/order

$C_3$ = 15% per year of the average inventory

$$= \frac{15}{100} \times 20 = \text{Rs. } 3 \text{ / part / year}$$

$$\text{EOQ} : Q^* = \sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2 \times 9,000 \times 15}{3}} = 300 \text{ units}$$

$$\text{and } N^* = \frac{D}{Q^*} = \frac{9,000}{300} = 30$$

$$T^* = \frac{1}{30} \text{ year}$$

$$\text{Minimum expected annual cost} = \sqrt{2 \times C_2 \times C_3 \times D}$$

$$= \sqrt{2 \times 15 \times 3 \times 9,000}$$

$$= \text{Rs. } 900$$

Suppose the company follows the policy of ordering every month.

Then the annual ordering cost (15 × 12) = 180

$$\text{Lot size of inventory (each month)} = \frac{9,000}{12} = 750$$

i.e., Q = 750


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Average inventory (at any time) $= \frac{750+0}{2} = 375$

Storage cost (at any time) $= 375 \times C_3$

$= 375 \times 3 = \text{Rs. } 1,125$

Total annual cost = Storage cost + Ordering cost

= Rs. 1,125 + 180

= Rs. 1,305

**P 5)** **A company buys in lots of 500 boxes which is a 3 month supply. The cost per box is Rs. 125.00 and the ordering cost is Rs. 150. The inventory carrying cost is estimated at 20% of unit value.**

**i) Determine the total annual cost of the existing inventory policy?**

**ii) How much money could be saved by employing the economic order quantity?**

***Sol. :*** Ordering cost/order = 150

$C_2$ = Rs. 150/order

Number of units/order = 500

Q = 500 units

Annual demand = 500 × 4 = 2,000 units

Demand rate (D) = 2,000/year

Carrying cost = 20% of unit value

$$C_3 = \frac{20}{100} \times 125 = \text{Rs. } 25$$

i) Total annual cost of the existing inventory policy

$$= \frac{D}{Q} C_2 + \frac{Q}{2} C_3$$

$$= \frac{2,000}{500} \times 150 + \frac{500}{2} \times 25 = \text{Rs. } 6,850$$



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ii) Economic order quantity $Q^* = \sqrt{\frac{2DC_2}{C_3}}$

$$= \sqrt{\frac{2 \times 2,000 \times 150}{25}}$$

$$= 155$$

The corresponding minimum annual cost $= \sqrt{2DC_2C_3}$

$$= \sqrt{2 \times 2,000 \times 150 \times 25} = \text{Rs. } 3,873$$

$\therefore$ By applying the economic order quantity, money saved by a company

= 6,850 – 3,873

= Rs. 2,977

P 6) **The following table gives the annual demand and unit price of 4 items,**

| Item | A | B | C | D |
|---|---|---|---|---|
| Annual Demand (units) | 800 | 400 | 392 | 13,800 |
| Unit Price (in Rs.) | 0.02 | 1.00 | 8.00 | 0.20 |

**Ordering cost is Rs. 5/order and holding cost is 10% of unit price.**

i) **Determine the EOQ in units.**

ii) **Total variable cost**

iii) **Compute EOQ in Rs.**

iv) **Compute EOQ in years of supply.**

v) **Number of orders per year.**


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*Sol. :* **For Item A**

Given,

$D = 800$ units/year

$C_2 = $ Rs. 5/order

$C_3 = 10\%$ of unit price

$$= \frac{10}{100} \times 0.02 = \text{Rs. } 0.002$$

$$EOQ = \sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2 \times 800 \times 5}{.002}} = 2{,}000 \text{ units}$$

Total variable cost $= \sqrt{2DC_2C_3}$

EOQ in Rs. $= 2{,}000 \times .02 = $ Rs. 40

EOQ in years of supply $= \frac{2{,}000}{800} = 2.5$ years

Number of orders per year (EOQ frequency)

$$= \frac{D}{Q^*} = \frac{800}{2000} = \frac{1}{2.5} = 0.4$$

**For Item B**

$D = 400$ units/year

$C_2 = $ Rs. 5/order

$C_3 = 10\%$ of unit Price

$$= \frac{10}{100} \times 1.00 = \frac{1}{10} = .1 \text{ Re.}$$

$$EOQ = \sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2 \times 400 \times 5}{0.1}} = 200 \text{ units}$$


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Total variable cost = $\sqrt{2DC_2C_3}$

$= \sqrt{2 \times 400 \times 5 \times 0.1}$ = Rs. 20

EOQ in Rs. = 200 × 1 = Rs. 200

EOQ in years of supply = $\frac{200}{400}$ = 0.5 year

Number of orders per year = $\frac{400}{200}$ = 2

**For Item C**

D = 392

$C_2$ = Rs. 5/order

$C_3$ = 10% of unit price

$= \frac{10}{100} \times 8 = 0.8$

EOQ = $\sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2 \times 392 \times 5}{0.8}}$ = 70 units

Total variable cost = $\sqrt{2DC_2C_3} = \sqrt{2 \times 392 \times 5 \times 0.8} = 56$

EOQ in Rs. = 70 × 8 = Rs. 560

EOQ in years of supply = $\frac{70}{392}$ = 0.18

Number of orders per year = $\frac{392}{70}$ = 5.6



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**For Item D**

$D = 13,800$

$C_2$ = Rs. 5/order

$C_3$ = 10% of unit price

$$= \frac{10}{100} \times 0.2 = 0.02$$

$$EOQ = \sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2 \times 13,800 \times 5}{0.02}} = 2,627 \text{ units}$$

$$\text{Total variable cost} = \sqrt{2DC_2C_3} = \sqrt{2 \times 13,800 \times 5 \times 0.02} = 52.54$$

EOQ in Rs. = 2,627 × 0.2 = 525.4

$$\text{EOQ in years of supply} = \frac{2,627}{13,800} = 0.19 \text{ year}$$

$$\text{Number of orders per year} = \frac{13,800}{2,627} = 5.26$$

**P 7)** **A company has to supply 1000 items per month at a uniform rate and each time a production run is started it costs Rs. 200. Cost of storing is Rs. 20 per item per month. The number of items to be produced per run has to be ascertained. Determine the total of set up cost and average inventory cost if the run size is 500, 600, 700, 800. Find the optimal production run size using EOQ formula.**

***Sol. :*** Given data,

Demand D = 1000/month

Set up cost $C_2$ = Rs. 200/order

Carrying cost $C_3$ = Rs. 20/item/month.


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| Run Size | Set Up Cost | Average Inventory Cost | Total Cost |
|---|---|---|---|
| 500 | $\frac{1,000}{500} \times 200 = 400$ | $\frac{500}{2} \times 20 = 5,000$ | 5,400 |
| 600 | $\frac{1,000}{600} \times 200 = 333.3$ | $\frac{600}{2} \times 20 = 6,000$ | 6,333.3 |
| 700 | $\frac{1,000}{700} \times 200 = 285.7$ | $\frac{700}{2} \times 20 = 7,000$ | 7,285.7 |
| 800 | $\frac{1,000}{800} \times 200 = 250$ | $\frac{800}{2} \times 20 = 8,000$ | 8,250 |

From the above table we conclude that, the total cost increases when the batch size increases.

$$\text{EOQ} : Q^* = \sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2 \times 1000 \times 200}{20}} = 141 \text{ units (approx.)}$$

**P 8) The demand rate for a particular item by a company is 10,000 units per year. The item may be obtained from either an outside supplier or subsidiary company. The following table gives the relevant data for the procurement of the item,**

| Costs | From Outside Supplier (in Rs.) | From Subsidiary Company (in Rs.) |
|---|---|---|
| Cost/unit | 12 | 13 |
| Cost of placing an order | 10 | 10 |
| Cost of receiving an order | 20 | 15 |
| Carrying cost | 2 | 2 |

**i) From which source would your recommend to procure.**

**ii) Determine the purchase quantity and the minimum total cost in that cost.**

*Sol. :* Given,

i) $D$ = 10,000 units/year

$C_1$ = 12/unit

Ordering cost $C_2$ = 10 + 20 = Rs. 30/order

Carrying cost $C_3$ = Rs. 2/unit/item

$$\text{EOQ} : Q^* = \sqrt{\frac{2DC_2}{2}} = \sqrt{\frac{2 \times 10,000 \times 30}{2}} = 548 \text{ units}$$

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Total annual cost corresponding to this size is

$$= \frac{1}{2} Q^* C_3 + \frac{D}{Q^*} C_2 + D\, C_1$$

$$= \left(\frac{1}{2}(548) \times 2\right) + \left(\frac{10,000}{548} \times 30\right) + (10,000 \times 12)$$

= Rs. 1,21,095.45

ii) Subsidiary company,

D = 10,000 units/year

$C_1$ = Rs. (10 + 15) = Rs. 25/order

$C_3$ = Rs. 2/unit/item

$$\text{EOQ}: Q^* = \sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2 \times 10,000 \times 25}{2}} = 500 \text{ units}$$

Total annual cost for this size $= \frac{1}{2} Q^* C_3 + \frac{D}{Q^*} C_2 + D\, C_1$

$$= \left(\frac{1}{2} \times 500 \times 2\right) + \left(\frac{10,000}{500} \times 25\right) + (10,000 \times 13)$$

= Rs. 1,31,000

From (i) and (ii), it can be seen that the total annual cost is minimum in case (i).

Hence the company should purchase the item from outside supplier.

Order should be 548 units.

Minimum total cost in this case = Rs. 1,21,095.45

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p 9) **A company wishes to manufacture 1,00,000 units of a product per year at a constant rate. Raw materials are supplied from outside supplier for its annual requirement. Ordering cost is Rs. 160 per order. Annual carrying inventory cost is equal to 20% of the item's cost. The product cost's Rs. 20 each. Determine,**

i) **The optimal order size.**

ii) **Total inventory cost.**

iii) **Number of orders placed in the next year.**

iv) **If the company decides to order for raw material only twice a month, how much of a cost penalty would it suffer.**

v) **Suppose that an ordering cost of only Rs. 80 had been used instead of the correct value of Rs. 160.**

a) **Determine the order size.**

b) **Find the net change in the total cost.**

*Sol. :* Given,

$D = 1,00,000$ units/year

$C_1 = $ Rs. 20/item

$C_2 = $ Rs. 160/order

$C_3 = 20\%$ of item's cost

$$= \frac{20}{100} \times 20 = \text{Rs. } 4 \text{ / unit / year}$$

i) Optimal order size $Q^* = \sqrt{\frac{2DC_2}{C_3}}$

$$= \sqrt{\frac{2 \times 1,00,000 \times 160}{4}}$$

$$= 2,828 \text{ units/order}$$

ii) Total inventory cost $TC^* = \sqrt{2D_2C_3}$

$$= \sqrt{2 \times 1,00,000 \times 160 \times 4}$$

$$= \text{Rs. } 11,313/\text{year}$$


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iii) Number of orders per year

$$N^* = \frac{D}{Q^*} = \frac{1,00,000}{2828} = 35 \text{ orders/year}$$

iv) For 2 orders/month,

$$\text{Number of units/order } (Q') = \frac{1,00,000}{24} \text{ / year}$$

$$= 4,167 \text{ units/year}$$

$$\text{Total inventory cost} = \left(\frac{1,00,000}{4,167} \times 160\right) + \left(\frac{4,167}{2} \times 4\right)$$

$$= \text{Rs. } 12,173.69$$

$$\text{Penalty cost} = \frac{12,173.69 - 11,312}{11,312} = 0.076 = 7.6\%$$

v) $D = 1,00,000$ units/year

$C_2 = $ Rs. 80/order

$C_3 = $ Rs. 4/unit/year

$$\text{New optimal size } Q^* = \sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2 \times 1,00,000 \times 80}{4}} = \text{Rs. } 2,000 \text{ units/order}$$

$$\text{Total inventory cost} = \frac{D}{Q^{**}}C_2 + \frac{Q^{**}}{2}C_3$$

$$= \left(\frac{1,00,000}{2,000} \times 160\right) + \left(\frac{2,000}{2} \times 4\right)$$

$$= \text{Rs. } 12000$$

Net change in the total cost = 12,000 – 11,312

= 6,888

= 68.8%



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## 1.1.3 Model I (b) : Demand Rate Non-uniform and Production Rate Infinite

In this model, all the assumptions are same as in Model I (a) except the demand is uniform (i.e.,) demand rate is different in different cycle. The total demand D is specified as demand during some long time period T.

The inventory costs are as follows,

$$\text{ordering cost} = \frac{D}{Q}C_2 \qquad \ldots (1)$$

$$\text{Carrying cost} = \frac{Q}{2}C_3T \qquad \ldots (2)$$

$$\text{Total inventory cost} = \frac{D}{Q}C_2 + \frac{Q}{2}C_3T$$

For calculating (EOQ) $Q^*$, equating (1) and (2), we get

$$\frac{D}{Q}C_2 = \frac{Q}{2}C_3T$$

$$\Rightarrow Q = \sqrt{\frac{2DC_2}{C_3T}}$$

$$\text{Thus, } Q^* = \sqrt{\frac{2C_2D/T}{C_3}}$$

$$\text{Minimum total yearly inventory cost} = \frac{D}{Q^*}C_2 + \frac{Q^*}{2}C_3T$$

$$= \frac{DC_2}{\sqrt{\frac{2C_2D}{C_3T}}} + \sqrt{\frac{2C_2D}{C_3T}}\frac{TC_3}{2}$$

$$TC^* = \sqrt{\frac{2DC_2C_3}{T}}$$

Here the uniform rate of demand is replaced by average rate of demand (i.e.,) D is replaced by $\frac{D}{T}$.

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## 5.5.1.1.4 Model (I b) Demand Rate Uniform Production Rate Finite (Manufacturing Model with no Shortages)

Here we make the following assumptions,

1) Demand rate D is a constant.

2) No shortage cost.

3) The cost coefficients $C_1$, $C_2$ and $C_3$ are constants.

4) Price per unit is constant i.e., no price discount is allowed.

5) Production rate R is finite (i.e.,) run sizes are constant.

6) New run will be started whenever inventory is zero.

7) Production rate is greater than the demand rate (R > D).

Since R > D, during the production period the inventory will be building up at the constant rate of (R - D) units per unit time. If there is no production (i.e.) after production stops, the inventory is decreasing at the constant demand rate.

If Q is the number of units produced per order cycle (Number of items/order), then the time of production run is given by $\frac{Q}{R}$.

As inventory is building up at the constant rate of (R – D), the quantity in inventory at the end of the run (which is the maximum inventory level) is given by,

Maximum inventory level $= \frac{Q}{R}(R-D)$

Minimum inventory level = 0

$\therefore$ Average inventory $= \frac{1}{2}\left[\frac{Q}{R}(R-D)+0\right]$

$= \frac{Q}{2R}(R-D)$

With ordering cost $C_2$ per order,

Annual ordering cost $= \frac{D}{Q}C_2$


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With carrying cost $C_3$ per item per year,

Total annual carrying cost = Average inventory $\times C_3 = \frac{Q}{2R}(R-D) \times C_3$

$\therefore$ Total annual inventory cost, $TC = \frac{D}{Q}C_2 + \frac{Q}{2R}(R-D)C_3$

For minimum total inventory cost,

$$\frac{d}{dQ}(TC) = 0$$

$$\Rightarrow -\frac{D}{Q^2}C_2 + \frac{1}{2R}(R-D)C_3 = 0$$

$$\Rightarrow Q^2 = \frac{2RDC_2}{(R-D)C_3}$$

$$\therefore Q = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{R}{R-D}}$$, which is the EOQ and is denoted by $Q^*$.

Thus optimal lot size, $Q^* = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{R}{R-D}}$

**Note :** Optimal number of production runs per year,

$$N^* = \frac{D}{Q^*}$$

$$= \frac{D}{\sqrt{\frac{2DC_2}{C_3} \cdot \frac{R}{R-D}}}$$

$$\therefore N^* = \sqrt{\frac{C_3D(R-D)}{2C_2R}}$$

Optimum length of each lot size production run $= \frac{Q^*}{R}$

$$= \frac{\sqrt{\frac{2DC_2}{C_3}\frac{R}{R-D}}}{R}$$

$$= \sqrt{\frac{2DC_2}{RC_3(R-D)}}$$



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Total minimum production inventory cost,

$$TC^* = \frac{D}{Q^*}C_2 + \frac{Q^*}{2R}(R-D)C_3$$

$$= \frac{D}{\sqrt{\frac{2DC_2}{C_3}\frac{R}{R-D}}}C_2 + \frac{1}{2R}\sqrt{\frac{2DC_2}{C_3}\frac{R}{R-D}}\cdot(R-D)C_3$$

$$= \sqrt{\frac{DC_2}{2R}}\sqrt{\frac{C_3(R-D)}{}} + \sqrt{\frac{DC_2(R-D)C_3}{2R}}$$

$$= 2\sqrt{\frac{DC_2(R-D)C_3}{2R}}$$

$$\therefore TC^* = \sqrt{\frac{2DC_2C_3(R-D)}{R}}$$

## 5.5.1.1.5 Solved Problems

**P 1)** **The demand rate for an item in a company is 18,000 units per year. The company can produce at the rate of 3000/month. The set up cost is Rs. 500 per order and the holding cost is 0.15 Re per unit per month. Calculate,**

i) **The optimum manufacturing quantity.**

ii) **The maximum inventory.**

iii) **The time between order.**

iv) **The number of orders per year.**

v) **The time of manufacture.**

vi) **The optimum annual cost is Rs. 2/unit.**


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Given,

Item cost $C_1$ = Rs. 2/unit

Setup cost $C_2$ = Rs. 500/order

Carrying cost $C_3$ = Re. 0.15/unit/month

Demand rade D = 18000 units/year

= 1,500 units/mongh

Production rate R = 3,000 units/month

i) Optimum manufacturing quantity $Q^* = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{R}{R-D}}$

$$= \sqrt{\frac{2 \times 1,500 \times 500}{0.15} \cdot \frac{3,000}{3,000 - 1,500}}$$

= 4,470 units

ii) The maximum inventory $= \frac{Q}{R}(R-D)$

$$= \frac{4,470}{3,000}(3,000 - 1,500) = 2,235 \text{ units}$$

iii) Time between order $= \frac{Q}{D} = \frac{4,470}{1,500}$ = 3 months (approx.)

iv) Number of orders/year $= \frac{12}{3} = 4$

v) The time of manufacture $= \frac{Q^*}{R} = \frac{4,470}{3,000}$ = 1.5 months

vi) The optimum annual cost

= Items cost + Ordering cost + Holding cost

$$= (18,000 \times 2) + \left(\frac{18,000}{4,470} \times 500\right) + \left(\frac{2,235}{36,000}(36,000 - 18,000) \times 1.8\right)$$

= Rs. 40,025.



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**P 2)** **A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that, when he starts a production run, he can produce 25,000 bearings per day. The holding cost of bearing in stock is Rs. 0.02 per year. Set up cost of a production run is Rs. 18. How frequently should production run be made.**

*Sol. :* Given,

Demand rate = D = 10,000 units/day

Production rate = R = 25,000 units/day

Set up cost $C_2$ = Rs. 18/order

Carrying cost $C_3$ = 0.02/unit/year

= Re. 0.000055/unit day

$$EOQ : Q^* = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{R}{R-D}}$$

$$= \sqrt{\frac{2 \times 10,000 \times 18}{0.000055} \cdot \frac{25,000}{25,000 - 10,000}}$$

= 1,04,447 bearings

$$\text{Time between orders} = \frac{Q^*}{D} = \frac{1,04,447}{10,000} = 10.4 \text{ days (approx)}$$

$$\text{Time of manufacture} = \frac{Q^*}{R} = \frac{1,04,447}{25,000} = 4 \text{ days (approx.)}$$

∴ The manufacture cycle (or) production cycle starts at an interval of 10.4 days and production continues for 4 days.

In each cycle a batch of 1,04,447 bearings are produced.

**P 3)** **An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. The ordering cost is Rs. 100/order and carrying cost is Re. 0.01 per unit per day. Determine the EOQ. for one run. Calculate the time of cycle and minimum total cost for one run.**

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*Sol. :* Given data,

Demand rate D = 25 items/day

Production rate R = 50 items/day

Ordering cost $C_2$ = Rs. 100/order

Carrying cost $C_3$ = Rs. 0.01/item/day

$$\text{E.O.Q for one run} = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{R}{R-D}}$$

$$= \sqrt{\frac{2 \times 25 \times 100}{0.01} \times \frac{50}{50-25}}$$

$$= 1,000 \text{ items}$$

$$\text{Time of cycle} = \frac{Q^*}{D} = \frac{1,000}{25} = 40 \text{ days}$$

$$\text{Minimum total cost (daily cost)} = \sqrt{\frac{2DC_2C_3(R-D)}{R}}$$

$$= \sqrt{\frac{2 \times 25 \times 100 \times 0.01 \times (50-25)}{50}} = \text{Rs. } 5$$

$\therefore$ Minimum total cost per run = 5 × 40

= Rs. 200

**P 4)** **The annual demand for a product is 1,00,000 units. The production rate is 2,00,000 units/year. The set up cost is Rs. 500 per production run. The variable production cost in Rs. 10 per item. The carrying cost per year per unit is 20% of its value. Determine the optimum production lot size and the length of the product run.**

*Sol. :* Given,

Demand rate D = 1,00,000 units

Product rate R = 2,00,000 units

Carrying cost $C_3$ = 20% of item cost

$$= \frac{20}{100} \times 10 = \text{Rs. } 2$$



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Optimum production lot size $Q^* = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{R}{R-D}}$

$$= \sqrt{\frac{2 \times 1,00,000 \times 500}{2} \cdot \frac{2,00,000}{2,00,000 - 1,00,000}}$$

$$= 10,000 \text{ units}$$

Length of production run $= \frac{Q^*}{R}$

$$= \frac{10,000}{2,00,000} = 0.05$$

### 5.5.1.2 Model II : EOQ Problem with Shortages

#### 5.5.1.2.1 Model (II a) Demand Rate Uniform, Production Rate Infinite, Shortages Allowed

In certain situations inventory shortages are permisible as they can be back ordered. The assumptions of this model are same as those of Model (I.a), except the shortages are allowed and consequently a shortage cost is incurred.

Let

D = Demand rate,

$C_2$ = Cost of setting up a production run,

$C_3$ = Cost of holding the item per unit time,

$C_4$ = Shortage cost per item per unit time,

Q = Number of items produced per production run,

t = Interval between runs.

Let the shortage be denoted by S for every cycle.


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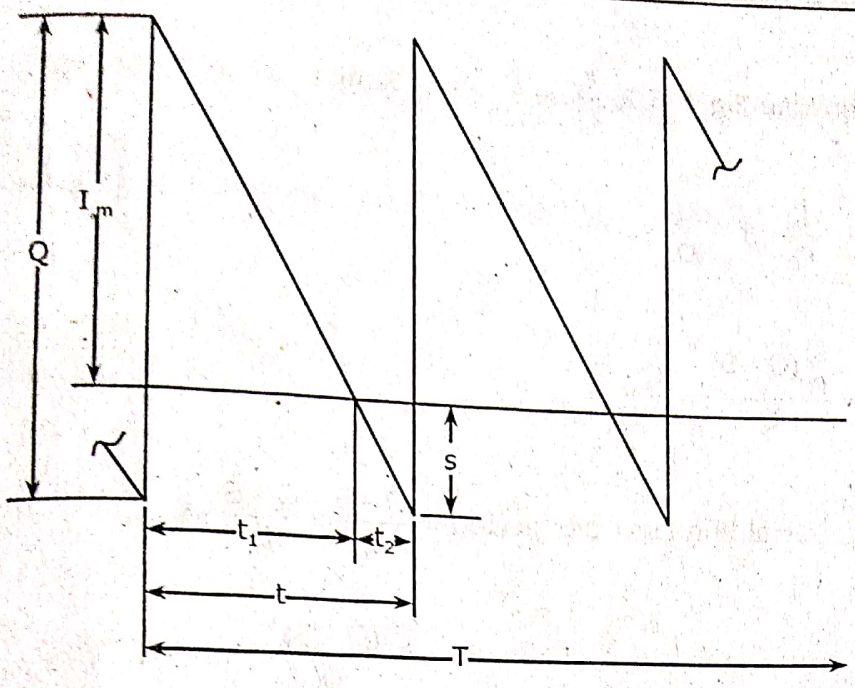



**Fig. 5.5.2**

Here the total cost per period = Item cost + Order cost + Holdig cost + Shortage cost

The total time period T is divided into n equal time intervals, say of interval t. This time interval t is further divided into two parts $t_1$ and $t_2$ such that $t = t_1 + t_2$.

where,

$t_1$ = The time interval during which items are drawn from inventory as needed,

$t_2$ = The time iterval during which orders for the item are being acumulated but not filled.

Let the maximum inventory be denoted by $I_m$ and is given by,

$I_m = Q - S$

Now item cost/period = $C_1Q$ (= Item cost × Number of items/period)

Order cost per period = $C_2$

Holding cost per period = Average inventory × $t_1$ × Holding cost/unit/unit time

$$= \frac{I_m}{2} \times t_1 \times C_3$$


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From the Fig. 5.5.2, $\frac{t_1}{I_m} = \frac{t}{Q}$ (Similar traignle concept)

$$t_1 = \frac{tI_m}{Q}, \quad t = \frac{Q}{D}$$

$$t_1 = \frac{\frac{Q}{D}(Q-S)}{Q} = \frac{Q-S}{D}$$

$\therefore$ Holding cost per period $= \frac{Q-S}{2} \times \frac{Q-S}{D} \times C_3$

$$= \frac{C_3(Q-S)^2}{2D}$$

Shortage cost per period = Average shortages/period $\times$ $t_2$ $\times$ Shortage cost

$$= \frac{S}{2} \times t_2 C_4$$

Form the Fig. 5.5.2, $\frac{t_2}{t} = \frac{Q-I_m}{Q}$

$$t_2 = \frac{tS}{Q} = \frac{\frac{Q}{D}S}{Q} = \frac{S}{D}$$

$\therefore$ Shortage cost per period $= \frac{S}{2} \times \frac{S}{D} \times C_4$

$$= \frac{C_4 S^2}{2D}$$

Hence the inventory cost per period $= C_1Q + C_2 + \frac{C_3(Q-S)^2}{2D} + \frac{Q_4S^2}{2D}$

$\therefore$ Inventory cost per unit time is obtained by dividing the above t (or) $\frac{Q}{D}$

$$C = C_1D + \frac{C_2D}{Q} + \frac{C_3(Q-S)^2}{2Q} + \frac{C_4S^2}{2Q}$$


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This average cost unit time is a function of two variables Q and S.

Optimum values of Q and S are obtained by equating $\frac{\partial C}{\partial Q}$ and $\frac{\partial C}{\partial S}$ to zero,

$$\frac{\partial C}{\partial Q} = -\frac{C_2 D}{Q^2} + \frac{C_3}{2} - \frac{C_3 S^2}{2Q^2} - \frac{C_4 S^2}{2Q^2}$$

$$\Rightarrow 0 = -\frac{C_2 D}{Q^2} + \frac{C_3}{2} - \frac{S^2}{2Q^2}(C_3 + C_4) \quad \ldots (1)$$

$$\Rightarrow \frac{\partial C}{\partial Q} = \frac{C_3}{2Q} 2(Q - S)(-1) + \frac{2C_4 S}{2Q}$$

$$\Rightarrow 0 = \frac{C_3}{Q}(S - Q) + \frac{C_4 S}{Q}$$

$$\therefore \frac{C_4 S}{Q} = \frac{C_3}{Q}(Q - S)$$

$$\Rightarrow C_4 S + C_3 S = C_3 Q$$

$$\Rightarrow S = \frac{C_3 Q}{C_3 + C_4}$$

Substituting this value of S in (1), we get

$$0 = \frac{-C_2 D}{Q^2} + \frac{C_3}{2} - \frac{C_3 + C_4}{2Q^2} \times \frac{C_3^2 Q^2}{(C_3 + C_4)^2}$$

$$= \frac{C_2 D}{Q^2} + \frac{C_3}{2} - \frac{C_3^2}{2(C_3 + C_4)}$$

$$\frac{C_2 D}{Q^2} = \frac{C_3}{2} - \frac{C_3^2}{2(C_3 + C_4)}$$

$$= \frac{C_3^2 + C_3 C_4 - C_3^2}{2(C_3 + C_4)} = \frac{C_3 C_4}{2(C_3 + C_4)}$$

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$$Q^2 = \frac{2(C_2 + C_4)}{C_3 C_4} C_2 D$$

$$Q = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{C_3 + C_4}{C_4}}$$

Thus the EOQ is,

$$Q^* = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{C_3 + C_4}{C_4}}$$

and $S^* = \frac{C_3 Q^*}{C_3 + C_4} = \sqrt{\frac{2DC_2}{C_4} \cdot \frac{C_3}{C_3 + C_4}}$

Optimal time interval t between run (i.e.,) time between receipt of orders (when to order) is,

$$t^* = \frac{Q^*}{D} = \sqrt{\frac{2C_2}{DC_4} \cdot \frac{C_3}{C_3 + C_4}}$$

### 5.5.1.2.2 Solved Problems

**P 1)** **The demand of an item is uniform at a rate of 25 units per month. The fixed cost is Rs. 30 each time a production is made. The production cost is Rs. 2 per item and the inventory carrying cost is 50 paise per unit per month. If the shortage cost is Rs. 3 per item per month, determine how often to make a production run and of what size?**

*Sol. :* Given,

Demand rate D = 25 units/month

Production cost $C_1$ = Rs. 2/unit

Carrying const $C_3$ = 0./5/unit/month

Shortage cost $C_4$ = 3/unit/month

production setup cost $C_2$ = Rs. 30/order



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i) The optimal EOQ is given by,

$$Q^* = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{C_3 + C_4}{C_4}}$$

$$= \sqrt{\frac{2 \times 25 \times 30}{0.5} \times \frac{0.5 + 3}{3}}$$

= 60 units (approx.)

ii) Optimal time between order is given by

$$t^* = \frac{Q^*}{D} = \frac{60}{25} = 2.4 \text{ months}$$

**P 2) The demand rate for a particular item is 18,000 units per year. The cost of one purchase is Rs. 400. The carrying cost is Rs. 1.2/unit/year. The cost of the item is Rs. 1 per item. The shortage cost is Rs. 5 per unit per year. Calculate,**

**i) The optimum order quantity.**

**ii) Optimum time between order.**

**iii) Number of orders per year.**

**iv) Optimum shortages.**

**v) The time of items being held.**

**vi) The optimum annual cost.**

*Sol. :* Given,

Demand rate D = 18,000/year

Item cost $C_1$ = Rs. 1/item

Ordering cost $C_2$ = Rs. 400/order

Carrying cost $C_3$ = Rs. 1.2/unit/year

Shortage cost $C_4$ = Rs. 5/year/unit


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i) The optimum order quantity is given by,

$$Q^* = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{C_3 + C_4}{C_4}}$$

$$= \sqrt{\frac{2 \times 18,000 \times 400}{1.2} \times \frac{1.2 + 5}{5}}$$

$$= 3,857 \text{ units}$$

ii) The optimum time between orders is given by,

$$t^* = \frac{Q^*}{D}$$

$$= \frac{3,857}{18,000} \text{ years}$$

$$= \frac{3,857 \times 12}{18,000} \text{ months} = 2.57 \text{ months}$$

iii) Number of orders per year is given by,

$$\text{No. of orders} = \frac{12}{2.57} = 4.66$$

iv) The optimum shortages is given by,

$$S^* = \sqrt{\frac{2DC_2}{C_4} \cdot \frac{C_3}{C_3 + C_4}}$$

$$= \sqrt{\frac{2 \times 18,000 \times 400}{5} \times \frac{1.2}{1.2 + 5}}$$

$$= 747 \text{ items}$$

v) The maximum inventory is,

$$I_m = Q^* - S = 3,857 - 747 = 3,110 \text{ units}$$


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vi) The time of items being held,

$$= \frac{\text{Maximum inventory}}{D}$$

$$= \frac{3,110}{18,000} \text{ year}$$

$$= \frac{3,110 \times 12}{18,000} \text{ month} = 2.07 \text{ months}$$

vii) Total cost per order = Item cost + Ordering cost + Carrying cost + Shortage cost

$$= C_1Q^* + C_2 + \frac{C_3(Q^* - S)^2}{2D} + \frac{C_4S^{*2}}{2D}$$

$$= (1 \times 3,857) + 400 + \frac{1.2(3,110)^2}{2(18,000)} + \frac{5(747)^2}{2(18,000)}$$

$$= 3,857 + 400 + 322.4 + 77.5$$

$$= \text{Rs. } 4,656.9$$

$\therefore$ Annual cost = 4.66 × 4,656.9 = 21,701.15

**P 3)** **A manufacturing company has to supply 3000 units per year to a customer who does not have enough space for storing the material. A penalty of Rs. 40 will be levied when the supplier fails to supply the material. The inventory carrying cost is Rs. 20/unit/month and the ordering cost is Rs. 400/run. Calculate the expected number of shortages at the end of each scheduling.**

***Sol. :*** Given,

Demand rate D = 3,000 units/year = 250 units/ month

Ordering cost $C_2$ = Rs. 400/order

Carrying cost $C_3$ = Rs. 20/unit/month

Shortage cost $C_4$ = Rs. 40/unit/month


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Number of shortages $= \sqrt{\frac{2DC_2}{C_4} \cdot \frac{C_3}{C_3 + C_4}}$

$$= \sqrt{\frac{2 \times 250 \times 400}{40} \cdot \frac{20}{20 + 40}}$$

= 41 units/(approx.)/period

**P 4) A particular item has a demand of 9,000 unit/year. The cost of one procurrement is Rs. 100 and the holding cost if Rs. 240/unit/year. The replacement is instantanesus and shortage is given as Rs. 5/unit/year. Determine,**

**i) The economic lot size.**

**ii) The number of orders per year.**

**iii) The time between orders.**

**iv) The total cost per year if the cost of one unit is Re. 1.**

*Sol. :* Given,

D = 9,000 units/year

$C_1$ = Re. 1/unit

$C_2$ = Rs. 100/order

$C_3$ = Rs. 2.40/unit/year

$C_4$ = Rs. 5/unit/year

i) $Q^* = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{C_3 + C_4}{C_4}}$

$$= \sqrt{\frac{(2 \times 9,000 \times 100)}{2.40} \frac{2.40 + 5}{5}}$$

= 1,053 units/run

ii) Number of orders/year $= \frac{9,000}{1,053} = 8.55$



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iii) The time between order $= \frac{Q^*}{D} = \frac{1,053}{9,000} = 0.117$ year

iv) Total cost / period $= C_1Q^* + C_2 + \frac{C_3(Q^* - S^*)^2}{2D} + \frac{C_4S^{*2}}{2D}$

$$S^* = \sqrt{\frac{2DC_2}{C_4} \cdot \frac{C_3}{C_3 + C_4}}$$

$$= \sqrt{\frac{2 \times 9,000 \times 100}{5} \cdot \frac{2.4}{2.4 + 5}} = \sqrt{1,16,756.3} = 342$$

$\therefore$ Total cost/period

$$= 1 \times 1,053 + 100 + \frac{2.4(1,053 - 342)^2}{2(9,000)} + \frac{5(342)^2}{2 \times 9,000}$$

$= 1,252.90$

Total annual cost = Number of orders/year × Total cost/period

$= 1,252.9 \times 8.55 =$ Rs. 10,712

**P 5)** **A commodity is to be supplied at a constant rate of 200 units per day. Supply of any amounts can be had at any required time, but each ordering costs Rs. 50.00, cost of holding and commodity in inventory is Rs. 2.00 per unit per day while the delay in the supply of the item induces a penalty of Rs. 10.00 per unit per delay of 1 day. Find the optimal policy (q, t) where t is the re-order cycle period and q is the inventory level after reorder. What would be the best policy, if the penalty cost becomes ∞.**

***Sol. :*** Given,

D = 200 units/day

$C_2$ = Rs. 50/order

$C_3$ = Rs. 2.00/unit/day

$C_4$ = Rs. 10.00/unit/day


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$$q = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{C_3 + C_4}{C_4}} = \sqrt{\frac{2 \times 200 \times 50}{2} \cdot \frac{2+10}{10}} = 109.5 \text{ units}$$

$$t = \frac{Q^*}{D} = \frac{109.5}{200} = 0.5475 = 0.55$$

(i.e.,) Optimum order quantity of 109.5 unity must be supplied after 0.55 days.

If the penalty costs become ∞,

(i.e.,) if $C_4 = \infty$, (shortages are prohibited) then

$$q = \sqrt{\frac{2DC_2}{C_3}} = \sqrt{\frac{2 \times 200 \times 50}{2}} = 100 \text{ units}$$

and $t = \frac{q}{D} = \frac{100}{2} = 0.5$ day

### 5.5.1.2.3 Model (II b) Demand Rate Uniform, Production Rate Infinite, Shortages Allowed, Time Interval Fixed

Let the time interval t be fixed (i.e.,) inventory is to be replenished (produced) after every fixed time period t. Assume that the items are supplied or produced at the rate of r units per unit of time during this fixed time interval t. All other assumptions of Model (II.a) hold good.

Total inventory during time $t = \frac{1}{2} I_m t_1$

Total inventory holding cost during time $t = \frac{1}{2} I_m t_1 \times C_3$

Total amount of shortages during time $t = \frac{1}{2}(Q - I_m)t_2$

Total shortage cost during time $t = \frac{1}{2}(Q - I_m)t_2 \times C_4$

Total cost during time $t = \frac{1}{2} I_m t_1 C_3 + \frac{1}{2}(Q - I_m)t_2 C_4$



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Total average cost per unit of time $= \frac{1}{t}\left[\frac{1}{2}I_m t_1 C_3 + \frac{1}{2}(Q - I_m)t_2 C_4\right]$

Using the relationships of similar triangle $t_1 = \frac{I_m t}{Q}$, $t_2 = \frac{Q - I_m}{Q}t$

$\therefore$ Total average cost per unit of time $= \frac{1}{t}\left[\frac{1}{2}I_m \frac{I_m t}{Q} C_3 + \frac{1}{2}(Q - I_m)\frac{Q - I_m}{Q} t C_4\right]$

$$TC = \frac{1}{2Q} I_m^2 \, C_3 + \frac{1}{2Q}(Q - I_m)^2 \, C_4$$

TC will be optimal if

$$\frac{d}{dI_m}(TC) = 0 \text{ and } \frac{d^2}{dI_m^2}(TC) > 0$$

$$\frac{d}{dI_m}(TC) = \frac{2}{2Q} I_m \, C_3 + \frac{1}{2Q} 2(Q - I_m)(-1)C_4$$

$$0 = \frac{I_m}{Q} C_3 + \frac{I_m - Q}{Q} C_4$$

$$C_3 I_m + C_4 I_m - C_4 Q = 0$$

$$(C_3 + C_4) I_m = C_4 Q$$

$$I_m = \frac{C_4}{C_3 + C_4} Q$$

$$\frac{d^2}{dI_m^2}(TC) = \frac{C_3}{Q} + \frac{C_4}{Q} = \frac{C_3 + C_4}{Q} > 0$$

$\therefore$ Optimum value $I_m$ is given by



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$$I_m = \frac{C_4}{C_3 + C_4} Q$$

$$I_m = \frac{C_4}{C_3 + C_4} rt$$

Minimum average cost/unit time

$$= \frac{1}{2Q} I_m^2 C_3 + \frac{1}{2Q} (Q - I_m)^2 C_4$$

$$= \frac{1}{2Q} \left( \frac{C_4}{C_3 + C_4} \right)^2 Q^2 C_3 + \frac{1}{2Q} \left( Q - \frac{C_4}{C_3 + C_4} Q \right)^2 C_4$$

$$= \frac{1}{2} C_3 Q \left( \frac{C_4}{C_3 + C_4} \right)^2 + \frac{1}{2} Q C_4 \left( \frac{C_3}{C_3 + C_4} \right)^2$$

$$= \frac{1}{2} Q \frac{C_3 C_4^2}{(C_3 + C_4)^2} + \frac{1}{2} Q \frac{C_4 C_3^2}{(C_3 + C_4)^2}$$

$$= \frac{1}{2} Q \frac{C_3 C_4 (C_3 + C_4)}{(C_3 + C_4)^2}$$

$$= \frac{1}{2} Q \frac{C_3 C_4}{C_3 + C_4}$$

$$TC^* = \frac{1}{2} \frac{C_3 C_4}{C_3 + C_4} rt$$

**Note :** In this model, the set-up cost $C_2$ is not considered, because of its being fixed as the time of one production run is fixed.



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### 5.5.1.2.4 Solved Problems

**P 1)** **A contractor has to supply diesel engines to a truck manufacturer at a rate of 20 engines per day. The penalty in the contract is Rs. 100/engine/day late for missing the scheduled delivery date. The cost of holding an engine in stock for one month is Rs. 150. His production process is such that each month he starts procuring a batch of engines through the agencies and all are available for supply after the end of the month. Determine the maximum inventory level at the beginning of each month.**

***Sol. :*** Given,

$C_3$ = Rs. 150/engine/month

= Rs. 150/engine/30 days = Rs. 3/engine/day

$C_4$ = 100/engine/day

r = 20 engines/day

Here time t is fixed, t = 30 days

$\therefore$ Maximum inventory level at the beginning of each month

$$= \frac{C_4}{C_3 + C_4} Q^* = \frac{C_4}{C_3 + C_4} rt$$

$$= \frac{100}{5+100} \times 20 \times 30 = 571 \text{ engines (approx.)}$$

**P 2)** **A contractor supplies diesel engines to a truct manufacturers at the rate of 25 per day. He has to pay a penalty of Rs. 10 per engine per day for missng the scheduled delivery date. He finds the cost of holding a complete engine in stock is Rs. 16/ month. The manufacturing of engines starts with the beginning of the month and is completed at the end of the month. What should be the inventory level at the beginning of each month.**

***Sol. :*** Given,

r = 25 engines/day

$C_3$ = Rs. 16/engine/month

$$= \text{Rs. } \frac{16}{30} \text{ / engine / day}$$


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$C_4$ = Rs. 10/engine/day

t = One month = 30 days

The inventory level at the beginning of each month

$$= \frac{C_4}{C_3 + C_4} rt$$

$$= \frac{10}{0.53 + 10} \times 25 \times 30$$

= 712 engines

### 5.5.1.2.5 Model (II c) Demand Rate Uniform, Production Rate Finite, Shortages Allowed

In this model all the assumptions are same as in model (II. a) except that the rate of replenishment of inventory is finite (i.e.,) production rate is finite.

Let

D = Number of items require per unit time,

R = Number of items produced per unit time,

$C_1$ = Cost of item,

$C_2$ = Cost of ordering per production run,

$C_3$ = Cost of holding the item per unit time,

Q = Number of items produced per production run,

Q = Dt,

t = Interval between runs,

$I_m$ = Number of items that form inventory at the beginning of time inverval t.

Each production run of length t is divided into two parts $t_1$ and $t_2$ which are further subdivided into two parts say $t_{11}$, $t_{12}$, $t_{21}$ and $t_{22}$.

$t_{11}$ = Time interval during which inventory is building up at a constant rate of R – D units per unit time.


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There is no replenishment during time $t_{12}$, inventory decreases at a constant rate of D per unit of time.

$t_{21}$ = Time interval during which shortage is building up at a constant rate of D per unit of time.

$t_{22}$ = Time interval during which shortages are filled immediately at the rate of R – D units per unit of time.

Total time taken during this cycle is $t = t_{11} + t_{12} + t_{21} + t_{22}$

The graphical representation of the situation is as shown in Fig. 5.5.3.

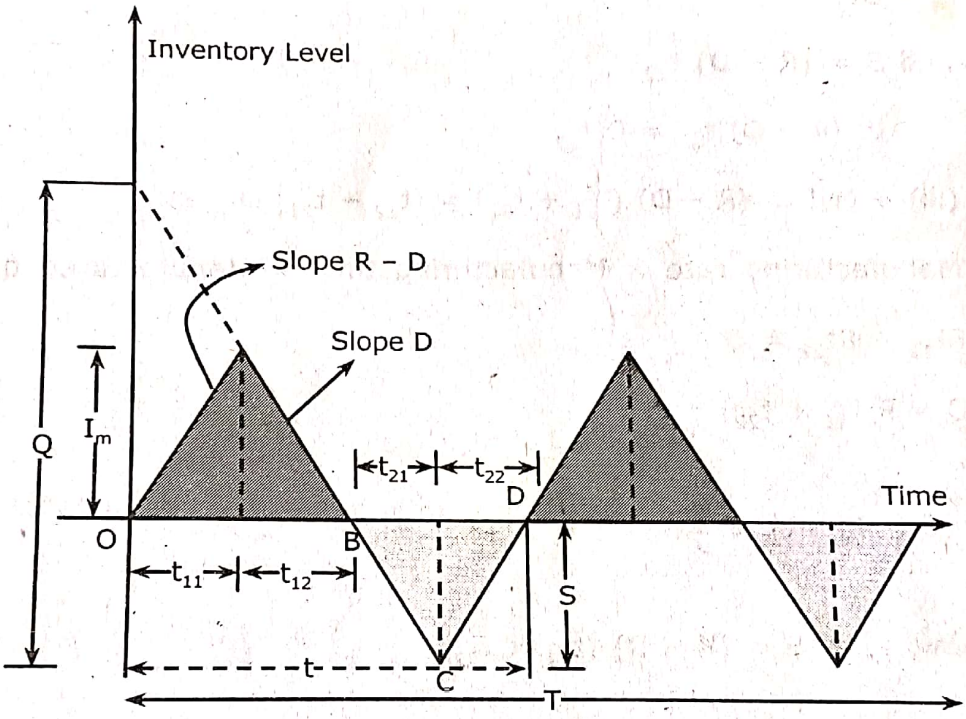



**Fig. 5.5.3**

Holding cost during time invterval $t = C_3(t_{11} + t_{12})\frac{I_m}{2}$

(= $C_3$ × area of $\Delta^{le}$ OAB)

Shortage cost during time interval $t = C_4(t_{21} + t_{22})\frac{S}{2}$

(= area of $\Delta^{le}$ BCD × $C_4$)

Set up cost during time interval t = $C_2$



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Total inventory cost per period t is

$$C_2 + C_3(t_{11} + t_{12})\frac{I_m}{2} + C_4(t_{21} + t_{22})\frac{S}{2} \quad \ldots (1)$$

Now $I_m = (R - D)\, t_{11}$ ... (i)

and $I_m = D\, t_{12}$ ... (ii)

$\therefore (R - D)\, t_{11} = D\, t_{12}$ ... (iii)

$S = Dt_{21}$ ... (iv)

and $S = (R - D)\, t_{22}$ ... (v)

$\therefore (R - D)\, t_{22} = D\, t_{21}$ ... (vi)

(iii) + (vi) $\Rightarrow (R - D)\,(t_{11} + t_{22}) = (t_{12} + t_{21})\, D$

Manufacturing rate × Manufacturing time = Manufactured quantity

$Rt_{11} + Rt_{22} = Q$

$Q = R(t_{11} + t_{22})$

$$\therefore t_{11} + t_{22} = \frac{Q}{R} \quad \ldots (vii)$$

Now, $I_m + S = (R - D)\,(t_{11} \times t_{22})$

$$= \frac{Q}{R}(R - D) \quad \text{from (vii)}$$

$$\therefore I_m = \frac{Q}{R}(R - D) - S \quad \ldots (viii)$$

From (i) and (ii), $t_{11} + t_{12} = \frac{I_m}{R - D} + \frac{I_m}{D}$

$$= \left[\frac{1}{R - D} + \frac{1}{D}\right]\left[\frac{Q}{R}(R - D) - S\right] \quad \text{from (viii)}$$


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From (iv) and (v),

$$t_{21} + t_{22} = \frac{S}{D} + \frac{S}{R-D}$$

$$= S\left[\frac{1}{D} + \frac{1}{R-D}\right] \quad \ldots \text{(ix)}$$

$\therefore t_{11} + t_{12} + t_{21} + t_{22}$

$$= \left(\frac{1}{R-D} + \frac{1}{D}\right)\left(\frac{Q}{R}(R-D) - S\right) + S\left(\frac{1}{D} + \frac{1}{R-D}\right)$$

$$= \left(\frac{1}{R-D} + \frac{1}{D}\right)\frac{Q}{R}(R-D)$$

$$= \frac{R}{D(R-D)}\frac{Q}{R}(R-D) = \frac{Q}{D}$$

(1) $\Rightarrow$ The total inventory cost per period t

$$= C_2 + C_3(t_{11} + t_{12})\frac{I_m}{2} + C_4(t_{21} + t_{22})\frac{S}{2}$$

$\therefore$ Total cost per unit of time

$$TC = \frac{1}{t}\left[C_3(t_{11} + t_{21})\frac{I_m}{2} + C_4(t_{21} + t_{22})\frac{S}{2}\right]$$

where $t = t_{11} + t_{12} + t_{21} + t_{22}$

Substituting the value of $I_M$, $t_{11} + t_{12}$, $t_{21} + t_{22}$ and $t_{11} + t_{12} + t_{21} + t_{22}$ and simplifying, we get

$$TC = \frac{1}{2Q}\frac{R}{R-D}\left[C_3\left\{Q \cdot \frac{R-D}{R} - S\right\}^2 + C_4S^2\right] + \frac{D}{Q}C_2$$



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This cost will be minimum if,

$$\frac{\partial}{\partial Q}(TC) = 0 \text{ and } \frac{\partial^2}{\partial Q^2}(TC) > 0$$

and $$\frac{\partial}{\partial S}(TC) = 0 \text{ and } \frac{\partial^2}{\partial S^2}(TC) > 0$$

Using these two conditions, the final results are given below,

Optimum (EOQ) $$Q^* = \sqrt{\frac{2DC_2}{C_3} \cdot \frac{R}{R-D} \cdot \frac{C_3 + C_4}{C_4}}$$

Optimum shortages $$S^* = \sqrt{\frac{2DC_2}{C_4} \cdot \frac{R-D}{R} \frac{C_3}{C_3 + C_4}}$$

Time between runs $$t^* = \frac{Q^*}{D} = \sqrt{\frac{C_3 + C_4}{C_4} \cdot \frac{R}{R-D} \cdot \frac{2C_2}{C_3 D}}$$

Total minimum production inventory cost,

$$TC^* = \sqrt{\frac{C_4}{C_3 + C_4} \cdot \frac{R-D}{R} 2C_2 C_3 D}$$

### 5.5.1.2.6 Solved Problems

**P 1) The demand for an item in a company is 18,000 units/year. The company can produce the item at the rate of 3,000/month. The set up cost is Rs. 500/set up and the holding cost is 15 paise/unit/month. The shortage cost is Rs. 20/unit/year. Determine,**

**i) The optimum manufacturing quantity.**

**ii) Number of shortages.**

**iii) Manufacturing time.**

**iv) The time between set ups.**


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*Sol. :* Given,

D = 18,000 units/year

= 1,500 units/month

R = 3,000 units/month

$C_2$ = Rs. 500/set up

$C_3$ = Re. 0.15/unit/month

$C_4$ = Rs. 20/unit/year

= Rs. 1.67/unit/month

i) $$Q^* = \sqrt{\frac{2DC_2}{C_3}\frac{R}{R-D}\frac{C_3+C_4}{C_4}}$$

$$= \sqrt{\frac{2\times 1,500\times 500}{0.15}\cdot\frac{3,000}{3,000-1,500}\frac{0.15+1.67}{1.67}}$$

= 4669 units

ii) $$S^* = \sqrt{\frac{2DC_2}{C_4}\frac{R-D}{R}\frac{C_3}{C_3+C_4}}$$

$$= \sqrt{\frac{2\times 1,500\times 500}{1.67}\cdot\frac{3,000-1,500}{3,000}\frac{0.15}{0.15+1.67}} = 193 \text{ units}$$

iii) Manufacturing time $= \frac{Q^*}{R} = \frac{4,669}{3,000} = 1.56$ months

iv) The time between set ups $= \frac{Q^*}{D} = \frac{4,669}{1,500} = 3.12$ months



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**P 2)** **A company has a demand of 12,000 units/year for an item and it can produce 2000 such items per month. The cost of one set up is Rs. 400 and the holding cost per unit per month is 0.15. The shortage cost of one unit is Rs. 20/year.**

**Calculate the optimum lot size.**

***Sol. :*** Given,

$D = 12,000$ units/year

$= 1,000$ units/month

$R = 2,000$ units/month

$C_2 = $ Rs. 400/set up

$C_3 = $ Re. 0.15/unit/month

$C_4 = $ Rs. 20/unit/year = Rs. 1.67/unit/month

i) $$Q^* = \sqrt{\frac{2DC_2}{C_3}\frac{R}{R-D}\frac{C_3+C_4}{C_4}}$$

$$= \sqrt{\frac{2\times 1,000\times 400}{0.15}\frac{2,000}{2,000-1,000}\frac{0.15+1.67}{1.67}}$$

$= 3,409$ units.

## 5.5.2 Probabilistic Inventory Model

So far we have discussed the models where the future demand was known precisely. In some situations the demand is not known exactly but the probability distribution of demand is somehow known. Here the demand is a random variable. It may be described by either a discrete or continuous probability distribution. The probability distribution of future demand is determined from the data collected from past experience.

Since the future demand is known only in terms of probability distribution, we choose the policies that minimize the expected cost rather than the actual inventory cost. The optimum order level will thus be derived by maximizing the total exptected cost rather than the actual total inventory cost. Expected costs are obtained by multiplying the actual costs for a particular situation with the probability of occurrence of that situation.


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## Model (III a) Discrete Probability Distribution of Demand

**(Instantaneous demand, setup cost zero, stock levels discrete and lead time zero).**

Here we shall use the following notations,

D = Discrete demand rate. (Number of units of an item demanded).

$P_D$ = Discrete probability distribution of the demand in a period.

$C_1$ = Unit purchase price.

$C_2$ = Set-up cost or ordering cost to place an order.

$C_3$ = Carrying cost per unit of item left unsold.

$C_4$ = Shortage cost for under stocking an item.

$C_5$ = Unit cost of over ordering (an opportunity loss associated with each unit left unsold at the end of the period)

$C_6$ = Unit cost of under ordering (an opportunity loss due to not meeting the demand which includes cost of loss good will etc.).

The objective is to determine the optimal number of units ($Q^*$) of an item with which to begin the period.

As D is a random variable, either $D < Q$ (or) $D > Q$.

If D is less than Q units, there will be surplus of units of item and we have to pay a penalty of having these unneeded units. If D is more than Q, there will be shortage of items and each short item will cause an opportunity loss of not meeting the demand.

The optimal order quantity $Q^*$ is given by,

$$G(Q^* - 1) \le \frac{C_6}{C_5 + C_6} \le G(Q^*) \qquad \ldots (1)$$

G(Q) is the cumulative probability distribution for the discrete probability distribution of demand.


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To calculate the optimal value of Q, we suggest the following steps,

1) Calculate the cost of over ordering and under ordering. (i.e.,) $C_5$ and $C_6$ using the following formulae,

$C_5 = C_1 + C_3 - V$

V = Salvage value (Resale value) for excess units.

$$C_6 = S - C_1 - \frac{C_3}{2} + C_4$$

S = Unit selling price.

2) Compute the cumulative probability distribution G(Q).

3) Take two values $(Q^* - 1)$ and $Q^*$ as stated in (I) and choose the maximum value as the optimal level of Q to start the period.

### 5.5.2.1.1 Solved Problems

**P 1) A newspaper boy buys papers for 5 paise each and sells them for 6 paise each. He cannot return unsold newspapers. Daily demand D for newspapers follows the distribution,**

| D | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|
| $P_D$ | 0.05 | 0.15 | 0.40 | 0.20 | 0.10 | 0.05 | 0.05 |

**If each days demand is independent of the previous day's, how many papers should be ordered each day.**

*Sol. :* Let Q be the number of newspapers ordered per day and D be the demand for it i.e., the number of newspapers actually sold per day.

Given,

$C_5$ = Re (0.05 + 0 − 0) = 0.05

$C_6$ = 0.06 − 0.05 − 0 + 0 = Re. 0.01

(Here S = 0.06, $C_1$ = 0.05)

$$\therefore \frac{C_6}{C_5 + C_6} = \frac{0.01}{0.05 + 0.01} = 0.167$$

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