

POWER SYSTEM STABILITY

Voltage stability $\rightarrow \pm 5\%$

Frequency stability $\rightarrow 1\%$

Angular stability $\rightarrow$

(a) steady state stability

(b) Transient stability

(c) Dynamic stability

* stability of a power system:- (IMP)

It is the ability of the power system to bring itself back to a stable condition. After being disturbed is called "stability of a power system".

— Based on the magnitude of disturbances the stabilities are classified into 3 types:-

(a) steady state stability

(b) Transient stability

(c) Dynamic stability

— (a) steady state stability:- It is the ability of the power system to bring itself back to a stable condition after small disturbances. The steady state stability is the consideration of variation of loads.

— (b) Transient stability:- It is the ability of the system to bring itself back to the stable condition after large disturbances. The transient stability is considered with sudden & large changes in N/w conditions.

* The large disturbances can occur due to sudden load changes of switching operation and faults etc.

(c) Dynamic stability:-

This is the extension of steady state stability. The dynamic stability is concerned with small disturbances existing for long time.

To differentiate steady state stability & dynamic stability, by using automatic control devices such as speed Governors & voltage regulators respectively.

- In dynamic stability, stability obtained by the use of automatic control devices, whereas in steady state stability, stability obtained without use of automatic control devices.

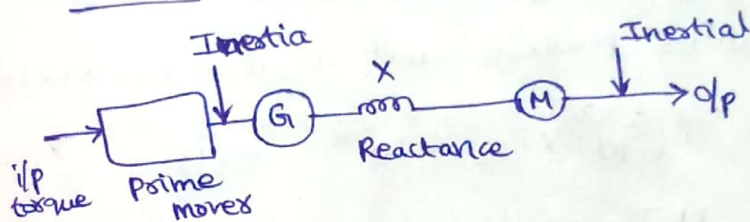
- stability limit:-

stability limit is the max. power that can be transferred from sending end to the receiving end without loss of synchronism.

* steady state stability limit:-

The maximum power that can be transferred without the system becoming unstable when a small disturbance occurs in a power system N/w.

→ Essential factors in a power system stability:-



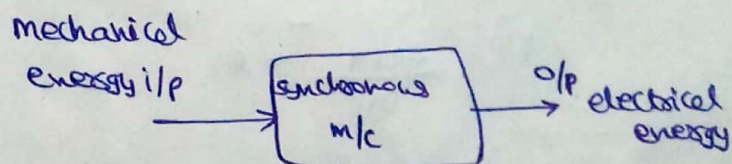
① Mechanical factors

- i/p torque to the prime mover & generators
- Inertia to the generators
- Inertia to the motors
- o/p torque to the motor

② Electrical factors

- Internal voltages of generator
- Reactance of T/m line.
- Internal voltages of motors

→ Constants/Dynamics of a synchronous machine



K.E of the rotor is, $KE = \frac{1}{2} J \omega_{sm}^2$ Joules

$$= \frac{1}{2} \left(J \omega_{sm}^2 \times \frac{10^6}{10^{-6}} \right) J$$

$$KE = \frac{1}{2} \left(J \omega_{sm}^2 \times 10^{-6} \right) MJ \quad \text{--- (1)}$$

where, $J \rightarrow$ moment of inertia of the rotor

$\omega_{sm} \rightarrow$ Angular speed of the rotor in mec. rad/sec

$$\omega_s = \left(\frac{P}{2} \right) \omega_{sm} \quad (\Theta_{ele} = \left(\frac{P}{2} \right) \Theta_{mec})$$

where, $P \rightarrow$ No. of poles &

$\omega_s \rightarrow$ Angular speed in electrical rad/sec

sub ② in ①, $K.E = \left[\frac{1}{2} J \left(\frac{2}{P} \right)^2 \omega_s^2 \times 10^{-6} \right] \text{ MJ}$

$$= \frac{1}{2} \left[J \left(\frac{2}{P} \right)^2 \omega_s \times 10^{-6} \right] \omega_s \text{ MJ}$$

$$KE = \frac{1}{2} M \omega_s \quad - (3)$$

where, $M \rightarrow$ Inertia constant & it is defined as power in MW required to produce unit angular acceleration.

where, $M = J \left(\frac{2}{P} \right)^2 \times \omega_s \times 10^{-6} \quad - (4)$

Units for 'M' is MJ-sec/electrical rad.

Another constant, "H" is defined as the ratio of the stored kinetic energy in MJ at synchronous speed to the rating of MVA.

$$H = \frac{\text{stored K.E in MJ}}{\text{Rating of the m/c}}$$

$$H = \frac{K.E}{G} \quad \text{where } G \rightarrow \text{rating of the m/c}$$

$$K.E = GH \quad - (5)$$

(MJ/MVA)

$$GH = \frac{1}{2} M \omega_s$$

$$M = \frac{2GH}{\omega_s} \quad - (6)$$

$$M = \frac{2GH}{2\pi f} = \frac{GH}{\pi f} \quad - (7)$$

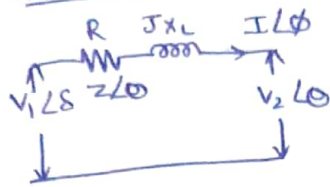
$$M = \frac{GH}{180f}$$

MJ/electrical rad.

$$- (8) \quad \text{MJ/electrical degree}$$

$$* M \text{ in p.u.} = \frac{H}{\pi f}$$

• TRANSFER REACTANCE



1- ϕ Transmission line current flowing through the line,

$$I = \frac{V_1 \angle \delta - V_2 \angle 0}{Z \angle \theta}$$

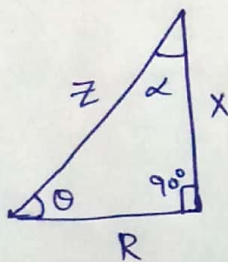
$$I = \frac{V_1}{Z} \angle \delta - \theta - \frac{V_2}{Z} \angle -\theta \quad \text{--- (1)}$$

Power received at the receiving end

$$\begin{aligned} P_R &= \text{Real} [V_2 I^*] \\ &= \text{Real} \left[V_2 \left\{ \frac{V_1}{Z} \angle \theta - \delta - \frac{V_2}{Z} \angle \theta \right\} \right] \\ &= \text{Real} \left[\frac{V_1 V_2}{Z} \angle \theta - \delta - \frac{V_2^2}{Z} \angle \theta \right] \end{aligned}$$

$$P_R = \frac{V_1 V_2}{Z} \cos(\theta - \delta) - \frac{V_2^2}{Z} \cos \theta \quad \text{--- (2)}$$

Impedance diagram :



from impedance Δ gle,

$$\left. \begin{aligned} \cos \alpha &= X/Z \\ \sin \alpha &= R/Z \\ \alpha + \theta &= 90^\circ \\ \theta &= 90^\circ - \alpha \end{aligned} \right\} \quad \text{--- (3)}$$

Sub (3) in (2)

$$\text{(2)} \Rightarrow P_R = \frac{V_1 V_2}{Z} \cos(90^\circ - \alpha - \delta) - \frac{V_2^2}{Z} \cos(90^\circ - \alpha)$$

$$P_R = \frac{V_1 V_2}{Z} \sin(\alpha + \delta) - \frac{V_2^2}{Z} \sin \alpha \quad \text{--- (4)}$$

To get max. power $\alpha + \delta = 90^\circ$

$$P_{R \text{ max}} = \frac{V_1 V_2}{Z} - \frac{V_2^2 R}{Z^2}$$

$$P_{\text{max}} = \frac{V_1 V_2}{\sqrt{R^2 + X^2}} - \frac{V_2^2}{2} \frac{R}{Z}$$

$$= \frac{V_1 V_2}{\sqrt{R^2 + X^2}} - \frac{V_2^2 R}{(R^2 + X^2)} \quad \text{--- (5)}$$

To transfer max. power from sending to receiving end

R is constant

X is varying

$$\frac{dP_{\text{max}}}{dX} = 0$$

$$\frac{dP_{\text{max}}}{dX} = 0$$

$$0 = -\frac{V_1 V_2}{2} \cdot \frac{2X}{(R^2 + X^2)^{3/2}} + \frac{V_2^2 R}{(R^2 + X^2)^2} \cdot 2X$$

Assume $V_1 = V_2$

$$\frac{V_2^2 \cdot X}{(R^2 + X^2)^{3/2}} = \frac{V_2^2 \cdot 2RX}{(R^2 + X^2)^2}$$

$$\frac{1}{(R^2 + X^2)^{3/2}} = \frac{2R}{(R^2 + X^2)^2}$$

By squaring on both sides

$$\frac{1}{(R^2 + X^2)^3} = \frac{4R^2}{(R^2 + X^2)^4}$$

$$R^2 + X^2 = 4R^2$$

$$3R^2 = X^2$$

$$X = \sqrt{3} R$$

To transfer max. power from sending end to receiving end & the transfer reactance $X = \sqrt{3} R$

Expression for steady state power / power in terms of ABCD parameters :-



To derive expression for steady state power in terms of ABCD parameters consider a 2-port N/w as shown in fig.

$$\begin{bmatrix} E_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_r \\ I_r \end{bmatrix}$$

$$E_s = AE_r + BI_r \quad \text{--- (1)}$$

$$I_s = CE_r + DI_r \quad \text{--- (2)}$$

from eqn (1)

$$BI_r = E_s - AE_r$$

$$I_r = \frac{E_s}{B} - \left(\frac{A}{B}\right) E_r \quad \text{--- (3)}$$

Let the parameters are,

$$A = |A| \angle \alpha$$

$$B = |B| \angle \beta$$

$$E_s = |E_s| \angle \delta$$

$$E_r = |E_r| \angle 0^\circ \text{ (reference)}$$

Substitute eqn (4) in eqn (3)

$$\text{(3)} \Rightarrow I_r = \frac{|E_s| \angle \delta}{|B| \angle \beta} - \frac{|A| \angle \alpha}{|B| \angle \beta} |E_r| \angle 0^\circ$$

$$I_r = \left| \frac{E_s}{B} \right| \angle \delta - \beta - \left| \frac{AE_r}{B} \right| \angle \alpha - \beta \quad \text{--- (5)}$$

Complex power received at the receiving end (Conjugate)

$$S = P_r + jQ_r = E_r I_r^*$$

$$S = E_r \left[\left| \frac{E_s}{B} \right| \angle -\delta + \beta - \left| \frac{AE_r}{B} \right| \angle \beta - \alpha \right]$$

$$= E_r \left[\left| \frac{E_s}{B} \right| \angle (\beta - \delta) - \left| \frac{AE_r}{B} \right| \angle \beta - \alpha \right]$$

$$S = \frac{E_s E_r}{B} \angle \beta - \delta - \left| \frac{A}{B} E_r^2 \right| \angle \beta - \alpha$$

By separating real & imaginary terms

$$P_x = \frac{E_s E_r}{B} \cos(\beta - \delta) - \left| \frac{A}{B} E_r^2 \right| \cos(\beta - \alpha) \quad - (6)$$

$$Q_x = \frac{E_s E_r}{B} \sin(\beta - \delta) - \left| \frac{A}{B} E_r^2 \right| \sin(\beta - \alpha) \quad - (7)$$

Consider, a short transmission line

$$A = D = 1 \angle 0^\circ, \quad C = 0, \quad B = X \angle 90^\circ \quad - (8)$$

Comparing eqn (7) & eqn (8)

$$\left. \begin{aligned} |A| &= 1, \quad \alpha = 0^\circ \\ |B| &= X, \quad \beta = 90^\circ \end{aligned} \right\} - (9)$$

Sub. eqn (9) in eqn (6)

$$P_x = \frac{E_s E_r}{X} \cos(90^\circ - \delta) - \left| \frac{1}{X} E_r^2 \right| \cos(90^\circ - 0^\circ)$$

$$P_x = \frac{E_s E_r}{X} \sin \delta \quad - (10)$$

Steady state power or power gle eqn

$$P_x = \frac{E_s E_r}{X} \sin \delta$$

Swing eqn :-

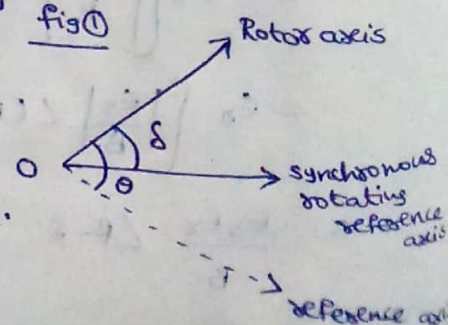
SWING EQUATION

Mid

Under normal operation conditions, The relative position of the rotor axis & synchronously rotating stator field is fixed.

The angle b/w these two

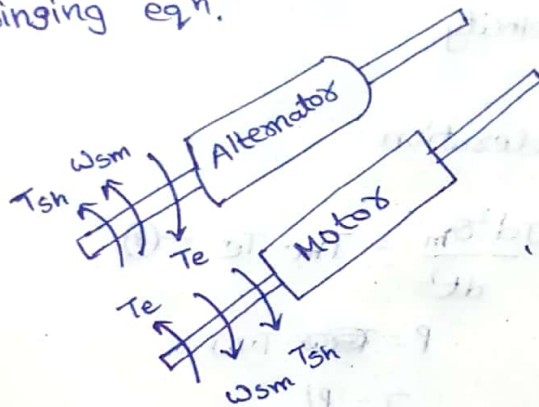
is called load angle / Torque angle.



→ It is represented with the letter ' δ '.

When load ↑, then the speed of the rotor decrease. When load ↓, then the speed ↑. That means, when load changes takes place there is change in speed of the rotor & a relative motion begins the rotor swinging w.r. to stator field.

→ The eqn describing the relative motion of the rotor w.r. to stator field as a function of time is known as "swinging eqn."



where, T_m is mechanical torque
 $= T_{sh}$ of the prime mover.

' T_e ' is electromagnetic torque.

The rotor of a synchronous machine is subjected to two torques both will be operating in opposite direction.

For generator, T_m & T_e both "ve".

For motor,

T_m & T_e , bothe '-ve'

* Under steady operating conditions, " T_m & T_e " are equal & the machine runs at constant speed called "synchronous speed".

If the difference b/w the two torque $T_m - T_e = T_a$ - ① then the rotor will have accelerating/decelerating torque & it is denoted by " T_a ".

$$T_m - T_e = T_a$$

* θ_m = is the angular displacement of the rotor w.r. to stationary reference axis.

* δ_m is the angular displacement of the rotor w.r. to synchronously rotating reference axis.

According to Newton's 2nd law of motion. The accelerating torque " T_a " is directly proportional to angular acceleration. The proportionality constant is, $T_a \propto \frac{d^2\theta_m}{dt^2}$

$$T_a = J \frac{d^2\theta_m}{dt^2} \quad \text{--- (2)}$$

J = Moment of inertia

θ_m = Angular displacement

$$\omega_{sm} = \frac{d\theta_m}{dt} = \text{angular velocity}$$

$$\alpha = \frac{d^2\theta_m}{dt^2} = \text{angular acceleration}$$

$$\text{Sub (1) in (2)} \Rightarrow (2) \Rightarrow J \frac{d^2\theta_m}{dt^2} = T_m - T_e \quad \text{--- (3)}$$

$$P = T \cdot \omega$$

$$T = P / \omega$$

$$(3) \Rightarrow J \frac{d^2\theta_m}{dt^2} = \frac{P_m}{\omega_{sm}} - \frac{P_e}{\omega_{sm}}$$

$$J \omega_{sm} \frac{d^2\theta_m}{dt^2} = P_m - P_e \quad \text{--- (4)}$$

The relation b/w θ_m & δ_m from fig ① can be written

$$\theta_m = \omega_{sm} t + \delta_m \quad \text{--- (5)}$$

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \quad \text{--- (6)}$$

Sub eqn (6) in (4)

$$(4) \Rightarrow J \omega_{sm} \frac{d^2\delta_m}{dt^2} = P_m - P_e \quad \text{--- (7)}$$

$$\omega = kT$$

$$H = \frac{K \cdot E}{G}$$

$$K \cdot E = G H$$

$$\frac{1}{2} J \omega_{sm}^2 = G_H$$

$$J \omega_{sm} = \frac{2G_H}{\omega_{sm}} \quad (8)$$

sub (8) in (7),

$$\frac{2G_H}{\omega_{sm}} \frac{d^2 \delta_m}{dt^2} = P_m - P_e \quad (9)$$

$$\left. \begin{aligned} \omega_s &= \left(\frac{P}{2}\right) \omega_{sm} \\ \delta &= \left(\frac{P}{2}\right) \delta_m \end{aligned} \right\} (10)$$

sub (10) in (9),

$$\frac{2G_H}{\left(\frac{2}{P}\right)\omega_s} \frac{d^2 \left(\frac{2}{P} \cdot \delta\right)}{dt^2} = P_m - P_e \quad (9) \Rightarrow \frac{2G_H}{\left(\frac{2}{P}\right)\omega_s} \left(\frac{2}{P}\right) \frac{d^2 \delta}{dt^2} = P_m - P_e$$

$$\frac{G_H}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e$$

$$\frac{G_H}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e \quad (11)$$

swing eqn in MW

P_m & P_e are in MW

$$\frac{H}{\pi f} \cdot \frac{d^2 \delta}{dt^2} = (P_m - P_e) \text{ in p.u.}$$

(Swing eqn is non-homogeneous second order differential eqn).

$$\text{Where, } M \text{ (p.u.)} = \frac{H}{\pi f}$$

Swing eqn is a second order non-linear differential eqn

It describes the rotor dynamics of a synchronous m/c

Machines running coherently (logical and consistent)

Consider the swing equation of two machines on a common system base

$$\frac{H_1}{\pi f} \frac{d^2 \delta_1}{dt^2} = P_{m1} - P_{e1} \quad \text{--- (1)}$$

$$\frac{H_2}{\pi f} \frac{d^2 \delta_2}{dt^2} = P_{m2} - P_{e2} \quad \text{--- (2)}$$

Since the machine rotors run together

$$\delta_1 = \delta_2 = \delta$$

Adding (1) + (2)

$$\frac{H_1}{\pi f} \frac{d^2 \delta}{dt^2} + \frac{H_2}{\pi f} \frac{d^2 \delta}{dt^2} = P_{m1} + P_{m2} - (P_{e1} + P_{e2})$$

$$\frac{H_{eq}}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e$$

$$\text{where } P_m = P_{m1} + P_{m2}$$

$$P_e = P_{e1} + P_{e2}$$

$$H_{eq} = H_1 + H_2$$

→ A 50 Hz four pole turbogenerator rated 100 MVA, 11 kV has an inertia constant of 8.0 MJ/MVA.

- (a) Find the stored energy in the rotor at synchronous speed
 (b) If the mechanical input is suddenly raised to 80 MW for an electrical load of 50 MW find rotor acceleration neglecting mechanical and electrical losses.

(a) Stored energy = $GH = 100 \times 8.0 = 800 \text{ MJ}$

(b) $P_a = P_m - P_e = 80 - 50 = 30 \text{ MW}$

$$P_a = M \frac{d^2 \delta}{dt^2}$$

$$M = \frac{GH}{180 \pi f} = \frac{100 \times 8}{180 \times 50} = \frac{4}{45} \text{ MJ-deg/elect deg.}$$

$$M \frac{d^2 \delta}{dt^2} = P_a = 30$$

$$\alpha = \frac{d^2 \delta}{dt^2} = \frac{30 \times 45}{4} = 337.5 \text{ elect deg/s}^2$$

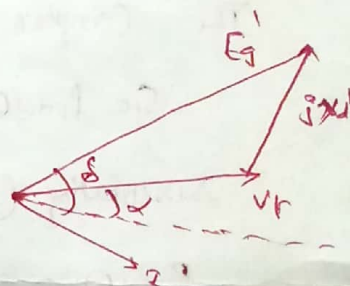
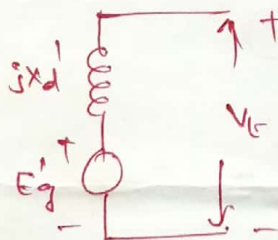
Power Angle Equation

7

The equation relating the electrical power generated (P_e) to the angular displacement of the rotor (δ) is called power angle equation.

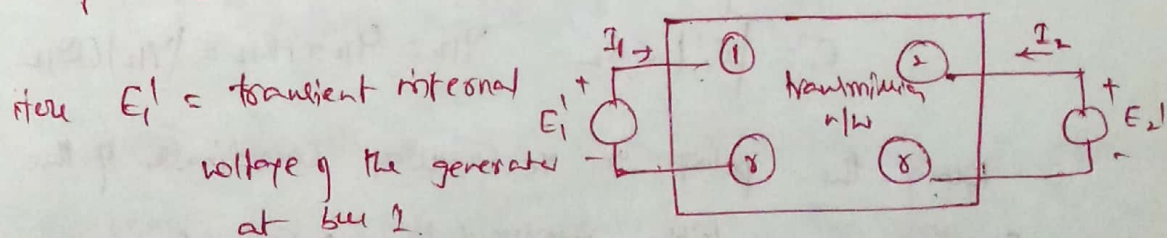
The power angle equation can be derived using the transient model of the generator. Beforehand for stability studies the transient model of the generator is used.

The transient model of the generator is shown in fig. It consists of the transient internal emf in series with transient reactance X_d' .



Consider a single generator supplying power through a transmission system to a load or to a large system at other end. Such a system can be represented by a box as shown in fig.

The rectangular box represents the linear passive elements (reactances) of the system including the transient reactance of the generator.



Here E_1' = transient internal voltage of the generator at bus 1.

E_2' = voltage at the receiving end (voltage at infinite bus (v))

transient internal voltage of synchronous motor at bus 2

The node basis matrix equation of 2-bus system can be written as,

$$I = Y_{bus} V \quad \text{--- (1)}$$

in the expanded form the above equation can be written as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1' \\ E_2' \end{bmatrix} \quad \text{--- (2)}$$

Where I_1, I_2 are the currents injected by the sources E_1', E_2'

$$\text{From (2)} \quad I_1 = Y_{11} E_1' + Y_{12} E_2' \quad \text{--- (3)}$$

The complex power S_1 injected at bus 1 is given by

$$S_1 = P_1 + jQ_1 = E_1' I_1^* \quad \text{--- (4)}$$

Substituting (3) in (4)

$$\begin{aligned} P_1 + jQ_1 &= E_1' (Y_{11} E_1' + Y_{12} E_2')^* \\ &= E_1' (Y_{11} E_1')^* + E_1' (Y_{12} E_2')^* \\ &= E_1' E_1'^* Y_{11}^* + E_1' E_2'^* Y_{12}^* \\ &= |E_1'|^2 Y_{11}^* + Y_{12}^* E_1' (E_2')^* \quad \text{--- (5)} \end{aligned}$$

The quantities $E_1', E_2', Y_{11}, Y_{12}$ in (5) are complex quantities so they can be expressed in polar coordinates as shown below.

$$\begin{aligned} \text{Let } E_1' &= |E_1'| \angle \delta_1 & Y_{11} &= G_{11} + jB_{11} = |Y_{11}| \angle \theta_{11} \\ E_2' &= |E_2'| \angle \delta_2 & Y_{12} &= G_{12} + jB_{12} = |Y_{12}| \angle \theta_{12} \end{aligned}$$

Using the polar or rectangular coordinates of the complex quantities involved in (5) can be written as

$$P_1 + jQ_1 = |E_1'|^2 (G_{11} - jB_{11}) + (Y_{12} \angle \theta_{12})^* |E_1'| \angle \delta_1 (|E_2'| \angle \delta_2)^*$$

$$= |E_1'|^2 (G_{11} - jB_{11}) + |Y_{12}| |E_1'| |E_2'| \angle -\delta_1 \angle \delta_2 \quad \text{--- (6)}$$

$$= |E_1'|^2 (G_{11} - jB_{11}) + |Y_{12}| |E_1'| |E_2'| \angle (\delta_1 - \delta_2 - \theta_{12})$$

$$= |E_1'|^2 (G_{11} - jB_{11}) + |E_1'| |E_2'| |Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12}) + j |E_1'| |E_2'| |Y_{12}| \sin(\delta_1 - \delta_2 - \theta_{12}) \quad \text{--- (7)}$$

On equating the real and imaginary parts

$$P_i = |E_1'|^2 G_{11} + |E_1'| |E_2'| |Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12}) \quad \text{--- (7)}$$

$$Q_i = -|E_1'|^2 B_{11} + |E_1'| |E_2'| |Y_{12}| \sin(\delta_1 - \delta_2 - \theta_{12}) \quad \text{--- (8)}$$

Equations (7) & (8) are equations of real and reactive power injected by a generator at bus 1.

$$\text{Let } \delta = \delta_1 - \delta_2$$

$$\gamma = \theta_{12} - \pi/2$$

$$P_c = |E_1'|^2 G_{11}$$

$$P_{max} = |E_1'| |E_2'| |Y_{12}|$$

$$P_i = P_c$$

Substituting above in eq (7)

$$P_c = P_c + P_{max} \cos(\delta - \theta_{12})$$

$$= P_c + P_{max} \cos(\delta - \gamma - \pi/2)$$

$$P_c = P_c + P_{max} \sin(\delta - \gamma) \quad \text{--- (9)}$$

Eq (9) is called power angle equation. In this equation the term P_c represents power loss in the system.

P_{max} represents the max. real power that can be delivered by the generator to an infinite bus.

Bus 2 is considered at infinite bus and so $\delta_2 = 0$

$$\therefore \delta_1 = \delta$$

The power angle equation can be further simplified by considering the n/w as purely reactive n/w by neglecting the resistance.

$$\therefore G_{11} = 0, \quad \Theta_{12} = \pi/2 \quad \text{and so } \gamma = 0.$$

On substituting these conditions in (9)

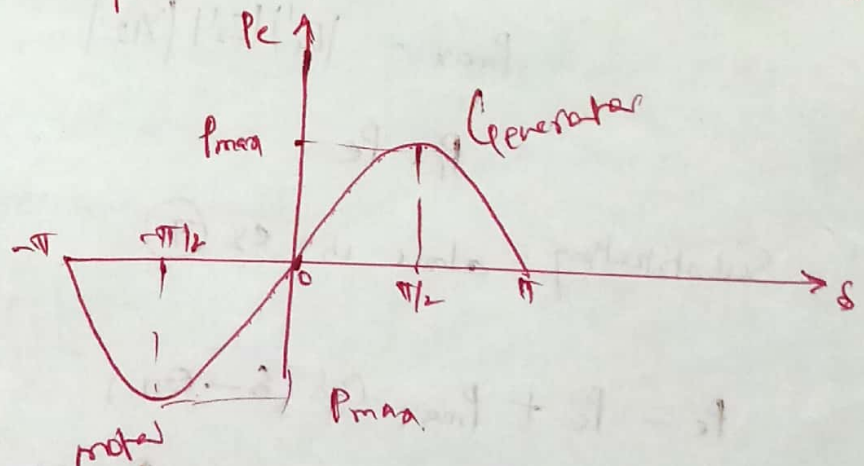
$$\boxed{P_e = P_{max} \sin \delta} \quad \text{--- (10)}$$

$$\text{Where } P_{max} = \frac{|E_1| |E_2|}{X_{12}}$$

X_{12} = Transfer reactance b/w bus 1 & 2.

$\hookrightarrow$ (10) is called simplified power angle equation.

The graph of P_e as function of δ is called power angle curve. The power angle curve is shown in Fig.



Steady state stability

In steady state every synchronous machine has a limit for power transfer to a receiving system.

The steady state limit of a machine or transmitting system is defined as the max power that can be transmitted to the receiving system without loss of synchronism.

Let us consider a single synchronous machine delivering power to a large system through a transmitting system.

Power angle equation can be used for the analysis of steady state stability if the transient emf are replaced by steady state emf.

Let $|E|$ = Magnitude of steady state internal emf of synchronous machine.

$|V|$ = Magnitude of voltage of receiving system.

X = Transfer reactance b/w the synchronous machine and receiving system.

$$P_{max} = \frac{|E||V|}{X} \quad \text{--- (1)}$$

Real power injected by machine to system

$$P_e = P_{max} \sin \delta \quad \text{--- (2)}$$

Let the system be operating with steady power transfer with a torque angle δ . In this operating condition the electrical power output be P_{eo} , mechanical power input equal to P_{eo} under ideal conditions.

With the power i/p P_m remaining same (let us assume)
 the electrical power P_e increased by a small amount
 Now the torque angle change by a small amount $\Delta\delta$.
 $\therefore$ new value of torque angle is $(\delta_0 + \Delta\delta)$

The electrical power P_e for this new torque angle can be obtained as

$$P_{e0} + \Delta P = P_{max} \sin(\delta_0 + \Delta\delta)$$

$$= P_{max} [\sin\delta_0 \cos\Delta\delta + \cos\delta_0 \sin\Delta\delta] \quad \text{--- (3)}$$

Since $\Delta\delta$ is a small incremental displacement from δ_0
 $\sin\Delta\delta \approx \Delta\delta$ and $\cos\Delta\delta \approx 1$ --- (4)

$$P_{e0} + \Delta P = P_{max} [\sin\delta_0 + \cos\delta_0 \Delta\delta]$$

$$P_{e0} + \Delta P = P_{max} \sin\delta_0 + (P_{max} \cos\delta_0) \Delta\delta \quad \text{--- (5)}$$

if $\delta = \delta_0$ then from (2)

$$P_e = P_{e0} = P_m \sin\delta_0 \quad \text{--- (6)}$$

from (5) $\Delta P = (P_{max} \cos\delta_0) \Delta\delta \quad \text{--- (7)}$

The non linear swing equation can be linearized about the operating point for steady state stability analysis. The non linear swing equation is

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{--- (8)}$$

WKT $M = \frac{H}{\pi f}$ P.U

$$M \frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{--- (9)}$$

for a torque of $\delta = \delta_0 + \Delta\delta$.

the electrical power in (9) should be replaced by $(P_{e0} + \Delta P_e)$

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$$M \frac{d^2}{dt^2}(\delta_0 + \Delta\delta) = P_m - (P_{e0} + \Delta P_e) \quad \text{--- (10)}$$

Since δ_0 is constant, $P_m = P_e$.

$$M \frac{d^2 \Delta\delta}{dt^2} = -\Delta P_e \quad \text{--- (11)}$$

Substituting (9) in (11)

$$M \frac{d^2 \Delta\delta}{dt^2} = -(P_{max} \cos \delta_0) \Delta\delta$$

$$M \frac{d^2 \Delta\delta}{dt^2} + P_{max} \cos \delta_0 \Delta\delta = 0 \quad \text{--- (12)}$$

$$\text{Let } \frac{d^2}{dt^2} = x^2, \quad P_{max} \cos \delta_0 = C$$

$$M x^2 \Delta\delta + C \Delta\delta = 0 \quad \text{--- (13)}$$

$$(M x^2 + C) \Delta\delta = 0 \quad \text{--- (14)}$$

$$\text{Since } \Delta\delta \neq 0, \quad M x^2 + C = 0 \quad \text{--- (15)}$$

Eq (15) is the characteristic equation of the system for small disturbances. The stability is determined by the roots of the characteristic equation.

$$x^2 = -\frac{C}{M} \quad \left[x = \pm \sqrt{-\frac{C}{M}} \right] \quad \text{--- (16)}$$

Case 1) When r is +ve. i.e. ($P_{max} \cos \delta_0 > 0$)

In this case the roots are purely imaginary and conjugate. Hence the system behaviour is oscillator about δ_0 .

In this analysis the resistance in the system have been neglected.

$\therefore$ In a practical system the system is stable for small increment in power provided, $P_{max} \cos \delta_0 > 0$
Case 2 When r is -ve (i.e. $P_{max} \cos \delta < 0$)

The roots will be real and equal in magnitude.

One of the root is +ve and the other is -ve.

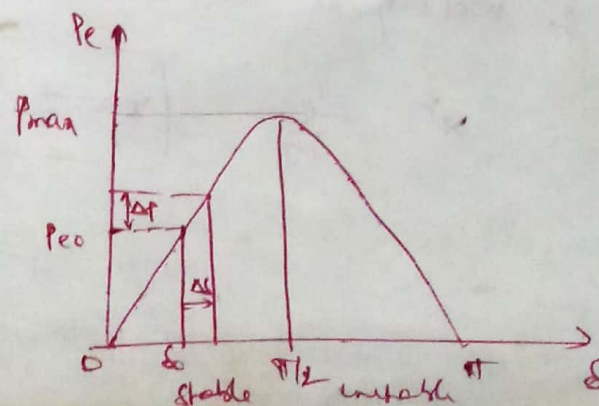
Due to the +ve root the torque angle increases without bound when there is a small increment in power and the machine will lose synchronism.

Hence the machine becomes unstable for small changes in power provided ($P_{max} \cos \delta_0 < 0$)

Steady State Limit

The term $P_{max} \cos \delta_0$ decides the steady state stability of the system and so it is called synchronizing coefficient or stiffness of synchronous machine.

From the power angle curve.



When $0 \leq \delta_0 \leq \pi/2$, $P_{\max} \cos \delta_0$ and

P_e are +ve.

When $\delta_0 = \pi/2$, $P_{\max} \cos \delta_0 = 0$,

$P_e = P_{\max}$.

When $\pi/2 < \delta_0 < \pi$, $P_{\max} \cos \delta_0$ is -ve and P_e is +ve.

From the above discussion, we can say that the synchronizing coefficient and real power injected to the system P_e are +ve when δ is in the range 0 to $\pi/2$.

$\therefore$ The max. power that can be transmitted without losing stability occurs for $\delta = \delta_0 = \pi/2 = 90^\circ$.

$\therefore$ The max power transmitted is $P_{\max}$.

$$P_{\max} = \frac{|E| |V|}{X}$$

In the stable operating region of the system the damping should be sufficient to reduce the oscillations developed due to small changes in loads.

If the oscillation exists for a long time then it may pose a problem to stability security.

Practically the system has to be operated below the steady state stability limit. This limit can be improved by reducing the reactance X or by increasing the voltage at sending end or at receiving end.

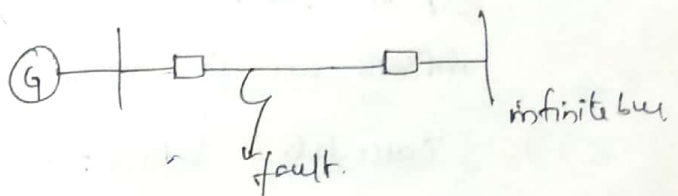
The reactance can be reduced by introducing series capacitors in transmission lines.

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The transient stability of a system is concerned with the study of system behaviour for large disturbances. The short circuits and switching heavy loads can be treated.

Consider a ~~single~~ machine system shown in fig feeding energy through a transmission line to an infinite bus. Let the ckt breaker shown in fig be autoreclosing type.

In this type the  ckt breaker will open its contacts open resulting a fault and after a small time it will close its contacts, if the fault still exist then again it will open its contact to permanently disconnect the faulty part.

This feature is useful in clearing transient fault. Because the transient fault exists for a small time and it gets cleared when the ckt is open. Then the ckt can be closed for normal operation.

Most of the autoreclosing ckt breaker will close the contacts twice before permanently disconnect the ckt. In the majority of faults the first reclosure will be successful.

Hence the system stability is improved by using autoreclosing ckt breaker.

Let a fault takes place on sending end of the transmission line in the system shown in fig.

Let the fault be transient in nature and cleared in the transient state. Hence the ckt breaker will close the ckt automatically after the fault is cleared.

9. The transient stability study for this situation involves the following steps.

1. From pre-fault loading determine the voltage behind the transient reactance and torque angle δ_0 of the machine with reference to the infinite bus.

2. Determine the equation for power during the fault condition. Let this equation be $P_e(\delta)$.

If the fault is 3- ϕ fault then power transferred to infinite bus is zero and the entire power goes to fault.

3. Calculate $\delta(t)$ for various time instant by solving the swing equation using a numerical technique.

The initial value of δ for this solution is δ_0 .

4. After clearance of the fault ^{once again} determine $P_e(\delta)$ and solve further for $\delta(t)$. In this case $P_e(\delta) = 0$ as when the fault is cleared the system gets disconnected.

5. After the transmission line is switched on again find $P_e(\delta)$ and continue to calculate $\delta(t)$.

6. If $\delta(t)$ goes through a maximum value and starts to reduce the system is regarded as stable.

It is unstable if $\delta(t)$ continues to increase. Calculation is ceased after a suitable length of time.

Equal Area Criterion

The transient stability analysis of simple system can be performed by using a simple criterion called equal area criterion.

During transient state of a power system we may come across the following two situations for change in δ .
W.r to time.

1. The δ may increase to a maximum value and then decrease to a stable value. The system is considered stable.
2. The δ may keep on increasing indefinitely. In this case the system is unstable.

The above facts can be treated as a stability criterion as given below:

- 1) The system is stable if $\frac{d\delta}{dt} = 0$ at some time instant.
- 2) The system is unstable if $\frac{d\delta}{dt} > 0$ for a sufficiently long time.

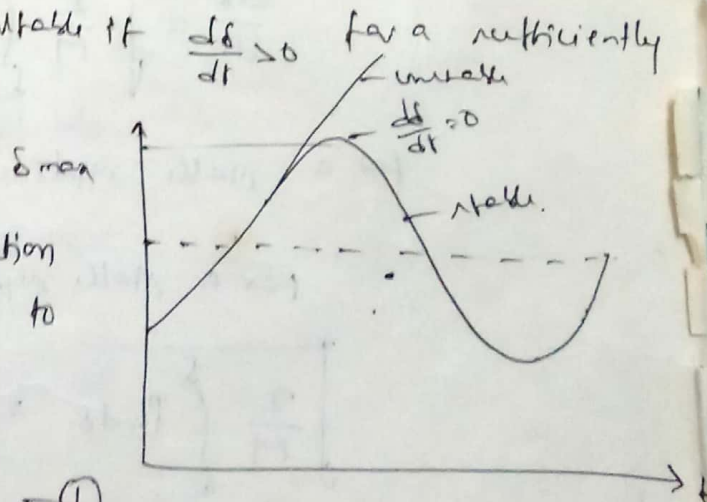
Consider the swing equation of a generator connected to infinite bus.

$$\frac{H}{\omega} \frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{--- (1)}$$

Let there be a change in P_e due to a large disturbance, with P_m remaining constant.

where, Now $P_m - P_e = P_a \quad \text{--- (2)}$

where P_a is the accelerating power.



$$M \frac{d^2 \delta}{dt^2} = P_a$$

$$\frac{d^2 \delta}{dt^2} = \frac{P_a}{M} \quad \text{--- (3)}$$

Multiplying (3) with $2 \frac{d\delta}{dt}$ we get

$$2 \frac{d\delta}{dt} \frac{d^2 \delta}{dt^2} = \frac{P_a}{M} 2 \frac{d\delta}{dt}$$

$$2 \frac{d\delta}{dt} \frac{d}{dt} \frac{d\delta}{dt} = \frac{2}{M} P_a \frac{d\delta}{dt}$$

But $\frac{d}{dt} \left(\frac{d\delta}{dt} \right)^2 = 2 \frac{d\delta}{dt} \frac{d^2 \delta}{dt^2}$

$$2 \frac{d}{dt} \left(\frac{d\delta}{dt} \right)^2 = \frac{2}{M} P_a \frac{d\delta}{dt}$$

$$2 d \left(\frac{d\delta}{dt} \right)^2 = \frac{2}{M} P_a d\delta \quad \text{--- (4)}$$

Integrating (4) we get

$$\left(\frac{d\delta}{dt} \right)^2 = \frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta \quad \text{--- (5)}$$

Where δ_0 is the initial value of torque angle, or rotor angle.

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta} \quad \text{--- (6)}$$

for a stable system $\frac{d\delta}{dt} = 0$

$\therefore$ for a stable system eq (6) can be written as

$$\sqrt{\frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta} = 0 \quad \text{--- (7)}$$

The above equation is zero if integral of P_a is zero.

Hence we can say that

$$\text{for } \frac{d\delta}{dt} = 0 \quad \boxed{\int_{\delta_0}^{\delta} P_a d\delta = 0} \quad \text{--- (8)}$$

The physical meaning of integration is the estimation of the area under the curve.

Hence the integral of $P_a = 0$ refers to zero area.

Hence the condition of stability can be stated as, the system is stable if the area under $P_a-\delta$ curve reduces to zero at some value of δ .

This is possible only if the +ve area (accelerating) area under $P_a-\delta$ curve is equal to the negative (decelerating) area under $P_a-\delta$ curve, for a finite change in δ .

Hence this stability criterion is called equal area criterion of stability.

The equal area criterion of stability can be applied to any type of disturbance that may occur on single machine connected to infinite bus bar. It is not applicable to multimachine systems.

from (8) can be split into

$$\int_{\delta_0}^{\delta_1} P_a d\delta + \int_{\delta_1}^{\delta_2} P_a d\delta = 0$$

$$\int_{\delta_0}^{\delta_1} P_a d\delta = - \int_{\delta_1}^{\delta_2} P_a d\delta$$

Where $A_1 = \int_{\delta_0}^{\delta_1} P_a d\delta$, $A_2 = \int_{\delta_1}^{\delta_2} P_a d\delta$

This condition is called the equal area criterion of stability

A synchronous generator having a reactance of 1 p.u. is connected to an infinite bus through a transmission line with a reactance of 0.7 p.u. The generator is running on no load with a voltage of 1.1 p.u. Take $H = 4.5$ MW-s/MVA. The voltage of infinite bus is 1.0 p.u. and its frequency is 50 Hz.

Calculate the frequency of natural oscillation if the machine is suddenly loaded to 60% (i) 75% of its max. power limit. Neglect the resistance and machine damping.

Sol

$$P_{\max} = \frac{|E||V|}{X}$$

$$\text{Given } |E| = 1.1 \text{ p.u.}, |V| = 1 \text{ p.u.}, X = (1 + 0.7) \text{ p.u.}$$

$$P_{\max} = \frac{1.1 \times 1}{1.7} = 0.6471 \text{ p.u.}$$

$$H = 4.5 \text{ MW-s}, f = 50 \text{ Hz}$$

$$M = \frac{H}{\pi f} = \frac{4.5}{\pi \times 50} = 0.0287 \text{ p.u.}$$

Case (i) 60% loading

The electrical power $P_e = P_{\max} \sin \delta$

for 60% loading let $\delta = \delta_1$, $P_e = P_{e1}$.

$$\frac{P_{e1}}{P_{\max}} = 0.6 = \sin \delta_1$$

$$\delta = \sin^{-1}(0.6) = 36.9^\circ \text{ (electrical)}$$

The characteristic equation of the system during small disturbance is given by $Mx^2 + C = 0$

$$\text{where } C = P_{\max} \cos \delta_1$$

The natural frequency of oscillation of the system for

Small disturbance is given by the value of characteristic equation.



$$\begin{aligned}
 x &= \pm \sqrt{\frac{-c}{M}} = \pm j \sqrt{\frac{P_{\max} \cos \delta_1}{M}} \\
 &= \pm j \sqrt{\frac{0.6471 \times \cos 26.9^\circ}{0.0287}} = \pm j 4.25
 \end{aligned}$$

The natural frequency of oscillation = 4.25 rad/sec

$$\frac{4.25}{2\pi} = \underline{\underline{0.6764 \text{ Hz}}}$$

Case 2 75% loading

$$P_e = P_{\max} \sin \delta$$

$$\text{Let } \delta = \delta_2 \quad P_e = P_{e2}$$

$$\frac{P_{e2}}{P_{\max}} = 0.75 = \sin \delta_2$$

$$\delta_2 = \sin^{-1}(0.75) = 48.6^\circ \text{ (r/e deg)}$$

$$Mx^2 + c = 0$$

$$c = P_{\max} \cos \delta_2$$

$$\begin{aligned}
 x &= \pm \sqrt{\frac{-c}{M}} = \pm j \sqrt{\frac{P_{\max} \cos \delta_2}{M}} \\
 &= \pm j \sqrt{\frac{0.6471 \times \cos 48.6^\circ}{0.0287}} = \pm j 3.86
 \end{aligned}$$

∴ Natural frequency of oscillation = 3.86 rad/sec

$$= \frac{3.86}{2\pi}$$

$$= \underline{\underline{0.6143 \text{ Hz}}}$$

→ A 2 pole 50 Hz, 11 kV turbo alternator has a rating of 100 MW. power factor 0.85 lagging. The rotor has a moment of inertia of 10000 kgm². calculate H & M.

Sol

$$p = 2, \quad f = 50 \text{ Hz}, \quad \text{P.f.} = 0.85 \text{ lag}$$

$$\text{rating } V = 11 \text{ kV}, \quad P = 100 \text{ MW}$$

$$J = 10,000 \text{ kgm}^2$$

$$\text{Synchronous speed in rps } n_s = \frac{2f}{p} = \frac{2 \times 50}{2} = 50 \text{ rps}$$

$$\begin{aligned} \text{Synchronous speed in rad/sec } \omega_s &= 2\pi n_s \\ &= 2\pi \times 50 = 314.16 \text{ ele rad/sec} \end{aligned}$$

$$\begin{aligned} \text{Inertia constant } M &= J \left(\frac{2}{p} \right)^2 \omega_s^2 \times 10^{-6} \text{ MJ-s/ele-rad} \\ &= 10000 \left(\frac{2}{2} \right)^2 \times 314.16^2 \times 10^{-6} \\ &= 3.1416 \text{ MJ-s/ele-rad} \end{aligned}$$

$$\text{Rating of the machine. } G = \frac{P}{\text{P.f.}} = \frac{100}{0.85} = 117.647 \text{ MVA}$$

$$KV_s = 11 \text{ kV}, \quad \text{MVA}_s = 117.647 \text{ MVA}$$

$$\text{Inertia constant } M \text{ in pu } M_{pu} = \frac{3.1416}{117.65} = 0.0267 \text{ pu}$$

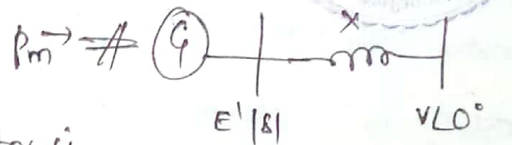
$$H = M \times \omega_s = 11 \times 50 \times 0.0267 = 4.197 \text{ MW-s/MVA}$$

Transient stability analysis for a sudden change in mechanical input.

12

Consider a single generator feeding energy to energy to infinite bus as shown in fig.

The electrical power



transmitted by the generator is

given by

$$P_e = \frac{|E'| |V|}{X} \sin \delta = P_{max} \sin \delta \quad \text{--- (1)}$$

$$\text{where, } P_{max} = \frac{|E'| |V|}{X}$$

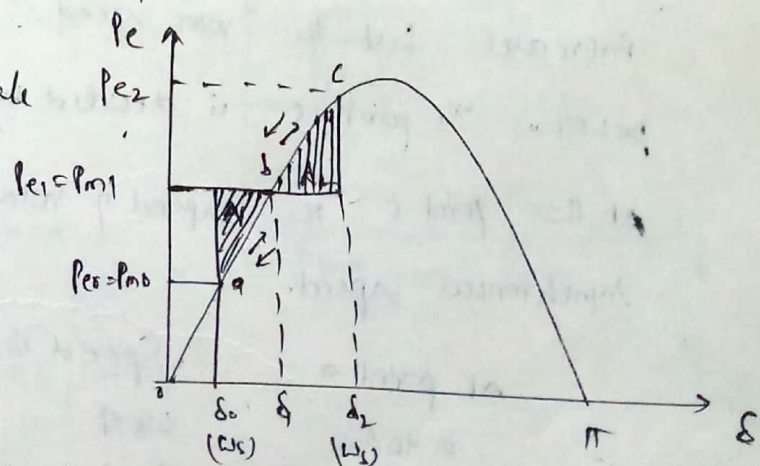
Let the generator be operating in steady state with a power angle δ_0 . At this condition the mechanical power input is P_{m0} and electrical power out is P_{e0} . Under this condition $P_{m0} = P_{e0}$

eq (1) can be written under this condition

$$P_{m0} = P_{e0} = P_{max} \sin \delta_0 \quad \text{--- (2)}$$

The power angle curve of the generator is shown in fig.

In this the steady state operating point can be described by eq (2) is the point a.



Let the mechanical input the generator rotor be suddenly increased to P_{m1} . Since the mechanical power is more than electrical power the generator will have an accelerating power P_a given by

$$P_a = P_{m1} - P_e \quad \text{--- (3)}$$

Where $P_e = P_{max} \sin \delta$.

Due to acceleratory power the rotor speed increases and so the rotor angle increases. This results in increased electrical power generation. The operating point will move upward along the power angle curve. At point b again the mechanical power P_m equals the electrical power P_e .

Where P_e is the electrical power corresponding to the torque angle δ_1 .

Now the rotor angle cannot stay at this point because the inertia of the rotor will make the rotor ~~because the inertia of the rotor~~ will make the rotor to oscillate w.r to point b.

Hence the torque angle will continue to increase till point c.

When the operating point moves from b to c the electrical power is more than mechanical power.

$\therefore$ P_a is negative and it is called deceleratory power.

In this region (from b to c) the rotor angle δ increases but the rotor speed decreases due to deceleratory power. The point c is decided by the damping of the system. At this point c the speed of rotor will be equal to synchronous speed.

At point a Speed is ω_s

a to b $\omega_s \uparrow$

b to c $\omega_s \downarrow$

Again at c speed is ω_s .

Thus the rotor oscillates b/w a and c before settling to point b.

Area A_1 is the accelerating area.

Area A_2 is the decelerating area.

The equal area criterion says that the system is stable if

$$\int_{\delta_0}^{\delta_1} P_a d\delta = 0 \quad \text{--- (4)}$$

In order to satisfy (4) $A_1 = A_2$.

$$P_{m1} = P_{e1} = P_{man} \sin \delta_1 \quad \text{--- (5)}$$

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_{e1}) d\delta \quad \text{--- (6)}$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_{e1} - P_{m1}) d\delta \quad \text{--- (7)}$$

For the system to be stable it should be possible to find δ_2 such that $A_1 = A_2$.
From the above discussion we can say that there is an upper limit for increase in mechanical power limit P_m .

As the mechanical power is increased a limiting condition

is finally reached at a point where $A_1 = A_2$ & δ_2 acquires the maximum value such that

under this condition δ_2

takes a max value P_{e1man}

δ_{2man} .

δ_2 at

$$\delta_{2man} = \pi - \delta_{1man}$$

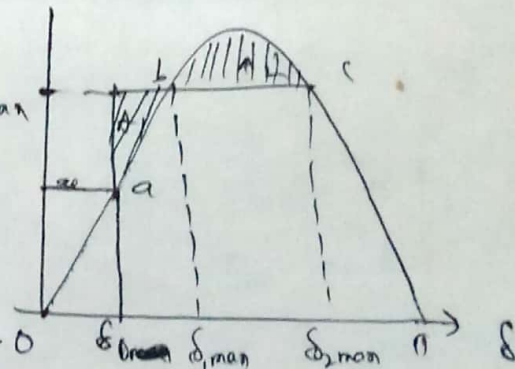
$$\delta_2 = \delta_{man} = \pi - \delta_1 \quad \text{--- (8)}$$

$$\text{from (7)} = \pi - \sin^{-1} \left(\frac{P_{m1}}{P_{man}} \right)$$

$$P_{e1man} = P_{m1,man} = P_{man} \sin \delta_{man}$$

$$\sin \delta_{man} = \frac{P_{m1,man}}{P_{man}}$$

$$\delta_{1man} = \sin^{-1} \left(\frac{P_{m1,man}}{P_{man}} \right) \quad \text{--- (9)}$$



from eq (9)

$$\delta_{2, \max} = \pi - \sin^{-1} \left(\frac{P_{m1, \max}}{P_{max}} \right) \quad \text{--- (10)}$$

from fig. ~~to~~ any further increase in P_{m1} will make the area A_2 is less than A_1 . The accelerating power is more than the decelerating power. Hence the system will have an excess P.E which causes it to ~~change~~ increase beyond point C.

If the δ increases beyond point C the decelerating power changes to accelerating power and the system becomes unstable.

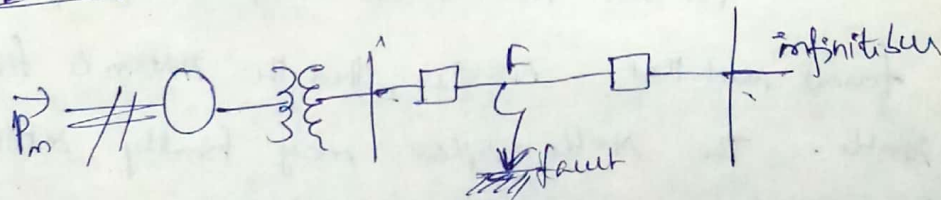
That means the accelerating power is more than the decelerating power.

Hence the system will have enough K.E. which causes δ to increase beyond point C.

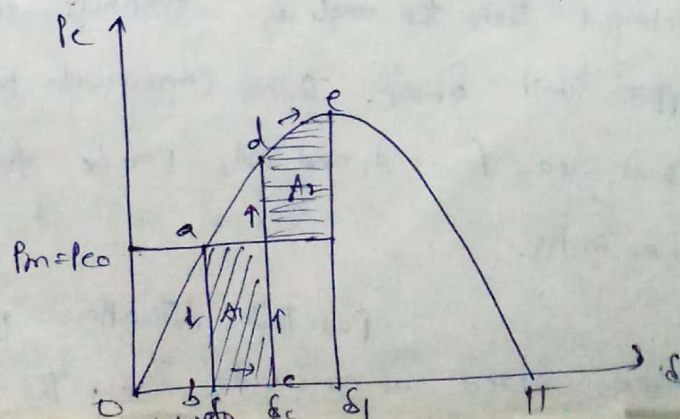
If δ increases beyond point C the decelerating power changes to accelerating power and so the system becomes unstable.

→ It is important to note that the system will remain stable even though the rotor may oscillate beyond $\delta = 90^\circ$ as long as the equal area criterion is met. Hence the condition of $\delta = 90^\circ$ is meant for use in steady state stability only and not for transient stability.

Effect of clearing time and clearing angle on stability.



Consider a single machine system shown in fig. Let the mechanical power input be P_m . The machine is operating in steady state with torque angle δ_0 . The power angle curve for this system is shown. The operating point is shown as point a.



Let a 2-g fault occur at a point f in the system. Now $P_f = 0$ and the operating point drops to b . That means the power transfer to the infinite bus is zero and the entire power generated is flowing through the fault. Now the operating point moves along bc .

Let this fault be transient in nature and so the fault is cleared by opening of the circuit breaker at point c where $\delta = \delta_c$ and the corresponding time is t_c . Here t_c is called clearing time and δ_c is called clearing angle.

It is assumed that the C.B. closes its contacts immediately after opening. Hence the normal operation is restored. Now the operating point shifts to d . The rotor now decelerates and the operating point moves along de .

For this transient state if an angle δ_1 can be found such that $A_1 = A_2$, then the system is found to be stable. The stable system may finally settle down to the steady operating ~~condition~~ point a in an oscillatory manner due to damping in the system.

In the above discussion it is assumed that the fault is cleared at δ_c but if the fault clearing is delayed then the angle δ_1 continues to increase to an upper limit δ_{max} . This corresponds to a point where equal area for A_1 and A_2 can be found for a given P_m shown in fig.

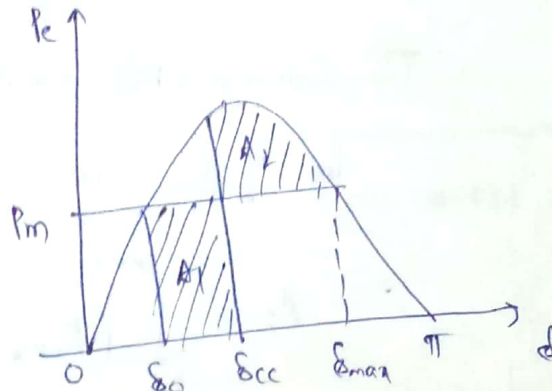
For this situation the fault would have been cleared at an angle δ_{cc} . The angle δ_{cc} is called

Critical clearing angle.

The time corresponding to this angle is called Critical clearing time t_{cc} .

If the fault is not cleared within critical time then δ , would increase to a value greater

than δ_{max} . In such a situation the area A_2 will be less than Area A_1 and the system could be unstable.



for a 3- ϕ fault in a simple power system the equation for δ_{cc} and t_{cc} can be obtained as follows.

From the above fig it is clear that

$$\delta_{max} = \pi - \delta_0 \quad \text{--- (1)}$$

Under steady state for a given δ_0 $P_m = P_e$

$$\therefore P_m = P_e = P_{max} \sin \delta_0 \quad \text{--- (2)}$$

$$P_a = P_m - P_e \quad \text{--- (3)}$$

When a 3- ϕ fault occurs $P_e = 0$

$$P_a = P_m = \text{constant} \quad \text{--- (4)}$$

The accelerating area A_1 can be evaluated by integrating P_a of (4) from $\delta = \delta_0$ to $\delta = \delta_{cc}$

$$\therefore A_1 = \int_{\delta_0}^{\delta_{cc}} P_m d\delta = P_m [\delta_{cc} - \delta_0] \quad \text{--- (5)}$$

When the power feeding is resumed after the fault

$$P_e = P_{max} \sin \delta$$

NOW $P_a = P_e - P_m = P_{max} \sin \delta - P_m$ — (6)

The decelerating area can be evaluated by integrating

P_a of (6) from $\delta = \delta_{cc}$ to $\delta = \delta_{max}$

$$\begin{aligned} \therefore A_2 &= \int_{\delta_{cc}}^{\delta_{max}} (P_{max} \sin \delta - P_m) d\delta \\ &= \left[-P_{max} \cos \delta - P_m \delta \right]_{\delta_{cc}}^{\delta_{max}} \\ &= -P_{max} \cos \delta_{max} - P_m \delta_{max} + P_{max} \cos \delta_{cc} + P_m \delta_{cc} \end{aligned}$$

$$A_2 = P_{max} (\cos \delta_{cc} - \cos \delta_{max}) - P_m (\delta_{max} - \delta_{cc})$$

— (7)

For a stable system $A_1 = A_2$

$$P_m (\delta_{cc} - \delta_0) = P_{max} (\cos \delta_{cc} - \cos \delta_{max}) - P_m (\delta_{max} - \delta_{cc})$$

$$\therefore P_{max} \cos \delta_{cc} = \cancel{P_m \delta_{cc}} - P_m \delta_0 + P_{max} \cos \delta_{max} + P_m \delta_{max} - \cancel{P_m \delta_{cc}}$$

$$P_{max} \cos \delta_{cc} = P_m (\delta_{max} - \delta_0) + P_{max} \cos \delta_{max}$$

$$\therefore \cos \delta_{cc} = \frac{P_m (\delta_{max} - \delta_0)}{P_{max}} + \cos \delta_{max}$$

$$\delta_{cc} = \cos^{-1} \left[\frac{P_m (\delta_{max} - \delta_0)}{P_{max}} + \cos \delta_{max} \right]$$

— (8)

es ⑧ can be used to estimate the value of critical clearing angle δ_{cc} .

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Let us consider the swing equation of a single machine system.

$$\frac{H}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e \quad \text{--- (9)}$$

during 3- ϕ fault $P_e = 0$

$$\frac{H}{\pi f} \frac{d^2 \delta}{dt^2} = P_m$$

$$\frac{d^2 \delta}{dt^2} = \frac{P_m \pi f}{H} \quad \text{--- (10)}$$

on integrating eq (10) twice

$$\frac{d\delta}{dt} = \frac{\pi f P_m}{H} t + C_1$$

$$\delta = \frac{\pi f P_m}{2H} t^2 + C_1 t + C_2 \quad \text{--- (11)}$$

where C_2 is the integral constant.

When $\delta = \delta_{cc}$, $t = t_{cc}$

at $t = t_{cc}$

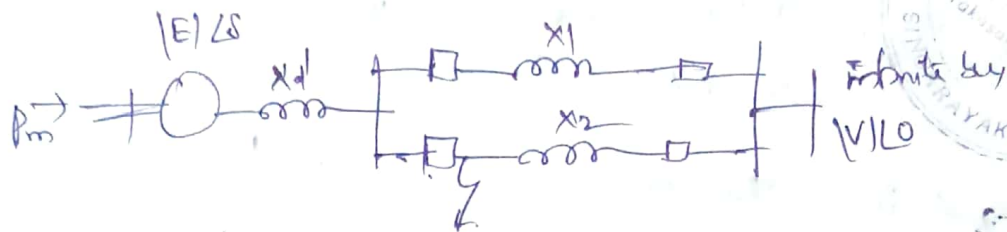
$$\delta_{cc} = \frac{\pi f}{2H} P_m t_{cc}^2 + C_2$$

$$\delta_{cc} - C_2 = \frac{\pi f}{2H} P_m t_{cc}^2$$

$$t_{cc} = \sqrt{\frac{2H (\delta_{cc} - C_2)}{\pi f P_m}} \quad \text{--- (12)}$$

es (12) can be used to estimate the value of critical clearing time t_{cc} .

Sudden Short ckt on one of parallel lines



Let us assume a short ckt at the generator end of the line 2 of a double ckt line shown in fig.

Assume a fault is a 3- ϕ fault.

Before occurrence of a fault the power angle curve is given by

$$P_{e1} = \frac{|E'| |V|}{x_d' + x_1 || x_2} \sin \delta = P_{max} \sin \delta$$

Upon occurrence of a 3- ϕ fault at the generator end of the line 2 the generator gets isolated from the power system for the purpose of power flow.

Thus during the period the fault lasts i

$$P_{e2} = 0$$

The rotor accelerates and angle δ increases. Synchronization will be lost unless the fault is cleared in time.

The ckt breakers at the two ends of the faulted line open at time t_c corresponding angle δ_c , the clearing time disconnected from the faulted line.

The power flow is now restored via the healthy

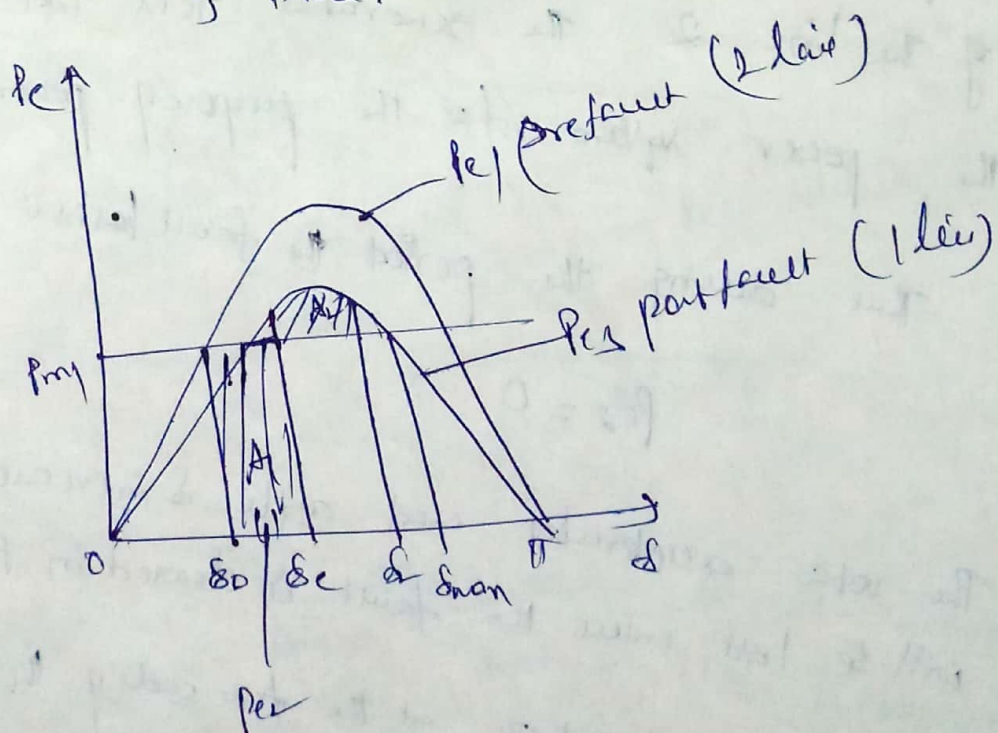
labeled with power angle curve.

$$P_{e3} = \frac{|E'| |V|}{X_d' + X_l} \sin \delta = P_{max3} \sin \delta$$

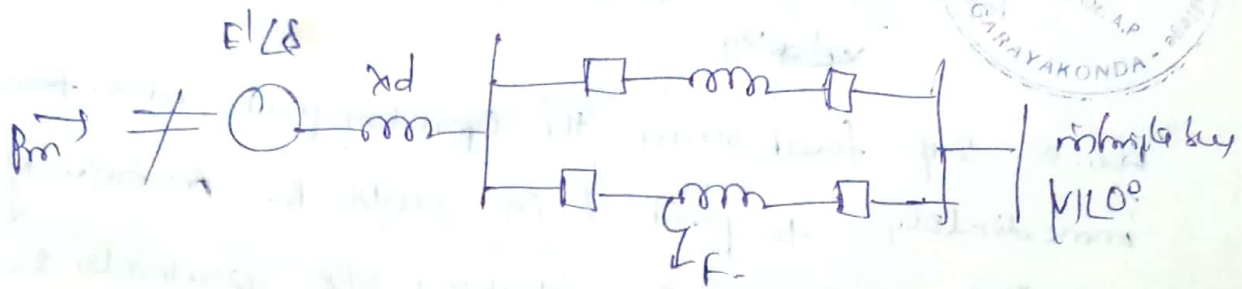
$P_{e3} < P_{e1}$ the motor now starts to decelerating again. The system is stable if a decelerating area A_2 can be found equal to acceleration area A_1 before δ_2 reaches the maximum allowable value δ_{max} .

As area A_1 depends upon clearing time t_c clearing time must be less than a certain value (critical clearing angle) for the system to be stable.

It is to be observed that the equal area criterion help to determine critical clearing angle and not critical clearing time.

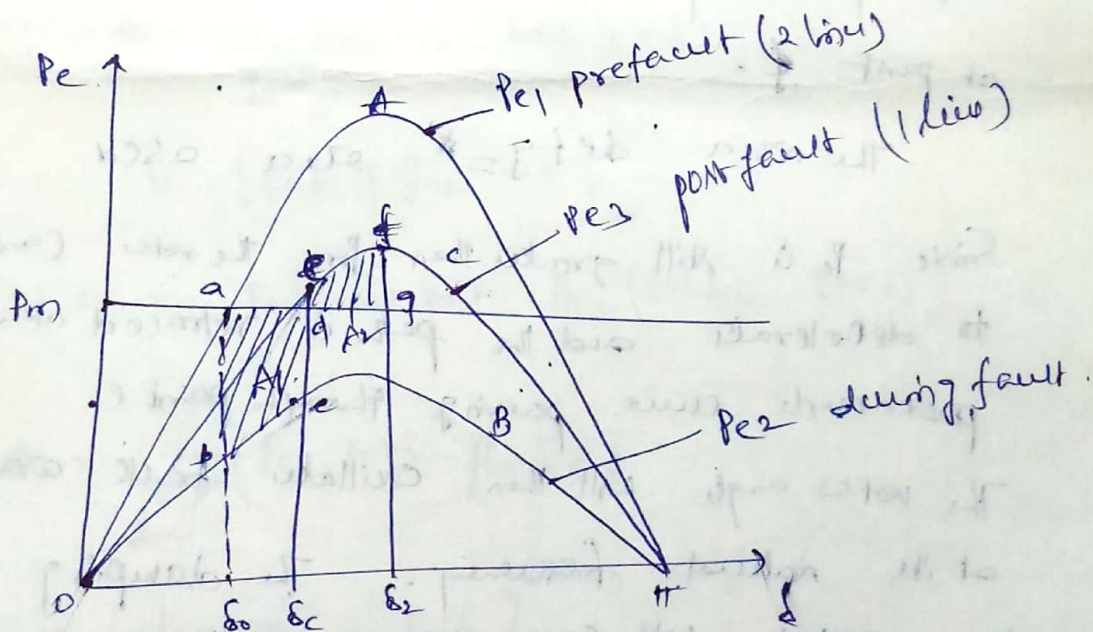


→ A generator is connected to an infinite busbar through two parallel lines.



When the fault occurs F at some distance away from sending end as shown in fig (say in middle of the line)

Assume that the power input is constant P_m and the machine is operating steadily delivering power to the system with a power angle δ_0 as shown in fig.



With fault location at f away from the sending end the equivalent transfer reactance b/w busbar is ~~increased~~ hence lowering the power transfer capability and the power angle curve is represented by curve B. Finally the curve C represents the post fault curve assuming the fault line is removed.

$$P_{e1} = \frac{E' |V|}{x_d' + x_1 \| x_2} \sin \delta = P_{max1} \sin \delta$$

$$P_{e2} = \frac{E' |V|}{x_d' + x_1 \| x_2} \sin \delta = P_{max2} \sin \delta$$

When a 3- ϕ fault occurs the operating point moves ~~from~~ immediately to point b in curve B . An excess of mechanical input over its electrical output accelerates the rotor, thereby storing excess K.E. and the angle δ increases. Assume that fault is cleared at δ_c by isolating the faulted line. This suddenly shifts a point e on curve C . The net power is decelerating and previously stored K.E. will be reduced to zero at point f .

The area $defg =$ area $abcd$.

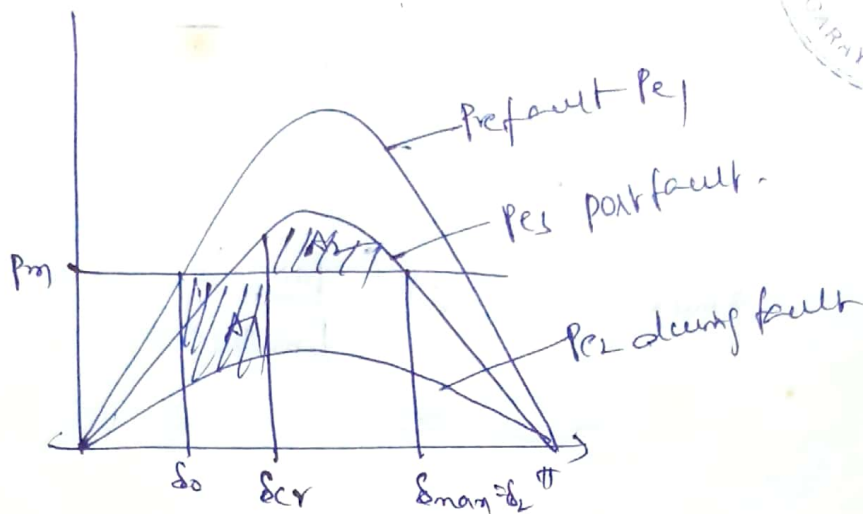
Since P_e is still greater than P_m , the rotor continues to decelerate and the path is retraced along the power angle curve passing through point e .

The rotor angle will then oscillate back around e at its natural frequency. The damping present in the machine will cause these oscillations to subside and a new steady state operation will be established at the intersection of P_m and curve C .

The critical clearing angle is reached when any increase in δ_c causes the area A_2 becomes less than area representing the accelerating energy A_1 .

This occurs at $\delta_{max} = \delta_c$

$A_2 = A_1$ for $\delta_c > \delta_{max}$



Applying equal area criterion to the case of critical clearing angle.

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{m2} \sin \delta) d\delta$$

$$= [P_m \delta + P_{m2} \cos \delta]_{\delta_0}^{\delta_{cr}}$$

$$A_1 = P_m(\delta_{cr} - \delta_0) + P_{m2}(\cos \delta_{cr} - \cos \delta_0)$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{e3} \sin \delta - P_m) d\delta$$

$$= [-P_{e3} \cos \delta - P_m \delta]_{\delta_{cr}}^{\delta_{max}}$$

$$A_2 = -P_{e3}(\cos \delta_{max} - \cos \delta_{cr}) - P_m(\delta_{max} - \delta_{cr})$$

$$A_1 = A_2$$

$$\cancel{P_m \delta_{cr}} - P_m \delta_0 + P_{m2} \cos \delta_{cr} - P_{m2} \cos \delta_0 = -P_{e3} \cos \delta_{max} + P_{e3} \cos \delta_{cr} - P_m \delta_{max} + \cancel{P_m \delta_{cr}}$$

$$\cos \delta_{cr} [P_{e3} - P_{e2}] = P_{e3} \cos \delta_{max} + P_m \delta_{max} - P_{m2} \cos \delta_0 - P_m \delta_0$$

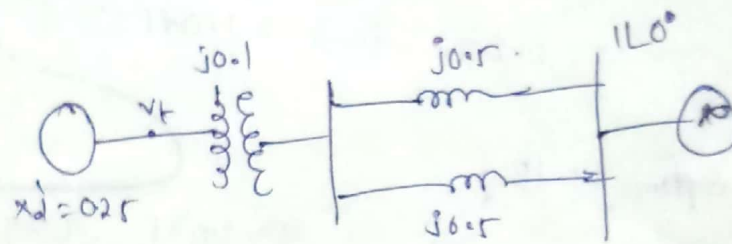
$$\cos \delta_{cr} = \frac{P_{e3} \cos \delta_{max} + P_m (\delta_{max} - \delta_0) - P_{m2} \cos \delta_0}{P_{e3} - P_{e2}}$$

$$\delta_{max} = \pi - \cos^{-1} \left(\frac{P_m}{P_{max III}} \right)$$

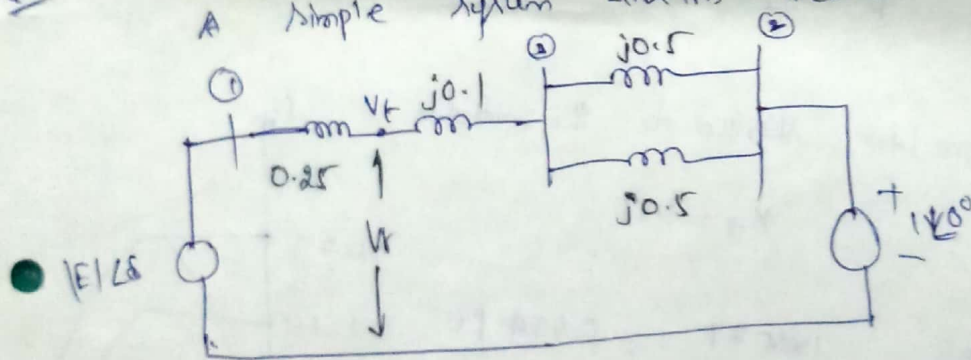
→ The generator is delivering 1.0 p.u power to the infinite bus 14.0 p.u
 with the generator terminal voltage of $|V_t| = 1.0$ p.u. Calculate the
 generator emf behind transient reactance. Find the
 maximum power that can be transferred under the following
 conditions

- System healthy
- one line shorted (b-g) in the middle
- one line open.

plot all the three power angle curves.



Ex 11
 A simple system with its reactance diagram is



Let $V_t = |V_t| \angle \alpha = 1 \angle \alpha$

$$X = j0.1 + \frac{j0.5 \times j0.5}{j1.0}$$

$$= j0.1 + \frac{j^2 0.25}{j}$$

$$= \underline{j0.35}$$

From power angle equation

$$\frac{|V_t| |V|}{X} \sin \alpha = P_e$$

$$\frac{1 \times 1}{0.25 + 0.1} \sin \alpha = 1$$

$$\underline{\alpha = 20.5^\circ}$$

current into infinite bus

$$I = \frac{V_t \angle \alpha - V \angle 0^\circ}{jX}$$

$$= \frac{1 \angle 20.5^\circ - 1 \angle 0^\circ}{j0.35}$$

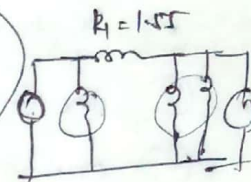
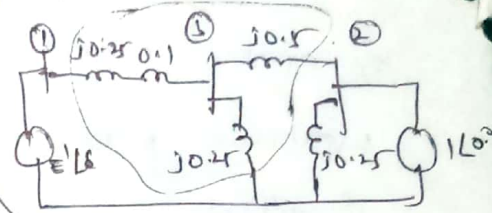
$$= 1 + j0.18 = 1.016 \angle 10.3^\circ$$

voltage behind transient reactance

$$E' = 1 \angle 0^\circ + j0.6 \times (1 + j0.18)$$

$$= 0.892 + j0.6 = 1.075 \angle 33.9^\circ$$

⑥ converting the star at the bus 2 to delta



$$X_{12} = 1.55$$

⑦ system healthy

$$P_{max} = \frac{V_1 E'}{X_{12}} = \frac{1 \times 1.075}{0.6} = 1.79 \text{ p.u.}$$

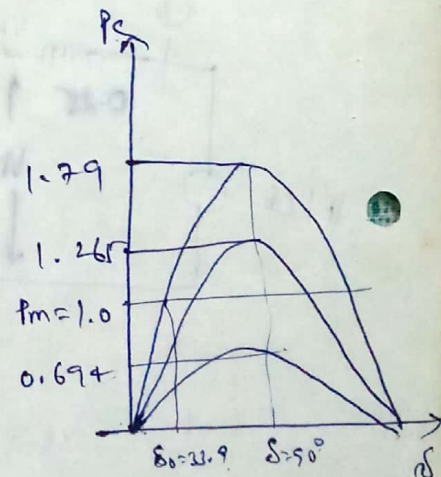
$$P_e = 1.79 \sin \delta \quad \text{--- (1)}$$

⑧ one line shorted in the middle

$$X_{12} = 1.55$$

$$P_{max} = \frac{1 \times 1.075}{1.55} = 0.694 \text{ p.u.}$$

$$P_e = 0.694 \sin \delta$$

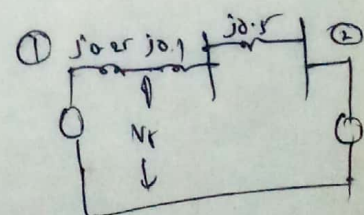


⑨ one line open

$$X_{12} = 0.25 + 0.1 + 0.5 = 0.85$$

$$P_{max} = \frac{1 \times 1.075}{0.85} = 1.265$$

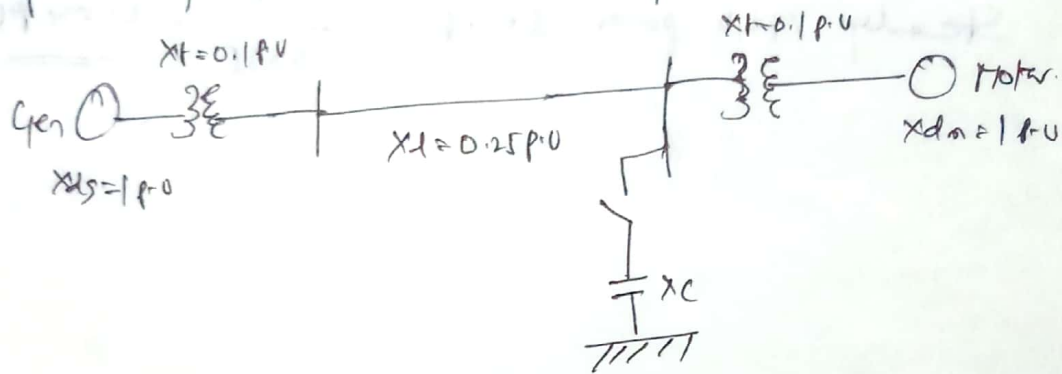
$$P_e = 1.265 \sin \delta$$



$$X_{12} = 0.25 + 0.1 + 0.5 = 0.85$$

→ In the system shown a 2-φ static capacitive reactor of reactance 1 p.u per phase is connected through a switch at motor bus. Calculate the level of steady state power with and without reactor switch closed.

Recalculate the power level with capacitive reactor replaced by an inductive reactor of the same value

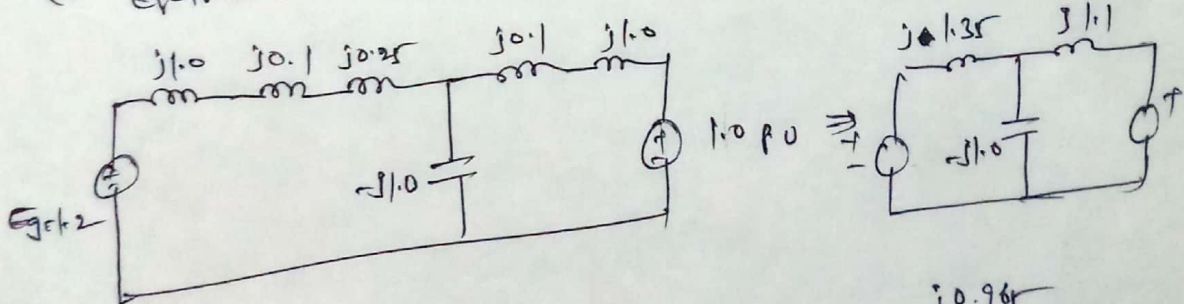


Assume the internal voltage of the generator to be 1.2 p.u and that of the motor to be 1.0 p.u

Q1) Steady state power level without reactor.

$$P = \frac{|E_g| |E_m|}{X_{total}} = \frac{1.2 \times 1}{1 + 0.1 + 0.25 + 0.1 + 1} = 0.491 \text{ p.u}$$

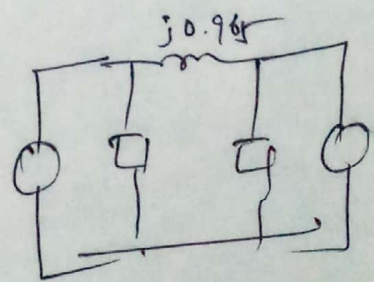
(2) Equivalent circuit with capacitive reactor



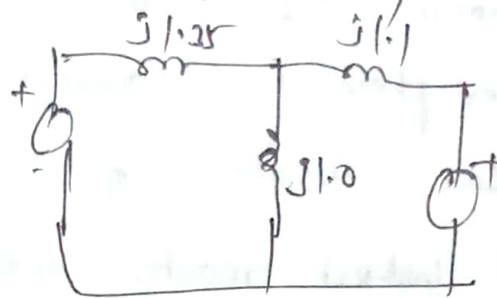
Converting star to delta

$$X = j0.965$$

$$\text{Steady state power level} = \frac{1.2 \times 1}{0.965} = 1.244 \text{ p.u}$$



③ Capacitive reactance replaced by Inductive reactance

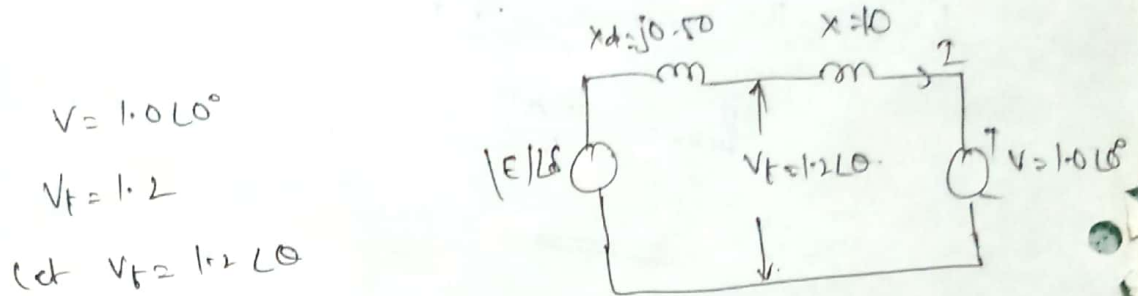


$$Q^* = \underline{j3.935}$$

$$\text{Steady state power level} = \frac{1.2 \times 1}{2.935} = \underline{\underline{0.409 \text{ W}}}$$

→ Find the steady state power limit of a system consisting of a generator equivalent reactance 0.50 pu connected to an infinite bus through a series reactance of 1.0 pu .
The terminal voltage of the generator is held at 1.2 pu and the voltage of the infinite bus is 1.0 pu .

Sol The system is shown in fig. Let the voltage of the infinite bus be taken as reference.



$$I = \frac{V_t - V}{jX} = \frac{1.2 \angle 0^\circ - 1.0}{j1.0}$$

$$\begin{aligned}
 E &= V_t + jX_d I = 1.2 \angle 0^\circ + j0.5 \left[\frac{1.2 \angle 0^\circ - 1.0}{j1} \right] \\
 &= 1.8 \angle 0^\circ - 0.5 \\
 &= 1.8 \cos 0^\circ + j1.8 \sin 0^\circ - 0.5 \\
 &= (1.8 \cos 0^\circ - 0.5) + j1.8 \sin 0^\circ
 \end{aligned}$$

The steady state power limit is reached when E has an angle of $\delta = 90^\circ$. its real part is zero.

$$1.8 \cos 0^\circ = 0.5$$

$$\cos 0^\circ = \frac{0.5}{1.8}$$

$$\theta = 73.87^\circ$$

$$\therefore \text{NOW } V_t = 1.2 \angle 73.87^\circ = 0.312 + j1.152$$

$$\therefore I = \frac{0.312 + j1.152 - 1}{j1} = 1.152 + j0.668$$

$$E = 0.332 + j1.152 + j0.2(1.152 + j0.668)$$

$$= -0.002 + j1.728 \approx 1.728 \angle 90^\circ$$

Steady state power limit is given by

$$P_{\max} = \frac{|E||V|}{X_d + X} = \frac{1.728 \times 1}{1.5} = \underline{\underline{1.152 \text{ p.u.}}}$$

If instead the generator emf is held at a value of 1.2 p.u. the steady state power limit would be

$$P_{\max} = \frac{1.2 \times 1}{1.5} = \underline{\underline{0.8 \text{ p.u.}}}$$