19/02/2020

Unit - V

Symmetrical components and Fault analysis:

The faults like LG, LL, 3-phase short circuits with different fault impedances will create a unbalance in the network.

An unbalanced system of 'n' phasors can be resolved into 'n' systems of balanced phasors. These unbalanced phasors are called as symmetrical

> The symmetrical components are mainly preffered to reduce the time while calculating the unbalanced electrical quantities.

> with reference to the 3-ϕ system the following balanced set of 3 components are identified and defined. Those are

1. positive sequence components.
2. Negative Sequence components
3. zero sequence components

Based on these symmetrical components for 3-ϕ system the unsymmetrical voltages and currents are represented as

$$V_R = V_{R1} + V_{R2} + V_{R0}$$

$$V_Y = V_{Y1} + V_{Y2} + V_{Y0}$$

$$V_B = V_{B1} + V_{B2} + V_{B0}$$

The currents at $I_R = I_{R1} + I_{R2} + I_{R0}$

$$I_Y = I_{Y1} + I_{Y2} + I_{Y0}$$

$$I_B = I_{B1} + I_{B2} + I_{B0}$$

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> Even though there are 9 symmetrical components. But they are balanced components and the components of Y-phase and B-phase will be expressed in terms of R-phase by using suitable notation (K)

> Where 'K' is a electrical quantity which has unity magnitude and 120° phase displacement in anti-clock wise direction.

$$K = 1\angle 120^\circ = 1\cos 120^\circ + j1\sin 120^\circ = -0.5 + j0.867$$

$$K^2 = K \times K = -0.5 - j0.867$$

$$K^3 = K \times K \times K = 1$$

$$K^4 = K^3 \cdot K = K$$

$$K^5 = K^3 \cdot K^2 = K^2$$

$$\boxed{K + K^2 + K^3 = 0}$$

$$\boxed{1 + K + K^2 = 0}$$

Positive Sequence components :-

> Whenever a short circuit fault occured on the power system network. then the stator side electrical quantities are becomes as unbalanced.

> However the direction of rotation of rotor flux will be (~~always clock wised direction~~). same as the direction of rotation of stator flux. when it is said to be a positive sequence.

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> These are the components having same magnitude and displaced by an angle of 120°.

> The phase sequence of these components is same as original phase sequency (R-Y-B)

Significance of positive sequence :-

1. The positive sequence components exists before fault and also in any type of the fault. Hence, these components are reference components in order to set the pick up value of relay for an circuit breaker.

2. The pick up value of the relay is defined as, it is a value beyond which the relay will be operate.

Negative sequence co

Negative Sequence components :-

> Whenever the direction of rotation of stator flux is opposite to that of direction of rotation of rotor flux the corresponding electrical quantities are known as negative sequence components (R-B-Y)

> In case of negative sequence the components are equal in magnitude with a displacement of 120°

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> But the phase sequence of the components are opposite to that original phase sequence of the network.

> In case of negative sequence, there is no induced voltage on stator winding ($E_{R2} = 0$).

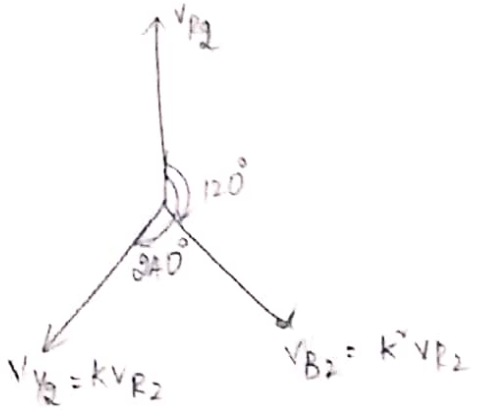
However due to unbalanced value of $V_{R2} \neq 0$

Significance :-

1. The negative sequence flux cut the rotor field winding such that an emf is induced on the rotor. and this induced emf will able to produce the circulating current in rotor.

2. An this circulating current will causes to failure of insulation.

In order to protect the synchronous generator against these conditions a negative sequence relay is used.



Zero sequence components :-

Zero sequence components are exists provided that fault must be a grounded fault.

These are the components having equal in magnitude and having without any phase displacement.

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since there will be a no phase ~~sequence~~ displacement then there is no phase sequence.

since the zero sequence components are having phase displacement of zero. so there is no production of rotational flux. such that it cannot effect on the rotor winding.

But due to these components, the fluxes are produced on the stator without any phase displacement. So, these components are in the form of leakage fluxes and it will be represented by leakage reactances.

$\longrightarrow V_{R_0}$

$\longrightarrow V_{Y_0} = V_{R_0}$

$\longrightarrow V_{B_0} = V_{R_0}$

Based on the above three phase sequence components, the unsymmetrical components V_R, V_Y, V_B can be represented as

$$\underbrace{\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}}_{\text{unsymmetrical matrix}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & K^2 & K \\ 1 & K & K^2 \end{bmatrix} \underbrace{\begin{bmatrix} V_{R_0} \\ V_{R_1} \\ V_{R_2} \end{bmatrix}}_{\text{symmetrical matrix}}$$

$$\begin{bmatrix} V_{R_0} \\ V_{R_1} \\ V_{R_2} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & K^2 & K \\ 1 & K & K^2 \end{bmatrix}}_{A}^{-1} \begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}$$

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$$A^{-1} = \frac{\text{Adj}\,A}{|A|}$$

$$\det A \Rightarrow |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & k^2 & k \\ 1 & k & k^2 \end{vmatrix}$$

$$= 1(k^4 - k^2) - 1(k^2 - k) + 1(k - k^2)$$

$$= k - k^2 - k^2 + k + k - k^2 \qquad \because k^4 = k$$

$$= 3k - 3k^2$$

$$|A| = 3(k - k^2)$$

<u>Cofactors:-</u>

$$1 = \begin{vmatrix} k^2 & k \\ k & k^2 \end{vmatrix} = k^4 - k^2 = k - k^2$$

$$1 = -\begin{vmatrix} 1 & k \\ 1 & k^2 \end{vmatrix} = -(k^2 - k) = -k^2 + k = k - k^2$$

$$1 = \begin{vmatrix} 1 & k^2 \\ 1 & k \end{vmatrix} = k - k^2$$

$$1 = -\begin{vmatrix} 1 & 1 \\ k & k^2 \end{vmatrix} = -(k^2 - k) = -k^2 + k = k - k^2$$

$$k^2 = +\begin{vmatrix} 1 & 1 \\ 1 & k^2 \end{vmatrix} = (k^2 - 1) \quad \cancel{= -k^2 + 1 = 1 - k^2}$$

$$k = -\begin{vmatrix} 1 & 1 \\ 1 & k^2 \end{vmatrix} = -(k^2 - 1) = 1 - k^2$$

$$1 = +\begin{vmatrix} 1 & 1 \\ k^2 & k \end{vmatrix} = k - k^2$$

$$k = -\begin{vmatrix} 1 & 1 \\ 1 & k^2 \end{vmatrix} = -(k^2 - 1) = -k^2 + 1 = 1 - k^2$$

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$$K^2 = \begin{vmatrix} 1 & 1 \\ 1 & K^2 \end{vmatrix} = K^2 - 1$$

Adj A = Transpose of co-factors.

$$= \begin{bmatrix} K-K^2 & K^2-K^2 & K-K^2 \\ K-K^2 & K^2-1 & 1-K \\ K-K^2 & 1-K & K^2-1 \end{bmatrix} = \begin{bmatrix} K-K^2 & K-K^2 & K-K^2 \\ K-K^2 & K^2-1 & 1-K \\ K-K^2 & 1-K & K^2-1 \end{bmatrix}$$

$$\begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} =$$

$$\frac{\text{Adj}A}{|A|} = \frac{K-K^2}{3(K-K^2)} = \frac{1}{3}$$

$$\frac{K-K^2}{3(K-K^2)} = \frac{1}{3}$$

$$\frac{K-K^2}{3(K-K^2)} = \frac{1}{3}$$

$$\frac{K-K^2}{3(K-K^2)} = \frac{1}{3}$$

$$\frac{K^2-1}{3(K-K^2)} = \frac{K(K^2-1)}{3K(K-K^2)} = \frac{K(K^2-1)}{3K^2-K^3} = \frac{K(K^2-1)}{3(K^2-1)} = \frac{K}{3}$$

$$\frac{1-K}{3(K-K^2)} = \frac{K^2(1-K)}{3(K-K^2)K^2} = \frac{K^2(1-K)}{3(K^3-K^4)} = \frac{K^2(1-K)}{3(1-K)} = \frac{K^2}{3}$$

$$\begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}$$

Similarly

$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}$$

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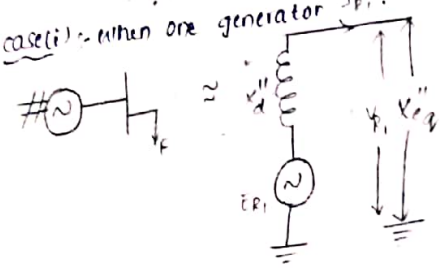
Positive Sequence sub transient reactance :-

For Synchronous Generator

case(i) :- when one generator is connected

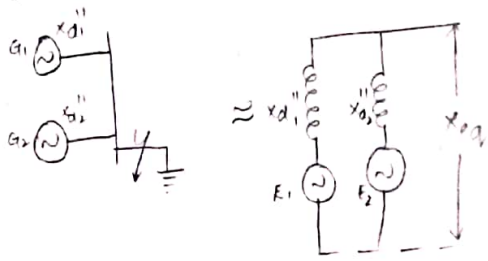


$$E_{R_1} = V_{R_1} + I_{R_1} X_d''$$

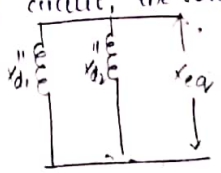
$$\boxed{V_{R_1} = E_{R_1} - I_{R_1} X_d''}$$

$$\boxed{X_{eq} = X_d''}$$

case(ii) :- Two generators are connected in parallel

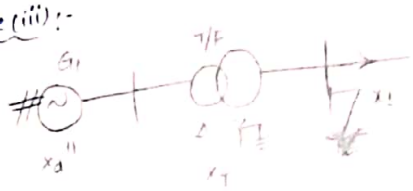


In the equivalent circuit, the voltage sources are short circuited

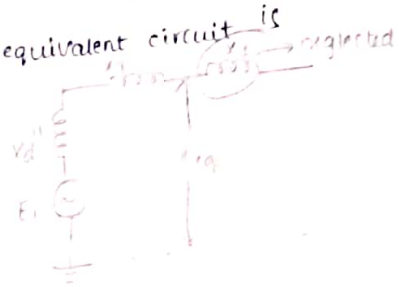


$$\boxed{X_{eq} = \frac{X_{d_1}'' * X_{d_2}''}{X_{d_1}'' + X_{d_2}''}}$$

case(iii) :-

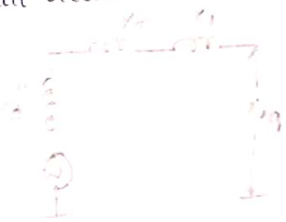


The equivalent circuit is



$$\boxed{X_{eq} = X_d'' + X_T}$$

→ If the fault occurs at end of the transmission line



$$\boxed{X_{eq} = X_d'' + X_T + X_L}$$

* If the fault occurs at middle of the transmission line

$$\boxed{X_{eq} = X_d'' + X_T + \frac{X_L}{2}}$$

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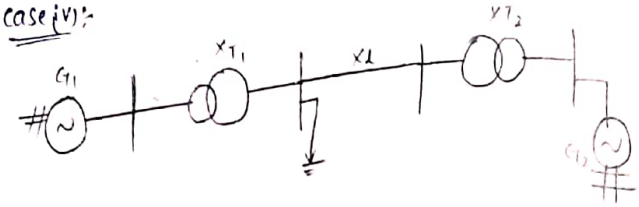
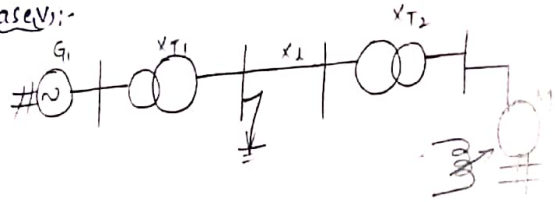
case(iv):-



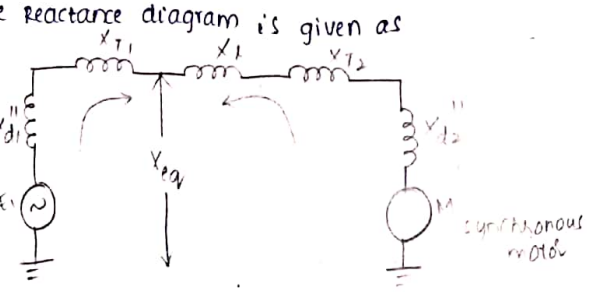
The Reactance diagram is given as

$$X_{eq} = (X_{d_1}'' + X_{T_1}) \,//^{lel}\, (X_L + X_{T_2} + X_{d_2}'')$$

case(v):-



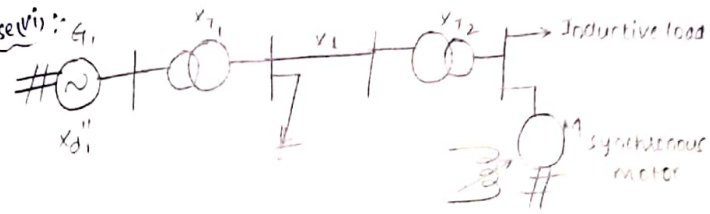
The Reactance diagram is given as



Here the synchronous motor is acts as a generator at some extent. Hence the X_{eq} is same as the above case

$$X_{eq} = (X_{d_1}'' + X_{T_1}) \,//^{lel}\, (X_L + X_{T_2} + X_{d_2}'')$$

case(vi):-



The Reactance diagram is given as

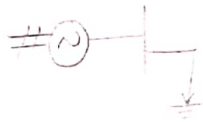
$$X_{eq} = (X_{d_1}'' + X_{T_1}) \,//^{lel}\, (X_L + X_{T_2} + X_{d_2}'')$$

Negative Sequence sub transient Reactance :-

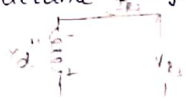
In Negative Sequence sub transient Reactance diagram, the values of induced voltage on stator are equal to zero. Because, the stator induced voltage is always depends upon the flux coming from the rotor side. Since the rotor always rotating in a clockwised manner, the corresponding Induced voltage on stator is E_{R_1}.

$\therefore E_{R_2} = 0$.

case(i):-



The Reactance diagram is given as



$V_{R_2} = -I_{R_2}(X_d'')$

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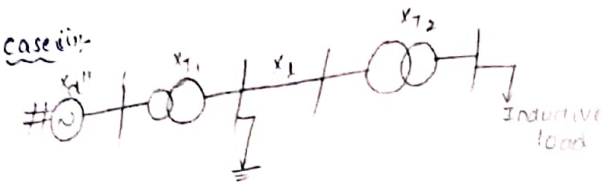
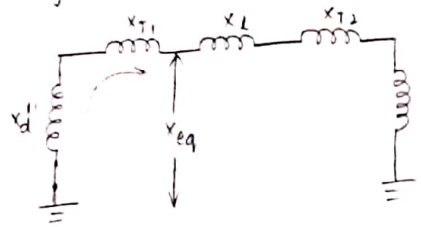
case(ii):-



The negative sequence reactance diagram is



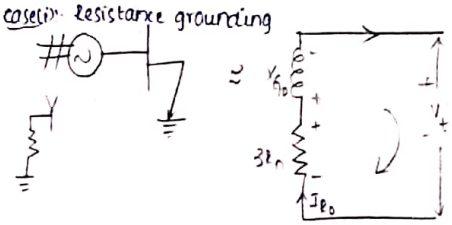
$$X_{eq} = X_d'' + X_{T_1}$$

~~case(iii)~~:-

Zero Sequence subtransient reactance:- 3rd order harmonics are considered as zero sequence Ex: 3, 9, 15, 21

Zero Sequence subtransient reactance diagram is drawn under the condition of grounded fault only.

case(i): Resistance grounding



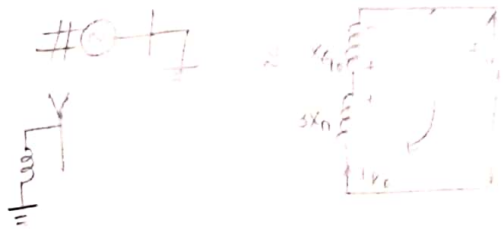
Terminal voltage

$$V_t = -I_{R_0}(3R_n) - jI_{R_0}X_{G_0}$$

$$V_t = -I_{R_0}(3R_n + jX_{G_0})$$

$$V_t = -I_{R_0}Z_n$$

case(ii):- Reactance grounding



Terminal voltage

$$V_t = -I_{R_0}(3X_n) - jI_{R_0}X_{G_0}$$

$$V_t = -I_{R_0}(3X_n + jX_{G_0})$$

$$V_t = -I_{R_0}$$

case(iii):- Solid grounding

Terminal voltage

$$V_t = -jI_{R_0}X_{G_0}$$

case(iv):- Isolated neutral

The circuit is open, the current flowing is zero, the Terminal voltage $V_t = 0$

$$V_t = 0$$

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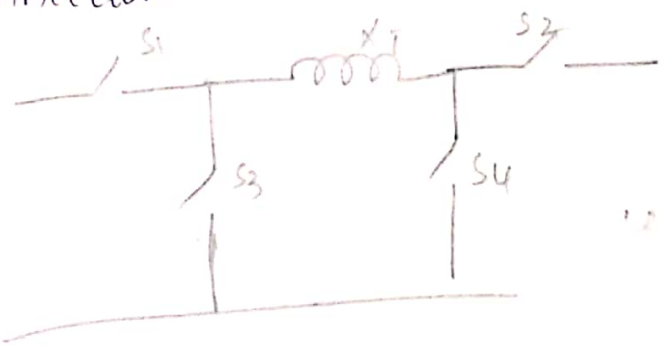
case vi: for delta connected winding

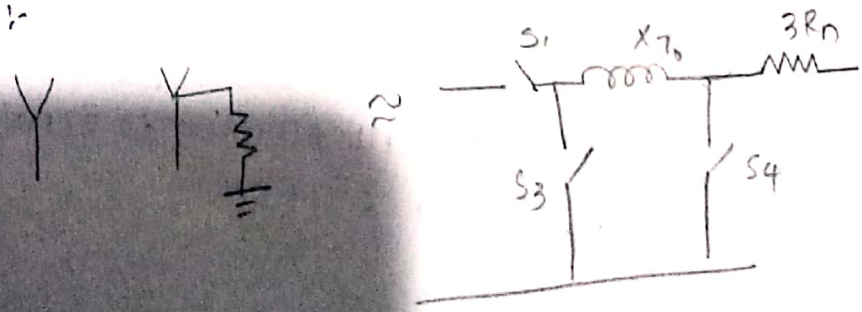
Representation of a zero sequence transformer :-

For a transformer, the zero sequence reactance diagram represented with the help of mechanical switches, S_1, S_2, S_3, S_4 as shown in below. where S_1, S_2 are operated for star connected and S_3, S_4 are used for delta connection.

For Δ S_3, S_4
Y S_1, S_2



case(i) :-



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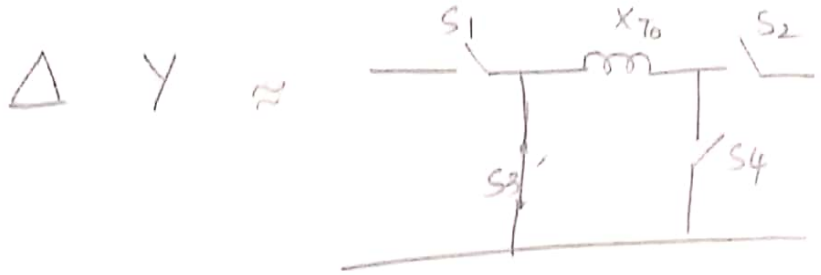
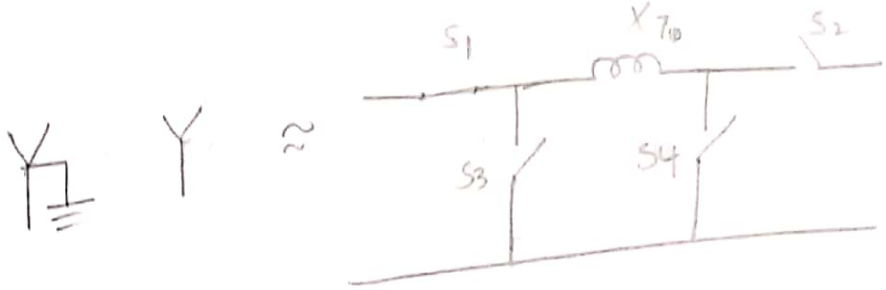
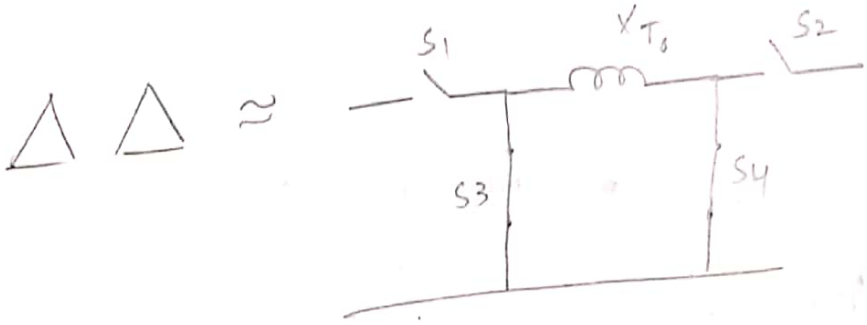
case(ii):



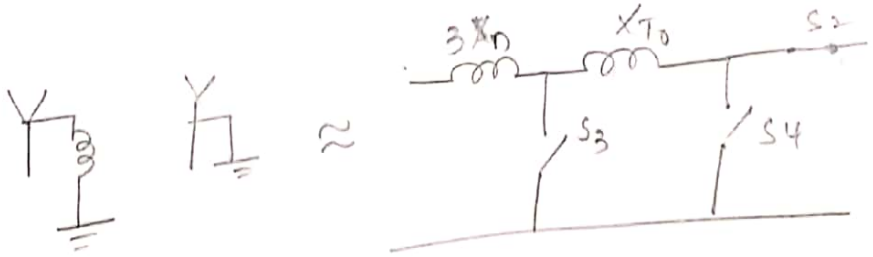
case(iii):-



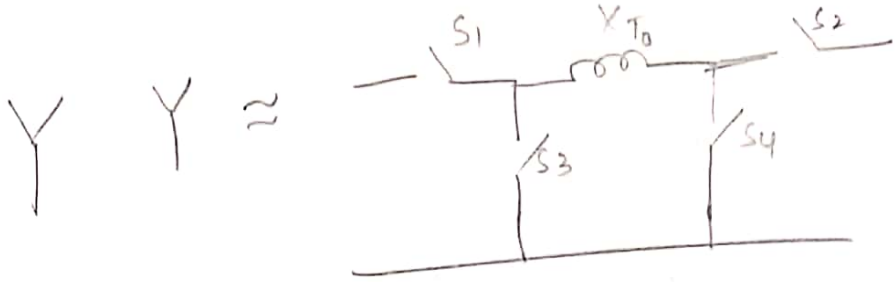
case(iv):-



case(v):-

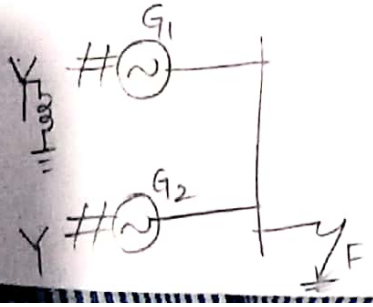


case(vi):-



Problem:-

1. Calculate the positive, negative, zero sequences for the network shown in below.



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$G_1 = G_2$

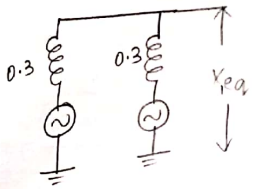
$X_d'' = 0.3$

$X_2 = 0.2$

$X_{G_0} = 0.08$

$X_n = 0.05$

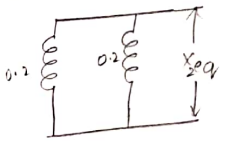
Sol:- positive sequence Reactance diagram :-



∴ The thevinen's Resistance equivalent

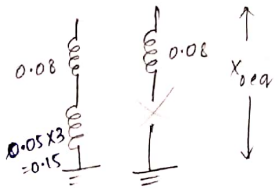
$$R_{th} = X_{eq} = \frac{1}{\frac{1}{0.3}+\frac{1}{0.3}} = \frac{0.3 \times 0.3}{0.3+0.3} = 0.15$$

Negative sequence Reactance diagram :



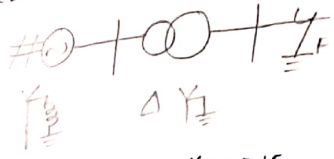
$$X_{eq} = \frac{0.2 \times 0.2}{0.2+0.2} = 0.1$$

Zero sequence Reactance diagram :



$X_{0eq} = 0.15 + 0.08 = 0.23$

2. Calculate the positive, negative, zero sequences for the network shown in below.



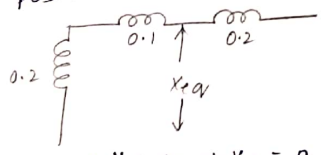
$X_d'' = 0.2$ $\quad X_2 = 0.15$ $\quad X_{G_0} = 0.08$ $\quad X_n = 0.05$

$X_{T_1} = X_{T_2} = X_{T_0} = 0.1$

$X_1 = X_2 = 0.2$

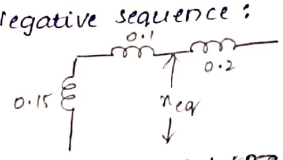
$X_0 = 0.6$

Sol:- positive Sequence Reactance diagram:



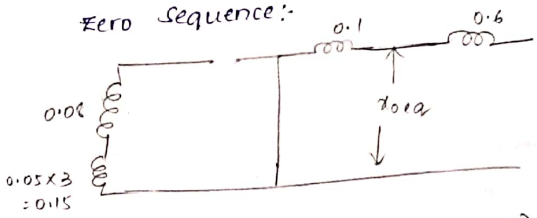
$X_{eq} = X_d'' + X_{T_1} + X_1 = 0.2 + 0.1 + 0.2 = 0.5$

Negative sequence :



$X_{eq} = 0.15 + 0.1 + 0.2 = 0.25$

Zero Sequence :-



$X_{eq} = 0.15 + 0.08 + 0.1 = 0.33$

$X_{eq} = 0.1$

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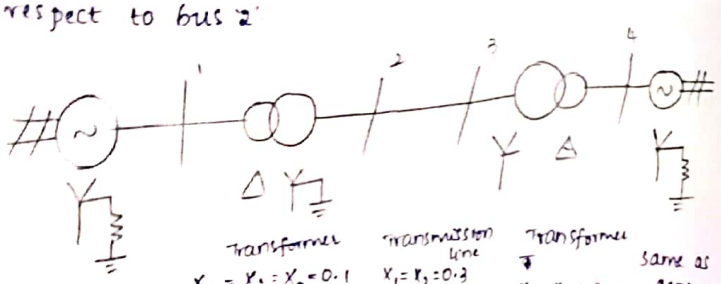
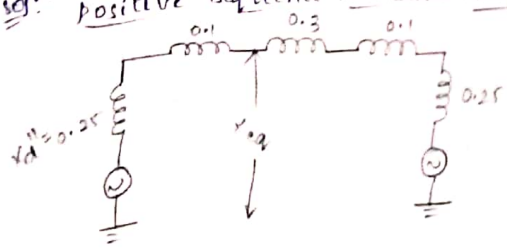
3. Calculate the equivalent sequence reactance with respect to bus 2



	Transformer	Transmission line	Transformer	Same as generator 1
$X_d'' = 0.25$	$X_1 = X_2 = X_0 = 0.1$	$X_1 = X_2 = 0.3$	$X_1 = X_2 = X_0 = 0.1$	$X_d'' = 0.25$
$X_2 = 0.2$		$X_0 = 0.9$		$X_2 = 0.2$
$X_{g_0} = 0.08$				$X_{G_0} = 0.08$
$R_n = 0.05$				$R_n = 0.05$

Sol:- Positive Sequence Reactance diagram.



0.25 is in series with 0.1

$0.25 + 0.1 = 0.35$

0.3, 0.1, 0.25 are in series

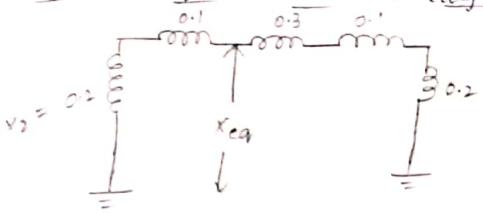
$0.3 + 0.1 + 0.25 = 0.65$

0.35 and 0.65 are in parallel

$$\frac{(0.35)(0.65)}{0.35 + 0.65} = 0.22$$

$X_{1eq} = 0.22$

Negative Sequence Reactance diagram:



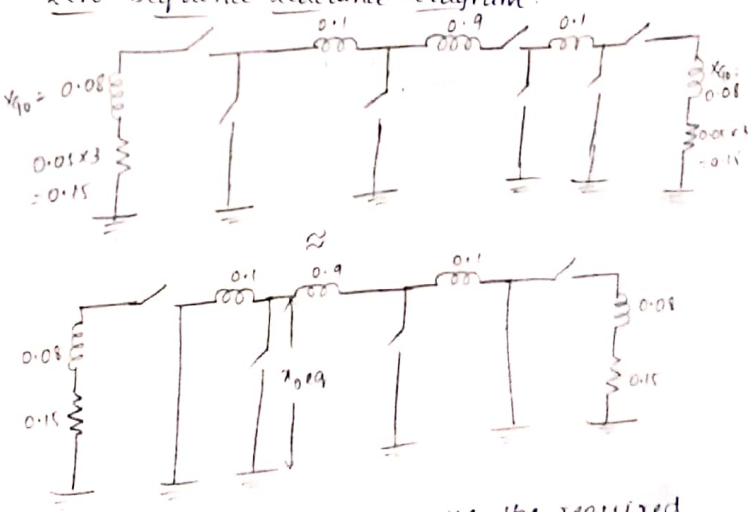
0.2 is in series with 0.1 = $0.2 + 0.1 = 0.3$

0.2, 0.1, 0.3 are in series $\Rightarrow 0.2 + 0.1 + 0.3 = 0.6$

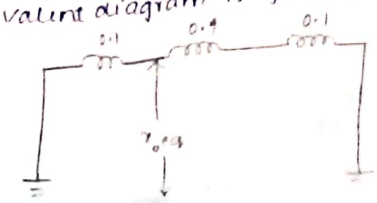
$$0.3 \,//\, 0.6 = \frac{(0.3)(0.6)}{0.3 + 0.6} = 0.2$$

$X_{2eq} = 0.2$

Zero sequence Reactance diagram.



By neglecting the open circuits, the required equivalent diagram is given as



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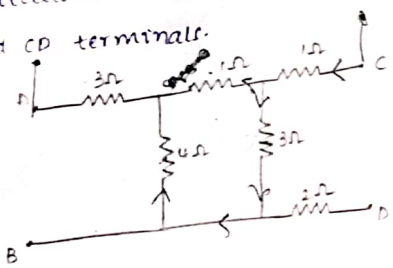
0.9 and 0.1 are in series = $0.9 + 0.1 = 1.0$

$0.1 // 1.0 \Rightarrow \frac{(0.1)(1.0)}{(0.1)+(1.0)} = 0.09$

$X_{eq} = 0.09$

* Calculate the equivalent resistance offered by AB and CD terminals.



$R_{AB} = 3\Omega // 4\Omega = \frac{3 \times 4}{3+4} = 1.714$

$1.714 + 1 = 2.714$

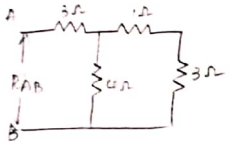
$2.714 // 3 = \frac{2.714 \times 3}{2.714 + 3} = 1.424$

1.424, 1, 2 are in series

$1.424 + 1 + 2 = 4.424$) X

$R_{CD} \Rightarrow$

$R_{AB} \Rightarrow$



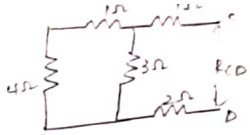
$1\Omega + 3\Omega = 4\Omega$

$\Rightarrow 4\Omega // 4\Omega = \frac{4 \times 4}{4+4} = 2\Omega$

$\Rightarrow 2\Omega + 3\Omega = 5\Omega$

$R_{AB} = 5\Omega$

$R_{CD} \Rightarrow$

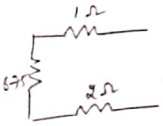


$\Rightarrow 4\Omega + 1\Omega = 5\Omega$

$\Rightarrow 5\Omega // 3\Omega$

$\frac{5 \times 3}{5+3} = \frac{15}{8} = 1.875\Omega$

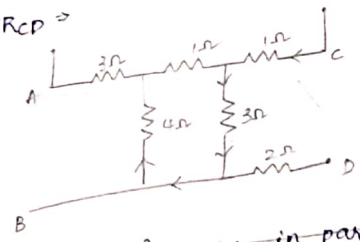
$\Rightarrow 1.875\Omega, 1\Omega, 2\Omega$ are in series



$1.875 + 1 + 2 = 4.875\Omega$

$R_{CD} = 4.875\Omega$

$R_{CD} \Rightarrow$



~~4Ω and 3Ω are in parallel~~

$\cancel{\frac{4 \times 3}{4+3} = \frac{12}{8} =}$

4Ω and 3Ω are in series

$4 + 3 = 7$

$7 // 1 \Rightarrow \frac{7 \times 1}{7+1} = \frac{7}{8} = 0.875\Omega$

0.875, 1Ω, 3Ω are in series

$0.875 + 1 + 3 = 4.875\Omega$

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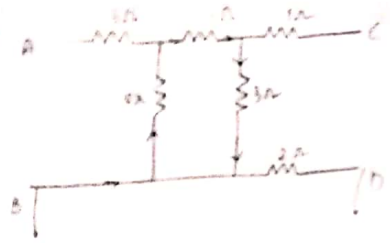
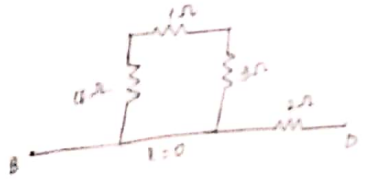
$R_{BD} = ?$



~~4Ω, 1Ω, 3Ω, 2Ω are in series~~

~~4 + 1 + 3 + 2 = 10Ω~~

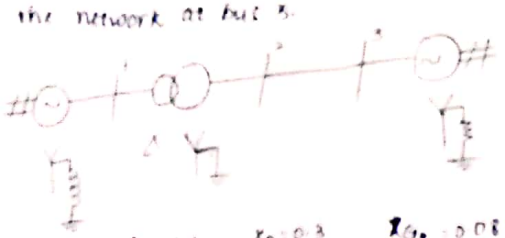


The current always chooses less resistance path

i.e., R = 0

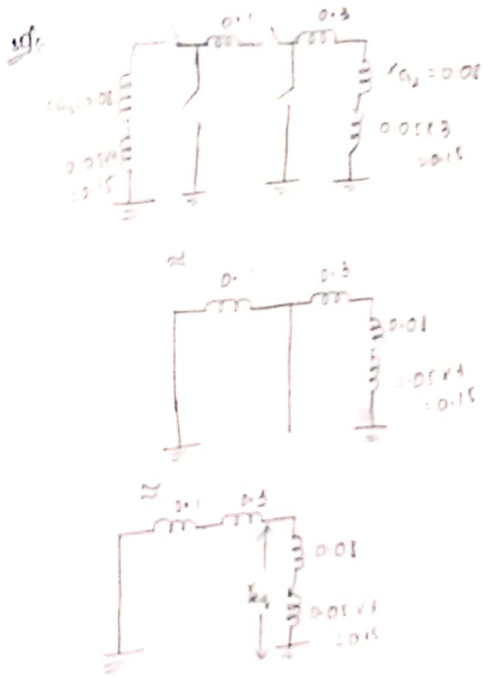
$R_{BD} = 2\Omega$

* Calculate the zero sequence reactance offered by the network at bus 3.



$X_{G_1} = 0.08$ $X_0 = 0.1$ $X_0 = 0.3$ $X_{G_2} = 0.08$

$X_n = 0.05$ $X_n = 0.05$



$X_{0eq} = 0.1 + 0.3$

$= 0.4$

0.08, 0.05 are in series

$0.08 + 0.15 = 0.23$

$0.4 // 0.23 = \frac{0.4 \times 0.23}{0.4 + 0.23} = 0.146$

$X_{0eq} = 0.146$

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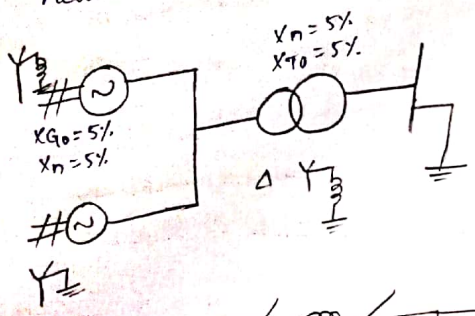
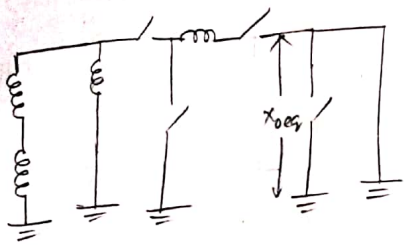
* calculate the zero sequence reactance for the given network.



Sol:



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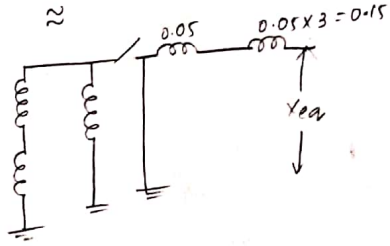

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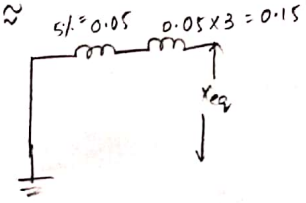


$X_{0eq} = 0.05 + 0.15 = 0.2$

$X_{0eq} = 0.2$

28/02/2020

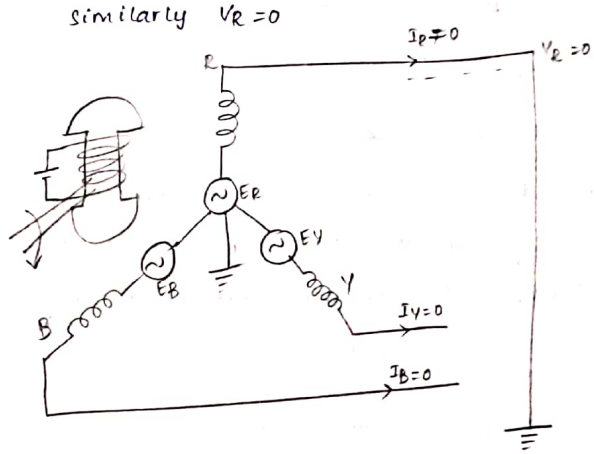
case(i):-

L-G Fault :- in solid grounding

consider an L-G Fault occured at one of the phase of alternator then the unsymmetrical components I_R, I_Y, I_B are related as

$I_Y = I_B = 0$ and $I_R \neq 0$

similarly $V_R = 0$



We know that the relationship between symmetrical and unsymmetrical current components are

$$\begin{bmatrix} I_{R_0} \\ I_{R_1} \\ I_{R_2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix} \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}$$

$I_Y = I_B = 0$

$$\begin{bmatrix} I_{R_0} \\ I_{R_1} \\ I_{R_2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix} \begin{bmatrix} I_R \\ 0 \\ 0 \end{bmatrix}$$

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$I_{R0} = \frac{1}{3}(I_R)$

$I_{R1} = \frac{1}{3} I_R$

$I_{R2} = \frac{1}{3} I_R$

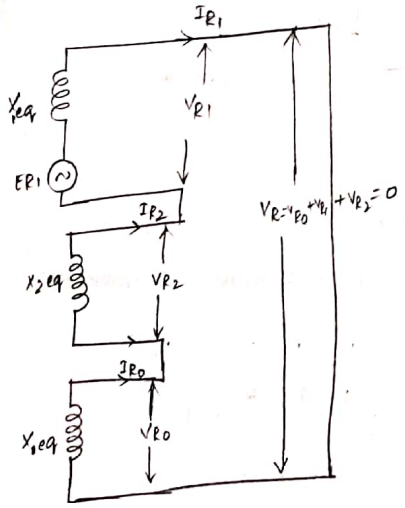
$I_{R0} = I_{R1} = I_{R2} = \frac{1}{3} I_R$ — (1)

From the diagram we know that

$V_R = 0$ (R-phase is short circuited)

$V_R = V_{R0} + V_{R1} + V_{R2} = 0$ — (2)

From eq (1) & (2) we observe that the three sequence component currents are equal such that all the three sequence networks are connected in series.



Calculation of fault current (I_f):-

The fault current is denoted with I_f.

From the diagram $I_f = I_R$

$V_R = 0$

$V_{R0} + V_{R1} + V_{R2} = 0$

$-I_{R0} X_{0eq} + E_{R1} - I_{R1} \cdot X_{1eq} - I_{R2} X_{2eq} = 0$

$-I_{R1} X_{0eq} + E_{R1} - I_{R1} X_{1eq} - I_{R1} X_{2eq} = 0$

$I_{R1}[X_{0eq} + X_{1eq} + X_{2eq}] = E_{R1}$

$$I_{R1} = \frac{E_{R1}}{X_{0eq} + X_{1eq} + X_{2eq}}$$

$I_{R0} = 3 I_{R1}$

$$\boxed{I_f = I_R = \frac{3 E_{R1}}{X_{0eq} + X_{1eq} + X_{2eq}}}$$

Actual fault current $\boxed{I_{F(act)} = I_{F\,P.U} \times I_{base}}$

Short circuit MVA :-

The rating of circuit breaker is in terms of braking current which is same as fault current.

In can also express in terms of braking capacity given by

$\boxed{\text{Short circuit MVA} = 3 \times V \times I_F}$

S.C MVA $= 3 V I_F$

$= 3 V I_{F(P.U)} \times I_{base}$

$= 3 \cdot V I_{base} \times I_{F.PU}$

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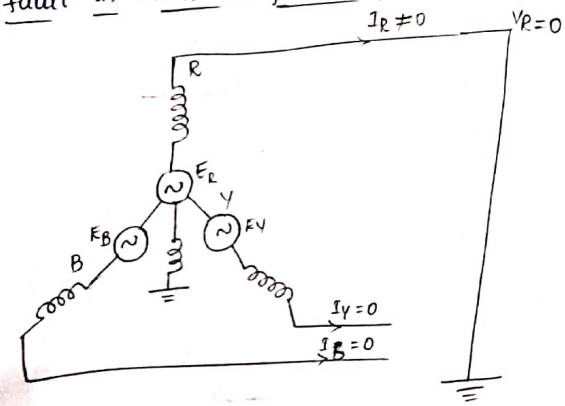
$$S.C.MVA = \text{Base MVA} \times \frac{3E_{R1}}{X_{1eq} + X_{2eq} + X_{0eq}}$$

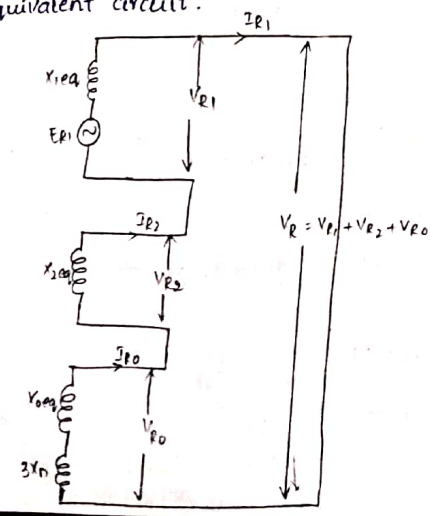
$$\boxed{S.C\,MVA = 3 \times \text{Base MVA} \times \frac{E_{R1}}{X_{1eq} + X_{2eq} + X_{0eq}}}$$

case-(ii):-

L-G fault in reactance grounding :-



Equivalent circuit :



During the fault

$I_R \neq 0,\ I_Y = 0,\ I_B = 0$

$V_R = 0$

$V_Y \neq 0$

$V_B \neq 0$

calculation of fault current (I_f)

$$V_R = 0$$

$$V_{R1} + V_{R2} + V_{R0} = 0$$

$$-I_{R0}X_{0eq} + E_{R1} - I_{R1} \cdot X_{1eq} - I_{R2}X_{2eq} - 3I_{R0}X_n = 0$$

$$E_{R1} - I_{R1} \cdot X_{1eq} - I_{R1}X_{2eq} - 3I_{R1}X_n = 0$$

$$I_{R1}\left[X_{0eq} + X_{1eq} + X_{2eq} + 3X_n\right] = E_{R1}$$

$$I_{R1} = \frac{E_{R1}}{X_{0eq} + X_{1eq} + X_{2eq} + 3X_n}$$

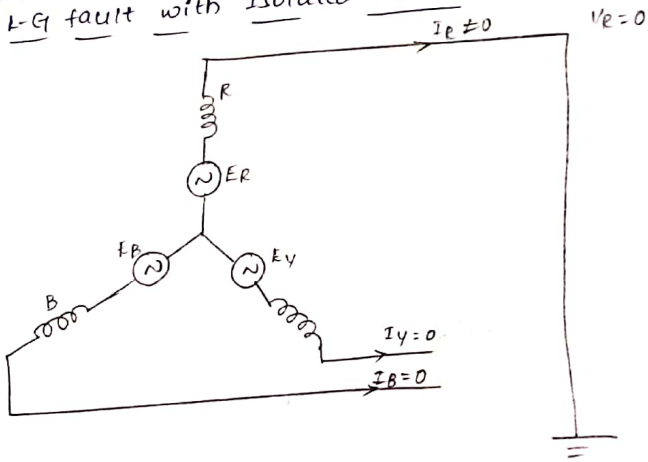
$$\boxed{I_f = I_R = \frac{3E_{R1}}{X_{0eq} + X_{1eq} + X_{2eq} + 3X_n}} \quad [\because I_{R_f} = 3I_{R_1}]$$

For resistance grounding

$$I_f = I_R = \frac{3E_{R1}}{X_{0eq} + X_{1eq} + X_{2eq} + 3R_n}$$

case-(iii):-

L-G fault with Isolated neutral :-



In case of isolated neutral, the positive sequence and negative sequence reactances are same. But the zero sequence reactance is infinite

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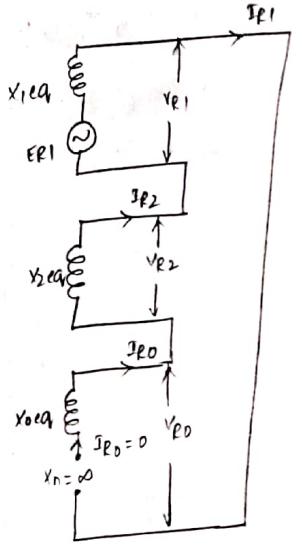
such that $I_{R0} = 0$

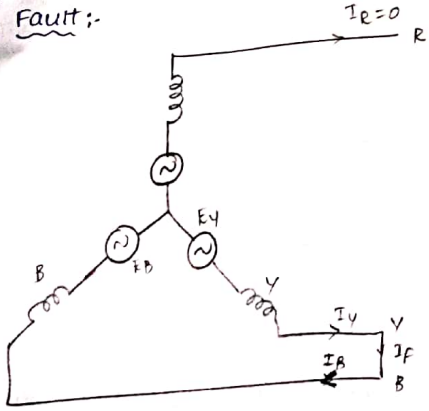
$$I_F = \frac{3 E_{R1}}{X_{1eq} + X_{2eq} + \infty}$$

$$\boxed{I_F = 0}$$



6/03/2020

L-L Fault :-



consider a L-L fault occured between the phases Y and B such that, before the fault

$I_R = 0$; $I_Y = 0$; $I_B = 0$ (At no load condition)

$V_R \neq 0$, $V_Y \neq 0$, $V_B \neq 0$

During the fault,

$I_R = 0$; $I_Y = -I_B$

$V_R \neq 0$; $V_Y = V_B$ — (3)

We know that the relationship between symmetrical and unsymmetrical current components are

$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix} \begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}$$

$$I_{R0} = \frac{1}{3}\left[I_R + I_Y + I_B\right] \qquad \because I_R = 0,\ I_Y = -I_B$$

$$I_{R0} = \frac{1}{3}\left[0 - I_B + I_B\right]$$

$$\boxed{I_{R0} = 0}$$

$$I_{R1} = \frac{1}{3}\left[I_R + K I_Y + K^2 I_B\right] \qquad \because I_R = 0,\ I_B = -I_Y$$

$$= \frac{1}{3}\left[0 + K I_Y - K^2 I_Y\right]$$

$$I_{R1} = \frac{I_Y}{3}\left[K - K^2\right] \quad — (1)$$

$$I_{R2} = \frac{1}{3}\left[I_R + K^2 I_Y + K I_B\right]$$

$$I_{R2} = \frac{I_Y}{3}\left[K^2 - K\right] \quad — (2)$$

From (1) and (2)

$$\boxed{I_{R1} = -I_{R2}} \quad — (4)$$

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From eq ③ ⇒ $V_Y = V_B$

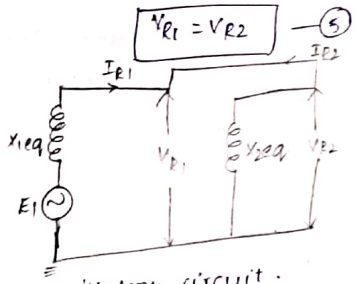
$$V_{Y0} + V_{Y1} + V_{Y2} = V_{B0} + V_{B1} + V_{B2}$$

$$\cancel{V_{R0}} + K^2 V_{R1} + K V_{R2} = \cancel{V_{R0}} + K V_{R1} + K^2 V_{R2}$$

$$K^2 V_{R1} - K V_{R1} = K^2 V_{R2} - K V_{R2}$$

$$V_{R1}\left[\cancel{K^2 - K}\right] = V_{R2}\left[\cancel{K^2 - K}\right]$$

$$\boxed{V_{R1} = V_{R2}} \quad \text{— ⑤}$$



From equivalent circuit;

$$V_{R1} = E_1 - X_{1eq} I_{R1}$$

$$V_{R2} = -X_{2eq} I_{R2}$$

From eqn ⑤

$$V_{R1} = V_{R2}$$

$$E_1 - X_{1eq} I_{R1} = -X_{2eq} I_{R2} \qquad \because -I_{R2} = I_{R1}$$

$$E_1 - I_{R1} X_{1eq} = I_{R1} X_{2eq}$$

$$E_1 = I_{R1} X_{2eq} + I_{R1} X_{1eq}$$

$$E_1 = I_{R1}\left[X_{1eq} + X_{2eq}\right]$$

$$\boxed{I_{R1} = \frac{E_1}{X_{1eq} + X_{2eq}}} \qquad I_{R2} = \frac{-E_1}{X_{1eq} + X_{2eq}} \qquad \because I_{R1} = I_{R2}$$

for calculating Fault current

From eq ①

$$I_F = I_Y = \frac{3 I_{R1}}{K - K^2}$$

$$= \frac{3\left[\frac{E_1}{X_{1eq} + X_{2eq}}\right]}{-0.5 + j0.86 - (-0.5 - j0.86)}$$

$\because K = 1\angle 120^\circ$
$K = -0.5 + j0.86$
$K^2 = 1\angle -120^\circ$
$K^2 = -0.5 - j0.86$

$$= \frac{3\left[\frac{E_1}{X_{1eq} + X_{2eq}}\right]}{j\sqrt{3}}$$

$$= \frac{3 I_{R1}}{j\sqrt{3}} = -j\sqrt{3}\, I_{R1}$$

$$\boxed{I_F = -j\sqrt{3}\, I_{R1}}$$

$$\boxed{I_F = I_B = j\sqrt{3}\, I_{R2}} \qquad (\because I_{R1} = -I_{R2})$$

$$I_F = -j\sqrt{3} \times \frac{E_1}{X_{1eq} + X_{2eq}}$$

$$|I_{F\,p.u}| = \frac{\sqrt{3}\, E_1}{X_{1eq} + X_{2eq}} \quad \text{(consider only magnitude)}$$

$$\boxed{I_{F(act)} = I_{F(p.u)} \times I_b}$$

Short circuit MVA $= 3V I_F$ $\qquad \because 3V I_b = \text{Base MVA}$

$$= 3V\, I_{F(p.u)} \times I_b \qquad \left(\because I_{F(p.u)} = \frac{\sqrt{3} E_1}{X_{1eq} + X_{2eq}}\right)$$

$$\boxed{\text{S.C MVA} = \text{Base MVA} \times \frac{\sqrt{3} E_1}{X_{1eq} + X_{2eq}}}$$

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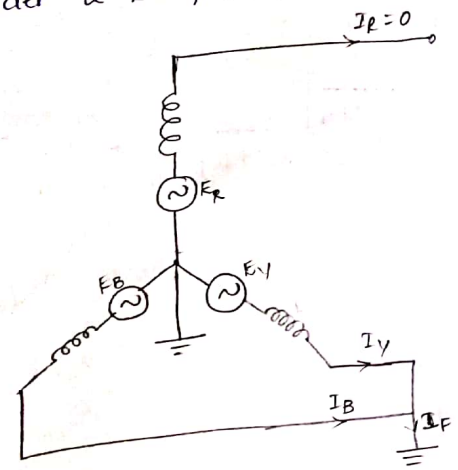
L-L-G Fault :-

Consider a L-L-G Fault occured in Y-phase and B-phase



Before Fault Condition (i.e; No load condition)

$I_R = I_Y = I_B = 0$

$V_R = V_Y = V_B \neq 0$

During Fault condition

$I_R = 0$, $I_Y + I_B = I_F$

$V_R \neq 0$, $V_Y + V_B = 0$

$$\begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}$$

$V_{R0} = \frac{1}{3}\left[V_R + V_Y + V_B\right]$ $\quad \because V_Y = V_B = 0$

$V_{R0} = \frac{1}{3}\left[V_R\right]$

$V_{R1} = \frac{1}{3}\left[V_R\right]$

$V_{R2} = \frac{1}{3}\left[V_R\right]$

$$\boxed{V_{R0} = V_{R1} = V_{R2} = \frac{1}{3}V_R} \quad \text{—(1)}$$

Fault current $I_F = I_Y + I_B$

$= I_{Y_0} + I_{Y_1} + I_{Y_2} + I_{B_0} + I_{B_1} + I_{B_2}$

$= I_{R0} + K^2 I_{R1} + K I_{R2} + I_{R0} + K I_{R1} + K^2 I_{R2}$

$= 2I_{R0} + I_{R1}\left[K^2 + K\right] + I_{R2}\left[K^2 + K\right]$ $\quad \left(\because 1 + K + K^2 = 0,\ K^2 + K = -1\right)$

$= 2I_{R0} + I_{R1}(-1) + I_{R2}(-1)$

$= 2I_{R0} - I_{R1} - I_{R2}$

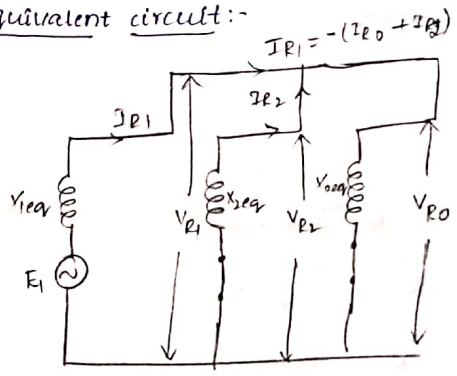
$\because I_{R1} + I_{R2} + I_{R0} = 0$

$I_{R1} = -(I_{R0} + I_{R2})$ —(2)

$I_F = 2I_{R0} - (-I_{R0})$

$$\boxed{I_F = 3I_{R0}}$$

Equivalent circuit :-



From eq (1) let $V_{R0} = V_{R1}$

$E_1 - I_{R1} X_{1eq} = -I_{R0} \cdot X_{0eq}$

$I_{R0} = -\dfrac{\left(E_1 - I_{R1} X_{1eq}\right)}{X_{0eq}}$

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From eq ①

$\Rightarrow \quad V_{R2} = V_{R1}$

$$-I_{R2} X_{2eq} = E_1 - I_{R1} \cdot X_{1eq}$$

$$I_{R2} = \frac{-\left[E_1 - I_{R1} X_{1eq}\right]}{X_{2eq}}$$

$$I_{R1} = -\left(I_{R0} + I_{R2}\right)$$

$$I_{R1} = \frac{E_1 - I_{R0} X_{1eq}}{X_{0eq}} + \frac{E_1 - I_{R1} X_{1eq}}{X_{2eq}}$$

$$I_{R1}\left[1 + \frac{X_{1eq}}{X_{0eq}} + \frac{X_{1eq}}{X_{2eq}}\right] = \frac{E_1}{X_{0eq}} + \frac{E_1}{X_{2eq}}$$

$$I_{R1}\left[\frac{X_{0eq} \cdot X_{2eq} + X_{1eq}\left[X_{2eq} + X_{0eq}\right]}{X_{0eq} X_{2eq}}\right] = E_1$$

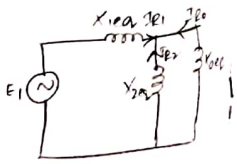
$$I_{R1} = \frac{E_1\left[X_{2eq} + X_{0eq}\right]}{X_{0eq} X_{2eq} + X_{1eq}\left[X_{2eq} + X_{0eq}\right]}$$

$$\boxed{I_{R1} = \frac{E_1}{X_{1eq} + \frac{X_{0eq} \cdot X_{2eq}}{X_{0eq} + X_{2eq}}}}$$

By current dividing rule

$$I_{R2} = -I_{R1} \times \frac{X_{0eq}}{X_{2eq} + X_{0eq}}$$

$$I_{R0} = -I_{R1} \times \frac{X_{2eq}}{X_{0eq} + X_{2eq}}$$



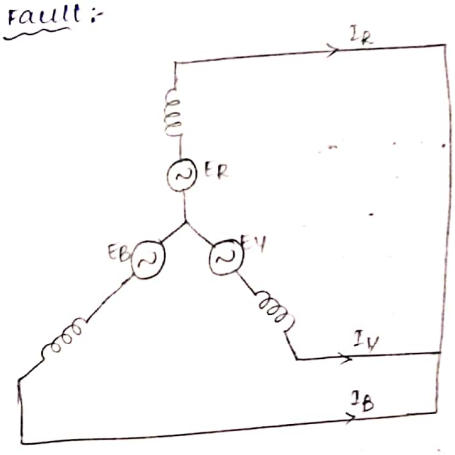
$I_{F(act)} = I_{F(PU)} \times I_{base}$

Short circuit MVA $= 3 V I_F$

$= 3 \cdot V \cdot I_{F(PU)} \cdot I_b$

S.C MVA = Base MVA $\times I_{F(PU)}$.

L-L-L Fault :-



During Fault (at no load)

$$I_R + I_Y + I_B = 0$$

$$V_R = V_Y = V_B$$

$$\begin{bmatrix} V_{R0} \\ V_{R1} \\ V_{R2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} V_R \\ V_Y \\ V_B \end{bmatrix}$$

$$V_{R0} = \frac{1}{3}\left[V_R + V_Y + V_B\right] \qquad \because V_R = V_Y = V_B$$

$$V_{R0} = \frac{3}{3} V_R = V_R$$

$$V_{R1} = \frac{1}{3}\left[V_R + K V_Y + K^2 V_B\right]$$

$$= \frac{1}{3}\left[V_R + K V_R + K^2 V_R\right] \qquad \because 1 + K + K^2 = 0$$

$$V_{R1} = \frac{V_R}{3}\left[1 + K + K^2\right]$$

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$V_{R1} = 0$

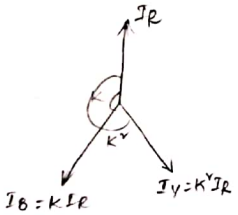
$V_{R2} = \frac{1}{3}\left[V_R + K^2 V_Y + K V_B\right]$

$= \frac{1}{3}\left[V_R + K^2 V_R + K V_R\right]$

$= \frac{V_R}{3}\left(1 + K^2 + K\right)$ $\quad \because 1 + K^2 + K = 0$

$V_{R2} = 0$

$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}$$



$I_R = \frac{1}{3}\left[I_R + I_Y + I_B\right] = 0$

$I_{R1} = \frac{1}{3}\left[I_R + K I_Y + K^2 I_B\right]$

$I_{R1} = \frac{1}{3}\left[I_R + K^3 I_R + K^3 I_R\right]$

$I_{R1} = I_R$

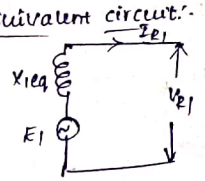
$I_{R2} = \frac{1}{3}\left[I_R + K^2 I_Y + K \cdot I_B\right]$

$= \frac{1}{3}\left[I_R + K^2 \cdot K^2 I_R + K^2 I_R\right]$

$I_{R2} = \frac{1}{3}\left[I_R + K \cdot I_R + K^2 I_R\right] = 0$

Fault current (I_F):-

Equivalent circuit:-



$V_{R1} = E_1 - I_{R1} X_{1eq}$

$0 = E_1 - I_{R1} X_{1eq}$

$$\boxed{I_F = I_{R1} = \frac{E_1}{X_{1eq}}}$$

* calculate the sequence components of following balanced line to neutral voltages.

$$V = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 220\angle 0^\circ \\ 220\angle -120^\circ \\ 220\angle +120^\circ \end{bmatrix}$$

Sol:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} 220\angle 0^\circ \\ 220\angle -120^\circ \\ 220\angle +120^\circ \end{bmatrix}$$

$V_{a0} = \frac{1}{3}\left(220\angle 0^\circ + 220\angle -120^\circ + 220\angle +120^\circ\right)$

$V_{a0} = 0$

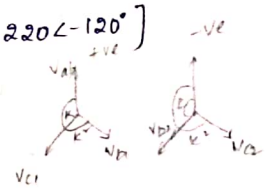
$V_{a1} = \frac{1}{3}\left[220\angle 0^\circ + K(220\angle -120^\circ) + K^2(220\angle +120^\circ)\right]$ $\quad \because K = 1\angle 120^\circ$

$V_{a1} = \frac{1}{3}\left[220\angle 0^\circ + (1\angle 120^\circ)(220\angle -120^\circ) + (1\angle 120^\circ)(220\angle 120^\circ)\right]$

$V_{a1} = 220^\circ$

$V_{a2} = \frac{1}{3}\left[220\angle 0^\circ + K^2(220\angle -120^\circ) + K(220\angle 120^\circ)\right]$

$= \frac{1}{3}\left[220\angle 0^\circ + 220\angle 120^\circ + 220\angle -120^\circ\right]$



$V_{a2} = 0$

$V_{b0} = V_{a0} = 0$

$V_{b1} = K^2 \cdot V_{a1} = (1\angle 120^\circ)(220) = 220\angle 120^\circ$

$V_{b2} = K \cdot V_{a2} = (1\angle 120^\circ)(0) = 0$

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$V_{c0} = V_{a0} = 0$

$V_{c1} = K^2 \cdot V_{a1} = (1\angle 120)^2 (220) = 220\angle -120^\circ$

$V_{c2} = K^2 \cdot V_{a2} = (1\angle 120^\circ)^2 (0) = 0$

* Find the symmetrical components for the given 3-ϕ currents

$I_a = 10\angle 0^\circ$

$I_b = 10\angle -90^\circ$

$I_c = 15\angle 135^\circ$

Sol:-

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} 10\angle 0^\circ \\ 10\angle -90^\circ \\ 15\angle 135^\circ \end{bmatrix}$$

$I_{a0} = \frac{1}{3}\left[10\angle 0^\circ + 10\angle -90^\circ + 15\angle 135^\circ\right]$

$= \frac{1}{3}\left[0.85\angle 135\right]$

$I_{a0} = 0.283\angle 135^\circ$

$I_{a1} = \frac{1}{3}\left[10\angle 0^\circ + K(10\angle -90^\circ) + K^2(15\angle 135^\circ)\right]$

$= \frac{1}{3}\left[10\angle 0^\circ + 10\angle 30^\circ + 15\angle 15^\circ\right]$

$I_{a1} = 11.43\angle 15$

$I_{a2} = \frac{1}{3}\left[10\angle 0^\circ + K^2(10\angle -90^\circ) + K(15\angle 135^\circ)\right]$

$= \frac{1}{3}\left[10\angle 0^\circ + 10\angle 150^\circ + 15\angle -105\right]$

$I_{a2} = 3.27\angle -105^\circ$

$I_{b0} = I_{a0} = 0.283\angle 135^\circ$

$I_{b1} = K^2 I_{a1} = (1\angle 120^\circ)^2 (11.4315) = 11.43\angle -105^\circ$

$I_{b2} = K I_{a2} = (1\angle 120^\circ)(3.27\angle -105^\circ) = 3.27\angle 15^\circ$

$I_{c0} = I_{a0} = 0.283\angle 135^\circ$

$I_{c1} = K^2 I_{a1} = (1\angle 120)^2 (11.43\angle 15) = 11.43\angle 135^\circ$

$I_{c2} = K I_{a2} = (1\angle 120)(3.27\angle -105^\circ) = 3.27\angle 135^\circ$

* In a fault study problem, the following currents are measured

$I_R = 0$

$I_Y = 10$

$I_B = -10$

Find the symmetrical components

Sol:- The above fault is a L-L fault

$$\begin{bmatrix} I_{R0} \\ I_{R1} \\ I_{R2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} I_R \\ I_Y \\ I_B \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & K & K^2 \\ 1 & K^2 & K \end{bmatrix}\begin{bmatrix} 0 \\ 10 \\ -10 \end{bmatrix}$$

$I_{R0} = \frac{1}{3}\left[0 + 10 - 10\right]$

$I_{R0} = 0$

$I_{R1} = \frac{1}{3}\left[0 + 10K - 10K^2\right]$

$= \frac{1}{3}\left[10\angle 120^\circ + 10\angle 60^\circ\right]$

$I_{R1} = 5.77\angle 90^\circ$

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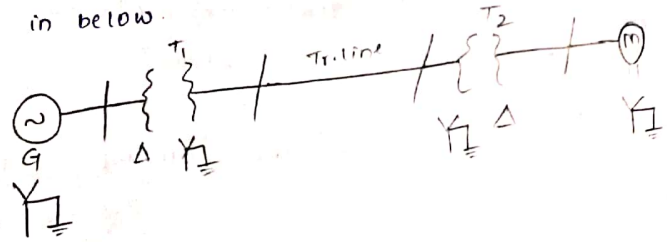
$$I_{R2} = \frac{1}{3}\left[0 + 10K^{2} - 10K\right]$$

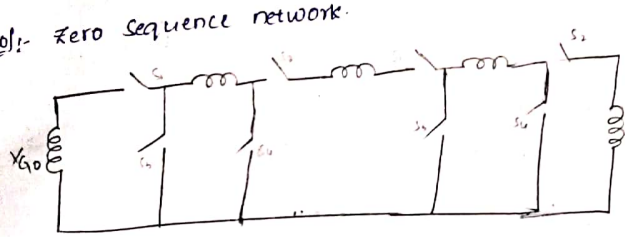
$$= \frac{1}{3}\left[10\angle -120 + 10\angle -60^{\circ}\right]$$

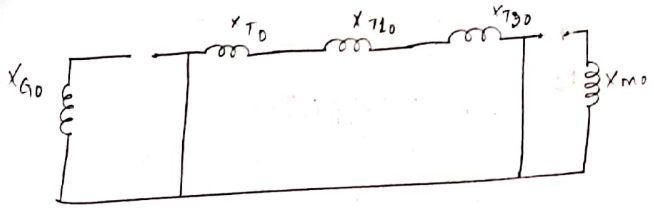
$$I_{R2} = 5.77\angle -90^{\circ}.$$

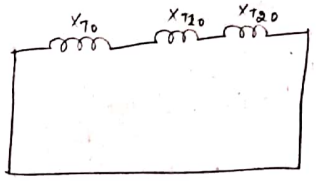
* Draw the zero sequence network for the system shown in below.



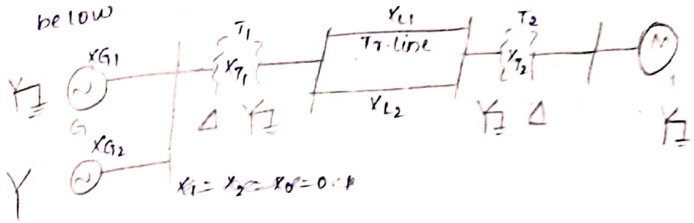
Sol:- Zero sequence network.





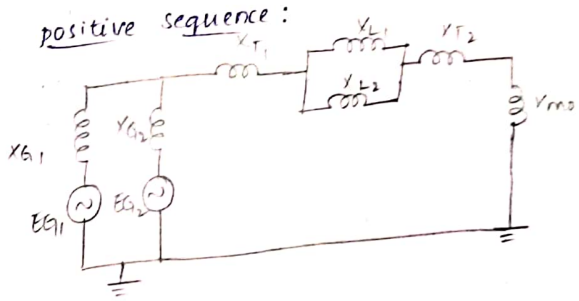


* Draw the sequence networks for the system shown in below

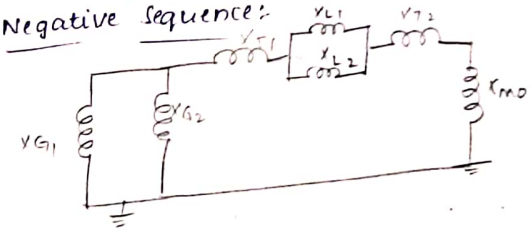


$X_1 = X_2 = X_0 = 0.4$

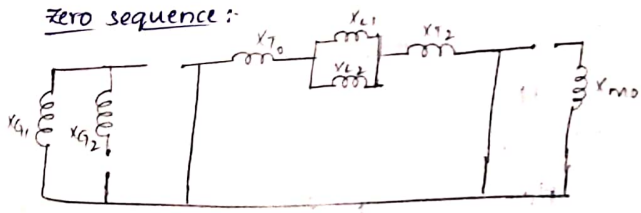
Sol:- positive sequence :

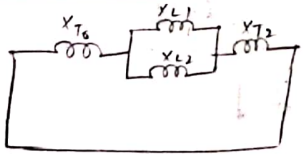


Negative sequence:-



Zero sequence :-





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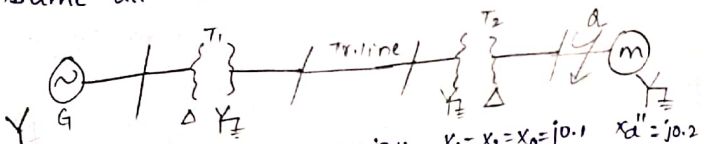
Problems:

* Calculate the sub-transient fault current in each phase for a dead short circuit on one phase to ground at bus 'Q' for the system shown in below. Assume all the reactances are in per unit



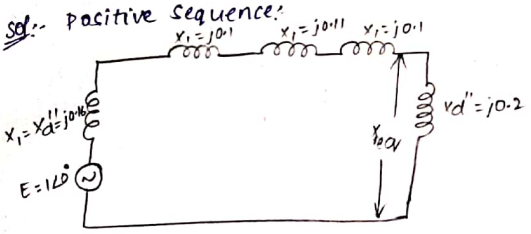
$E = 1\angle 0°$ $\quad X_1 = X_2 = j0.1$ $\quad X_1 = X_2 = j0.11$ $\quad X_1 = X_2 = X_0 = j0.1$ $\quad X_d'' = j0.2$

$X_d'' = j0.16$ $\quad X_0 = 0.1j$ $\quad X_0 = j0.33$ $\quad X_2 = j0.22$

$X_2 = j0.17$ $\quad X_0 = j0.15$

$X_0 = j0.06$

Sol:- Positive sequence:



j0.16 is in series with j0.1, j0.11, j0.1

$j0.16 + j0.1 + j0.11 + j0.1$

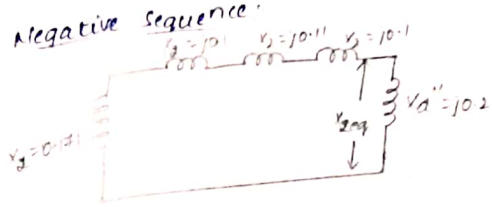
$= j0.47$

j0.19 ||el with j0.2

$$\frac{(j0.19)(j0.2)}{j0.19 + j0.2} = 0.1029j$$

$X_{1eq} = 0.1029j$

Negative sequence:



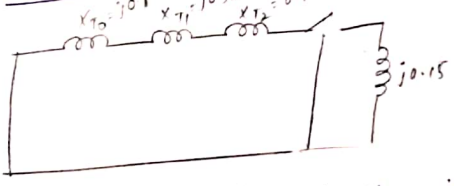
$j0.17 + j0.1 + j0.11 + j0.1$

$= 0.48j$

0.48j ||el with j0.2

$$X_{2eq} = \frac{(0.48j)(j0.2)}{0.48j + 0.2j} = \frac{j(0.096)}{j0.68} = 0.151j$$

Zero sequence



$X_{0eq} = \cancel{0.1j + 0.33j + 0.1j = 0.53j}$ $\quad X_{0eq} = j0.15$

$$\text{Fault current} = \frac{E}{X_{1eq} + X_{2eq} + X_{0eq}}$$

$$= \frac{1\angle 0°}{0.1029j + 0.15 + 0.15j}$$

Fault current $= -2.2j$

* From the data in above problem, if a line to line fault occurs then calculate the sequence components of fault current.

Sol:- We know that, the fault current when the network is subjected to LL fault is given by

$I_F = j\sqrt{3}\, I_{F2}$

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$I_F = -j\sqrt{3} I_{R1}$

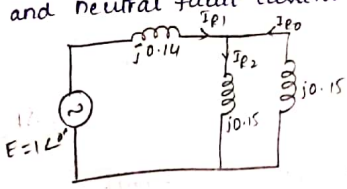
So, the sequence components of fault currents are I_{R1} and I_{R2}.

Where $I_{R1} = \dfrac{E_1}{X_{1eq} + X_{2eq}}$

$= \dfrac{1\angle 0^\circ}{j0.14 + j0.15} = -3.44j$

$I_{R2} = -\dfrac{E_1}{X_{1eq} + X_{2eq}} = \dfrac{-1\angle 0^\circ}{j0.14 + j0.15} = 3.44j$

If the line to line fault in above example takes place by involving ground with no fault impedance. Now determine sequence components of fault current and neutral fault current.



$I_{R1} = \dfrac{E_1}{X_{1eq} + \dfrac{X_{0eq} \cdot X_{2eq}}{X_{0eq} + X_{2eq}}}$

$= \dfrac{1\angle 0^\circ}{j0.14 + j0.075}$

$I_{R1} = -4.65j$

$I_{R0} = I_{R2} = -I_{R1} \times \dfrac{j0.15}{j0.3}$

$= -4.65j \times \dfrac{0.15j}{0.3j}$

$I_{R0} = I_{R2} = -2.325j$

neutral fault current

$I_N = j3 I_{R0} = j3(-2.325j) = -6.96j^2$

$I_N = -6.96$

* A 20MVA 6.6KV star connected generator has positive, negative and zero sequence reactances of 30% and 25% at 7% respectively. A reactor with 5% reactance based on the rating of generator is placed in the neutral to ground connection. A line to line fault occurs at the terminals of the generator when it is operating at rated voltage. Find the initial symmetrical line to ground rms fault current. Find also the line to line voltage.

Sol. $X_{1eq} = j0.3$ $X_{2eq} = j0.25$ let $E_1 = 1\angle 0^\circ$

$I_{R1} = \dfrac{E_1}{X_{1eq} + X_{2eq}}$

$I_{R1} = \dfrac{1}{j0.3 + j0.25} = -1.81j$

Actual current I_{R1} = Perunit current × Base current

$= \cancel{-1.81j}$

$S = \sqrt{3} V_L I_L$

$I_L = \dfrac{S}{\sqrt{3} V_L} = \dfrac{20 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 1749.5$

$I_{R1} = -1.81j \times 1749.5$

$I_{R1} = -3166.5j$

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