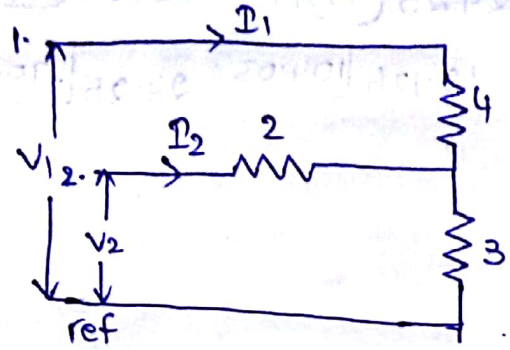
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Unit-3

Z-Bus formulation

Problem-1

Calculate the Z-Bus matrix



Z-Bus

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

Z_{11} making $I_2 = 0$

$V_1 = I_1 \times R$

$= I_1 \times 7$

$\frac{V_1}{I_1} = 7\Omega$ $\quad \boxed{Z_{11} = 7\Omega}$

$V_1 = 3 I_1$

$\boxed{Z_{21} = 3\Omega}$

Z_{22} and Z_{21} making $I_1 = 0$

$V_1 = 3 I_2$

$\boxed{Z_{12} = 3\Omega}$

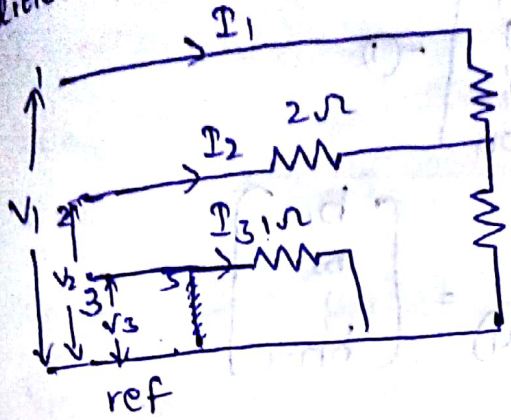
$V_2 = 5 I_2$

$\boxed{Z_{22} = 5\Omega}$

$$Z_{BUS} = \begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix}$$

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Addition of element from new bus to reference



Calculation of Z_{11}, Z_{21}, Z_{31} making I_2 & $I_3 = 0$

$V_1 = 7I_1$ $\quad \frac{V_1}{I_1} = Z_{11} = 7\Omega$

$V_2 = 3I_1$ $\quad \frac{V_2}{I_1} = Z_{21} = 3\Omega$

$V_3 = 0I_1$ $\quad \frac{V_3}{I_1} = Z_{31} = 0\Omega$

Calculation of Z_{12}, Z_{22}, Z_{32} making I_1 & $I_3 = 0$

$V_1 = 3I_2$ $\quad Z_{12} = 3\Omega$

$V_2 = 5I_2$ $\quad Z_{22} = 5\Omega$

$V_3 = 0I_2$ $\quad Z_{32} = 0\Omega$

Calculation of Z_{13}, Z_{23}, Z_{33} making I_1 & $I_2 = 0$

$V_1 = 0I_3$ $\quad Z_{13} = 0$

$V_2 = 0I_3$ $\quad Z_{23} = 0$

$V_3 = 1I_3$ $\quad Z_{33} = 1$

$$Z_{BUS} = \begin{bmatrix} 7 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Krons reduction method

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & \cdots & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & \cdots & \cdots & a_{nn} \end{bmatrix}_{n\times n} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n = 0 \end{bmatrix}_{n\times 1}$$

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$$\left[\begin{array}{c|c} A' & A_n^{col} \\ \hline A_n^{row} & a_{nn} \end{array}\right]\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \quad -(1)$$

$$\underset{(n-1)\times n}{\left[A' \,|\, A_n^{col}\right]} \cdot \underset{(n\times 1)}{\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \end{bmatrix}$$

$$\underset{(n-1)\times(n-1)}{[A']}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \end{bmatrix} + \underset{(n-1)\times 1}{[A_n^{col}]}\underset{1\times 1}{[x_n]} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \end{bmatrix}$$

From (1)

$$\underset{1\times(n-1)}{[A_n^{row}]}\underset{(n-1)\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}} + a_{nn}[x_n] = 0$$

$$x_n = -\frac{[A_n^{row}]\begin{bmatrix} x_1 \\ \vdots \\ x_{n-1} \end{bmatrix}}{a_{nn}}$$

$$[A']\begin{bmatrix} x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} - \frac{[A_n^c][A_n^r]}{a_{nn}}\begin{bmatrix} x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_{n-1} \end{bmatrix}$$

$$\left[[A'] - \frac{1}{a_{nn}}[A_n^c][A_n^r]\right]\begin{bmatrix} x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_{n-1} \end{bmatrix}$$

- $[A']$ → oldmatrix
- a_{nn} → new added Element
- A_n^c → nth column element matrix
- A_n^r → nth row element matrix

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Addition of z-bus when a new element is added Z_b from new bus (K) to reference (r)

We know that

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \cdots Z_{1n}I_n.$$

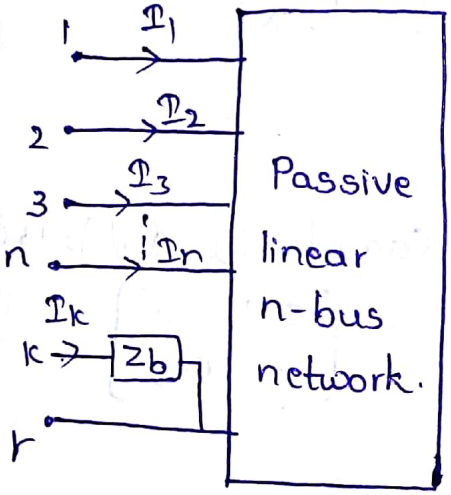
$$V_2 = Z_{21}I_1 + Z_{22}I_2 + \cdots Z_{2n}I_n.$$

$$V_n = Z_{n1}I_1 + Z_{n2}I_2 + \cdots Z_{nn}I_n.$$

from above eqn

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

Consider an element Z_b which is connected in between new bus k to reference r



From the above diagram the Voltage at new bus V_k

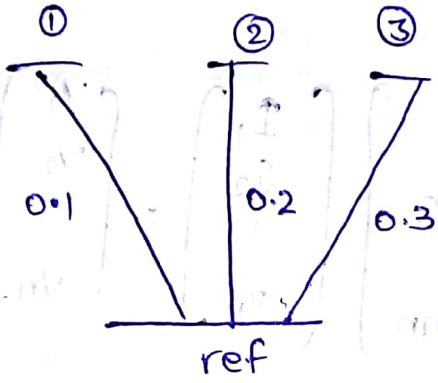
$$V_k = I_k \times Z_b$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline V_k \end{bmatrix} = \left[\begin{array}{ccc|c} Z_{11} & \cdots & Z_{1n} & 0 \\ \vdots & & \vdots & \vdots \\ Z_{n1} & \cdots & Z_{nn} & 0 \\ \hline 0 & \cdots & 0 & Z_b \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_k \end{bmatrix}$$

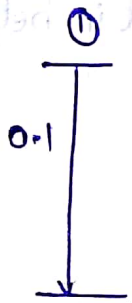
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$$Z_{BUS\ modified} = \left[\begin{array}{c|c} Z_{bus\ old} & \begin{matrix}0\\ \vdots \\ 0\end{matrix} \\ \hline \begin{matrix}0 & 0 & \cdots & 0\end{matrix} & Z_b \end{array}\right]$$

Calculate the Z-bus matrix by using Z-bus building algorithm.



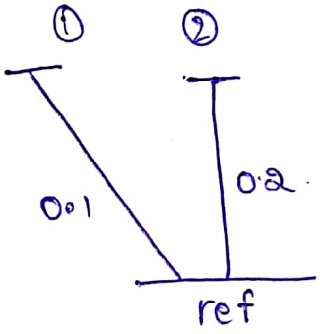
Step-1



Consider 1st

Matrix is $[0.1]_{1\times 1}$

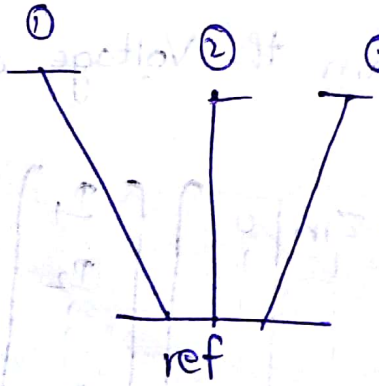
Step-2



Consider ① & ② buses

$$\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$$

Step-3



Consider ① & ② & ③ buses

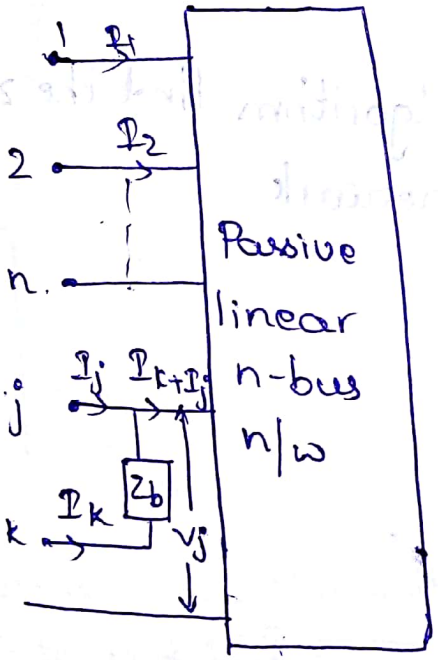
$$\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}$$

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case(i) Modification of z-bus when a new element is added z_b from new bus k to old bus j'

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

Consider a new element Z_b added from new bus k to old bus J as shown in diagram.

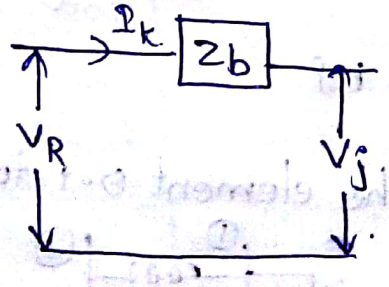


$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \cdots Z_{1n}I_n$$

$$V_j = Z_{j1}I_1 + Z_{j2}I_2 + \cdots Z_{jj}[I_j + I_k] + \cdots Z_{jn}I_n$$

$$V_k = I_k Z_b + V_j$$

V_{R_5}



$$V_R = I_k Z_b + Z_{j1}I_1 + \cdots Z_{jj}(I_J + I_k) + Z_{jn}I_n$$

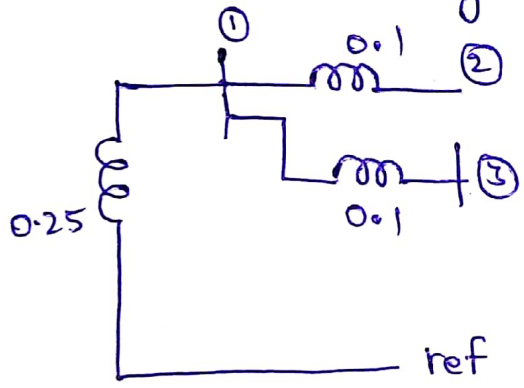
$$V_K = Z_{j1}I_1 + \cdots Z_{jj}I_j + \cdots Z_{jn}I_n + I_k[Z_b + Z_{jj}]$$

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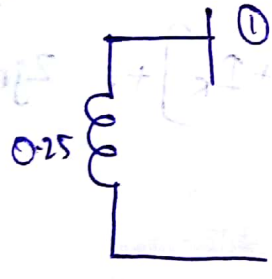
$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline V_k \end{bmatrix} = \left[\begin{array}{ccc|c} Z_{11} & \cdots & Z_{1n} & Z_{1j} \\ \vdots & & \vdots & \vdots \\ Z_{n1} & \cdots & Z_{nn} & Z_{nj} \\ \hline Z_{j1} & Z_{j2} \cdots & Z_{jn} & Z_b + Z_{jj} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_k \end{bmatrix}$$

$$Z_{Bus\ modified} = \left[\begin{array}{c|c} Z_{Bus\ old} & \begin{matrix} Z_{1j} \leftarrow \text{old bus number} \\ \vdots \\ Z_{nj} \end{matrix} \\ \hline Z_{j1} \cdots Z_{jn} & Z_{jj} + Z_b \end{array}\right]$$

By using jth bus building algorithm find the z-bus matrix for the following network.



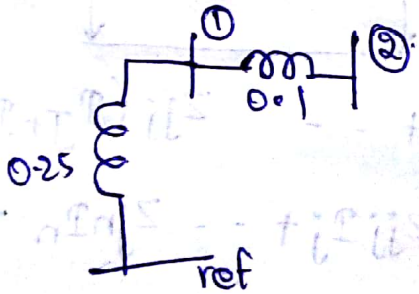
Step-1 Order is 1X1 j=1 K=1



$$Z_{bus} = 1\left[0.25\right]$$

Step-2 Add the element 0.1 ~~to 0.25~~ in b/w old bus to new bus;

j=1 K=2



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$$= \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.25+0.1 \end{bmatrix} = \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.35 \end{bmatrix}$$

Step 3 Adding an element 0.1 in b/w new bus (3) to old bus (1)

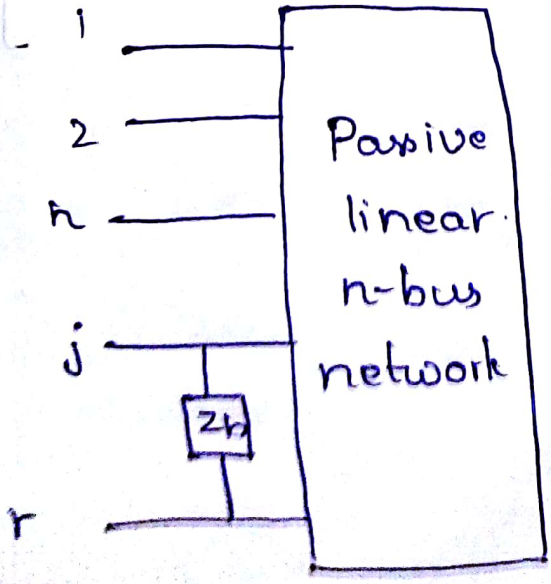
$j=1 \quad k=3$

$$\begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix}$$

Case+iii Addition of element Z_b from reference to old bus J

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & & & \vdots \\ Z_{n1} & \cdots & & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

Consider a new element Z_b connected from reference bus r. to old bus J as shown in diagram.



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$$\begin{bmatrix} V_1 \\ \vdots \\ V_j \\ V_n \\ V_{k=0} \end{bmatrix} = \left[\begin{array}{c|c} Z_{bus\,old} & \begin{matrix} Z_{1j} \\ \vdots \\ Z_{nj} \end{matrix} \\ \hline \begin{matrix} Z_{j1} & \cdots & Z_{jn} \end{matrix} & Z_{jj}+Z_b \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_n \\ \hline I_k \end{bmatrix}$$

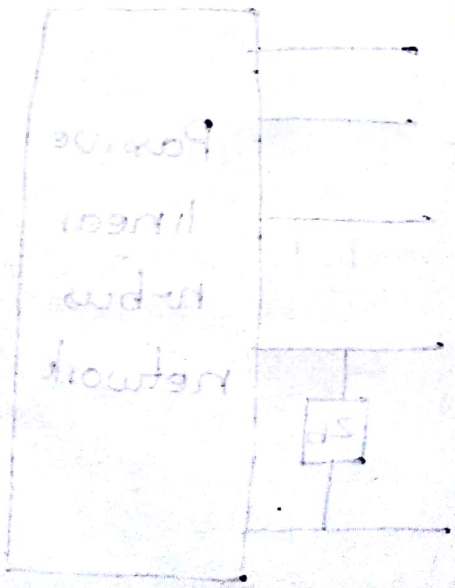
$$\begin{bmatrix} V_1 \\ \vdots \\ V_n \end{bmatrix} = \left[\begin{array}{c|c} Z^r & Z_n^c \\ \hline & nx(n-1) \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_n \\ \hline I_k \end{bmatrix}_{(n+1)x1}$$

$$= [Z^r]_{nxn} \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix}_{nx1} + [Z_n^c]_{nx1} [I_k]_1 \quad - (1)$$

$$0 = [Z_n^r] \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} + (Z_{jj} + Z_b) I_k$$

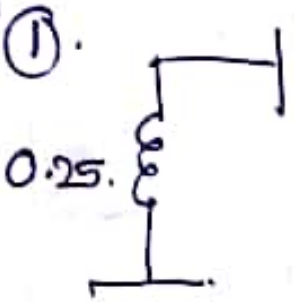
$$I_k = -\frac{1}{Z_{jj}+Z_b} [Z_n^r] \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} \quad - (2)$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \left[(Z_{bus}^{old})_{nxn} - \frac{1}{Z_{jj}+Z_b} [Z_n^c]_{(nx1)} [Z_n^r]_{(1xn)} \right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$



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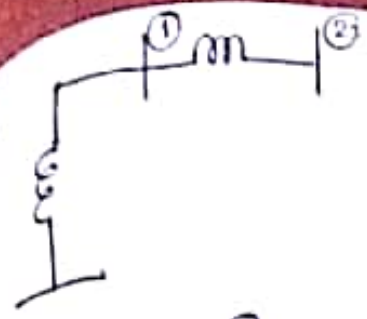
Sol)



$$Z_{Bus} = [0.25]_{1\times1}$$

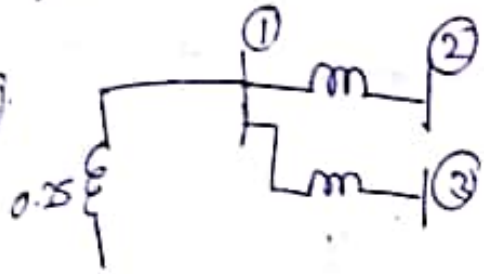
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2)



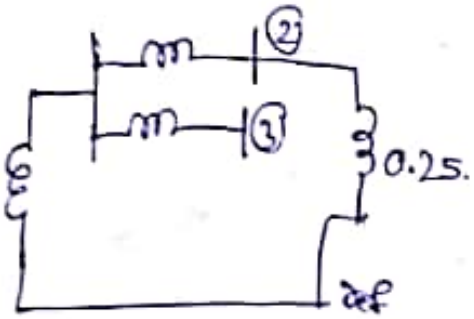
$$Z_{bus}^{mod} = \left[\begin{array}{cc|c} 0.25 & 0.25 & \\ 0.25 & 0.35 & \end{array}\right]$$

3.



$$Z_{bus}^{mod} = \left[\begin{array}{cc|c} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ \hline 0.25 & 0.25 & 0.35 \end{array}\right]$$

4)



$$Z_{bus}^{mod} = \left[\begin{array}{ccc|c} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 & 0.35 \\ 0.25 & 0.25 & 0.35 & 0.25 \\ \hline 0.25 & 0.35 & 0.25 & 0.6 \end{array}\right]_{4\times4}$$

$$Z_{bus}^{New} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \frac{1}{0.6} \begin{bmatrix} 0.25 \\ 0.35 \\ 0.25 \end{bmatrix}_{3\times1} \begin{bmatrix} 0.25 & 0.35 & 0.25 \end{bmatrix}_{1\times3}$$

$$= \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \frac{1}{0.6} \begin{bmatrix} 0.0625 & 0.0875 & 0.0625 \\ 0.0875 & 0.1225 & 0.0875 \\ 0.0625 & 0.0875 & 0.0625 \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \begin{bmatrix} 0.1041 & 0.14 & 0.1 \\ 0.14 & 0.2 & 0.14 \\ 0.10 & 0.14 & 0.10 \end{bmatrix}$$

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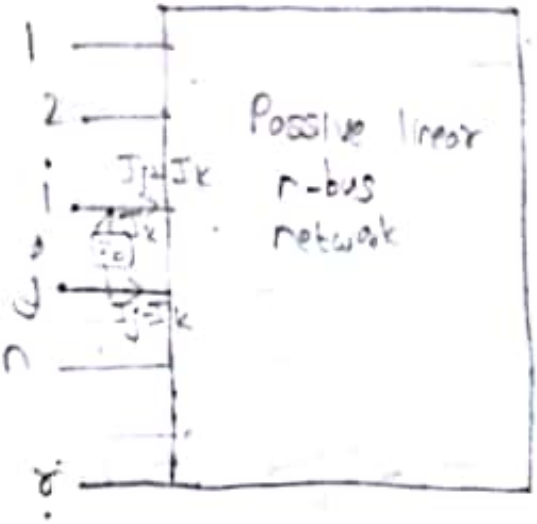
$$\Rightarrow \begin{bmatrix} 0.15 & 0.11 & 0.15 \\ 0.11 & 0.15 & 0.11 \\ 0.15 & 0.11 & 0.25 \end{bmatrix}$$

Case (iv) :-

Addition of element z_b from two old buses i, j

We know that

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \cdots & Z_{1n} \\ Z_{21} & Z_{22} \cdots & Z_{2n} \\ \vdots & & \vdots \\ Z_{n1} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$



By adding new element z_b in blw two old buses i,j the modified equations are

$$V_i = Z_{i1}I_1 + Z_{i2}I_2 + \cdots + Z_{in}I_n$$

$$V_j = Z_{j1}I_1 + Z_{j2}I_2 + \cdots + Z_{jn}I_n$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \cdots Z_{1n}I_n$$

$$V_n = Z_{n1}I_1 + Z_{n2} \cdots Z_{nn}I_n$$

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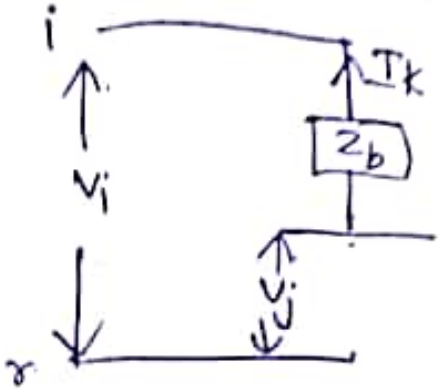
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For element :-

$V_i = z_{i1} I_1 + z_{i2} I_2 + \cdots z_{ii}(I_i + I_k) + z_{ij}(I_j - I_k) + \cdots z_{in} I_n$

$V_j = z_{j1} I_1 + \cdots z_{ji}(I_i + I_k) + z_{jj}(I_j - I_k) + \cdots z_{jn} I_n$

Applying KVL to the loop



$V_i = V_j + z_b I_k$

$(V_j - V_i) + z_b I_k = 0$

$I_1(z_{j1} - z_{i1}) + I_2(z_{j2} - z_{i2}) + I_i(z_{ji} - z_{ii}) + I_j(z_{jj} - z_{ij}) + I_k(z_{ji} - z_{jj} - z_{ii} + z_{ij} + z_b) + \cdots I_n(z_{jn} - z_{in}) = 0$

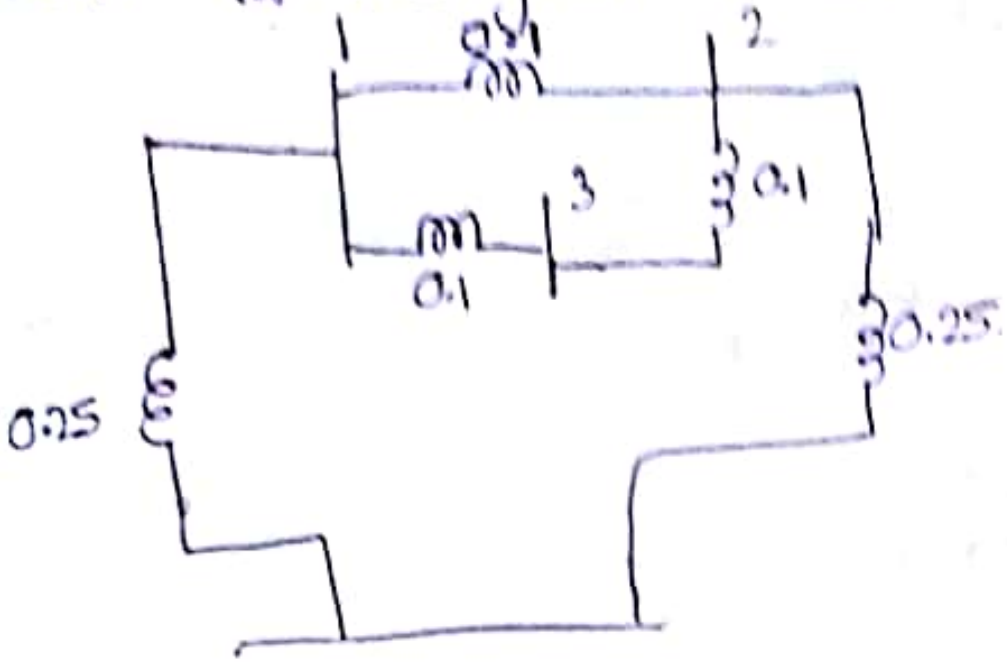
$$\begin{bmatrix} V_1 \\ \vdots \\ V_n \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} Z_{Bus}^{old} & \begin{matrix} z_{1j} - z_{1i} \\ z_{2j} - z_{2i} \\ \vdots \end{matrix} \\ \hline z_{ji} - z_{i1} + \cdots & z_{ji} - z_{jj} - z_{ii} + z_b \end{array}\right] \begin{bmatrix} I_1 \\ \vdots \\ I_n \\ \hline I_k \end{bmatrix}$$

By using Kron's reduction method

$$Z_{Bus}^{new} = Z_{Bus}^{old} - \frac{1}{z_{ji} - z_{ji} - z_{ij} + z_b} \begin{bmatrix} Z_{new}^{c} \end{bmatrix} \begin{bmatrix} Z_{new}^{row} \end{bmatrix}$$

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* By using 2 bus building algorithm, calculate the Z bus matrix for the given n.w ckt.



$Z_{ij} - Z_{ii}$

$Z_{ij} - Z_{ij}$

From above problem

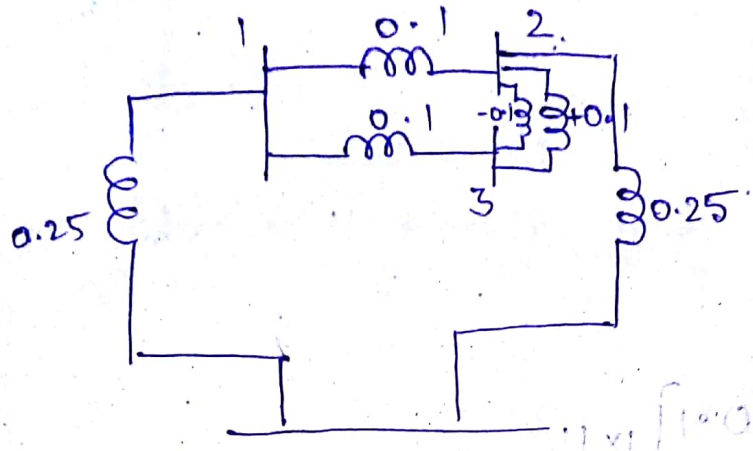
$$N \quad Z_{Bus} = \begin{bmatrix} 0.145 & 0.104 & 0.145 \\ 0.104 & 0.145 & 0.104 \\ 0.145 & 0.104 & 0.245 \end{bmatrix} - \frac{1}{Z_{ii} - Z_{ji} - Z_{ij} + Z_{jj}} \begin{bmatrix} Z_{1i} - Z_{1j} \\ Z_{2i} - Z_{2j} \end{bmatrix}$$

Here $i = 2, j = 3$.

$$\Rightarrow \begin{bmatrix} 0.145 & 0.104 & 0.145 \\ 0.104 & 0.145 & 0.104 \\ 0.145 & 0.104 & 0.245 \end{bmatrix} - \frac{1}{2 \times 0.104 - 0.145 - 0.245 + 0.1} \begin{bmatrix} 0.041 \\ -0.041 \\ 0.141 \end{bmatrix}_{3 \times 1} \begin{bmatrix} 0.041 & -0.041 & 0.141 \end{bmatrix}_{1 \times 3}$$

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for the above problem open the element which is connected in between Bus2 and Bus3.
Calculate the modified z-Bus matrix



$$Z_{BUS} = \begin{bmatrix} 0.139 & 0.11 & 0.125 \\ 0.11 & 0.139 & 0.124 \\ 0.125 & 0.124 & 0.175 \end{bmatrix}$$

$$Z_{BUS}^{Mod} = \left[\begin{array}{ccc|c} 0.139 & 0.11 & 0.125 & 0.015 \\ 0.11 & 0.139 & 0.124 & -0.015 \\ 0.125 & 0.124 & 0.175 & 0.05 \\ \hline 0.015 & -0.015 & 0.05 & -0.034 \end{array}\right]$$

$= Z_{ii} + Z_{jj} + Z_b - 2Z_{ij}$

$= 0.139 + 0.175 - 0.1 - 2(0.124)$

$= -0.034$

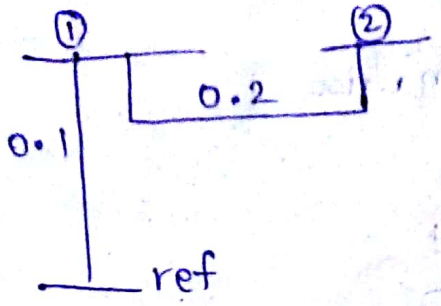
$$Z_{BUS}^{N} = \begin{bmatrix} 0.139 & 0.11 & 0.125 \\ 0.11 & 0.139 & 0.124 \\ 0.125 & 0.124 & 0.175 \end{bmatrix} - \frac{1}{-0.034} \begin{bmatrix} 0.015 \\ -0.015 \\ 0.05 \end{bmatrix}_{5\times1} \begin{bmatrix} 0.015 & -0.015 & 0.05 \end{bmatrix}$$

$$Z_{BUS}^{N} = \begin{bmatrix} 0.145 & 0.104 & 0.145 \\ 0.104 & 0.1458 & 0.1042 \\ 0.145 & 0.1042 & 0.245 \end{bmatrix}$$

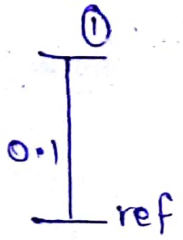
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Formulate the z-bus matrix for the given network.

1)

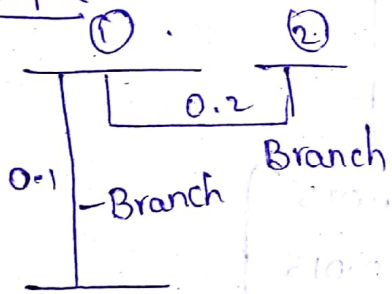


Step+1



$[0.1]_{1\times1}$

Step2



$Z_{BUS}^{N} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}$ Z_{ij} → 2×2

where j is old bus $j=1$

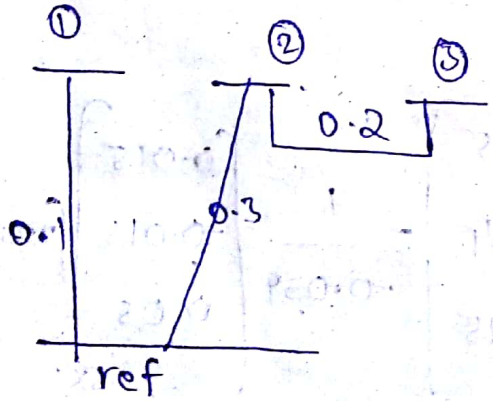
$= Z_{11}$

$Z_{jj} + Z_b$

$= Z_{11} + Z_b$

$= 0.1 + 0.2 = 0.3$

2)



Step+1 Addition of element 0.1 from reference to new bus ①

$Z_{BUS} = 1\begin{bmatrix} 0.1 \end{bmatrix}_{1\times1}$ Type+1

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Step 2 Addition of element 0.3 from reference to new bus (2)

Type 1

$$\begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}_{2\times 2}$$

Step 3 Addition of element from new bus (3) to old bus (2)

z_{ij} $= z_{12}$

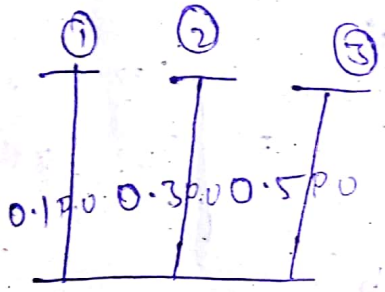
$= z_{22}$ Type 2

$$\left[\begin{array}{cc|c} 0.1 & 0 & 0 \\ 0 & 0.3 & 0.3 \\ \hline 0 & 0.3 & 0.5 \end{array}\right] \rightarrow z_{bb}+z_{jj}$$

$0.2+0.3$

$= 0.5$

3)



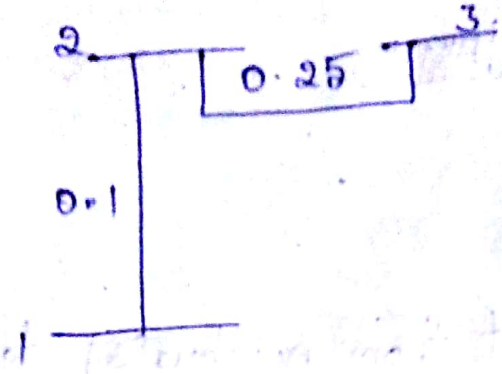
Type 1 Addition of element from reference to new bus

Therefore Order is 3x3

$$Z_{BUS} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}_{3\times 3}$$

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4)



Step+1 Addition of element from bus 1 to bus 2

$$Z_{BOS} = \begin{matrix} & 2 \\ 2 & \begin{bmatrix} 0.1 \end{bmatrix} \end{matrix}$$

Step+2 Addition of element from old bus ② to new bus ③.

$$Z_{BOS}^{N} = \left[\begin{array}{c|c} 0.1 & 0.1 \\ \hline 0.1 & 0.35 \end{array}\right] \quad Z_{2j} = Z_{22}$$

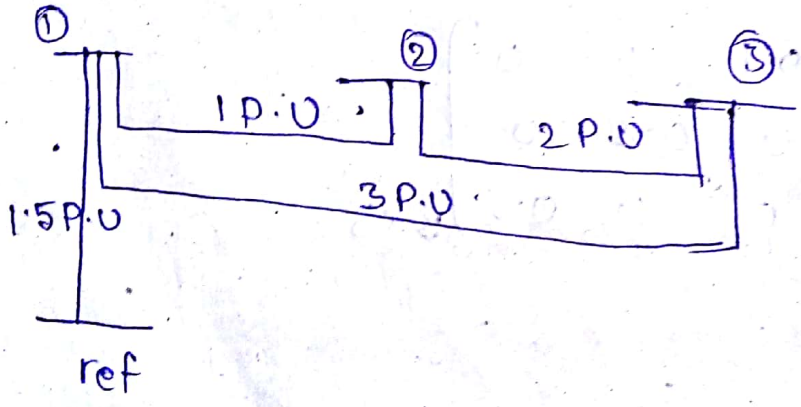
$$= \begin{bmatrix} Z_{22} & Z_{23} \cdots \\ Z_{32} & Z_{33} \cdots \\ \vdots & \end{bmatrix}$$

$$Z_{jj} + Z_b$$

$$= Z_{22} + 0.25$$

$$= 0.1 + 0.25$$

5)



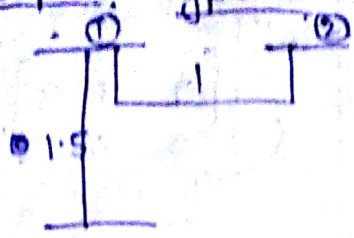
Step+1 Addition of element from bus 1 to ref

Type+1

Now Z_{bus} $\begin{matrix} & 1 \\ 1 & \begin{bmatrix} 1.5 \end{bmatrix} \end{matrix}_{1\times 1}$

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Step-2 Type-2

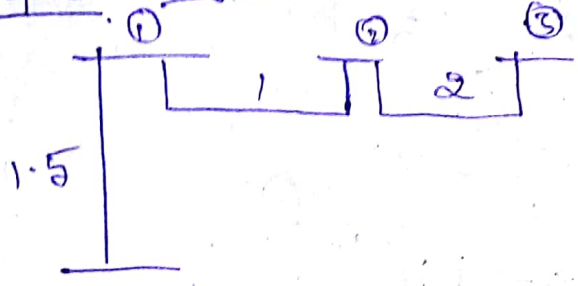


$$Z_{BUS}^{modified} = \left[\begin{array}{cc|c} 1.5 & 1.5 & \\ 1.5 & 2.5 & \end{array}\right] \quad \begin{matrix} \\ \\ Z_{1j} \end{matrix}$$

$Z_{jj} = Z_{11} + Z_b$

$= 1.5 + 1 = 2.5$

Step-3 Type-2



$$Z_{BUS}^{modified} = \left[\begin{array}{cc|c} 1.5 & 1.5 & 1.5 \\ 1.5 & 2.5 & 2.5 \\ \hline 1.5 & 2.5 & 4.5 \end{array}\right] \quad \begin{matrix} \\ = Z_{12} \\ = Z_{22} \\ \\ Z_{1j} \end{matrix}$$

$\rightarrow Z_{jj} = Z_b + Z_{jj}$

$= 2 + 2.5$

$= 4.5$

Step-4 Type-4 In between two old buses

$$Z_{BUS}^{mod} = \left[\begin{array}{ccc|c} 1.5 & 1.5 & 1.5 & 0 \\ 1.5 & 2.5 & 2.5 & 1 \\ 1.5 & 2.5 & 4.5 & 3 \\ \hline 0 & 1 & 3 & 6 \end{array}\right]$$

$j = 3 \quad i = 1$

$Z_{ii} + Z_{jj} + Z_b - 2[Z_{ij}]$

$= 1.5 + 4.5 + 3 - 2 \times 1.5 = 6$

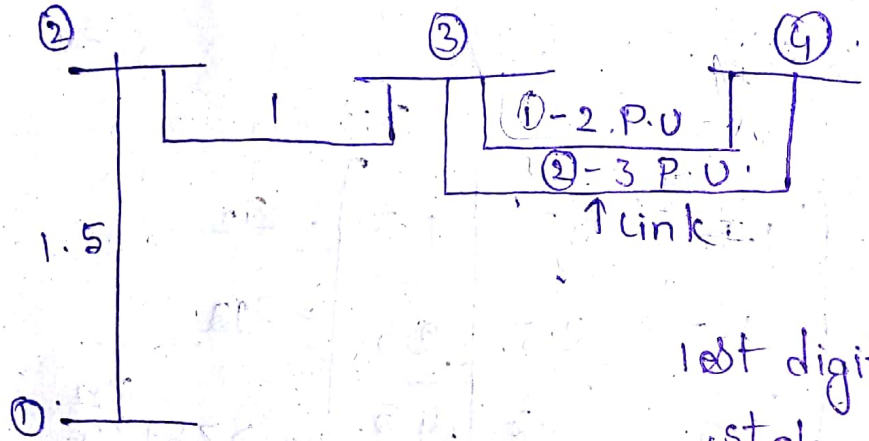
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$$Z_{BUS}^{NEW} = \begin{bmatrix} 1.5 & 1.5 & 1.5 \\ 1.5 & 2.5 & 2.5 \\ 1.5 & 2.5 & 4.5 \end{bmatrix} - \frac{1}{6} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}_{3\times1} \begin{bmatrix} 0 & 1 & 3 \end{bmatrix}_{1\times3}$$

$$= \begin{bmatrix} 1.5 & 1.5 & 1.5 \\ 1.5 & 2.5 & 2.5 \\ 1.5 & 2.5 & 4.5 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.166 & 0.5 \\ 0 & 0.5 & 1.5 \end{bmatrix}$$

$$Z_{BUS}^{New} = \begin{bmatrix} 1.5 & 1.5 & 1.5 \\ 1.5 & 2.334 & 2 \\ 1.5 & 2 & 3 \end{bmatrix}$$

6.



1st digit indicates 1st element

$$Z_{BUS}^{old} = \left[\begin{array}{ccc|c} 1.5 & 1.5 & 1.5 & 0 \\ 1.5 & 2.5 & 2.5 & 0 \\ 1.5 & 2.5 & 4.5 & 2 \\ \hline 0 & 0 & 2 & 5 \end{array}\right] \quad \begin{matrix} & & 2 & 3 & 4 & \\ & & & & & \end{matrix}$$

$= Z_{ii} + Z_{jj} + Z_b - 2(Z_{ij})$

$= Z_{33} + Z_{44} + Z_b - 2(Z_{34})$

$= 2.5 + 4.5 + 3 - 2(2.5)$

$= 5$

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$$Z_{Bus}^{New} = \begin{bmatrix} 1.5 & 1.5 & 1.5 \\ 1.5 & 2.5 & 2.5 \\ 1.5 & 2.5 & 4.5 \end{bmatrix} - \frac{1}{5}\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}\begin{bmatrix} 0 & 0 & 2 \end{bmatrix}$$

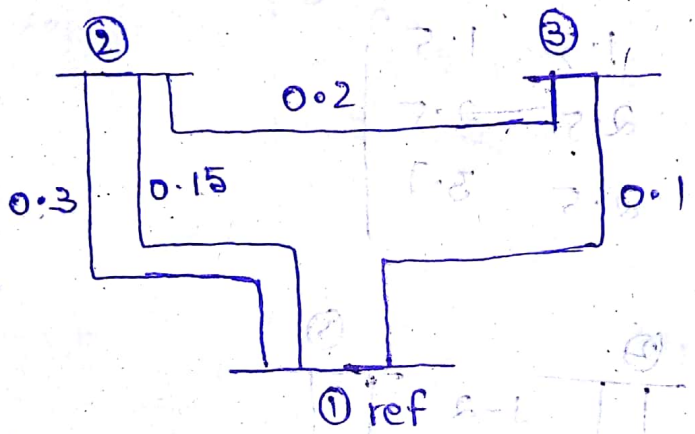
$$= \begin{bmatrix} 1.5 & 1.5 & 1.5 \\ 1.5 & 2.5 & 2.5 \\ 1.5 & 2.5 & 4.5 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}$$

$$Z_{Bus}^{New} = \begin{bmatrix} 1.5 & 1.5 & 1.5 \\ 1.5 & 2.5 & 2.5 \\ 1.5 & 2.5 & 3.7 \end{bmatrix}$$

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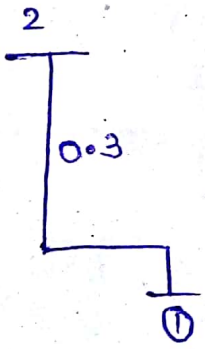
Develop z-bus Using building algorithm for the power system whose element data is given in the following table:

Element	Connected b/w bus number	Self reactance In (per Unit)
1	1-2	0.3
2	1-3	0.1
3	2-3	0.2
4	1-2	0.15



Step-1

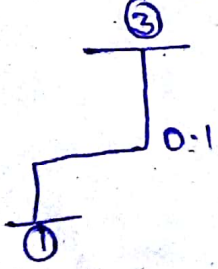
① to ② [reference to new] Type-1



$$^{2}[0.3]_{1\times1}^{2}$$

Step-2

① to ③ [reference to new]



$$\left[\begin{array}{c|c} 0.3 & 0 \\ \hline 0 & 0.1 \end{array}\right]$$

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Step-3

2 to 3 (old to old) Type-4.

$i = 2, \quad j = 3$

$$Z_{BUS}^{Mod} = \begin{array}{c} \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \left[\begin{array}{cc|c} 0.3 & 0 & -0.3 \\ 0 & 0.1 & +0.1 \\ \hline -0.3 & 0.1 & 0.6 \end{array}\right] \end{array} \begin{array}{l} \leftarrow Z_{2j} - Z_{2i} \\ \leftarrow Z_{3j} - Z_{3i} \end{array}$$

$$Z_{ii} + Z_{jj} + Z_b - 2Z_{ij}$$

$$= Z_{22} + Z_{33} + Z_b - 2Z_{23}$$

$$= 0.3 + 0.1 + 0.2 - 2(0)$$

$$= 0.6$$

$$Z_{BUS}^{N} = Z_{BUS}^{old} - \frac{1}{+0.6}\left[Z_{BUS}^{c}\right]\left[Z_{BUS}^{r}\right]$$

Step-4

① to ② reference to old (Type-3)

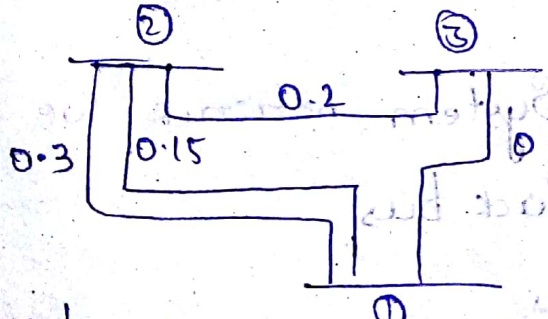
By using krons reduction.

$$= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix} - \frac{1}{0.6}\begin{bmatrix} -0.3 \\ 0.1 \end{bmatrix}\begin{bmatrix} -0.3 & 0.1 \end{bmatrix}$$

$$Z_{BUS}^{N} = \begin{bmatrix} 0.15 & 0.05 \\ 0.05 & 0.08 \end{bmatrix}$$

Step-4

① to ② reference to old (Type-3)



$$Z_{BUS}^{Mod} = \begin{array}{c} \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \left[\begin{array}{cc|c} 0.15 & 0.05 & 0.15 \\ 0.05 & 0.08 & 0.05 \\ \hline 0.15 & 0.05 & 0.3 \end{array}\right] \end{array} \begin{array}{l} Z_{2j} \\ Z_{3j} \end{array}$$

$$Z_{jj} + Z_b = Z_{22} + Z_b$$

$$0.15 + 0.15 = 0.3$$

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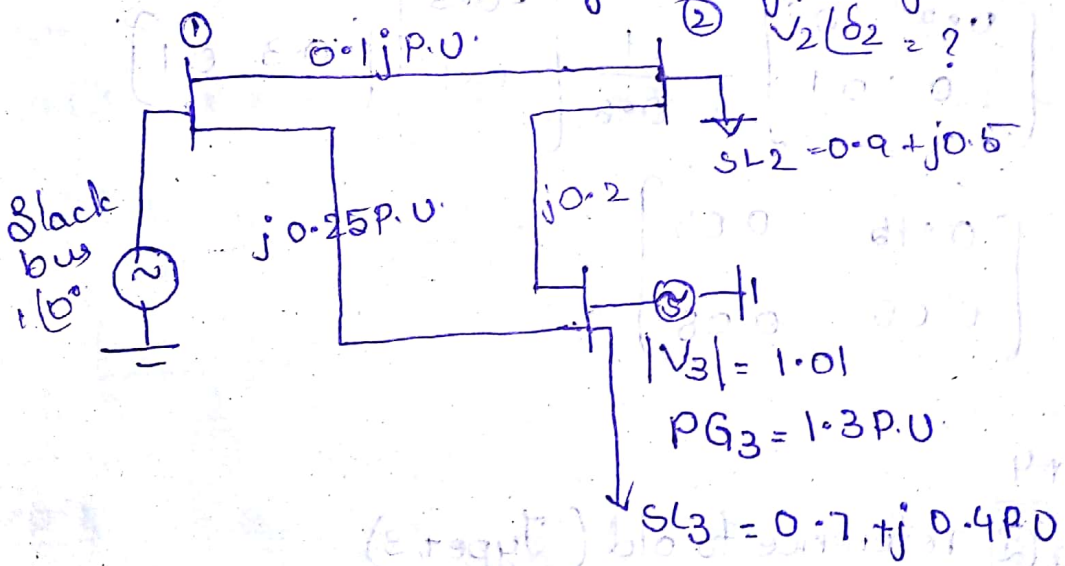
By using krons reduction method:

$$Z_{BUS}^{N} = Z_{BUS}^{old} - \frac{1}{Z_{jj}+Z_b}\begin{bmatrix} Z_{BUS}^{c} \end{bmatrix}\begin{bmatrix} Z_{BUS}^{r} \end{bmatrix}$$

$$= \begin{bmatrix} 0.15 & 0.05 \\ 0.05 & 0.08 \end{bmatrix} - \frac{1}{0.3}\begin{bmatrix} 0.15 \\ 0.05 \end{bmatrix}\begin{bmatrix} 0.15 & 0.05 \end{bmatrix}$$

$$= \begin{bmatrix} 0.069 & 0.02 \\ 0.02 & 0.069 \end{bmatrix}$$

For the given power system network determine the unknown states of the system by using N-R method



From the above given power system network we observe that Bus 1 is a Slack bus

Bus 2 is a PQ bus Or load bus

Bus 3 is a PV bus

Now Unknown State Variables are δ_2, δ_3, V_2

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Step-1 Calculation of Y-bus

$Y_{11} = y_{11} + y_{12} + y_{13}$

$\therefore Y_{11} = \frac{1}{0.1j} + \frac{1}{0.25j}$

$Y_{11} = -14j$

$$Y_{BUS} = \begin{bmatrix} -14j & 10j & 4j \\ 10j & -15j & 5j \\ -4j & 5j & -9j \end{bmatrix}$$

Since bus 1 is a Slack bus no Calculations required

We know that by Using N-R method the Unknown State Variables are as follows

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial V_2} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial V_2} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial V_2} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_2 \end{bmatrix}$$

known residual matrix — Jacobian matrix — Unknown residual matrix

$\Delta P_2 = P_{2(spe)} - P_{2(cal)}$

$\Delta P_3 = P_{3(spe)} - P_{3(cal)}$

$\Delta Q_2 = Q_{2(spe)} - Q_{2(cal)}$

From the given data

$P_{2spe} = 0.9$

$P_{3spe} = 1.3 - 0.7 = P_G - P_D$

$= 0.6$

$Q_{2spe} = -0.5$

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$P_2(cal) = \sum_{i=1}^{n} |V_i||V_k||Y_{ki}| \cos(\delta_i - \delta_k + \theta_{ki})$

$= |V_1||V_2||Y_{21}| \cos(\delta_1 - \delta_2 + \theta_{21}) + |V_2||V_2||Y_{22}| \cos(\delta_2 - \delta_2 + \theta_{22}) + |V_3||V_2||Y_{23}| \cos(\delta_3 - \delta_2 + \theta_{23})$

$= |1||1||10| \cos(0 - 0 + 90) + |1||1||15| \cos(-90) + |1.01||1||5| \cos(90)$

$= 0$

$P_3(cal) = 0$

$Q_2 = -|V_1||V_2||Y_{21}| \sin(\delta_1 - \delta_2 + \theta_{21}) - |V_2||V_2||Y_{22}| \sin(\delta_2 - \delta_2 + \theta_{22}) - |V_3||V_2||Y_{23}| \sin(\delta_3 - \delta_2 + \theta_{23})$

$= -|1||1||10| \sin(90) - |1||1||15| \sin(-90) - |1.01||1||5| \sin(0 - 0 + 90)$

$= -10 + 15 - 5.05$

$= -0.05$

$\Delta P_2 = P_{2spe} - P_{2cal}$

$= -0.9 - 0 = -0.9$

$\Delta P_3 = P_{3spe} - P_{3cal}$

$= 0.6 - 0$

$= 0.6$

$\Delta Q_2 = -0.5 + 0.05$

$= -0.45$

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$$\frac{\partial P_2}{\partial \delta_2} = |V_1||V_2||Y_{21}|\sin(\delta_1-\delta_2+\theta_{21}) + |V_2||V_2||Y_{22}|\sin(\theta) + |V_3||V_2||Y_{23}|\sin(\delta_3-\delta_2+\theta_{23})$$

$$= |1||1||10|\sin(0-0+90) + |1.01||1||5|\sin(90)$$

$$= 10+5.05$$

$$= 15.05$$

$$\frac{\partial P_2}{\partial \delta_3} = |V_3||V_2||Y_{23}|\sin(\delta_3-\delta_2+\theta_{23})$$

$$= -|1.01||1||5|\sin(0-0+90)$$

$$= -5.05$$

$$\frac{\partial P_2}{\partial V_2} = |V_1||Y_{21}|\cos(\delta_1-\delta_2+\theta_{21}) + 2|V_2||Y_{22}|\cos(\delta_2-\delta_2+\theta_2$$

$$+ |V_3||Y_{23}|\cos(\delta_3-\delta_2+\theta_{23})$$

$$= |1||10|\cos(90) + 2|1||5|\cos(-90) + |1.01||\cos(90$$

$$= 0$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_2||V_3||Y_{23}|\sin(\delta_2-\delta_3+\theta_{23})$$

$$= -|1||1.01||5|\sin(0-0+90)$$

$$= -5.05$$

$$\frac{\partial P_3}{\partial \delta_3} = +|V_3||V_1||Y_{31}|\ \text{Sin Cos}(\delta_1-\delta_3+\theta_{31}) +$$

$$|V_2||V_3||Y_{32}|\sin(\delta_2-\delta_3+\theta_{23})$$

$$= |1.01||1||4|\sin(90) + |1||1.01||6|\sin(90)$$

$$= 4.04+5.05$$

$$= 9.09.$$

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$$\frac{\partial P_2}{\partial V_2} = |V_3||Y_{23}|\cos(\delta_2 - \delta_3 + \theta_{23})$$

$$= |1.01||5|\cos(90)$$

$$= 0$$

$$\frac{\partial Q_2}{\partial \delta_2} = +|V_1||V_2||Y_{21}|\cos(\delta_1 - \delta_2 + \theta_{21})$$

$$= 0$$

$$\frac{\partial Q_2}{\partial \delta_3} = 0$$

$$\frac{\partial Q_2}{\partial V_2} = -|V_1||Y_{21}|\sin(\delta_1 - \delta_2 + \theta_{21}) + 2|V_2||Y_{22}|$$
$$\sin(\delta_2 - \delta_2 + \theta_{22}) - |V_3||Y_{23}|\sin(\delta_3 - \delta_2 + \theta_{23})$$

$$\frac{\partial Q_2}{\partial V_2} = -|1||10|\sin(90) - 2|1||15|\sin(-90) -$$
$$|1.01||5|\sin(0 - 0 + 90)$$

$$= -10 + 30 - 5.05$$

$$\frac{\partial Q_2}{\partial V_2} = -15.05 + 30$$

$$= 14.95$$

$$[J] = \begin{bmatrix} 15.05 & -5.05 & 0 \\ -5.05 & 9.09 & 0 \\ 0 & 0 & 14.95 \end{bmatrix}$$

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$$\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \Delta V_2\end{bmatrix} = \begin{bmatrix}15.05 & -5.05 & 0\\ -5.05 & 9.09 & 0\\ 0 & 0 & 14.95\end{bmatrix}^{-1}\begin{bmatrix}-0.9\\ 0.6\\ -0.45\end{bmatrix}$$

$$= \begin{bmatrix}0.08 & 0.04 & 0\\ 0.04 & 0.135 & 0\\ 0 & 0 & 0.066\end{bmatrix}\begin{bmatrix}-0.9\\ 0.6\\ -0.45\end{bmatrix}_{3\times 1}$$

$$\begin{bmatrix}\Delta\delta_2\\ \Delta\delta_3\\ \Delta V_2\end{bmatrix} = \begin{bmatrix}-0.048 & 0.04 & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}-0.048\\ 0.045\\ -0.029\end{bmatrix}$$

$V_2 = 1 - 0.029 \quad 1 - 0.029$

$= \quad = 0.971$

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