22/11/19

Unit-I

Per Unit Representation & Topology

Per Unit System:

The per unit System is defined as the ratio of actual Value to the base Value i.e z per unit is equal to $\frac{Z_{actual}}{Z_{base}}$

we know that $Z_b = \frac{V_b}{I_b}$

we know $S_b = V_b \cdot I_b$

$$I_b = \frac{S_b}{V_b}$$

$$Z_b = \frac{V_b}{\frac{S_b}{V_b}}$$

$$Z_b = \frac{(V_b)^2}{S_b}$$

$$Z_{p.u} = \frac{Z_{actual}}{\frac{(V_b)^2}{S_b}} = Z_{actual} \times \frac{S_{base}}{(V_{base})^2}$$

$$Z_{p.u} = Z_{actual} \times \frac{S_{base} \times 10^6}{(V_{base})^2 \times 10^6}$$

$$Z_{p.u} = Z_{actual} \times \frac{(MVA)_{base}}{(kV)^2_{base}}$$

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Calculate the per unit Value of X (reactance) when S=100MVA V=11KV and X=4Ω

Sol: $X_{PU} = X_{actual} \times \frac{(MVA)_{base}}{(KV)^2_{base}}$

$= 4 \times \frac{100}{(11)^2}$

$X_{PU} = 4 \times \frac{100}{121} = 3.3$

$\boxed{X_{P.U} = 3.3}$

2) Calculate the ohmic Value of reactance of an alternator if S=250MVA and V=13.2KV and X=1.5

$X_{PU} = X_{actual} \times \frac{(MVA)_{base}}{(KV)^2_{base}}$

$1.5 = X_{actual} \times \frac{250}{(13.2)^2}$

$\boxed{X_{actual} = 1.04\Omega}$

3) A Synchronous generator is rated as 250MVA, 13.2KV, X=2 then per unit Calculate the per unit reactance of generator at 500MVA and 22KV

Sol: $X_{PU} = X_{actual} \times \frac{(MVA)_{base}}{(KV_{base})^2}$

$2 = X_{actual} \times \frac{250}{(13.2)^2}$

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$X_{actual} = 1.39$

$X_{P.U_N} = 1.39 \times \frac{500}{(22)^2}$

$X_{P.U_N} = 1.3$

$$X_{PU\,N} = X_{P.U.O} \times \left(\frac{MVA_N}{MVA\,O}\right) \times \left(\frac{KV_{old}}{KU_N}\right)^2$$

4) Calculate the per unit quantities of the following by Considering 100V and 20A are the base Values. i) $130\angle 20$ V ii) 30A iii) $5.4 + j6$ KVA.

iv) $21 + j15\,\Omega$ 4 KW at 0.8 lagging P.F

Sol: i) Given $V_{actual} = 130\angle 20$ V

$V_{base} = 100$V

$$V_{P.U} = \frac{V_{act}}{V_{base}} = \frac{130\angle 20}{100} = 1.3\angle 20 \text{ P.U}$$

ii) Given $I_{actual} = 30$A

$I_{base} = 20$A

$$I_{P.U} = \frac{I_{act}}{I_{base}} = \frac{30}{20} = 1.5 \text{ P.U.}$$

iii) Given $S_{actual} = (5.4 + j6)$ KVA

$S_b = V_b \cdot I_b$

$S_b = 100 \times 20 = 2000\text{VA} = 2\text{KVA}$

$$S_{P.U} = \frac{S_{act}}{S_{base}} = \frac{5.4 + j6}{2000} =$$

$$S_{P.U} = 2.7 \times 10^{-3} + j3 \times 10^{-3}$$

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$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{100}{20}$$

$$Z_{base} = 5$$

$$Z_{p.u} = \frac{Z_{act}}{Z_{base}} = \frac{21+j15}{5} = 4.2 + 3j$$

ii) 4kw at 0.8 pf lagging.

$$\cos\phi = \frac{P}{S}$$

$$0.8 = \frac{4\times10^3}{S}$$

$$S = \frac{4\times10^3}{0.8} = 5\,KVA$$

$$\boxed{S = 5\,KVA}$$

$$S_{p.u} = \frac{S_{actual}}{S_{base}} = \frac{5\times10^3}{2\times10^3} = 2.5$$

5) Calculate the actual values for the following if $Z_b = 10\Omega$ and $V_b = 300V$

1) Impedance of $0.8 + j0.9$.
2) Voltage of 0.9 at 2°
3) Current of I of 1.4
4) Complex power of 2.2 at 0.8 lagging.

Sol: $Z_{actual} = Z_{p.u} \times Z_{base}$

$$= (0.8 + j0.9)(10)$$

$$Z_{actual} = 8 + 9j$$

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ii) $V_{actual} = V_{p.u} \times V_{base}$.

$= 0.9 \angle 2^\circ \times 300$

$V_{act} = 269.8 + 9.4j\ V$

iii) $I_{actual} = I_{p.u} \times I_{base}$.

$I_{act} = 1.4 \times 30 = 42V$

$Z_b = \frac{V_b}{I_b} = \frac{300}{10} = 30$

$I_b = 30$

$I_{act} = 4.2V$

iv) Given.

$S_{p.u} = 2.2 \angle$

$\cos\phi = \frac{P}{S}$

$S_{actual} = S.P.U \times S_{base}$

$S_{base} = V_b I_b$

$= 300 \times 30$

$= 9000\ VA = 9KVA$

$= 2.2 \times 9 \times 10^3$

$S_{act} = 19,800\ VA$

6) The ratings of 4 cables are given below. Determine which cable is most efficient by Using normal or actual Values and per Unit Values

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Cable	V	I	R
A	210V	70A	1Ω
B	500V	100A	1.5Ω
C	330V	40A	2Ω
D	1000V	100A	3.5Ω.

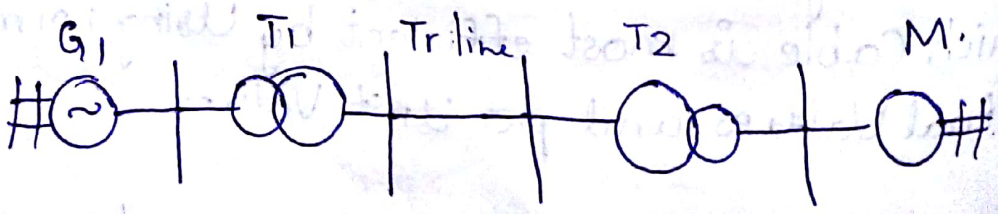
Normal approach

Cable	V	I	R	$S = V_b I_b$	loss I^2R	% loss = $\frac{\text{loss}}{S}$
A	210V	70A	1Ω	14700	4900	0.33
B	500 V	100A	1.5Ω	50000	15000	0.3
C	330V	40A	2Ω	13,200	3200	0.24
D	1000V	100A	3.5Ω	1,00pco	35000	0.35

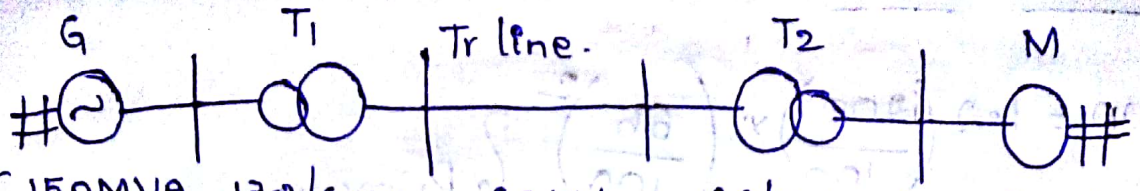
Per Unit approach.

Cable	V	I	Ract	$R_{base} = \frac{V_b}{I_b}$	$R_{PU} = \frac{R_{act}}{R_{base}}$
A	210V	70A	1Ω	3Ω	0.33
B	500V	100A	1.5Ω	5Ω	0.3
C	300V	40A	2Ω	8.25Ω	0.24
D	1000V	100A	3.5Ω	10Ω.	0.35

7) Calculate the new per Unit Values for the given Single line diagram by Considering base Values of 200MVA & 20KV



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	G	T_1	Tr line	T_2	M
old	150MVA, 13.2 KV, X = 2.0	13.2/66 KV, 100MVA, X = 1.2	66KV, 100MVA, X = 20Ω	66/6.6 KV, 100MVA, X = 1.2	6.6 KV, 125 MVA, X = 1.5
New	200 MVA, 20KV	200 MVA, 20/$\frac{20\times66}{132}$ = 100KV, 13.2 → 66, 20 → ?	200MVA, 100KV	200MVA, 100/10KVA, 66 → 6.6, 100 → ?	200 MVA, 10KVA

For G_1

$$X_{PUN} = X_{PUO} \times \frac{MVA_N}{MVA_O} \times \left(\frac{KV_O}{KV_N}\right)^2$$

$$X_{PU} = 2 \times \left(\frac{200}{150}\right) \times \left(\frac{13.2}{20}\right)^2$$

$$X_{PU} = 1.16$$

For T_1

$$X_{PUN} = 12 \times \left(\frac{200}{100}\right) \times \left(\frac{13.2}{20}\right)^2 \Big\} \text{ Primary}$$

$$X_{PUN} = 1.04$$

$$X_{PUN} = 1.2 \times \left(\frac{200}{100}\right) \times \left(\frac{66}{100}\right)^2 \Big\} \text{ Seconday}$$

$$X_{PUN} = 1.04$$

For Transmission line

$$X_{PU} = X_{act} \times \frac{(MVA)}{(KV_{base})^2}$$

$$= 20 \times \frac{200}{(100)^2} = 0.4$$

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For Transform T_2

$$X_{PUN} = 1.2\left(\frac{200}{100}\right)\times\left(\frac{66}{100}\right)^2 \Big\} \text{ Primary}$$

$$X_{PUN} = 1.04$$

$$X_{PUN} = 1.2\left(\frac{200}{100}\right)\times\left(\frac{6.6}{10}\right)^2 \Big\} \text{ Secondary}$$

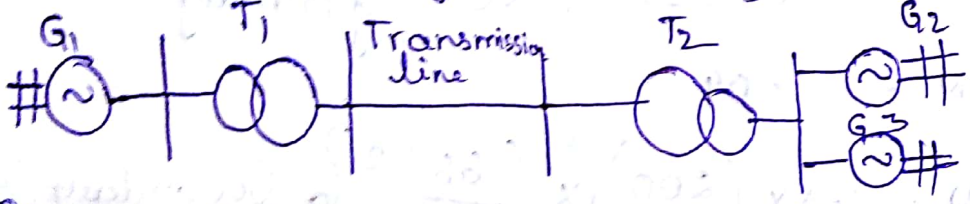
$$X_{PUN} = 1.04$$

For Motor

$$X_{PUN} = 1.5 \times \left(\frac{200}{125}\right) \times \left(\frac{6.6}{10}\right)^2$$

$$X_{PUN} = 1.54$$

Draw the reactance network by representing per Unit Values for the given Single line diagram choose the base MVA as generator capacity with a voltage of 50KV



200MVA	300MVA	300MVA	300MVA		
11KV	11/132KV	132KV	132/66KV		
X=1.5	X=0.8	X=40Ω	X=0.8		
200	200	200	200	200	200
50KV	50/600	600KV	600/300KV	300KV	300

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For Generator G_1

$$X_{PU_{new}} = X_{PU_{old}} \times \frac{MVA_N}{MVA_0} \times \left(\frac{kV_0}{kV_N}\right)^2$$

$$= 1.5 \times \frac{200}{200} \times \left(\frac{11}{50}\right)^2$$

$$X_{PUN} = 0.07$$

For Transformer T_1

$$X_{PUN} = X_{PU} \cdot 0.8 \times \frac{200}{300} \times \left(\frac{11}{50}\right)^2 \Big\} \text{Primary}$$

$$= 0.025$$

$$X_{PUN} = 0.8 \times \frac{200}{300} \times \left(\frac{132}{600}\right)^2 \Big\} \text{Secondary}$$

$$= 0.025$$

For Transmission line.

$$X_{PUN} = X_{act} \times \frac{MVA}{(kV_{bg})^2}$$

$$= 40 \times \frac{200}{(600)^2} = 0.02$$

For Transformer T_2

$$X_{PUN} = 0.8 \times \frac{200}{300} \times \left(\frac{132}{600}\right)^2 \Big\} \text{Primary}$$

$$X_{PUN} = 0.025$$

$$X_{PUN} = 0.8 \times \frac{200}{300} \times \left(\frac{66}{300}\right)^2 \Big\} \text{Secondary}$$

$$X_{PUN} = 0.025$$

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For Generator G_2

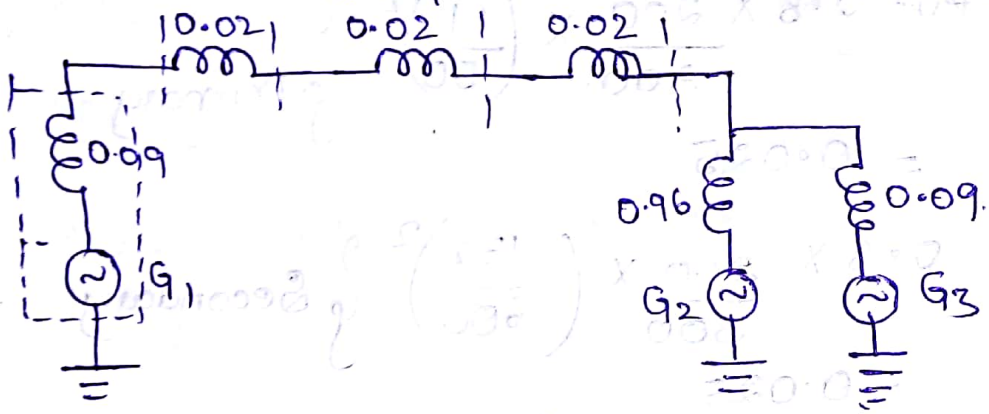
$$X_{PUN} = 10 \times \left(\frac{200}{100}\right) \times \left(\frac{66}{300}\right)^2$$

$$X_{PUN} = 0.9.$$

For Generator G_3

$$X_{P.U.N} = 0.5 \times \left(\frac{200}{50}\right) \times \left(\frac{66}{300}\right)^2$$

$$X_{P.U.N} = 0.09.$$



Reactance diagram

A 3 bus System is given below the ratings of Various components are listed below.

Generator-1 - 50MVA, 13.8KV, $X'' = 0.15$ P.U

Generator-2 - 40MVA, 13.2KV, $X'' = 0.20$ P.U.

Generator-3 - 30MVA, 11KV, $X'' = 0.25$ P.U

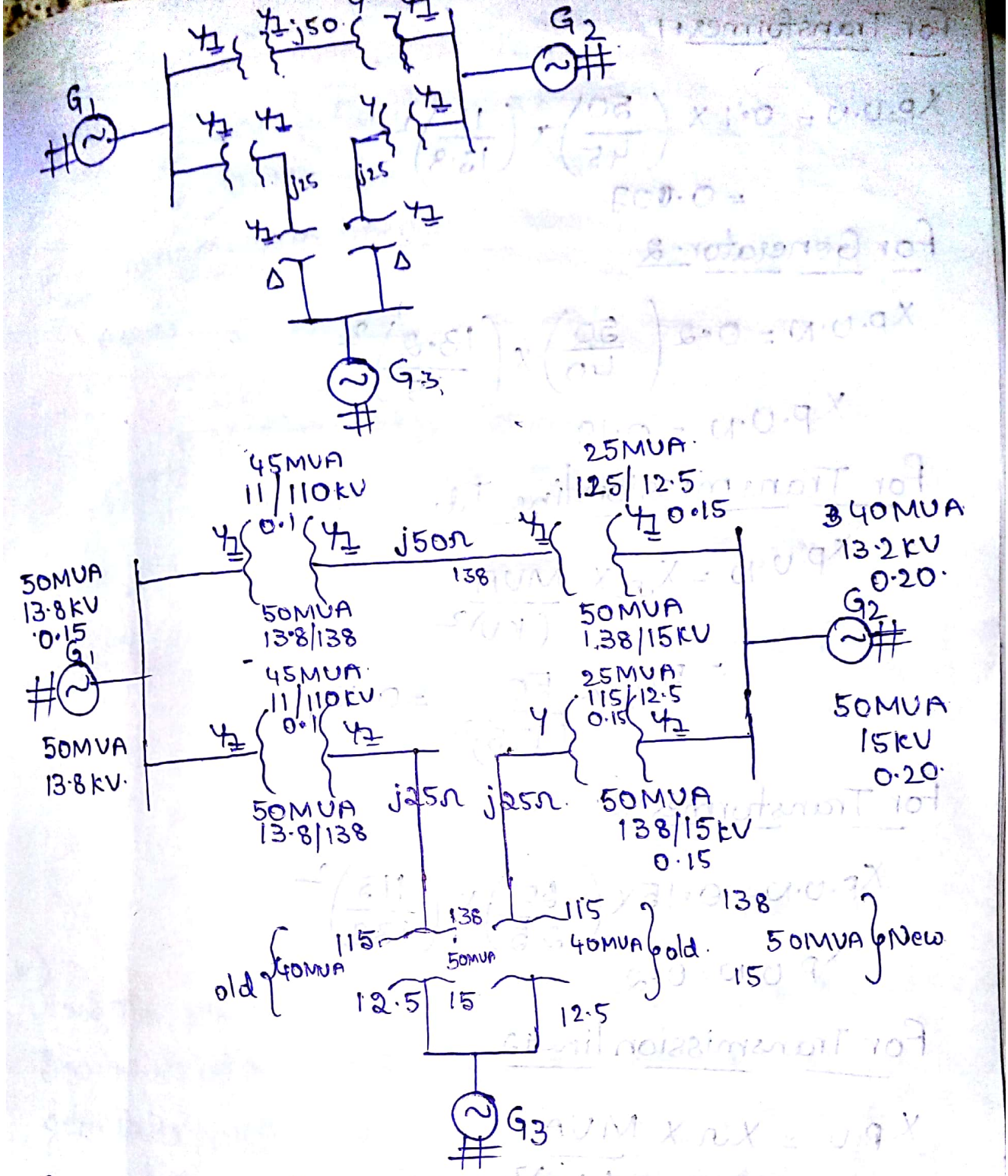
Transformer-1 - 45MVA, 11KV/110KV $X = 0.1$

Transformer-2 - 25MVA, 12.5/115KV $X = 0.15$ P.U

Transformer-3 - 40MVA, 12.5/115KV, $X = 0.15$ P.U

The line impedances are shown in fig. Determine the reactance diagram based on 50MVA and 13.8KV as base quantities in G_1

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from the given base Values the new power and Voltages for all elements is given by

for Generator 1

$$X_{P.U.N} = X_{P.U.O} \times \frac{MVA_N}{MVA_O} \times \left(\frac{kV_O}{kV_N}\right)^2$$

$$= 0.15 \times \frac{50}{50} \times \left(\frac{13.8}{13.8}\right)^2$$

$$X_{P.U.N} = 0.15$$

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For Transformer-1

$$X_{p.u.N} = 0.1 \times \left(\frac{50}{45}\right) \times \left(\frac{11}{13.8}\right)^2$$

$$= 0.007$$

For Generator-2

$$X_{p.u.N} = 0.2 \left(\frac{50}{40}\right) \times \left(\frac{13.2}{15}\right)^2$$

$$X_{p.u.N} = 0.19$$

For Transmission line TL1

$$X_{p.u.N} = X_{\Omega} \times \frac{MVA}{(KV)^2}$$

$$= 50j \times \frac{50}{(13.8)^2} = 0.13j$$

For Transformer-2

$$X_{p.u.N} = 0.15 \times \left(\frac{50}{25}\right) \times \left(\frac{115}{138}\right)^2$$

$$X_{p.u.N} = 0.2$$

For Transmission line-2

$$X_{p.u} = X_{\Omega} \times \frac{MVA}{(KV)^2}$$

$$= 25j \times \frac{50}{(138)^2}$$

$$= 0.06j$$

For Generator-3

$$X_{p.u.N} = 0.25 \times \left(\frac{50}{30}\right) \left(\frac{11}{15}\right)^2$$

$$X_{p.u.N} = 0.22$$

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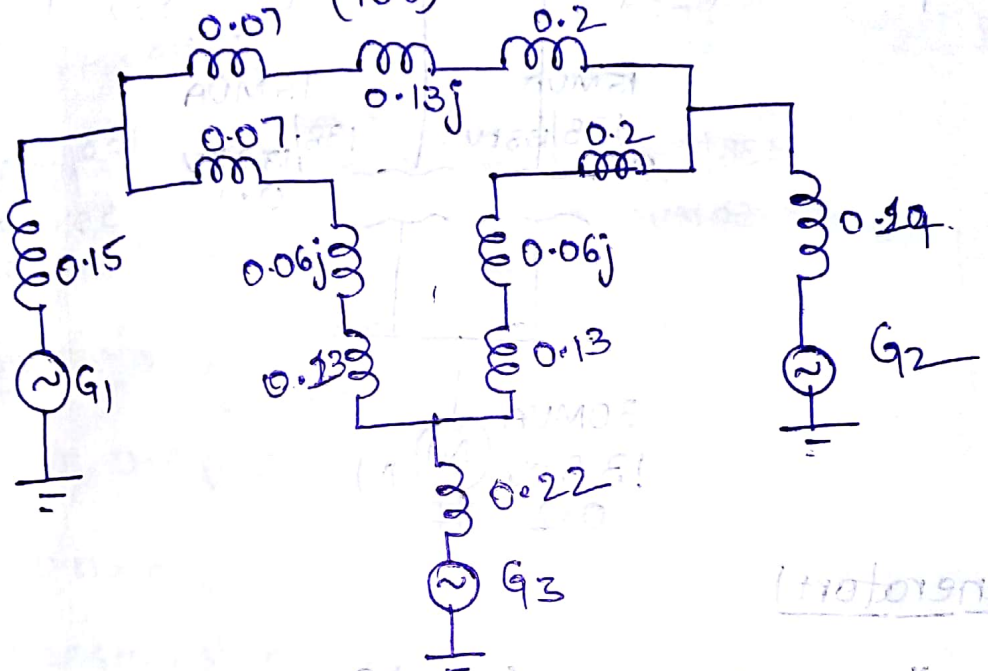
For Transformer+3

$$X_{p.u.N} = 0.15 \times \left(\frac{50}{40}\right) \times \left(\frac{115}{138}\right)^2$$

$$= 0.13$$

For Transmission line+3

$$X_{p.u.N} = 25j \times \frac{50}{(138)^2} = 0.06j$$



10) Draw the reactance diagram for the power Systems shown in below and the ratings of Transformer, generator and motor are also given below neglect the resistance and use a base of 50MVA 138KV in 40Ω line.

For Generator+1 20MVA 18KV and $x'' = 20\%$ =

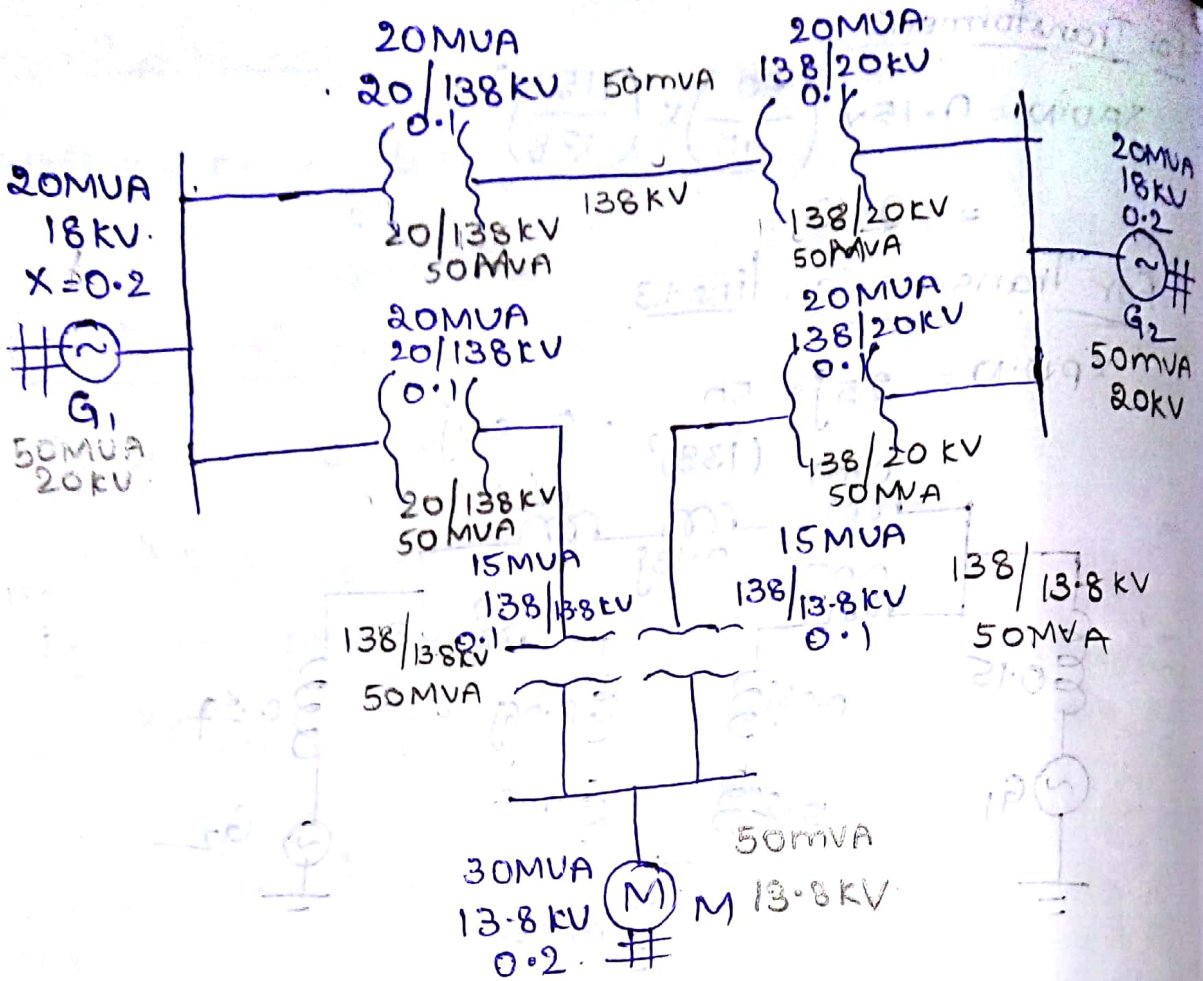
for Generator2 20MVA 18KV and $x'' = 20\%$.

Synchronousmotor 30MVA 13.8KV and $x'' = 20\%$.

3phase Y-Y Trans 20MVA 138/20KV and $X = 10\%$

3phase Y-Δ Trans 15MVA 138/138KV and $X = 10\%$

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For Generator:1)

$$X_{p.u.N} = X_{p.u.o} \times \frac{MVA_N}{MVA_o} \times \left(\frac{kV_o}{kV_N}\right)^2$$

$$= 0.2 \times \frac{50}{20} \times \left(\frac{18}{20}\right)^2$$

$$X_{p.u.N} = 0.4$$

for 2 Transformer:1) Y-Y Connected.

$$X_{p.u.N} = X_{p.u.o} \times \frac{MVA_N}{MVA_o} \times \left(\frac{kV_o}{kV_N}\right)^2 \Big\} \text{Primary.}$$

$$= 0.1 \times \left(\frac{50}{20}\right)^2 \times \left(\frac{20}{20}\right)^2$$

$$= 0.25$$

$$X_{p.u.N} = X_{p.u.o} \times \frac{MVA_N}{MVA_o} \times \left(\frac{kV_o}{kV_N}\right)^2$$

$$= 0.1 \times \left(\frac{50}{20}\right)^2 \times \left(\frac{138}{138}\right)^2$$

$$= 0.25$$

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for Transformer-2 Y-Δ Connected

$$X_{p.u.N} = X_{p.u.o} \times \frac{MVA_N}{MVA_o} \times \left(\frac{kV_o}{kV_N}\right)^2$$

$$= 0.1 \times \left(\frac{50}{15}\right) \times \left(\frac{138}{138}\right)^2$$

$$X_{p.u.N} = 0.3$$

for Generator-2

$$X_{p.u.N} = 0.2 \times \left(\frac{50}{20}\right) \times \left(\frac{18}{20}\right)^2$$

$$X_{p.u.N} = 0.40$$

for Generator-3 Motor

$$X_{p.u.N} = 0.2 \times \left(\frac{50}{30}\right) \times \left(\frac{13.8}{13.8}\right)^2$$

$$X_{p.u.N} = 0.33$$

for Transmission line-1

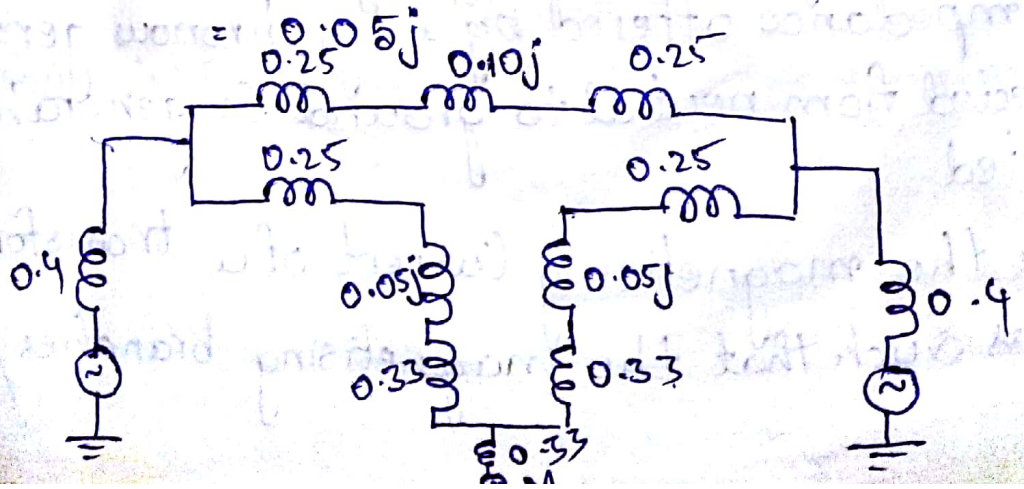
$$X_{p.u.N} = X_{actual} \times \frac{MVA}{(kV)^2}$$

$$= j40 \times \frac{50}{(138)^2} = 0.10j$$

for Transmission line-2

$$X_{p.u.N} = j20 \times \frac{50}{(138)^2}$$

$$= 0.05j$$

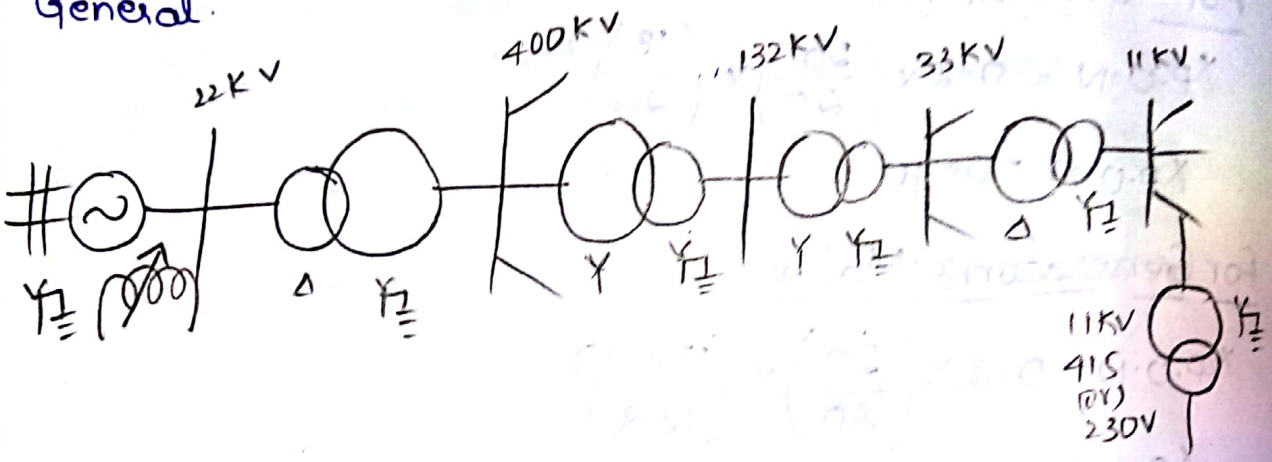


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Single line diagram.

Eventhough the existing power System network is three phase. Since all the 3 phases are identical Such that the 3 phase transmission line can be represented as phase manner or physically it can be represented as Single line diagram

General.



Impedance diagram

→ The impedance diagram is mainly drawn from the Single line diagram of power System network.

→ The impedance diagram is mainly used at load flow Studies.

→ The following are Some of the assumptions made while drawing the impedance diagram.

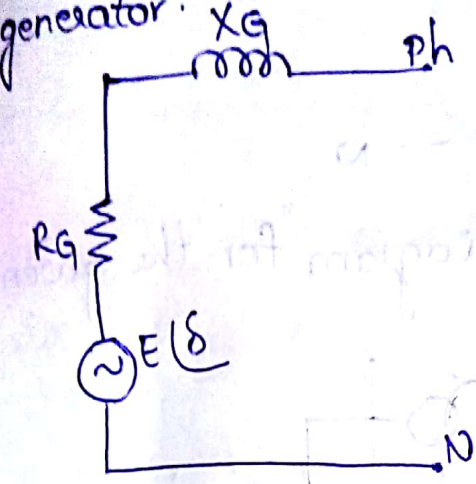
i) The impedance offered by a Synchronous generator Connected from neutral to ground is generally neglected.

2) Since the magnetising Current of a transformer is less Such that the magnetising branches

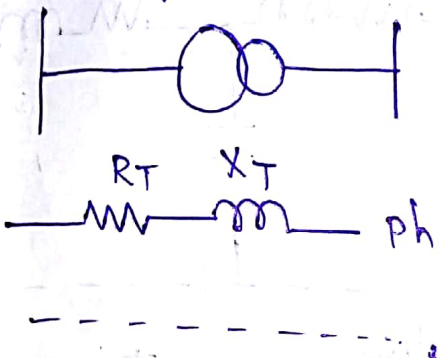
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Shunt branches of transformer are neglected.

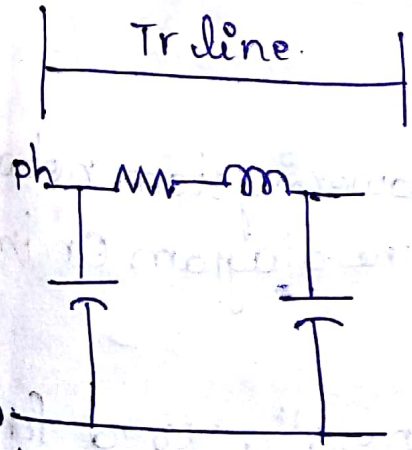
The equivalent impedance diagram for a Synchronous generator.



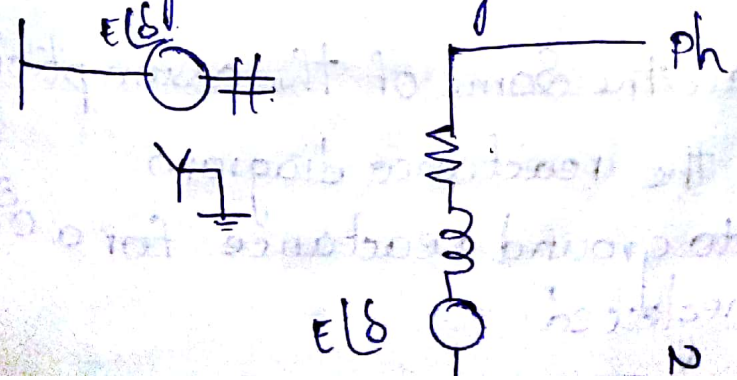
Equivalent impedance diagram for a transformer



Equivalent impedance diagram for a Transmission line.

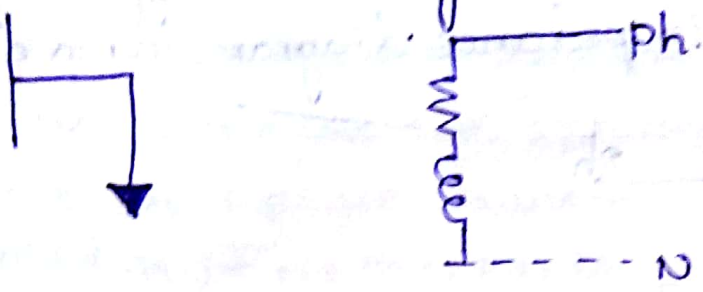


Equivalent impedance diagram for Synchronous motor.

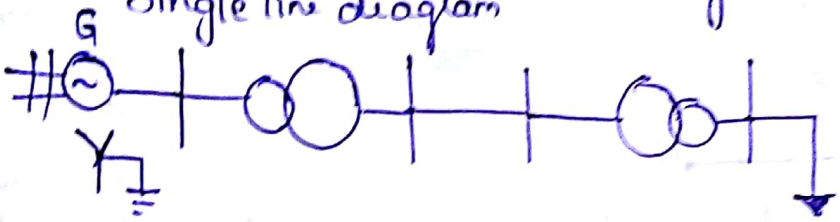


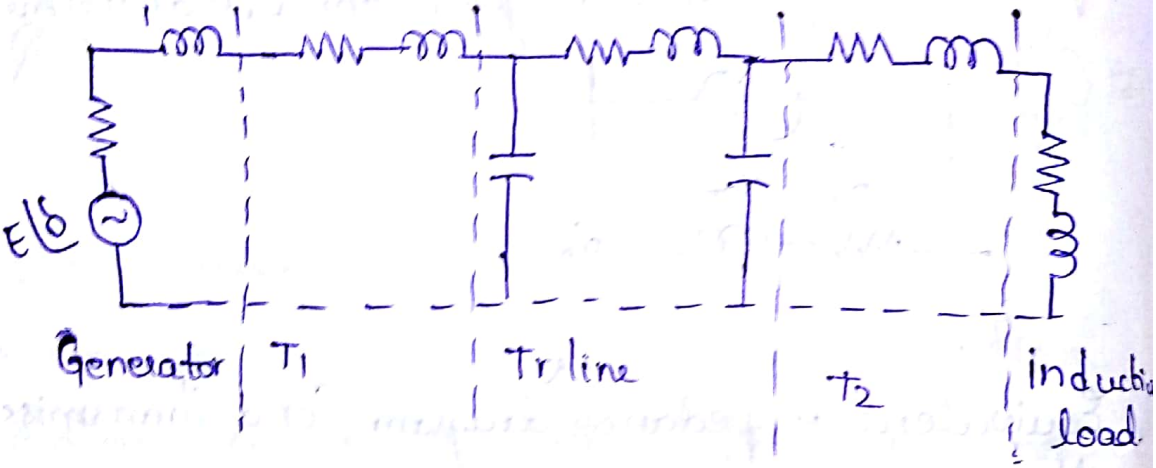
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Equivalent impedance diagram for inductive load



Example: Draw the impedance diagram for the given Single line diagram





Reactance diagram.

→ The reactance diagram of a power System network can be drawn by using Single line diagram or impedance diagram.

→ The reactance diagram is generally used for fault calculations.

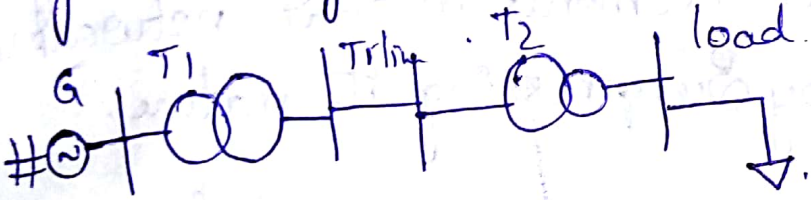
The following are the Some of the assumptions made while drawing the reactance diagram

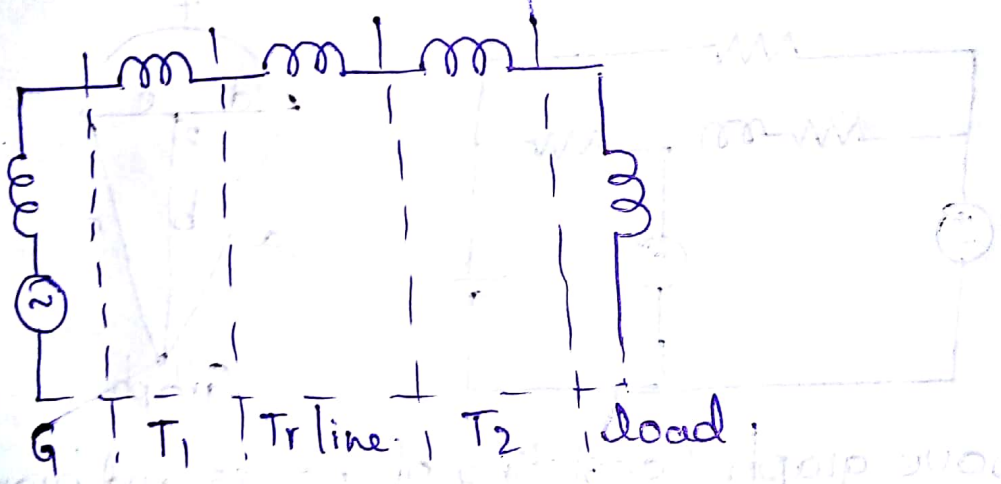
1. The neutral to ground reactance for a Synchronous generator is neglected.

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2. The shunt branch impedances of a transformer are neglected.
3. The line to ground capacitances are also neglected
4. The resistances in the equivalent circuits of various components of the system are omitted

Draw the reactance diagram for the following single line diagram.





Advantages of per Unit System.

1) The per Unit impedances offered for a transformer is same with refer to primary and secondary.
2) Since the entire power system network has common base values the required characterstic equations are less while compare with the actual representation of network.
3) Some of the manufacturers are representing the impedance values of an electrical equipment either in percentage or per unit format

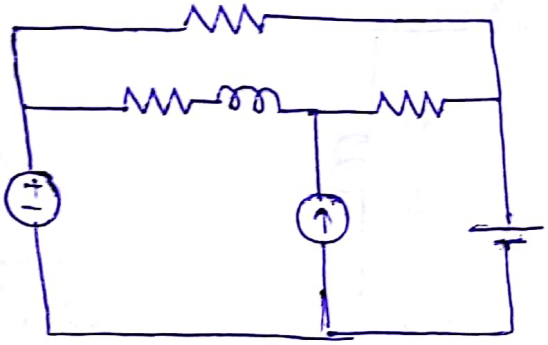
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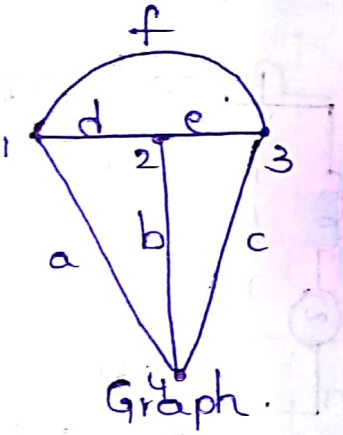
4. By using per unit representation the percentage of losses for a specific transmission line is directly calculated equal to per unit resistance of particular transmission line.

Graph theory:-

The ⋆ Graph is defined as any electrical circuit or network can be represented by lines by replacing all the elements in that network with a thick line by suppressing its nature.

Example:-





Graph.

The above graph consisting of nodes and branches. The nodes can be denoted with numbers and branches are denoted with letters

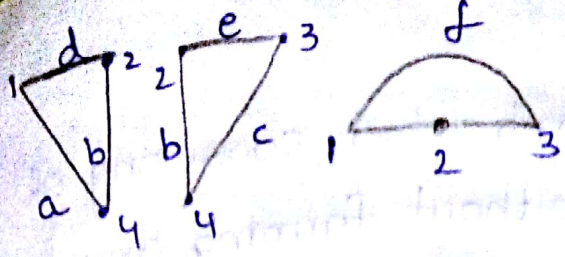
Node: It is a point of intersection of two or more elements

Branch: It is a two terminal element drawn b/w two nodes and represented by a thick line.

Sub graph: A graph is said to be a sub graph of main graph if it contains some of the nodes and branches of main graph.

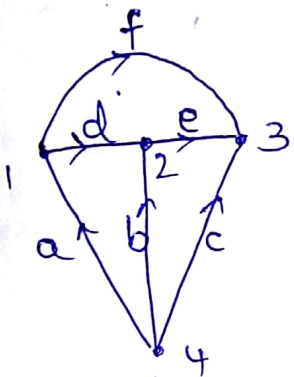
Example:

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Oriented graph:

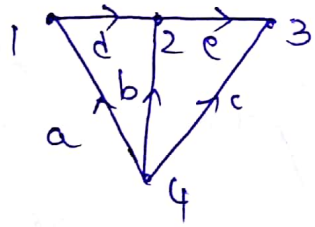
It is defined as the branches of a main graph can be assigned with particular direction



Connected graph or just Connected graph

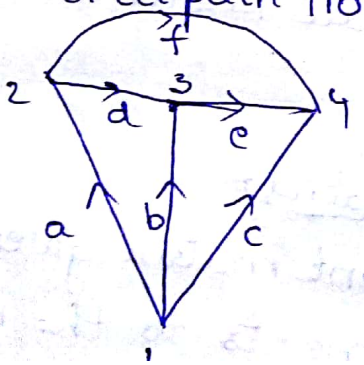
A graph is said to be connected if there exists at least one path from one node to another anode.

Example:



Completely Connected graph.

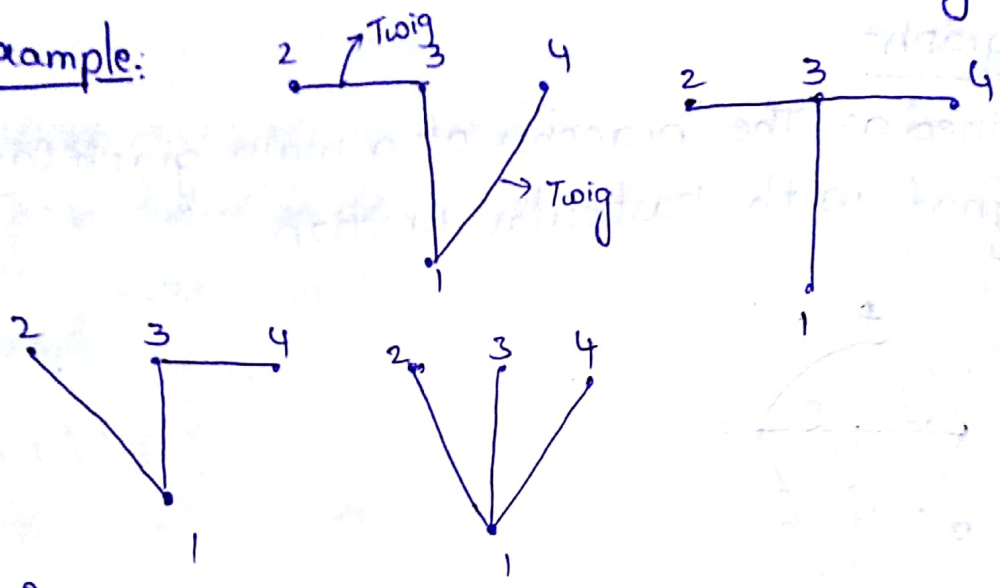
A graph is said to be Completely Connected if there exist direct path from one node to every other node.



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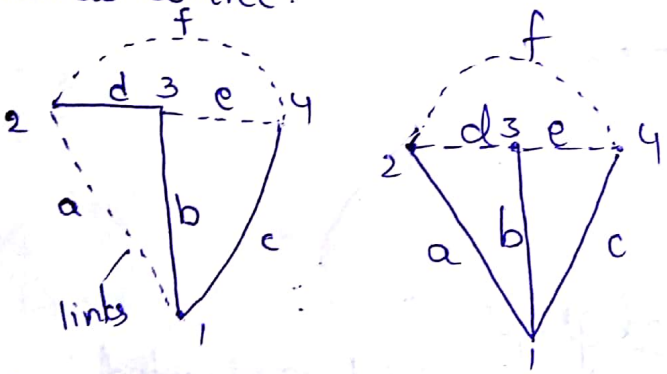
Tree:-
It is a subgraph of a main graph which consisting of all number of nodes and without forming any loop

Example:



Twigs:-
The branches of tree are known as twigs which are indicated with thick lines.

Co tree:-
The set of branches other than tree branches are known as Co-tree.



Links:-
The branches of Cotree are known as links indicated with dotted lines

element node incident matrix:-
The element node incident matrix gives the relation between number of elements to the nodes in a

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oriented graph. It is indicated with letter $\bar{A}$

The Order of $\bar{A}$ is given by exn.

where e is no of elements

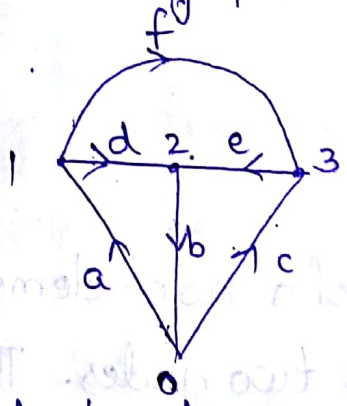
n is no of nodes

The elements or entries in the incident matrix can be given by $\bar{A} = [a_{ij}] = +1$ if ith element incident on jth node and oriented away

$[a_{ij}] = -1$ if ith element incident on jth node towards it and Oriented towards it

$[a_{ij}] = 0$ if ith element not at all incident on jth node.

Example: Construct an element, node incident matrix for the given Oriented graph.



Incident matrix exn = 6 X 4.

	0	1	2	3
a	+1	-1	0	0
b	-1	0	+1	0
c	+1	0	0	-1
d	0	+1	-1	0
e	0	0	-1	+1
f	0	+1	0	-1

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Bus incident matrix

The bus incident matrix can be obtained by eliminating elements or entries related to a reference node from the element node incident matrix

The bus incident matrix is indicated with letter A

The Order of A is ex(n-1)

	2	3	4
a	-1	0	0
b	0	-1	0
c	0	0	-1
d	1	-1	0
e	0	1	-1
f	1	0	-1

1st node is taken as reference node.

primitive network:-

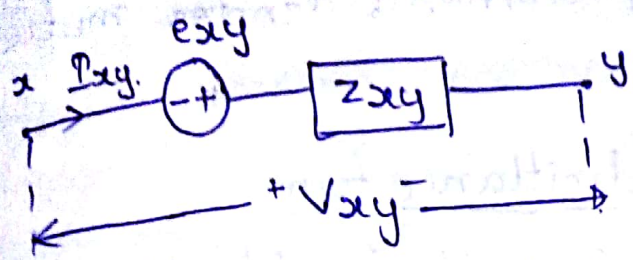
A power System network Consists of a no of elements and each element is incident between two nodes. This element may be a passive or active or Combination of both.

Each element Can be represented by a decoupled partial network known as the primitive network

This network Can be either in impedance form or admittance form.

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Impedance form of primitive network.



consider an element which is connected in b/w two nodes x and y.

Let exy is active source of an element connected in between x and y.

And ixy is the current flowing through the element x and y

Zxy is the self impedance of the element x and y.

Vxy is the voltage across the element x and y

From the Diagram according to KVL.

$$I_{ij} Z_{ij} - V_{ij}$$

$$I_{xy} Z_{xy} - V_{xy} - e_{xy} = 0$$

$$V_{xy} + e_{xy} = Z_{xy} I_{xy}$$

where V, e, I are the column matrices with a size of ex1

and z is a square matrix of size exe

matrix z is known as primitive impedance matrix.

In general z matrix is a diagonal matrix provided the elements are not mutually coupled with the other elements.

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So the diagonal elements of z denotes Self impedance while the half of diagonal elements denotes mutual impedances.

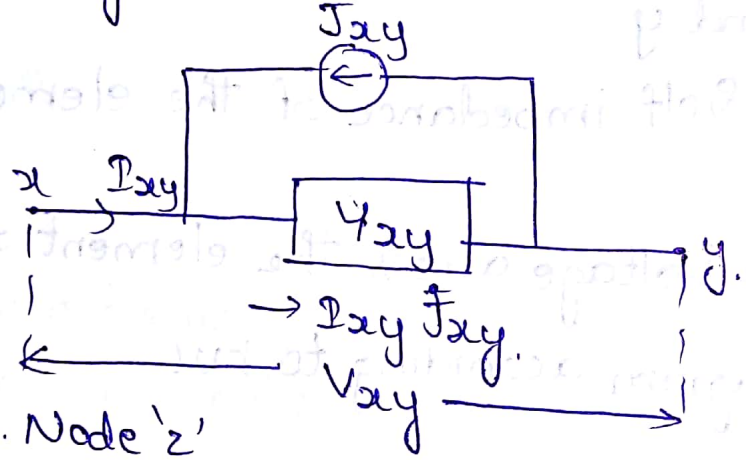
primitive network in Admittance form.

Consider an elements connected in b/w the nodes x and y. J_{xy} is the active current Source of a element x, y

Y_{xy} is the Self admittance of an element

V_{xy} is the Voltage across the elements x and y

By Using KCL.



From. Node 'z'

$$I_{xy} + J_{xy} = I_{xy} + J_{xy}$$

$$I_{xy} + J_{xy} = V_{xy} \cdot Y_{xy}$$

$$I + J = V_{xy}\ V_Y$$

where I, J and V are the Column matrices and Y is the primitive admittance matrices with a Size of exe

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Ybus formulation:

The Y_{bus} or admittance matrix can be formulated by using two methods.

i) Singular transformation method.

ii) Direct inspection method.

Singular transformation method:

The formulation of Y_{bus} in Singular transformation method can be obtained by determining the relation between the Variables and parameters of the primitive admittance network.

we know that by using primitive admittance network

$$I + J = VY$$

Multiplying by transpose of bus incidence matrix

then $[A^T]i + [A^T]j = [A^T]VY$ - ①

we know that from kirchoff's current law the algebric Sum of Currents at any node is equal to Zero.

Such that $[A^T]i = 0$ - ②

In Similar

The algebric Sum of Source Current at each bus must be equal to the Current impressed at the bus.

$$[A^T]j = I_{BUS} \text{ - ③} \quad - ⑥$$

Substitute ② & ③ in ①

$$0 + I_{BUS} = [A^T]VY.$$

$$I_{BUS} = A^T YV \text{ - ②}$$

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In bus frame the power in the network is given by

$$P_{BUS} = [I_{BUS}^*]^T \cdot V_{BUS} \quad -③$$

From the primitive admittance matrix

$$P = [J^*]^T \cdot V \quad -④$$

From eqn ③ & ④.

$$[I_{BUS}^*]^T V_{BUS} = [J^*]^T \cdot V \quad -⑤$$

From eqn ②

Take the Conjugate and transpose on both sides

$$[I_{BUS}^*]^T = [[A^T]^T]^* [j^*]^T$$

$$[I_{BUS}^*]^T = [j^*]^t \cdot A \quad -⑦$$

Substitute eqn ⑦ in eqn ⑤

$$[j^*]^t A V_{BUS} = [J^*]^t V$$

$$\boxed{V = A V_{BUS}} \quad -⑧$$

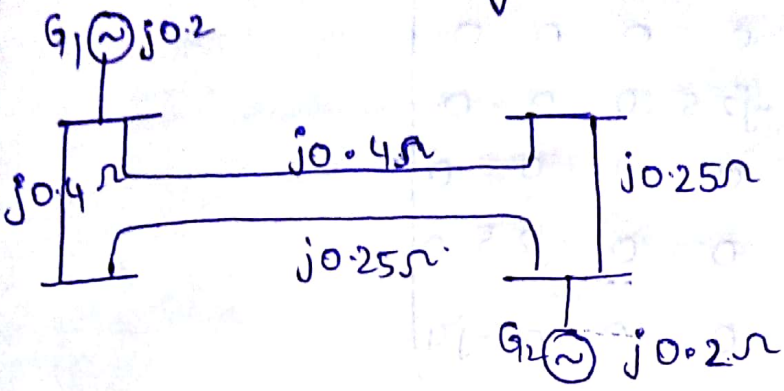
From ② & ⑧.

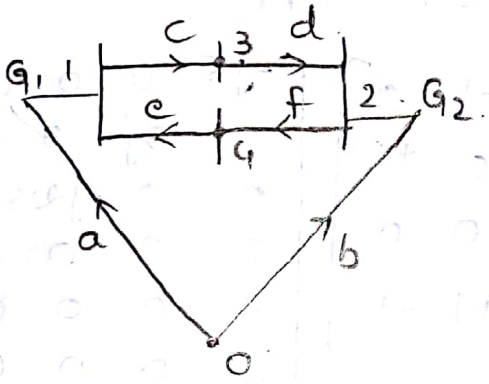
$$I_{BUS} = A^T \cdot Y \cdot A \cdot V_{BUS}$$

$$I_{BUS} = V_{BUS} Y_{BUS} \qquad A^T \cdot Y \cdot V = Y_{BUS}$$

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Form the 4 bus by using Singular transformation for the network shown in fig





node incident matrix exn

6 × 5.

	0	1	2	3	4
a	+1	-1	0	0	0
b	+1	0	-1	0	0
c	0	+1	0	-1	0
d	0	0	-1	+1	0
e	0	-1	0	0	+1
f	0	0	+1	0	-1

To obtain bus incident matrix neglect reference node.

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ +1 & 0 & -1 & 0 \\ 0 & -1 & +1 & 0 \\ -1 & 0 & 0 & +1 \\ 0 & +1 & 0 & -1 \end{bmatrix}$$

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$$Y = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} -j5 & 0 & 0 & 0 & 0 & 0 \\ 0 & -j5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -j2.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j4 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j2.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -j4 \end{bmatrix} \end{array}$$

We know that $Y_{BUS} = A^T \cdot Y \cdot A$

$$Y_{BUS} = \begin{bmatrix} -1 & 0 & +1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 & 0 & +1 \\ 0 & 0 & -1 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & -1 \end{bmatrix} \begin{bmatrix} -j5 & 0 & 0 & 0 & 0 & 0 \\ 0 & -5j & 0 & 0 & 0 & 0 \\ 0 & 0 & -j2.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j4 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j2.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -j4 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

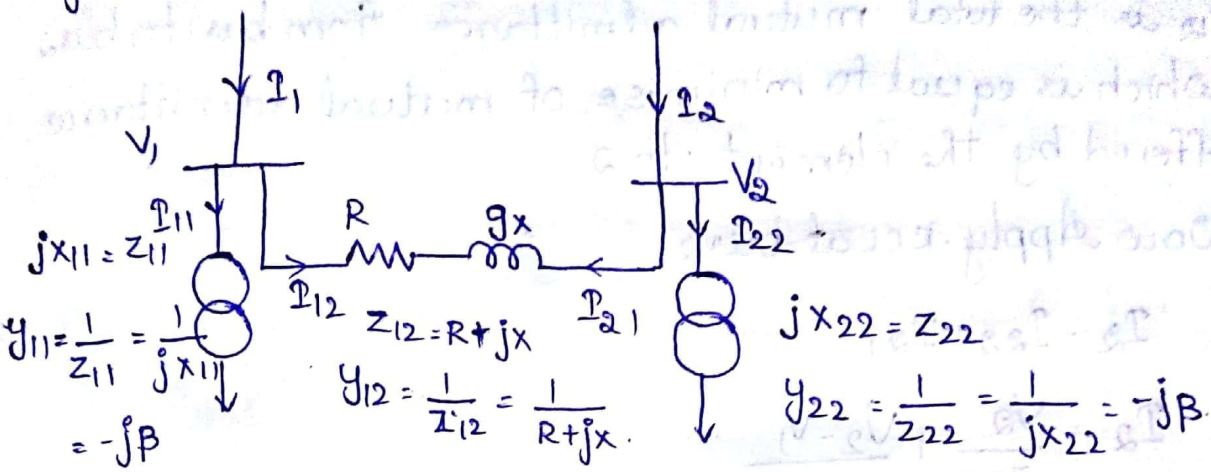
$$= \begin{bmatrix} +j5 & 0 & -j2.5 & 0 & j2.5 & 0 \\ 0 & +5j & 0 & j4 & 0 & -j4 \\ 0 & 0 & j2.5 & -j4 & 0 & 0 \\ 0 & 0 & 0 & 0 & -j2.5 & j4 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -j5-j2.5-j2.5 & 0 & j2.5 & j2.5 \\ 0 & -5j-4j-4j & j4 & -j4 \\ j2.5 & j4 & -j2.5-j4 & 0 \\ j2.5 & j4 & 0 & -j2.5+j4 \end{bmatrix} = \begin{bmatrix} -10j & 0 & j2.5 & j2.5 \\ 0 & -13j & j4 & -j4 \\ j2.5 & j4 & -6.5j & 0 \\ j2.5 & j4 & 0 & -1.5j \end{bmatrix}$$

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Direct-Inspection Method:

Consider a two bus Systems a bus Voltages of V_1 & V_2 and injected Currents I_1, I_2



From the diagram $y_{12} = G - jB$

Z_{12} is the impedance offered by an element which is connected in b/w

Z_{11} is the Self impedance at bus 1

Z_{22} is the Self impedance at bus 2

V_1 and V_2 are the Voltages at the buses 1 and 2 respectively

I_{12} is the Current flowing through the element from bus 1 to bus 2.

Now Apply KCL at bus 1

$$I_1 = I_{11} + I_{12}$$

$$I_1 = \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_{12}} \cdots$$

$$I_1 = V_1 y_{11} + (V_1 - V_2) y_{12}$$

$$I_1 = V_1 [y_{11} + y_{12}] + V_2 [-y_{12}]$$

$$I_1 = V_1 y_{11} + V_2 y_{12} - (1)$$

From eqn (1) y_{11} is the total Self admittance at

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bus1 which is equal to Sum of Self admittance and mutual admittance

Y_{12} is the total mutual admittance from bus1 to bus2 which is equal to minimise of mutual admittance offered by the element 1 to 2

Now Apply KCL at bus2

$$I_2 = I_{22} + I_{21}$$

$$I_2 = \frac{V_2}{Z_{22}} + \frac{V_2 - V_1}{Z_{21}}$$

$$I_2 = V_2 y_{22} + (V_2 - V_1) y_{21}$$

$$I_2 = V_1[-y_{21}] + V_2[y_{22} + y_{21}]$$

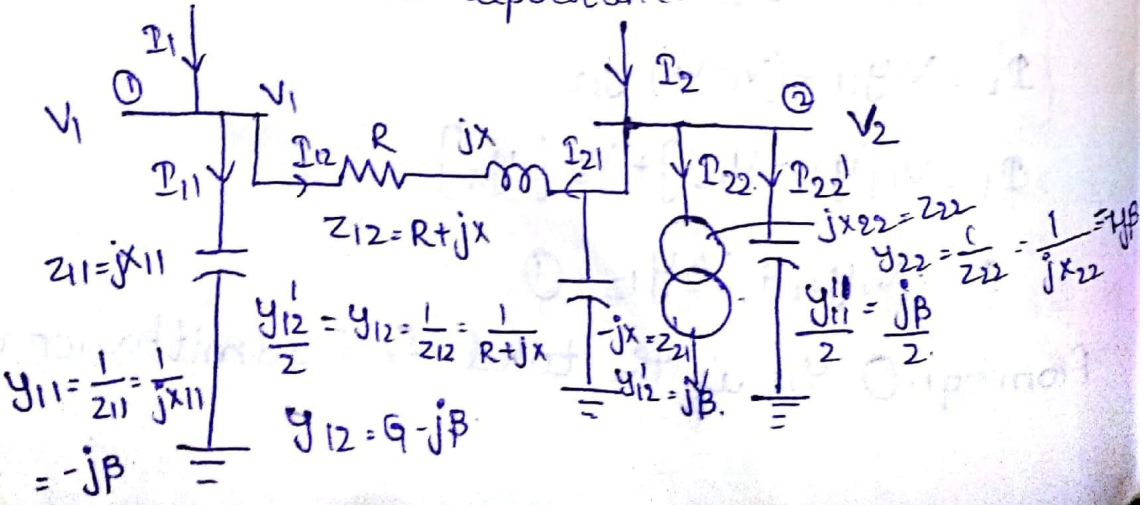
$$I_2 = V_1 Y_{21} + V_2 Y_{22} \quad -(2)$$

From eqn (1) & (2)

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Y-Bus with line charge admittance

Consider the line charge admittance Value y'_{12} at the transmission line will be replaced as having line charge admittance at the buses in Order to give the effect of shunt Capacitance



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Apply KCL at bus 1

$$I_1 = I_{11} + I_{12} + I_{11}'$$

$$I_1 = \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_{12}} + \frac{V_1}{Z_{11}'}$$

$$I_1 = V_1 y_{11} + (V_1 - V_2) y_{12} + V_1 \left(\frac{y_{12}'}{2}\right)$$

$$I_1 = V_1 \left[y_{11} + y_{12} + \frac{y_{12}'}{2}\right] + V_2 \left[-y_{12}\right]$$

Apply KCL at bus '2'

$$I_2 = I_{22} + I_{21} + I_{22}'$$

$$= \frac{V_2}{Z_{22}} + \frac{V_2 - V_1}{Z_{21}} + \frac{V_2}{Z_{22}'}$$

$$= V_2 y_{22} + (V_2 - V_1) y_{21} + \frac{V_2 y_2'}{2}$$

$$I_2 = V_1 \left[-y_{21}\right] + V_2 \left[y_{22} + y_2 + \frac{y_{12}'}{2}\right]$$

$$I_2 = V_1 Y_{21} + V_2 Y_{22} \quad -(2)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Construct the 4 bus matrix for the given following data.

Bus Code	line impedance per Unit	line charge admittance (Y_{pq}')
1-2	$Z_{12} = 0.2 + j0.4$	$Y_{12}' = 0.8j$
1-3	$Z_{13} = 0.1 + j0.2$	$Y_{13}' = 0.4j$
2-3	$Z_{23} = 0.05 + j0.1$	$Y_{23}' = 0.2j$

We know that

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}_{3\times 3}$$

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Calculation of Y_{11}

$$Y_{11} = y_{11} + y_{12} + y_{13} + \frac{y'_{12}}{2} + \frac{y'_{13}}{2}$$

$y_{11} = 0$

$$y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.2+j0.4} = 1-2j$$

$$y_{13} = \frac{1}{Z_{13}} = \frac{1}{0.1+j0.2} = 2-4j$$

$$\frac{y'_{12}}{2} = \frac{0.8j}{2} = \frac{0.4j}{}$$

$$\frac{y'_{13}}{2} = \frac{0.4j}{2} = 0.2j$$

$$Y_{11} = 0 + 1-2j + 2-4j + 0.4j + 0.2j$$

$$Y_{11} = 3-5.4j$$

Calculation of Y_{22}

$$Y_{22} = y_{21} + y_{22} + y_{23} + \frac{y'_{21}}{2} + \frac{y'_{23}}{2}$$

$$y_{21} = \frac{1}{Z_{21}} = \frac{1}{0.2+j0.4} = 1-2j$$

$$y_{22} = \frac{1}{Z_{22}} = 0$$

$$y_{23} = \frac{1}{Z_{23}} = \frac{1}{0.05+j0.1} = 4-8j$$

$$\frac{y'_{21}}{2} = \frac{0.8j}{2} = 0.4j$$

$$\frac{y'_{23}}{2} = \frac{0.2j}{2} = 0.1j$$

$$Y_{22} = 0 + (1-2j) + (4-8j) + 0.4j + 0.1j$$

$$Y_{22} = 5-9.5j$$

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$$Y_{33} = y_{31} + y_{32} + y_{33} + \frac{y_{31}'}{2} + \frac{y_{32}'}{2}$$

$y_{33} = 0$

$$y_{31} = \frac{1}{0.1+j0.2} + \frac{1}{0.05+j0.1} + (0) + 0.02j + 0.1j$$

$y_{33} = 6 - j11.4$.

$Y_{12} = Y_{21} = -y_{12} = -1+2j$

$Y_{13} = Y_{31} = -y_{13} = -2+4j$

$Y_{23} = Y_{32} = -y_{23} = -4+8j$

$$Y_{Bus} = \begin{bmatrix} 3-5.4j & -1+2j & -2+4j \\ -1+2j & 5-9.5j & -4+8j \\ -2+4j & -4+8j & 6-11.4j \end{bmatrix}_{3\times3}$$

Formulate the 4-bus matrix

Bus Code.	line impedance in per unit	line charge admittance ($Y_{pq}/2$)
1-2	$z_{12} = j0.2$	$y_{12}' = j0.8$
1-3	$z_{13} = j0.1$	$y_{13}' = j0.4$
2-3	$z_{23} = j0.5$	$z_{23}' = j1$

Sol: Calculate

$$Y_{11} = y_{11} + y_{12} + y_{13} + \frac{y_{12}'}{2} + \frac{y_{13}'}{2}$$

$y_{11} = 0$

$$y_{12} = \frac{1}{z_{12}} = \frac{1}{j0.2} = -5j$$

$$y_{13} = \frac{1}{z_{13}} = \frac{1}{j0.1} = -10j$$

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$\frac{y_{12}'}{2} = j0.8$, $\frac{y_{13}'}{2} = j0.4$

$y_{11} = (-5j) + (-10j) + (j0.8) + (j0.4)$

$y_{11} = -13.8j$

Calculate y_{22}.

$y_{22} = y_{21} + y_{22} + y_{23} + \frac{y_{21}'}{2} + \frac{y_{23}'}{2}$

$= \frac{1}{0.2j} + 0 + \frac{1}{j0.5} + j0.8 + j1$

$= -5.2j$

Calculate y_{33}

$y_{33} = y_{31} + y_{32} + y_{33} + \frac{y_{31}'}{2} + \frac{y_{32}'}{2}$

$= \frac{1}{0.1j} + \frac{1}{0.5j} + 0 + j0.4 + j1$

$= -10.6j$

$y_{12} = -y_{12} = 5j$

$y_{13} = -y_{13} = 10j$

$y_{23} = -y_{23} = 2j$

$y_{31} = 5j$

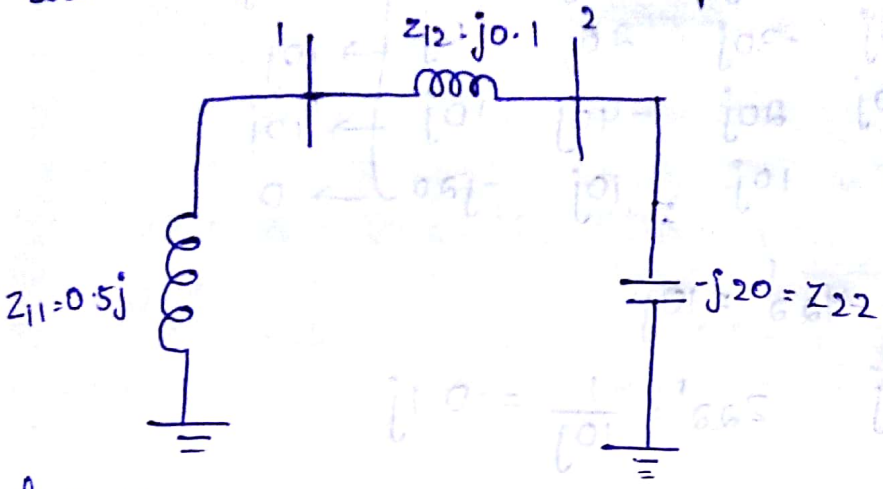
$y_{32} = 2j$

$$Y_{BUS} = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$$

$$Y_{BUS} = \begin{bmatrix} -13.8j & 5j & 10j \\ 5j & -5.2j & 2j \\ 5j & 2j & -10.6j \end{bmatrix}$$

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Construct the Y-Bus of the System shown in below all the Specified Values are per unit



Sol: $Y_{11} = y_{11} + y_{12}$

$$= \frac{1}{0.5j} + \frac{1}{0.1j} = -12j$$

$$Y_{12} = -y_{12} = -\frac{1}{z_{12}} = -\frac{1}{0.1j} = 10j$$

$$Y_{21} = Y_{12}$$

$$Y_{22} = y_{21} + y_{22}$$

$$= \frac{1}{0.1j} + \frac{1}{-20j}$$

$$Y_{22} = -9.95j$$

$$Y_{Bus} = \begin{bmatrix} -12j & 10j \\ 10j & -9.95j \end{bmatrix}_{2\times 2}$$

For the given Y-Bus matrix draw the reactance diagram and indicate the per Unit Values

$$Y_{Bus} = \begin{bmatrix} -40j & 10j & 20j & 0 \\ 10j & -30j & j20 & j10 \\ 20j & 20j & -40j & 10j \\ 0 & j10 & 10j & -j20 \end{bmatrix}_{4\times 4}$$

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$$Y_{Bus} = \begin{bmatrix} -40j & 10j & 20j & 0 \\ 10j & -30j & 20j & 10j \\ 20j & 20j & -40j & 10j \\ 0 & 10j & 10j & -j20 \end{bmatrix} \begin{matrix} \rightarrow -10j \\ \rightarrow 10j \\ \rightarrow 10j \\ \rightarrow 0 \end{matrix}$$

$Y_{11}' = -10j$ $\quad Y_{22}' = 10j$

$Z_{11}' = \frac{1}{-10j} = 0.1j$ $\quad Z_{22}' = \frac{1}{10j} = -0.1j$

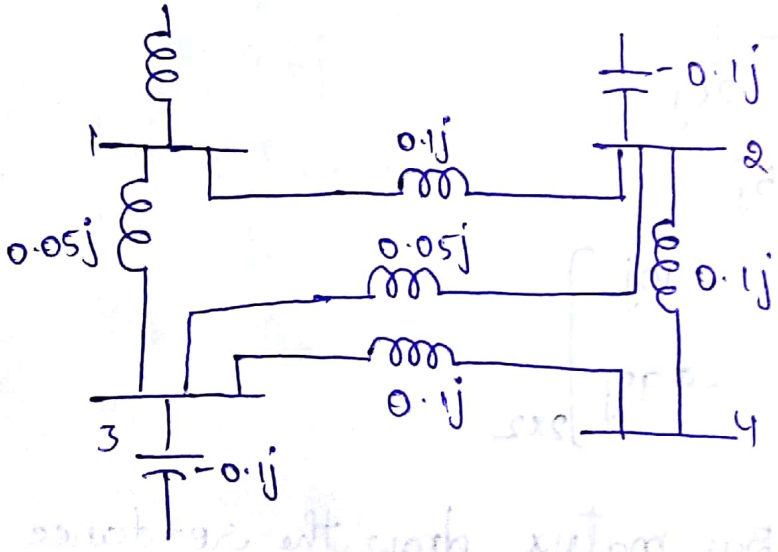
$Y_{33}' = 10j$ $\quad Y_{44}' = 0$

$Z_{33}' = \frac{1}{10j} = -0.1j$ $\quad Z_{44}' = 0$

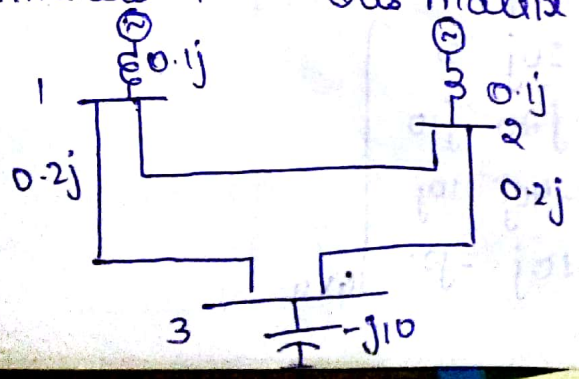
$Y_{12} = 10j$ $\quad Y_{13} = -y_{13} = 20j$ $\quad Y_{23} = -y_{23} = 20j$

$-y_{12} = 10j$ $\quad Z_{13} = \frac{-1}{20j}$ $\quad Z_{23} = 0.05j$

$Z_{12} = 0.1j$



Formulate the 4-Bus matrix to the network shown below



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Sol: $Y_{11} = y_{11} + y_{12} + y_{13} + \frac{y'_{12}}{2} + \frac{y'_{13}}{2}$

$= \frac{1}{0.1j} + \frac{1}{0.2j} + \frac{1}{0.2j} + 0 + 0$

$Y_{11} = -20j$

$Y_{22} = y_{21} + y_{22} + y_{23} + \frac{y'_{12}}{2} + \frac{y'_{23}}{2}$

$= \frac{1}{0.2j} + \frac{1}{0.1j} + \frac{1}{0.2j} + 0 + 0$

$Y_{22} = -20j$

$Y_{33} = y_{31} + y_{32} + y_{33} + \frac{y'_{31}}{2} + \frac{y'_{32}}{2}$

$= \frac{1}{j0.2} + \frac{1}{j0.2} + \frac{1}{-j10}$

$Y_{33} = -9.9j$

$Y_{12} = -y_{12} = \frac{1}{0.2j} = -5j$

$Y_{21} = Y_{12} = 5j$

$Y_{13} = -y_{13} = \frac{1}{Z_{13}} = \frac{1}{0.2j} = 5j$

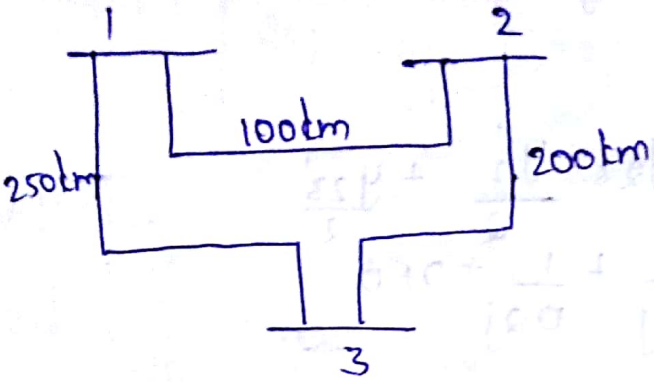
$Y_{31} = Y_{13} = 5j$

$Y_{23} = -y_{23} = \frac{1}{Z_{23}} = \frac{1}{0.2j} = 5j$

$$Y_{Bus} = \begin{bmatrix} -20j & 5j & 5j \\ 5j & -20j & 5j \\ 5j & 5j & 9.9j \end{bmatrix}$$

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The transmission line having an impedance of j0.001 per Unit/km Now formulate the 4-Bus matrix for the network shown in below



Given Impedance per Unit = 0.001j for 1km.

$$Z_{12} = 100 \times 0.001j = 0.1j$$

$$Z_{13} = 250 \times 0.001j = 0.25j$$

$$Z_{23} = 200 \times 0.001j = 0.2j$$

$$Y_{11} = y_{11} + y_{12} + y_{13}$$

$$= 0 + \frac{1}{0.1j} + \frac{1}{0.25j}$$

$$\boxed{Y_{11} = -14j}$$

$$Y_{12} = -\frac{1}{Z_{12}} = -\frac{1}{0.1j} = 10j$$

$$Y_{13} = -\frac{1}{0.25j} = 4j$$

$$Y_{23} = -\frac{1}{0.2j} = 5j$$

$$Y_{22} = y_{21} + y_{22} + y_{23}$$

$$= \frac{1}{0.1j} + 0 + \frac{1}{0.2j}$$

$$Y_{22} = -15j$$

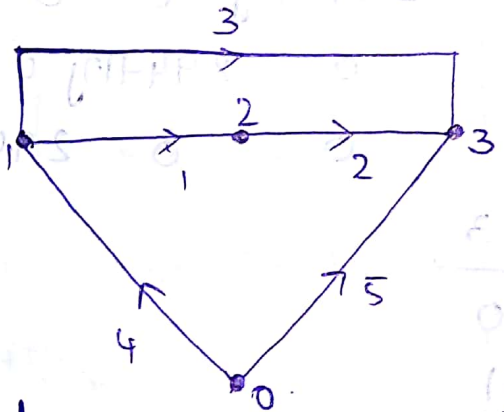
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$Y_{33} = -9j$

$$Y_{BUS} = \begin{bmatrix} -14j & 10j & 4j \\ 10j & -15j & 5j \\ 4j & 5j & -9j \end{bmatrix}$$

$$Y_{BUS} = \begin{bmatrix} -14j & 10j & 4j \\ 10j & -15j & 5j \\ 4j & 5j & -9j \end{bmatrix}$$

Consider the linear graph shown in below which represents a four bus Transmission System with all the shunt admittance lumped together Each line as a Series impedance of $0.02+j0.08$ and half line charging admittance of $j0.02$. Compute the Y_{BUS} by Using Singular transformation & Direct inspection method



Singular Transformation method

We know that by Using Singular transformation method the y-bus matrix can be obtained as follows

$$Y_{BUS} = A^T y A$$

Calculation of bus incidence matrix

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	0	1	2	3
1	0	+1	-1	0
2	0	0	+1	-1
3	0	+1	0	-1
4	+1	-1	0	0
5	+1	0	0	-1

Primitive admittance matrix Y.

$$y = \frac{1}{z} = \frac{1}{0.02 + j0.083}$$

$$y = 2.94 - 11.7j$$

$$Y = \begin{bmatrix} 2.94-11.7j & 0 & 0 & 0 & 0 \\ 0 & 2.94-11.7j & 0 & 0 & 0 \\ 0 & 0 & 2.94-11.7j & 0 & 0 \\ 0 & 0 & 0 & 2.94-11.7j & 0 \\ 0 & 0 & 0 & 0 & 2.94-11.7j \end{bmatrix}$$

$$A = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & -1 & 0 \\ 2 & 0 & 0 & 1 & -1 \\ 3 & 0 & 1 & 0 & -1 \\ 4 & 1 & -1 & 0 & 0 \\ 5 & 1 & 0 & 0 & -1 \end{array}$$

$$Y_{BUS} = A^T Y A$$

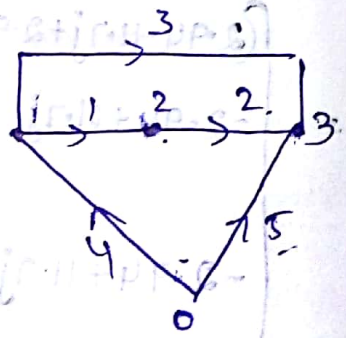
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$$\begin{bmatrix} 5.88-22.76j & -2.94+j11.75 & 0 & -2.94j+11.7j \\ -2.94+j11.75 & 8.82-j35.1 & -2.94+j11.75 & -2.94+j11.75 \\ 0 & -2.94+j11.75 & 5.88-22.76j & -2.94+j11.75 \\ -2.94+11.7j & -2.94+11.7j & -2.94+11.75j & 8.82-j35 \end{bmatrix}$$

$$\therefore Y_{BUS} = \begin{bmatrix} 5.88-22.76j & -2.94+j11.75 & 0 & -2.94+11.75j \\ -2.94+j11.75 & 8.82-j35.1 & -2.94+j11.75 & -2.94+j11.75 \\ 0 & -2.94+j11.75 & 5.88-22.76j & -2.94+j11.75 \\ -2.94+j11.75 & -2.94+11.7j & -2.94+11.75j & 8.82-j35 \end{bmatrix}$$

$Y_{11} = y_{11} + y_{12} + y_{13} + \frac{y'_{12}}{2} + \frac{y'_{13}}{2} + \frac{y'_{14}}{2}$

$$\begin{bmatrix} y_{00} & y_{01} & y_{02} & y_{03} \\ y_{10} & y_{11} & y_{12} & y_{13} \\ y_{20} & y_{21} & y_{22} & y_{23} \\ y_{30} & y_{31} & y_{32} & y_{33} \end{bmatrix}$$



$$y_{00} = y_{00} + y_{01} + y_{02} + y_{03} + \frac{y'_{01}}{2} + \frac{y'_{02}}{2} + \frac{y'_{03}}{2}$$

$y_{00} = y_{01} + y_{02} + y_{03}$

$2.94 + j11.4 + 0 + 2.94 - j11.7 + 0.02j + 0$

$0.02j = 5.88 - 22.76j$

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$y_{01} = -y_{01} = -2.94 + 11.7j$

$y_{02} = -y_{02} = 0$

$y_{03} = -y_{03} = -2.94 + 11.7j$

$y_{10} = y_{10} + y_{11} + y_{12} + y_{13} + \frac{y'_{01}}{2} + \frac{y'_{12}}{2} + \frac{y'_{13}}{2}$

$= 2.94 - 11.7j + 0 + 2.94 + 11.7j +$

$y_{11} = y_{11} + y_{10} + y_{12} + y_{13} + \frac{y'_{10}}{2} + \frac{y'_{12}}{2} + \frac{y'_{13}}{2}$

$y_{11} = 0 + 2.94 - 11.4j + 2.94 - 11.7j + 2.94 - 11.7j + 0.02j +$

$0.02j + 0.02j$

$y_{11} = 8.82 - j35.1$

$y_{10} = -y_{10} = y_{01} = -2.94 + 11.7j$

$y_{12} = -y_{12} = -(-2.94 + 11.7j) = -2.94 -$

$y_{13} = -y_{13} = -2.94 + 11.7j$

$y_{22} = y_{21} + y_{22} + y_{23} + y_{20} + \frac{y'_{20}}{2} + \frac{y'_{29}}{2} + \frac{y'_{21}}{2}$

$y_{22} = 2.94 - 11.7j + 0 + 2.94 - 11.7j + 0 + 0 + 0.02j + 0.02j$

$y_{22} = 5.88 - 22.76j$

$y_{33} = y_{31} + y_{30} + y_{32} + y_{33} + \frac{y'_{30}}{2} + \frac{y'_{31}}{2} + \frac{y'_{32}}{2}$

$y_{33} = 2.94 + 11.7j + 2.94 - 11.7j + 2.94 - 11.7j + 0 + 0.02j + 0.02j$

$= 8.82 - 35.04j \qquad + 0.02j$

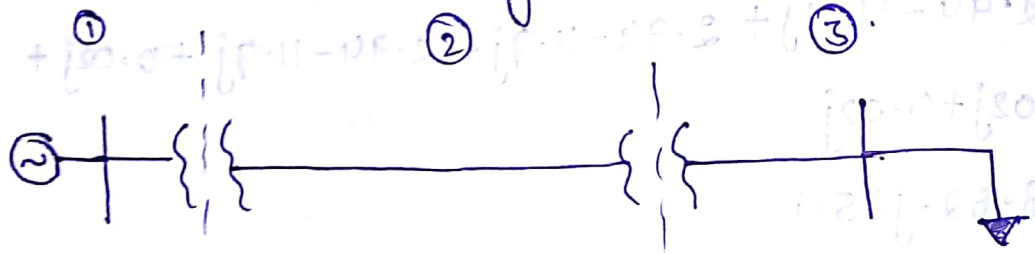
$y_{30} = -y_{30} = -2.94 + 11.7j; \quad y_{31} = -y_{13} = -2.94 + 11.7j$

$y_{32} = -y_{32} = -2.94 + 11.7j$

$$Y_{BUS} = \begin{bmatrix} 5.88 - 22.76j & -2.94 + 11.7j & 0 & -2.94 + 11.7j \\ -2.94 + 11.7j & 8.82 - j35.1 & -2.94 + 11.7j & -2.94 + 11.7j \\ 0 & -2.94 + 11.7j & 5.88 - 22.76j & -2.94 + 11.7j \\ -2.94 + 11.7j & -2.94 + 11.7j & -2.94 + 11.7j & 8.82 - 35.04j \end{bmatrix}$$

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3 Zones of 1-ϕ Circuit are identified in the following figure The Zones are connected by transformers T_1 and T_2 whose ratings are also shown Using base Values of 30KVA and 240V in zone 1 Draw the per unit Circuit and determine the per Unit impedance and per Unit Source Voltage. calculate the load Current both in per Unit and Amperes. Assume the transformer Winding resistance and shunt admittance are neglected.



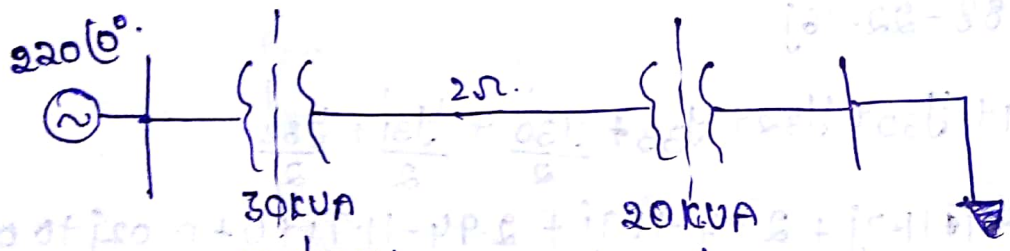
For Generator $V = 220\angle 0^\circ V$

For T_1 30KVA, 240/480V $x = 0.1PU$.

For Transmission line 2Ω.

For T_2 20KVA 460/115V $x = 0.1PU$.

$Z_{load} = 0.9 + j0.2\Omega$.



30KVA	30KVA	30KVA	30KVA	30KVA
240V.	240/480V	480V	480/120V	120V.
	X=0.1PU.			

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For Transformer 1

$x_{pu} = 0.1 \text{ p.u}$

For transmission line.

$$x_{p.u} = x_{act} \times \frac{VA}{(V)^2}$$

$$= 2 \times \frac{30,000}{(480)^2} = 0.26 \text{ p.u.}$$

For transformer-2.

$$x_{p.u.N} = 0.1 \times \frac{30,000}{20,000} \times \left(\frac{11.5}{120}\right)^2 = 0.13 \text{ p.u}$$



For load

$$x_{p.u.N} = (0.9 + j0.2) \times \frac{30,000}{(120)^2}$$

$$= 1.8 + j0.419 \text{ p.u.}$$

For Generator

$$V_{p.u} = \frac{V_{act}}{V_{base}} = \frac{220}{240} = 0.91$$

For Transformer 1

$$V_{p.u} = \frac{240}{240} = 1$$

For transmission line.

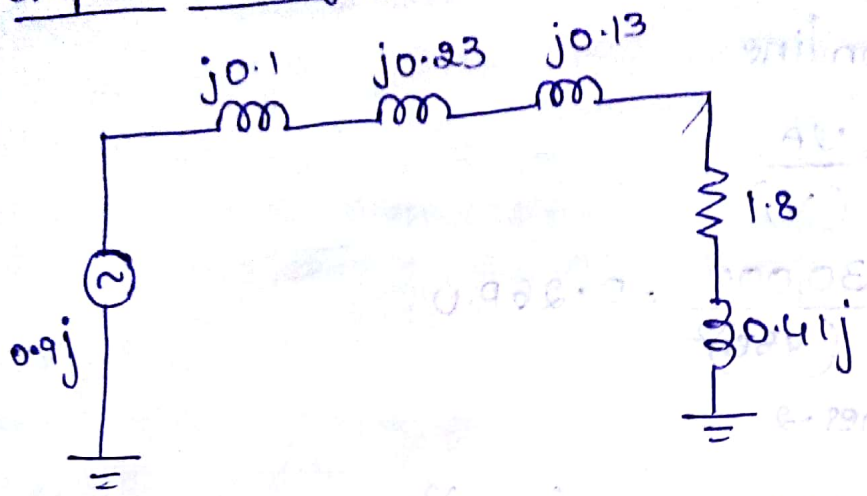
$$V_{p.u.} = \frac{480}{480} = 1$$

For Transformer 2

$$V_{p.u} = \frac{460}{480} = \frac{115}{120} = 0.75$$

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Impedance diagram.



$$I_{per\ unit} = \frac{V_{per\ unit}}{z_{per\ unit}} = \frac{0.91}{1.8 + j0.41}$$

$$= 049 \angle -26.31$$

$$I_{act} = I_{p.u} \times I_{base}$$

$$I_b = \frac{V_b}{Z_b} = \frac{240}{\frac{V_b}{S_b}} =$$

$$I_b = \frac{S_b}{V_b} = 30,000$$

$$I_b = \frac{V_b}{Z_b}$$

$$= \frac{240}{\frac{V_b}{(I_b)}} = \frac{V_b}{\frac{(V_b)^2}{S}}$$

$$= \frac{V_b}{V_b^2} \times S$$

$$= \frac{240}{(240)^2} \times 30,000$$

$$= 250$$

$$I_{act} = I_{p.u} \times I_{base}$$

$$= 0.49 \angle -26.31 \times 250$$

$$= 112.5 \angle -26.31 \text{ Amp}$$

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prove that the per Unit Calculations for two winding transformer is same in both HV and LV Side.

Consider a two winding transformer by neglecting the magnetising branch parameters

Assume Z_1 = Total impedance offered by the transform refers to the primary

Z_2 = Total impedance offered by the transform refers to Secondary.

we know that

$$Z_1 = Z_2 \left(\frac{N_1}{N_2}\right)^2$$

Similarly $Z_2' = Z_1 \left[\frac{N_2}{N_1}\right]^2$

N_1 = primary no of turns

N_2 = Secondary no of turns

$$Z_1 P.U = \frac{Z_1 \Omega}{Z_1 b}$$

$$= Z_1 \Omega \times \frac{I_1 b}{V_1 b}$$

$$Z_1 P.U = Z_2 \left(\frac{N_1}{N_2}\right)^2 \times \frac{I_1 b}{V_1 b}$$

$$= Z_2 \left(\frac{V_1 b}{V_2 b}\right)^2 \times \frac{I_1 b}{V_1 b}$$

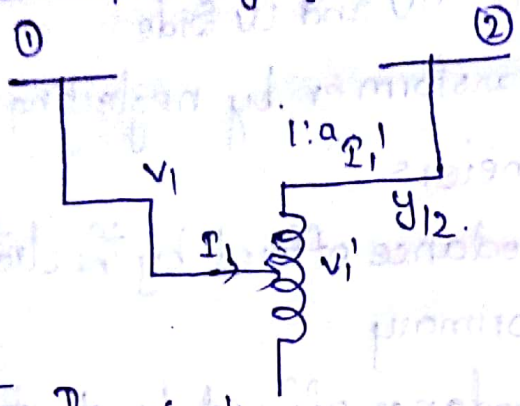
$$Z_1 P.U = Z_2 \times \frac{V_1 b \times I_1 b}{V_2^2 b} \quad (V_1 I_1 = V_2 I_2)$$

$$Z_1 P.U = Z_2 \times \frac{V_2 b \times I_2 b}{V_2^2 b}$$

$$Z_1 P.U = \frac{Z_2}{\left(\frac{V_2 b}{I_2 b}\right)} \quad \therefore Z_1 PU = Z_2 P.U$$

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Effect of tap changing transformer on Y-bus



$$a = \frac{V_1'}{V_1}$$

$$a = \frac{I_1}{I_1'}$$

Case-1

$$I_1 = (V_1' - V_2)\, y_{12}$$

$$I_1 = aI_1' = a[V_1' - V_2]\, y_{12}$$

$$= a[aV_1 - V_2]\, y_{12}$$

$$\boxed{I_1 = a^2 V_1 y_{12} - a y_{12} V_2}$$

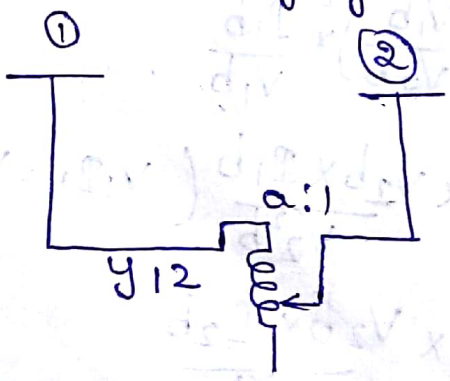
$$I_2 = (V_2 - V_1')\, y_{12}$$

$$\boxed{I_2 = V_2 y_{12} - a V_1 y_{12}}$$

$$Y_{BUS} = \begin{bmatrix} a^2 y_{12} & -a y_{12} \\ -a y_{12} & y_{12} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

(Y_{11}, Y_{12}, Y_{21}, Y_{22})

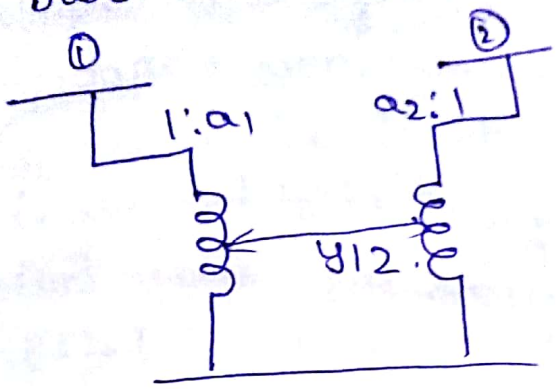
Case-2 The tap changing transformer is at bus 2



$$Y_{BUS} = \begin{bmatrix} y_{12} & -a y_{12} \\ -a y_{12} & a^2 y_{12} \end{bmatrix}$$

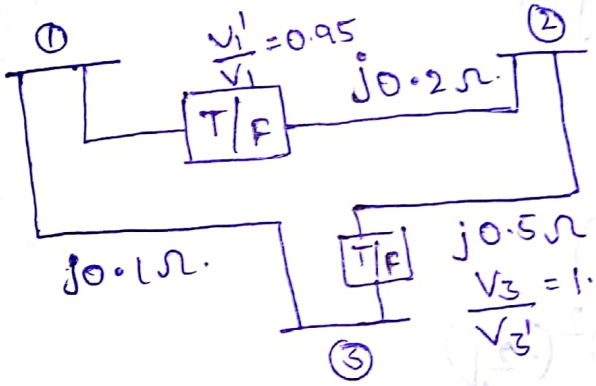
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Case-3 The tap changing transformer is at both the buses



$$Y_{Bus} = \begin{bmatrix} a_1^2 y_{12} & -a_1 a_2 y_{12} \\ -a_1 a_2 y_{12} & a_2^2 y_{12} \end{bmatrix}$$

Construct the Y-bus for the following network.



$$Y_{11} = y_{11} + y_{12} + y_{13}$$

$$Y_{11} = y_{12} + y_{13}$$

$$= a^2 y_{12} + y_{13}$$

$$= (0.95)^2(-j5) + (-j10) = -4.51j - 10j = -14.5j$$

$$Y_{12} = -a y_{12}$$

$$Y_{12} = (-0.95)(-5j)$$

$$Y_{12} = 4.75j$$

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$Y_{13} = -y_{13} = j10$

$Y_{21} = Y_{12} = 4.75j$

$Y_{22} = y_{21} + y_{23}$

$= -j5 - j2$

$= -j7$

$Y_{23} = -a y_{23}$

$Y_{23} = -0.95 \times (-j2)$

$Y_{23} = 1.9j$

$Y_{31} = -y_{31} = Y_{13} = j10$

$Y_{32} = Y_{23} = 1.9j$

$Y_{33} = y_{31} + a^2(y_{32})$

$= -j10 + (0.95)^2(-2j)$

$Y_{33} = -1.8j - 10j = -11.8j$

$Y_{33} = -11.8j$

$$Y_{BUS} = \begin{bmatrix} -14.5j & 4.75j & j10 \\ 4.75j & -7j & 1.9j \\ j10 & 1.9j & -11.8j \end{bmatrix}$$

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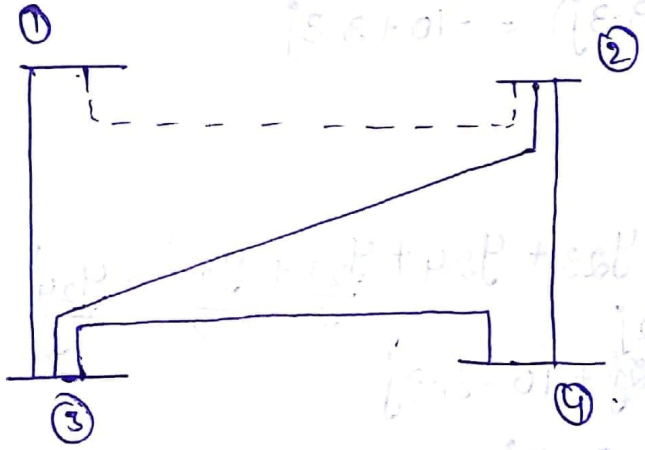
The diagrams shown in below is a one line diagram of a simple 4 bus system and the table gives the line impedances identified by the buses on which this terminales. The shunt admitance at all the buses is assumed to be negligible.

a) Find ~~4×4 bus~~ Y_{BUS} assuming that the line shown dotted is not connected.

b) what modifications need to be carried out in 4 bus if the line shown dotted is connected.

c) Modify Y_{BUS} of the system by include a regulating transformer in the line 3 to 4. near bus 3

$\frac{V_3}{V_3'} = 1.04.$



Bus to Bus	Resistance in P.U	X P.U
1 - 2	0.05	0.15
1 - 3.	0.1	0.3.
2 - 3	0.15	0.45
2 - 4	0.1	0.3
3 - 4.	0.05	0.15

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Bus to Bus	$\frac{1}{R} = G$	$\frac{1}{X} = B$
1 - 2	+20	$-6.66j$
1 - 3	+10	$-3.3j$
2 - 3	+6.6	$-2.2j$
2 - 4	+10	$-3.3j$
3 - 4	+20	$-6.6j$

$Y_{11} = y_{11} + y_{12} + y_{13} + y_{14} + \frac{y_{12}'}{2} + \frac{y_{13}'}{2} + \frac{y_{14}'}{2}$. (y_{12}, $\frac{y_{12}'}{2}$, $\frac{y_{13}'}{2}$, $\frac{y_{14}'}{2}$ is neglected)

$Y_{11} = 20 - 6.6 \quad 10 - 3.3j$

$Y_{12} = 0$

$Y_{13} = -y_{13} = (10 - 3.3j) = -10 + 3.3j$

$Y_{14} = 0$.

$Y_{22} = y_{22} + y_{21} + y_{23} + y_{24} + \frac{y_{21}'}{2} + \frac{y_{23}'}{2} + \frac{y_{24}'}{2}$

$6.6 - 2.2j$

$= 6.6 - 2.2j + 10 - 3.3j$

$= 16.6 - 5.5j$

$Y_{21} = Y_{12} = 0$.

$Y_{23} = -y_{23} = -6.6 + 2.2j$

$Y_{24} = -y_{24} = -10 + 3.3j$

$Y_{33} = y_{31} + y_{32} + y_{33} + y_{34} + \frac{y_{31}'}{2} + \frac{y_{32}'}{2} + \frac{y_{34}'}{2}$

$= 10 - 3.3j + 6.6 - 2.2j + 20 - 6.6j$

$Y_{33} = 36.6 - 52j$

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$y_{31} = -y_{31} = -10+3.3j$

$y_{32} = -y_{32} = -6.6+2.2j$

$y_{34} = -y_{34} = -20+6.6j$

$$Y_{BUS} = \begin{bmatrix} 10-3.3j & 0 & -10+3.3j & 0 \\ 0 & 16.6-5.5j & -6.6+2.2j & -10+3.3j \\ -10+3.3j & -6.6+2.2j & 36.6-5.52j & -20+6.6j \end{bmatrix}$$

$y_{44} = y_{41} + y_{42} + y_{43} + y_{44}$

$y_{44} = 0 + 10 - 3.3j + 20 - 6.6j + 0$

$y_{44} = 30 - 9.9j$

$y_{41} = 0$; $y_{42} = y_{24} = -10+3.3j$

$y_{43} = y_{34} = -20+6.6j$

$$Y_{BUS} = \begin{bmatrix} 10-3.3j & 0 & -10+3.3j & 0 \\ 0 & 16.6-5.5j & -6.6+2.2j & -10+3.3j \\ -10+3.3j & -6.6+2.2j & 36.6-12j & -20+6.6j \\ 0 & -10+3.3j & -20+6.6j & 30-9.9j \end{bmatrix}$$

b)

$y_{11} = y_{12} + y_{13}$

$= 20-6.66j + 10-3.3j$

$y_{11} = 30-9.99j$

$y_{22} = y_{21} + y_{23} + y_{24}$

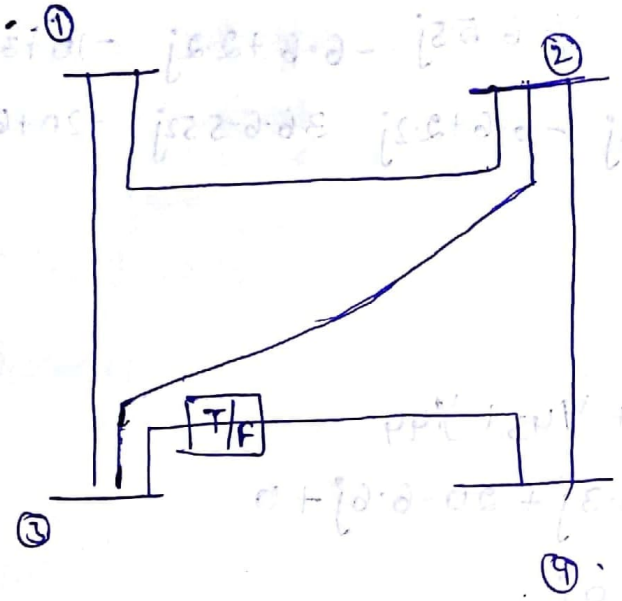
$= 20-6.6j + 6.6-2.2j+10-3.3j = 36.6-12.1j$

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$Y_{12} = -y_{12} = -20+6.6j$: $Y_{BUS} = \begin{bmatrix} 30-9.9j & -20+6.6j & -10+3.3j & 0 \\ -20+6.6j & 36.6-12.1j & -6.6+2.2j & -10+3.3j \\ -10+3.3j & -6.6+2.2j & 36.6-12.1j & -20+6.6j \\ 0 & -10+3.3j & -20+6.6j & 30-9.9j \end{bmatrix}$

$Y_{12} = Y_{21}$

c) when transformer is connected to the line 3-4.



$Y_{33} = y_{31} + y_{32} + y_{33} + y_{34}$

$Y_{33} = Y_{33}(old) - y_{34} + a^2 y_{34}$ (or)

$Y_{33} = a^2 y_{34} + y_{31} + y_{32}$

$Y_{33} = (0.96)^2(20-6.6j) + 10-3.3j + 6.6-2.2j$

$Y_{33} = 18.43 - 6.08j + 10 - 3.3j + 6.6 - 2.2j$

$= 34.6 - 11.58j$

$Y_{34} = -a y_{34}$

$= -(0.96)(20-6.6j)$

$= -19.2 + 6.33j$

$Y_{34} = Y_{43} = -19.2 + 6.33j$

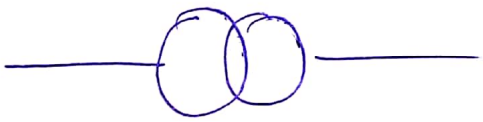
$Y_{44} = 30 - 9.9j$

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$$Y_{BUS} = \begin{bmatrix} 30-9.9j & -20+6.6j & -10+3.3j & 0 \\ -20+6.6j & 36.6-12.1j & -6.6+2.2j & -10+3.3j \\ -10+3.3j & -6.6+2.2j & 34.6-12.5j & -19.2+6.3j \\ 0 & -10+3.3j & -19.2+6.3j & 30-9.9j \end{bmatrix}$$

A Single phase two winding transformer is rated 20KVA, 480/120V, 60Hz. The equivalent leakage impedance of transformer refer to the 120V Winding denoted Winding2 is $Z_{eq2} = 0.0525\angle 78.13\,\Omega$ Using the transformer rating as base values determine the per Unit leakage impedance refers to Winding 2 & refered to Winding 1

Sol: Given.



20KVA

480/120V.

$$Z_{eq2} = 0.0525\angle 78.13^\circ\,\Omega.$$

$$Z_{p.u.2} = Z_\Omega \times \frac{V_A}{(V)^2}$$

$$Z_{p.u2} = 0.0525\angle 78.13 \times \frac{20000}{(120)^2}.$$

$$Z_{p.u.2} = 0.01 + 0.071j \Omega.$$

$$Z_{eq} = Z_{eq2} \times \left(\frac{N_1}{N_2}\right)^2$$

$$Z_{eq1} = Z_{eq2} \times \left(\frac{V_1}{V_2}\right)^2$$

$$= 0.0525\angle 78.13 \times \left(\frac{480}{120}\right)^2$$

$$Z_{eq1} = 0.84\angle 78.13$$

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$$Z_{p.u.1} = 0.84\left(78.13 \times \left(\frac{20,000}{(480)^2}\right)\right)$$

$$= 0.014 + 0.07j$$

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