4. FIR Filters

A filter is freq. selective sky. Digital filters are classified as finite duration unit impulse response (FIR) filters & infinite duration unit impulse response (IIR) filters. depending on the form of unit impulse response of the sky. So in the FIR sky the impulse response sequence is of finite duration. The IIR sky has an infinite no. of non-zero terms. IIR filters are usually implemented using recursive structures (feedback poles & zeros) & FIR filters are usually implemented using non-recursive structures (no F.B only zeros).

The response of the FIR filter depends only on the present and past i/p samples. Whereas for the IIR filter the present response is a function of the present and past values of the excitation as well as past values of the response.

Advantages of FIR filters over IIR filters:-

→ FIR filters are always stable.

→ FIR filters with exactly linear phase can easily be design.

→ FIR filters can be realized in both recursive and non-recursive structures.

→ FIR filters are free of limit cycle oscillations when implemented on a finit wordlength digital sky.

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→ 3. excellent design methods are available for various kinds of FIR filter.

Disadvantages of FIR filter are as follow:-

→ The implementation of narrow transition band FIR filter is very costly as it requires more arithmetic operation & hardware components such as multiplier, adders, delay element.

→ Memory requirment & execution time are very high.

Comparision of IIR & FIR filter :-

IIR filter	FIR filter
→ All the infinite sample of impulse response are consider.	→ Only finite no. of sample of impulse response are considered.
→ The impulse response can't be directly converted to digital filter T.F.	→ The impulse response can be converted to digital filter T.F.
→ Linear phase characteristics can't be achieve.	→ Linear phase filters can be easily designed.
→ IIR filters are easily realized recursively.	→ FIR filters can be realized recursively & non-recursively.
→ The specifications include the desired characteristic for magnitude response only.	→ The specification include the derived characteristic for both magnetude & phase response.
→ The design involves design of analog filter & then transforming analog filter to digital filter.	→ The digital filter can be directly design to achieve the desired specifications.

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The round of noise in IIR → Errors due to round o
filters is more. noise are less serious in FIR
filters mainly becau FIR
not used.

Characteristics of FIR filters with linear phase

The T.F a FIR Causal filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

Where $h(n)$ is the impulse response of the filter.

Now the freq response [F.T of $h(n)$] is given by

$$H(\omega) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n} = H(e^{j\omega})$$

Since $H(\omega)$ is Complex it can be expressed as

$$H(\omega) = |H(\omega)| \angle H(\omega)$$

Where $|H(\omega)|$ is the magnitude response and

$$H(\omega) = |H(\omega)| e^{j\theta(\omega)}$$

Here $\theta(\omega)$ is phase response

We define the phase delay τ_p & group delay τ_g of a filter as

$$\tau_p = \frac{-\theta(\omega)}{\omega}, \quad \tau_g = \frac{-d\theta(\omega)}{d\omega}$$

For FIR filters with linear phase we can define

$$\theta(\omega) = -\alpha\omega; \quad -\pi \le \omega \le \pi$$

$$\tau_p = \frac{+\alpha\omega}{\omega} = +\alpha, \quad \tau_g = \frac{-d(-\alpha\omega)}{d\omega} = \alpha$$

Here $\tau_p = \tau_g = \alpha$ which means that α is independ of frequency

$$H(\omega) = \pm |H(\omega)| e^{j\theta(\omega)}$$

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$$\sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \pm |H(\omega)| e^{-j\alpha\omega}$$

$$\sum_{n=0}^{N-1} h(n)[\cos\omega n - j\sin\omega n] = \pm |H(\omega)| [\cos\alpha\omega - j\sin\alpha\omega] \quad \text{---(1)}$$

On equating the real part & imaginary part of the above eq'n we get

$$\sum_{n=0}^{N-1} h(n)\cos\omega n = \pm |H(\omega)| \cos\alpha\omega \quad \text{---(2)}$$

$$\sum_{n=0}^{N-1} h(n)\sin\omega n = \pm |H(\omega)| \sin\alpha\omega. \quad \text{---(3)}$$

On dividing eq'n (3) by (2)

$$\frac{\sum_{n=0}^{N-1} h(n)\sin\omega n}{\sum_{n=0}^{N-1} h(n)\cos\omega n} = \frac{\pm |H(\omega)| \sin\alpha\omega}{\pm |H(\omega)| \cos\alpha\omega}$$

$$\tan\omega n = \tan\alpha\omega$$

$$\sum_{n=0}^{N-1} h(n)\sin\omega n \cos\alpha\omega = \sum_{n=0}^{N-1} h(n)\sin\alpha\omega \cos\omega n.$$

$$\sum_{n=0}^{N-1} h(n)\cos\omega n \sin\alpha\omega - \sum_{n=0}^{N-1} h(n)\sin\omega n\cos\alpha\omega = 0.$$

$$\sum_{n=0}^{N-1} h(n)[\sin\alpha\omega\cos\omega n - \cos\alpha\omega\sin\omega n] = 0.$$

$$\sum_{n=0}^{N-1} h(n)\sin(\alpha\omega - \omega n) = 0.$$

$$\boxed{\sum_{n=0}^{N-1} h(n)\sin(\alpha - n)\omega = 0} \quad \text{---(4)}$$

One solution of eq'n (4) exists when $\alpha = \frac{N-1}{2}$

and $h(n) = h(N-1-n)$ for $0 \le n \le N-1$

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$h(n) \sin(\alpha - n)\omega$.

$= h(n) \sin\left(\left(\frac{N-1}{2} - n\right)\omega\right)$

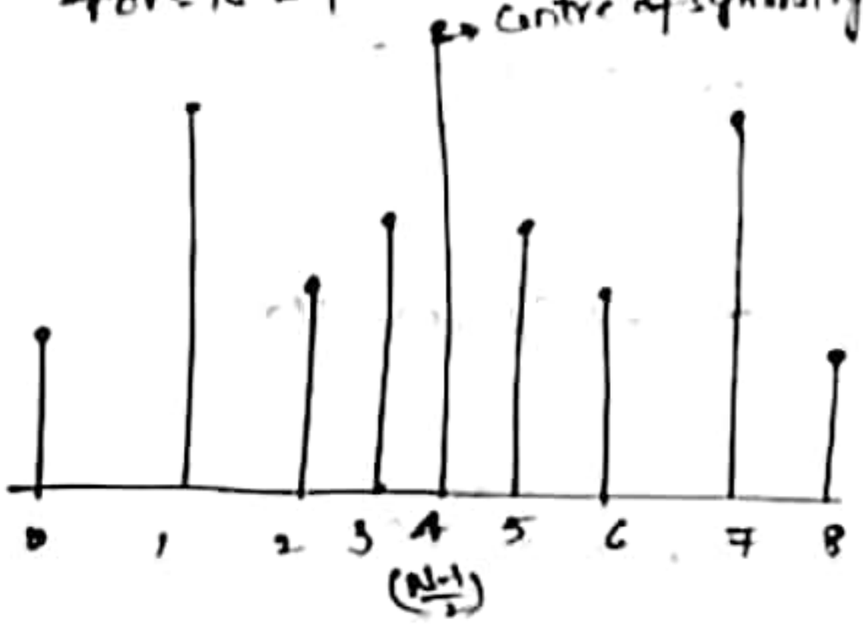
$= h(n) \sin\left[\frac{N-1-n-n}{2}\omega\right]$

$= h(n) \sin\left[\frac{n-n}{2}\omega\right]$

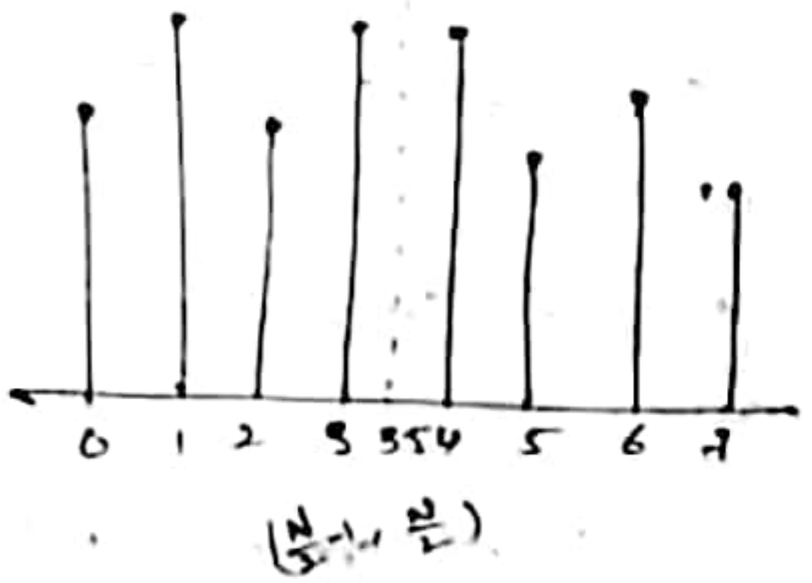
$= 0$.

This shows that FIR filters will have Constant phase and group delay when impulse response is symmetrical about $\alpha = \frac{N-1}{2}$. The impulse response satisfying the symmetric condition $h(n) = h(N-1-n)$ for odd, even values of N is shown in fig. when $N=9$ the center of the symmetry of sequence occurs at 4th sample & when $N=8$ the filter delay is 3 1/2 samples.

for $N = 9$ (odd)



for $N = 8$ even



Case ii:- If only Constant group delay is required & not the phase delay

$\theta(\omega) = \beta - \alpha\omega$

$\tau_p = -\frac{\theta(\omega)}{\omega} = \frac{-(\beta-\alpha\omega)}{\omega} = \alpha - \beta/\omega$; It is not a Constant

$\tau_g = -\frac{d\,\theta(\omega)}{d\omega} = \frac{-d(\beta-\alpha\omega)}{d\omega} = \alpha$; It is Constant

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$$H(\omega) = \pm |H(\omega)| e^{j\theta(\omega)}$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n) e^{-j\omega n} = \pm |H(\omega)| e^{j(\beta-\alpha\omega)}$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n) [\cos\omega n - j\sin\omega n] = \pm |H(\omega)| [\cos(\beta-\alpha\omega) + j\sin(\beta-\alpha\omega)]$$

equating real and imaginary parts

$$\sum_{n=0}^{N-1} h(n) \cos\omega n = \pm H(\omega) \cos(\beta-\alpha\omega) \longrightarrow (6)$$

$$-\sum_{n=0}^{N-1} h(n) \sin\omega n = \pm H(\omega) \sin(\beta-\alpha\omega) \longrightarrow (7)$$

Eq (7)/(6) $\Rightarrow$

$$\frac{-\sum_{n=0}^{N-1} h(n) \sin\omega n}{\sum_{n=0}^{N-1} h(n) \cos\omega n} = \frac{\pm H(\omega) \sin(\beta-\alpha\omega)}{\pm H(\omega) \cos(\beta-\alpha\omega)}$$

$$-\sum_{n=0}^{N-1} h(n) \sin\omega n [\cos(\beta-\alpha\omega)] = \sum_{n=0}^{N-1} h(n) \cos\omega n [\sin(\beta-\alpha\omega)]$$

$$\sum_{n=0}^{N-1} h(n) \cos\omega n \sin(\beta-\alpha\omega) + \sum_{n=0}^{N-1} h(n) \sin(\omega n) \cos(\beta-\alpha\omega) = 0$$

$$\sum_{n=0}^{N-1} h(n) [\cos(\omega n) \sin(\beta-\alpha\omega) + \sin(\omega n) \cos(\beta-\alpha\omega)] = 0 \qquad \because \sin(A+B)$$

$$\sum_{n=0}^{N-1} h(n) \sin(\omega n + \beta - \alpha\omega) = 0$$

$$\boxed{\sum_{n=0}^{N-1} h(n) \sin(\omega(n-\alpha)+\beta) = 0} \quad (\text{or}) \quad \sum_{n=0}^{N-1} h(n) \sin(\beta-(\alpha-n)\omega) = 0$$

If $\beta = \pi/2$

$$\sum_{n=0}^{N-1} h(n) \sin(\pi/2 - (\alpha-n)\omega) = 0 \Rightarrow \boxed{\sum_{n=0}^{N-1} h(n) \cos(\alpha-n)\omega = 0}$$

This eqn will be satisfy when $\alpha = \frac{N-1}{2}$ and $h(n) = \pm h(N-1-n)$

This shows that FIR filters have constant group delay $\tau_g = \alpha$ and

τ_p = not constant when the impulse response is anti symmetrical about

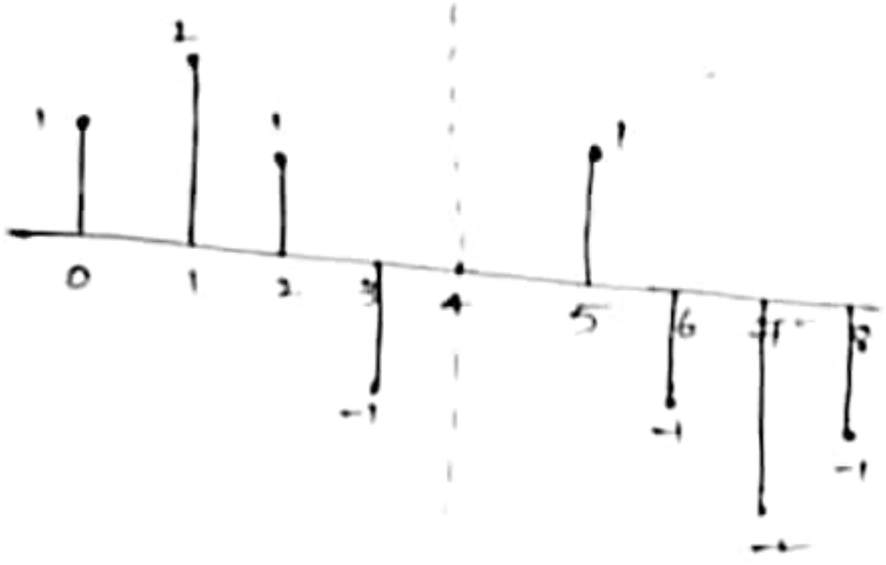
$\alpha = \frac{N-1}{2}$.

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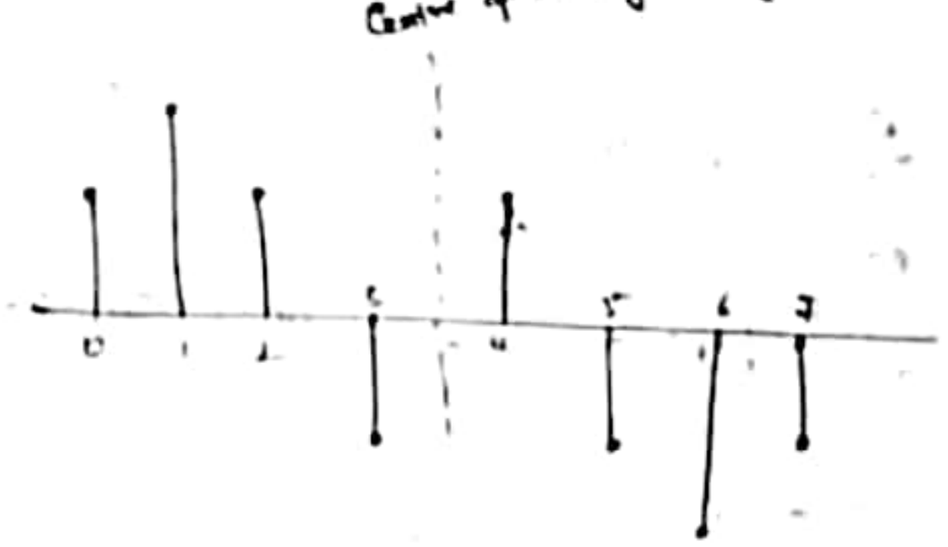
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Ex: The impulse response satisfying the anti symmetric condition is shown in the following figures when N=7 the centre of anti symmetry occures at 4th sample and when N=8 the centre of anti symmetry occures at 3½ sample.

For N=7



For N=8



Frequency response of linear phase FIR filter:-

→ The frequency response of the filter in the F.T of its impulse response.

→ If h(n) is the impulse response of the sly then the freq response of the sly is denoted by $H(e^{jw})(\omega)$ $+H(\omega)$

→ $H(\omega)$ is a Complex fu'n of freq ω and the Complex it can be expressed as magnitude fu'n $|H(j\omega)|$ & phase fu'n $\angle H(j\omega)$

→ Depending on the value of N[odd(a) even] and the type of symmetry response of the filter impulse response sequence (symmetry (Anti symmetry), there are following 4 types of impulse response for Linear phase FIR filter

→ freq response of linear phase FIR filter when impulse response is symmetrical and N is odd:-

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Let $h(n)$ be the impulse response of the sly. the freq. response of the sly $H(\omega)$ is given by

$$H(\omega) = H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{N} h(n) e^{-j\omega n}$$

when N odd $h(n)$ is symmetric centre of symmetry is at $\frac{N-1}{2}$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}$$

$m = N-1-n \quad ; \quad n = N-1-m$

$n = \frac{N+1}{2} \quad ; \quad m = N-1-\frac{N+1}{2} = \frac{N-3}{2}$

$n = N-1 \quad ; \quad m = N-1-(N-1) = 0$

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-m) e^{-j\omega(N-1-m)}$$

put $m = n$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n) e^{-j\omega(N-1-n)}$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} e^{j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega(N-1-n)} e^{j\omega\left(\frac{N-1}{2}\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} + e^{-j\omega\left(N-1-n-\left(\frac{N-1}{2}\right)\right)} \right] \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} + e^{-j\omega\left(\frac{N-1}{2}-n\right)} \right] \right]$$

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$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n)\left[2\cos\omega\left[\frac{N-1}{2} - n\right]\right]\right]$$

Let $k = \frac{N-1}{2} - n$; $n = \frac{N-1}{2} - k$.

$n = 0$; $k = \frac{N-1}{2}$; $n = \frac{N-3}{2}$, $k = \frac{N-1}{2} - \frac{N-3}{2} = \frac{N-1-N+3}{2} = 1$.

$$= e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{k=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2} - k\right]\cos\omega k\right]$$

put $k = n$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2} - n\right]\cos\omega n\right]$$

The magnitude fu'n of $H(\omega)$ is given by

$$|H(\omega)| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right)\cos\omega n$$

The phase fu'n of $H(\omega)$ is given by

$$\angle H(\omega) = -\omega\left(\frac{N-1}{2}\right) = -\omega\alpha.$$

freq. response of Linear phase FIR filter when impulse response is Symmetrical and N even/~~odd~~ :-

Proof :- freq. response of Linear phase FIR filter, with impulse response $h(n)$ of N Samples.

$$H(\omega) = H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n}.$$

for symmetric impulse response with even no. of Samp the centre of symmetry Lies b/n. $n = \frac{N-1}{2}$ & $n = \frac{N}{2}$

Hence $H(\omega)$ is expressed as

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h(n)e^{-j\omega n}.$$

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Let $m = N-1-n$; $n = N-1-m$

$H(\omega)$: $n = \frac{N}{2}$, $m = N-1-\frac{N}{2} = \frac{N-2}{2} = \frac{N}{2}-1$

$n = N-1$, $m = N-1-N+1 = 0$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} + \sum_{m=0}^{\frac{N}{2}-1} h(N-1-m) e^{-j\omega(N-1-m)}$$

put $m = n$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} h(N-1-n) e^{-j\omega(N-1-n)}$$

The impulse response is symmetric $h(N-1-n) = h(n)$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} e^{j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega(N-1-n)} e^{j\omega\left(\frac{N-1}{2}\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{+j\omega\left(\frac{N-1}{2}+n\right)} + e^{-j\omega\left[N-1-n-\left(\frac{N-1}{2}\right)\right]} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} + e^{-j\omega\left(\frac{N-1}{2}-n\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n)\, 2\cos\omega\left(\frac{N-1}{2}-n\right)$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n)\, 2\cos\omega\left(\frac{N}{2}-n-\frac{1}{2}\right)$$

Let $k = \frac{N}{2}-n$; $n = \frac{N}{2}-k$

$n = 0$; $k = \frac{N}{2}$.

$n = \frac{N}{2}-1$; $k = \frac{N}{2}-\frac{N}{2}+1 = 1$

Put $k = n$.

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=1}^{\frac{N}{2}} h\left(\frac{N}{2}-n\right) 2\cos\omega\left[n-\frac{1}{2}\right] \right]$$

$$|H(\omega)| = \sum_{n=0}^{\frac{N}{2}} 2h\left(\frac{N}{2}-n\right) \cos\omega\left[n-\frac{1}{2}\right]$$

$$\angle H(\omega) = -\omega\left(\frac{N-1}{2}\right) = -\omega\alpha \quad \text{where } \alpha = \frac{N-1}{2}.$$

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- frequency response of Linear phase FIR-filter when impulse response is - Antisymmetric and N is Odd :-

Proof:- freq. response of linear phase FIR-filter with impulse response h(n) of N-samples

$$H(\omega) = H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n}$$

antisymm.

- for a symmetric response with N odd samples, the centre of symmetry is at $\frac{N-1}{2}$.

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}.$$

If impulse response is anti symmetric $h\left(\frac{N-1}{2}\right) = 0$.

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}$$

Let $m = N-1-n$, $n = N-1-m$.

$n=0 \Rightarrow m = N-1$

$n = \frac{N+1}{2} \Rightarrow m = N-1-\frac{N+1}{2} = \frac{N-3}{2}$

$n = N-1 \Rightarrow m = 0$

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + \sum_{m=0}^{\frac{N-3}{2}} h(N-1-m) e^{-j\omega(N-1-m)}$$

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n) e^{-j\omega(N-1-n)}$$

The impulse response is anti symmetry. $h(N-1-n) = -h(n)$

$$H(\omega) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} - \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N-3}{2}} h(n) \left[e^{+j\omega\left[\frac{N-1}{2}-n\right]} - e^{-j\omega\left[N-1-n-\left(\frac{N-1}{2}\right)\right]} \right] \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N-1}{2}} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} - e^{-j\omega\left(\frac{N-1}{2}-n\right)} \right]$$

Let $k = \frac{N-1}{2} - n$; $n = \frac{N-1}{2} - k$.

$n=0 \Rightarrow k = \frac{N-1}{2}$

$n = \frac{N-3}{2}$, $k = \frac{N-1}{2} - \frac{N-3}{2} = 1$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{k=1}^{\frac{N-1}{2}} h\left(\frac{N-1}{2}-n\right) \left[2j \sin\omega\left(\frac{N-1}{2}-n\right) \right]$$

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→ . System properties.

→ Impulse response, step response.

→ DIT & FFT

→ Circular Convolution

→ Difference bet analog & digital filters,

→ direct form 1.

$$|H(j\omega)| = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) j \sin\omega n$$

$$j = e^{j\pi/2}$$

$$H(\omega) = e^{j\pi/2} e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \sin\omega n.$$

$$= e^{j\left[\frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right)\right]} \sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2} - n\right] \sin\omega n.$$

$$|H(\omega)| = \sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2} - n\right] \sin\omega n.$$

$$\angle H(\omega) = \frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right) = \beta - \omega\alpha$$

where $\beta = \frac{\pi}{2}$, $\alpha = \frac{N-1}{2}$

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Frequency response of Linear phase FIR filter when impulse response is Antisymmetric and N is even :-

Proof :- Freq. Rupone of Linear phase FIR-filter with impulse response $h(n)$ in N Sample.

$$H(\omega) = \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n}$$

Impulse response is Anti Symmetric and N-even the Centre of antisymmetry lies b/n $\frac{N}{2}-1$ & $\frac{N}{2}$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h(n)\, e^{-j\omega n}$$

Let $m = N-1-n$; $n = N-1-m$

$n = \frac{N}{2}$; $m = N-1-\frac{N}{2} = \frac{N}{2}-1$

$n = N-1$; $m = 0$.

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{-j\omega n} + \sum_{m=0}^{\frac{N}{2}-1} h(N-1-m)\, e^{-j\omega(N-1-m)}$$

put $m = n$.

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} h(N-1-n)\, e^{-j\omega(N-1-n)}$$

The impulse response is Anti Symmetric $h(N-1-n) = -h(n)$

$$H(\omega) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\, e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} -h(n)\, e^{-j\omega(N-1-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} - e^{-j\omega\left(N-1-n-\left(\frac{N-1}{2}\right)\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} - e^{-j\omega\left(\frac{N-1}{2}-n\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n)\, 2j \sin\omega\left[\frac{N-1}{2}-n\right]$$

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$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n)\, 2j \sin\left[\frac{N}{2} - n - \frac{1}{2}\right]$$

Let $k = \frac{N}{2} - n \Rightarrow n = \frac{N}{2} - k$.

$n = 0 ; \quad k = \frac{N}{2}$

$n = \frac{N}{2} - 1 ; \quad k = 1$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{k=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - k\right) j \sin\left[k - \frac{1}{2}\right]$$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) e^{j\pi/2} \sin\left[n - \frac{1}{2}\right]$$

$$H(\omega) = e^{j\left[\frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right)\right]} \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \sin\omega\left(n - \frac{1}{2}\right)$$

$$\therefore |H(\omega)| = \sum_{n=1}^{\frac{N}{2}} 2h\left(\frac{N}{2} - n\right) \sin\omega\left(n - \frac{1}{2}\right)$$

$$\angle H(\omega) = \frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right) = \beta - \omega x.$$

where $\beta = \frac{\pi}{2}$, $x = \frac{N-1}{2}$.

Design Techniques for Linear phase FIR filters:

The well known methods of designing FIR filters are as follows.

1. Fourier series method.
2. Window method.
3. Frequency sampling method.

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Fourier Series method:-

The freq. response $H(e^{jw})$ or $H(w)$ of a system is periodic in ω. From F.S analysis we know that any periodic fn can be expressed as a Linear Combination of complex exponential.

∴ The desired freq. response of an FIR filter can be represented by the F.S

$$H_d(e^{jw}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-jwn} \quad \text{——(1)}$$

where the Fourier Coefficients $h_d(n)$ are the desired impulse response sequence of the system. The Samples of $h_d(n)$ can be determine by using the eq'n (which is inverse FT $H_d(e^{jw})$)

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{jw}) e^{jwn} dw \quad \text{——(2)}$$

The impulse response $h_d(n)$ from eq'n (2) is an infinite duration sequence.

For FIR filters we required that an infinite impulse response to a finite duration sequence of length 'N' where 'N' is odd

$$h(n) = \begin{cases} h_d(n) & \text{: for } n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N-1}{2}\right) \\ 0 & \text{, otherwise} \end{cases}$$

Taking ZT of $h(n)$ we get

$$ZT[h(n)] = H(z) = \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} h(n) z^{-n}$$

$$= \sum_{n=-\frac{N-1}{2}}^{-1} h(n) z^{-n} + h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) z^{-n} \quad \text{——(3)}$$

$$= \sum_{n=1}^{\frac{N-1}{2}} h(n) z^{n} + h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) z^{-n} \quad \text{——(4)}$$

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Impulse response is symmetric

$$h(-n) = h(n)$$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right] \quad \text{——— (5)}$$

This transfer fun of the filter $H(z)$ represents a non-causal filter (due to the presence of +ve power subject).

So, the transfer fun $H(z)$ is not physically realizable

Realizability can be brought by multiplying eq'n (5) by where $\frac{N-1}{2}$ is delay in samples.

Hence we see that causality is brought about by multiplying the transfer fun by the delay factor $\alpha = \frac{N-1}{2}$

this modification doesn't effect the amplitude response of the amplitude response of the filter.

However the abupt truncation of the FS results in oscillations in the pass & stop band

These oscillations are due to the slow convergence of the particularly near the point of discontinuity. This effect is known as gibbs phenomenon the undesirable oscillations can be reduced by multiplying the desired freq. response coefficient by an appropriate window fun.

Design procedure for digital FIR filter by Fourier series method:

→ The specifications of Digital FIR filter are

* The desired freq. response $H_d(j\omega)$
* The cut-off freq. ω_c for Lowpass & HPF and
  ω_{c1} & ω_{c2} for bandpass & band stop filters

Note:- If analog filter cut-off freq. F_c and sampling freq. F_s are specified then calculate the cut-o

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freq. of digital filter ω_c using the eqn $\omega_c = \frac{2\pi f_c}{f_s}$

* The no. of samples of impulse response 'N'

→ Determine the desired impulse response $h_d(n)$ by

$h_d(n) = \frac{1}{2\pi}$ taking inverse F.T of the desired freq. response $h_d(e^{j\omega})$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) d\omega$$

→ Calculate N samples of $h_d(n)$ for $n = -\left(\frac{N-1}{2}\right)$ to $\left(\frac{N-1}{2}\right)$

And from the impulse response $h(n) = h_d(n)$ $-\left(\frac{N-1}{2}\right)$ to $\left(\frac{N-1}{2}\right)$ with $n = 0$ and so

Note: The impulse response is symmetric $h(-n) = h(n)$

Hence it is sufficient we calculate $h(n)$ for $n = 0$ to $\frac{N-1}{2}$

→ Take Z.T of impulse response to get the non-causal transfer function of FIR filter $H(z) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} h(n) z^{-n}$

$$= h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) \left[z^{-n} + z^{n}\right]$$

→ Convert the non-causal T.F $H(z)$ to causal T.F $H'(z)$ by multiplying $H(z)$ by $z^{-\left(\frac{N-1}{2}\right)}$

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} \left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) \left[z^{n} + z^{-n}\right] \right]$$

→ Draw a suitable structure for realisation of FIR filter.

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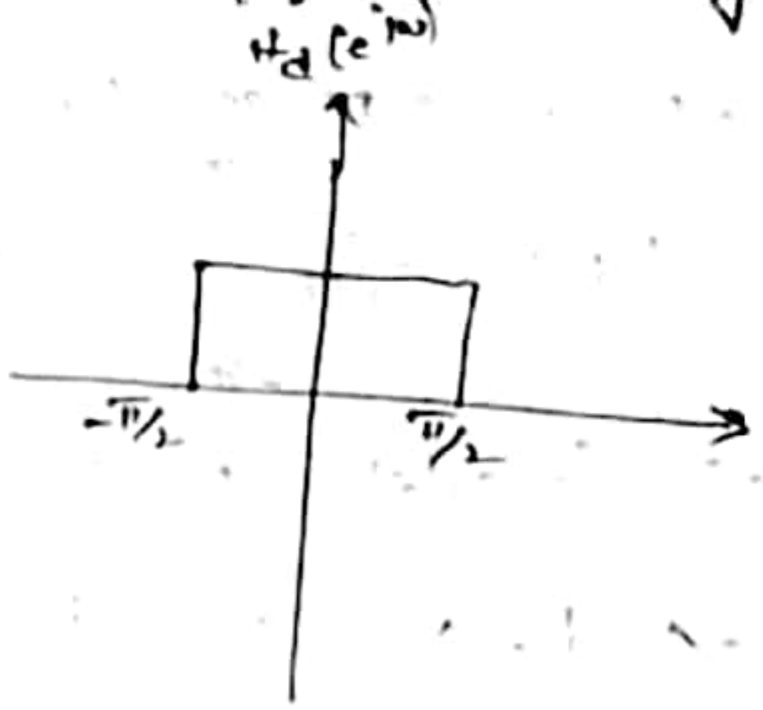
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* Design an ideal LPF with freq. response

$$H_d(e^{j\omega}) = \begin{cases} 1 \text{ for } & -\pi/2 \le \omega \le \pi/2 \\ 0 \text{ for } & \pi/2 \le |\omega| \le \pi \end{cases}$$

, find the value of $h(n)$ for $N = 11$, find $H(z)$ & plot the magnitude response.

Sol:-



$$H_d(e^{j\omega}) = \begin{cases} 1 \text{ for } & -\pi/2 \le \omega \le \pi/2 \\ 0 \text{ for } & \pi/2 \le |\omega| \le \pi \end{cases}$$

$N = 11$.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi} \left(\frac{e^{j\frac{\pi n}{2}} - e^{-j\frac{\pi n}{2}}}{jn} \right)$$

$$\boxed{h_d(n) = \frac{1}{\pi n} \sin\left(\frac{\pi n}{2}\right)}$$

$n = -\infty$ to ∞.

$\alpha = \frac{N-1}{2}$

$$h(n) = h_d(n) \Big|_{n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N-1}{2}\right) = -5 \text{ to } 5}$$

$h(n) = h(-n)$

$h(0) = \frac{0}{0}$ = undetermined

By using L. hospital rule $\left(\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \right)$

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$$h(0) = h_d(0) = \lim_{n \to 0} \frac{\sin \frac{\pi n}{2}}{\pi n} = \frac{1}{2}$$

$$h(1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} = 0.318 = h(-1)$$

$$h(2) = \frac{\sin \pi}{2\pi} = 0 = h(-2)$$

$$h(3) = \frac{\sin \frac{3\pi}{2}}{3\pi} = \frac{-1}{3\pi} = -0.106 = h(-3)$$

$$h(4) = \frac{\sin \frac{4\pi}{2}}{4\pi} = 0 = h(-4)$$

$$h(5) = \frac{\sin\left(\frac{5\pi}{2}\right)}{5\pi} = 0.065 = h(-5)$$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^n + z^{-n}\right]$$

$$= \frac{1}{2} + h(1)\left[z^1 + z^{-1}\right] + h(2)\left[z^2 + z^{-2}\right] + h(3)\left[z^3 + z^{-3}\right] +$$
$$h(4)\left[z^4 + z^{-4}\right] + h(5)\left[z^5 + z^{-5}\right]$$

$$= \frac{1}{2} + 0.318\left(z^1 + z^{-1}\right) + 0 - 0.106\left[z^3 + z^{-3}\right] + 0.0636\left[z^5 + z^{-5}\right]$$

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$

$$= z^{-5} H(z)$$

$$= z^{-5}\left[0.5 + 0.318\left(z^1 + z^{-1}\right) - 0.106\left[z^3 + z^{-3}\right] + 0.0636\left[z^5 + z^{-5}\right]\right]$$

$$= 0.5z^{-5} + 0.318z^{-4} + 0.318z^{-6} - 0.106z^{-2} - 0.106z^{-8}$$
$$+ 0.0636 + 0.0636z^{-10}$$

$$= 0.0636 - 0.106z^{-2} + 0.318z^{-4} + 0.5z^{-5} + 0.318z^{-6}$$
$$- 0.106z^{-8} + 0.0638z^{-10}$$

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h(n) = filter Coefficients.

$h(0) = h(10) = 0.0636$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.106$

$h(3) = h(7) = 0$

$h(4) = h(6) = 0.318$

$h(5) = h(5) = 0.5$

$h(n) = h(N-1-n)$

odd N=11

Magnitude response of $H(e^{j\omega})$

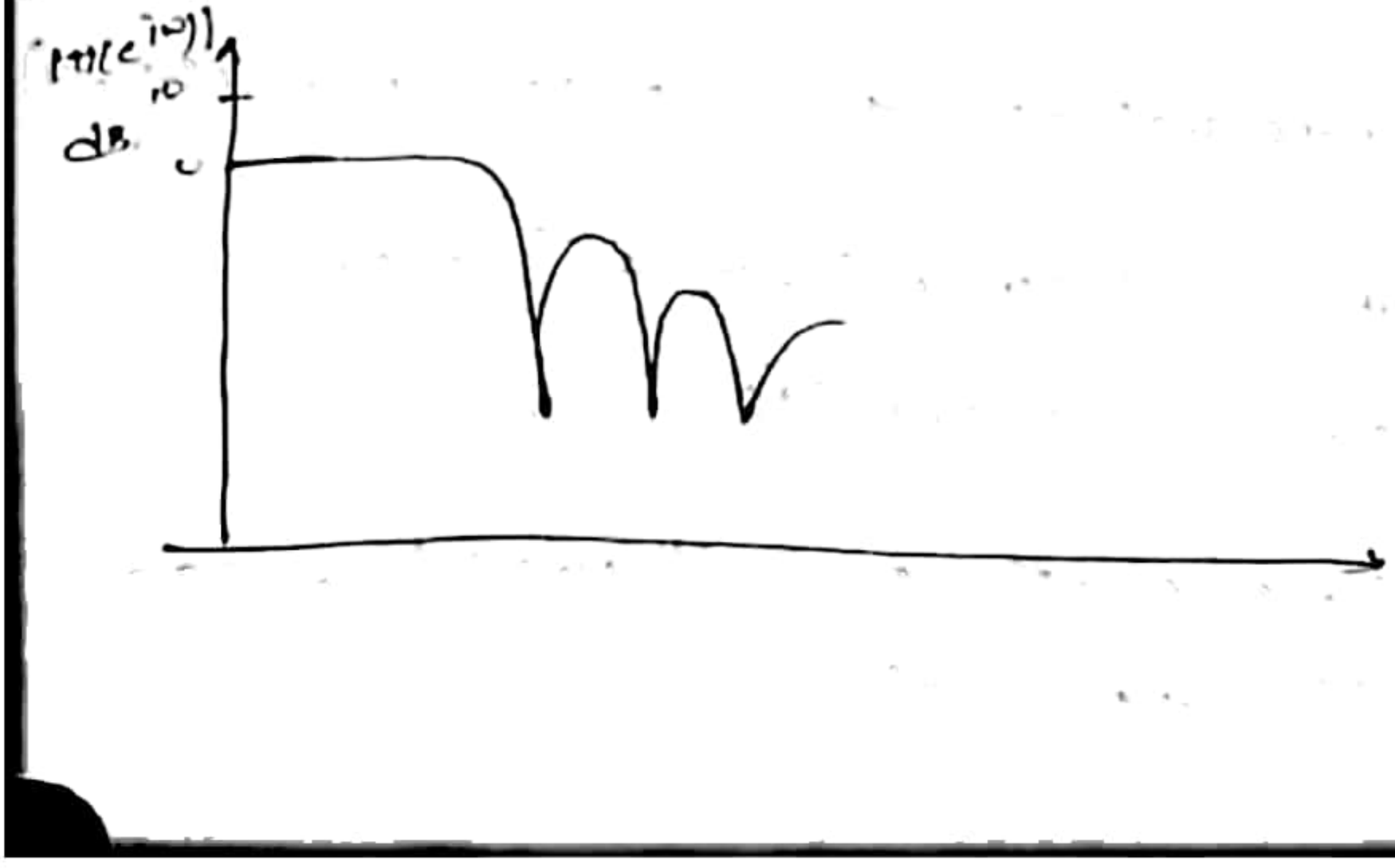
$$|H(e^{j\omega})| = |H(\omega)| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right)\cos\omega n$$

$$|H(e^{j\omega})| = h(5) + 2h(4)\cos\omega + 2h(3)\cos 2\omega + 2h(2)\cos 3\omega + 2h(1)\cos 4\omega + 2h(0)\cos 5\omega$$

$$= 0.5 + 2(0.318)\cos\omega + 2(-0.106)\cos 3\omega + 2(0.0636)\cos 5\omega$$

$$= 0.5 + 0.636\cos\omega - 0.212\cos 3\omega + 0.1272\cos 5\omega$$

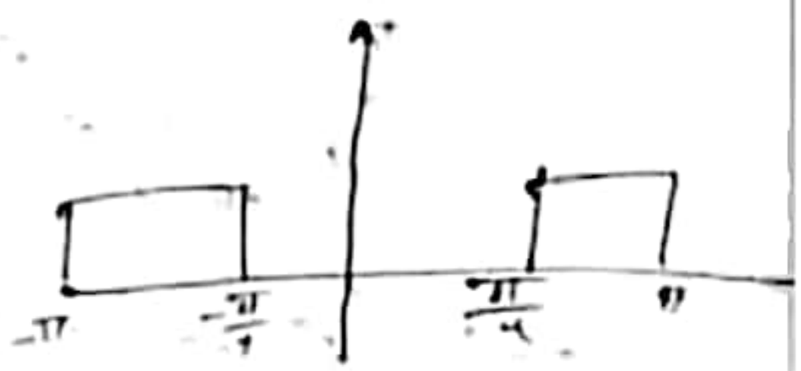
ω	0	10	20	30	40	50	60	70	80	90	100	110	120
$\|H(e^{j\omega})\|$	2.05	0.056	1.07		-0.21	-0.242	0.47	0.2	1.79	-6	-1.456	-3.18	-20.6



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* Design an ideal HPF with freq. response $H_d(e^{j\omega}) = 1$ for $\frac{\pi}{4} \le |\omega| < \pi$, 0 for $|\omega| \le \pi/4$. Find the values of $h(n)$ for $N = 11$. Find $H(e)$. plot the magnitude response.



Sol:-

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{-\pi/4} e^{j\omega n} d\omega + \int_{\pi/4}^{\pi} \right]$$

$$* \left[= \frac{1}{2\pi} \left(\frac{e^{j\omega n}}{jn} \right)_{\pi/4}^{\pi} = \frac{1}{2\pi} \left[\frac{e^{j\pi n} - e^{j\frac{\pi}{4} n}}{jn} \right] \right.$$

$$= \frac{1}{2\pi} \left(\frac{e^{j\omega n}}{jn} \Big|_{-\pi}^{-\frac{\pi}{4}} \right) + \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \Big|_{\pi/4}^{\pi} \right]$$

$$= \frac{1}{2\pi jn} \left(e^{-j\frac{\pi n}{4}} - e^{-j\pi n} \right) + \frac{1}{2\pi jn} \left(e^{j\pi n} - e^{j\frac{\pi n}{4}} \right)$$

$$= \frac{1}{2\pi jn} \left(e^{j\pi n} + e^{-j\frac{\pi n}{4}} - e^{j\frac{\pi n}{4}} - e^{-j\pi n} \right)$$

$$= \frac{1}{2\pi jn} \left(2j \sin \pi n + 2j \sin\left(\frac{\pi n}{4}\right) \right)$$

$$= \frac{1}{\pi n} \left(\sin \pi n + \sin\left(\frac{\pi n}{4}\right) \right) \quad n = -\infty \text{ to } \infty$$

$$h(n) = h_d(n) \Big|_{n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N-1}{2}\right) = -5 \text{ to } 5}$$

$$h(n) = h(-n)$$

$$* \left| h(0) = \frac{1}{\pi(0)} \right| \sin = \lim_{n \to 0} \left(\frac{\sin \pi n + \sin\left(\frac{\pi n}{4}\right)}{n\pi} \right)$$

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$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} 1\,d\omega + \frac{1}{2\pi}\int_{-\pi/4}^{+\pi/4} d\omega$$

$$= \frac{1}{2\pi}\left[\omega\Big|_{-\pi}^{-\pi/4}\right] + \frac{1}{2\pi}\left[\omega\Big|_{-\pi/4}^{\pi/4}\right]$$

$$= \frac{1}{2\pi}\left[\frac{-\pi}{4} + \pi\right] + \frac{1}{2\pi}\left[\pi - \frac{\pi}{4}\right] = \frac{3}{4}$$

$$h(1) = h(-1) = \frac{1}{\pi}\left[\sin\pi - \sin\frac{\pi}{4}\right] = -0.225$$

$h(2) = h(-2) = -0.159$

$h(3) = h(-3) = -0.075$

$h(4) = h(-4) = -0.056 = 0$

$h(5) = h(-5) = -0.045 = 0.045$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]$$

$$= \frac{3}{4} + h(1)\left[z^{1} + z^{-1}\right] + h(2)\left[z^{2} + z^{-2}\right] + h(3)\left[z^{3} + z^{-3}\right]$$

$$+ h(4)\left[z^{4} + z^{-4}\right] + h(5)\left[z^{5} + z^{-5}\right]$$

$$= \frac{3}{4} - 0.225\left[z + z^{-1}\right] - 0.159\left[z^{2} + z^{-2}\right] - 0.075\left[z^{3} + z^{-3}\right]$$

$$0.045\left[z^{5} + z^{-5}\right]$$

Realizable filter T.F :-

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z).$$

$$= z^{-5}\left[H(z)\right]$$

$$= 0.75z^{-5} - 0.225z^{-4} - 0.225z^{-6} - 0.15z^{-3} - 0.159z^{-7} - 0.075z^{-2}$$

$$-0.075z^{-8} + 0.045 + 0.045z^{-10}$$

$$= 0.045 - 0.075z^{-2} - 0.159z^{-3} - 0.225z^{-4} + 0.75z^{-5}$$

$$-0.225z^{-6} - 0.159z^{-7} - 0.075z^{-8} + 0.045z^{-10}$$

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Causal filter coefficients are

$h(0) = h(10) = 0.021$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.095$

$h(3) = h(7) = -0.159$

$h(4) = h(6) = -0.225$

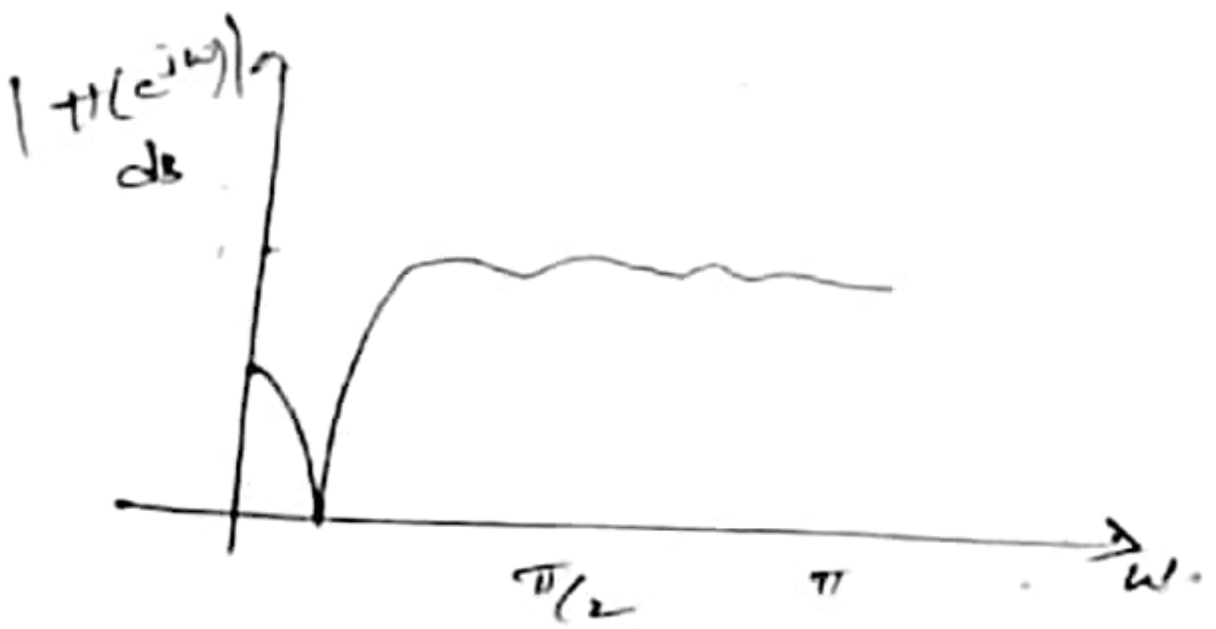
$h(5) = 0.797$

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2} - n\right]\cos\omega n$$

$$= h(5) + 2h(4)\cos\omega + 2h(3)\cos 2\omega + 2h(2)\cos 3\omega + 2h(1)\cos 4\omega + 2h(0)\cos 5\omega$$

$$= 0.797 - 0.45\cos\omega - 0.318\cos 2\omega - 0.15\cos 3\omega + 0.09\cos 4\omega$$

ω	0	10	20	30	40	50	60	70	80	90	100
$\lvert H(e^{j\omega})\rvert$	-22dB	-23.62	-41.3	-18.2	-9.16	-4.2	-1.1	-0.504	0.95	0.157	-0



* Design an ideal band pass filter with frequency response $H_d(e^{j\omega}) = 1$ for $\frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4}$ $= 0$ otherwise. Find the values of $h(n)$ for $n = 11$ & plot frequency response.

Sol: $$h_d(n) = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{-\frac{\pi}{4}} 1\cdot e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{\pi/4}^{\frac{3\pi}{4}} e^{j\omega n}\,d\omega$$

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$$= \frac{1}{2\pi}\left[\frac{e^{jw n}}{jn}\Big|_{-3\pi/4}^{\pi/4}\right] + \frac{1}{2\pi}\left[\frac{e^{jwn}}{jn}\Big|_{\pi/4}^{3\pi/4}\right]$$

$$h_d(n) = \frac{1}{2\pi}\left(\frac{e^{-j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n}}{jn}\right) + \frac{1}{2\pi jn}\left[e^{j\frac{3\pi}{4}n} - e^{j\frac{\pi}{4}n}\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n} + e^{j\frac{3\pi}{4}n} - e^{j\frac{\pi}{4}n}\right]$$

$$= \frac{1}{2\pi jn}\left[2j\sin\left(\frac{3n\pi}{4}\right) - 2j\sin\left[\frac{n\pi}{4}\right]\right]$$

$$= \frac{1}{\pi n}\left[\sin\left(\frac{3n\pi}{4}\right) \cdot \sin\left(\frac{n\pi}{4}\right)\right]$$

$$h(n) = h_d(n)\Big|_{n=-\left(\frac{N-1}{2}\right)+0\left(\frac{N-1}{2}\right)} = -1+0+1$$

$$h(0) = \lim_{n\to 0}\frac{1}{\pi n}\frac{\sin\frac{3n\pi}{4}}{\frac{3n\pi}{4}\cdot\frac{4}{3}} - \lim_{n\to 0}\frac{\sin\frac{n\pi}{4}}{\frac{n\pi}{4}\times 4}$$

$$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$h(1) = h(-1) = \frac{1}{\pi}\left[\sin\frac{3\pi}{4} - \sin\frac{\pi}{4}\right] = 0$$

$$h(2) = h(-2) = -0.3183$$

$$h(3) = h(-3) = +0$$

$$h(4) = h(-4) = 0$$

$$h(5) - h(-5) = 0$$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]$$

$$= 0.5 + h(2)\left[z^{n} + z^{-n}\right) = 0.5 - 0.3183\left[z^{-n} + z^{n}\right)$$

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)}H(z)$$

$$= z^{-2}\left[0.5 - 0.3183\left(z^{-2} + z^{2}\right)\right)$$

$$= 0.5z^{-2} - 0.3183z^{-4} - 0.3183z^{-6}$$

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$= -0.318 z^{-3} + 0.318 z^{-5} 0.5 z^{-7} - 0.318 z^{-9}$

$h(0) = h(10) = 0.5$

$h(1) = h(9) = 0$

$h(2) = h(8) = 0$

$h(3) = h(7) = 0.318$

$h(4) = h(6) = 0$

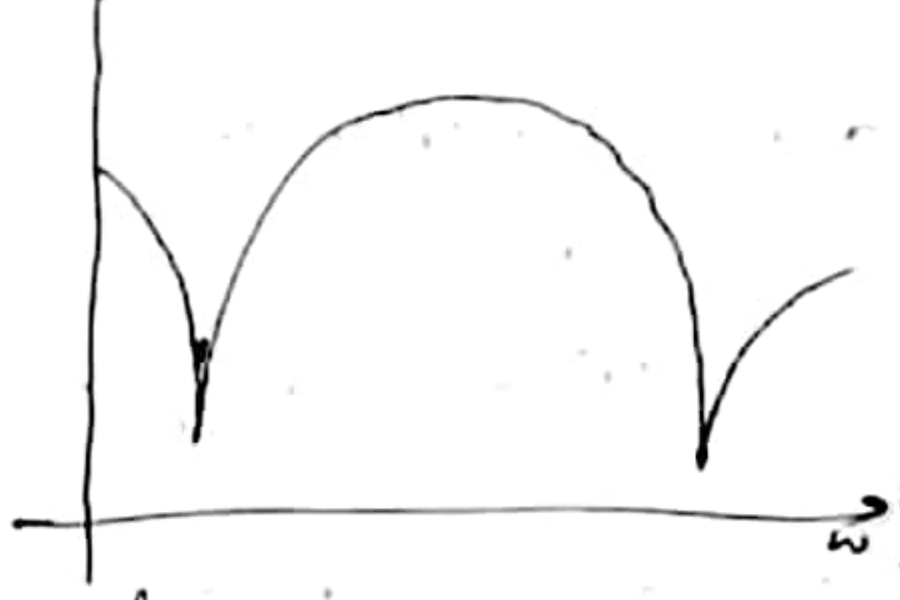
$h(5) = 0.5$

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right) \cos\omega n.$$

$= 0.5 + 2h(3)\cos 2\omega.$

$|H(e^{j\omega})| = 0.5 - 0.6366 \cos 2\omega.$

ω	0	20	30	40	60	75	90	105	120	135	150	160	180
$20\log\|H(e^{j\omega})\|$	-17.3	-35.17	-14.8	-10.02	-1.94	0.4348	1.11	0.4348	-1.94	6.02	14.8	$\frac{\infty}{17}$	-17.3



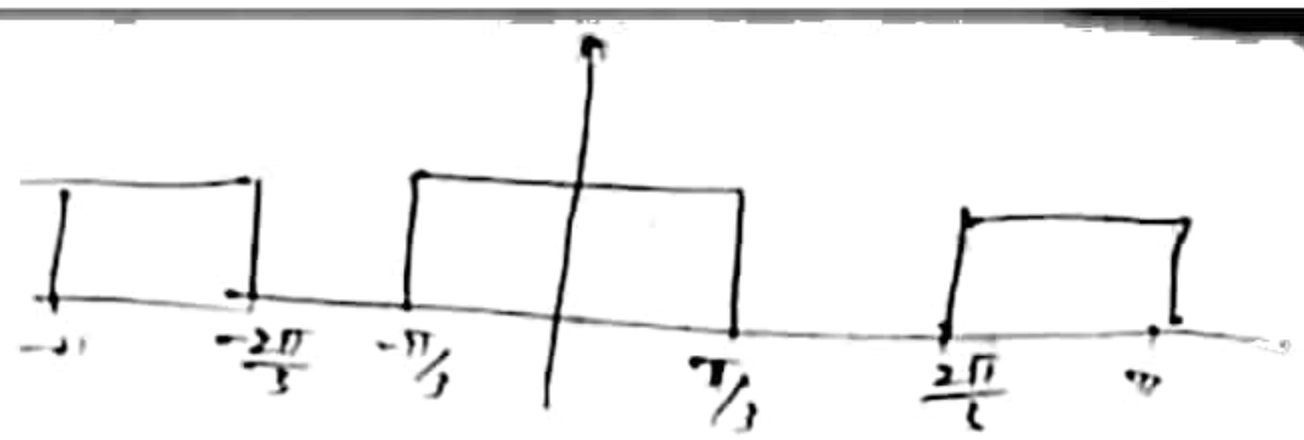
* Design an ideal band Reject filter with a desired freq. response $H(e^{j\omega}) = 1$ for $|\omega| \le \pi/3$ & $|\omega| \ge 2\pi/3$
$= 0$ for otherwise.
find $h(n)$ for $N = 11$. find $H(z)$ plot the magnitude response.

Sol:- $h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$

$\|\omega\| \le \pi/3$	$\|\omega\| \ge 2\pi/3$
$\omega \le \pi/3$	$\omega \ge 2\pi/3$
$-\omega \le \pi/3$	$-\omega \ge 2\pi/3$
$\omega \ge -\pi/3$	$\omega \ge -2\pi/3$

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$$= \frac{1}{2\pi}\int_{-\pi}^{-\frac{2\pi}{3}} e^{jwn}dw + \frac{1}{2\pi}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} e^{jwn}dw + \frac{1}{2\pi}\int_{\frac{2\pi}{3}}^{\pi} e^{jwn}dw.$$

$$= \frac{1}{2\pi}\left[\frac{e^{jwn}}{jn}\Big|_{-\pi}^{-\frac{2\pi}{3}} + \frac{e^{jwn}}{jn}\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} + \frac{e^{jwn}}{jn}\Big|_{\frac{2\pi}{3}}^{\pi}\right]$$

$$= \frac{1}{2\pi jn}\left[\left(e^{jn\pi} - e^{-j\frac{2n\pi}{3}}\right) + \left(e^{j\frac{n\pi}{3}} - e^{-j\frac{n\pi}{3}}\right) + \left(e^{jn\pi} - e^{j\frac{2n\pi}{3}}\right)\right]$$

$$= \frac{1}{n\pi}\left[\sin n\pi + \sin\frac{n\pi}{3} - \sin\frac{2n\pi}{3}\right)$$

Step2 $h(n) = h(-n)$

$N = 11$

$$h(n) = h_d(n)\Big|_n = -\left(\frac{N-1}{2}\right) \text{ to } \left(\frac{N+1}{2}\right) = -5 \text{ to } 5.$$

$$h(0) = \lim_{n\to 0}\left[\frac{1}{\pi n}\left(\sin n\pi + \sin\frac{n\pi}{3} - \sin\frac{2n\pi}{3}\right)\right]$$

$$= 1 + \frac{1}{3} - \frac{2}{3} = \frac{2}{3}.$$

$h(1) = h(-1) = 0$

$h(2) = h(-2) = 0.275$

$h(3) = h(-3) = 0$

$h(4) = h(-4) = -0.319$

$h(5) = 0.$

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Steps: Non-Caus.

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right)$$

$$H(z) = h(0) + h(1)\left[z^{-1} + z^{1}\right] + h(2)\left[z^{2} + z^{-2}\right]$$

$$+ h(3)\left[z^{3} + z^{-3}\right) + h(4)\left[z^{4} + z^{-4}\right) + h(5)\left[z^{5} + z^{-5}\right]$$

$$H(z) = 0.66 + 0.275\left[z^{-2} + z^{2}\right) + 0 - 0.317\left[z^{-4} + z^{4}\right) + 0$$

$$H(z) = 0.66 + 0.275\left(z^{-2} + z^{2}\right) - 0.317\left(z^{-4} + z^{4}\right)$$

Steps Caus.

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z)$$

$$H'(z) = z^{-5}\left[0.66 + 0.275\left(z^{-2} + z^{2}\right) - 0.317\left(z^{-4} + z^{4}\right)\right.$$

$$= -0.317z^{-1} + 0.275z^{-3} + 0.66z^{-5} + 0.275z^{-7} - 0.317z^{-9}$$

Steps: Caus filter coefficient

$$h(n) = h(N-1-n)$$

$$h(0) = h(10) = 0$$

$$h(1) = h(9) = -0.317$$

$$h(2) = h(8) = 0$$

$$h(3) = h(7) = 0.275$$

$$h(4) = h(6) = 0$$

$$h(5) = 0.66.$$

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step 6:- Magnitude Response

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left[\frac{N-1}{2} - n\right] \cos\omega n.$$

$$= h(3) + 2h(4)\cos\omega + 2h(2)\cos 2\omega + 2h(1)\cos 3\omega + 2h(0)\cos 4\omega.$$

$$|H(e^{j\omega})| = 0.66 + 0.5\cos\omega - 0.24\cos 4\omega$$

b. Design of FIR filters Using window.

The procedure for designing FIR filter using Windows is

→ Choose the desired freq. Response of the filter $H_d(e^{j\omega})$

→ Take I.F.T of $H_d(e^{j\omega})$ to obtain the desired impulse Response $h_d(n)$

→ Choose a window sequence $w(n)$ & multiply $h_d(n)$ by $w(n)$ to convert the infinite duration impulse response the finite duration

$$h(n) = h_d(n)\, w(n)$$

→ The TF $H(z)$ of the filter is obtained by taking $ZT[h(n)]$.

Window Sequence for FIR filter design:-

Name of the window.	Window Sequence.
→ Rectangular window	$w_R(n) = \begin{cases} 1 ; & -(N-1) \le n \le \left[\frac{N-1}{2}\right] \\ 0 ; & \text{otherwise} \end{cases}$
→ Triangular window	$w_T(n) = \begin{cases} 1 - \frac{2\lvert n\rvert}{N-1} ; & -\frac{N-1}{2} \text{ to } \frac{N-1}{2} \\ 0 ; & \text{otherwise.} \end{cases}$

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→ Hamming window $\omega_H(n) = \begin{cases} 0.54 + 0.46\cos\frac{2\pi n}{N-1}; & -\left(\frac{N-1}{2}\right) \text{ to } \frac{N-1}{2} \\ 0 & ; \text{otherwise} \end{cases}$

→ Hanning window $\omega_{hn}(n) = \begin{cases} 0.5 + 0.5\cos\frac{2\pi n}{N-1} & -\left(\frac{N-1}{2}\right) \text{ to } \frac{N-1}{2} \\ 0 & ; \text{otherwise} \end{cases}$

→ Blackman window $\omega_B(n) = \begin{cases} 0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1}; & -\left(\frac{N-1}{2}\right) \text{ to } \frac{N-1}{2} \\ 0 & ; \text{otherwise.} \end{cases}$

→ Kaiser window $\omega_K(n) = \frac{I_0\left(\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{I_0(\alpha)}$ $\frac{N-1}{2}$ to $\frac{N}{2}$

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P. Design an ideal highpass filter with frequency respo

$$H_d(e^{j\omega}) = \begin{cases} 1 & ; \frac{\pi}{4} \le |\omega| \le \pi \\ 0 & ; |\omega| \le \frac{\pi}{4} \end{cases}$$

find the value of h(n) for

n=11 find H(z) and plot the mag response having

hanning window.

Sol: step1

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$h_d(n) = \frac{1}{2\pi} \left[\int_{-\pi}^{-\pi/4} e^{j\omega n} d\omega + \int_{-\pi/4}^{\pi/4} e^{j\omega n} d\omega + \int_{\pi/4}^{\pi} e^{j\omega n} d\omega \right]$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\frac{-\pi}{4}} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{\pi/4}^{\pi} e^{j\omega n} d\omega.$$

$$h_d(n) = \frac{1}{n\pi} \left[\sin n\pi - \sin \frac{n\pi}{4} \right].$$

step2 Hamming window.

$$-\frac{N-1}{2} \le n \le \frac{N-1}{2}$$

$$w_{hm}(n) = \begin{cases} 0.5 + 0.5 \cos \frac{2n\pi}{N-1} & ; \\ 0 & ; \text{otherwise} \end{cases}$$

$$w_{hm}(n) = \begin{cases} 0.5 + 0.5 \cos \frac{n\pi}{5} & ; -5 \le n \le 5 \\ 0 & ; \text{otherwise} \end{cases}$$

N=11

step 3 $h(n) = h_d(n) \cdot w_{hm}(n)$ $-5 \le n \le 5$

$h_d(0) = 3/4 = 0.75$

$h_d(1) = h_d(-1) = -0.225$

$h_d(2) = h_d(-2) = -0.159$

$h_d(3) = h_d(-3) = -0.094$

$h_d(4) = h_d(-4) = 0$

$h_d(5) = h_d(-5) = 0.045$

$w_{hm}(0) = 1$

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$w_{hn}(1) = w_{hn}(-1) = 0.5 + 0.5\cos\pi/5 = 0.904$

$w_{hn}(2) = w_{hn}(-2) = 0.5 + 0.5\cos 2\pi/5 = 0.654$

$w_{hn}(3) = w_{hn}(-3) = 0.5 + 0.5\cos 3\pi/5 = 0.346$

$w_{hn}(4) = w_{hn}(-4) = 0.5 + 0.5\cos 4\pi/5 = 0.096$

$w_{hn}(5) = w_{hn}(-5) = 0.5 + 0.5\cos 5\pi/5 = 0$

$h_n(n) = h_d(n) \cdot w_{hn}(n)$

$h(0) = h_d(0) \cdot w_{hn}(0) = 0.25 \times 1 = 0.25$

$h(1) = h(-1) = h_d(1)\, w_{hn}(1) = -0.225 \times 0.904 = -0.204$

$h(2) = h(-2) = h_d(2)\, w_{hn}(2) = -0.159 \times 0.654 = -0.104$

$h(3) = h(-3) = h_d(3)\, w_{hn}(3) = -0.071 \times 0.346 = -0.026$

$h(4) = h(-4) = h_d(4)\, w_{hn}(4) = 0 \times 0.09 = 0$

$h(5) = h(-5) = h_d(5)\, w_{hn}(5) = 0.045 \times 0 = 0$

Step 4: T.F of the Patterson filter given by

$$H(z) = h(0) + \sum_{n=1}^{5} h(n)\left[z^{-n} + z^{n}\right]$$

$$H(z) = 0.25 + h(1)\left[z^{1} + z^{-1}\right] + h(2)\left[z^{2} + z^{-2}\right] + h(3)\left[z^{3} + z^{-3}\right] + h(4)\left[z^{4} + z^{-4}\right] + h(5)\left[z^{5} + z^{-5}\right]$$

$$= 0.25 - 0.204\left[z^{1} + z^{-1}\right] - 0.104\left[z^{2} + z^{-2}\right] - 0.026\left[z^{3} + z^{-3}\right]$$

$$= 0.25 - 0.204z^{+1} - 0.204z^{-1} - 0.104z^{2} - 0.104z^{-2} - 0.026z^{3} - 0.026z^{-3}$$

Step 5: T.F of Realizable filter is

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z) = z^{-5} H(z)$$

$$H'(z) = 0.25z^{-5} - 0.204z^{-6} - 0.204z^{-4} - 0.104z^{-7} - 0.104z^{-3} - 0.026z^{-8} - 0.026z^{-2}$$

Step 6: Count filter coefficients

$$h(n) = h(N-1-n)$$

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$$H(z) = -0.026z^{-2} - 0.104z^{-3} - 0.204z^{-4} + 0.75z^{-5} - 0.204z^{-6} - 0.104z^{-7} - 0.026z^{-8}$$

$h(n) = h(N-1-n)$

$h(0) = h(10) = 0$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.026$

$h(3) = h(7) = -0.104$

$h(4) = h(6) = -0.204$

$h(5) = 0.75$

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - n\right)\cos\omega n$$

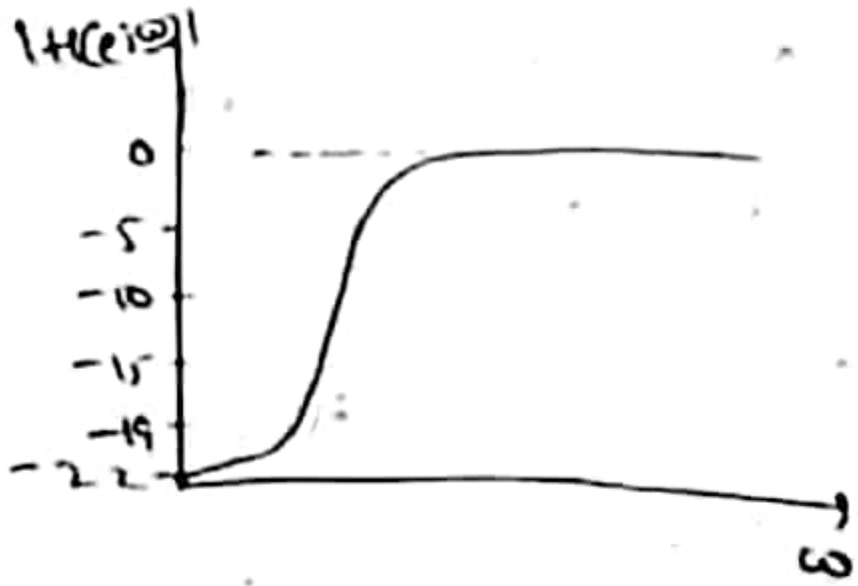
$$= h(5) + \sum_{n=1}^{5} 2h(5-n)\cos\omega n$$

$$= h(5) + 2h(4)\cos\omega + 2h(3)\cos 2\omega + 2h(2)\cos 3\omega + 2h(1)\cos 4\omega + 2h(0)\cos 5\omega$$

$$= 0.75 + 2(-0.204)\cos\omega + 2(-0.104)\cos 2\omega + 2(-0.026)\cos 3\omega$$

$$|H(e^{j\omega})| = 0.75 - 0.408\cos\omega - 0.208\cos 2\omega - 0.052\cos 3\omega$$

ω	0	15	30	45	60	75	90	105	120	135	150	165	180
$\|H(e^{j\omega})\|$ in dB	21.72	17.14	10.67	6.05	3.07	1.253	0.3726	0.0037	0.052	0.015	0	0	0.017



$h_d(0) = 0.75$

$h_d(1) = h_d(-1) = -0.225$

$h_d(2) = h_d(-2) = -0.159$

$h_d(3) = h_d(-3) = -0.075$

$$\begin{cases} w_h(n) = 0.54 + 0.46\cos\left(\frac{2n\pi}{N-1}\right) \\ = 0, \text{ otherwise} \end{cases}$$

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$h_d(4) = h_d(-4) = 0$

$h_d(5) = h_d(-5) = 0.045$

$w_h(n) = w_h(-n)$

$w_h(0) = 0.54 + 0.46 = 1$

$w_h(1) = w_h(-1) = 0.54 + 0.46 \cos(\pi/5) = 0.912$

$w_h(2) = w_h(-2) = 0.54 + 0.46 \cos(2\pi/5) = 0.682$

$w_h(3) = w_h(-3) = 0.54 + 0.46 \cos(3\pi/5) = 0.398$

$w_h(4) = w_h(-4) = 0.54 + 0.46 \cos(4\pi/5) = 0.168$

$w_h(5) = w_h(-5) = 0.54 + 0.46 \cos(\pi) = 0.08$

The filter coefficients using hamming window sequence are

$$h(n) = h_d(n) \cdot w_h(n) \quad ; -5 \le n \le 5$$

$$0 \qquad \text{; otherwise}$$

$h(0) = h_d(0) w_h(0) = 0.75 \times 1 = 0.75$

$h(1) = h(-1) = h_d(1) w_h(1) = -0.225 \times 0.912 = -0.2052$

$h(-2) = h(2) = h_d(2) w_h(2) = -0.159 \times 0.682 = -0.1084$

$h(-3) = h(3) = h_d(3) w_h(3) = -0.075 \times 0.398 = -0.03$

$h(-4) = h(4) = h_d(4) w_h(4) = 0.$

$h(-5) = h(5) = h_d(5) w_h(5) = 0.045 \times 0.08 = 0.0036$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n) [z^n + z^{-n}]$$

$= 0.75 + h(1)[z^1 + z^{-1}] + h(2)[z^2 + z^{-2}] + h(3)[z^3 + z^{-3}]$

$+ h(4)[z^4 + z^{-4}] + h(5)[z^5 + z^{-5}]$

$= 0.75 - 0.2052 z^1 - 0.2052 z^{-1} - 0.1084 z^2 - 0.1084 z^{-2} - 0.03 z^3 - 0.03 z^{-3}$

$+ 0.0036 z^5 + 0.0036 z^{-5}$

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Realizable T.F

$$H'(z) = z^{-\left(\frac{N-1}{2}\right)} H(z) = z^{-5} H(z)$$

$$= 0.75z^{-5} - 0.205z^{-4} - 0.205z^{-6} - 0.105z^{-3} - 0.105z^{-7}$$
$$- 0.03z^{-2} - 0.03z^{-8} + 0.0036z^{0} + 0.0036z^{-10}$$

$$= 0.0036 - 0.03z^{-2} - 0.105z^{-3} - 0.205z^{-4} + 0.75z^{-5}$$
$$- 0.205z^{-6} - 0.105z^{-7} - 0.03z^{-8} + 0.0036z^{-10}$$

$h(n) = h(-n)$

$\omega_h(n) = \omega_h(-n)$

Causal filter coefficients.

$h(n) = h(N-1-n)$.

$h(0) = h(10) = 0.0036$

$h(1) = h(9) = 0.$

$h(2) = h(8) = 0.03$

$h(3) = h(7) = -0.105$

$h(4) = h(6) = -0.205$

$h(5) = 0.75$.

Magnitude fu'n of $|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + \sum_{n=1}^{\frac{N-1}{2}} h\left(\frac{N-1}{2}-n\right)\cos\omega n$

$$= h(5) + \sum_{n=1}^{5} 2h(5-n)\cos\omega n$$

$$= 0.75 + 2h(4)\cos\omega + 2h(3)\cos 2\omega + 2h(2)\cos 3\omega$$
$$+ 2h(1)\cos 4\omega + 2h(0)\cos 5\omega$$

$= 0.75 - 0.410\cos\omega - 0.21\cos 2\omega - 0.06\cos 3\omega + 0.007\cos$

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* The normalized Ideal (desired)-freq. response for FIR filter design using window.

→ Low pass filter

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\omega_c \le \omega \le \omega_c \\ 0 & ; -\pi \le \omega \le -\omega_c \\ 0 & ; \omega_c \le \omega \le \pi \end{cases}$$

→ High pass filter :-

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\pi \le \omega \le -\omega_c \\ e^{-j\alpha\omega} & ; \omega_c \le \omega \le \pi \\ 0 & ; -\omega_c < \omega < \omega_c \end{cases}$$

→ Band pass filter :-

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\omega_{c2} \le \omega \le -\omega_{c1} \\ e^{-j\alpha\omega} & ; \omega_{c1} \le \omega \le \omega_{c2} \\ 0 & ; -\pi \le \omega < -\omega_{c2} \\ 0 & ; -\omega_{c1} < \omega < \omega_{c1} \\ 0 & ; \omega_{c2} < \omega \le \pi \end{cases}$$

→ Band stop filter :-

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\pi \le \omega \le -\omega_{c2} \\ e^{-j\alpha\omega} & ; -\omega_{c1} \le \omega \le \omega_{c1} \\ e^{-j\alpha\omega} & ; \omega_{c2} < \omega < \pi \\ 0 & ; -\omega_{c2} \le \omega \le -\omega_{c1} \\ 0 & ; \omega_{c1} < \omega < \omega_{c2} \end{cases}$$

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(5). Design a HPF using hamming window with a cut off freq. of 1.2 rad/sec and N=9.

Sol:- The desired freq. response $H_d(e^{j\omega})$ for a HPF is

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\pi \le \omega < \omega_c \\ e^{-j\alpha\omega} & ; \omega_c \le \omega \le \pi \\ 0 & ; -\omega_c < \omega < \omega_c \end{cases}$$

$N = 9.$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} H_d(e^{j\omega})\, e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{-j\alpha\omega}\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{-j\alpha\omega}\, e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{j\omega(n-\alpha)}\, d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{j\omega(n-\alpha)}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{n-\alpha}\right]_{-\pi}^{-\omega_c} + \frac{1}{2\pi}\left[\frac{e^{j\omega(n-\alpha)}}{n-\alpha}\right]_{\omega_c}^{\pi}$$

$$= \frac{1}{2\pi}\,\frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)}}{j(n-\alpha)} + \frac{1}{2\pi}\,\frac{e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{j(n-\alpha)}$$

$$= \frac{1}{2\pi j(n-\alpha)}\left[\left[e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}\right] + \left[e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)}\right]\right]$$

$$= \frac{1}{2\pi j(n-\alpha)}\left[-2j\sin\omega_c(n-\alpha) + 2j\sin\pi(n-\alpha)\right]$$

$$h_d(n) = \frac{1}{\pi(n-\alpha)}\left[-\sin\omega_c(n-\alpha) + \sin\pi(n-\alpha)\right], \quad n \neq \alpha = \frac{N-1}{2}$$ for other values

$n = \alpha$. By using L-Hospital rule

$$h_d(n) = \lim_{n\to\alpha} \frac{1}{\pi(n-\alpha)}\left[\sin\omega_c(n-\alpha) + \sin\pi(n-\alpha)\right]$$

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$$= \lim_{n\to\alpha} \frac{\sin\pi(n-\alpha)}{\pi(n-\alpha)} - \lim_{n\to\alpha} \frac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)} \Bigg]$$

$$h_d(n) = 1 - \frac{\omega_c}{\pi} \qquad n = \alpha = \frac{N-1}{2} = \frac{9-1}{2} = 4.$$

(P) Design a band stop filter to reject freq. in the range 1 to 2 rad/sec. using rectangular window with $N=7$.

Sol:- $\omega_{c1} = 1$ rad/sec, $\omega_{c2} = 2$ rad/sec

$$\alpha = \frac{N-1}{2} = 3$$

The desired freq. response for BSF.

$$H(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; -\pi \le \omega \le -\omega_{c2}, -\omega_{c1} \le \omega \le \omega_{c1}, \omega_{c2} < \omega < \\ 0 & ; \text{other values} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\omega_{c2}} e^{-j\alpha\omega} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{-\omega_{c1}}^{\omega_{c1}} e^{-j\alpha\omega} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{\omega_{c2}}^{\pi} e^{-j\alpha\omega} e^{j\omega n} d\omega$$

$$= \frac{1}{\pi(n-\alpha)} \left[\sin(n-\alpha)\omega_{c1} + \sin(n-\alpha)\pi - \sin(n-\alpha)\omega_{c2} \right].$$

$$0.$$

At $n = \alpha$.

$$h_d(n) = 1 + \frac{\omega_{c1}}{\pi} - \frac{\omega_{c2}}{\pi} \quad \text{for} \quad h_d(3)$$

(D) Design a Band pass FIR filter for the following specification Cutoff freq. = 400 Hz & 800 Hz. Sampling freq. 2000 Hz. $N = 11$.

Sol:- Cut off freq. $f_{c1} = 400$ Hz

$f_{c2} = 800$ Hz

$f_s = 2000$ Hz.

Normalized Cut off freq. $\omega_{c1} = \frac{2\pi f_{c1}}{f_s} = \frac{2\pi \times 4}{2\pi}$

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$$\omega_2 = \frac{2\pi(800)}{2000} = \frac{4\pi}{5}$$
$$10$$

Frequency Sampling method

→ Design of FIR filter by freq. Sampling method.

Procedure for Type-I

→ Choose the ideal (desired) freq response $H_d(e^{j\omega})$

→ Sample $H_d(e^{j\omega})$ at N points by taking $\omega = \omega_k = \frac{2\pi k}{n}$

where $k = 0, 1, 2, \ldots N-1$. to generate the sequence

→ $\tilde{H}(k)$ (Tild).

$$\therefore \tilde{H}(k) = \underset{(\omega)}{H_d(e^{j\omega})}\Big|_{\omega = \frac{2\pi k}{N}} \quad H_d(\omega)$$

→ Compute the N-sample of $h(n)$ using the following eqn.

Case I: N is odd

$$h(n) = \frac{1}{N}\left[\tilde{H}(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[\tilde{H}(k)\, e^{\frac{j2\pi nk}{N}}\right]\right]$$

Case II N is even

$$h(n) = \frac{1}{N}\left[\tilde{H}(0) + 2\sum_{k=0}^{\frac{N}{2}-1} Re\left[\tilde{H}(k)\, e^{\frac{j2\pi nk}{N}}\right]\right]$$

→ Take z T of the impulse response of $h(n)$

to get the T.F $H(z)$

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}.$$

$$\tilde{H}(k) = |H(k)| = |H(k)|\, e^{j\theta}$$

for Linear phase filter $\theta = -\alpha\omega$

$$\theta = -\left(\frac{N-1}{2}\right)\frac{2\pi k}{N}$$

$$\theta = -\left(\frac{N-1}{N}\right)\pi k \quad ; \quad k = 0, 1, \ldots N-1$$

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The filter coefficients $h(n)$ can be obtained by finding IDFT of $H(k)$

$$h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k)\, e^{\frac{j2\pi kn}{N}} \quad ; n = 0, 1, 2 \cdots N-1$$

If $h(n)$ the impulse response of the filter is to be realized using the freq samples $H(k)$ must satisfy the symmetry requirment for N odd or even

$$H(N-k) = H^*(k) \quad ; k = 0, 1, 2 \cdots N-1$$

In addition for N even $h\left(\frac{N}{2}\right) = 0$.

With the freq response $H(k)$ the magnitude response is an even function.

$$|H(k)| = |H(N-k)| \quad ; k = 0, 1, \cdots N-1.$$

The phase is an odd function: we represent phase function

$$\theta(k) = -\theta(N-k)$$

$$\theta(N-k) = -\left(\frac{N-1}{N}\right)\pi(N-k)$$

$$= -\left(\frac{N-1}{N}\right)(\pi N - \pi k)$$

$$= -\left(\frac{N-1}{N}\right)\pi N + \left(\frac{N-1}{N}\right)\pi k$$

$$= -(N-1)\pi + \left(\frac{N-1}{N}\right)\pi k$$

$\theta(k)$ for N odd is given by θk

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k \quad ; \quad k = 0, 1, 2 \cdots N-1$$

$$= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k \quad ; \text{for } k = \frac{N+1}{2} \cdots N$$

$\theta(k)$ for N even in given by

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k \quad ; \quad n = 0, 1, \cdots \frac{N}{2} - 1$$

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$= 0 \quad ; \quad K = \frac{N}{2}$

$= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k \quad ; \quad K = \frac{N}{2}+1 \quad ; \cdots N-1$

$\theta(k) = -\theta(N-k)$

for N odd

$H(k) = |H(k)|\, e^{-j\left(\frac{N-1}{N}\right)\pi k}$ for $k = 0, 1, \cdots \frac{N-1}{2}$

$= |H(k)|\, e^{+j\left[(N-1)\pi - \left(\frac{N-1}{N}\right)\pi k\right]}$ for $k = \frac{N+1}{2} \cdots N+1$

for N even

$H(k) = |H(k)|\, e^{-j\left(\frac{N-1}{N}\right)\pi k} \quad ; \quad k = 0, 1, \cdots \frac{N}{2} - 1$

$= |H(k)| \quad ; k = \frac{N}{2}.$

$= |H(k)|\, e^{j\left[(N-1)\pi - \left(\frac{N-1}{N}\right)\pi k\right]} \quad ; \quad k = \frac{N}{2}+1, \cdots N-1.$

for N odd

$$h(n) = \frac{1}{N}\sum_{k=0}^{N-1}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}}\right]\right)$$

for N even

$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{k=1}^{\frac{N}{2}-1} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}}\right]\right)$$

Once the filter coefficient $h(n)$ have been determine

the sly fair filter is given by

$$\Rightarrow H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

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D Determine the filter coefficients $h(n)$ obtained by

Sampling $H_d(n) = H_d(e^{jw}) = e^{-j(N-1)w_k}$; $0 \le |\omega| < \pi/2$

$= 0$; $\pi/2 \le |\omega| < \pi$

for $N = 7$.

Sol: $\omega_k = \frac{2\pi}{N} k$, $k = 0, 1 \ldots N-1$.

$H(k) = H(k) = |H(k)| e^{j\theta(k)}$

$\omega_k = \frac{2\pi}{N} k$ $\quad k = 0, 1, 2, 3, 4, 5, 6$

$\omega_0 = 0$, $k = 0$ $\quad \frac{2}{7} = 0.28$

$\omega_1 = \frac{2\pi}{7}$, $k = 1$ $\quad \frac{4}{7} = 0.57$

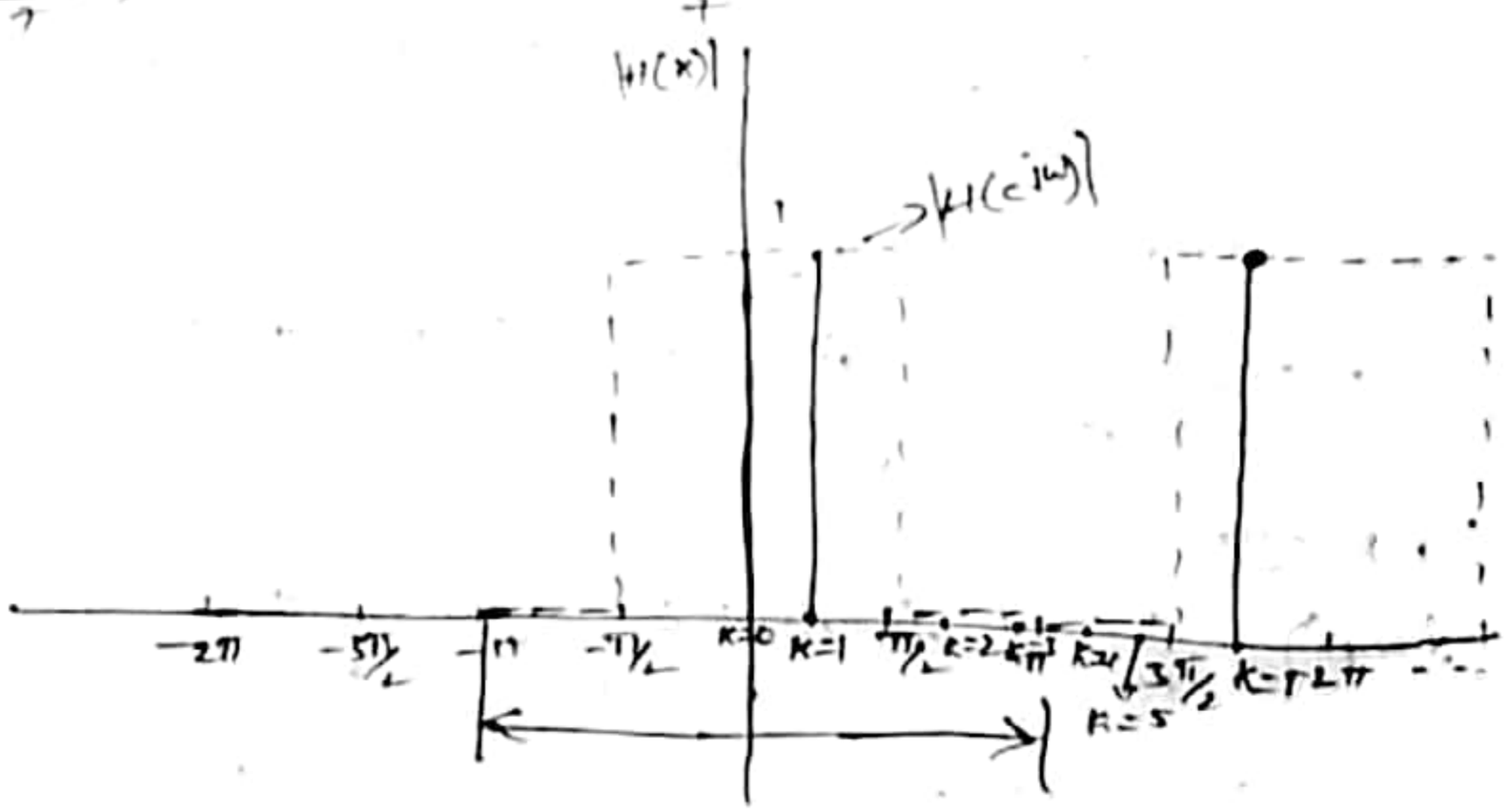
$\omega_2 = \frac{4\pi}{7}$, $k = 2$ $\quad \frac{6}{7} = 0.857$

$\omega_3 = \frac{6\pi}{7}$, $k = 3$ $\quad \frac{8}{7} = 1.14$

$\omega_4 = \frac{8\pi}{7}$, $k = 4$ $\quad \frac{10}{7} = 1.42$

$\omega_5 = \frac{10\pi}{7}$, $k = 5$ $\quad \frac{12}{7} = 1.71$.

$\omega_6 = \frac{12\pi}{7}$, $k = 6$



$|H(k)| = 1$; $k = 0, 1, 6$

$= 0$; $k = 2, 3, 4, 5$

Neven

$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k$; $k = 0, 1, 4, 3$

$= -\frac{6}{7}\pi k$; $k = 0, 1, 2, 3$.

Odd

$\theta(k) = (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k$

$= 6\pi - \frac{6\pi}{7} k$; $k = 4, 5, 6$

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$$H(k) = |H(k)| e^{j\theta(k)}$$

$$H(k) = 1\, e^{-j6/7\,\pi k} \quad ; k = 0,1$$

$$0 \quad , k = 2,3,4,5$$

$$= 1\, e^{j\left[6\pi - \frac{6}{7}\pi k\right]} \quad ; \quad k = 6.$$

N odd

$$h(n) = \frac{1}{N}\left[H(0) + 2 \sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}} \right] \right]$$

$$= \frac{1}{7}\left[1 + 2 \sum_{k=1}^{3} Re\left[H(k)\, e^{\frac{j2\pi nk}{N}} \right] \right]$$

$$= \frac{1}{7}\left[1 + 2\, Re\left[e^{-j\frac{6}{7}\pi(1)}\, e^{\frac{j2\pi n(1)}{7}} \right] \right]$$

$$h(n) = \frac{1}{7}\left[1 + 2\, Re\left[e^{-j\,3\pi/7 (n-3)} \right] \right]$$

$$= \frac{1}{7}\left[1 + 2 \cos\frac{2\pi}{7}(n-3) \right]$$

$$h(n) = h(N-1-n)$$

$$h(n) = h(6-n)$$

$$h(0) = h(6) = \frac{1}{7}\left[1 + 2\cos\frac{6\pi}{7} \right) = -0.1145.$$

$$h(1) = h(5) = \frac{1}{7}\left(1 + 2\cos\frac{4\pi}{7} \right) = 0.099.$$

$$h(2) = h(4) = \frac{1}{7}\left[1 + 2\cos\frac{2\pi}{7} \right) = 0.321$$

$$h(3) = h(3) = \frac{1}{7}\left[1 + 2\cos(0) \right] = 0.4285.$$

Using freq. Sampling method Design a BPF with the following specification.

→ Sampling freq $f = 8000$ Hz

→ cut off freq $f_{c1} = 1000$ Hz, $f_{c2} = 3000$ Hz.

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Determine the filter coefficient $N=7$.

$\omega_k = \frac{2\pi}{N}k$

$\omega_0 = 0$, $k=0$ $\omega_6 = 12/7\pi$ $k=6$

$\omega_1 = \frac{2}{7}\pi$, $k=1$

$\omega_2 = \frac{4}{7}\pi$, $k=2$

$\omega_3 = 6/7\pi$ $k=3$.

$\omega_4 = 8/7\pi$ $k=4$,

$\omega_5 = 10/7\pi$, $k=5$.

$\omega_{c1} = \frac{2\pi f_{c1}}{f_s} = \frac{2\pi 1000}{8000} = \pi/4$.

$\omega_{c2} = \frac{2\pi 3000}{8000} = 3\pi/4$.

$|H(k)| = 1$; $k=1,2,5,6$

0 ; $k=0,3,4$.

$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k$; $k=0,1,2,3$

$= -6/7\pi k$; $k=0,1,2,3$

$\theta(k) = (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k$; $k=4,5,6$

$= 6\pi - 6/7\pi k$; $k=4,5,6$.

$H(k) = |H(k)| e^{j\theta(k)}$

$H(k) = 0$; $k=0,3,4$.

$= 1 \quad e^{-j6/7\pi k}$; $k=1,2$

$= 1 \quad e^{j(0\pi - 6/7\pi k)}$; $k=5,6$.

$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k) \quad e^{j\frac{2\pi nk}{N}}\right]\right]$

$= \frac{1}{7}\left[0 + 2\sum_{k=1}^{3} Re\left[H(k) e^{\frac{j2\pi nk}{N}}\right]\right]$

$= 1/7\left[-Re\left[e^{-j6/7\pi} e^{\frac{j2\pi n}{7}}\right] + 2Re\left[e^{-j12/7\pi} e^{j\pi}\right]\right.$

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$$= \frac{1}{7}\left[2\,Re\left[e^{\frac{j2\pi}{7}(n-3)}\right] + 2\,Re\left[e^{j4\pi/7(n-3)}\right]\right]$$

$$h(n) = \frac{1}{7}\left[2\cos\frac{2\pi}{7}(n-3) + 2\cos\frac{4\pi}{7}(n-3)\right)$$

$$h(n) = h(N-1-n)$$

$$h(0) = h(6) = \frac{1}{7}\left[2\cos 6\pi/7 + 2\cos 12/7\pi\right) = -0.0892.$$

$$h(1) = h(5) = \frac{1}{7}\left[2\cos 5\pi/7 + 2\cos\frac{\pi}{7\pi}\right) = -0.321$$

$$h(2) = h(4) = \frac{1}{7}\left[2\cos\frac{2\pi}{7} + 2\cos 4\pi/7\right) = 0.1145$$

$$h(3) = \frac{1}{7}\left[2\cos 0 + 2\cos 0\right) = 4/7 = 0.57$$

Structures for realization of FIR Sly's :-

The diff types of structures for realizing the FIR

Sly are 1

1. direct form realization.
2. Transposed form realization.
3. Cascade realization
4. Lattice structure realization,
5. Linear phase realization

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Theorem of FIR sly is

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{N-1} b_k z^{-k}$$

$$= b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{N-1} z^{-(N-1)}$$

$$H(z) = Z[h(n)] = \sum_{n=0}^{N-1} h(n) z^{-n}$$

change index n to k

$$= \sum_{k=0}^{N-1} h(k) z^{-k} = h(0) + h(1) z^{-1} + \cdots h(N-1) z^{-(N-1)}$$

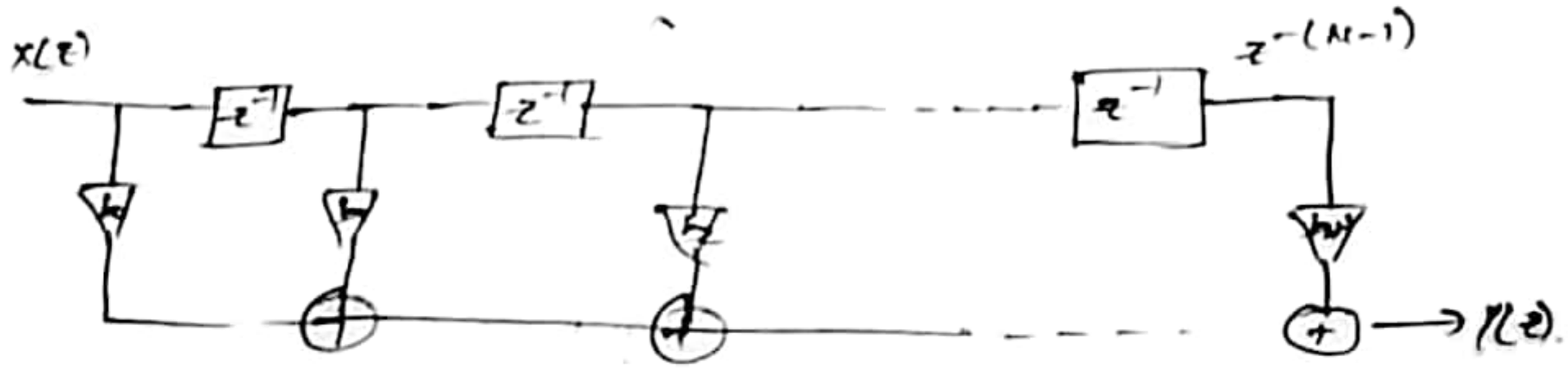
$b_0 = h(0)$, $b_1 = h(1)$ ... $b_k = h(k)$

Direct form realization of FIR sly :-

The direct form structure can be obtained from the general eq'n $Y(z)$ of the FIR sly

$$Y(z) = \sum_{k=0}^{N-1} b_k z^{-k} X(z)$$

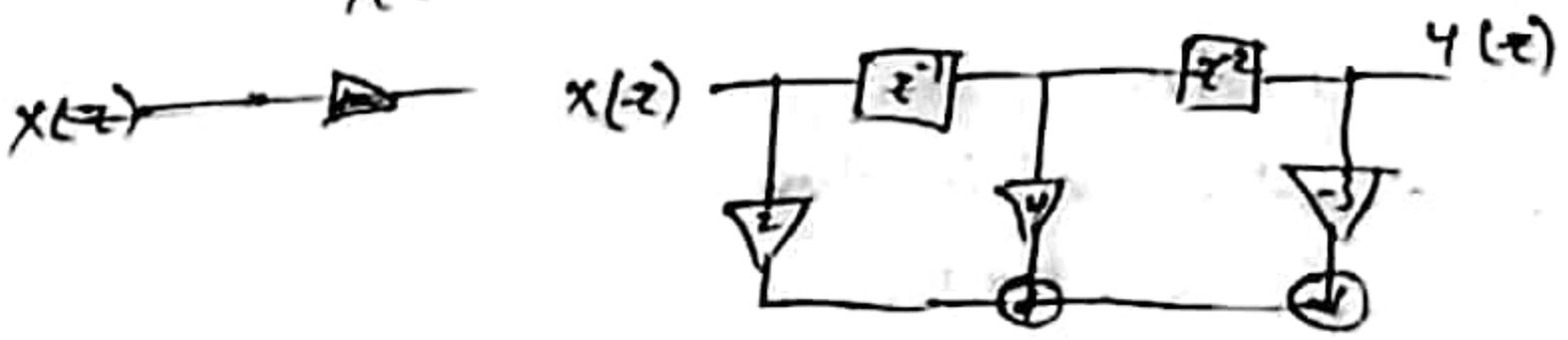
$$= b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + \cdots b_{N-1} z^{-(N-1)} X(z)$$



(P). Realize the second order FIR sly $y(n) = 2x(n) + 4x(n-1) - 3x(n-2)$ using direct form structure.

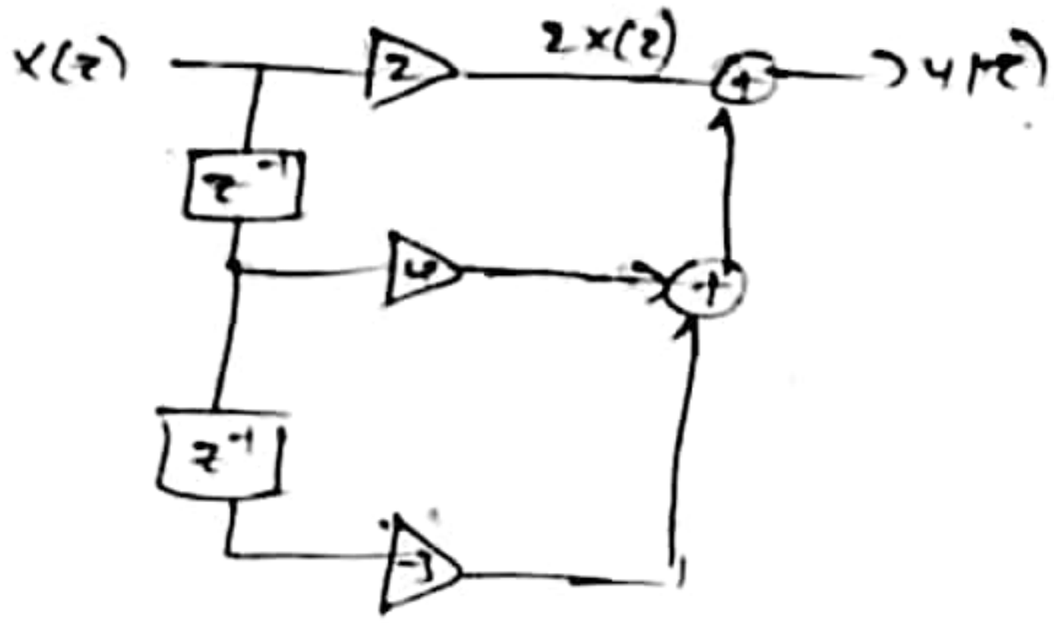
Sol:- Given $y(n) = 2x(n) + 4x(n-1) - 3x(n-2)$

$$Y(z) = 2X(z) + 4z^{-1}X(z) - 3z^{-2}X(z)$$



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Lattice structure Realization of FIR Systems:-

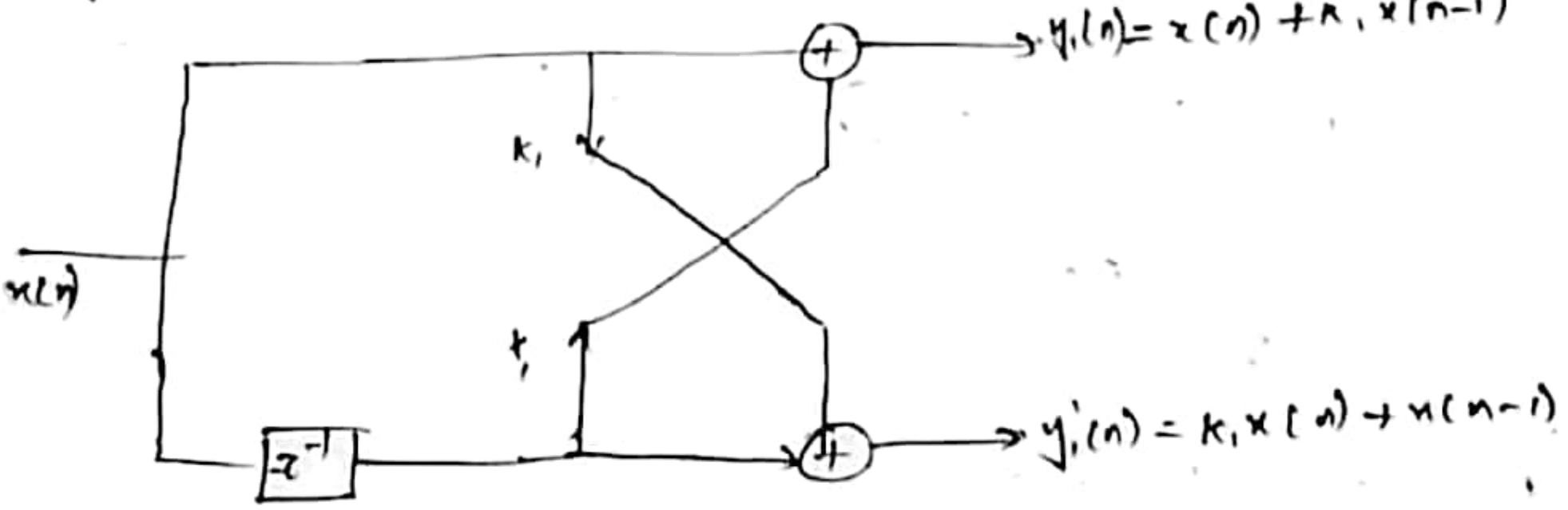
The lattice structure consists of two diff. paths through which the i/p $x(n)$ is passed. Hence the lattice structure has two diff. output setups $y(n)$ & $y'(n)$.

$y(n)$ is the real o/p & $y'(n)$ is the supporting o/p. which offers support for obtaining the o/p for the next stage.

→ A single stage lattice structure shown in the following fig. where in k is reflection coefficient.

$y_1(n) = x(n) + k_1 x(n-1)$

$y_1'(n) = k_1 x(n) + x(n-1)$.



(P). ~~Realize the sly~~

Procedure to realize the lattice structure of the FIR sly:-

→ If the Coeff of the present i/p $x(n)$ is not unity. Convert it to unity by taking Common of the Coefficient of the present i/p

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→ Find the order of the diff. eq'n and Compare the coeff. of the given diff eq'n with the coeffs. of the same order Lattice structure o/p involving the reflection coeff. $K_1, K_2, K_3 \cdots$

→ Assign the calculated values of $K_1, K_2 \cdots$ and Constr the structure.

(P). Realize a nly with $G + H(z) = 5 + 3z^{-1}$ by using Latti Structure.

Sol:- Given $+1(z) = 5 + 3z^{-1}$

$$\frac{Y(z)}{X(z)} = 5 + 3z^{-1}$$

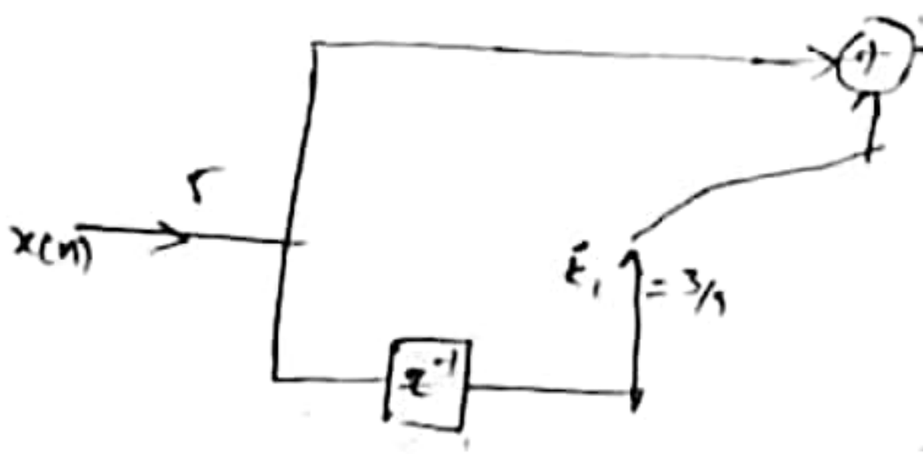
$$Y(z) = 5\,X(z) + 3z^{-1}X(z)$$

$$y(n) = 5\,x(n) + 3\,x(n-1)$$

$$= 5\left[x(n) + \frac{3}{5}\,x(n-1)\right]$$

$$y_1(n) = x(n) + K_1\,x(n-1)$$

$$K_1 = 3/5$$



$\rightarrow y(n) = 5\left[x(n) + \frac{3}{5}\,x(n-1)\right]$.

Two stage Lattice structure :—

The output from the second stage of the Lattice structure which is the second order FIR sky is given as follows.

$$y_2(n) = y_1(n) + K_2\,y_1'(n-1)$$

$$y_2'(n) = K_2\,y_1(n) + y_1'(n-1)$$

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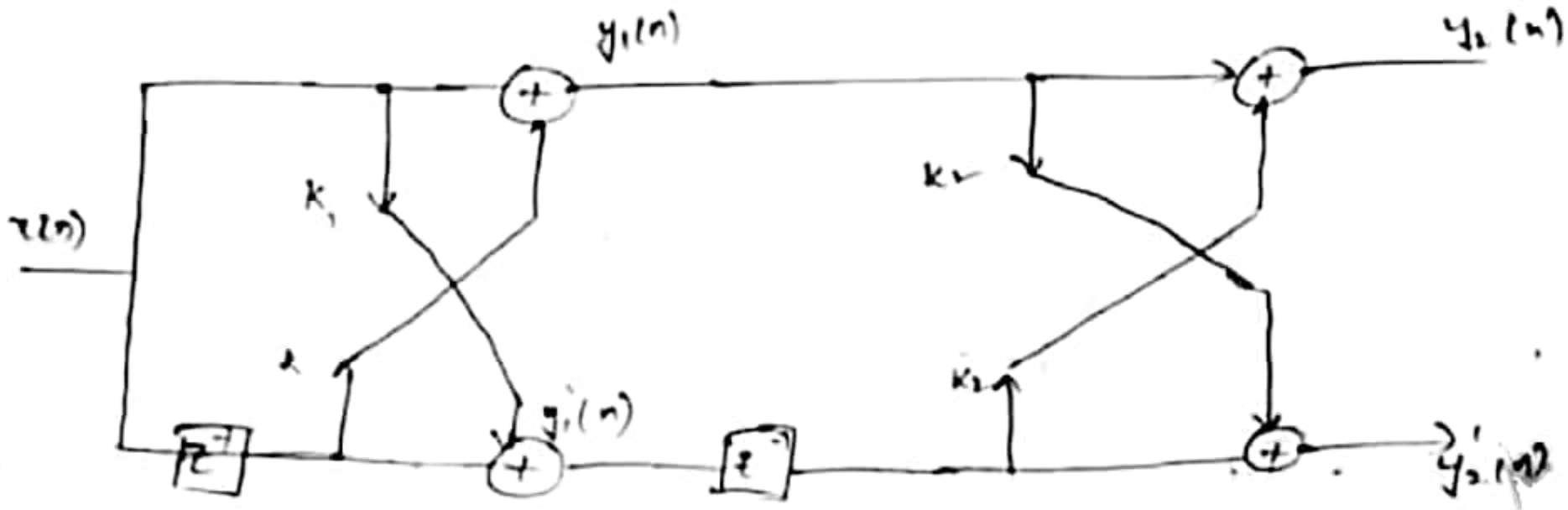
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$$y_2(n) = [x(n) + k_1 x(n-1)] + k_2 [k_1 x(n-1) + x(n-2)]$$

$$= x(n) + k_1 x(n-1)[1 + k_2] + k_2 x(n-2)$$

$$y_2'(n) = k_2 [x(n) + k_1 x(n-1)] + [k_1 x(n-1) + x(n-2)]$$

$$= k_2 x(n) + k_1 x(n-1)[1 + k_2] + x(n-2)$$



① Determine the lattice coeff. Corresponding to the FIR filter with the filter $H(z) = 1 + \frac{7}{9} z^{-1} + \frac{3}{5} z^{-2}$ and realize it.

Sol:-

$$H(z) = 1 + \frac{7}{9} z^{-1} + \frac{3}{5} z^{-2}$$

$$Y(z) = X(z) + \frac{7}{9} z^{-1} X(z) + \frac{3}{5} z^{-2} X(z)$$

$$y(n) = x(n) + \frac{7}{9} x(n-1) + \frac{3}{5} x(n-2) \quad \text{---} (2)$$

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