21/12/17

Unit : 3

i) Review of Z-Transforms

2. Realization of Digital filter structures:

Digital filters can be realized by the following two realization techniques:

1. Recursive Realization. (or) IIR Digital filter structures.
2. Non-Recursive Realization (or) FIR Digital filter structures.

1. Recursive Realization:

for recursive realization the current o/p $y(n)$ is a function of present i/p, past i/p and past o/p samples.

$$y(n) = \{x(n), x(n-1), x(n-2), \ldots \& y(n-1), y(n-2), \ldots \}$$

This realization corresponds to IIR Digital filter structures.

2. Non Recursive Realization:

for Non Recursive realization, the current o/p $y(n)$ is a function present & past i/p samples only. This realization corresponds to FIR Digital filter structures.

$$y(n) = \{x(n), x(n-1), x(n-2), x(n-3) \ldots\}$$

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IIR Digital filter structures (or) Recursive Realization techniques:

There are seven well known to realize IIR digital filter.

i) Direct form - I
ii) Direct form - II (canonic) min. no. of memory locations required realization technique.
iii) Transposed Realization.
iv) parallel cascade Realization.
v) parallel Realization.
vi) Lattice Realization
vii) Ladder Realization.

i) Direct form-I Realization technique:

Let us consider a system (IIR Digital filter) described by a Nth order LCCDE (linear constant coefficient difference eqn) is given by

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \quad \text{---} (1)$$

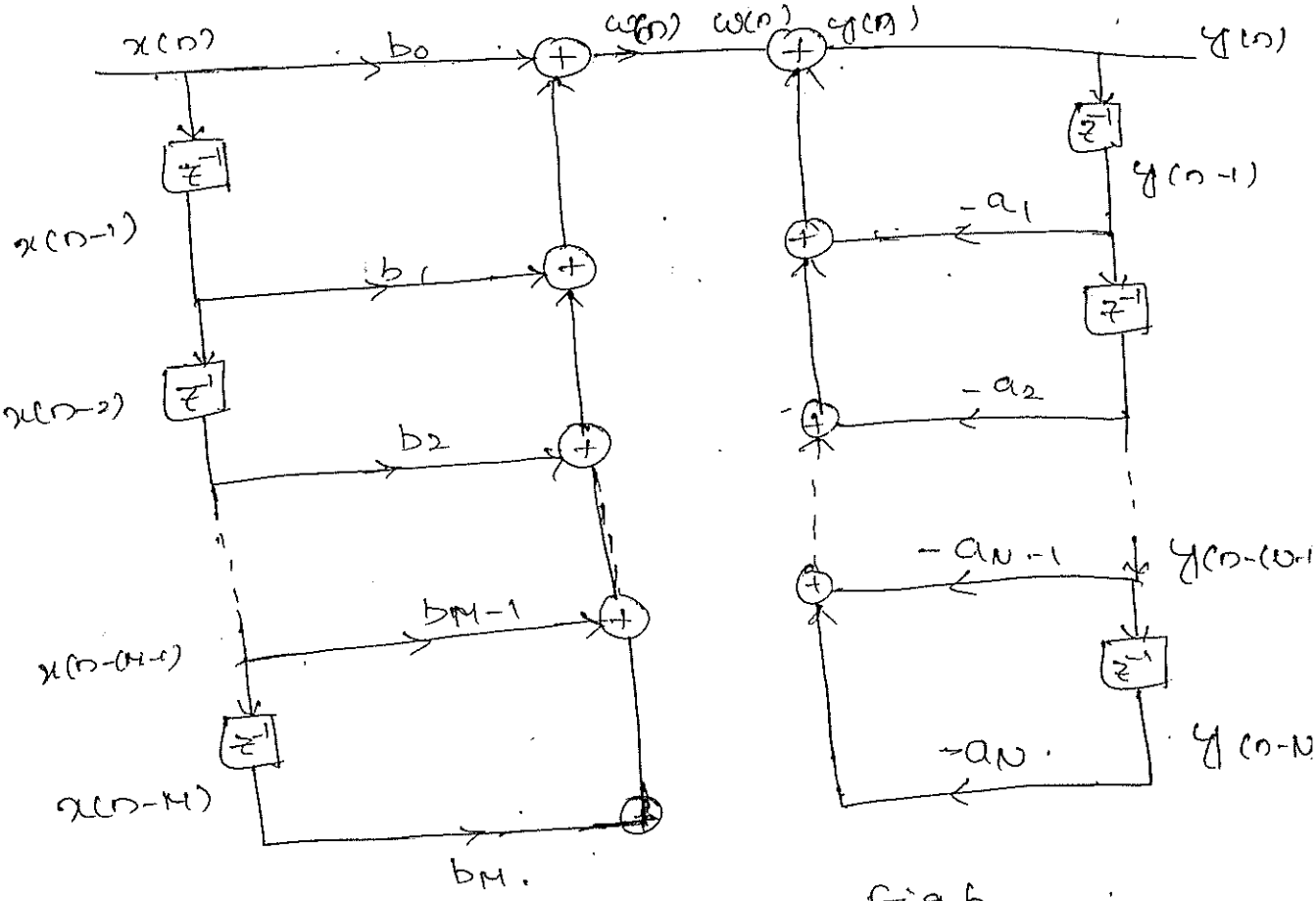
$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - \cdots - a_{N-1} y(n-(N-1)) - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \cdots + b_{M-1} x(n-(M-1)) + b_M x(n-M) \quad \text{---} (2)$$

Consider $w(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \cdots + b_{M-1} x(n-(M-1)) + b_M x(n-M)$ ---- (3).

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$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - \cdots - a_{N-1} y(n-(N-1)) - a_N y(n-N) + w(n) \quad \text{---- (4)}$$

Eqn (3) can be realized by using direct form 1 realization as shown in fig(a).



fig(a).

fig(b).

fig 5: Direct Form-I structure

Eqn (4) can be realized as shown in fig(b).

The above structure is called Direct form-I IIR Digital filter structure, which used separate delays for both input and output as shown in above fig 5.

This realization requires, total no. of multiplications = M+N+1.

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The total no. of additions required = M+N.

The total no. of memory locations required = M+N.

Direct Form II (or) canonic (or) Minimum no. of memory locations realization:

Let us consider a system (IIR digital filter) described by N^{th} order LCCDE

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \quad \text{---(1)}$$

Apply z - Transforms.

$$Y(z) = -\sum_{k=1}^{N} a_k Y(z) z^{-k} + \sum_{k=0}^{M} b_k X(z) z^{-k}$$

$$Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = \sum_{k=0}^{M} b_k X(z) z^{-k}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{Y(z)}{W(z)} = \sum_{k=0}^{N} b_k z^{-k}$$

$$Y(z) = W(z)\left[b_0 z^0 + b_1 z^{-1} + b_2 z^{-2} + \cdots\right]$$

$$y(n) = w(n) b_0 + w(n-1) b_1 + \cdots \quad \text{---(2)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

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$$W(z)\left[1+\sum_{k=1}^{N} a_k z^{-k}\right] = X(z).$$

$$w(z) \neq x(z)$$

$$w(z)+\sum_{k=1}^{N} a_k z^{-k} w(z) = x(z)$$

Apply I.z.T, we get

$$I.z.T\left[w(z)+\sum_{k=1}^{N} a_k z^{-k} w(z)\right] = IZT\left[x(z)\right]$$

By linearity property of z.T, we get

$$IZT[w(z)]+\sum_{k=1}^{N} a_k\, I.z.T\left[z^{-k} w(z)\right] = I\,z\,T[x(z)]$$

we know:

$$x(n) \xleftrightarrow{z.T} X(z)$$

$$w(n) \longleftrightarrow W(z)$$

$$w(n-k) \longleftrightarrow z^{-k} w(z)$$

$$y(n) \longleftrightarrow y(z).$$

$$w(n)+\sum_{k=1}^{N} a_k\, w(n-k) = x(n).$$

$$w(n) = x(n) - \sum_{k=1}^{N} a_k\, w(n-k).$$

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-1) - \cdots - a_{N-1}$$

$$w(n-(N-1)) - a_N w(n-N) \quad \text{---------- (1)}.$$

$$\frac{y(z)}{w(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$y(z) = \sum_{k=0}^{M} b_k z^{-k} w(z).$$

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Apply IZT & linearity property.

$$\mathcal{Z}^{-1}\left[Y(z)\right] = \sum_{k=0}^{M} b_k \mathcal{Z}^{-1}\left[z^{-k} W(z)\right]$$

$$y(n) = \sum_{k=0}^{M} b_k w(n-k).$$

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \cdots + b_{m-1} w(n-(m-1)) + b_m w(n-m) \quad \text{---(II)}$$

eqn (I) can be realized using direct form-I structure as shown in below fig.

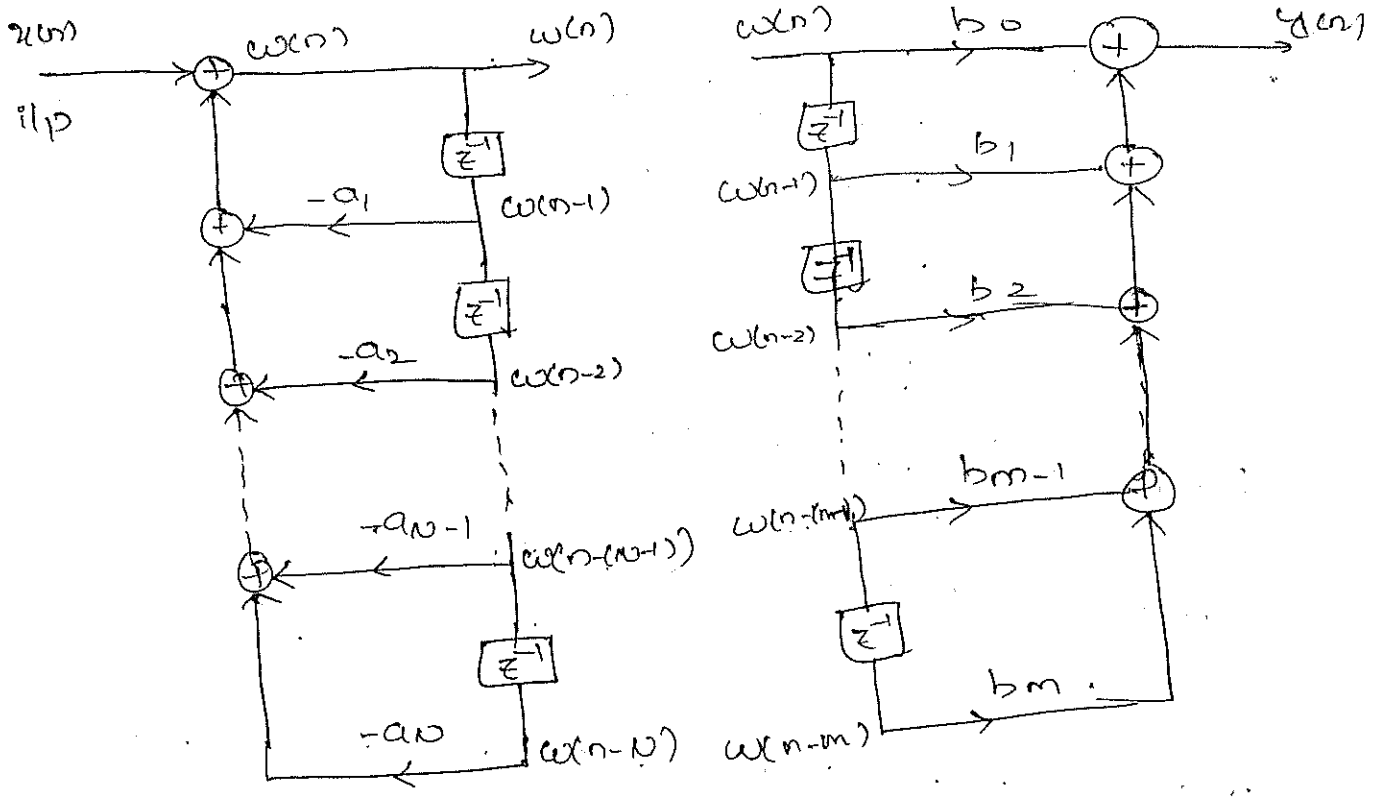
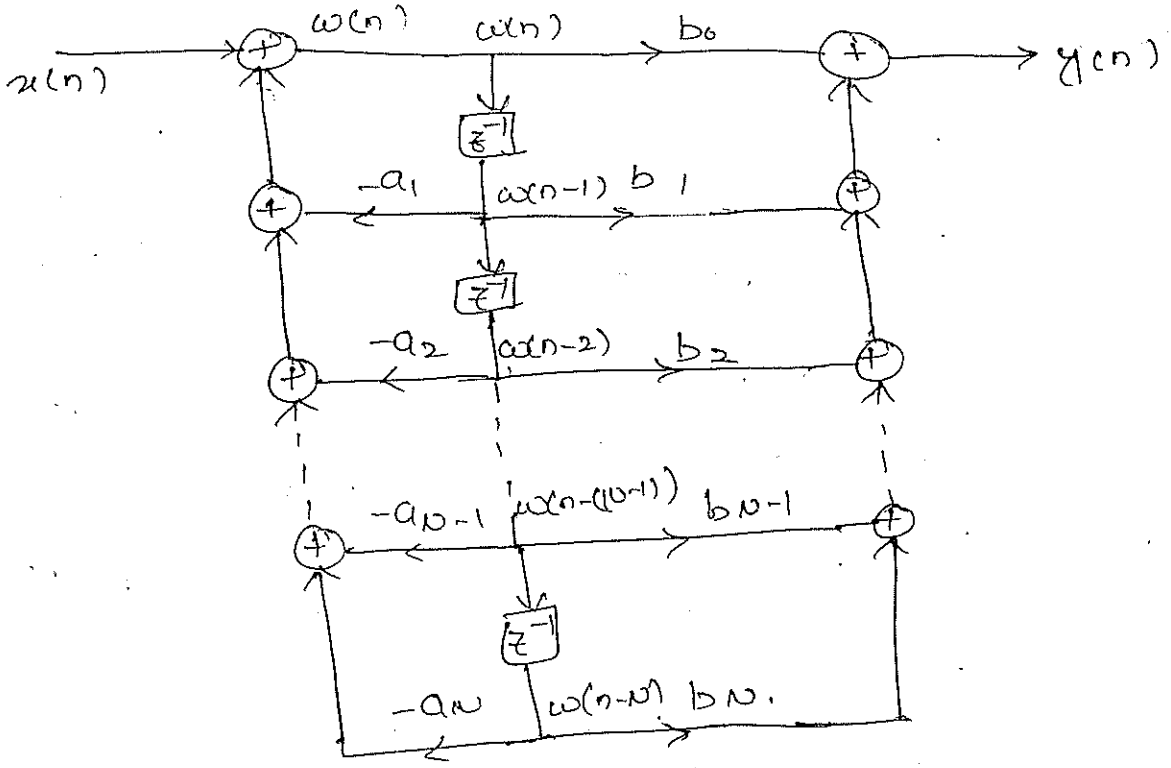


fig 9

Eqn (2) can be realized as show in fig.6.

for N=M

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Nth order

fig (a): Direct form - II structure for Nth order LCCDE.

The above structure is called Direct form - II structure, which used common delay factors both i/p & o/p side. By using this realization, we can reduce half of delay terms (or) memory locations. Compare with direct form - I structure. Therefore this realization is also known as canonic (or) min. no. of memory locations required realization. For this realization, the total no. of multiplier required = M+N.

total no. of adders = M+N.

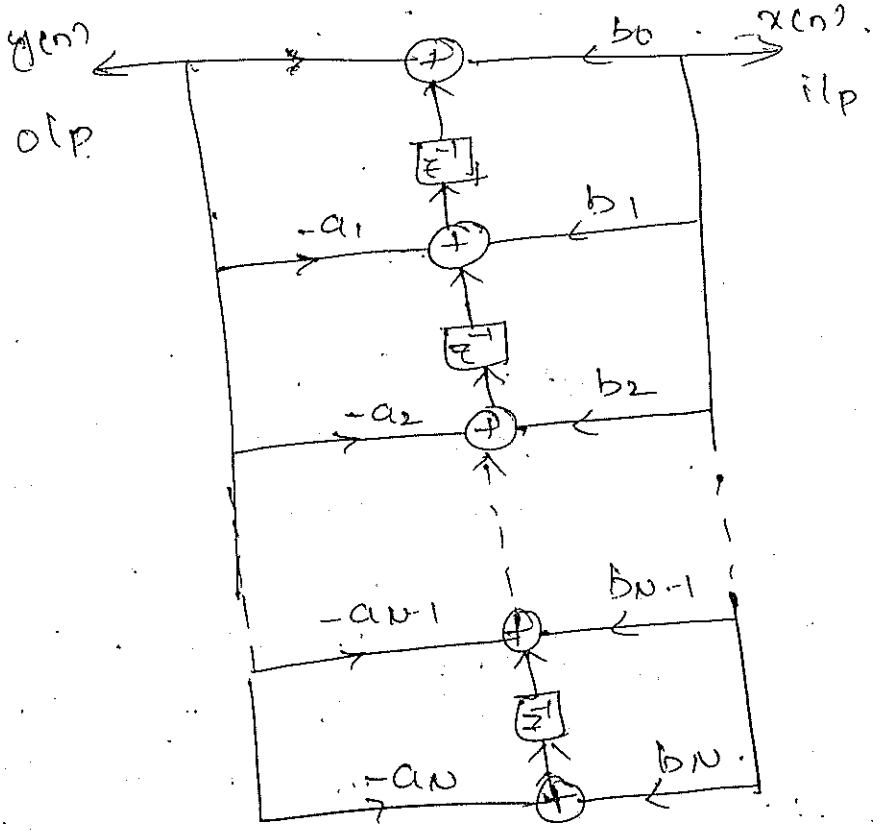
total no. of memory locations = max(M,N).

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(iii) Transposed structure:

Procedure steps:

1. from the given difference eqn, we have to implement Direct form - II structure.
2. Interchange i/ps and o/ps
3. Reverse the direction of flow of signals in the direct form II structure.
4. Interchange nodes and summing points



(iv) Cascade Realization Technique:

Cascade Realization is implemented by attaching all the factors of system function H(z) is in series that is cascad

Let us consider the Nth order system is described by system function

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$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

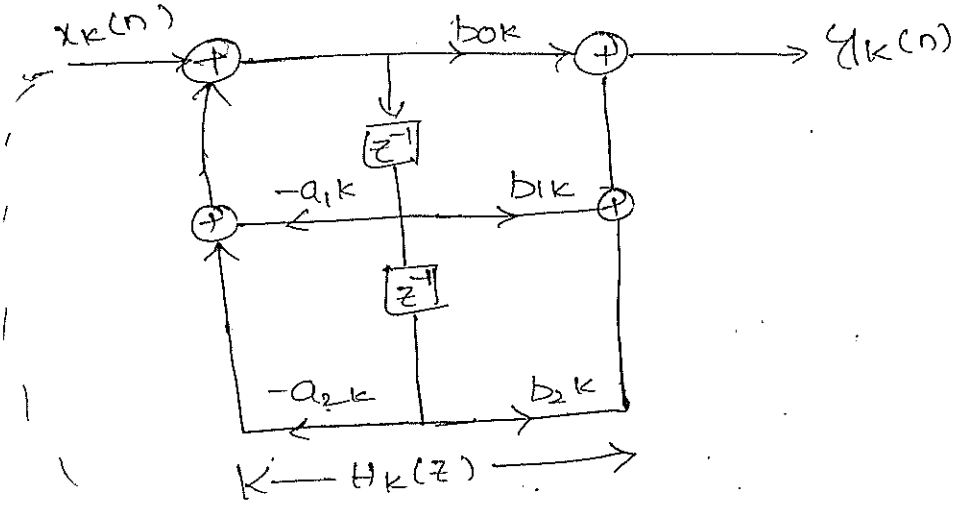
Let the system function $H(z)$ is represented in cascade of subsystems, each high-ested subsystem's highest degree is '2'.

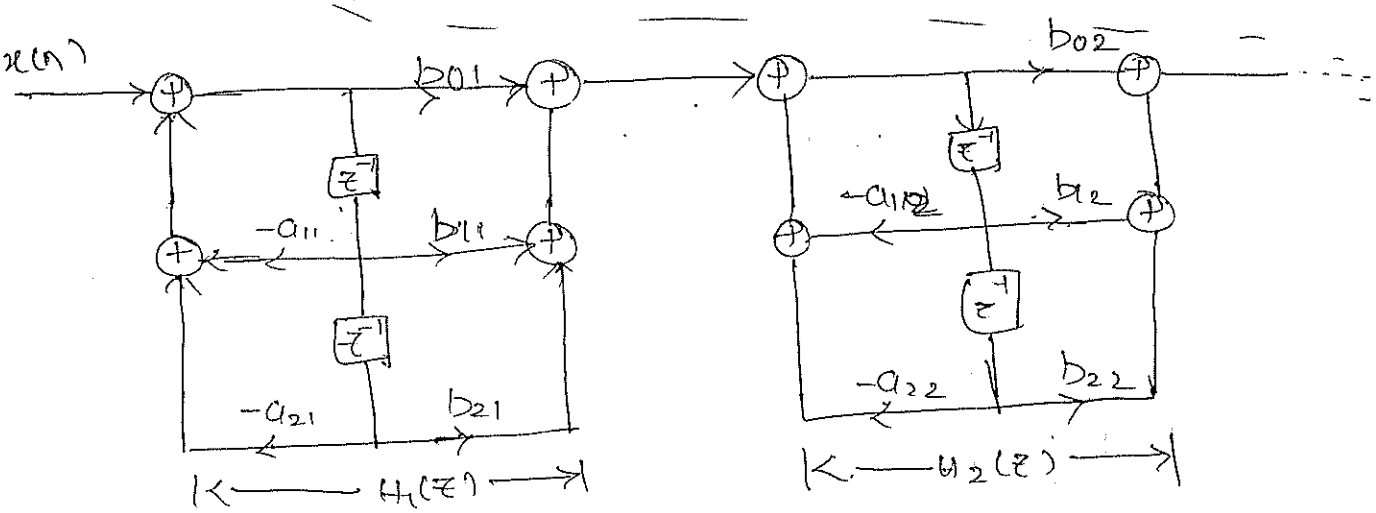
i.e., $H(z) = \prod_{k=1}^{N} H_k(z) = H_1(z) \cdot H_2(z) \cdots H_N(z)$

Here, N - Integer part of $\left(\frac{N+1}{2}\right)$

Π - product of subsystem of $H(z)$

$$H_k(z) = \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 + a_{1k} z^{-1} + a_{2k} z^{-2}}$$







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v) parallel form Realization technique:

parallel form realization is obtained by making use of partial fraction expansion of system function $H(z)$.

Let us consider N^{th} order system which is described by $H(z)$.

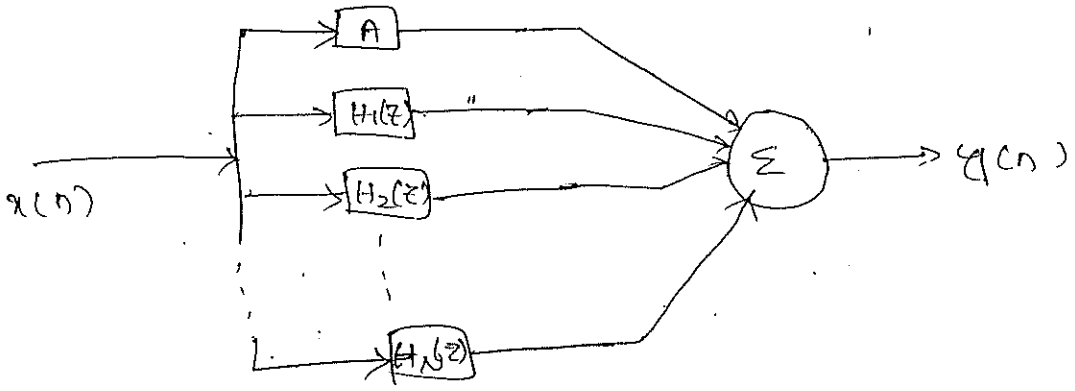
i.e.,

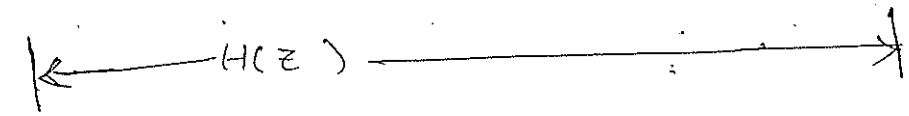
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{m} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} = A + \sum_{k=1}^{N} \frac{A_k}{1 - a_k z^{-1}}$$

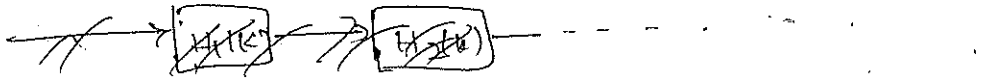
$$= A + \sum_{k=1}^{N} H_k(z)$$

$$H(z) = A + \sum_{k=1}^{N} H_k(z) = A + H_1(z) + H_2(z) + \cdots + H_N(z)$$

Here, $A = \frac{b_m}{a_N}$

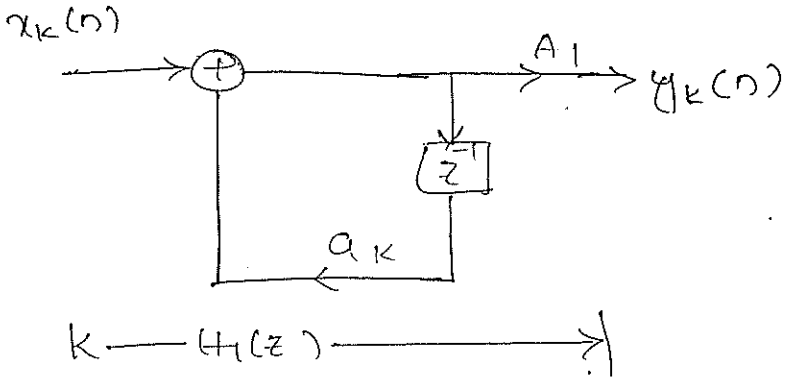


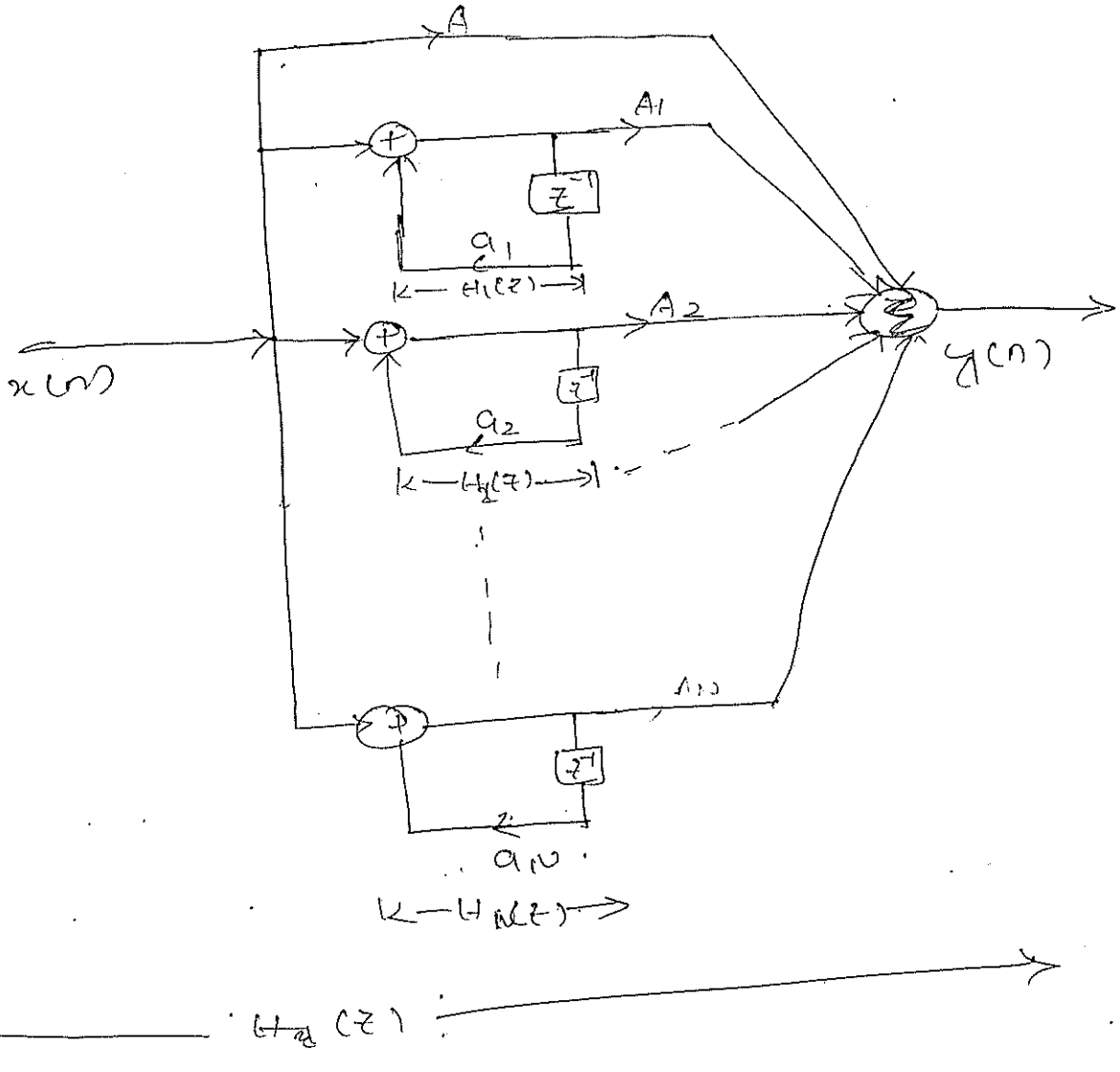




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$$H_K(z) = \frac{A_K}{1 - a_K z^{-1}}$$





K ← $H_2(z)$ →

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Problems:

1. Realize the following systems.

i. $y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$. by using

i) Direct form-I, Direct form II & transposed structure.

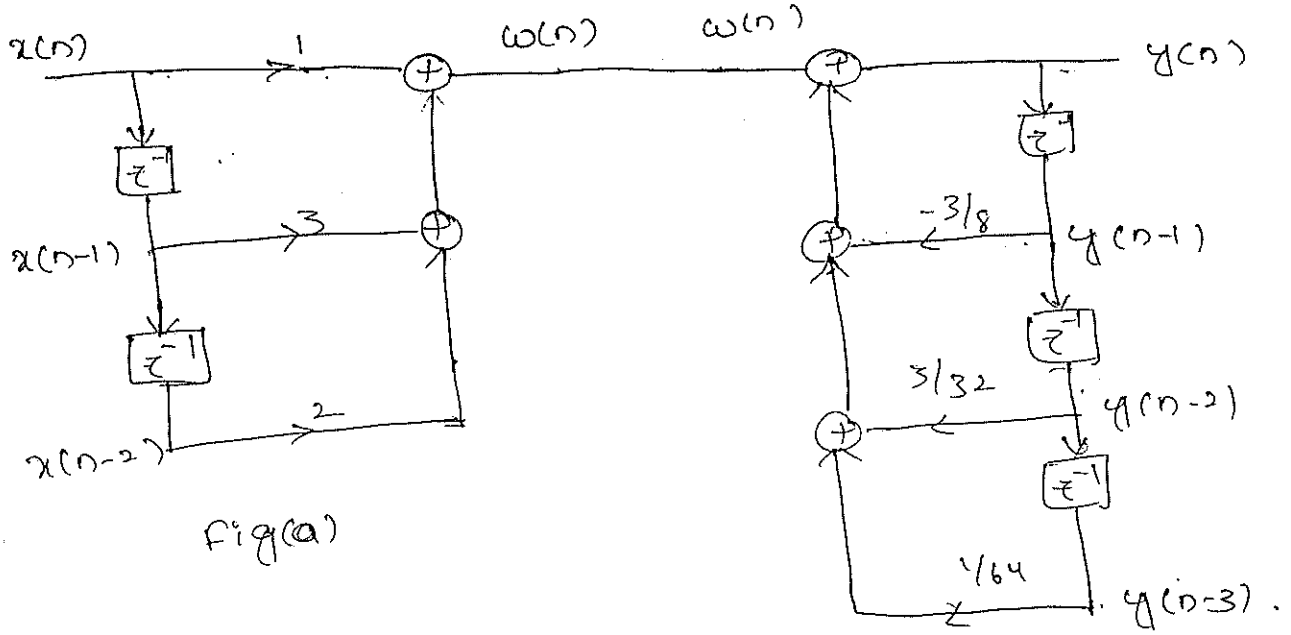
i) Direct form I

$$y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + x(n) + 3x(n-1) + 2x(n-2).$$

Let $w(n) = x(n) + 3x(n-1) + 2x(n-2)$, then. ——(1)

$$y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + w(n) \quad \text{——(11)}.$$

Eqn can be realized as show in figa. & eqn (2) as shown in fig(b)



fig(a)

fig(b).

fig(c).

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This realization requires

Total no. of multipliers = M+N+1
= 2+3+1
= 6.

Summers = M+N
= 2+3 = 5

Memory location = M+N
= 2+3
= 5.

Direct form II :-

$$y(n) = -\frac{3}{8}y(n-1) + \frac{3}{32}y(n-2) + \frac{1}{64}y(n-3) + x(n) + 3x(n-1) + 2x(n-2)$$

Applying Z.T on both sides, we get

$$Y(z) = -\frac{3}{8}z^{-1}Y(z) + \frac{3}{32}z^{-2}Y(z) + \frac{1}{64}z^{-3}Y(z) + X(z) + 3z^{-1}X(z) + 2z^{-2}X(z)$$

$$Y(z)\left[1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3}\right] = X(z)\left[1 + 3z^{-1} + 2z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1} + 2z^{-2}}{1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3}}$$

$$= \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} - \frac{1}{64}z^{-3}}$$

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$$W(z)\left[1+3/8\,z^{-1}-\frac{3}{32}z^{-2}-\frac{1}{64}z^{-3}\right]=X(z).$$

$$W(z)=X(z)-3/8\,z^{-1}W(z)+\frac{3}{32}z^{-2}W(z)+\frac{1}{64}z^{-3}W(z)$$

Applying Inverse z-Transform.

$$w(n)=x(n)-3/8\,w(n-1)+\frac{3}{32}w(n-2)+\frac{1}{64}w(n-3) \quad \text{——— (1)}.$$

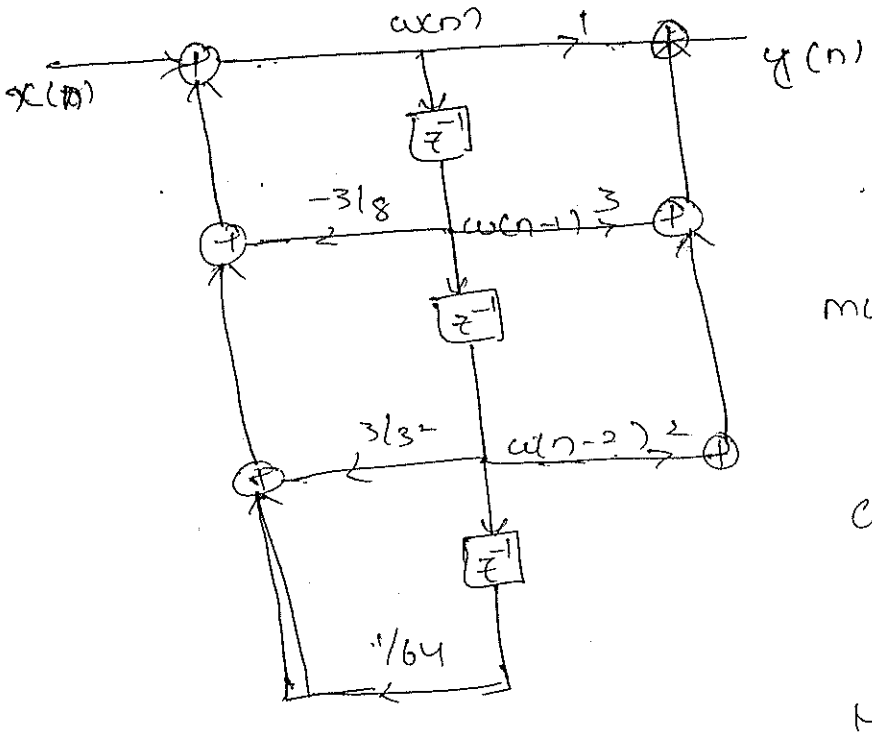
$$\frac{Y(z)}{W(z)}=1+3z^{-1}+2z^{-2}$$

$$Y(z)=\left[1+3z^{-1}+2z^{-2}\right]W(z)$$

$$=W(z)+3z^{-1}W(z)+2z^{-2}W(z).$$

Apply IZT.

$$y(n)=w(n)+3w(n-1)+2w(n-2) \quad \text{——— (2)}$$



multipliers = M+N+1

= 2+3+1

= 6.

adders = M+N

= 2+3

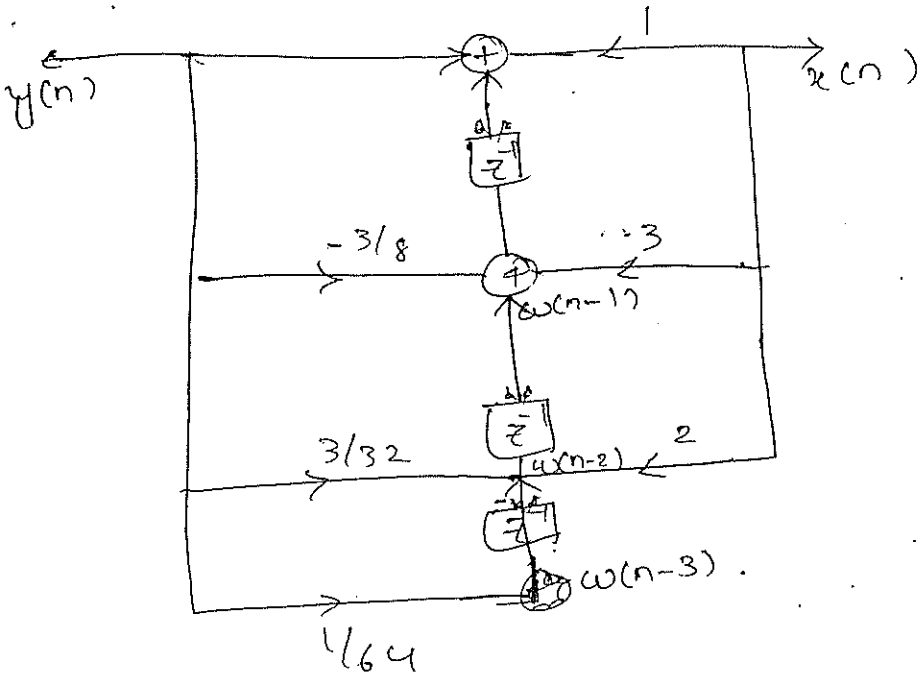
= 5.

Memory = max[2, 3]

= 3.

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Transposed structure:



multipliers =

2. Realise the following system using Direct form 2, Direct form 2 & transposed structure.

$$H(z) = \frac{8z^3 - 4z^2 + 11z - 2}{(z - 1/4)(z^2 - z + 1/2)}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-3}\left[8z^3 - 4z^2 + 11z - 2\right]}{z^{-3}\left[(z - 1/4)(z^2 - z + 1/2)\right]}$$

$$= \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - z^{-1} + \frac{z^{-2}}{2} - \frac{1}{4}z^{-1} + \frac{z^{-2}}{4} - \frac{1}{8}z^{-3}}$$

$$\frac{Y(z)}{X(z)} = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - 5/4\,z^{-1} + 3/4\,z^{-2} - \frac{1}{8}z^{-3}}$$

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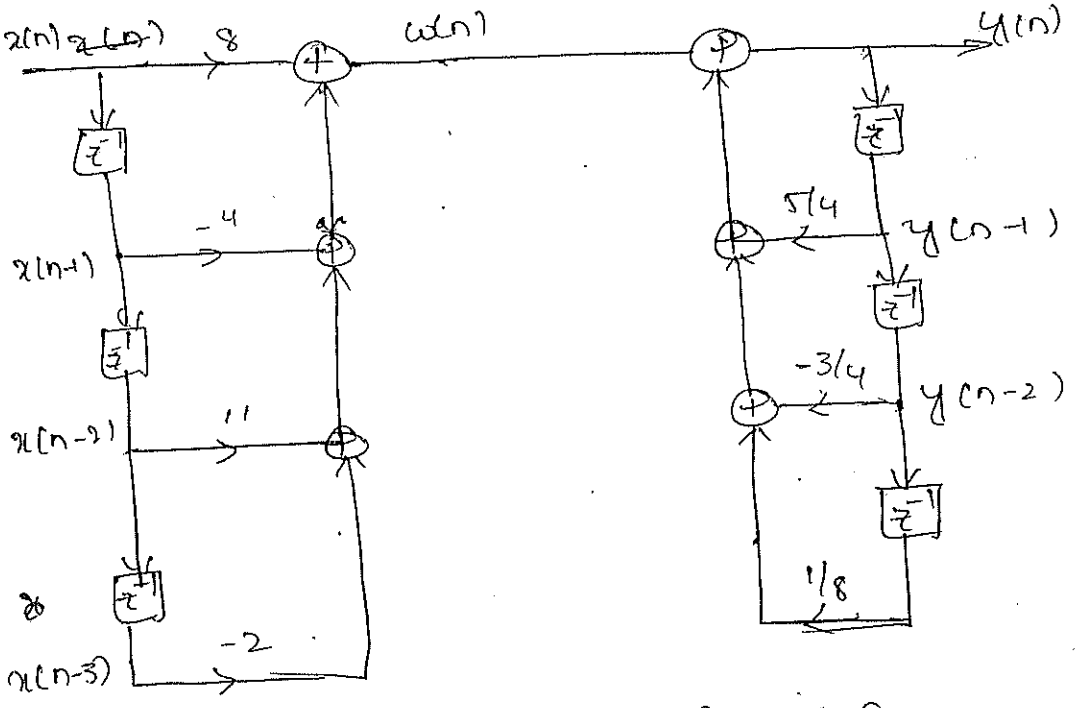
$$Y(z)\left[1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}\right] = \left(8 - 4z^{-1} + 11z^{-2} - 2z^{-3}\right)X(z)$$

Apply Z-T.

$$y(n) = \frac{5}{4}y(n-1) - \frac{3}{4}y(n-2) + \frac{1}{8}y(n-3) + 8x(n) - 4x(n-1) + 11x(n-2) - 2x(n-3)$$

$$w(n) = 8x(n) - 4x(n-1) + 11x(n-2) - 2x(n-3).$$

$$y(n) = \frac{5}{4}y(n-1) - \frac{3}{4}y(n-2) + \frac{1}{8}y(n-3) + w(n)$$



Direct form I.

adders = M+N+2

= 3+3+2 = 8.

multipliers = M+N+1 = 7.

Memory = M+N

= 6.

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ii)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - 5/4\, z^{-1} + 3/4\, z^{-2} - \frac{1}{8} z^{-3}}$$

$$= \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)},$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 - 5/4\, z^{-1} + \frac{3}{4} z^{-2} - \frac{1}{8} z^{-3}}$$

$$W(z)\left[1 - 5/4\, z^{-1} + \frac{3}{4} z^{-2} - \frac{1}{8} z^{-3}\right] = X(z).$$

Apply I.Z.T

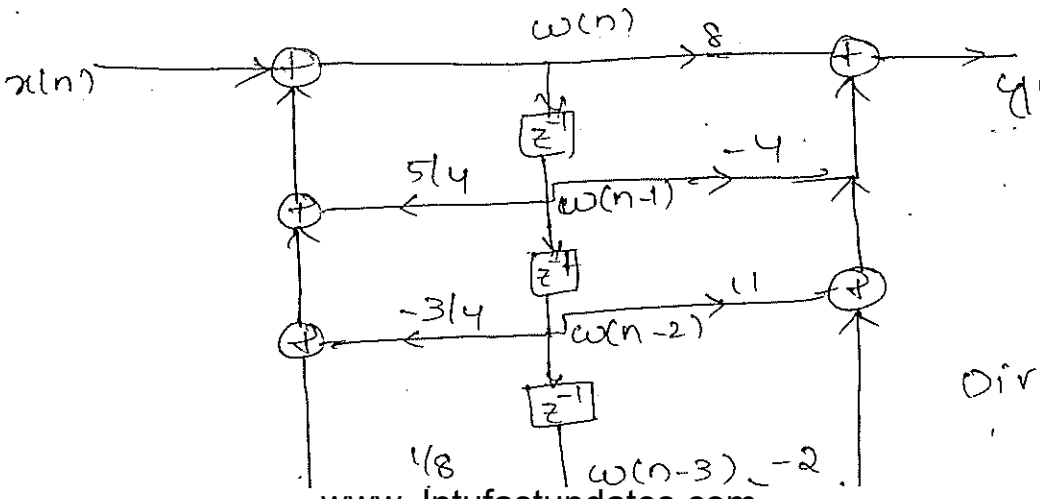
$$w(n) = x(n) + 5/4\, w(n-1) - 3/4\, w(n-2) + \frac{1}{8} w(n-3)$$

$$\frac{Y(z)}{W(z)} = 8 - 4z^{-1} + 11z^{-2} - 2z^{-3}.$$

$$Y(z) = (8 - 4z^{-1} + 11z^{-2} - 2z^{-3})\, W(z).$$

Apply I.Z.T.

$$y(n) = 8w(n) - 4w(n-1) + 11w(n-2) - 2w(n-3)$$



Direct form II

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multipliers = M+N+1

= 3+3+1 = 7.

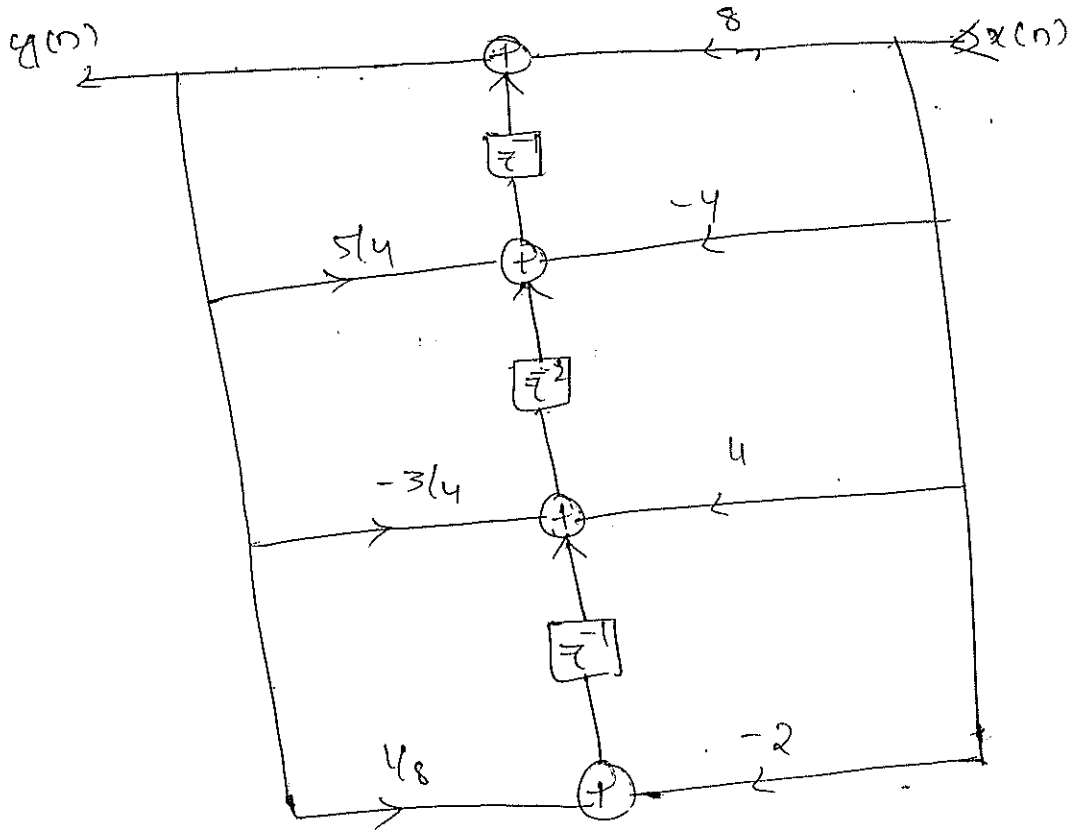
Adders = M+N

= 3+3 = 6.

Memory = max [M, N]

= 3.

Transposed structure:



* Realize the following system.

$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3\,x(n) + 3.6\,x(n-1) + 0.6\,x(n-2)$. by using i) Direct form I ii) Direct form II iii) Canonic. Transposed iv) cascade v) parallel form realisations.

Sol:-

Cascade Realization:

$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3x(n) + 3.6x(n-1) + 0.6\,x(n-2)$,

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Applying z-transform on both sides

$$Y(z) = -0.1 z^{-1} Y(z) + 0.2 z^{-2} Y(z) + 3X(z) + 3.6 z^{-1} X(z) + 0.6 z^{-2} X(z)$$

$$Y(z)\left[1 + 0.1 z^{-1} - 0.2 z^{-2}\right] = \left[3 + 3.6 z^{-1} + 0.6 z^{-2}\right] X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6 z^{-1} + 0.6 z^{-2}}{1 + 0.1 z^{-1} - 0.2 z^{-2}} \times \frac{z^2}{z^2}$$

$$= \frac{3z^2 + 3.6z + 0.6}{z^2 + 0.1z - 0.2}$$

$$= \frac{(3z + 0.6)(z+1)}{(z+0.5)(z-0.4)}$$

$$H(z) = \left[\frac{3z+0.6}{z+0.5}\right]\left[\frac{z+1}{z-0.4}\right] = \left[\frac{3 + 0.6 z^{-1}}{1 + 0.5 z^{-1}}\right]\left[\frac{1 + z^{-1}}{1 - 0.4 z^{-1}}\right]$$

$$= H_1(z) \cdot H_2(z)$$

$$H_k(z) = \frac{3 + 0.6 z^{-1}}{1 + 0.5 z^{-1}} \cdot \frac{A_k + b_k z^{-1}}{1 + a_k z^{-1}} = \frac{Y_k(z)}{X_k(z)} = \frac{Y_k(z)}{W_k(z)} \cdot \frac{W_k(z)}{X_k(z)}$$

$$\frac{W_k(z)}{X_k(z)} = \frac{1}{1 + a_k z^{-1}} \Rightarrow W_k(z) + W_k(z) a_k z^{-1} = X_k(z)$$

$$W_k(z) = X_k(z) - a_k z^{-1} W_k(z)$$

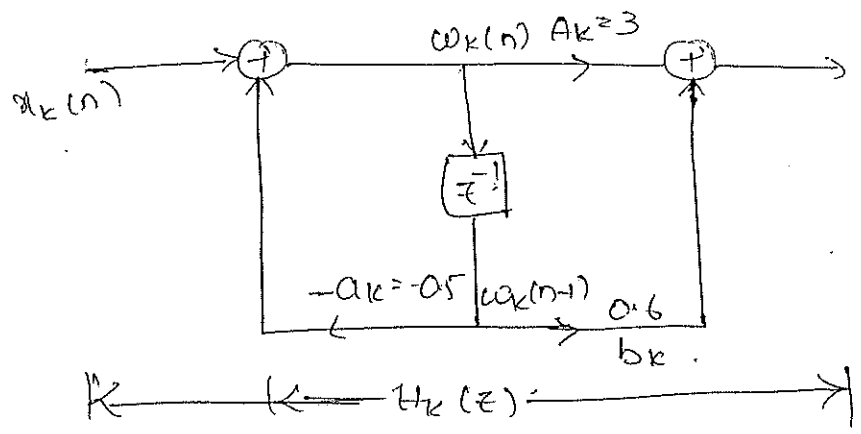
$$W_k(n) = X_k(n) - a_k W_k(n-1) \quad \longrightarrow (1)$$

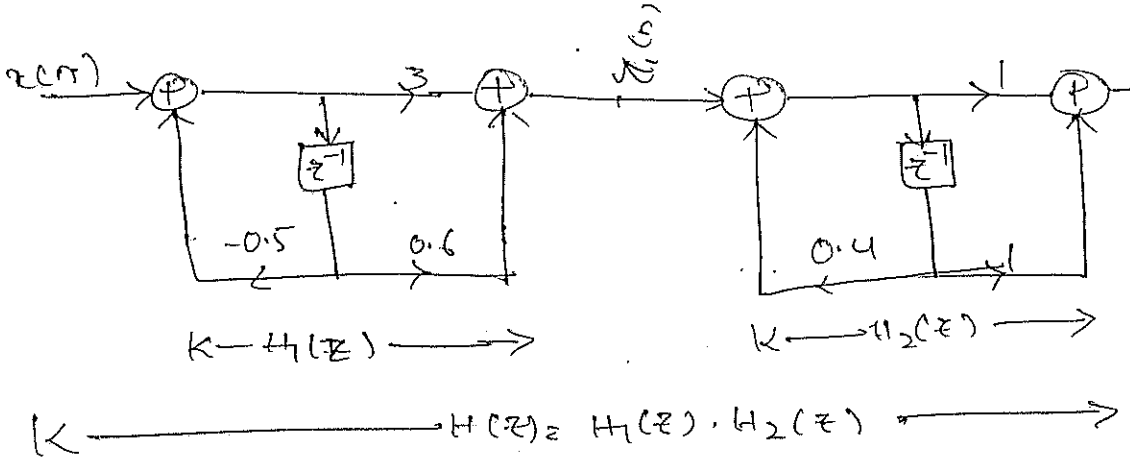
$$\frac{Y_k(z)}{W_k(z)} = A_k + b_k z^{-1} \Rightarrow Y_k(z) = A_k W_k(z) + b_k z^{-1} W_k(z)$$

$$y_k(n) = A_k W_k(n) + b_k W_k(n-1)$$

$\longrightarrow (2)$.

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Parallel form Realization:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

$$= \frac{3z^2+3.6z+0.6}{z^2+0.1z-0.2} = \frac{(3z+0.6)(z+1)}{(z+0.5)(z-0.4)}$$

$$= 3 + \frac{3.3z+1.2}{z^2+0.1z-0.2}$$

$$z^2+0.1z-0.2 \;\big|\; 3z^2+3.6z+0.6 \;\big|\; 3$$
$$3z^2+0.3z-0.6$$
$$3.3z+1.2$$

$$= 3 + \frac{3.3z+1.2}{(z+0.5)(z-0.4)}$$

$$\frac{H(z)}{z} = \frac{3z+0.6}{z(z^2+0.1z-0.2)} \cdot \frac{3z^2+3.6z+0.6}{z(z+0.5)(z-0.4)}$$

$$= \frac{A}{z} + \frac{B}{z+0.5} + \frac{C}{z-0.4}$$

$$\frac{H(z)}{z} = \frac{-3}{z} + \frac{-1}{z+0.5} + \frac{4}{z-0.4}$$

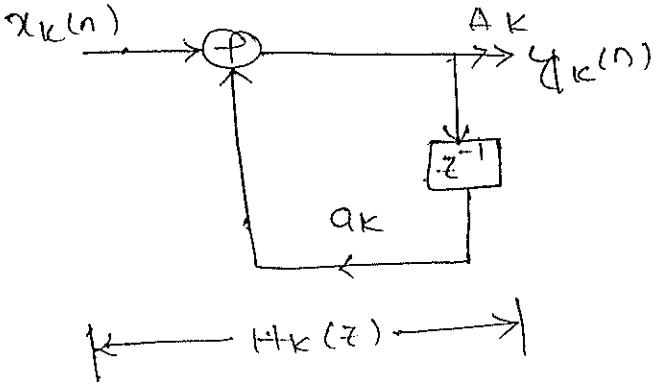
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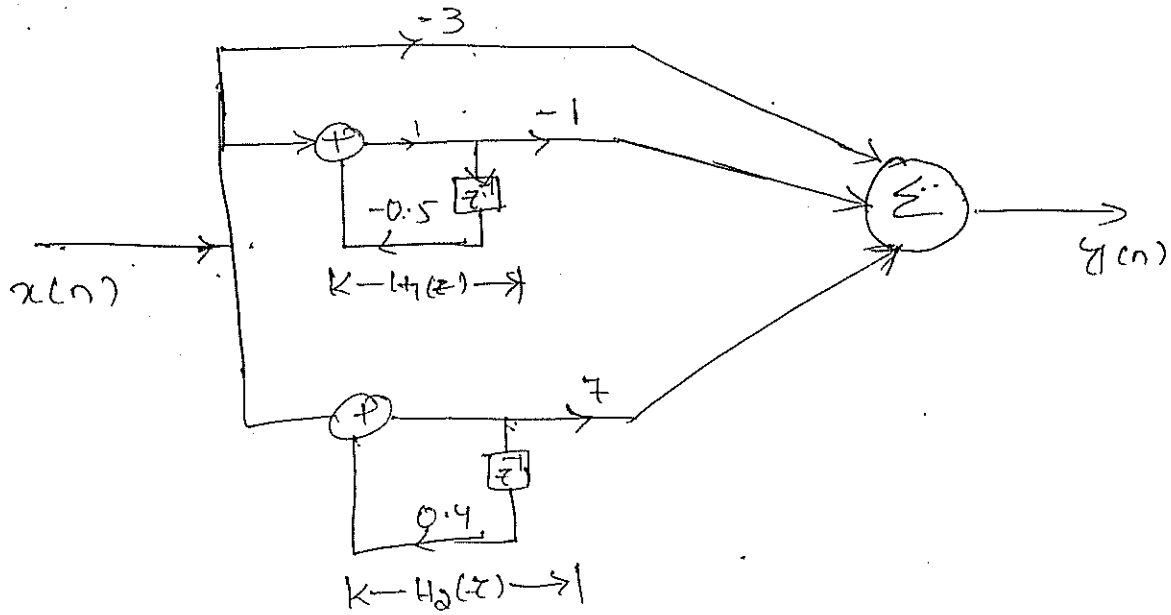
$$H(z) = -3 + \frac{-z}{z+0.5} + \frac{7z}{z-0.4}$$

$$H(z) = -3 + \frac{-1}{1+0.5z^{-1}} + \frac{7}{1-0.4z^{-1}}$$

$$= A + H_1(z) + H_2(z).$$

$$H_K(z) = \frac{A_K}{1-a_K z^{-1}} = \frac{Y_K(z)}{X_K(z)}$$





FIR Digital filters structures (or) Non Recursive Realization techniques:

There are 3 well known techniques to realize FIR digital filters.

1. Direct form realization (or) Transversal realization (or) Tapped delay line filter.

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2. Cascade Realization

3. Linear phase Realization (or) Min no of multiplier realization technique.

1. Direct form Realization technique:

If the system having finite duration impulse response sys sequence then it is called FIR system.

If the length of impulse response is N

i.e.,

$h(n) = \{h(0), h(1), h(2), \ldots h(N-1)\}$;

length

$$T.f = H(z) = \frac{Y(z)}{X(z)} = Z.T\left[h(n)\right] = \sum_{n=0}^{N-1}\left[h(n)\right]z^{-n}.$$

$$= \sum_{k=0}^{N-1}\left[h(k)\right]z^{-k}.$$

$$Y(z) = \sum_{k=0}^{N-1} h(k)\, z^{-k} X(z).$$

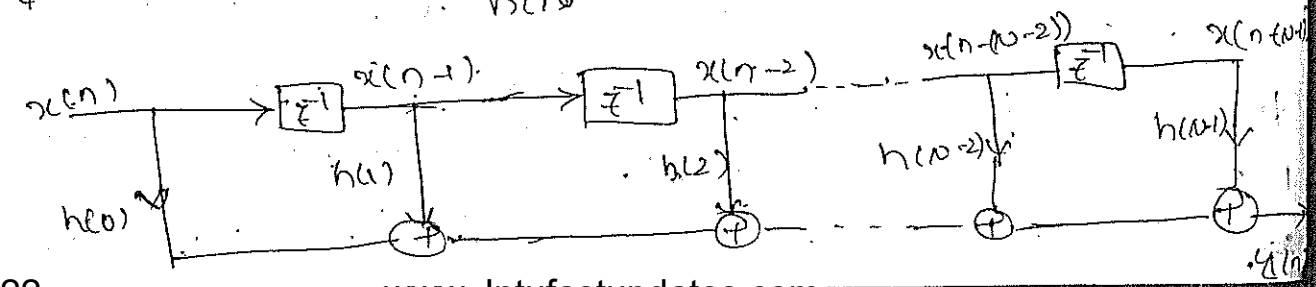
Applying I.Z.T & linearity property, we get.

$$I.Z.T\left[Y(z)\right] = I.Z.T\left[\sum_{k=0}^{N-1} h(k) z^{-k} X(z)\right]$$

$$y(n) = \sum_{k=0}^{N-1} h(k)\, I.Z.T\left[z^{-k}X(z)\right]$$

$$= \sum_{k=0}^{N-1} h(k)\, x(n-k).$$

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots + h(N-1)x(n-(N-1)) \quad \text{---- (1)}.$$



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This realization requires,

total no. of multipliers = N.

total no. of adders = N-1.

Memory locations = N-1.

Cascade Realization:

This realization is implemented all the subsystems of system function H(z) can be connected in series i.e., cascade.

Case(i): If 'N' is odd

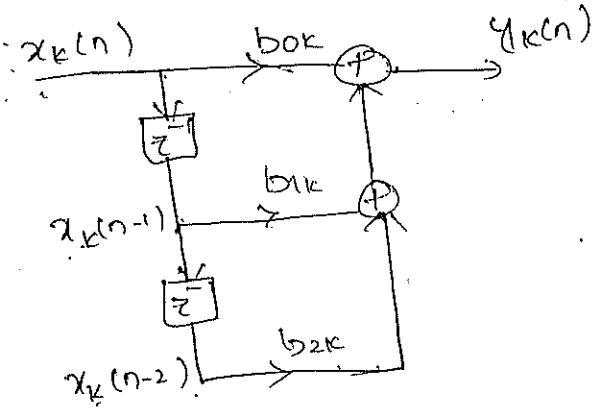
$$H(z) = \prod_{k=1}^{\frac{N-1}{2}} H_k(z) = \prod_{k=1}^{\frac{N-1}{2}} \left[b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2} \right]$$

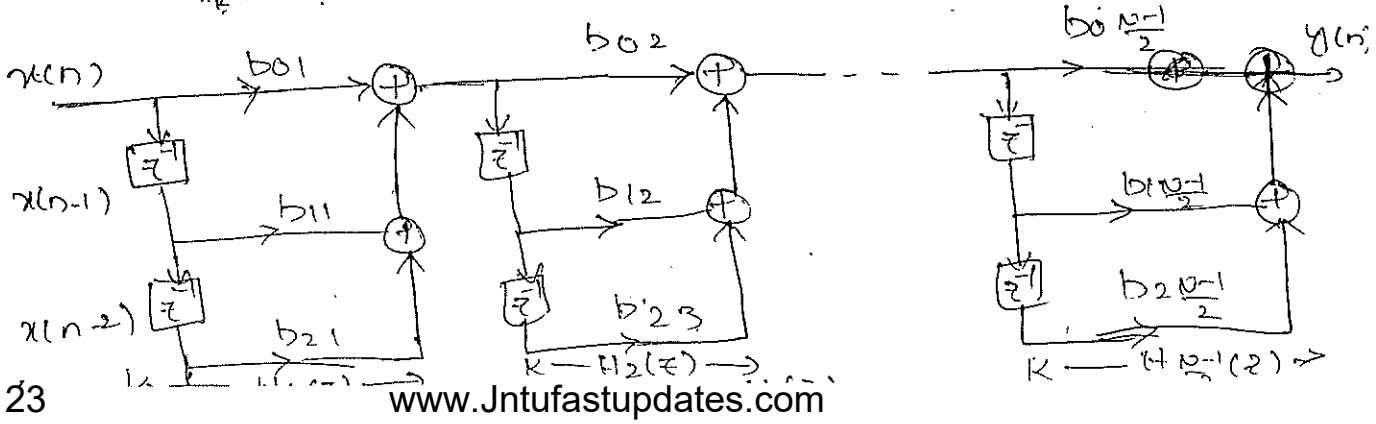
$$= H_1(z) \cdot H_2(z) \cdot \ldots H_{\frac{N-1}{2}}(z)$$

N = odd, $\frac{N-1}{2}$ = even & H(z) have $\left(\frac{N-1}{2}\right)$ second order systems.

$$H_k(z) = \frac{Y_k(z)}{X_k(z)} = b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}$$

$$y_k(n) = b_{0k} x_k(n) + b_{1k} x_k(n-1) + b_{2k} x_k(n-2) \quad \text{---(1)}$$





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If N is even, it has one first order system and $\frac{N}{2}-1$ second order systems.

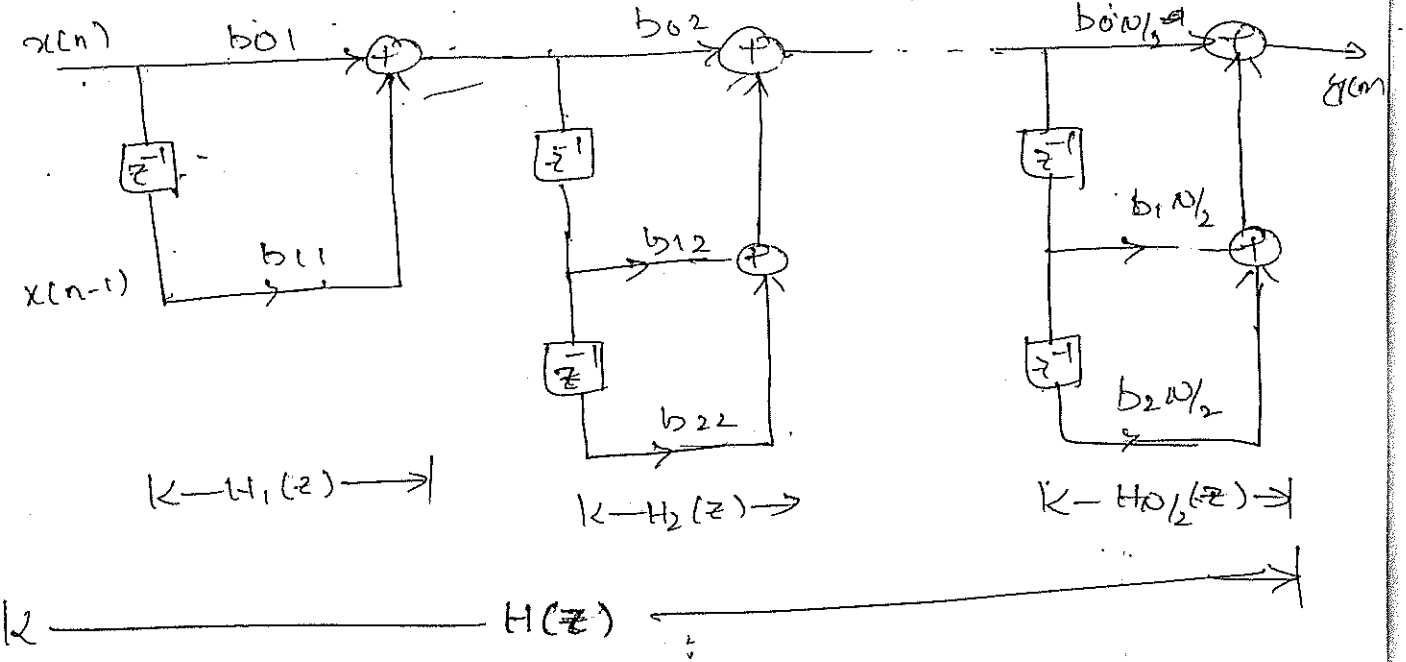
$$H(z) = (b_{01} + b_{11}z^{-1}) \cdot \prod_{k=2}^{N/2} H_k(z)$$

$$= (b_{01} + b_{11}z^{-1}) \prod_{k=2}^{N/2} [b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}]$$

$$= (b_{01} + b_{11}z^{-1}) H_2(z) \cdot H_3(z) \cdots H_{\frac{N}{2}}(z).$$

$$H_k(z) = \frac{Y_k(z)}{X_k(z)} = b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}$$

$$y_k(n) = b_{0k}x_k(n) + b_{1k}x_k(n-1) + b_{2k}x_k(n-2)$$



* Realize the following system using direct form realization.

$$h(n) = \delta(n) - 2\delta(n-1) + 3\delta(n-2) + 2\delta(n-4) + 5\delta(n-6) - \delta(n-7).$$

Sol:-

$$h(n) = \{1, -2, 3, 0, 2, 0, 5, -1\}$$
$$\quad\quad\quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$$

$$H(z) = Z.T[h(n)] = \sum_{n=-\infty}^{\infty} [h(n)]z^{-n}$$

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$$= \sum_{n=0}^{7} h(n) z^{-n}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} + h(6) z^{-6} + h(7) z^{-7}.$$

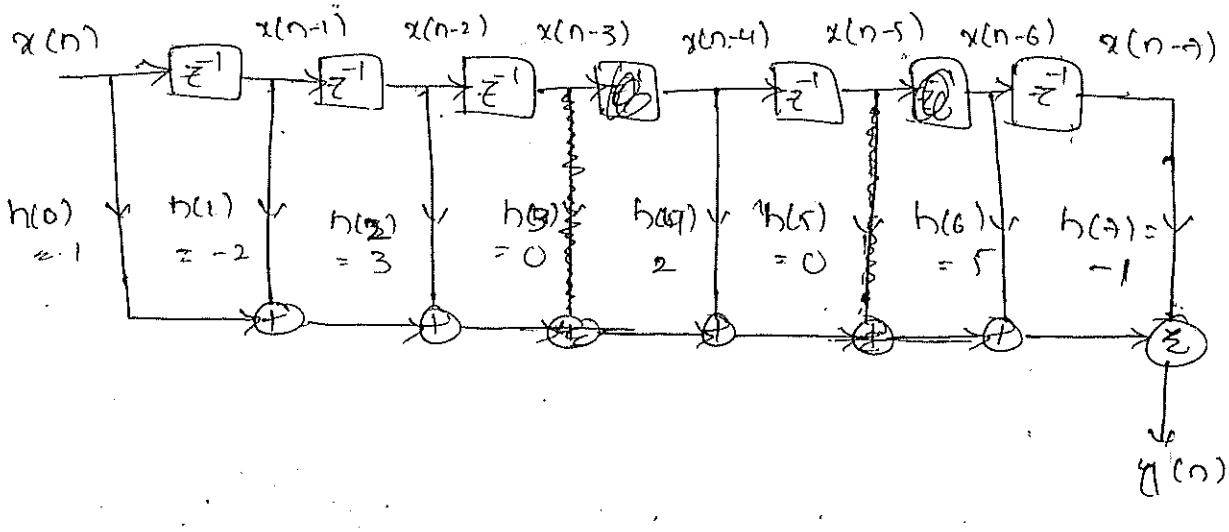
$$H(z) = \frac{Y(z)}{X(z)} = 1 - 2z^{-1} + 3z^{-2} + 2z^{-4} + 5z^{-6} - z^{-7}.$$

$$Y(z) = X(z) - 2z^{-1} X(z).$$

$$Y(z) = X(z) - 2z^{-1}X(z) + 3z^{-2}X(z) + 2z^{-4}X(z) + 5z^{-6}X(z) - z^{-7}X(z).$$

Apply I.Z.T & linearity property of Z.T, we get.

$$y(n) = x(n) - 2x(n-1) + 3x(n-2) + 2x(n-4) + 5x(n-6) - x(n-7) \quad \text{---} \quad (1).$$



* Realize the following system using

1. Cascade Realization
2. Direct form Realization.

$$H(z) = (1 - z^{-1} + 2z^{-2} + 3z^{-3} - z^{-4})(2 - z^{-1} - 4z^{-3} + 5z^{-5})$$

$$H(z) = H_1(z) \cdot H_2(z).$$

$$H_1(z) = 1 - z^{-1} + 2z^{-2} + 3z^{-3} - z^{-4} = \frac{Y_1(z)}{X_1(z)}$$

$$H_2(z) = 2 - z^{-1} - 4z^{-3} + 5z^{-5} = \frac{Y_2(z)}{X_2(z)}$$

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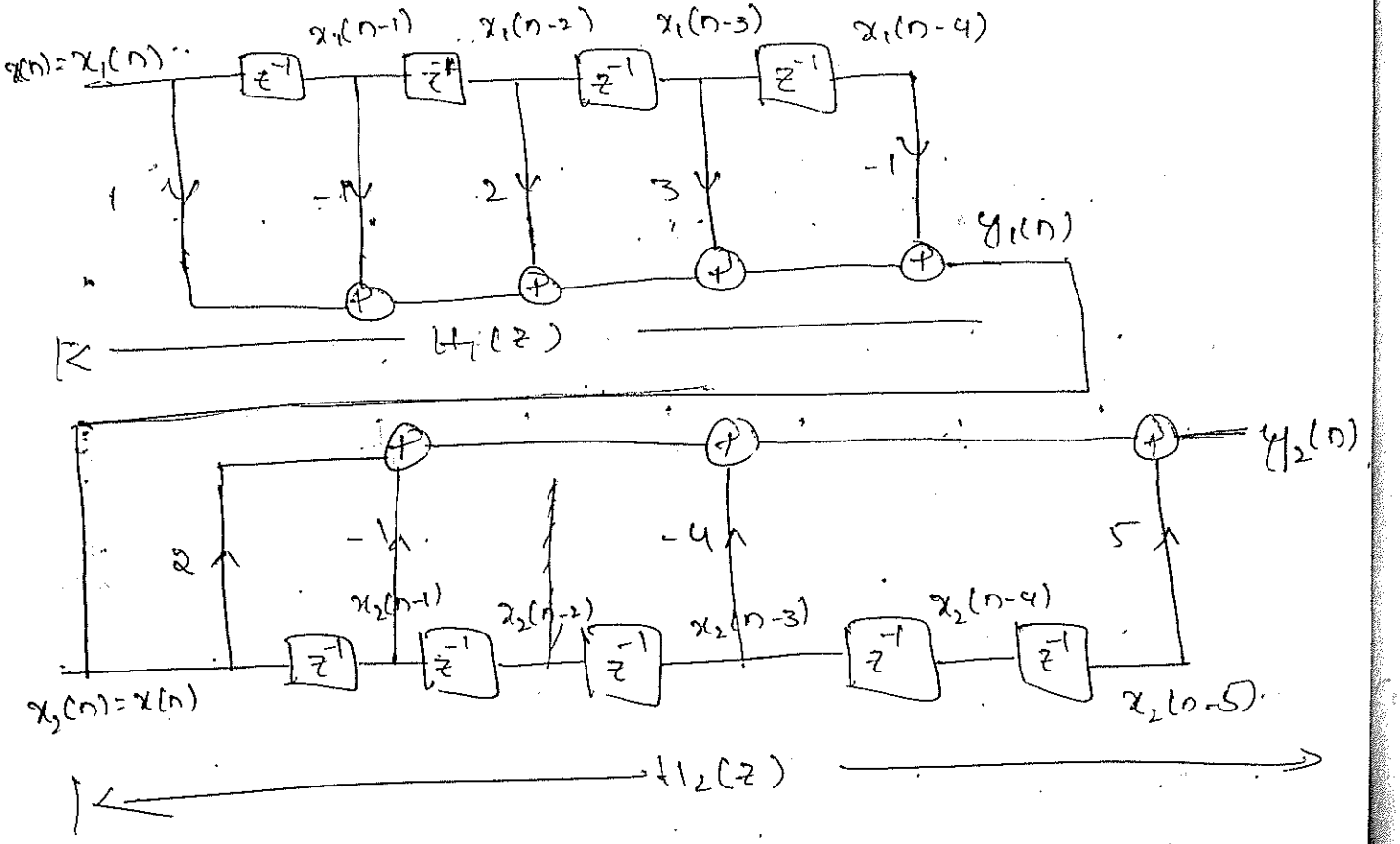
$y_1(n) =$

$$Y_1(z) = X_1(z) - z^{-1}X_1(z) + 2z^{-2}X_1(z) + 3z^{-3}X_1(z) - X_1(z)z^{-4}.$$

$$y_1(n) = x_1(n) - x_1(n-1) + 2x_1(n-2) + 3x_1(n-3) - x_1(n-4) \quad \text{---(1)}$$

$$Y_2(z) = 2X_2(z) - X_2(z)z^{-1} - 4z^{-3}X_2(z) + 5z^{-5}X_2(z)$$

$$y_2(n) = 2x_2(n) - x_2(n-1) - 4x_2(n-3) + 5x_2(n-5) \quad \text{---(2)}$$



Direct form:

$$H(z) = (1 - z^{-1} + 2z^{-2} + 3z^{-3} - z^{-4})(2 - z^{-1} - 4z^{-3} + 5z^{-5})$$

$$= 2 - z^{-1} - 4z^{-3} + 5z^{-5} - 2z^{-1} + z^{-2} + 4z^{-4} - 5z^{-6} + 4z^{-2} - 2z^{-3} - 8z^{-5} + 10z^{-7} + 6z^{-3} - 3z^{-4} - 12z^{-6} + 15z^{-8} - 2z^{-4} + z^{-5} + 4z^{-7} - 5z^{-9}$$

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$= 2 - 3z^{-1} + 5z^{-2} + z^{-4}$

$X_1(z)$

$z-3)$

$X_2(z)$

$z-5)$

$z^{-5})$

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Linear phase Realization technique / Minimum no. of multipliers required realization:-

for linear phase FIR digital filter, impulse response satisfies the following condition.

$$h(n) = h(N-1-n) \quad \forall\ 'n'.$$

N - length of $h(n)$

$$h(n) = \{h(0), h(1), h(2), \ldots h(N-1)\}.$$

$$T.f = Z.T[h(n)] = H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + \ldots + h(N-1)z^{-(N-1)}$$

Case(i) :-

If 'N' is even

$$T.f = Z.T[h(n)] = H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} [h(n)] z^{-n} + \sum_{n=\frac{N}{2}}^{N-1} [h(n)] z^{-n}$$

put $n = N-1-m$.

$m = N-1-n$.

$n = N/2$; $m \to (N-1) - \frac{N}{2} = \frac{N}{2} - 1$.

$n = N-1$; $m \to (N-1) - (N-1) = 0$.

$$\ldots z^{-n} + \sum_{m=\frac{N}{2}-1}^{0} h(N-1-m) z^{-(N-1-m)}$$

put $m = n$.

$$= \sum_{n=0}^{N/2-1} h(n) z^{-n} + \sum_{n=0}^{N/2-1} h(N-1-n) z^{-(N-1-n)}$$

$\because h(n) = h(N-1-n)$.

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$$= \sum_{n=0}^{N/2-1} h(n) z^{-n} + \sum_{n=0}^{N/2-1} h(n) z^{-(N-1-n)}$$

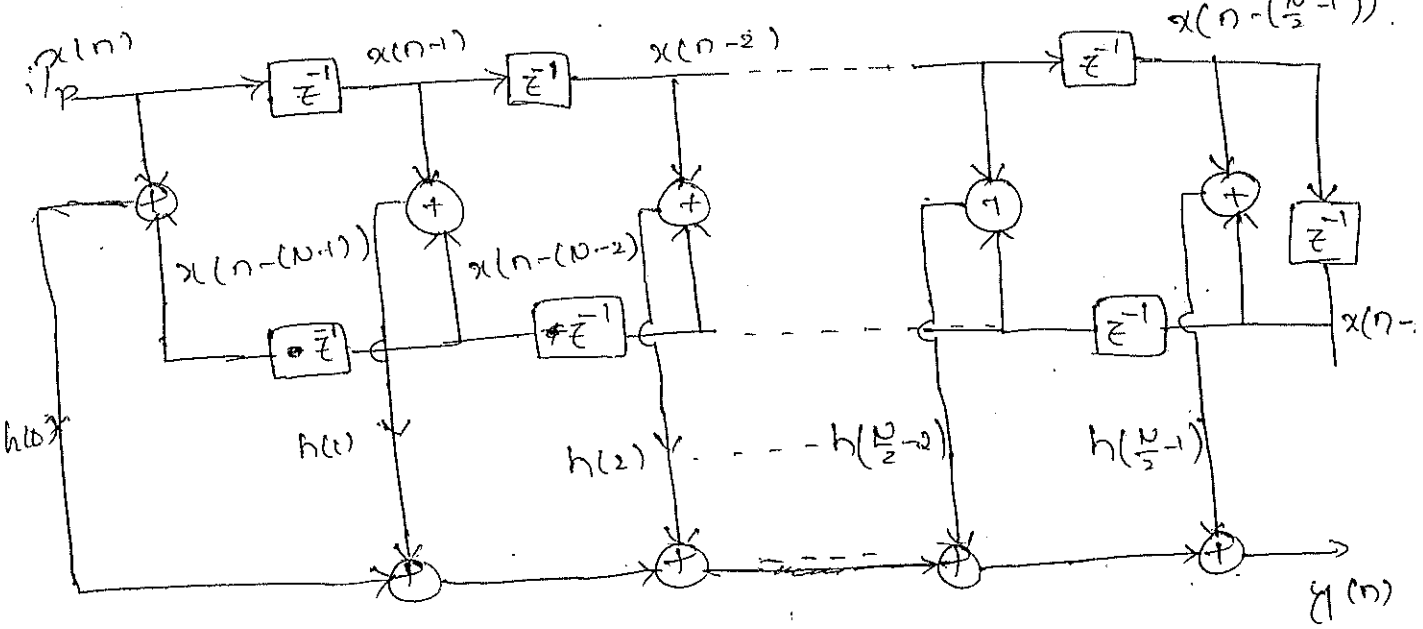
$$H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N/2-1} h(n) \left[z^{-n} + z^{-(N-1-n)} \right]$$

$$Y(z) = \sum_{k=0}^{N/2-1} h(k) \left[z^{-k} X(z) + z^{-(N-1-k)} X(z) \right]$$

Applying linearity property.

$$y(n) = \sum_{k=0}^{N/2-1} h(k) \left[x(n-k) + x(n-(N-1-k)) \right]$$

$$y(n) = h(0)\left[x(n) + x(n-(N-1))\right] + h(1)\left[x(n-1) + x(n-(N-2))\right] + \cdots + h\left(\frac{N}{2}-1\right)\left[x\left(n-\left(\frac{N}{2}-1\right)\right) + x\left(n-\left(\frac{N}{2}\right)\right)\right] \quad \text{---(1)}$$



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for Direct form realization, total no. of multi-pliers required = N.

for linear phase realization & N is even, total no. of multipliers required = $N/2$.

Case(ii) :-

If N is odd number.

$$T.F = Z.T[h(n)] = H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-n} + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) z^{-n}$$

put $n = N-1-m$

$m = N-1-n$.

$n \to \frac{N+1}{2} \; ; \; m \to (N-1)-\left(\frac{N+1}{2}\right) = \frac{N-3}{2}$

$n \to N-1 \; ; \; m = 0$

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-n} + \sum_{m=\frac{N-3}{2}}^{0} h(N-1-m) z^{-(N-1-m)}$$

put $m = n$

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-n} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n) z^{-(N-1-n)}$$

$\because h(n) = h(N-1-n)$

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-n} + \sum_{n=0}^{\frac{N-3}{2}} h(n) z^{-(N-1-n)}$$

$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}$

$$= h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n) \left[z^{-n} + z^{-(N-1-n)} \right]$$

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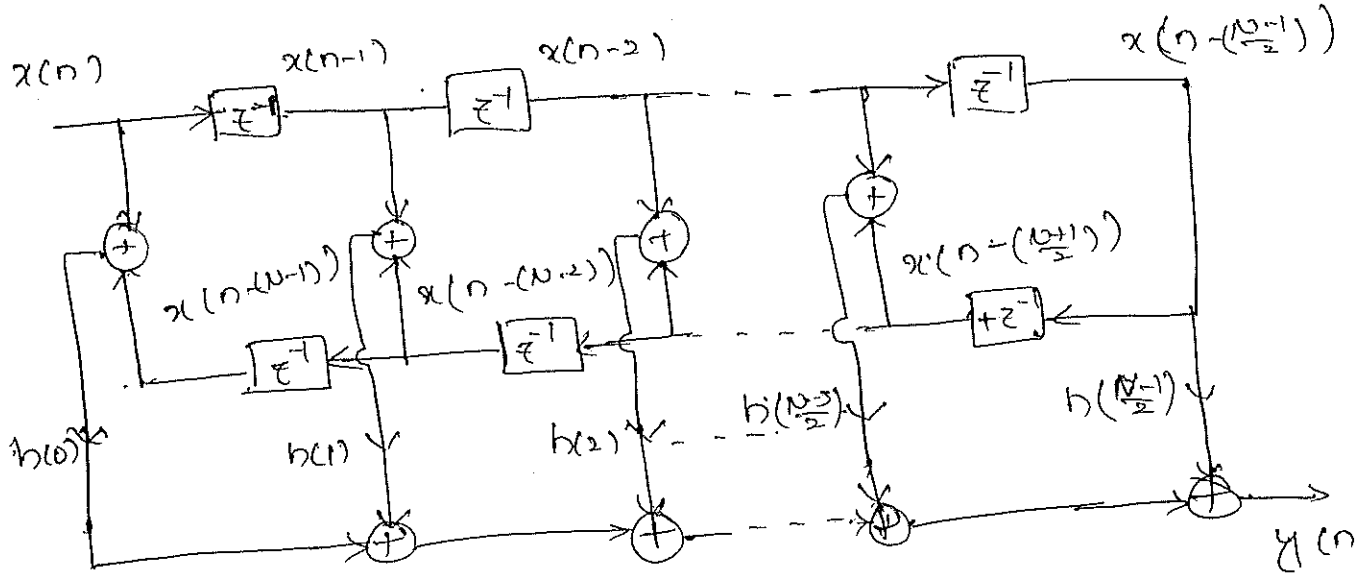
$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{\frac{N-3}{2}} h(k)\left[z^{-k} + z^{-(N-1-k)}\right] + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)}$$

$$Y(z) = \sum_{k=0}^{\frac{N-3}{2}} h(k)\left[z^{-k}X(z) + z^{-(N-1-k)}X(z)\right] + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} X(z).$$

Applying L.T.I & linearity prop.

$$y(n) = \sum_{k=0}^{\frac{N-3}{2}} h(k)\left[x(n-k) + x(n-(N-1-k))\right] + h\left(\frac{N-1}{2}\right) x\left(n-\left(\frac{N-1}{2}\right)\right)$$

$$y(n) = h(0)\left[x(n) + x(n-(N-1))\right] + h(1)\left[x(n-1) + x(n-(N-2))\right] + \cdots + h\left(\frac{N-3}{2}\right)\left[x\left(n-\left(\frac{N-3}{2}\right)\right) + x\left(n-\left(\frac{N+1}{2}\right)\right)\right] + h\left(\frac{N-1}{2}\right) x\left(n-\left(\frac{N-1}{2}\right)\right) \quad \text{--- (1)}$$



for N = odd.

no. of multipliers = $\frac{N+1}{2}$.

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* Realise the following system using...

i) Direct form ii) linear phase iii) ~~cascade~~ R.T.

$$h(n) = \frac{1}{2}\delta(n) + \frac{1}{3}\delta(n-1) + \delta(n-2) + \frac{1}{4}\delta(n-3) + \delta(n-4) + \frac{1}{3}\delta(n-5) + \frac{1}{2}\delta(n-6).$$

Sol:- i) Direct form:-

$$h(n) = \left\{\frac{1}{2}, \frac{1}{3}, 1, \frac{1}{4}, 1, 1/3, 1/2\right\}$$

length of $h(n) = N = 7$.

$$H(z) = Z.T\left[h(n)\right] = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{6} h(n) z^{-n}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}.$$

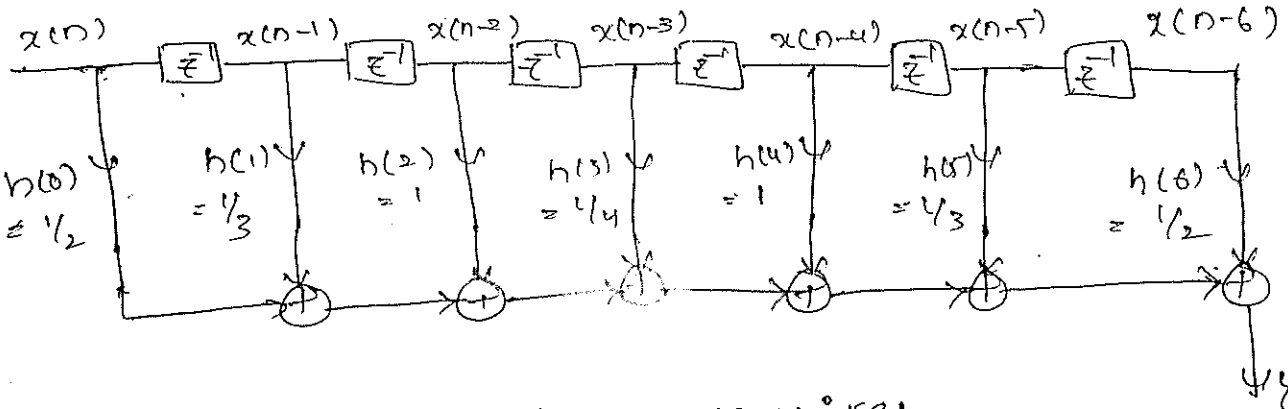
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + 1/3\, z^{-5} + 1/2\, z^{-6}.$$

$$Y(z) = \frac{1}{2}X(z) + \frac{1}{3}z^{-1}X(z) + z^{-2}X(z) + \frac{1}{4}z^{-3}X(z) + z^{-4}X(z) + \frac{1}{3}z^{-5}X(z) + \frac{1}{2}z^{-6}X(z)$$

Applying 1.z.7 & linearity prop.

$$y(n) = \frac{1}{2}x(n) + \frac{1}{3}x(n-1) + x(n-2) + \frac{1}{4}x(n-3) + x(n-4) + \frac{1}{3}x(n-5) + \frac{1}{2}x(n-6) \quad \text{---} (1),$$

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This realization requires,

no. of multipliers = 8 → 7.
no. of adders = 7 − 1 = 6.
no. of memory elements locations = 7 − 1 = 6.

ii) Linear phase:

We know condition for linear phase filter,

$$h(n) = h(N-1-n), \forall n.$$

$N = 7$; $h(n) = h(6-n)$.

$n = 0$; $h(0) = h(6) = 1/2$.

$n = 1$; $h(1) = h(5) = 1/3$.

$n = 2$; $h(2) = h(4) = 1$.

$n = 3$; $h(3) = 1/4 = h(3)$.

∴ Given system satisfies linear phase condition.

$$H(z) = ZT[h(n)] = \sum_{n=0}^{6} h(n) z^{-n}.$$

$$H(z) = \frac{Y(z)}{X(z)} = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}.$$

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$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(2)z^{-4} + h(1)z^{-5} + h(0)z^{-6}.$$

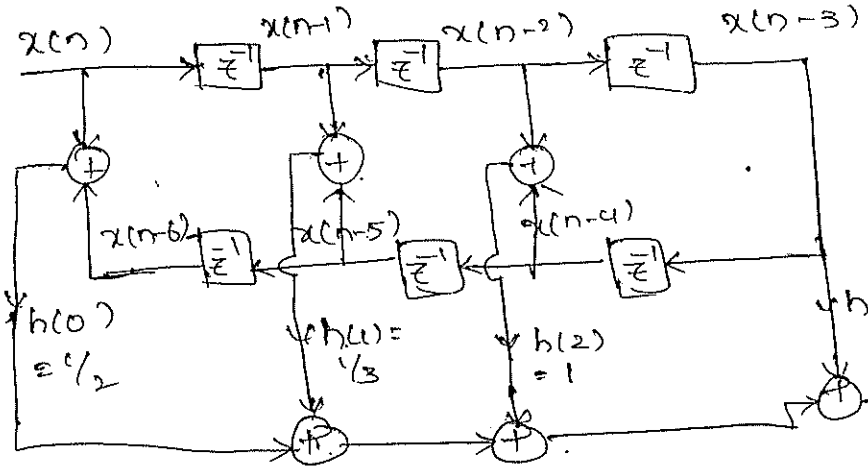
$$h(0)\left[1+z^{-6}\right] + h(1)\left[z^{-1}+z^{-5}\right] + h(2)\left[z^{-2}+z^{-4}\right] + h(3)z^{-3}$$

$$= \cdots + h(1)\left[z^{-1}+z^{-5}\right] + h(2)\left[z^{-2}+z^{-4}\right] + h(3)z^{-3}$$

$$Y(z) = h(0)\left[X(z) + z^{-6}X(z)\right] + h(1)\left[z^{-1}X(z) + z^{-5}X(z)\right] + h(2)\left[z^{-2}X(z) + z^{-4}X(z)\right] + h(3)z^{-3}X(z).$$

Applying 1.z.t & linearity prop.

$$y(n) = h(0)\left[x(n) + x(n-6)\right] + h(1)\left[x(n-1) + x(n-5)\right] + h(2)\left[x(n-2) + x(n-4)\right] + h(3)x(n-3) \quad \text{---- (1)}$$



no. of multipliers = $\frac{N+1}{2} = 4$.

adders = $\frac{N-1}{2} = 3$.

memory = 6.

2. $h(n) = \frac{1}{2}\delta(n) + \frac{1}{3}\delta(n-1) + \delta(n-2) + \frac{1}{4}\delta(n-3) + \frac{1}{4}\delta(n-4) + \delta(n-5) + \frac{1}{3}\delta(n-6) + \frac{1}{2}\delta(n-7)$.

Sol:- Linear phase:

$h(n) = \left\{\underset{0}{\frac{1}{2}}, \underset{1}{\frac{1}{3}}, \underset{2}{1}, \underset{3}{1/4}, \underset{4}{1/4}, \underset{5}{1}, \underset{6}{1/3}, \underset{7}{1/2}\right\}$.

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$N = 8$.

Condition for linear phase.

$h(n) = h(N-1-n)$ $\forall n$.

$n=0$; $h(7-0) = h(0) = h(7) = 1/2$.

$n=1$; $h(1) = h(6) = 1/3$.

$n=2$; $h(2) = h(5) = 1$.

$n=3$; $h(3) = h(4) = 1/4$.

∴ Given system satisfies linear phase condition.

$$H(z) = Z[h(n)] = \sum_{n=0}^{7} h(n) z^{-n}.$$

$$H(z) = \frac{Y(z)}{X(z)} = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6} + h(7)z^{-7}.$$

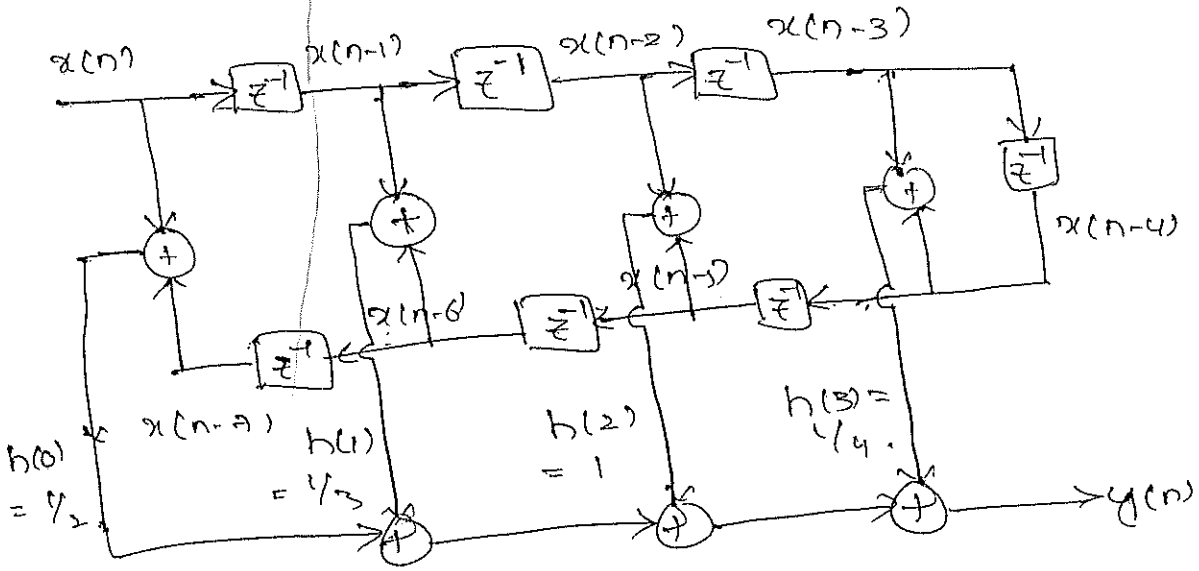
$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(0)z^{-6} + h(1)z^{-5} + h(2)z^{-4} + h(3)z^{-3}$$

$$= h(0)\left[1 + z^{-7}\right] + h(1)\left[z^{-1} + z^{-6}\right] + h(2)\left[z^{-2} + z^{-5}\right] + h(3)\left[z^{-3} + z^{-4}\right]$$

$$Y(z) = h(0)\left[X(z) + z^{-7}X(z)\right] + h(1)\left[X(z)z^{-1} + z^{-6}X(z)\right] + h(2)\left[z^{-2}X(z) + z^{-5}X(z)\right] + h(3)\left[z^{-3}X(z) + z^{-4}X(z)\right]$$

Applying L.Z.T and linearity prop

$$y(n) = h(0)\left[x(n) + x(n-8)\right] + h(1)\left[x(n-1) + x(n-6)\right] + h(2)\left[x(n-2) + x(n-5)\right] + h(3)\left[x(n-3) + x(n-4)\right] \quad \text{---(2)}$$



Total no. of multipliers used = $\frac{8}{2} = 4$.

adders = $N-1 = 8$.

memory elements = 8.

Cascade) —

3. $H(z) = \left(1 + \frac{1}{2}z^{-1} + z^{-2}\right)\left(1/2 + z^{-1} + 1/2\, z^{-2}\right)$.

$= H_1(z)\, H_2(z)$.

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} = 1 + \frac{1}{2}z^{-1} + z^{-2}$$

$$Y_1(z) = X_1(z) + \frac{1}{2}z^{-1}X_1(z) + X_1(z)z^{-2}.$$

$$y_1(n) = x_1(n) + \frac{1}{2}x(n-1) + x(n-2).$$

$$= \left[x_1(n) + x_1(n-2)\right] + \frac{1}{2}x_1(n-1) \quad \text{---(1)}$$

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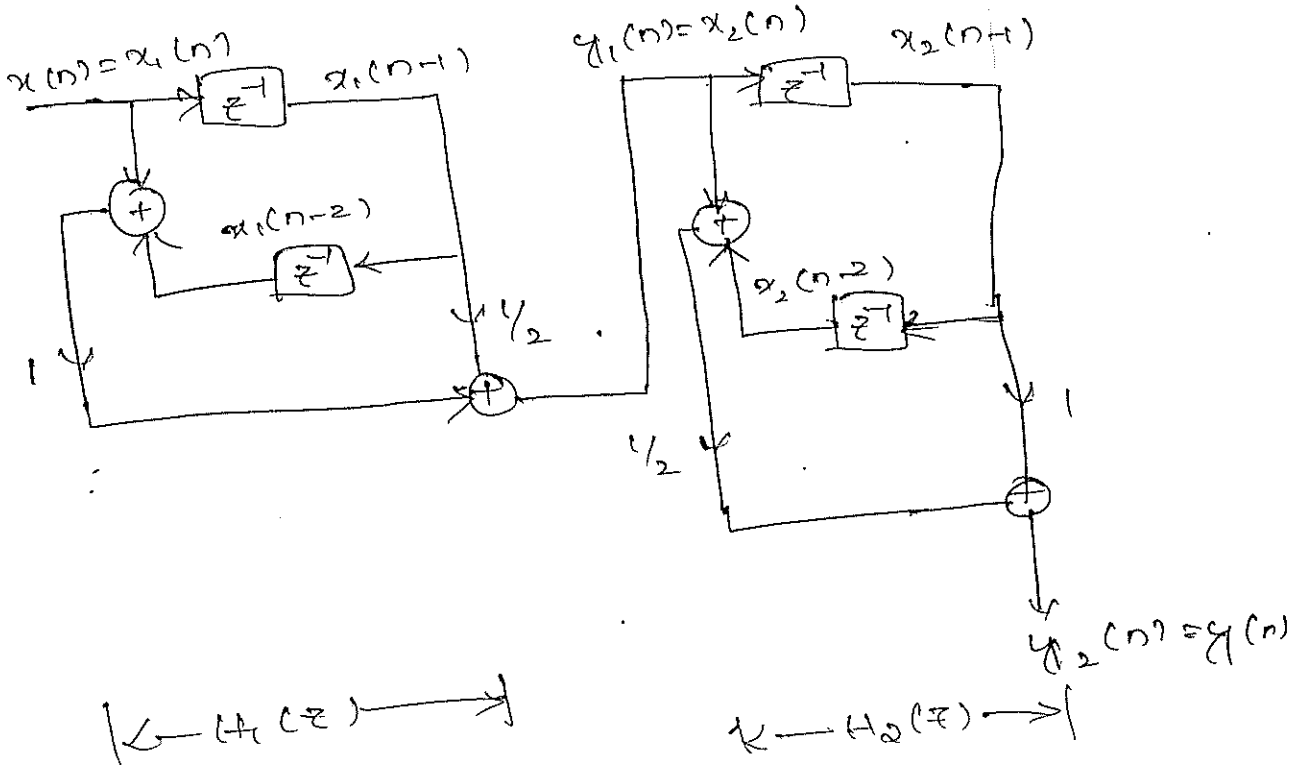
Hey.

$$H_2(z) = \frac{Y_2(z)}{X_2(z)} = \frac{1}{2} + z^{-1} + \frac{1}{2}z^{-2}$$

$$Y_2(z) = \frac{1}{2}X_2(z) + z^{-1}X_2(z) + \frac{1}{2}z^{-2}X_2(z)$$

$$y_2(n) = \frac{1}{2}x_2(n) + x_2(n-1) + \frac{1}{2}x_2(n-2)$$

$$= \frac{1}{2}\left[x_2(n) + x_2(n-2)\right] + x_2(n-1) \quad \text{---(2)}$$



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