Unit-II

Discrete fourier Series

Consider a Periodic Sequence $x_p(n)$ with a period N.

i.e $x_p(n) = x_p(n+ln)$

where l is an Integer.

The Periodic Sequence can be represented as weighted sum of complex exponential whose frequencies are integer multiples of fundamental frequency $2\pi/N$.

$\therefore e^{j2\pi kn/N} = e^{j2\pi k(n+ln)/N}$

$\therefore$ $x_p(n)$ can be defined as $\boxed{x_p(n) = 1/N \sum_{k=0}^{N-1} X_p(k) e^{j2\pi kn/N}}$

where $X_p(k)$ are discrete fourier series co-efficient for

$k = 0$ to $N-1$
$= 0, 1, 2, 3, \ldots, N-1$

$$\boxed{X_p(k) = \sum_{n=0}^{N-1} x_p(n) e^{-j2\pi kn/N}}$$

$\therefore$ The above two equation are the discrete fourier series pa

it can be represented as $DFS[x_p(n)] = X_p(k)$

Properties:

1) Linearity Property:

If $DFS[x_1p(n)] = X_1p(k)$

$DFS[x_2p(n)] = X_2p(k)$

Then $DFS[a x_1p(n) + b x_2p(n)] = a X_1p(k) + b X_2p(k)$

2) Time shifting property

If $DFS[x_p(n)] = X_p(k)$

Then $DFS[x_p(n-m)] = e^{-j2\pi km/N} X_p(k)$

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Periodic Convolution Property:-

If $DFS\ [x_{1p}(n)] = X_{1p}(k)$

$DFS\ [x_{2p}(n)] = X_{2p}(k)$

Then $DFS[x_{1p}(n) \circledast x_{2p}(n)] = X_{1p}(k)\ X_{2p}(k)$

Symmetry Property:-

If $DFS\ [x_p(n)] = X_p(k)$

$DFS\ [x_p^*(n)] = X_p^*(-k)$

$DFS\ [x_p^*(-n)] = X_p^*(k)$

Then

$$DFS\ [Re\ x_p(n)] = DFS\left[\frac{x_p(n) + x_p^*(n)}{2}\right] = \frac{X_p(k) + X_p^*(-k)}{2} = X_{pe}(k)$$

$$DFS\ [j Im\ x_p(n)] = DFS\left[\frac{x_p(n) - x_p^*(n)}{2}\right] = \frac{X_p(k) - X_p^*(-k)}{2} = X_{po}(k)$$

Consider: $x_p(n) = x_{pe}(n) + x_{po}(n)$

then $x_{pe}(n) = \frac{x_p(n) + x_p^*(-n)}{2}$

$x_{po}(n) = \frac{x_p(n) - x_p^*(-n)}{2}$

$$DFS\ [x_{pe}(n)] = DFS\left[\frac{x_p(n) + x_p^*(-n)}{2}\right] = \frac{X_p(k) + X_p^*(k)}{2} = Re\ X_p(k)$$

$$DFS\ [x_{po}(n)] = DFS\left[\frac{x_p(n) - x_p^*(-n)}{2}\right] = \frac{X_p(k) - X_p^*(k)}{2} = j Im\ X_p(k)$$

Discrete fourier Transform (DFT):-

The discrete fourier transform of sequence $x(n)$ is given by

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N} \quad \text{and}$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

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Let us (Consider) define a term $W_N = e^{-j2\pi/N}$ & called twiddle factor.

$$W_N = e^{-j2\pi/N} = \cos 2\pi/N - j \sin 2\pi/N$$

The magnitude of twiddle factor will be

$$|W_N| = |\cos 2\pi/N - j \sin 2\pi/N| \text{ and}$$

Phase $\omega_N = \angle e^{-j2\pi/N}$

* In terms of twiddle factor the sequence of Discrete fourier transform of $x(n)$ is defined as

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-nk}$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$$

∴ The representation of discrete fourier Transform is

$$DFT[x(n)] = X(k)$$

$$x(n) = IDFT[X(k)]$$

1) Find the DFT of a sequence $x(n) = \{1, 1, 0, 0\}$ and IDFT of the sequence $Y(k) = \{1, 0, 1, 0\}$

Sol Given Sequence $x(n) = \{1, 1, 0, 0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

Given N and $L \Rightarrow N = L = 4$

$$\text{Consider } X(0) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi(0)n/4}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 0 + 0$$

$$= 2.$$

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$$x(1) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi(1)n/4}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\pi n/2}$$

$$= x(0)\, e^{-j\pi(0)/2} + x(1)\, e^{-j\pi(1)/2} + x(2)\, e^{-j\pi(2)/2} + x(3)\, e^{-j\pi(3)/2}$$

$$= 1.1 + 1.e^{-j\pi/2} + 0 + 0$$

$$= 1 + 1\,[\cos\pi/2 - j\sin\pi/2]$$

$$= 1 + [0 - j.1]$$

$$= 1 - j.$$

$$x(2) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi(2)n/4}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= x(0)\, e^{-j\pi(0)} + x(1)\, e^{-j\pi(1)} + x(2)\, e^{-j\pi(2)} + x(3)\, e^{-j\pi(3)}$$

$$= 1.1 + 1.e^{-j\pi} + 0 + 0$$

$$= 1 + [\cos\pi - j\sin\pi]$$

$$= 1 + [-1 - j(0)]$$

$$= 1 - 1 = 0$$

$$x(3) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi(3)n/4}$$

$$= \sum_{n=0}^{3} x(n)\, e^{-j3\pi n/2}$$

$$= x(0)\, e^{-j3\pi(0)/2} + x(1)\, e^{-j3\pi(1)/2} + x(2)\, e^{-j3\pi(2)/2} + x(3)\, e^{-j3\pi(3)/2}$$

$$= 1 + 1.e^{-j3\pi/2}$$

$$= 1 + 1.[\cos 3\pi/2 - j\sin 3\pi/2]$$

$$= 1 + [0 - j(-1)]$$

$$= 1 + j.$$

$$X(k) = \{2,\ 1-j,\ 0,\ 1+j\}$$

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* Given Sequence $y(k) = \{1, 0, 1, 0\}$

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} y(k)\, e^{j 2\pi k n / N}$$

$$y(0) = \frac{1}{4} \sum_{k=0}^{3} y(k)\, e^{j 2\pi (0) n / 4}$$

$$= \frac{1}{4} \sum_{k=0}^{3} y(k) \cdot 1$$

$$= \frac{1}{4} [y(0) + y(1) + y(2) + y(3)]$$

$$= \frac{1}{4} [1 + 0 + 1 + 0]$$

$$= \frac{1}{4}(2) = \frac{1}{2}$$

$$y(1) = \frac{1}{4} \sum_{k=0}^{3} y(k)\, e^{j 2\pi (1) n / 4}$$

$$= \frac{1}{4} [y(0) e^{j 2\pi (0)/4} + y(1) e^{j 2\pi/4} + y(2) e^{j 2\pi (2)/4} + y(3) e^{j 2\pi (3)/4}]$$

$$= \frac{1}{4} [1 \cdot 1 + 0 + 1 \cdot e^{j\pi} + 0]$$

$$= \frac{1}{4} [1 + \cos\pi + j \sin\pi] = \frac{1}{4} [1 + (-1) + 0] = 0$$

$$y(2) = \frac{1}{4} \sum_{k=0}^{3} y(k)\, e^{j 2\pi (2) n / 4}$$

$$= \frac{1}{4} \sum_{k=0}^{3} y(k)\, e^{j n \pi}$$

$$= \frac{1}{4} [y(0) e^{j(0)\pi} + y(1) e^{j(1)\pi} + y(2) e^{j(2)\pi} + y(3) e^{j(3)\pi}]$$

$$= \frac{1}{4} [1 \cdot 1 + 0 + 1 \cdot e^{j 2\pi} + 0]$$

$$= \frac{1}{4} [1 + (1 + j(0)]$$

$$= \frac{1}{4}(2) = \frac{1}{2}$$

$$y(3) = \frac{1}{4} \sum_{k=0}^{3} y(k)\, e^{j 2\pi (3) n / 4}$$

$$= \frac{1}{4} \left[\sum_{k=0}^{3} y(k) e^{j 3\pi n/2} \right]$$

$$= \frac{1}{4} [y(0) e^{j 3\pi (0)/2} + y(1) e^{j 3\pi (1)/2} + y(2) e^{j 3\pi (2)/2} + y(3) e^{j 3\pi (3)/2}]$$

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$= \frac{1}{4}[1.1 + 0 + 1.e^{j3\pi} + 0]$

$= \frac{1}{4}[1 + (\cos 3\pi + j \sin 3\pi)]$

$= \frac{1}{4}[1 + (-1 + j(0))]$

$y(3) = \frac{1}{4}(0) = 0$

$y(n) = \{1/2, 0, 1/2, 0\}$

8) Find 8 point DFT of the sequence $x(n) = \{1,1,1,1,1,1,0,0\}$

Sol) given sequence is $x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$N = 8$

$N - 1 = 8 - 1 = 7$

$$X(0) = \sum_{n=0}^{7} x(n) e^{-j2\pi (0)n/8}$$

$$= \sum_{n=0}^{7} x(n) \cdot 1$$

$= x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7)$

$= 1 + 1 + 1 + 1 + 1 + 1 + 0 + 0$

$= 6$

$$X(1) = \sum_{n=0}^{7} x(n) e^{-j2\pi (1)n/8}$$

$$= \sum_{n=0}^{7} x(n) e^{-j\pi n/4}$$

$X(1) = x(0) e^{-j\pi(0)/4} + x(2) e^{-j\pi(1)/4} + x(2) e^{-j\pi(2)/4} + x(3) e^{-j\pi(3)/4}$

$+ x(4) e^{-j\pi(4)/4} + x(5) e^{-j\pi(5)/4} + x(6) e^{-j\pi(6)/4}$

$+ x(7) e^{-j\pi(7)/4}$

$X(1) = 1 + 1.e^{-j\pi/4} + 1.e^{-j\pi/2} + 1.e^{-j3\pi/4} + 1.e^{-j\pi} + 0 + 0$

$+ 1.e^{-j5\pi/4}$

$= 1 + 1[\cos \pi/4 - j \sin \pi/4] + [\cos \pi/2 - j \sin \pi/2] +$

$[\cos 3\pi/4 - j \sin 3\pi/4] + [\cos \pi - j \sin \pi] + [\cos 5\pi/4 - \sin 5\pi/4$

$X(1) = 1 + 1/\sqrt{2} - j\,1/\sqrt{2} + 0 - j - 1/\sqrt{2} - j/\sqrt{2} - 1 - 0 - 1/\sqrt{2} + j/\sqrt{2}$

$= 1 + 1/\sqrt{2} - j/\sqrt{2} - j - 1/\sqrt{2} - j/\sqrt{2} - 1 - 1/\sqrt{2} + j/\sqrt{2}$

$= -1/\sqrt{2} - j/\sqrt{2} - j$

$= -1/\sqrt{2} - j(1 + 1/\sqrt{2})$

$X(1) = -0.707 - j(1.707)$

$X(2) = \sum_{n=0}^{7} x(n) e^{-j2\pi(2)n/8}$

$= \sum_{n=0}^{7} x(n) e^{-jn\pi/2}$

$= x(0) e^{-j\pi(0)/2} + x(1) e^{-j\pi/2} + x(2) e^{-j\pi(2)/2} + x(3) e^{-j\pi(3)/2}$
$+ x(4) e^{-j\pi(2)/2} + x(5) e^{-j\pi(5)/2} + x(6) e^{-j\pi(6)/2} +$
$x(7) e^{-j\pi(7)/2}$

$= 1 + e^{-j\pi/2} + 1 \cdot e^{-j\pi} + e^{-j3\pi/2} + e^{-j2\pi} + e^{-j5\pi/2} + 0 + 0$

$= 1 + [\cos\pi/2 - j\sin\pi/2] + [\cos\pi - j\sin\pi] + [\cos 3\pi/2 - j\sin 3\pi/2]$
$+ [\cos 2\pi - j\sin 2\pi] + [\cos 5\pi/2 - j\sin 5\pi/2]$

$= 1 + 0 - j - 1 - 0 + 0 + j + 1 - 0 + 0 - j$

$= 1 - j$

$X(3) = \sum_{n=0}^{7} x(n) e^{-j2\pi(3)n/8}$

$= \sum_{n=0}^{7} x(n) e^{-j3\pi(n)/4}$

$= x(0) e^{-j3\pi(0)/4} + x(1) e^{-j3\pi/4} + x(2) e^{-j3\pi(2)/4} + x(3) e^{-j3\pi(3)/4}$
$+ x(4) e^{-j3\pi(4)/4} + x(5) e^{-j3\pi(5)/4} + x(6) e^{-j3\pi(6)/4} + x(7) e^{-j3\pi(7)/4}$

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$x(3) = 1 + e^{-j3\pi/4} + e^{-j3\pi/2} + e^{-j9\pi/4} + e^{-j3\pi} + e^{-j5\pi/4} + 0 + 0$

$= 1 + [\cos 3\pi/4 - j\sin 3\pi/4] + [\cos 3\pi/2 - j\sin 3\pi/2] +$
$[\cos 9\pi/4 - j\sin 9\pi/4] + [\cos 3\pi - j\sin 3\pi] +$
$[\cos 15\pi/4 - j\sin 15\pi/4]$

$x(3) = 1 + [-1/\sqrt{2} - j(1/\sqrt{2})] + [0 - j(-1)] + [1/\sqrt{2} - j(1/\sqrt{2})]$
$+ (-1 - j(0)) + (1/\sqrt{2} - j(-1/\sqrt{2}))$

$= 1 - 1/\sqrt{2} - j/\sqrt{2} + 0 + j + 1/\sqrt{2} - j/\sqrt{2} - 1 - 0 + 1/\sqrt{2} + j/\sqrt{2}$

$= 1/\sqrt{2} - j/\sqrt{2} + j$

$= 1/\sqrt{2} + j(1 - 1/\sqrt{2})$

$= 0.707 + 0.293j$

$x(4) = \sum_{n=0}^{7} x(n) e^{-j2\pi(4)n/8}$

$= \sum_{n=0}^{7} x(n) e^{-jn\pi}$

$= x(0)e^{-j(0)\pi} + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi} + x(4)e^{-j4\pi}$
$+ x(5)e^{-j5\pi} + x(6)e^{-j6\pi} + x(7)e^{-j7\pi}$

$= 1 + [\cos\pi - j\sin\pi] + [\cos 2\pi - j\sin 2\pi] + [\cos 3\pi - j\sin 3\pi]$
$+ [\cos 4\pi - j\sin 4\pi] + [\cos 5\pi - j\sin 5\pi] + 0 + 0$

$= 1 + [-1 - j(0)] + [1 - j(0)] + [-1 - j(0)] + [1 - j(0)]$
$+ [(-1) - j(0)]$

$= 1 - 1 + 1 - 1 + 1 - 1$

$= 0$

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$$X(5) = \sum_{n=0}^{7} x(n)\, e^{-j2\pi(5)n/8}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-j5\pi n/4}$$

$$= x(0)e^{-j5\pi(0)/4} + x(1)e^{-j5\pi(1)/4} + x(2)e^{-j5\pi(2)/4} + x(3)e^{-j5\pi(3)/4} + x(4)e^{-j5\pi(4)/4} + x(5)e^{-j5\pi(5)/4} + x(6)e^{-j5\pi(6)/4} + x(7)e^{-j5\pi(7)/4}$$

$$= 1 + e^{-j5\pi/4} + e^{-j5\pi/2} + e^{-j15\pi/4} + e^{-j5\pi} + e^{-j25\pi/4} + 0 + 0$$

$$= 1 + [\cos 5\pi/4 - j\sin 5\pi/4] + [\cos 5\pi/2 - j\sin 5\pi/2] + [\cos 15\pi/4 - j\sin 15\pi/4] + [\cos 5\pi - j\sin 5\pi] + [\cos 25\pi/4 - j\sin 25\pi/4]$$

$$= 1 + [-1/\sqrt{2} - j(-1/\sqrt{2})] + [0 - j] + [1/\sqrt{2} + j/\sqrt{2}] + [-1 - j(0)] + [1/\sqrt{2} - j(1/\sqrt{2})]$$

$$= 1 - 1/\sqrt{2} + j/\sqrt{2} - j + 1/\sqrt{2} + j/\sqrt{2} - 1 + 1/\sqrt{2} - j/\sqrt{2}$$

$$= j/\sqrt{2} - j + 1/\sqrt{2}$$

$$= 0.707 - j(1 - 1/\sqrt{2})$$

$$X(5) = 0.707 - j(0.293)$$

$$X(6) = \sum_{n=0}^{7} x(n)\, e^{-j2\pi(6)n/8}$$

$$= \sum_{n=0}^{7} x(n)\, e^{-j3\pi n/2}$$

$$= x(0)e^{-j3\pi(0)/2} + x(1)e^{-j3\pi(1)/2} + x(2)e^{-j3\pi(2)/2} +$$

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$x(3)e^{-j3\pi(3)/2} + x(4)e^{-j3\pi(4)/2} + x(5)e^{-j3\pi(5)/2} +$

$x(6)e^{-j3\pi(6)/4} + x(7)e^{-j3\pi(7)/2}$

$X(6) = 1 + e^{-j3\pi/2} + 1e^{-j3\pi} + e^{-j9\pi/2} + 1e^{-j6\pi} + e^{-j15\pi/2} + 0 + 0$

$= 1 + [\cos 3\pi/2 - j\sin 3\pi/2] + [\cos 3\pi - j\sin 3\pi] +$

$[\cos 9\pi/2 - j\sin 9\pi/2] + [\cos 6\pi - j\sin 6\pi] +$

$[\cos 15\pi/2 - j\sin 15\pi/2]$

$= 1 + (0 - j(-1)) + (-1 - j(0)) + (0 - j(1)) + (1 - j(0)) +$

$(0 - j(-1))$

$X(6) = 1 + j - 1 - j + 1 + j$

$= 1 + j$

$X(7) = \sum_{n=0}^{7} x(n) e^{-j2\pi(7)n/8}$

$= \sum_{n=0}^{7} x(n) e^{-j7\pi n/4}$

$= x(0)e^{-j7\pi(0)/4} + x(1)e^{-j7\pi/4} + x(2)e^{-j7\pi(2)/4} +$

$x(3)e^{-j7\pi(3)/4} + x(4)e^{-j7\pi(4)/4} + x(5)e^{-j7\pi(5)/4} +$

$x(6)e^{-j7\pi(6)/4} + x(7)e^{-j7\pi(7)/4}$

$= 1 + e^{-j7\pi/4} + e^{-j7\pi/2} + e^{-j21\pi/4} + e^{-j7\pi} + e^{-j35\pi/4} + 0 + 0$

$= 1 + [\cos 7\pi/4 - j\sin 7\pi/4] + [\cos 7\pi/2 - j\sin 7\pi/2] +$

$[\cos 21\pi/4 - j\sin 21\pi/4] + [\cos 7\pi - j\sin 7\pi] +$

$[\cos 35\pi/4 - j\sin 35\pi/4]$

$= 1 + [1/\sqrt{2} - j(-1/\sqrt{2})] + [0 - j(-1)] + [-1/\sqrt{2} - j(-1/\sqrt{2})] +$

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$(-1-j(0)) + (-1/\sqrt{2} - j(1/\sqrt{2}))$

$= -1 + 1/\sqrt{2} + j/\sqrt{2} + j - 1/\sqrt{2} + j/\sqrt{2} - 1 - 1/\sqrt{2} - j/\sqrt{2}$

$= j - 1/\sqrt{2} + j/\sqrt{2}$

$X(7) = -0.707 + j(1.707)$

$X(k) = \{6, -0.707 - j1.707, 1-j, 0.707 + 0.293j, 0,$
$0.707 - 0.293j, 1+j, -0.707 + j1.707\}$

3) Find IDFT of sequence $X(k) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0\}$

Sol given Sequence $X(k) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0\}$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N}$$

$N = 8$

$N-1 = 8-1 = 7$

$$x(0) = \frac{1}{8} \sum_{k=0}^{7} X(k) e^{j2\pi(0)k/8}$$

$$= \frac{1}{8} \sum_{k=0}^{7} X(k) \cdot 1$$

$x(0) = \frac{1}{8}[X(0) + X(1) + X(2) + X(3) + X(4) + X(5) + X(6) + X(7)]$

$= \frac{1}{8}[5 + 0 + 1 - j + 0 + 1 + 0 + 1 + j + 0]$

$= \frac{1}{8}[8]$

$= 1$

$$x(1) = \frac{1}{8} \sum_{k=0}^{7} x(k)\, e^{j2\pi k(1)/8}$$

$$= \frac{1}{8} \sum_{k=0}^{7} x(k) e^{j\pi k/4}$$

$= \frac{1}{8}[X(0)e^{j\pi(0)/4} + X(1)e^{j\pi(1)/4} + X(2)e^{j\pi(2)/4} + X(3)e^{j\pi(3)/4}$
$+ X(4)e^{j\pi(4)/4} + X(5)e^{j\pi(5)/4} + X(6)e^{j\pi(6)/4} + X(7)e^{j\pi(7)/4}]$

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$$X(1)=\frac{1}{8}\left[5+0+(1-j)e^{j\pi/2}+0\times e^{j3\pi/4}+e^{j\pi}+0\cdot e^{j5\pi/4}+(1+j)e^{j3\pi/2}+0\cdot e^{j7\pi/4}\right]$$

$$=\frac{1}{8}\left[5+(1-j)\left[\cos\pi/2+j\sin\pi/2\right]+\left[\cos\pi+j\sin\pi\right]+(1+j)\left[\cos 3\pi/2+j\sin 3\pi/2\right]\right]$$

$$=\frac{1}{8}\left[5+(1-j)(0+j)+(-1+j(0))+(1+j)(0+j(-1))\right]$$

$$=\frac{1}{8}\left[5+j-j^2-1+(1+j)(-j)\right]$$

$$=\frac{1}{8}\left[5+j-j^2-1-j^2-j\right]$$

$$=\frac{1}{8}\left[5-1+1+1\right]$$

$$=6/8=3/4=0.75$$

$$X(2)=\frac{1}{8}\sum_{K=0}^{7}x(k)\,e^{-j2\pi k(2)/8}$$

$$=\frac{1}{8}\sum_{K=0}^{7}x(k)\,e^{j\pi k/2}$$

$$X(2)=\frac{1}{8}\left[x(0)e^{j\pi(0)/2}+x(1)e^{j\pi(1)/2}+x(2)e^{j\pi(2)/2}+x(3)e^{j\pi(3)/2}+x(4)e^{j\pi(4)/2}+x(5)e^{j5\pi/2}+x(6)e^{j\pi(6)/2}+x(7)e^{j7\pi/2}\right]$$

$$=\frac{1}{8}\left[5+0+(1-j)e^{j\pi}+0+e^{j2\pi}+0+(1+j)e^{j3\pi}+0\right]$$

$$=\frac{1}{8}\left[5+(1-j)\left[\cos\pi+j\sin\pi\right]+(\cos 2\pi+j\sin 2\pi)+(1+j)(\cos 3\pi+j\sin 3\pi)\right]$$

$$=\frac{1}{8}\left[5+(1-j)(-1+j(0))+(1+j(0))+(1+j)(-1+j(0))\right]$$

$$=\frac{1}{8}\left[5-1+j+1+(1+j)(-1)\right]$$

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$= \frac{1}{8}[4+j+1-1-j]$

$= 4/8 = 1/2 = 0.5$

$X(3) = \frac{1}{8}\sum_{k=0}^{7} x(k) e^{j2\pi k(3)/8}$

$= \frac{1}{8}\sum_{k=0}^{7} x(k) e^{j3\pi k/4}$

$= \frac{1}{8}\left[x(0)e^{j3\pi(0)/4} + x(1)e^{j3\pi(1)/4} + x(2)e^{j3\pi(2)/4} + x(3)e^{j3\pi(3)/4} + x(4)e^{j3\pi(4)/4} + x(5)e^{j3\pi(5)/4} + x(6)e^{j3\pi(6)/4} + x(7)e^{j3\pi(7)/4}\right]$

$x(3) = \frac{1}{8}\left[5+0+(1-j)e^{j3\pi/2} + 0 + e^{j3\pi} + 0 + (1+j)e^{j9\pi/2} + 0\right]$

$= \frac{1}{8}\left[5+(1-j)[\cos 3\pi/2 + j\sin 3\pi/2] + (\cos 3\pi + j\sin 3\pi) + (1+j)(\cos 9\pi/2 + j\sin 9\pi/2)\right]$

$= \frac{1}{8}\left[5+(1-j)(0+j(-1)) + ((-1)+j(0)) + (1+j)(0+j)\right]$

$= \frac{1}{8}\left[5+(1-j)(-j) + (-1) + (1+j)(j)\right]$

$= \frac{1}{8}\left[5-j+j^2-1+j+j^2\right]$

$= \frac{1}{8}\left[4-1-1\right] = 1/8(2) = 1/4 = 0.25$

$x(4) = \frac{1}{8}\sum_{k=0}^{7} x(k) e^{j2\pi k(4)/8}$

$= \frac{1}{8}\sum_{k=0}^{7} x(k) e^{j\pi k}$

$= \frac{1}{8}\left[x(0)e^{j\pi(0)} + x(1)e^{j\pi(1)} + x(2)e^{j\pi(2)} + x(3)e^{j\pi(3)} +\right.$

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$x(4)e^{j4\pi} + x(5)e^{j5\pi} + x(6)e^{j6\pi} + x(7)e^{j7\pi}]$

$x(4) = \frac{1}{8}[5+0+(1-j)(\cos 2\pi + j\sin 2\pi) + 0 + [\cos 4\pi + j\sin 4\pi)$
$+ (1+j)(\cos 6\pi + j\sin 6\pi)]$

$= \frac{1}{8}[5+(1+j)(1+j(0)) + (1+j(0)) + (1+j)(1+j(0))]$

$= \frac{1}{8}[5+1-j+1+1+j]$

$= 8(1/8) = 1$

$x(5) = \frac{1}{8}\sum_{k=0}^{7} X(k)e^{j7\pi k(5)/8}$

$= 1/8 \sum_{k=0}^{7} X(k)e^{j5\pi k/4}$

$x(5) = \frac{1}{8}[X(0)e^{j5\pi(0)/4} + x(1)e^{j5\pi(1)/4} + X(2)e^{j5\pi(2)/4} +$
$X(3)e^{j5\pi(3)/4} + X(4)e^{j5\pi(4)/4} + X(5)e^{j5\pi(5)/4}$
$+ X(6)e^{j5\pi(6)/4} + X(7)e^{j5\pi(7)/4}]$

$= \frac{1}{8}[5+0+(1-j)e^{j5\pi/2} + 0 + e^{j5\pi} + 0 + (1+j)e^{j15\pi/2} + 0]$

$= \frac{1}{8}[5+(1-j)[\cos 5\pi/2 + j\sin 5\pi/2] + (\cos 5\pi + j\sin 5\pi)$
$+ (1+j)(\cos 15\pi/2 + j\sin 15\pi/2)]$

$= \frac{1}{8}[5+(1-j)(0+j(1)) + (-1+j(0)) + (1+j)(0+j(-1))]$

$= \frac{1}{8}[5+j-j^2-1-j-j^2]$

$= \frac{1}{8}[4-2j^2] = \frac{1}{8}[4+2] = 6/8$

$x(5) = 0.75$

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$$x(6)=\frac{1}{8}\sum_{k=0}^{7}x(k)\,e^{j2\pi k(6)/8}$$

$$=\frac{1}{8}\sum_{k=0}^{7}x(k)\,e^{j3\pi k/2}$$

$$=\frac{1}{8}\left[x(0)e^{j3\pi(0)/2}+x(1)e^{j3\pi/2}+x(2)e^{j3\pi(2)/2}+x(3)e^{j3\pi(3)/2}\right.$$
$$\left.+x(4)e^{j3\pi(4)/2}+x(5)e^{j3\pi(5)/2}+x(6)e^{j3\pi(6)/2}+\right.$$
$$\left.x(7)\,e^{j3\pi(7)/2}\right]$$

$$=\frac{1}{8}\left[5+0+(1-j)\,e^{j3\pi}+0+e^{j6\pi}+0+(1+j)\,e^{j9\pi}+0\right]$$

$$=\frac{1}{8}\left[5+(1-j)\left[\cos 3\pi+j\sin 3\pi\right]+\left[\cos 6\pi+j\sin 6\pi\right]+\right.$$
$$\left.(1+j)\left[\cos 9\pi+j\sin 9\pi\right]\right]$$

$$=\frac{1}{8}\left[5+(1-j)\left(-1+j(0)\right)+\left(1+j(0)\right)+(1+j)\left(-1+j(0)\right)\right]$$

$$=\frac{1}{8}\left[5-1+j+1-j-1\right]$$

$$=4/8=1/2=0.5$$

$$x(7)=\frac{1}{8}\sum_{k=0}^{7}x(k)\,e^{j2\pi k(7)/8}$$

$$=\frac{1}{8}\sum_{k=0}^{7}x(k)\,e^{j7\pi k/4}$$

$$=\frac{1}{8}\left[x(0)e^{j7\pi(0)/4}+x(1)e^{j7\pi(1)/4}+x(2)e^{j7\pi(2)/4}+\right.$$
$$x(3)\,e^{j7\pi(3)/4}+x(4)e^{j7\pi(4)/4}+x(5)e^{j7\pi(5)/4}$$
$$\left.+x(6)e^{j7\pi(6)/4}+x(7)e^{j7\pi(7)/4}\right]$$

$$=\frac{1}{8}\left[5+0+(1-j)\,e^{j7\pi/2}+0+e^{j7\pi}+(1+j)e^{j21\pi/2}+0\right.$$
$$\left.+0\right]$$

$= \frac{1}{8}\left[5+(1-j)\left[\cos 7\pi/2 + j\sin 7\pi/2\right] + \left[\cos 7\pi + j\sin 7\pi\right)\right.$
$\left. + (1+j)\left[\cos 21\pi/2 + j\cdot \sin 21\pi/2\right]\right]$

$= \frac{1}{8}\left[5+(1-j)\left(0+j(-1)\right) + \left(-1+j(0)\right) + (1+j)\left(0+j(1)\right)\right]$

$= \frac{1}{8}\left[5+(1-j)(-j) + (-1) + (1+j)(j)\right]$

$= \frac{1}{8}\left[5-j+j^2-1+j+j^2\right]$

$= \frac{1}{8}\left[4+2j^2\right] = \frac{1}{8}\left[4-2\right] = 1/8\,[2] = 1/4 = 0.25$

$x(n) = \{1, 0.75, 0.5, 0.25, 1, 0.75, 0.5, 0.25\}$

Zero Padding:-

Consider a Sequence $x(n)$ having length 'L' is given by $x(n) = \{x(0), x(1), x(2), \ldots x(L-1)\}$ to improve the frequency Resolution of discrete fourier transform $(2\pi/N)$. Zero padding is require that means adding no. of zero's to the given Sequence $x(n)$.

1) find the DFT of a Sequence $x(n) = \{1$ for $0 \le n \le 2$
0 otherwise
for $N=4$ & $N=8$

Sol for $N=4$

$x(n) = \{1, 1, 1, 0\}$

$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/4}$

$X(0) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi(0)n/N}$

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$$X(0) = \sum_{n=0}^{3} x(n) \cdot 1$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 1 + 0$$

$$X(0) = 3$$

$$X(1) = \sum_{n=0}^{3} x(n) e^{-j2\pi(1)n/4}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\pi n/2}$$

$$= x(0) e^{-j\pi(0)/2} + x(1) e^{-j\pi/2} + x(2) e^{-j\pi(2)/2} + x(3) e^{-j\pi(3)/2}$$

$$= 1 + e^{-j\pi/2} + e^{-j\pi} + 0$$

$$= 1 + [\cos\pi/2 - j\sin\pi/2] + [\cos\pi - j\sin\pi]$$

$$= 1 + [0 - j] + [-1 - j(0)]$$

$$= 1 - j - 1$$

$$= -j$$

$$X(2) = \sum_{n=0}^{3} x(n) e^{-j2\pi(2)n/4}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\pi n}$$

$$= x(0) e^{-j\pi(0)} + x(1) e^{-j\pi} + x(2) e^{-j2\pi} + x(3) e^{-j3\pi}$$

$$= 1 + 1[\cos\pi - j\sin\pi] + [\cos 2\pi - j\sin 2\pi] + 0$$

$$= 1 + [-1 - j(0)] + [1 - j(0)]$$

$$= 1 - 1 + 1 = 1$$

$$X(3) = \sum_{n=0}^{3} x(n) e^{-j2\pi(3)n/4}$$

$$= \sum_{n=0}^{3} x(n) e^{-j3\pi n/2}$$

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$= x(0)e^{-j3\pi(0)/2} + x(1)e^{-j3\pi(1)/2} + x(2)e^{-j3\pi(2)/2} + x(3)e^{-j3\pi(3)/2}$

$= 1 + [\cos 3\pi/2 - j\sin 3\pi/2] + [\cos 3\pi - j\sin 3\pi] + 0$

$= 1 + [0 - j(-1)] + [-1 - j(0)]$

$= 1 + j - 1$

$= j$

$x(k) = \{3, -j, 1, j\}$

(ii) for $N = 8$

$x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$

$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/8}$

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Circular Convolution:-

The linear Convolution of two sequences $x(n)$ of 'L' no of samples and $h(n)$ of 'M' no of samples produce a result $y(n)$ which contains $\boxed{N=L+M-1}$ incase of circular convolution if $x(n)$ contains 'L' no. of samples and $h(n)$ has 'M' no of samples then we can perform Circular Convolution bw the two sequence using

$$N=\max(L,M)$$

To find the Circular Convolution of two sequence their are two methods are use

1) Concentric Circle method
2) Matrix multiplication method

(1) Concentric Circle method:-

Given two sequences $x_1(n)$ and $x_2(n)$ the Circular Convolution of these two sequence $x_3(n)$ $x_3(n) = x_1(n) \textcircled{N} x_2(n)$ can be found by using the following steps.

1. Graph 'n' samples of $x_1(n)$ at equally spaced points around an outer circle in counter clock -wise direction.
2. Start at the same point as $x_1(n)$ graph 'n' samples of $x_2(n)$ as equally spaced points around as inner circle in clockwise direction.
3. Multiply the corresponding samples on the two circles and sum the products to produce an outputs.

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4. Rotate the inner circle one sample at a time in count clockwise direction and go to step3 to obtain the next value of output.

5. Repeat step4 until the inner circle first sample lines up with the first sample of the exterior circle once again.

(2) Matrix Multiplication Method:-

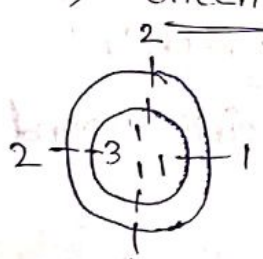
In this method, the Circular convolution of two sequence $x_1(n)$ & $x_2(n)$ can be obtained by representing the Sequence in matrix form that is the sequence $x_2(n)$ is repeated viva circular shift of samples and represented in NxN matrix form the sequence $x_1(n)$ is represented as column matrix. the multiplication of this two matrices gives the sequences $x_3(n)$.

1) find Circular convolution of two Sequence $x_1(n)=\{1,2,2,1\}$, $x_2(n)=\{1,2,3,1\}$ using concentric Circle method, matrix multiplication method.

A) Given $x_1(n)=\{1,2,2,1\}$

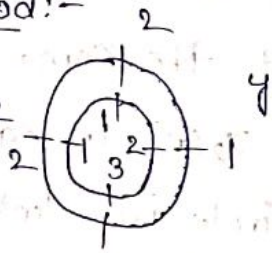
$x_2(n)=\{1,2,3,1\}$

1) Concentric Circle method:-



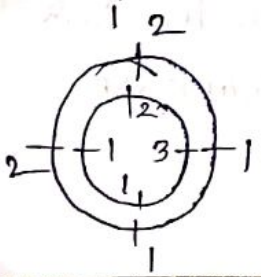
$y(0)=1+2+6+2$

$=1$



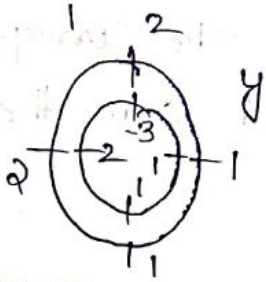
$y(1)=2+2+2+3$

$=9$



$y(2)=3+4+2+1$

$=10$



$y(3)=1+6+4+1$

$=12$

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$y(n) = \{11, 9, 10, 12\}$

2) Matrix Multiplication method

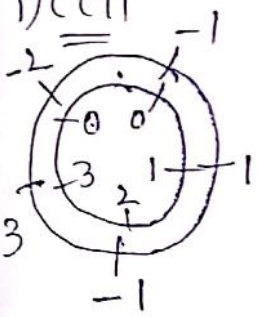
$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2+6+2 \\ 2+2+3+1 \\ 2+4+3+1 \\ 1+4+6+1 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 10 \\ 12 \end{bmatrix}$$

3) find the Circular Convolution of two finite duration of Sequence $x_1(n) = \{1, -1, -2, 3, -1\}$ and $x_2(n) = \{1, 2, 3\}$

A) given $x_1(n) = \{1, -1, -2, 3, -1\}$

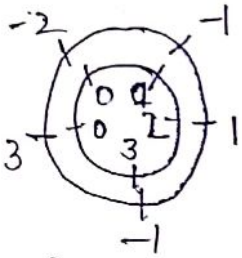
$x_2(n) = \{1, 2, 3, 0, 0\}$

1) CCM



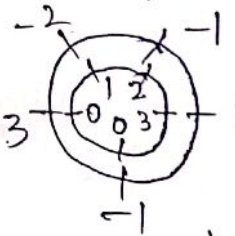
$y(0) = 1+0+9-2+0$

$= 8$



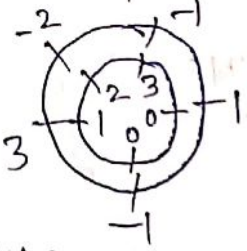
$y(1) = 2-1+0+0-3$

$= -2$

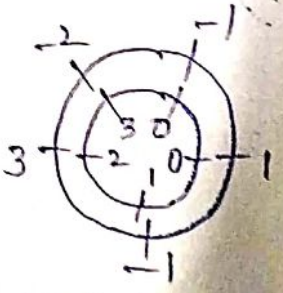


$y(2) = 3-2-2+0+0$

$= -1$



$y(3) = 0-3-4+3+0 = -4$



$y(4) = 0+0-6+6-1$

$= -1$

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$y(n) = \{8, -2, -1, -4, -1\}$

2) HHH

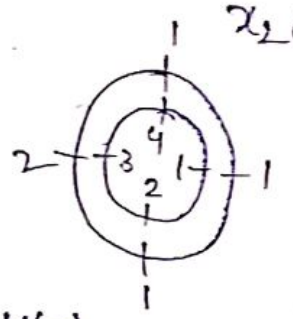
$$\begin{bmatrix} 1 & -1 & 3 & -2 & -1 \\ -1 & 1 & -1 & 3 & -2 \\ -2 & -1 & 1 & -1 & 3 \\ 3 & -2 & -1 & 1 & -1 \\ -1 & 3 & -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1-2+9+0+0 \\ -1+2-3+0+0 \\ -2-2+3+0+0 \\ 3-4-3+0+0 \\ -1+6-6+0+0 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ -1 \\ -4 \\ -1 \end{bmatrix}$$

$y(n) = \{8, -2, -1, -4, -1\}$

3) Perform the circular convolution of the following sequence (i) $x_1(n) = \{1, 1, 2, 1\}$, $x_2(n) = \{1, 2, 3, 4\}$, (ii) $x_1(n) = \{1, 2, 3, 1\}$ and $x_2(n) = \{4, 3, 2, 2\}$

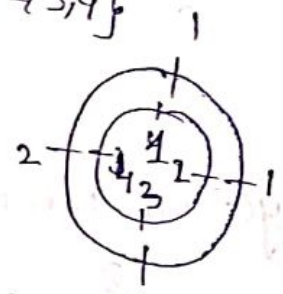
Sol (i) $x_1(n) = \{1, 1, 2, 1\}$

$x_2(n) = \{1, 2, 3, 4\}$



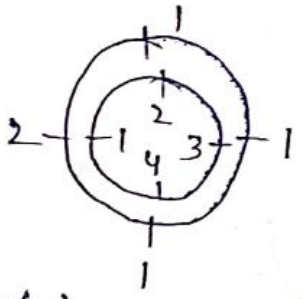
$y(0) = 1+4+6+2$

$= 13$



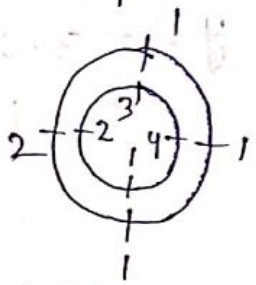
$y(1) = 2+1+8+3$

$= 14$



$y(2) = 3+2+2+4$

$= 11$



$y(3) = 4+3+4+1$

$= 12$

$y(n) = \{13, 14, 11, 12\}$

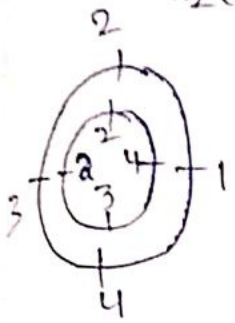
2) MMM

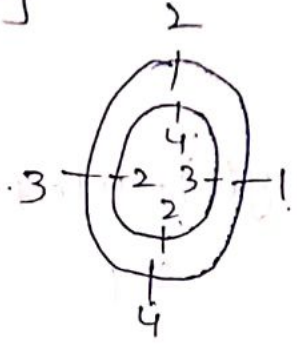
$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+2+6+4 \\ 1+2+3+8 \\ 2+2+3+4 \\ 1+4+3+4 \end{bmatrix} = \begin{bmatrix} 13 \\ 14 \\ 11 \\ 12 \end{bmatrix}$$

$y(n) = \{13, 14, 11, 12\}$

(ii) $x_1(n) = \{1, 2, 3, 1\}$

$x_2(n) = \{4, 3, 2, 2\}$



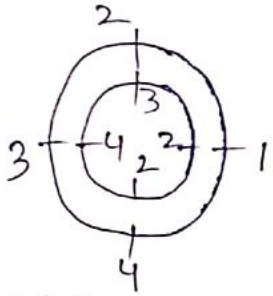


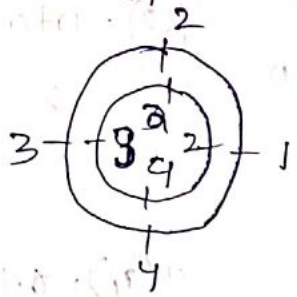
$y(0) = 4+4+6+3$

$= 17$

$y(1) = 3+8+6+8$

$= 19$





$y(2) = 2+6+12+2$

$= 22$

$y(3) = 2+4+9+4$

$= 19$

$y(n) = \{17, 19, 22, 19\}$

(ii) $\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4+3+6+4 \\ 8+3+2+6 \\ 12+6+2+2 \\ 4+9+4+2 \end{bmatrix} = \begin{bmatrix} 17 \\ 19 \\ 22 \\ 19 \end{bmatrix}$

$y(n) = \{17, 19, 22, 19\}$

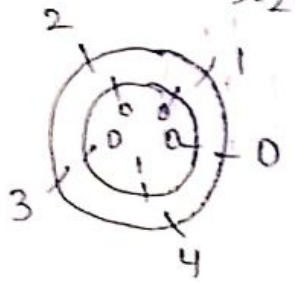
4) Consider the Sequence $x_1(n) = \{0, 1, 2, 3, 4\}$,

$x_2(n) = \{0, 1, 0, 0, 0\}$

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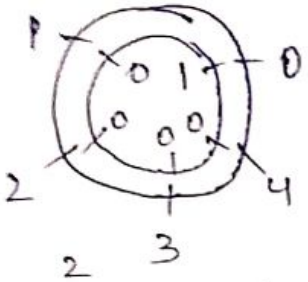
A) given $x_1(n) = \{0, 1, 2, 3, 4\}$

$x_2(n) = \{0, 1, 0, 0, 0\}$



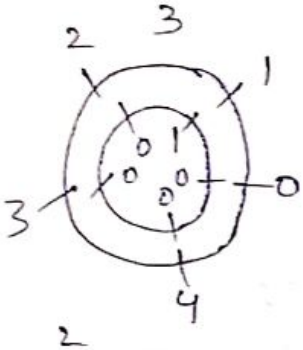
$y(0) = 0+0+0+0+4$

$= 4$



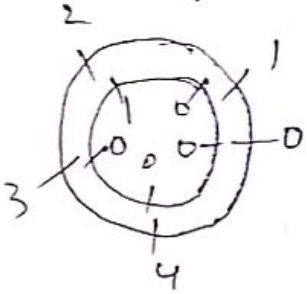
$y(1) = 0+0+0+0+0$

$= 0$



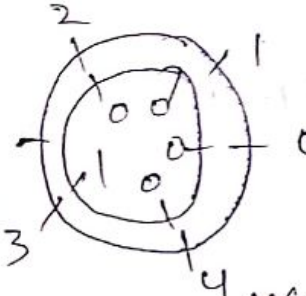
$y(2) = 0+0+1+0+0$

$= 1$



$y(3) = 0+0+2+0+0$

$= 2$



$y(4) = 0+0+0+3+0$

$= 3$

$y(n) = \{4, 0, 1, 2, 3\}$.

(2) MMM

$$\begin{bmatrix} 0 & 4 & 3 & 2 & 1 \\ 1 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+4+0+0+0 \\ 0+0+0+0+0 \\ 0+1+0+0+0 \\ 0+0+2+0+0 \\ 0+0+0+3+0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

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Properties of DFT :-

1) Linearity property :-

If DFT $[x(n)] = X(k)$

If DFT $[x_1(n)] = X_1(k)$

DFT $[x_2(n)] = X_2(k)$ Then

$$DFT\left[a x_1(n) + b x_2(n)\right] = a X_1(k) + b X_2(k)$$

Proof:-

$$DFT\left[a x_1(n) + b x_2(n)\right] = \sum_{n=0}^{N-1} \left[a x_1(n) + b x_2(n)\right] e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} a x_1(n) e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} b x_2(n) e^{-j2\pi kn/N}$$

$$= a \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi kn/N} + b \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi kn/N}$$

$$= a X_1(k) + b X_2(k)$$

$$\therefore DFT\left[a x_1(n) + b x_2(n)\right] = a X_1(k) + b X_2(k)$$

2) Time Shifting Property:-

If DFT $[x(n)] = X(k)$

then DFT $[x(n-m)] = e^{-j2\pi km/N} \cdot X(k)$

Proof:-

$$DFT\left[x(n-m)\right] = \sum_{n=0}^{N-1} x(n-m) e^{-j2\pi kn/N}$$

Let $n-m = l$

$n = l+m$

$$= \sum_{l=0}^{N-1} x(l) e^{-j2\pi k(l+m)/N}$$

$$= \sum_{l=0}^{N-1} x(l) e^{-j2\pi kl/N} \cdot e^{-j2\pi km/N}$$

$$= e^{-j2\pi km/N} \sum_{l=0}^{N-1} x(l) e^{-j2\pi kl/N}$$

$$= e^{-j2\pi km/N} \cdot X(k)$$

$$\therefore DFT\left[x(n-m)\right] = e^{-j2\pi km/N} \cdot X(k)$$

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Circular Convolution:-

If DFT $[x_1(n)] = X_1(k)$

DFT $[x_2(n)] = X_2(k)$

Then DFT $[x_1(n) \textcircled{N} x_2(n)] = X_1(k)\, X_2(k)$

Proof:

Multiplication of Two Sequences:-

If DFT $[x_1(n)] = X_1(k)$

DFT $[x_2(n)] = X_2(k)$

Then DFT $[x_1(n) \cdot x_2(n)] = \frac{1}{N} [X_1(k) \textcircled{N} X_2(k)]$

Parseval's Theorem:-

If DFT $[x(n)] = X(k)$

DFT $[y(n)] = Y(k)$

Then $\sum_{n=0}^{N-1} x(n) y^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) Y^*(k)$

Time Reversal Property:

If DFT $[x(n)] = X(k)$

Then DFT $[x(-n)] = X((-k))_N = X(N-k)$ $\quad x((-n))_N = x(N-n)$

Proof:

~~DFT $[x(n)] = \sum_{n=0}^{N-1} x(-n) e^{-j2\pi kn/N}$~~

let $-n =$

DFT $[x((-n))_N] =$ DFT $[x(N-n)]$

$= \sum_{n=0}^{N-1} x(N-n) e^{-j2\pi kn/N}$ $\quad$ let $N-n = m$, $n = N-m$

$= \sum_{m=0}^{N-1} x(m) e^{-j2\pi k(N-m)/N}$

$= \sum_{m=0}^{N-1} x(m) e^{-j2\pi kN/N} \cdot e^{j2\pi km/N}$

$= \sum_{m=0}^{N-1} x(m) e^{-j2\pi k} e^{j2\pi km/N}$ $\quad e^{j2\pi k} = 1,\ k = 0, 1, 2, \ldots$

$= \sum_{m=0}^{N-1} x(m) e^{j2\pi km/N}$

$= X((-k))_N = X(N-k)$

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Fast Fourier Transforms:

~~The DFT of a Sequence Can~~

Direct evaluation of DFT: The DFT of a sequence can be evaluated using the formula $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$

Substituting $W_N = e^{-j2\pi/N}$

$$\therefore X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$= \sum_{n=0}^{N-1} [\mathrm{Re}\, x(n) + j\, \mathrm{Im}\, x(n)] [\mathrm{Re}\, W_N^{kn} + j\, \mathrm{Im}\, W_N^{kn}] \rightarrow (1)$$

$$\Rightarrow \sum_{n=0}^{N-1} \mathrm{Re}\, x(n)\, \mathrm{Re}\, W_N^{kn} - \sum_{n=0}^{N-1} \mathrm{Im}\, x(n)\, \mathrm{Im}\, W_N^{kn} + j\left[\sum_{n=0}^{N-1} \mathrm{Re}\, x(n)\, \mathrm{Im}\, W_N^{kn} + \sum_{n=0}^{N-1} \mathrm{Re}\, W_N^{kn}\, \mathrm{Im}\, x(n)\right]$$

* From eq (1) we can see that to evaluate one value of $x(k)$ the no. of complex multiplication required 'N' ∴ to evaluate all 'N' values of $x(k)$ the no. of complex multiplication is $N \times N = N^2$.

* In the same way to evaluate one value of $x(k)$ the no. of complex additions required is 'N-1' ∴ to evaluate all 'N' values of $x(k)$ the no. of complex additions required is $N(N-1)$

* From eq (2) ∴ to evaluate all values of $x(k)$ the required no. of complex multiplications $4N^2$.

* Similarly to evaluate $x(k)$ for all values of 'k' then required no. of complex addition required is $4 * (N-1)$

* The direct evaluation of DFT is basically inefficiency because it doesn't use symmetry and periodicity properties of twiddle factor W_N.

Symmetry property $\rightarrow W_N^{k+N/2} = -W_N^{k}$

Periodicity property $\rightarrow W_N^{k+N} \rightarrow W_N^{k}$

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Fast Fourier Transform :

The FFT algorithms can be used the disadvantages of DFT can be overcome i.e

Two Basic Properties of triple factor.

Reduce the no. of complex multiplications & Additions.

FFT algorithms are based on the fundamental principle of decomposing the computation of (discrete) DFT of a sequence of a length 'N' into successively smaller DFT

There are basically two FFT algorithms they are.

1. Decimation in Time
2. Decimation in frequency.

→ In Decimation in Time the FFT algorithm the time samples of the DFT are decomposed into smaller and ~~some~~ small sub sequences.

→ In Decimation in frequency approach the frequency samples of the DFT are decomposed into smaller and smaller sub-sequences

1) Decimation in Time Algorithm.

This algorithm is also known as Radix-2-d which means the no of O/P points 'N' can be expressed as a '2' i.e

Let $x(n)$ is an n point sequence where 'n' is assumed to be a power of '2' decimate ~~this~~ sequence into two sequences of length $N/2$ where one sequence consisting of the even index value of x^n and the other of odd indexed value of

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The DFT of a sequence $x(n)$ is given by $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} x(2n) W_N^{2kn} + \sum_{n=0}^{N/2-1} x(2n+1) W_N^{k(2n+1)}$$

$$= \sum_{n=0}^{N/2-1} x(2n) W_{N/2}^{kn} + \sum_{n=0}^{N/2-1} x(2n+1) W_N^{2kn} \cdot W_N^{k}$$

$$= \sum_{n=0}^{N/2-1} x(2n) W_{N/2}^{kn} + W_N^{k} \sum_{n=0}^{N/2-1} x(2n+1) W_N^{2kn}$$

The above equation the first sum $N/2$ DFT of the even indexed sequence and the second sum being the $N/2$ DFT of the odd indexed sequence and $X_e(k)$ and $X_o(k)$ are periodic in k with period $N/2$.

for $k \geq N/2$ by using symmetry property

$$X(k) = X_e(k - N/2) - W_N^{k} X_o(k - N/2)$$

For $N=8$ the values of $X_e(k)$ and $X_o(k)$ are 4 point DFT's of even indexed sequence $x_e(n)$ and odd indexed sequence $x_o(n)$ respectively where

$x_e(0) = x(0)$ $\quad$ $x_o(0) = x(1)$

$x_e(1) = x(2)$ $\quad$ $x_o(1) = x(3)$

$x_e(2) = x(4)$ $\quad$ $x_o(2) = x(5)$

$x_e(3) = x(5)$ $\quad$ $x_o(3) = x(7)$

By substituting different values of 'k' i.e $k=0$ to 7

$X(0) = X_e(0) + W_8^0 X_o(0)$ $\quad$ $X(4) = X_e(0) - W_8^0 X_o(0)$

$X(1) = X_e(1) + W_8^1 X_o(1)$ $\quad$ $X(5) = X_e(1) - W_8^1 X_o(1)$

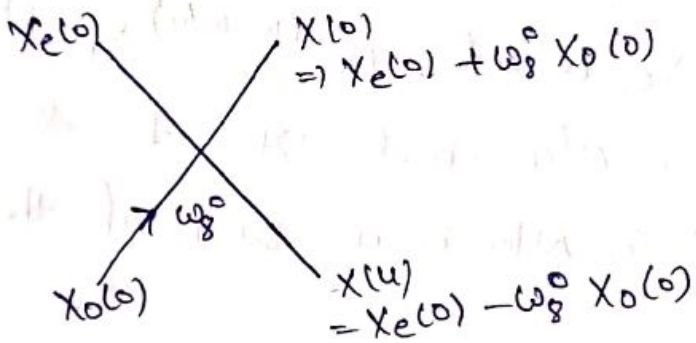
$X(2) = X_e(2) + W_8^2 X_o(2)$ $\quad$ $X(6) = X_e(2) - W_8^2 X_o(2)$

$X(3) = X_e(3) + W_8^3 X_o(3)$ $\quad$ $X(7) = X_e(3) - W_8^3 X_o(3)$

~~$X(4) = X_e(4)$~~

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From the above set of equations we can find that $x(0)$ and $x(4)$, $x(1)$ and $x(5)$, $x(2)$ and $x(6)$, $x(3)$ and $x(7)$ have same inputs. $X(0)$ is obtained by multiplying $X_0(0)$ with ω_8^0 and adding the product to $X_e(0)$. Similarly $X(4)$ is obtained by multiplying $X_0(0)$ with ω_8^0 and subtracting the product from $X_e(0)$. This operation can be represented by a "butterfly" diagram as shown in fig below.



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* The 'N/2' point DFT is can be express as a combination of 'N/4' point DFT's that is $X_e(k) = X_{ee}(k) + W_N^{2k} X_{eo}(k)$

* $X_{ee}(k+N/4) = W_N^{2(k+N/4)} X_{eo}(k+N/4)$ where $X_{ee}(k)$ is 'N/4' point DFT of the even numbers of $x_e(n)$ and $X_{eo}(k)$ is N/4 point DFT of the odd numbers of $x_e(n)$

Similarly $X_o(k) = X_{oe}(k) + W_N^{2k} X_{oo}(k)$

$= X_{oe}(k+N/4) - W_N^{2(k+N/4)} X_{oo}(k+N/4)$

where $X_{oe}(k)$ is N/4 point DFT of the even numbers of $x_o(n)$ and $X_{oo}(k)$ is N/4 point DFT of the odd numbers of $x_o(n)$

* for N=8 the sequence $x_e(n)$ can be divided into even and odd indexed sequences.

$x_{ee}(0) = x_e(0)$

$x_{ee}(1) = x_e(2)$

$x_{eo}(0) = x_e(1)$

$x_{eo}(1) = x_e(3)$

$X_e(0) = X_{ee}(0) + W_8^0 X_{eo}(0)$

$X_e(1) = X_{ee}(1) + W_8^2 X_{eo}(1)$

$X_e(2) = X_{ee}(0) - W_8^0 X_{eo}(0)$

$X_e(3) = X_{ee}(1) - W_8^2 X_{eo}(1)$

* Similarly the odd sequence $x_o(n)$ is divided into even and odd indexed values.

$x_{oe}(0) = x_o(0)$

$x_{oe}(1) = x_o(2)$

$x_{oo}(0) = x_o(2)$

$x_{oo}(1) = x_o(3)$

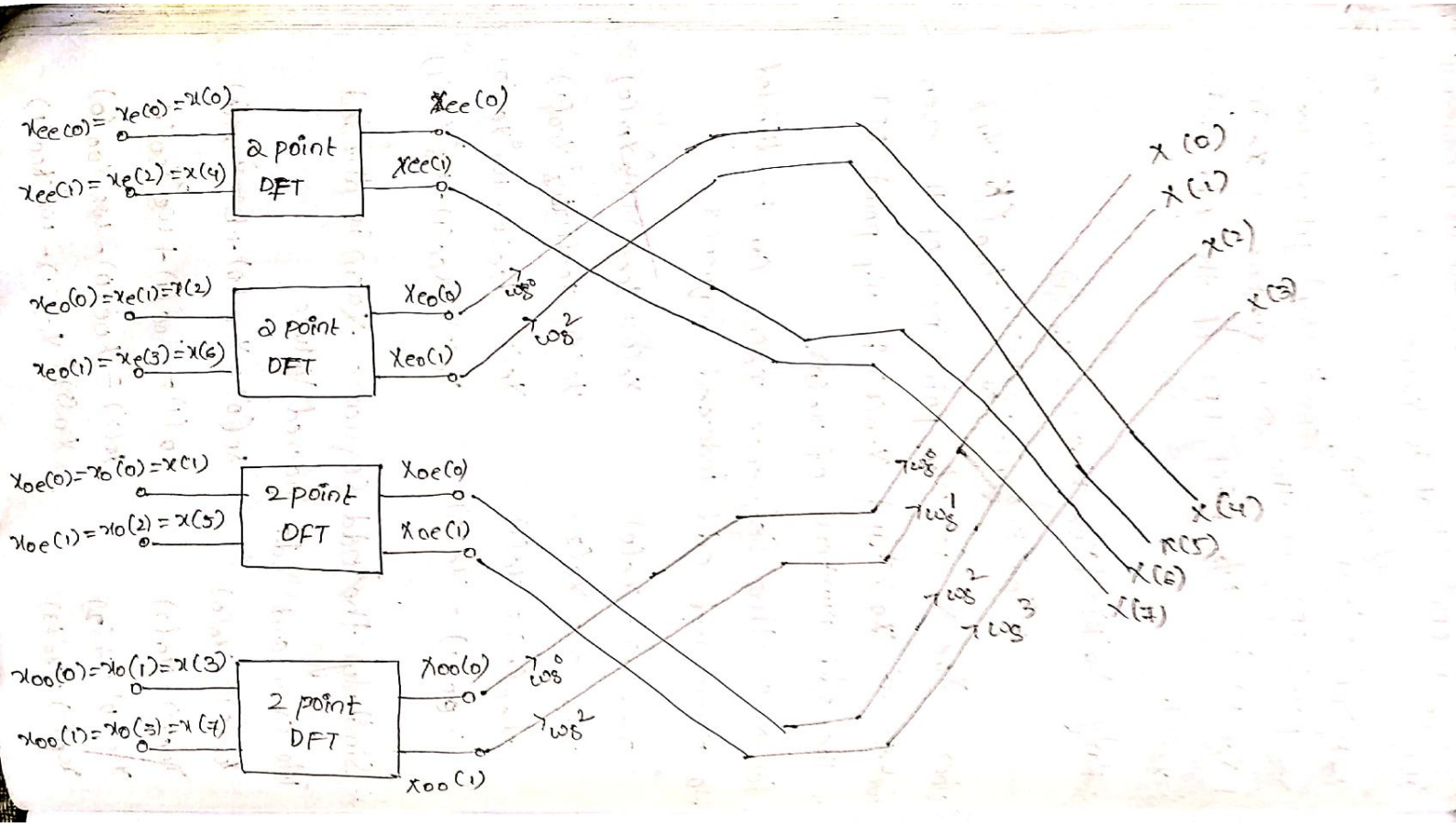
$X_o(0) = X_{oe}(0) + W_8^0 X_{oo}(0)$

$X_o(1) = X_{oe}(1) + W_8^2 X_{oo}(1)$

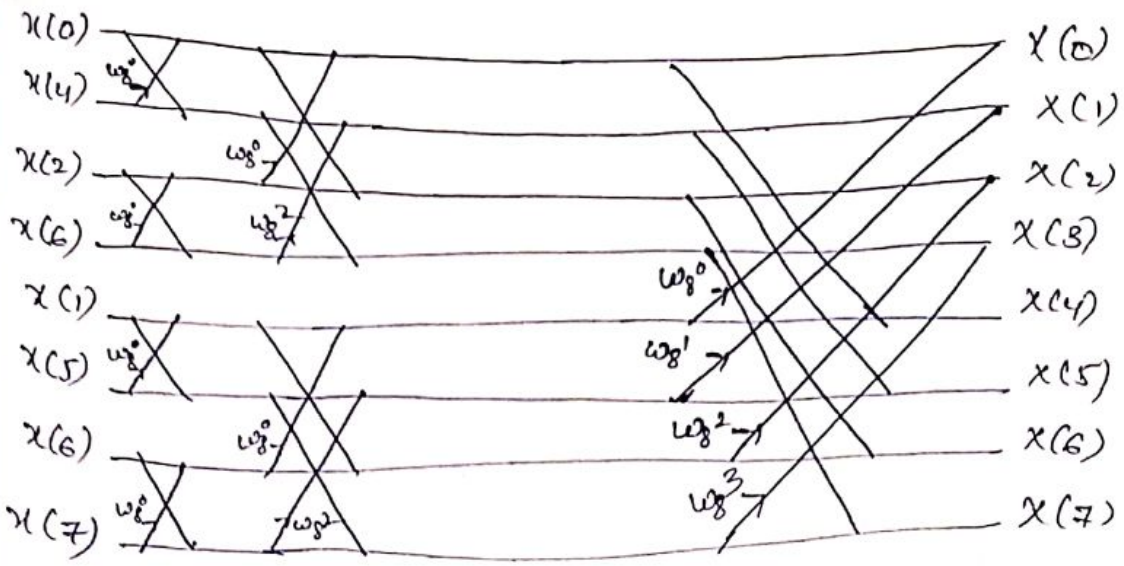
$X_o(2) = X_{oe}(0) - W_8^0 X_{oo}(0)$

$X_o(3) = X_{ee}(1) - W_8^2 X_{oo}(1)$

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i/p Sample	Binary rep of i/p sample	Bit reversal Binary	Bit Reversal sample Index
0	0 0 0	0 0 0	0
1	0 0 1	1 0 0	4
2	0 1 0	0 1 0	2
3	0 1 1	1 1 0	6
4	1 0 0	0 0 1	1
5	1 0 1	1 0 1	5
6	1 1 0	0 1 1	3
7	1 1 1	1 1 1	7

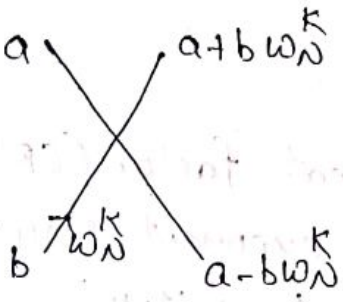
In DFT Algorithm we can find that the output sequences to be in natural order that X(k) where k = 0 to m-1. the input sequence has to be stored in a bit reversal order for an 8-point DFT Algorithm the input sequence is in order x(0), x(4), x(2), x(6), x(1), x(5), x(3), x(7).

for n=8 the bit Reversal process is shown in the table.

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Basic operation:-

The Basic operation of DIT Algorithm is represents by using butterfly in which two inputs are combine to give two outputs at the input a and b are the given values and the output are shown in the figure.



Steps of Radix-2 Decimation in time FFT:-

1. The no of i/p samples $N=2^M$. where 'M' is an interger.
2. The i/p sequence is shuffled through bit reversal.
3. The no of stages in the flow graph is given by $M=\log_2 N$.
4. Each stage consists of $N/2$ butterflys.
5. i/p or outputs for each butterfly are seperate by 2^{m-1}, where 'm' represents the stage index. that is for first stage $m=1$, & for second stage $m=2$, and so on.
6. The no of complex multiplications is given by $N/2 \log_2 N$.
7. The no of complex Additions is given by $N\log_2 N$.

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8. The twiddle factor exponents are the function of stage 'm' index and is given by

$$K = \frac{Nt}{2^m}, \quad t = 0, 1, \ldots\ldots 2^{m-1} - 1$$

9. The no of sets of butterflyes in each stage given by 2^{M-m}

10. The exponent Repeat factor (ERF) which is the no of times the exponent sequence associated with 'm' is repeated by '2^{M-m}'

1) Find DFT of sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using decimation in time algorithm.

Sol $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$

$N = 8$

$M = \log_2 N = \log_N 2^3 = 3$

$W_8^0 = (e^{-j2\pi/N})^0 = 1$

$W_8^1 = (e^{-j2\pi/8})^1$

$= e^{-j\pi/4} = \cos\pi/4 - j\sin\pi/4 = 1/\sqrt{2} - j\,1/\sqrt{2}$

$= 0.707 - 0.707j$

$W_8^2 = (e^{-j2\pi/8})^2$

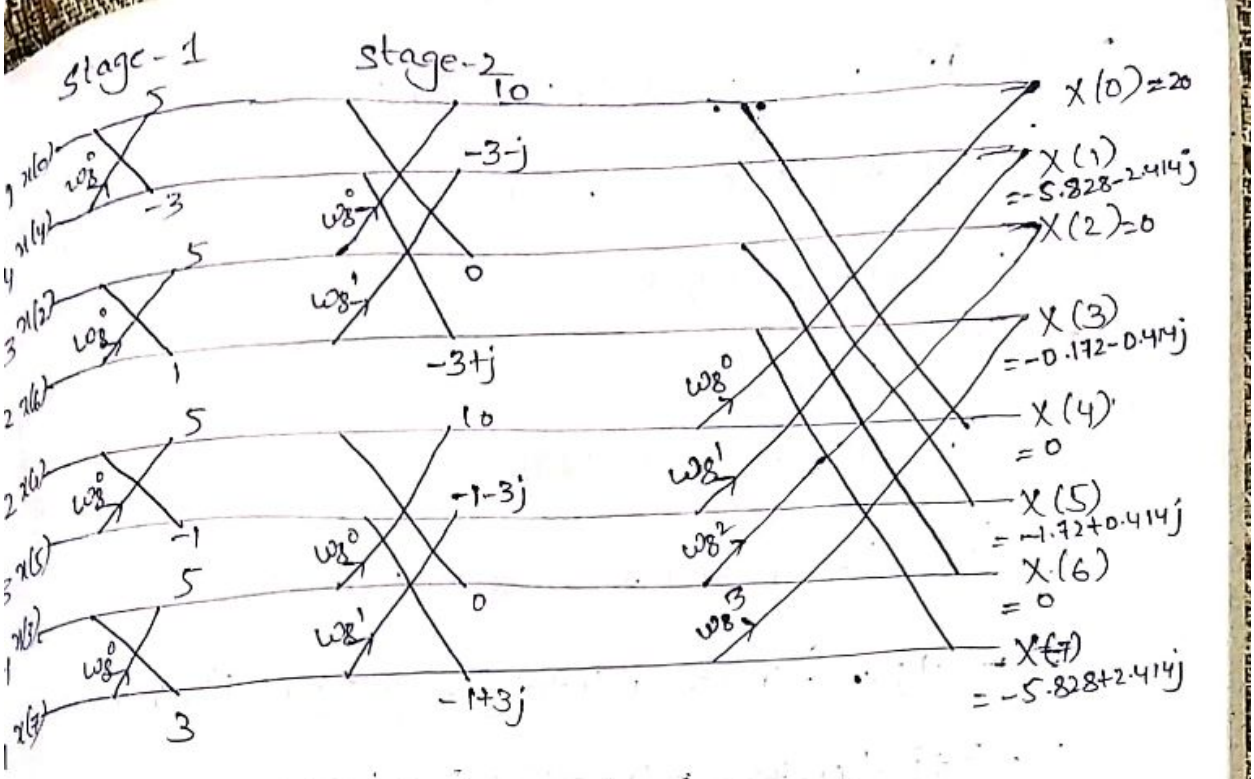
$= e^{-j2\pi/8 \cdot 2} = e^{-j\pi/2} = \cos\pi/2 - j\sin\pi/2 = 0 - j = -j$

$W_8^3 = (e^{-j2\pi/8})^3$

$= e^{-j2\pi \cdot 3/8\,4} = e^{-j3\pi/4}$

$= \cos 3\pi/4 - j\sin 5\pi/4 = -0.707 - 0.707j$

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$$X(1) = (-3-j) + (0.707 - 0.707j)(-1-3j)$$

$$= -3 - j - 0.707 + j0.707 - 3xj0.707 + 3j^2 x 0.707$$

$$= -3 - j - 0.707 + j0.707 - 2.12j - 2.12$$

$$= -5.83 + j(-1 + 0.707 - 2.12)$$

$$= -5.83 - 2.41j$$

$$X(5) = (-3-j) - (0.707 - j0.707)(-1-3j)$$

$$= -3 - j - (-0.707 + j0.707 - 3x0.707j + 3j^2 x 0.707)$$

$$= -3 - j + 0.707 - j0.707 + 3jx0.707 + 3x0.707$$

$$= -3 - j + 0.707 - 0.707j + 2.12j + 2.12$$

$$= -0.17 - j(1 + 0.707 - 2.12)$$

$$= -0.17 + 0.41j$$

$$X(2) = 0 + W_8^2(0) = 0$$

$$X(6) = 0 - W_8^2(0) = 0$$

$$X(3) = (-3+j) + W_8^3(-1+3j)$$

$$= (-3+j) + (-0.707 + j0.707)(-1+3j)$$

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$= -3+j+0.707-0.707j-2.12j-2.12$

$= -4.41+j(1-0.707-2.12)$

$= -4.41+j(-1.83)$

$= -4.41-1.83j$

$X(7) = (-3+j) - W_8^3(-1+3j)$

$= (-3+j) - (-0.707+j0.707)(-1+3j)$

$= -3+j-(-0.707-0.707j-2.12j+3j^2\times 0.707)$

$= -3+j+0.707+0.707j+2.12j+2.12$

$= -5.828+j2.414$

$X(k) = \{20, -5.828-2.414j, 0, -0.172-j0.414, 0, -1.72+j0.414, 0, -5.828+j2.414\}$

2) Compute the 8-point DFT of the sequence $x(n) = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$ using decimation in time.

Sol

$W_8^0 = (e^{-j2\pi/8})^0 = 1$

$W_8^1 = (e^{-j2\pi/8})^1 = \cos\pi/4 - j\sin\pi/4 = 0.707-j0.707$

$W_8^2 = (e^{-j2\pi/8})^2 = \cos\pi/2 - j\sin\pi/2 = 0-j = -j$

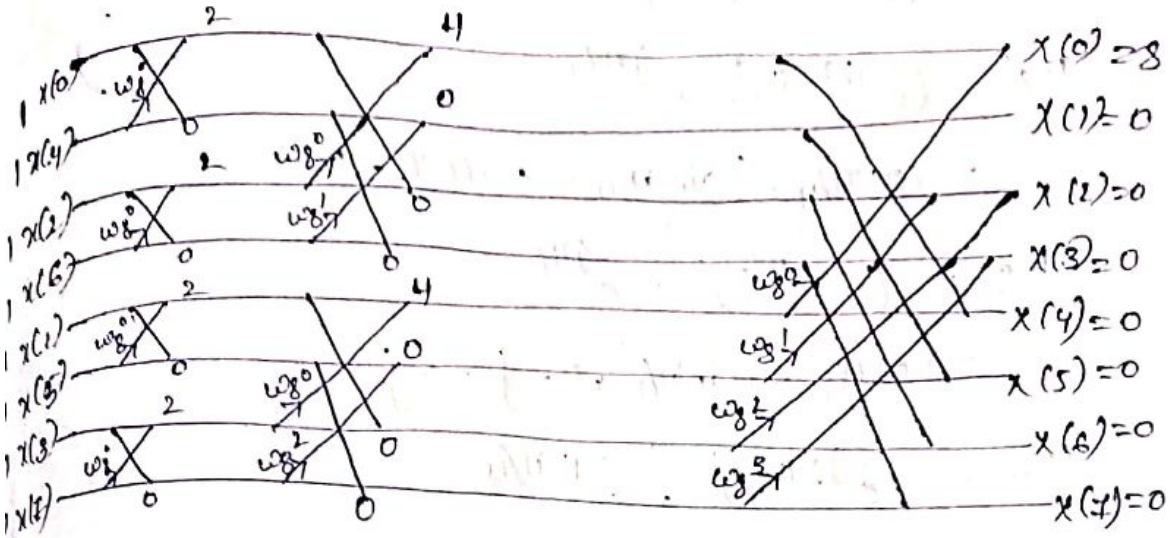
$W_8^3 = (e^{-j2\pi/8})^3 = \cos 3\pi/4 - j\sin 3\pi/4 = -0.707-0.707j$

$N = 8$

$M = \log_2^N = \log_2^8$

$= \log_2^{2^3} = 3$

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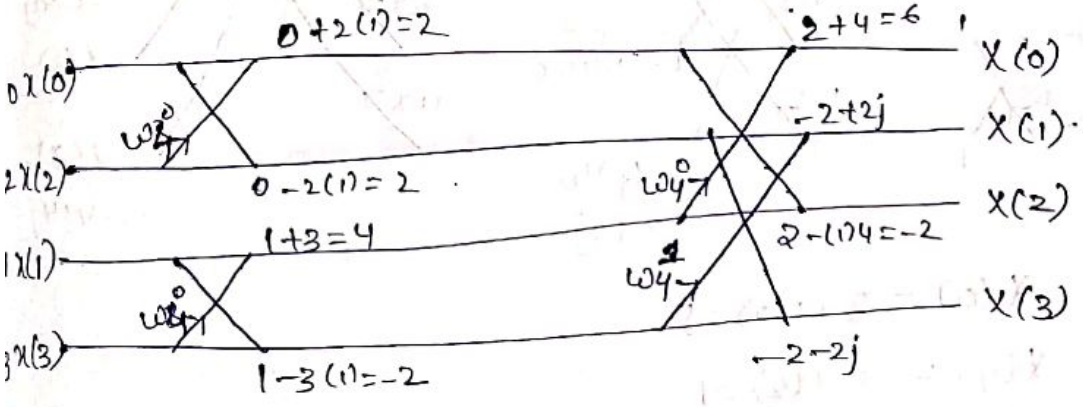


$X(k) = \{8, 0, 0, 0, 0, 0, 0, 0\}$

3) Compute 4-point DFT of the sequence $x(n) = \{0; 1, 2, 3\}$ using decimation in time Algorithm.

$\omega_4^0 = (e^{-j2\pi/4})^0 = 1$

$\omega_4^1 = (e^{-j2\pi/4})^1 = e^{-j\pi/2} = \cos\pi/2 - j\sin\pi/2 = 0 - j = -j$



$X(k) = \{6, -2+2j, -2, -2-2j\}$

4) find 8-point DFT of Sequence $x(n) = \{1, 1, 1, 1, 1, 0, 0, 0\}$ using Decimation in time algorithm.

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Sol $W_8^0 = (e^{-j2\pi/N})^0 = 1$

$W_8^1 = (e^{-j2\pi/8})^1 = e^{-j\pi/4}$

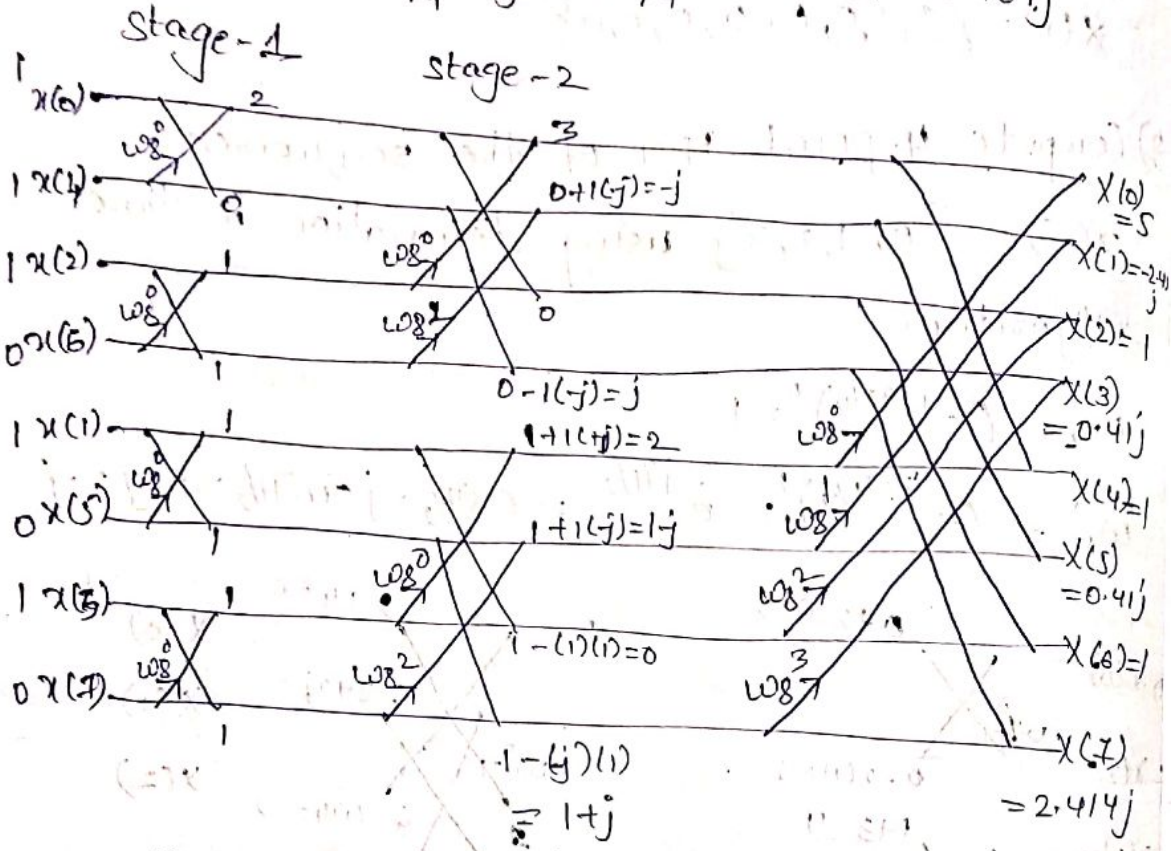
$= \cos\pi/4 - j\sin\pi/4 = 0.707 - 0.707j$

$W_8^2 = (e^{-j2\pi/8})^2 = e^{-j\pi/2}$

$= \cos\pi/2 - j\sin\pi/2 = 0 - j = -j$

$W_8^3 = (e^{-j2\pi/8})^3 = e^{-j3\pi/4}$

$= \cos 3\pi/4 - j\sin 3\pi/4 = -0.707 - 0.707j$



$X(0) = 3+1(2) = 5$

$X(4) = 3-1(2) = 1$

$X(1) = -j + (0.707 - j0.707)(1-j)$

$= -j + (0.707 - j0.707 - 0.707j + j^2 0.707)$

$= -j + 0.707 - 1.41j - 0.707$

$= -2.41j$

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$X(5) = -j - (0.707 - 0.707j)(1-j)$

$= -j - (0.707 - 0.707j - 0.707j + 0.707j^2)$

$= -j - (0.707 - 1.41j - 0.707)$

$= -j + 1.41j = 0.41j$

$X(2) = 1 + (j)(0) = 1$

$X(6) = 1 - (-j)(0) = 1$

$X(3) = j + (-0.707 - 0.707j)(1+j)$

$= j + (-0.707 - 0.707j - 0.707j - j^2 0.707)$

$= j + (0.707 - 1.41j + 0.707j)$

$= j - 1.41j = -0.41j$

$X(7) = j - (-0.707 - j0.707)(1+j)$

$= j - (-0.707 - 0.707j - 0.707j - j^2 0.707)$

$= j - (-0.707 - 1.41j + 0.707)$

$= j + 1.41j$

$= 2.414j$

$X(k) = \{5, -2.41j, 1, -0.41j, 1, 0.414j, 1, 2.414j\}$

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⑤ Draw the signal flow graph of 16 point DIT and FFT

No. of Input Samples is 16

$N = 16 = 2^4$

→ The I/P Sequence bit Reversal

$N = 16 = 2^4$

0 – 0000	→ 0000 – 0	9 – 1001	→ 1001 – 9
1 – 0001	→ 1000 – 8	10 – 1010	→ 0101 – 5
2 – 0010	→ 0100 – 4	11 – 1011	→ 1101 – 13
3 – 0011	→ 1100 – 12	12 – 1100	→ 0011 – 3
4 – 0100	→ 0010 – 2	13 – 1101	→ 1011 – 11
5 – 0101	→ 1010 – 10	14 – 1110	→ 0111 → 7
6 – 0110	→ 0110 – 6	15 – 1111	→ 1111 → 15
7 – 0111	→ 1110 – 14		
8 – 1000	→ 0001 – 1		

Steps:

No. of stages $M = \log_2 N$

$= \log_2 16 = 4$.

Each stage have $\frac{N}{2}$ butterflies $\Rightarrow \frac{16}{2} = 8$

Spacing b/w butterflies is $2^{M-1} = 2^0 = 1$

→ No. of Complex multiplication $= N/2 \log_2 N$

$= \frac{16}{2} \log_2 16 = 8 \log_2 16$

$= 8 \log_2 2^4$

$= 8 \times 4 \log_2 2 = 32$

→ Nitik Sachdeva

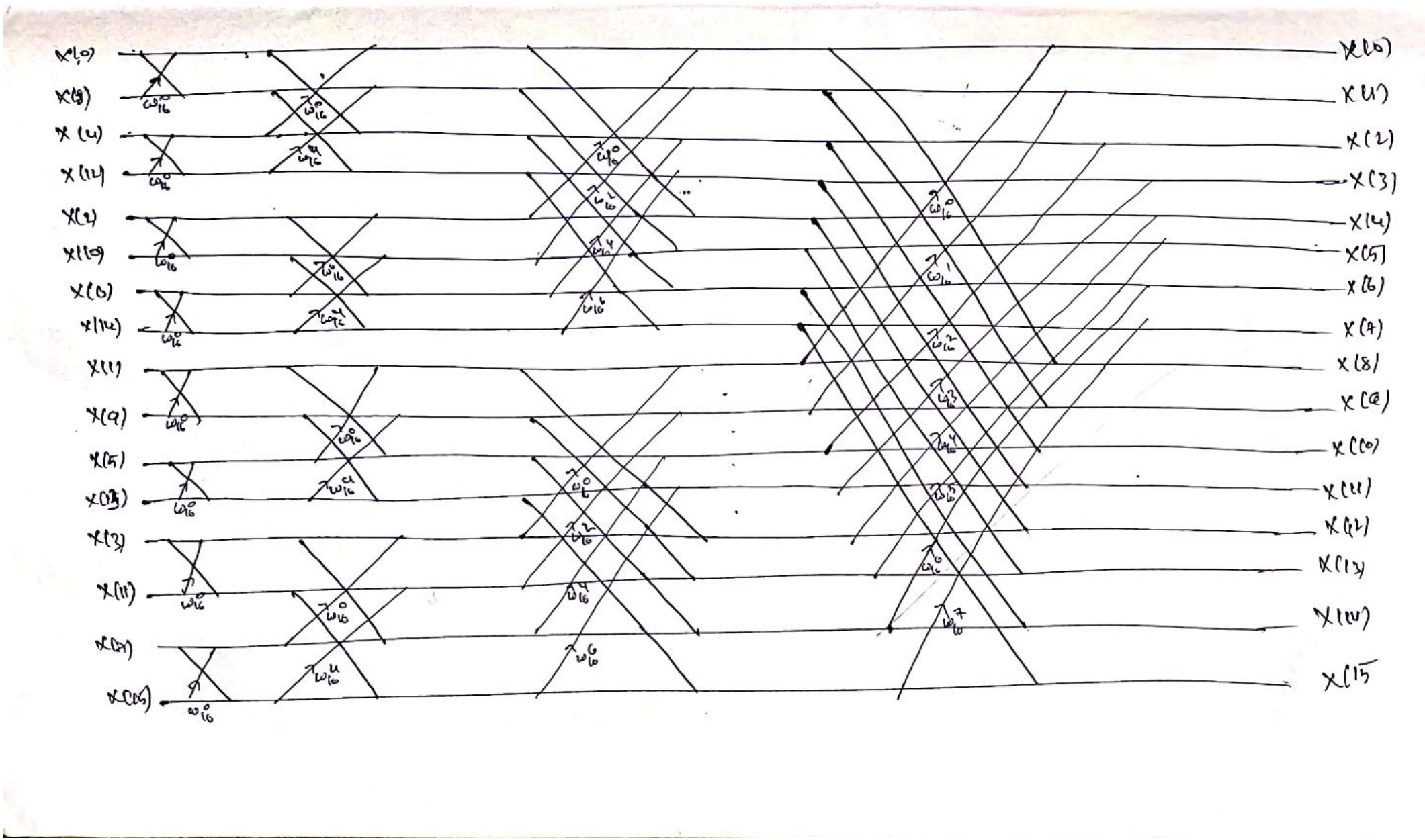
$K = \frac{Nt}{2m}$ $\quad t \rightarrow 0 \text{ to } 2^{M-1} - 1 \quad m = 1$

$= \frac{16(0)}{2}$ $\quad 0 \text{ to } 2^{4-1} - 1$

$= 0 \text{ to } 0$

$K = 0$

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Decimation in Frequency Fast Fourier Transform Algorithm:-

In Decimation in frequency Algorithm the output Sequence $X(k)$ is divided into smaller and Smaller Sub Sequence.

In this Algorithm the input sequence $x(n)$ is partion into two Sequence each of length $N/2$ Samples. the first Sequence $x_1(n)$ consists of first $N/2$ Samples of $x(n)$. and Second Sequence $x_2(n)$ Consists of the last $N/2$ Samples of $x(n)$.

i.e., $x_1(n) = x(n)$ for $n = 0, 1, \ldots N/2 - 1$

$x_2(n) = x(n + N/2)$ for $n = 0, 1, \ldots N/2 - 1$

The N-point DFT of $x(n)$ Can be written as

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} x(n)\, W_N^{kn} + \sum_{n=N/2}^{N-1} x(n)\, W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} x_1(n)\, W_N^{kn} + \sum_{n=0}^{N/2-1} x_2(n)\, W_N^{k(n+N/2)}$$

$$= \sum_{n=0}^{N/2-1} x_1(n)\, W_N^{kn} + W_N^{kN/2} \sum_{n=0}^{N/2-1} x_2(n)\, W_N^{kn}$$

$$W_N^{kN/2} = e^{-j\frac{2\pi}{N}k\frac{N}{2}}$$

$$= e^{-j\pi k}$$

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$$X(k) = \sum_{n=0}^{N/2-1} x_1(n)\, W_N^{kn} + e^{-j\pi k} \sum_{n=0}^{N/2-1} x_2(n)\, W_N^{kn}$$

for Even Values of k $e^{-j\pi k} = 1$

$$\therefore X(2k) = \sum_{n=0}^{N/2-1} x_1(n)\, W_N^{kn} + \sum_{n=0}^{N/2-1} x_2(n)\, W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} [x_1(n) + x_2(n)]\, W_N^{2kn}$$

$$= \sum_{n=0}^{N/2-1} f(n)\, W_N^{2kn}$$

where $f(n) = x_1(n) + x_2(n)$

* f(n) is obtained by adding the first half and last half of the input sequence x(n).

when k is odd $e^{-j\pi k} = -1$

$$\therefore X(2k+1) = \sum_{n=0}^{N/2-1} x_1(n)\, W_N^{kn} - \sum_{n=0}^{N/2-1} x_2(n)\, W_N^{kn}$$

$$= \sum_{n=0}^{N/2-1} [x_1(n) - x_2(n)]\, W_N^{(2k+1)n}$$

$$= \sum_{n=0}^{N/2-1} [x_1(n) - x_2(n)]\, W_N^{n}\, W_N^{2kn}$$

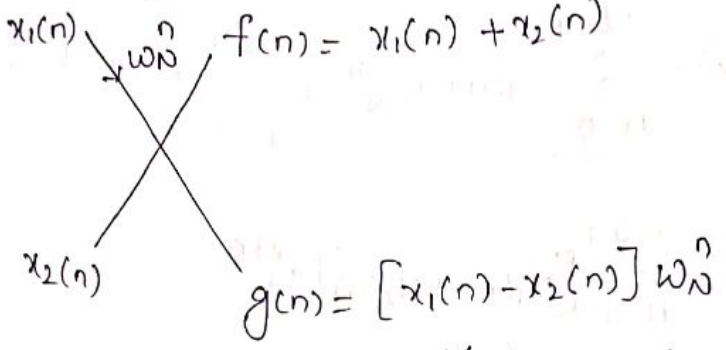
$$= \sum_{n=0}^{N/2-1} g(n)\, W_N^{2kn}$$

where $g(n) = [x_1(n) - x_2(n)]\, W_N^{n}$

*g(n) is obtained the Subtracting the second half of Sequence from first half of the input Sequence and that multiplying (results with) relevant Sequence W_N^{n}.

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from the above two sequences of $f(n)$ and $g(n)$ the basic operation can be represented by butterfly as shown in figure.



For $n=8$ we have $X(0) = \sum_{n=0}^{N/2-1} f(n) W_N^{2kn}$

$$X(0) = \sum_{n=0}^{3} f(n) W_8^{2(0)n}$$

$$= \sum_{n=0}^{3} f(n) \cdot 1$$

$$= f(0) + f(1) + f(2) + f(3)$$

$$X(2) = \sum_{n=0}^{3} f(n) W_8^{2(1)n}$$

$$= \sum_{n=0}^{3} f(n) W_8^{2n}$$

$$= f(0) W_8^{2(0)} + f(1) W_8^{2(1)} + f(2) W_8^{2(2)} + f(3) W_8^{2(3)}$$

$$= f(0) + f(1) W_8^{2} + f(2) W_8^{4} + f(3) W_8^{6}$$

$$W_8^4 = -1$$

$$W_8^8 = 1$$

$$W_8^6 = -W_8^2$$

$$X(2) = f(0) + f(1) W_8^2 - W_8 f(2) - f(3) W_8^2$$

$$X(4) = \sum_{n=0}^{3} f(n) W_8^{2(2)n}$$

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$$X(4) = \sum_{n=0}^{3} f(n)\, W_8^{4n}$$

$$= \sum_{n=0}^{3} f(n)\,(-1)^n$$

$$= f(0)(-1)^0 + f(1)(-1)^1 + f(2)(-1)^2 + f(3)(-1)^3$$

$$X(4) = f(0) - f(1) + f(2) - f(3)$$

$$X(6) = \sum_{n=0}^{3} f(n)\, W_8^{2(3)n}$$

$$= \sum_{n=0}^{3} f(n)\, W_8^{6n}$$

$$= \sum_{n=0}^{3} f(n)\,(-W_8^2)^n$$

$$= f(0)(-W_8^2)^0 + f(1)(-W_8^2)^1 + f(2)(-W_8^2)^2 + f(3)(-W_8^2)^3$$

$$= f(0) - f(1)W_8^2 - f(2) + f(3)W_8^2$$

$$X(1) = \sum_{n=0}^{N/2-1} g(n)\, W_N^{2kn}$$

$$= \sum_{n=0}^{3} g(n)\, W_8^{2(0)n}$$

$$= \sum_{n=0}^{3} g(n) \cdot 1$$

$$= g(0) + g(1) + g(2) + g(3)$$

$$X(3) = \sum_{n=0}^{3} g(n)\, W_8^{2(1)n}$$

$$= \sum_{n=0}^{3} g(n)\, W_8^{2n}$$

$$= \sum_{n=0}^{3} g(0)W_8^{2(0)} + g(1)W_8^{2(1)} + g(2)W_8^{2(2)} + g(3)W_8^{2(3)}$$

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$$X(3) = g(0) + g(1)\, \omega_8^2 + g(2)\, \omega_8^4 + g(3)\, \omega_8^6$$

$$X(3) = g(0) + g(1)\, \omega_8^2 - g(2) - g(3)\, \omega_8^2$$

$$X(5) = \sum_{n=0}^{3} g(n)\, \omega_N^{2(2)n}$$

$$= \sum_{n=0}^{3} g(n)\, \omega_8^{4n}$$

$$= \sum_{n=0}^{3} g(n)\, (-1)^n$$

$$= g(0)(-1)^0 + g(1)(-1)^1 + g(2)(-1)^2 + g(3)(-1)3$$

$$= g(0) - g(1) + g(2) - g(3)$$

$$X(7) = \sum_{n=0}^{3} g(n)\, \omega_N^{2(3)n}$$

$$= \sum_{n=0}^{3} g(n)\, \omega_8^{6n}$$

$$= \sum_{n=0}^{3} g(n)(\omega_8^2)^n$$

$$= g(0)(-\omega_8^2)^0 + g(1)(-\omega_8^2)^1 + g(2)(-\omega_8^2)^2 + g(3)(-\omega_8^2)^3$$

$$= g(0) - g(1)\omega_8^2 + g(2)(-1) - g(3)(\omega_8^6)$$

$$= g(0) - g(1)\omega_8^2 - g(2) + g(3)\omega_8^2$$

We have seen that the even indexed samples of $X(k)$ can be obtained from the four point DFT of the sequences $f(n)$. where $f(n) = x_1(n) + x_2(n)$.

$f(0) = x_1(0) + x_2(0)$

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$f(1) = x_1(1) + x_2(1)$

$f(2) = x_1(2) + x_2(2)$

$f(3) = x_1(3) + x_2(3)$

The odd indexed values of $X(k)$ can be obtain from the 4-point DFT of Sequence $g(n)$.

$$g(n) = [x_1(n) - x_2(n)]\, W_N^n$$

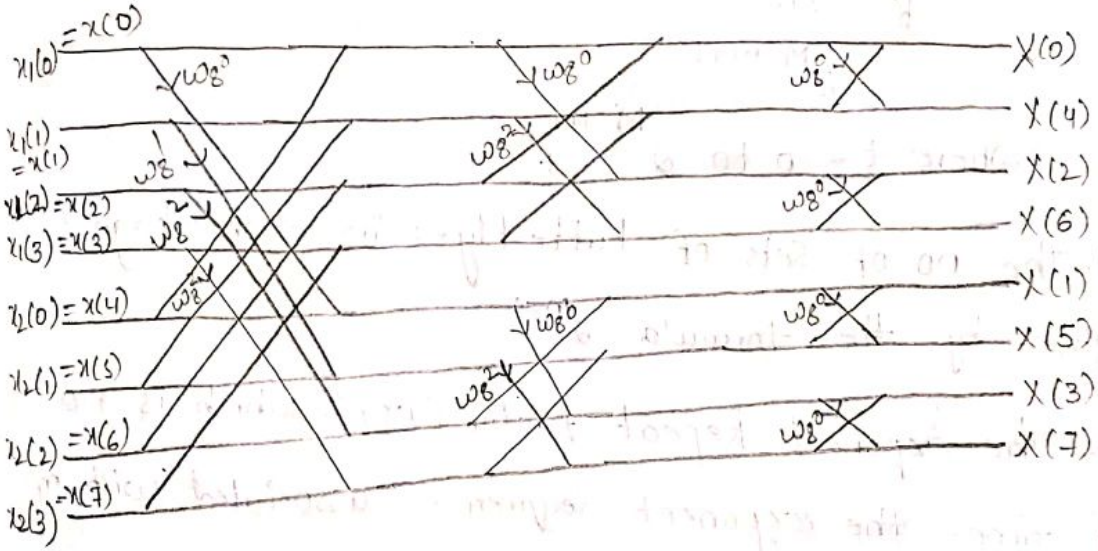
$$g(n) = [x_1(n) - x_2(n)]\, W_8^n$$

$$g(0) = [x_1(0) - x_2(0)]\, W_8^0$$

$$g(1) = [x_1(1) - x_2(1)]\, W_8^1$$

$$g(2) = [x_1(2) - x_2(2)]\, W_8^2$$

$$g(3) = [x_1(3) - x_2(3)]\, W_8^3$$



Steps for Radix-2 Decimation in Frequency FFT

Algorithm:-

1. The no of input Samples $N = 2^M$ where M is the no of stages.
2. The input Sequence is in natural order.
3. The no of stages in the flow graph is given by

$$M = \log_2 N$$

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4. Each stage consits of $N/2$ butterflyes.

5. inputs (or) output for each butterfly are seperated by 2^{M-m} Samples.

where 'm' represents the stage index that is for first stage $M=1$ and Second

6. the no of complex multiplications is given by

$N/2 \log_2 N$.

7. The no of complex Additions is given by $N \log_2 N$

8. The twiddle factor Components (or) a function of Stage index and is given by

$$K = \frac{Nt}{2^{M-m+1}}$$

Where $t = 0$ to $2^{M-m} - 1$

9. The no of sets of butterflyes in each stage is given by the formula 2^{m-1}

10. The exponent Repeat factor (ERF) which is no of times the exponent Sequence associated with 'm' repeated is given by 2^{m-1}

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Difference And Similarities Between DIT and DIF

Difference

1. For Decimation in time the input is bit reversal while the output is natural order. where as for Decimation in frequency the i/p is in natural order while the output is bit Reversal order.
2. The DIF butterfly is Slightly different from DIT where in DIF the complex multiplication takes place after the add Subtract operation.

Similarities:-

1. Both Algorithms require $N \log_2 N$ operations to calculate the DFT.
2. Both Algorithms can be done in place and both need to perform bit Reversal at some place during the calculation

1) Find the DFT of the Sequence $x(n)=\{1, 2, 3, 4, 4, 3, 2, 1\}$ using Decimation in frequency Algorithm.

Sol given $W_8^0 = (e^{-j2\pi/8})^0 = 1$

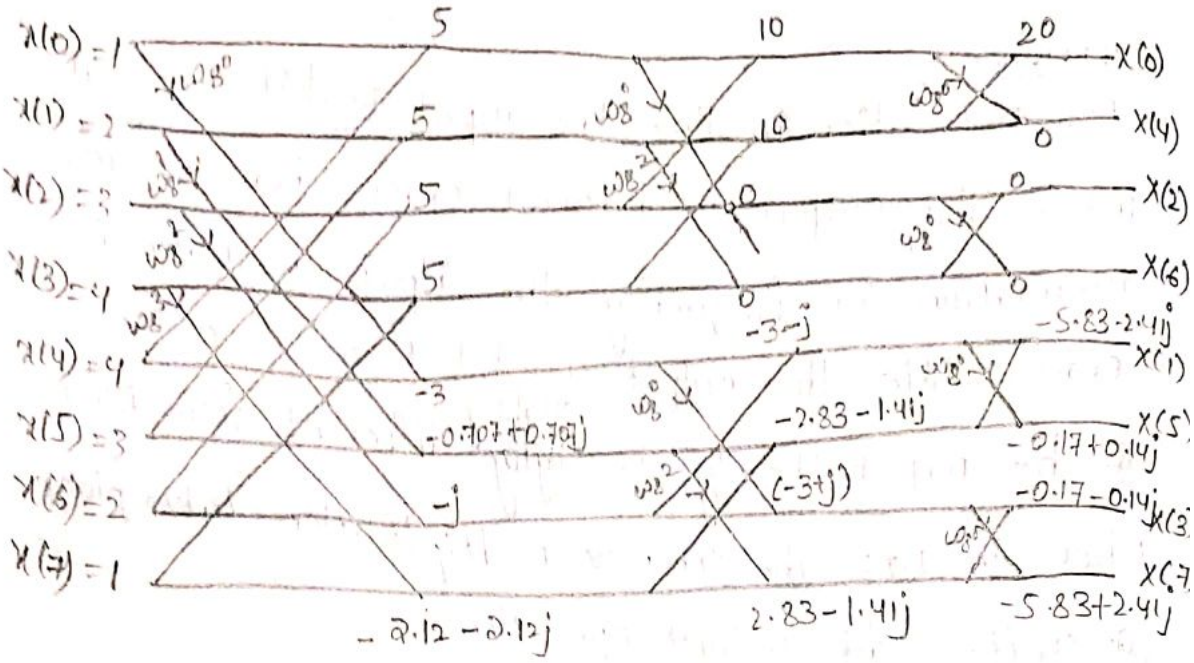
$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$

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$x(-3+j)(1)$

$-0.707+0.707j - 2.12 - 2.12j$

$-0.707+0.707j + 2.12 + 2.12j$

$(1.41 + 2.83j)(-j)$

$-1.41j + 2.83$

$-3-j-2.83-1.41j$

$-5.83 - 2.41j$

$-3-j+2.83+1.41j$

$(0.17 + 0.41j)$

$-3+j+2.83-1.41j$

$-0.17 - 0.41j$

$(-3+j-2.83+1.41j)$

$X(K) = \{20, -5.83-2.41j, 0, -0.17-0.14j, 0, -0.17+0.14j, 0, -5.83+2.41j\}$

2) Find the Calculate the 8-point DFT of the Sequence $x(n) = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$ by using Decimation in time & Decimation in frequency Algorithms.

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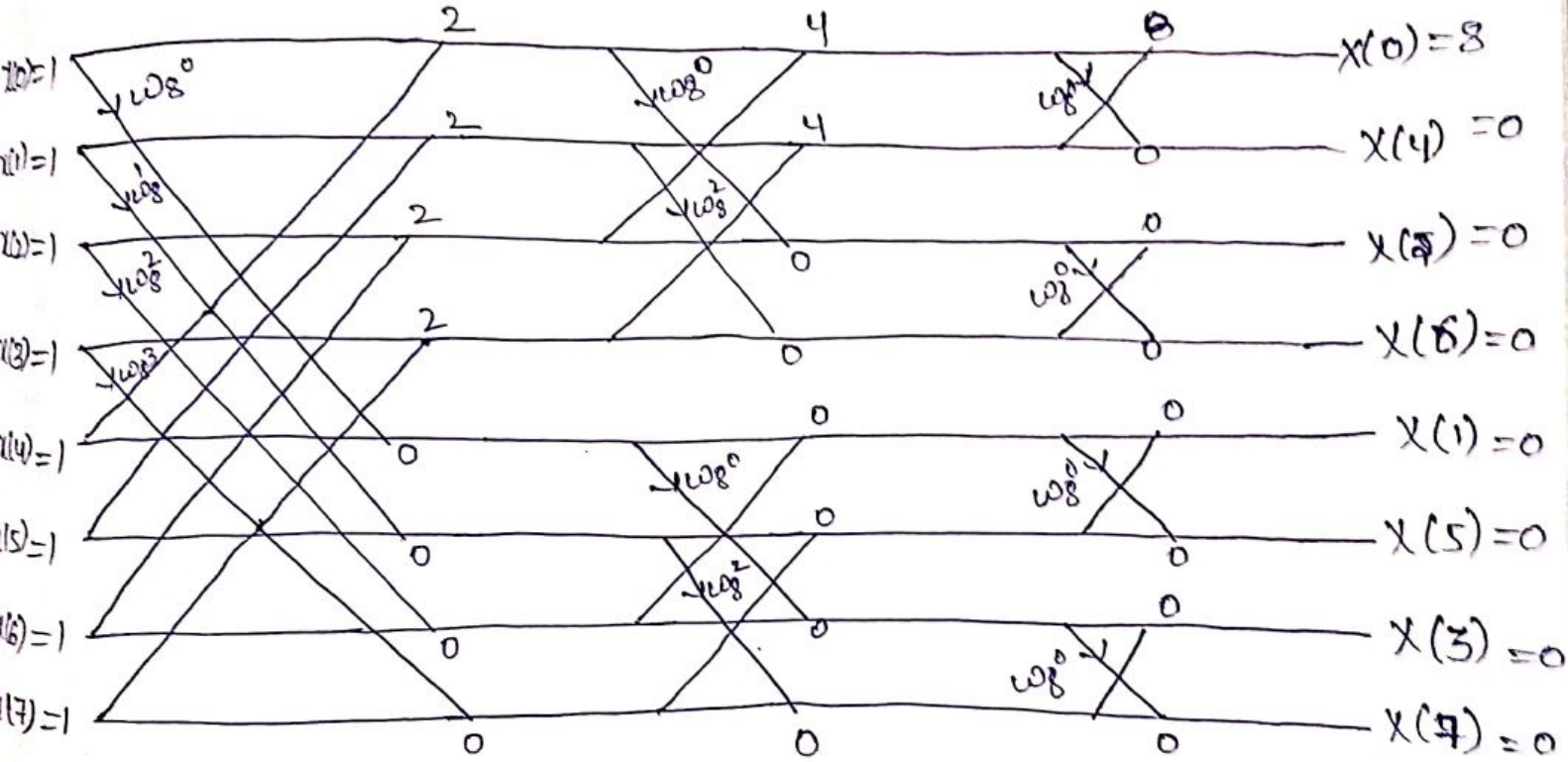
Sol) Decimation in Frequency

$W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$



$X(K) = \{8, 0, 0, 0, 0, 0, 0, 0\}$

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