Unit-I

Introduction to DSP

Signal:- Anything that carries some information is called Signal. (OR)

It is a physical quantity that varies with time, space and so on.

For ex:- Voice Signals, Speech Signal, ECG Signal.

There are basically 2 types of Signals.

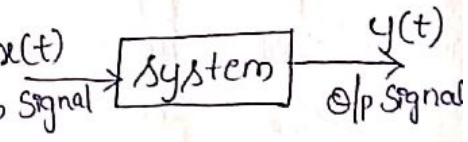
1. Continuous time Signals
2. Discrete time Signals

System:- Interconnection of components

It is a physical device that generates response or O/P for a given i/p.

Basically there are 2 types of systems.

1. Continuous time System
2. Discrete time System



Continuous time System

Continuous time System is one it can operates in continuous time Signal and produces continuous time Signal as O/p.

Let x(t) & y(t) represents the i/p and O/p of the continuous time system.



$$y(t) = T[x(t)]$$

Discrete time System:- The system is defined as it can operates on discrete time and produces discrete time signal as the O/p.

Let x(n) and y(n) are the i/p & O/p of the System.

$$y(n) = T[x(n)]$$

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Continuous time Signal:- The Signals that are defined for every instant of time are known as continuous time Signals. It can be represented as $x(t)$.

Discrete time Signals:- The Signals that are defined for discrete instant of time are known as discrete time Signals. It can be represented as $x(n)$

These are discrete in time, continuous in amplitude

Digital Signals:- These Signals are quantized in amplitude and discrete in time are known as digital Signals.

Elementary discrete time Sequences

1. Unit step Sequence
2. Unit ramp Sequence
3. Unit impulse Sequence
4. Exponential
5. Sinusoidal
6. complex Exponential

Unit step Sequence

The discrete time Unit step Sequence can be defined as $u(n) = 1$ for $n \geq 0$

$= 0$ for $n < 0$

The graphical representation of $u(n)$ may be represented as follows.

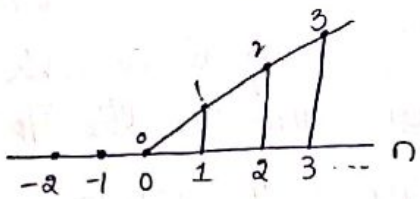


Unit ramp Sequence

The discrete time Unit ramp Sequence can be represented as $r(n) = n$ for $n \geq 0$

$= 0$ for $n < 0$

The graphical representation of $r(n)$ may be represented as



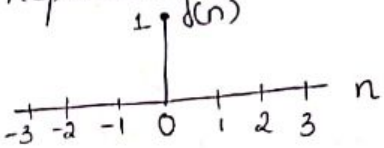
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Unit impulse Sequence

The discrete time unit impulse sequence can be defined as $\delta(n) = 1$ for $n = 0$

$= 0$ for $n \neq 0$

The graphical representation of $\delta(n)$ can be represented as



Exponential Sequence

The Exponential signal can be defined as $x(n) = a^n$ for all 'n'. This exponential sequence can be dependend on the value of a i.e., $a > 1$ (or) $0 < a < 1$ (or) $a < 0$

Sinusoidal Sequence

The discrete time Sinusoidal Signal can be defined as $x(n) = A \cos(\omega_0 n + \phi)$. Where ω_0 is frequency measured in rad / samples.

$\phi \rightarrow$ phase (radians)

By using Euler's identify, the above expression can be written as

$$x(n) = \frac{A}{2} e^{j\phi} e^{jn\omega_0} + \frac{A}{2} e^{-j\phi} e^{-jn\omega_0}$$

Complex Exponential Sequence

The complex exponential Sequence can be defined as $x(n) = a^n e^{j(\omega_0 n + \phi)}$

$$= a^n \left[\cos(\omega_0 n + \phi) + j \sin(\omega_0 n + \phi)\right]$$

Representation of discrete time Signals

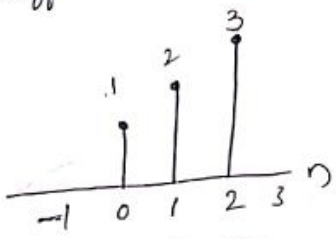
There are 4 different ways to represent the discrete time Signals.

1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequence representation

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Graphical representation:- Let us consider a discrete time Signal $x(n)$ having different values i.e.,

$x(-1) = 0$
$x(0) = 1$
$x(1) = 2$
$x(2) = 3$



Functional representation:- The discrete Time Signal $x(n)$ can be represented in the form of functional representation.

$$x(n) = \begin{cases} -2 & \text{for } n=2 \\ -1 & \text{for } n=1 \\ 0 & \text{for } n=0 \\ 1 & \text{for } n=-1 \\ 2 & \text{for } n=-2 \end{cases}$$

Tabular representation:- The discrete time Signal $x(n)$ can be represented in tabular form.

n	-1	-2	0	1	2
x(n)	0	1	2	3	4

Sequence representation:- The finite duration Sequence $x(n)$ the time origin for $n=0$ can be represented by an arrow ↑.

For ex:- $x(n) = \{1, 2, -1, -2\}$
↑

An infinite duration Sequence $x(n)$ the time origin can be represented as $x(n) = \{\ldots\ldots -1, 0, 1, 2, 3, \ldots\ldots\}$
↑

Classification of discrete time Signals

1. Energy and power Signals
2. Periodic and Aperiodic Signals
3. Symmetric and Asymmetric Signals
4. Casual and non-casual Signals

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Energy and power Signals

The Energy Signal can be defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

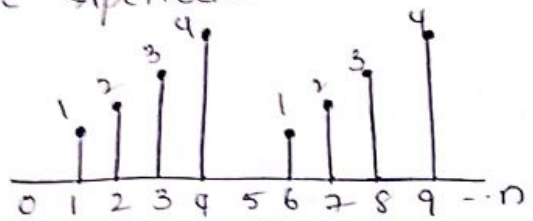
The power Signal can be defined as

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

Periodic and Aperiodic Signals

A Signal is said to be period must satisfies the condition $x(n) = x(n+N)$. otherwise the Signal is said to be aperiodic.

for N=4



Symmetric and Asymmetric Signals

A Signal is said to be Symmetric Signals or even Symmetry must satisfies the condition

$$x(n) = x(-n)$$

for ex:- $A \cos \omega n$

A Signal is said to be odd Symmetry or Asymmetric Signal must satisfies the condition is

$$-x(n) = x(-n)$$

(or)

$$x(n) = -x(-n)$$

A discrete time Signal $x(n)$ can be represented as

$$x(n) = x_e(n) + x_o(n)$$

In the above eqn n is replaced by '$-n$' then

$$x(-n) = x_e(-n) + x_o(-n)$$

According to definition $x(-n) = x_e(n) - x_o(n)$

Combining $x(n)$ & $x(-n)$ then we can get

$$x(n) = x_e(n) + x_o(n)$$
$$x(-n) = x_e(n) - x_o(n)$$
$$\overline{x(n) + x(-n) = 2x_e(n)}$$

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$$x_e(n) = \frac{x(n)+x(-n)}{2}$$

Similarly Subtracting $x(n)$ & $x(-n)$ then we can get

$$x(n) = x_e(n) + x_o(n)$$
$$x(-n) = x_e(n) - x_o(n)$$

$$x(n) - x(-n) = 2x_o(n)$$
$$x_o(n) = \frac{x(n)-x(-n)}{2}$$

Casual and non-casual Signals

A Signal is said to be casual Signal if its value '0' for $n<0$. otherwise the signal is said to be non-casual [for $n \geq 0$].

Ex:- for casual Signal is $x(n) = a^n u(n)$

for non-casual Signal $x(n) = a^n u(-n+1)$

Classification of discrete time Systems

1) Static and dynamic Systems
2) casual and non-casual systems
3) Linear and non-linear systems
4) Time invariant and Time variant Systems
5) FIR and IIR Systems
6) Stable and unstable Systems

Static and dynamic Systems

A discrete time System is called static if its output at any instant of time depends on the input sample at the same time but not on past and Future samples of the input, in any other case the System is said be dynamic System.

The examples are

$y(n) = a\,x(n)$ for static System

$y(n) = ax(n-1) + x(n+2)$ for dynamic System.

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Casual and non-casual Systems

A System is said to be casual if the output of the system at any time depends only at present and past inputs but does not depend on future inputs. It can be represented as

$$y(n) = f[x(n), x(n-1), x(n-2) \ldots]$$

If the output of a system depends on future inputs the system is said to be non-casual. It can be represented as $y(n) = x[2n] + x(n+1)$.

Linear and non-linear Systems

A System that satisfies Super position principles is said to be a linear System. The Super position principle states that the response of the system to a weighted sum of signals should be equal to the corresponding weighted sum of the outputs of the systems to each of the individual input signal.

A system is linear if and only if

$$T[ax_1(n) + bx_2(n)] = a\,T[x_1(n)] + b\,T[x_2(n)]$$

Here a and b are arbitary constants.

A System that does not satisfy the super position principle is called non-linear System.

Time invariant and Time variant Systems

A System is said to be time invariant or shift invariant if the characteristics of the system does not change with time.

For a time invariant System if $y(n)$ is the response of the system to the input $x(n)$ then response of the System to the input $x(n-k)$ is $y(n-k)$.

$$y(n-k) = T[x(n-k)]$$

$y(n-k) \neq T[x(n-k)]$ then the System is said to be time variant.

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A linear time invariant System (LTI) satisfies both linearities in the time invariant properties.

FIR and IIR Systems

If the impulse response of the system is of finite duration, then the system is called FIR System.

Ex:- $h(n) = \begin{cases} 1 & \text{for } n=-1 \\ 2 & \text{for } n=0 \\ 3 & \text{for } n=1 \end{cases}$

A System has an impulse response for infinite dura is called IIR System.

Ex:- $h(n) = a^n u(n)$

Stable and unstable Systems

An LTI System is stable if it produces a boun O/p sequence for every bounded I/p sequence.

If for Some bounded I/p sequence $x(n)$ the O/p is unbounded the System is called unstable. The necessary and sufficient condition for stability is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Problems

1] Test the stability of the system whose impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$.

Sol:- To test the stability the given condition is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Given data

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$= \sum_{n=-\infty}^{\infty} \left| \left[\left(\frac{1}{2}\right)^n u(n) \right] \right|$$

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$$= \sum_{n=-\infty}^{0} \left[\left(\frac{1}{2}\right)^n u(n)\right] + \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n u(n)\right]$$

$$= 0 + \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n u(n)\right]$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n u(n) \qquad [\because u(n) = 1 \text{ for } n \geq 0]$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

$$= 1 + 1/2 + (1/2)^2 + (1/2)^3 + \cdots + \infty$$

$$= 1 + x + x^2 + x^3 + \cdots + \infty = \frac{1}{1-x}$$

$$= \frac{1}{1-1/2} = 2 < \infty$$

∴ The given System is stable.

2] Test the given System for time invariant or not
$y(n) = n\, x(n)$.

Sol:- Given data

$$y(n) = n\, x(n)$$

$$y(n-k) \neq T\,[x(n-k)]$$

∴ The given system is time variant.

3] Determine wheather the following system given by $y(n) = \log_{10}[x(n)]$ is casual or not.

Sol:- Given data

$$y(n) = \log_{10}[x(n)]$$

for $n = 0$

$$y(0) = \log_{10} x(0)$$

for $n = -1$

$$y(-1) = \log_{10} x(-1)$$

∴ The given system is casual system.

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4] Determine wheather the system defined by $y(n) = x(-n^2 - 2)$ is time variant or not.

Sol:- Given data

$$y(n) = x(-n^2 - 2)$$

$$y(n-k) = x\left[-(n-k)^2 - 2\right]$$

∴ The given system is time variant.

5] Test the following systems for time invariant

$$y(n) = n\,x^2(n).$$

Sol:- Given data

$$y(n) = n\,x^2(n)$$

$$y(n-k) \neq T[x(n-k)]$$

∴ The given system is time variant.

6] Test wheather the following Signal is periodic or no If periodic find the fundamental period $\sin \sqrt{2}\pi t$.

Sol:- Given $\sin \sqrt{2}\pi t$

periodic Signal $\Rightarrow x(t) = x[t+T]$

Let $x(t) = \sin \sqrt{2}\pi t = \sin \omega_0 t$

$$\omega_0 = \sqrt{2}\pi$$

$$2\pi f = \sqrt{2}\pi$$

$$\frac{2\pi}{T} = \sqrt{2}\pi$$

$$T = \frac{2}{\sqrt{2}}$$

$$T = \sqrt{2}$$

7] Test wheather the following Signal is periodic or not. If periodic find the fundamental period

$\sin 20\pi t + \sin 5\pi t$.

Sol:- Given Data

$$x(t) = \sin 20\pi t + \sin 5\pi t = \sin \omega_1 t + \sin \omega_2 t$$

$$\omega_1 = 20\pi \qquad \omega_2 = 5\pi$$

$$\frac{2\pi}{T_1} = 20\pi \qquad \frac{2\pi}{T_2} = 5\pi$$

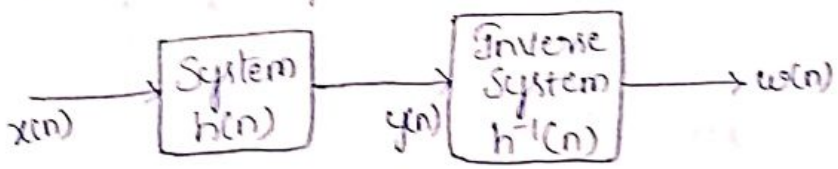
$$T_1 = \frac{2}{10} = \frac{1}{10} = 0.1 \qquad T_2 = \frac{2}{5} = 0.4$$

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$$\therefore T = \frac{T_1}{T_2} = \frac{0.1}{0.4} = 0.25$$

Invertability (or) Inverse System

- A System is said to be invertable we can generate the i/p Signal $x(n)$ from the o/p of a System $y(n)$.
- It can be generated by using cascading of inverse System with the Original System.
- It can be represented as



$$w(n) = y(n) * h^{-1}(n)$$

$$= [x(n) * h(n)] * h^{-1}(n)$$

$$= x(n) * h(n) * h^{-1}(n)$$

$$= x(n)$$

* Determine the convolution of two sequences

$x(n) = \{3, 2, 1, 2\}$ $\quad h(n) = \{1, 2, 1, 2\}$ (↑ at 2)

Sol:- Given Sequences

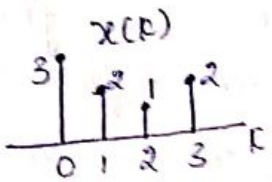
$x(k) = \{3, 2, 1, 2\}$ $\quad h(k) = \{1, 2, 1, 2\}$ (↑ at 2)

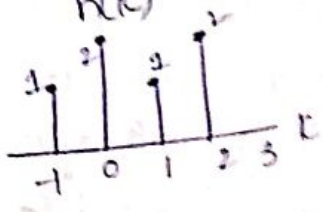
Length of the Sequence $x(k) = 4 = N_1$

Length of the Sequence $h(k) = 4 = N_2$

Total o/p Sequence $= N_1 + N_2 - 1 = 4 + 4 - 1 = 8 - 1 = 7$

$n_1 = 0$ & $n_2 = -1 \Rightarrow n_1 + n_2 = 0 - 1 = -1$





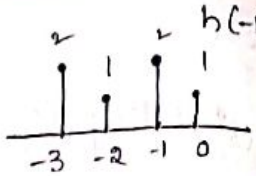
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y(n) = \sum_{k=-1}^{5} x(k)\, h(n-k) \qquad [k = -1, 0, 1, 2, 3, 4, 5]$$

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for $n=-1 \Rightarrow y(-1) = \sum x(k)\,h(-1-k)$

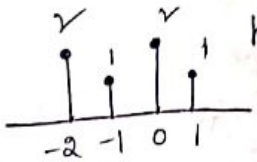
$= 1\times 3 = 3$



$y(0) = \sum x(k)\,h(-k)$

$= (3\times 2)+(1\times 2)$

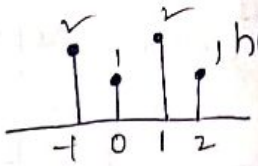
$= 6+2 = 8$



$y(1) = \sum x(k)\,h(1-k)$

$= (1\times 3)+(2\times 2)+(1\times 1)$

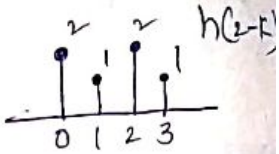
$= 3+4+1 = 8$



$y(2) = \sum x(k)\,h(2-k)$

$= (3\times 2)+(1\times 2)+(2\times 1)+(1\times 2)$

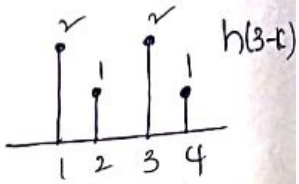
$= 12$



$y(3) = \sum x(k)\,h(3-k)$

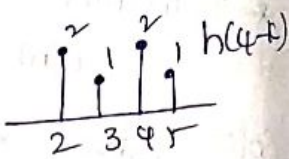
$= (2\times 2)+(1\times 1)+(2\times 2)$

$= 9$



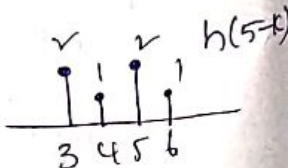
$y(4) = \sum x(k)\,h(4-k)$

$= 2+2 = 4$



$y(5) = \sum x(k)\,h(5-k)$

$= 4$



$y(n) = \{3, 8, 8, 12, 9, 4, 4\}$

Convolution

- For a linear time invariant System with the input Sequence $x(n)$ and impulse response $h(n)$ are given we can find the O/P $y(n)$ by using the equation

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$ which is known as convolution Sum and can be represented as $y(n) = x(n) * h(n)$.

where $*$ represents the convolution operation.

The convolution Sum of two Sequences can be found by using the following steps.

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Step 1:- Choose an initial value of n, the starting time for Evaluating the O/p Sequence y(n). If x(n) starts at $n=n_1$ and h(n) starts at $n=n_2$ then $n=n_1+n_2$ is considerable choice.

Step 2:- Express both Sequences interms of the index k.

Step 3:- Fold h(k) about k=0 to Obtained h(-k) and shift by 'n' to the right if n is positive and left if n is negative to Obtain h(n-k).

Step 4:- Multiply the two Sequences x(k) and h(n-k) Element by Element and Sum of the products to get y(n).

Step 5:- Increment the index n, Shift the Sequence h(n-k) to right by one Sample and do step 4.

Step 6:- Repeat Step 5 until the Sum of products is Zero for all the remaining values of 'n'.

Properties of convolution

1. Commutative law

$$x(n) * h(n) = h(n) * x(n)$$

2. Associative law

$$(x(n) * h_1(n)) * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

3. Distributive law

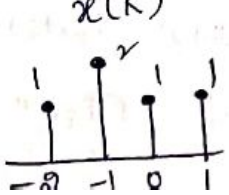
$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

* Find the convolution of two Sequences $x(n)=\{1, 2, 1, 1\}$ (↑ at 1)
$h(n) = \{1, -1, 1, -1\}$

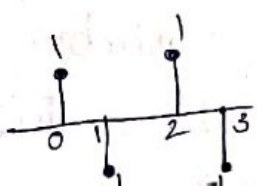
Sol:- $n_1=-2$ and $n_2=0$

$n = n_1+n_2 = -2+0 = -2$

x(k)



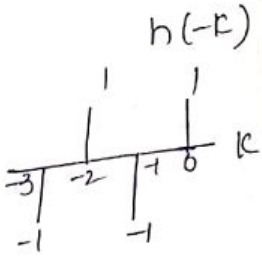
h(k)



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$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$y(n) = \sum_{k=-2}^{2} x(k)\, h(n-k)$$

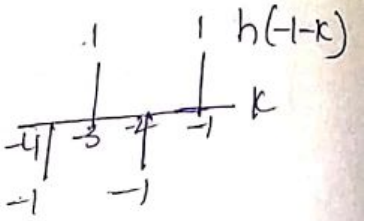


for $n=0$:- $y(0) = \sum x(k)\, h(-k)$

$= 1-2+1 = 0$

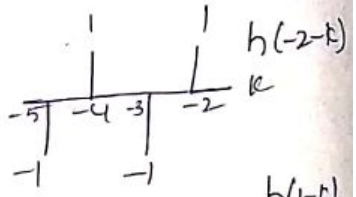
$y(-1) = \sum x(k)\, h(-1-k)$

$= -1+2 = 1$



$y(-2) = \sum x(k)\, h(-2-k)$

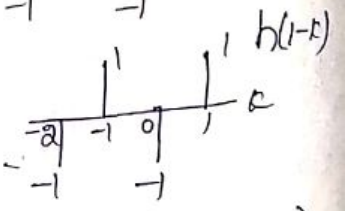
$= 1$



$y(1) = \sum x(k)\, h(1-k)$

$= -1+2-1$

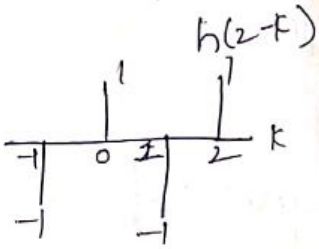
$= 1$



$y(2) = \sum x(k)\, h(2-k)$

$= -2+1-1$

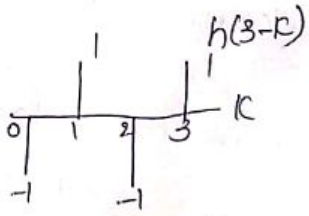
$= -2$



$y(3) = \sum x(k)\, h(3-k)$

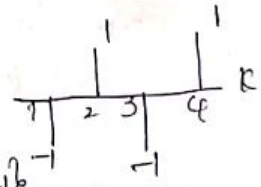
$= -1+1-1$

$= -1$



$y(4) = \sum x(k)\, h(4-k)$

$= -1$



$y(n) = \{1, 1, 0, 1, -2, 0, -1\}$

↑

Z-Transform

The z-transform of a discrete time signal $x(n)$ is defined as $X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$.

Where z is a complex variable in polar form $z = re^{j\omega}$. Where r is the radius of the circle.

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The z-value Substitute in the above equation

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, (re^{j\omega})^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, r^{-n}.\, e^{-j\omega n}$$

- The value of r is consider to be 1 then

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

∴ The z-transform is converted into fourier transform whenever $r=1$.

- This is also called as two sided z-transform.
- The discrete time signal $x(n)$ is considered to be a casual Signal then.

$$X_T(z) = \sum_{n=0}^{\infty} x(n)\, z^{-n}$$, this is called One Sided z-transform.

Region of convergence (ROC)

The set of all values of z for which $X(z)$ gets finite value is called ROC of $X(z)$.

Right hand Sequence:- A sequence is said to be right hand sequence for which $x(n)=0$ for $n < n_0$. where n_0 may be +ve or −ve but finite. The ROC of right hand sequence will be entire z-plane except $z=0$.

For Ex:- $x(n) = \{1, 0, -2, 3, 4\}$ (↑ at 1)

$$X(z) = \sum_{n=0}^{\infty} x(n)\, z^{-n}$$

$$= x(0)\, z^{0} + x(1)\, z^{-1} + x(2)\, z^{-2} + x(3)\, z^{-3} + x(4)\, z^{-4}$$

$$= 1 - 2z^{-2} + 3z^{-3} + 4z^{-4}$$

The ROC of $X(z)$ will be Entire z-plane Except $z=0$

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Left hand Sequence :- A Sequence is said to be left hand Sequence for which $x(n) = 0$ for $n \geq n_0$. Where n_0 may be +ve or -ve but finite value.

- The ROC of left hand sequence will be Entire z-plane Except $z = \infty$.

For Ex :- $x(n) = \{-3, -2, -1, 0\}$
$\uparrow_{-n}$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= x(-3) z^3 + x(-2) z^2 + x(-1) z^1 + x(0) z^0$$

$$= -3z^3 - 2z^2 - z$$

The ROC of $X(z)$ will be Entire z-plane Except $z = \infty$

Two Sided Sequence :- A Signal that has finite duration on both left and right hand sides is known as two Side Sequence for Such type of Sequence the ROC is Entire z-plane except $z = 0$ and $z = \infty$.

For Ex :- $x(n) = \{2, -1, 3, 2, 1, 0, 2, 3, -1\}$. Find $X(z)$.
$\uparrow_{-n}$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= x(-4) z^4 + x(-3) z^3 + x(-2) z^2 + x(-1) z^1 + x(0) z^0 + x(1) z^{-1} + x(2) z^{-2} + x(3) z^{-3} + x(4) z^{-4}$$

$$= 2z^4 - z^3 + 3z^2 + 2z + 1 + 2z^{-2} + 3z^{-3} - z^{-4}$$

$$X(z) = 2z^4 - z^3 + 3z^2 + 2z + 1 + 2z^{-2} + 3z^{-3} - z^{-4}$$

- The ROC of $X(z)$ will be Entire z-plane except $z = 0$ and $z = \infty$.

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properties of ROC

1] The ROC is a ring/disk in z-plane centered at origin.
2] The ROC cannot contain any poles.
3] The ROC of a casual Sequence will be Entire z-plane except $z=0$.
4] The ROC of a non-casual Sequence will be Entire z-plane except $z=\infty$.
5] The ROC of finite duration two Sided Sequence will be Entire z-plane except $z=0$ and $z=\infty$.
6] The ROC of in-finite duration two Sided sequence will be ring in the z-plane.
7] The ROC of LTI Stable System contain Unit Circle.
8] The ROC must be a connected region.

properties of z-transform

1] Linearity property:-

If $Z[x_1(n)] = X_1(z)$

$Z[x_2(n)] = X_2(z)$

then $Z[a x_1(n) + b x_2(n)] = a X_1(z) + b X_2(z)$

Proof:- $Z[a x_1(n) + b x_2(n)] = \sum_{n=-\infty}^{\infty} [a x_1(n) + b x_2(n)] z^{-n}$

$= \sum_{n=-\infty}^{\infty} a x_1(n) z^{-n} + \sum_{n=-\infty}^{\infty} b x_2(n) z^{-n}$

$= a \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} + b \sum_{n=-\infty}^{\infty} x_2(n) z^{-n}.$

$$\boxed{Z[a x_1(n) + b x_2(n)] = a X_1(z) + b X_2(z)}$$

2] Time shifting property

If $Z[x(n)] = X(z)$

then $Z[x(n-m)] = z^{-m} X(z)$

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Proof:- $\mathcal{Z}[x(n-m)] = \sum_{n=-\infty}^{\infty} x(n-m) z^{-n}$

Let $n-m = l$

$n = l+m$

$= \sum_{n=-\infty}^{\infty} x(l) z^{-(l+m)}$

$= \sum_{n=-\infty}^{\infty} x(l) \cdot z^{-l} \cdot z^{-m}$

$= z^{-m} \sum_{n=-\infty}^{\infty} x(l) \cdot z^{-l}$

$$\boxed{\mathcal{Z}[x(n-m)] = z^{-m} X(z)}$$

3] Time Reversal property

If $\mathcal{Z}[x(n)] = X(z)$

then $\mathcal{Z}[x(-n)] = X(z^{-1})$

Proof:- $\mathcal{Z}[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$

Let $-n = l$

$= \sum_{l=-\infty}^{\infty} x(l) z^{l}$

$= \sum_{l=-\infty}^{\infty} x(l) (z^{-1})^{-l}$

$$\boxed{\mathcal{Z}[x(n)] = X(z^{-1})}$$

4) Differentiation property :- If z-transform of $\mathcal{Z}[x(n)] = X(z)$ th

$$\mathcal{Z}[n x(n)] = -z \frac{d}{dz} X(z)$$

Proof:- $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$$\frac{d}{dz} X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot n \cdot z^{-n-1}$$

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$$\frac{d}{dz}X(z) = \sum_{n=-\infty}^{\infty} nx(n)\, z^{-n}\left(\frac{-1}{z}\right)$$

Multiplying $-z$ on both sides of eqn.

$$-z\frac{d}{dz}X(z) = -z\sum_{n=-\infty}^{\infty} nx(n)\, z^{-n}\left(\frac{-1}{z}\right)$$

$$-z\frac{d}{dz}X(z) = \sum_{n=-\infty}^{\infty} nx(n)\, z^{-n}$$

$$= z\,[nx(n)]$$

$$z[nx(n)] = -z\frac{d}{dz}X(z)$$

5. Convulation property :-

If $z[x(n)] = X(z)$ and $z[h(n)] = H(z)$ then

$z[x(n) * h(n)] = X(z)H(z)$

proof :- convulation of two sequences

$$x(n) * h(n) = y(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

$$z[x(n) * h(n)] = \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} x(k)h(n-k)\, z^{-n}$$

Interchange the order of summation.

$$= \sum_{k=-\infty}^{\infty} x(k) \sum_{n=-\infty}^{\infty} h(n-k)\, z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x(k) \sum_{l=-\infty}^{\infty} h(l)\, z^{-l} z^{-k}$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, z^{-k} \sum_{l=-\infty}^{\infty} h(l)\, z^{-l}$$

$$= X(z)H(z)$$

6. Initial value theorem :-

If $z[x(n)] = X_+(z)$ then

$x(0) = \lim_{z\to\infty} X_+(z)$ where

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$X_+(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$ in this eqn.

Sub in $z = +\infty$ all the values are vanish except $x(0)$.

7. Final value theorem :-

If $z[x(n)] = X_+(z)$ then $x(\infty) = \lim_{z \to 1} (z-1) X_+(z)$

System Functions :- In general the system is described by a linear constant co-efficient differential equation of the form

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{m} b_k x(n-k)$$

$$a_0 y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

Let $a_0 = 1$

$$\boxed{y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)}$$

Taking z- transform on b.s & applying time shifting property we can get

$$y(z) = -\sum_{k=1}^{N} a_k Y(z) z^{-k} + \sum_{k=0}^{M} b_k X(z) z^{-k}$$

$$Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = \sum_{k=0}^{M} b_k X(z) z^{-k}.$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k \cdot z^{-k}}{1 + \sum_{k=1}^{N} a_k \cdot z^{-k}}$$

$$\boxed{H(z) = \frac{\sum_{k=0}^{M} b_k \cdot z^{-k}}{1 + \sum_{k=1}^{N} a_k \cdot z^{-k}}}$$

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* Here $H(z)$ is known as transfer function (or) system function.

* $Y(z)$ is the z-transform of the o/p sequence $y(n)$ and $X(z)$ is the z-transform of the i/p sequence $y(n)$.

* By using the system function we can generate poles & zeroes. i.e., the zeros of the system function $H(z)$ are the values of 'z' for which $H(z)=0$

* The poles of the system function are the values of 'z' for which $H(z)=\infty$.

* The system function contains both poles & zeroes and hence the corresponding system is called pole zero system with 'n' poles & 'm' zeroes.

(P) Find the system function and impulse response of the system describe by the difference eqn $y(n)=\frac{1}{5}y(n-1)+x(n)$

Sol $y(n)=\frac{1}{5}y(n-1)+x(n)$

Applying z-transform on b.s

$$Y(z)=\frac{1}{5}Y(z)z^{-1}+X(z)$$

$$Y(z)\left[1-\frac{1}{5}z^{-1}\right]=X(z) \Rightarrow \frac{X(z)}{Y(z)}=\frac{1}{1-\frac{1}{5}z^{-1}}$$

$$\boxed{H(z)=\frac{1}{1-\frac{1}{5}z^{-1}}}$$

Applying inverse z-transform on b.s

$$\boxed{h(n)=\left(\frac{1}{5}\right)^n u(n)}$$

(P) Find a z-transform of response $x(n)=\left(\frac{1}{3}\right)^{n-1}u(n-1)$

Sol Given data

$$x(n)=\left(\frac{1}{3}\right)^{n-1}u(n-1)$$

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$x(n) = (1/3)^n u(n)$

$$X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}$$

By using time shifting property

$$Z\left[\left(\frac{1}{3}\right)^{n-1} u(n-1)\right] = z^{-1}X(z)$$

$$= z^{-1}\frac{z}{z - \frac{1}{3}}$$

$$\boxed{= \frac{1}{z - \frac{1}{3}}}$$

(P) Find z- transform of sequence $x(n) = n \cdot a^n u(n)$

Sol Given data,

$$x(n) = na^n u(n)$$

$$Z\left[a^n u(n)\right] = \frac{1}{1 - az^{-1}} = \frac{z}{z - a}$$

By using differenciate property

$$Z[n x(n)] = -z\frac{d}{dz}X(z)$$

$$= -z\frac{d}{dz}\left(\frac{z}{z-a}\right)$$

$$= -z\left[\frac{z - a - z}{(z-a)^2}\right]$$

$$\boxed{Z[n x(n)] = az/(z-a)^2}$$

Time Response Analysis of Discrete time systems :-

* The difference eqn of n^{th} order discrete time system can be written as

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$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

* If the co-efficient $a_0 \neq 1$ the solution of the difference eqn. can be apressed as some of two parts given by

$$y(n) = y_h(n) + y_p(n)$$

Where $y_h(n)$ is known as homogeneous solution and $y_p(n)$ is called particular solution.

Natural Response (or) Zero i/p Response :-

* The natural response is the solution of the above difference equation with $x(n) = 0$

∴ For a discrete time system the natural response is the solution of homogeneous equation.

i.e., $$\sum_{k=0}^{N} a_k y(n-k) = 0$$

* The solution of this equation is of the form $y_h(n) = \lambda^n$

sub. these value in the above eqn. we can get

$$\sum_{k=0}^{N} a_k \lambda^{n-k} = 0$$

$$a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \cdots + a_{N-1}\lambda^{n-(N-1)} + a_N\lambda^{n-N} = 0$$

Let, $a_0 = 1$

$$\lambda^{n-N}\left[\lambda^N + a_1\lambda^{N-1} + \cdots + a_{N-1}\lambda + a_N\right] = 0$$

Which gives,

$$\lambda^N + a_1\lambda^{N-1} + \cdots + a_{N-1}\lambda + a_N = 0.$$

The above eqn. is characteristic eqn. of the system

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∴ The n^{th} order characteristic eqn. can be expressed in characterised form as

$$(\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\lambda_N)=0$$

* Where, $\lambda_1, \lambda_2, \lambda_3 \ldots \lambda_N$ are called roots of the characteristic equation (or) Eigen values of the system.

* The nature of the roots will be real, imaginary and complex roots.

* If the roots $\lambda_1, \lambda_2 \ldots \lambda_N$ are distinct in that case the general solution of homogeneous eqn.

$$y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n + c_3\lambda_3^n + \ldots + c_N\lambda_n^N$$

Where, $c_1, c_2, c_3 \ldots c_N$ are orbitary constants.

* If the roots are repeated for example if the roots of the characteristic eqn. $\lambda_1=-2$ & $\lambda_2=-2$ and $\lambda_3=2$ then the solution will be

$$y_h(n) = [c_1+c_2 n]\lambda_1^n + c_3\lambda_3^n$$

* If the roots are complex then $\lambda_1 = a+ib$ $\lambda_2 = a-ib$

$$y_h(n) = r[A_1\cos\omega\theta + A_2\sin\omega\theta]$$

$$r = \sqrt{a^2+b^2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Where A_1 & A_2 are constants.

*** (P) Find the natural response of the system described by the difference equation $y(n)+2y(n-1)+y(n-2)=x(n)+x(n-1)$ with initial conditions $y(-1)=y(-2)=1$

Sol Given diff. eqn,

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$y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-1)$

$y(n) + 2y(n-1) + y(n-2) = 0$

$\lambda^n + 2\lambda^{n-1} + \lambda^{n-2} = 0$

$\lambda^{n-2}[\lambda^2 + 2\lambda + 1] = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$(\lambda+1)^2 = 0$

$\lambda_1 = -1, \quad \lambda_2 = -1$

$y(n) = [c_1 + c_2 n](-1)^n$

By applying initial conditions

$y(0) = c_1$

$y(1) = (c_1 + c_2)(-1)$

$= -c_1 - c_2$

$y(n) + 2y(n-1) + y(n-2) = 0$

Sub $y(n) = 0$

$y(0) + 2y(0-1) + y(0-2) = 0$

$y(0) + 2 + 1 = 0$

$y(0) = -3$

$y(1) + 2y(0) + y(-1) = 0$

$y(1) - 6 + 1 = 0$

$y(1) = 5$

$\boxed{c_1 = -3}$

$5 = -c_1 - c_2$

$5 = 3 - c_2$

$\boxed{c_2 = -2}$

$\boxed{y_h(n) = [-3 + (-2)n](-1)^n u(n)}$

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⑩ Find the natural response of the system described by the difference equation $y(n)-4y(n-1)+4y(n-2)=x(n)-x(n-1)$ with initial conditions $y(-1)=y(-2)=1$

Sol Given diff. eqn

$$y(n)-4y(n-1)+4y(n-2)=x(n)-x(n-1)$$

$$y(n)-4y(n-1)+4y(n-2)=0$$

$$\lambda^n-4\lambda^{n-1}+4\lambda^{n-2}=0$$

$$\lambda^{n-2}[\lambda^2-4\lambda+4]=0$$

$$\lambda^2-4\lambda+4=0$$

$$(\lambda-2)^2=0$$

$$\lambda_1=2 \; ; \lambda_2=2.$$

$$y(n)=[c_1+c_2n](\lambda)^n$$

$$y(n)=[c_1+c_2n](2)^n$$

By applying initial conditions

$$y(0)=[c_1](2)^0=c_1$$

$$y(1)=[c_1+c_2](2)^1$$

$$=[c_1+c_2](2)$$

$$y(n)-4y(n-1)+4y(n-2)=0$$

$$y(0)-4y(-1)+4y(-2)=0$$

$$y(0)-4+4=0$$

$$y(0)=0$$

$$y(1)-4y(0)+4y(-1)=0$$

$$y(1)-0+4(1)=0$$

$$y(1)=-4$$

$$c_1=0$$

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$2(C_1+C_2) = -4$

$0+2C_2 = -4$

$\boxed{C_2 = -2}$

$$\boxed{y_h(n) = [-3+(-2)^n](-1)^n u(n)}$$

Forced Response :-

The forced Response of the system is obtained by suming the particular solution and homogenous solution and finding the coefficients the homogeneous solution so that the Combined response $y_h(n) + y_p(n)$ satisfys the zero initial Conditions.

General form of particular solution for several types of inputs as shown in table.

i/p signal	$y_p(n)$ particular solution
$\delta(n)$	0
A(step i/p)	k
AM^n	kM^n
An^m	$Kon^m + K_1 n^{m-1} + K_2 n^{m-2} + \cdots + KM$
$A^n N^m$	$A^n[Kon^m + K_1 n^{m-1} + \cdots + km]$
$A\cos\omega n$ $A\sin\omega n$	$C_1\cos\omega n + C_2 \sin\omega n$

where A, M, N and C_1 & C_2 are Constants.

1) find the forced response of the system described by the difference equation

$y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-1)$ for a given i/p

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$x(n) = \left(\frac{1}{2}\right)^n u(n)$

Sol given data the difference eqn is

$y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-1)$

$y_f(n) = y_h(n) + y_p(n)$

$y_h(n) = [c_1 + c_2](-1)^n$

$x(n) = \left(\frac{1}{2}\right)^n u(n)$

$y_p(n) = k\left(\frac{1}{2}\right)^n u(n)$

$y_f(n) = [c_1 + c_2](-1)^n + k\left(\frac{1}{2}\right)^n u(n)$

$k\left(\frac{1}{2}\right)^n + 2k\left(\frac{1}{2}\right)^{n-1} + k\left(\frac{1}{2}\right)^{n-2} = \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1}$

$n-2=0$
$n=2$

$k\left(\frac{1}{2}\right)^2 + 2k(1/2) + k = (1/2)^2 + 1/2$

$k(1/4) + k + k = \frac{1}{4} + 1/2$

$k\left(\frac{1}{4} + 2\right) = 3/4$

$k = 1/3$

$y_p(n) = 1/3\left(\frac{1}{2}\right)^n u(n)$

Inorder to determine the values of c_1 & c_2 by applying inital Conditions.

$y_f(n) = [c_1 + c_2 n](-1)^n + k(1/2)^n u(n)$

$y(0) = [c_1](1) + 1/3(1)$

$y(0) = c_1 + 1/3$

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$y(1) = (c_1+c_2)(-1)+(1/3)(1/2)$

$y(1) = -c_1 - c_2 + 1/6$

$y(n) + 2y(n-1) + y(n-2) = x(n) + x(n-1)$

$y(0) + 2y(-1) + y(-2) = x(0) + x(-1)$

$y(0) + 2(0) = 1 + 0$

$y(0) = 1$

$c_1 + 1/3 = 1$

$c_1 = 1 - 1/3 = 2/3$

$y(1) + 2y(0) + y(-1) = x(1) + x(0)$

$y(1) + 2(1) + 0 = 1/2 + 1$

$y(1) + 2 = 3/2$

$y(1) = -1/2$

$-c_1 - c_2 + 1/6 = -1/2$

$-c_2 = -1/2 - 1/6 + 2/3$

$c_2 = 0$

$\therefore \boxed{y_f(n) = 2/3 + 1/3(1/2)^n u(n)}$

2) find the forced response of the system described by the difference equation $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ when the i/p is $x(n) = (-1)^n u(n)$.

Sol given difference equation

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

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$$y_f(n) = y_h(n) + y_p(n)$$

$$y_h(n) = [C_1 + C_2 n](2)^n$$

$$y_p(n) = K(-1)^n u(n)$$

$$y_f(n) = [C_1 + C_2 n](2)^n + K(-1)^n u(n)$$

$$K(-1)^n - 4K(-1)^{n-1} + 4K(-1)^{n-2} = (-1)^n - (-1)^{n-1}$$

$$n-2 = 0$$

$$n = 2$$

$$K(-1)^2 - 4K(-1)^1 + 4K(-1)^0 = (-1)^2 - (-1)^1$$

$$K + 4K + 4K = 1 + 1$$

$$9K = 2$$

$$K = 2/9$$

$$y_p(n) = \frac{2}{9}(-1)^n u(n)$$

Inorder to determine the values of C_1 & C_2 by applying initial Conditions

$$y_f(n) = [C_1 + C_2 n](2)^n + K(-1)^n u(n)$$

$$y(0) = [C_1 + 0](2)^0 + \frac{2}{9}(-1)^0 \cdot 1$$

$$y(0) = C_1 + \frac{2}{9}$$

$$y(1) = (C_1 + C_2)(2) + \frac{2}{9}(-1)^1$$

$$y(1) = (C_1 + C_2)(2) - \frac{2}{9}$$

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

$$y(0) - 4y(-1) + 4y(-2) = x(0) - x(-1)$$

$$y(0) = 1$$

$C_1 + \frac{2}{9} = 1$

$C_1 = 1 - 2/9$

$C_1 = 7/9$

$y(1) - 4y(0) + 4y(-1) = x(1) - x(0)$

$y(1) - 4 + 0 = (-1) - 1$

$y(1) = -2 + 4$

$y(1) = 2$

$2C_1 + 2C_2 - 2/9 = 2$

$2(7/9) + 2C_2 = 2 + 2/9$

$\frac{14}{9} + 2C_2 = 20/9$

$2C_2 = \frac{20}{9} - \frac{14}{9} = 6/9$

$C_2 = 1/3$

$$y_f(n) = \left[\frac{7}{9} + \frac{1}{3}n\right](2)^n + \left(\frac{2}{9}\right)(-1)^n u(n)$$

Frequency Response Analysis of Discrete time Systems:-

The output $y(n)$ of any linear time invariant system to an input signal $x(n)$ can be obtained by using Convolution sum $y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$.

Where $h(n)$ impulse response of the system.

Let us Consider a complex exponential signal $x(n) = e^{j\omega n}$ as input to the system then the output is given by

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$$y(n) = \sum_{k=-d}^{d} h(k)\, e^{j\omega(n-k)}$$

$$= \sum_{k=-d}^{d} h(k)\, e^{j\omega n - j\omega k}$$

$$= \sum_{k=-d}^{d} h(k)\, e^{j\omega n} \cdot e^{-j\omega k}$$

$$y(n) = e^{j\omega n} \sum_{k=-d}^{d} h(k)\, e^{-j\omega k}$$

$$y(n) = e^{j\omega n} H(e^{j\omega})$$

where $H(e^{j\omega}) = \sum_{k=-d}^{d} h(k)\, e^{-j\omega k}$

The quantity $H(e^{j\omega})$ is called frequency response of the system. The frequency response is a complex valued function it can be expressed in polar form as $H(e^{j\omega}) = |H(e^{j\omega})|\, e^{j\theta(\omega)}$.

where $|H(e^{j\omega})|$ is called the magnitude response and it is an even function of 'ω'.

i.e., $|H(e^{j\omega})| = |H(e^{-j\omega})|$

* $\theta(\omega)$ is called phase response of the system and it is an odd function of 'ω'.

i.e., $\theta(\omega) = -\theta(-\omega)$

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