

Thermal Radiation

Radiation: Radiation heat transfer is defined as the transfer of energy across a system boundary by means of an electromagnetic mechanism due to temperature difference.

Surface emission properties:

The rate of emission of radiation by a body depends upon the following factors:

- (i) The temperature of the surface
- (ii) The nature of the surface
- (iii) The wavelength or frequency of radiation

Parameters of surface emission properties:

Total emissive power (E): The emissive power is defined as the total amount of radiation emitted by a body per unit area and time.

units: W/m^2

$$E_b = \sigma T^4 \quad W/m^2$$

$$E_b = \sigma A T^4 \quad W$$

σ - Stefan-Boltzmann constant = $5.67 \times 10^{-8} W/m^2 K^4$

Monochromatic emissive power (E_λ): The rate of energy radiated per unit area of the surface per unit wavelength.

The total emissive power is $E = \int_0^\infty E_\lambda d\lambda \quad W/m^2$

The emissive power from a real surface

$$E = \epsilon \sigma A T^4 \quad W$$

ϵ - Emissivity of the material

Emissivity (ϵ) : It is defined as the ability of the of a body to radiate heat.

(or)

It is defined as the ratio of emissive power of any body to the emissive power of a black body of equal temperature

$$\epsilon = \frac{E}{E_b}$$

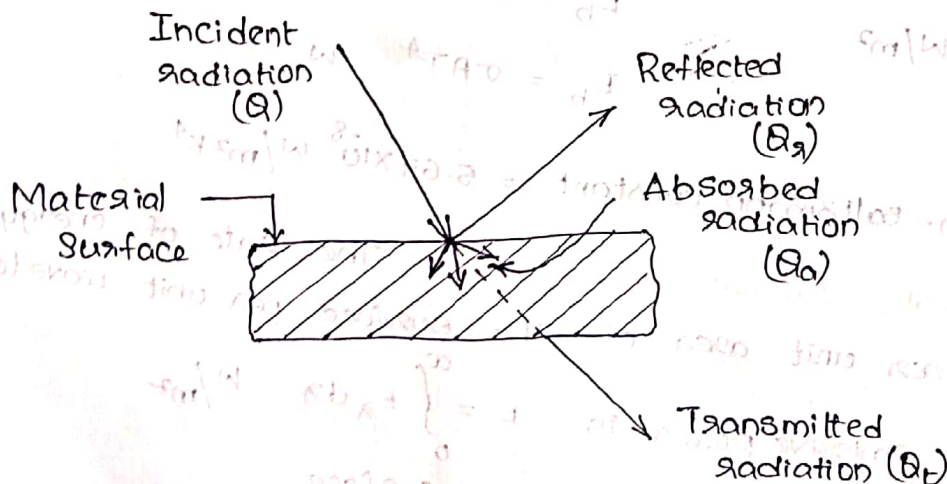
Emissivity varies from 0 to 1

$\epsilon = 0 \Rightarrow$ for white body

$\epsilon = 1 \Rightarrow$ for black body

Absorptivity, Reflectivity And Transmissivity :

When incident radiation (Q) also called irradiation impinges on a surface, three things happens; a part is reflected back (Q_r), a part is transmitted through (Q_t) and remainder is absorbed (Q_a).



By conservation of energy principle

$$Q_a + Q_r + Q_t = Q$$

Dividing both sides by Q

$$\frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = 1$$

$$\alpha + \rho + \tau = 1$$

α = absorptivity $\Rightarrow$ fraction of incident radiation² absorbed.

ρ = reflectivity $\Rightarrow$ fraction of incident radiation reflected

τ = transmissivity $\Rightarrow$ fraction of incident radiation transmitted.

Black body: For perfectly absorbing body, $\alpha=1$, $\rho=0$, $\tau=0$

Such a body is called a black body.

Eg: Black body

Opaque body: When no incident radiation is transmitted through the body it is called opaque body.

Eg: Glasses and liquids

$$\tau=0, \alpha+\rho=1$$

White body: If all the incident radiation falling on the body are reflected it is called white body

$$\rho=1, \alpha=0 \text{ and } \tau=0$$

Eg: hydrogen, oxygen, nitrogen

Gray body: If the radiative properties α, ρ, τ of a body are assumed to be uniform over the entire wavelength spectrum

Law's of Radiation:

There are four laws of radiation. They are:

1. The Stefan-boltzmann Law
2. Kirchoff's Law
3. Planck's Law
4. Wien's Displacement Law

1. Stefan - Boltzmann Law:

The Law states that the emissive power of a black body is directly proportional to the fourth power of its absolute temperature.

$$E_b \propto T^4$$

$$E_b = \sigma T^4$$

σ = Stefan - Boltzmann constant

$$= 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

E_b = Emissive power of a black body

2. Kirchhoff's Law:

The Law states that at any temperature the ratio of total emissive power E to the total absorptivity α is a constant for all substances which are in thermal equilibrium with their environment.

$$E_b = \frac{E}{\alpha} = \frac{E_b}{\alpha} = \frac{E}{\alpha}$$

Let us consider a large radiating body of surface area A which encloses a small body of surface area A_1 .

The energy absorbed by body (1) must be equal to the energy emitted

$$A_1 E_1 = \alpha_1 A_1 E_b$$

If body (1) is replaced by body (2) having absorptivity α_2

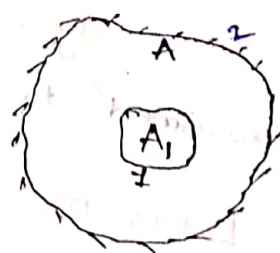
$$A_2 E_2 = \alpha_2 A_2 E_b$$

Considering generality of bodies,

$$E_b = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E}{\alpha}$$

we know,

$$E = \frac{E}{E_b} \Rightarrow E_b = \frac{E}{\alpha}$$



$$\epsilon = \alpha$$

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Thus, Kirchhoff's Law also states that the emissivity of a body is equal to the absorptivity when the body remains in thermal equilibrium with its surroundings.

Planck's Law:

This law gives relation between monochromatic emissive power of black body and wavelength.

$$(E_\lambda)_b = \frac{2\pi c^2 h \lambda^{-5}}{\exp\left(\frac{ch}{\lambda kT}\right) - 1}$$

$$c_1 = 2\pi c^2 h = 3.742 \times 10^8 \text{ W m}^2/\text{m}^2$$

$$c_2 = \frac{ch}{k} = 1.4388 \times 10^4 \text{ W m K}$$

$$(E_\lambda)_b = \frac{c_1 \lambda^{-5}}{\exp\left(\frac{c_2}{\lambda T}\right) - 1}$$

$(E_\lambda)_b$ - Monochromatic emissive power of a black body

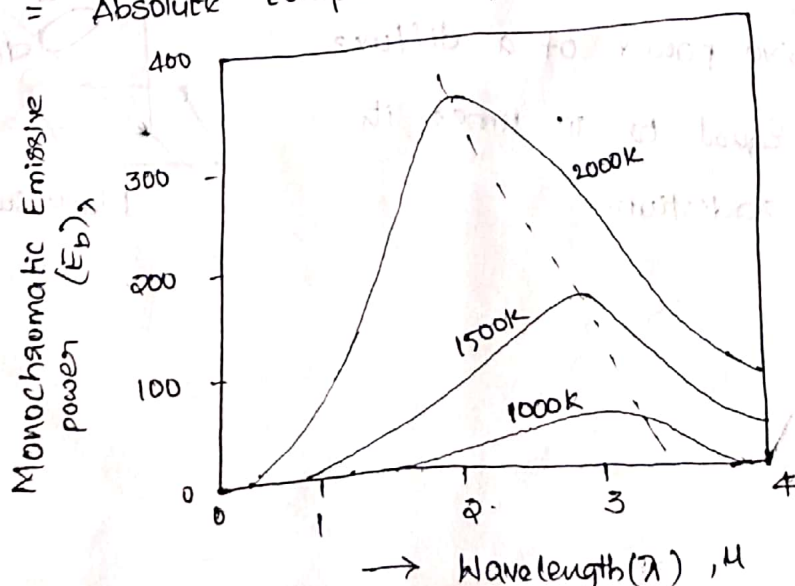
c = velocity of light in vacuum $\Rightarrow 3 \times 10^8 \text{ m/s}$

h = Planck's constant $= 6.625 \times 10^{-34} \text{ J.s}$

λ = wavelength

k = Boltzmann constant $= 1.3805 \times 10^{-23} \text{ J/K}$

T = Absolute temperature, K



Wien's displacement Law :

Wien's displacement law states that the product of λ_{max} and T is constant i.e.,

$$\lambda_{\text{max}} T = \text{constant} = 2898 \mu\text{mK}$$

i.e. Wien's displacement Law established relationship between the temperature of a black body and the wavelength at which the maximum value of monochromatic emissive power occurs.

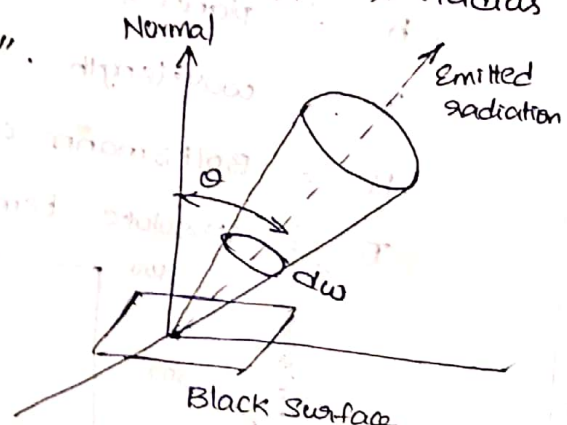
Intensity of radiation:

The intensity of radiation (I) is defined as the rate of energy leaving a surface in a given direction per unit solid angle per unit area of emitting surface normal to the mean direction in space. $W/\text{sr m}^2$

Solid Angle: A solid angle is defined as a portion of the space inside a sphere enclosed by a conical surface with the vertex of the cone at the centre of the radius of the sphere. Represented by " ω ".

Unit: steradian (sr)

Total emissive power of a diffuse surface is equal to π times its intensity of radiation.



St's cosine Law:

The law states that the total emissive power E_θ from a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission.

$$E_\theta \propto \cos\theta$$

$$E_\theta = E_n \cos\theta$$

E_n = Total emissive power of the radiating surface in the direction of its normal.

θ = Angle subtended by the normal to the radiating surface and the direction vector of emission of receiving surface.

Heat exchange between black bodies:

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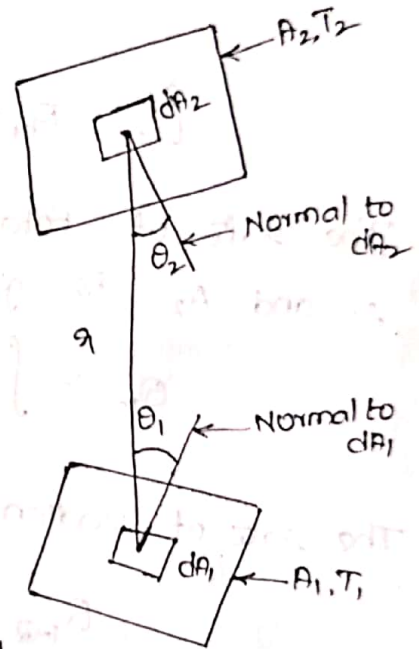
Let us consider heat exchange b/w elementary areas dA_1 and dA_2 of two black radiating bodies, separated by a non absorbing medium, having areas A_1 and A_2 and temperatures T_1 and T_2 respectively.

The elementary areas dA_1 & dA_2 are at a distance r apart and normals to these areas make angles θ_1 and θ_2 with the line joining them.

Each elemental area subtends a solid angle at the centre of the other.

Let $d\omega_1$ be subtended at dA_1 by dA_2 and $d\omega_2$ subtended at dA_2 by dA_1

$$\text{Then } d\omega_1 = \frac{dA_2 \cos \theta_2}{r^2} \quad d\omega_2 = \frac{dA_1 \cos \theta_1}{r^2}$$



Energy leaving dA_1 in direction given by the angle per unit solid angle = $I_{b1} dA_1 \cos \theta_1 d\omega_1$

$$dQ_{1-2} = \frac{I_{b1} dA_1 \cos \theta_1 \cos \theta_2 dA_2}{r^2}$$

This quantity of energy radiated by dA_2 and absorbed by dA_1 is given by

$$dQ_{2-1} = \frac{I_{b2} \cos \theta_2 \cos \theta_1 dA_2 dA_1}{r^2}$$

The net rate of transfer of energy between dA_1 and dA_2 is

$$\begin{aligned} dQ_{12} &= dQ_{1-2} - dQ_{2-1} \\ &= \frac{I_{b1} \cos \theta_1 \cos \theta_2 dA_1 dA_2}{r^2} - \frac{I_{b2} \cos \theta_1 \cos \theta_2 dA_1 dA_2}{r^2} \\ &= \frac{\cos \theta_1 \cos \theta_2 dA_1 dA_2}{r^2} (I_{b1} - I_{b2}) \end{aligned}$$

$$dQ_{12} = \frac{\sigma dA_1 dA_2 \cos\theta_1 \cos\theta_2}{r^2} (E_{b1} - E_{b2})$$

$$\left[\because I_{b1} = \frac{E_{b1}}{\pi} \quad I_{b2} = \frac{E_{b2}}{\pi} \right]$$

$$dQ_{12} = \frac{\sigma dA_1 dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2} (T_1^4 - T_2^4)$$

$$\left[\because E_{b1} = \sigma T_1^4 \quad E_{b2} = \sigma T_2^4 \right]$$

The rate of total net heat transfer for the total areas A_1 and A_2 is given by

$$Q_{12} = \int dQ_{12} = \sigma (T_1^4 - T_2^4) \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{r^2}$$

The rate of radiant energy emitted by A_1 that falls on A_2

$$Q_{1 \rightarrow 2} = I_{b1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{r^2}$$

$$Q_{12} = \sigma T_1^4 \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{\pi r^2}$$

The rate of total energy radiated by A_1 is given by

$$Q_1 = A_1 \sigma T_1^4$$

Hence fraction of the rate of energy leaving A_1 and impinging on area A_2 is given by

$$\frac{Q_{1 \rightarrow 2}}{Q} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{\pi r^2}$$

$$\frac{Q_{1 \rightarrow 2}}{Q} = F_{1 \rightarrow 2}$$

$F_{1 \rightarrow 2}$ is known as 'configuration factor' (or) surface factor (or) view factor between the two radiating surfaces.

Factor: It is defined as the fraction of radiative energy that is diffused from one surface element and strike the other surface directly with no intervening reflections.

$$Q_{1-2} = F_{1-2} A_1 \sigma T_1^4$$

For the calculation of shape factors for specific geometries and for analysis of radiant heat exchange between surfaces, following properties are used:

1. Shape factor is purely a function of geometric parameters only.
2. When two bodies are exchanging radiant energy with each other, the shape factor relation is given by:

$$A_1 F_{1-2} = A_2 F_{2-1} \Rightarrow \text{Reciprocity theorem}$$

3. When all radiation emitting from convex surface 1 is intercepted by the enclosing surface 2 the shape factor of convex surface with respect to the enclosure F_{1-2} is unity.
i.e. $F_{1-2} = 1$

From reciprocity theorem: $A_1 F_{1-2} = A_2 F_{2-1}$

$$F_{2-1} = \frac{A_1}{A_2}$$

4. A concave surface has a shape factor with itself because the radiant energy coming out from one part of surface is intercepted by another part of the same surface.
shape factor with itself: F_{1-1}
5. For a flat (or) convex surface, the shape factor with respect to itself is zero (i.e. $F_{1-1} = 0$)

6. If two surfaces A_1 and A_2 are parallel and long radiation occurs across the gap between them so that $A_1 = A_2$ and all radiation emitted by one falls on the other then

$$F_{1-2} = F_{2-1} = 1$$

7. If the enclosure comprises n surfaces, then the energy radiated from one surface is always intercepted by the other $(n-1)$ surfaces and the surface itself if it is a concave one. This is called Principle of conservation.

$$F_{1-1} + F_{1-2} + F_{1-3} + \dots + F_{1-n} = 1$$

$$F_{2-1} + F_{2-2} + F_{2-3} + \dots + F_{2-n} = 1$$

$$F_{n-1} + F_{n-2} + F_{n-3} + \dots + F_{n-n} = 1$$

Heat exchange between Non-Black bodies:

The black body concept is an idealization which serves as a standard for the performance of a real body. In Engineering applications, most surfaces encountered do not behave like black bodies which absorb the entire incident radiation. The real surfaces do not absorb the whole of the incident radiation, a part is reflected back to the radiating surface and this back and forth reflections between the surfaces may go on several times. The surfaces are mostly assumed as gray bodies.

Infinite parallel planes:

Interchange factor for the radiation from surface 1 to surface 2

$$f_{1-2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$\epsilon_1 \Rightarrow$ Emissivity of surface 1

$\epsilon_2 \Rightarrow$ Emissivity of surface 2

Energy transfer from body 1 to body 2

$$Q = f_{1-2} A_1 \epsilon_1 \sigma T_1^4$$

Heat transfer from body 2 to body 1

$$Q_2 = f_{1-2} A_2 F_{2-1} \sigma T_2^4$$

Net heat exchange between two bodies

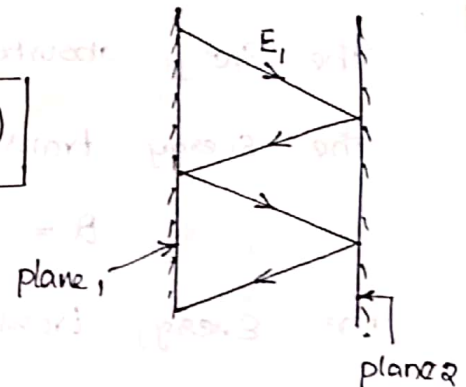
$$Q_{1,2} = f_{1-2} A_1 F_{1-2} \sigma T_1^4 - f_{1-2} A_2 F_{2-1} \sigma T_2^4$$

$$A_1 F_{1-2} = A_2 F_{2-1}$$

for infinite parallel plates $F_{1-2} = F_{2-1} = 1$

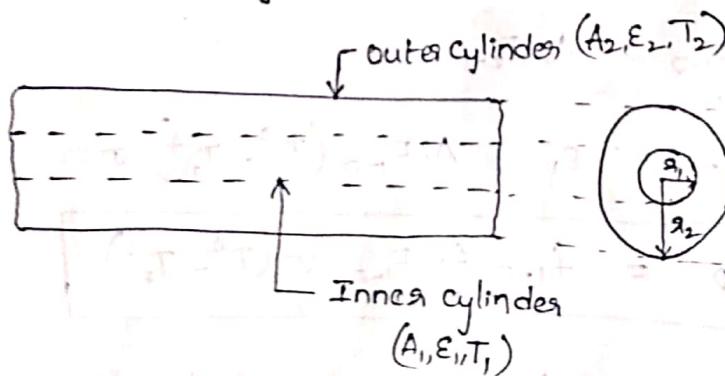
$$A_1 = A_2$$

$$\therefore Q_{1,2} = f_{1-2} A \sigma (T_1^4 - T_2^4)$$



Infinite Long Concentric cylinders:

Consider two concentric cylinders of areas A_1 and A_2 with emissivities ϵ_1 and ϵ_2 and their surfaces maintained at temperatures T_1 and T_2 respectively.



Interchange factor $f_{1-2} = \frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left(\frac{1}{\epsilon_2} - 1 \right)}$

Net heat exchange between two bodies

$$Q_{1,2} = f_{1-2} A_1 \sigma (T_1^4 - T_2^4)$$

Small gray bodies:

Consider two small gray bodies 1 and 2 having emissivities ϵ_1 and ϵ_2 (or) absorptivities α_1, α_2 .

The radiant energy emitted by body 1 is partly absorbed by body 2.

$$\text{The energy emitted by body 1} = A_1 \epsilon_1 \sigma T_1^4$$

$$\text{The energy emitted by body 2} = A_2 \epsilon_2 \sigma T_2^4$$

$$\text{The energy absorbed by body 2} = \alpha_2 \epsilon_1 A_1 F_{1-2} T_1^4 \sigma$$

The energy transfer from body 1 to body 2

$$Q = \epsilon_1 \epsilon_2 A_1 F_{1-2} \sigma T_1^4 \quad [\because \alpha_2 = \epsilon_2]$$

The energy transfer from body 2 to body 1

$$Q = \epsilon_1 \epsilon_2 A_2 F_{2-1} \sigma T_2^4$$

The net heat exchange between two bodies is

$$Q_{12} = \epsilon_1 \epsilon_2 A_1 F_{1-2} \sigma T_1^4 - \epsilon_1 \epsilon_2 A_2 F_{2-1} \sigma T_2^4$$

$$A_1 F_{1-2} = A_2 F_{2-1}$$

$$Q_{12} = \epsilon_1 \epsilon_2 A_1 F_{1-2} (T_1^4 - T_2^4) \sigma$$

$$Q_{12} = f_{1-2} A_1 F_{1-2} \sigma (T_1^4 - T_2^4)$$

$\therefore$ Interchange factor for small gray bodies is

$$f_{1-2} = \epsilon_1 \epsilon_2$$

Small body in a large enclosure:

Consider a small body placed in a large enclosure.

The large enclosure acts like a black body, absorbing all radiation incident upon it and reflecting negligibly small energy back to small gray body.

$$F_{1-2} = 1$$

Electrical Network Analogy for Thermal Radiation Systems:

Electrical analogy is an approach for analysing radiation heat exchange between gray (or) black surfaces. In this approach the two terms commonly used are irradiation and radiosity.

Irradiation (G): It is defined as the total radiation incident up on a surface per unit time per unit area.

Radiosity (J): This term is used to incident the total radiation leaving a surface per unit time per unit area.

$$J = E + \rho G$$

$$= \epsilon E_b + \rho G \quad \text{--- (1)}$$

E_b = Emissive power of a perfect black body

$$\alpha + \rho + \tau = 1$$

$$\alpha + \rho = 1$$

$$\rho = (1 - \alpha)$$

[$\tau = 0$, as surface is opaque]

$$J = \epsilon E_b + (1 - \alpha) G$$

$$\alpha = \epsilon$$

By Kirchhoff's law

$$J = \epsilon E_b + (1 - \epsilon) G \quad \text{--- (2)}$$

$$J - \epsilon E_b = (1 - \epsilon) G$$

$$G = \frac{J - \epsilon E_b}{(1 - \epsilon)} \quad \text{--- (3)}$$

The net energy leaving a surface is the difference between its radiosity and irradiation.

$$\frac{Q_{\text{net}}}{A} = J - G$$

$$= J - \frac{J - \epsilon E_b}{(1 - \epsilon)}$$

$$= \frac{\epsilon (E_b - J)}{1 - \epsilon}$$

Energy emitted by the small body 1 = $A_1 \epsilon_1 \sigma T_1^4$

Energy emitted by the enclosure 2 = $A_2 \epsilon_2 \sigma T_2^4$

Energy incident upon small body 1 = $F_{2-1} A_2 \epsilon_2 \sigma T_2^4$

Energy absorbed by body 1
= $\alpha_1 F_{2-1} A_2 \epsilon_2 \sigma T_2^4$
= $\epsilon_1 \epsilon_2 F_{2-1} A_2 \sigma T_2^4$

Net heat exchange between the small body 1 and outer large enclosure 2.

$$Q_{12} = \epsilon_1 A_1 \sigma T_1^4 - \epsilon_1 \epsilon_2 A_2 F_{2-1} \sigma T_2^4$$

$$A_1 F_{1-2} = A_2 F_{2-1}$$

$$A_1 = A_2 F_{2-1} \quad [\because F_{1-2} = 1]$$

$$Q_{12} = \epsilon_1 A_1 F_{2-1} (T_1^4 - T_2^4) \sigma$$

$$Q_{12} = f_{1-2} A_1 F_{2-1} \sigma (T_1^4 - T_2^4)$$

Interchange factor for a small body in a large enclosure

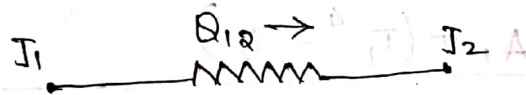
$$f_{12} = \epsilon_1$$

$$Q_{\text{net}} = \frac{A (E_b - J)}{(1 - \epsilon / \epsilon)}$$

$$Q_{\text{net}} = \frac{E_b - J}{\left(\frac{1 - \epsilon}{\epsilon A} \right)}$$

Here $\frac{1 - \epsilon}{\epsilon A}$ is known as "Surface resistance"

Now consider the Exchange of radiant energy between the two surfaces (non-black) 1 and 2.



$$Q_{12} = J_1 A_1 F_{1-2} - J_2 A_2 F_{2-1}$$

of the total radiation which leaves surface 1, the amount that reaches surface 2 is $J_1 A_1 F_{1-2}$.

heat radiated by surface 2 and received by surface 1 is $J_2 A_2 F_{2-1}$

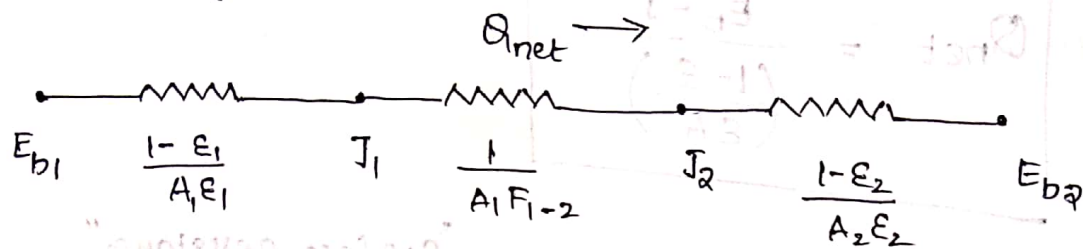
$$A_1 F_{1-2} = A_2 F_{2-1}$$

$$Q_{12} = A_1 F_{1-2} (J_1 - J_2)$$

$$Q_{12} = \frac{J_1 - J_2}{\left(\frac{1}{A_1 F_{1-2}} \right)}$$

$\frac{1}{A_1 F_{1-2}}$ is known as "Space resistance".

If the surface resistance of two bodies and space between them is considered then net heat flow can be represented by an electric circuit.



$$Q_{net} = \frac{E_{b1} - E_{b2}}{\frac{1 - \epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{1-2}} + \frac{1 - \epsilon_2}{A_2 \epsilon_2}}$$

$$= \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1 - \epsilon_1}{\epsilon_1} + \frac{1}{F_{1-2}} + \frac{1 - \epsilon_2}{\epsilon_2} \left(\frac{A_1}{A_2} \right)}$$

$$Q_{net} = F_{1-2} A_1 \sigma (T_1^4 - T_2^4)$$

$$Q_{net} = f_{1-2} A_1 \sigma (T_1^4 - T_2^4)$$

$$f_{1-2} = \frac{1}{\frac{1 - \epsilon_1}{\epsilon_1} + \frac{1}{F_{1-2}} + \frac{1 - \epsilon_2}{\epsilon_2} \left(\frac{A_1}{A_2} \right)}$$

This is known as gray body factor.

Special cases:

1. When the radiating surfaces are infinite parallel planes:

$$A_1 = A_2 \quad F_{1-2} = 1$$

$$f_{1-2} = \frac{1}{\frac{1 - \epsilon_1}{\epsilon_1} + 1 + \frac{1 - \epsilon_2}{\epsilon_2}} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

8. When the radiating bodies are concentric cylinders/spheres:

$$F_{1-2} = 1$$

$$f_{1-2} = \frac{1}{\frac{1-\epsilon_1}{\epsilon_1} + 1 + \frac{1-\epsilon_2}{\epsilon_2} \frac{A_1}{A_2}}$$

3. When a small body lies inside a large enclosure:

$$F_{1-2} = 1 \quad A_1 \ll A_2 \Rightarrow \frac{A_1}{A_2} \rightarrow 0$$

$$f_{1-2} = \frac{1}{\frac{1-\epsilon_1}{\epsilon_1} + 1}$$