

UNIT - III

Convective Heat Transfer

Convection: Convection is the mode of heat transfer between a surface and a fluid moving over it.

The energy transfer in convection is predominantly due to bulk motion of the fluid particles.

Convection is of two types: (i) ^{free} convection (ii) forced convection.

Natural convection: If this motion of fluid particles is mainly due to density variations associated with temperature gradients within the fluid, the mode of heat transfer is said to be due to free convection.

Forced convection: If fluid motion is mainly due to density differences principally produced by some superimposed velocity field (like a fan, blower or a pump), the energy transport is due to forced convection.

Dimensional analysis :

Dimensional analysis is a mathematical technique which makes use of study of the dimensions for solving Engineering problems.

Methods of dimensional analysis :

With the help of dimensional analysis the equation of a physical phenomenon can be developed in terms of dimensionless groups (or) parameters and thus reducing the number of variables. Following are the methods of dimensional analysis

- (i) Buckingham's π method
- (ii) Rayleigh method

Buckingham's π -Method / theorem:-

The Buckingham's π -theorem states as follows:

"If there are n variables (dependent & independent variables) in a dimensionally homogeneous Equation and if these contain s fundamental dimensions (such as $M, L, T, \dots$) then the variables are arranged into $(n-s)$ dimensionless teams. These dimensionless teams are called π -teams".

Mathematically if any variable x_1 depends on independent variables $x_2, x_3, x_4, \dots$ functional Equation can be written as

$$x_1 = f(x_2, x_3, \dots)$$

Each dimensionless π -team is formed by combining s variables out of the total n variables with one of the remaining $(n-s)$ variables i.e. each π -team containing $(s+1)$ variables. These are variables which appear repeatedly in each π -team are called repeating variables.

$$\pi_1 = x_2^{a_1} \cdot x_3^{b_1} \cdot x_4^{c_1} \dots x_1$$

$$\pi_2 = x_2^{a_2} \cdot x_3^{b_2} \cdot x_4^{c_2} \dots x_5$$

$\vdots$

where $a_1, b_1, c_1 ; a_2, b_2, c_2$ etc.

are constants, which are determined by dimensional homogeneity.

After obtaining $\pi_1, \pi_2, \pi_3, \dots$ teams final general Equation may then be obtained by expressing any one of the π -term as a function of others as

$$\pi_1 = \phi(\pi_2, \pi_3, \dots)$$

$$\pi_2 = \phi(\pi_1, \pi_3, \dots)$$

Dimensional Analysis applied to Forced convection: (Buckingham's π -Method)

Let us assume that the heat transfer coefficient in a fully developed forced convection in a tube is a function of the following variables:

$$D, \rho, \mu, C_p, k, h$$

The physical quantities with their dimensions are as under:

S.NO.	Variables	Symbols	Dimensions
1	Heat transfer coefficient	h	$\frac{M}{t^3 T}$
2	Fluid density	ρ	$\frac{M}{L^3}$
3	Tube diameter	D	L
4	Fluid velocity	U	$\frac{L}{t}$
5	Fluid viscosity	μ	$\frac{M}{L t}$
6	Specific heat	C_p	$\frac{L^2}{t^2 T}$
7	Thermal conductivity	k	$\frac{ML}{t^3 T}$

Total no. of variables $n = 7$

Fundamental dimensions in the problem $x = 4$

No. of dimensionless π -terms $= n - x = 7 - 4 = 3$

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

We choose D, ρ, μ, k as the core group. The groups are to be formed are now represented as the following π -groups.

$$\pi_1 = D^a \rho^b \mu^c k^d U$$

$$\pi_2 = D^a \rho^b \mu^c k^d C_p$$

$$\pi_3 = D^a \rho^b \mu^c k^d h$$

We write π , dimensionally as follows:

$$M^0 L^0 T^0 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{LT}\right)^c \left(\frac{ML}{t^3 T}\right)^d \left(\frac{L}{t}\right)$$

Equating the sum of the exponents of each basic dimension to zero, we get the following set of Equations:

$$L : a - 3b - c + d + 1 = 0$$

$$M : b + c + d = 0$$

$$t : -c - 3d - 1 = 0$$

$$T : -d = 0$$

solving these Equations we get,

$$\boxed{d = 0}$$

$$-c - 3d - 1 = 0$$

$$-c - 0 - 1 = 0$$

$$\boxed{c = -1}$$

$$b + c + d = 0$$

$$b - 1 + 0 = 0$$

$$\boxed{b = 1}$$

$$a - 3b - c + d + 1 = 0$$

$$a - 3 + 1 + 0 + 1 = 0$$

$$a - 3 + 1 + 0 + 1 = 0$$

$$\boxed{a = 1}$$

$$\text{Now, } \pi_1 = D^a \rho^b \mu^c K^d U$$

$$\pi_1 = D^1 \rho^1 \mu^{-1} K^0 U$$

$$\pi_1 = \frac{\rho U D}{\mu} = \text{Reynolds number } (Re_D)$$

Similarly for π_2 ,

$$\pi_2 = D^a g^b \mu^c k^d c_p$$

$$\pi_2 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{ML}{t^3 T}\right)^d \left(\frac{L^2}{t^2 T}\right)$$

$$L : a - 3b - c + d + 2 = 0$$

$$M : b + c + d = 0$$

$$t : -c - 3d - 2 = 0$$

$$T : -d - 1 = 0$$

$$\boxed{d = -1}$$

$$-c - 3d - 2 = 0$$

$$-c + 3 - 2 = 0$$

$$\boxed{c = 1}$$

$$b + c + d = 0$$

$$b + 1 - 1 = 0$$

$$\boxed{b = 0}$$

$$a - 3b - c + d + 2 = 0$$

$$a - 0 - 1 - 1 + 2 = 0$$

$$\boxed{a = 0}$$

$$\pi_2 = D^0 g^0 \mu^1 k^{-1} h$$

$$\pi_2 = D^0 g^0 \mu^1 k^{-1} h$$

$$\pi_2 = \frac{\mu c_p}{K} = \text{prandtl number } (Pr)$$

Similarly for π_3 ,

$$\pi_3 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{ML}{t^3 T}\right)^d \left(\frac{M}{t^3 T}\right)$$

$$L : a - 3b - c + d = 0$$

$$M : b + c + d + 1 = 0$$

$$t : -c - 3d - 3 = 0$$

$$T : -d - 1 = 0$$

$$\boxed{d = -1}$$

$$-c - 3d - 3 = 0$$

$$-c + 3 - 3 = 0$$

$$\boxed{c = 0}$$

$$b + c + d + 1 = 0$$

$$b + 0 - 1 + 1 = 0$$

$$\boxed{b = 0}$$

$$a - 3b - c + d = 0$$

$$a - 0 - 0 - 1 = 0$$

$$\boxed{a = 1}$$

$$\begin{aligned}\pi_3 &= D^a g^b \mu^c k^d h \\ &= D^1 g^0 \mu^0 k^{-1} h\end{aligned}$$

$$\pi_3 = \frac{hD}{k} \Rightarrow \text{Nusselt number (Nu)}$$

According to Buckingham π -method:

$$f(\pi_1, \pi_2, \pi_3) \text{ as}$$

$$\pi_3 = \phi(\pi_1, \pi_2)$$

$$Nu = \phi(Re, Pr)$$

$$\boxed{Nu = C Re^m Pr^n}$$

If we choose D, g, μ, c_p as the core group, the π group obtained would have Re, Pr and a non-dimensional form of heat transfer coefficient which is designated as Stanton number st , and is expressed as

$$st = \frac{Nu}{Re Pr} = \frac{h}{g \mu c_p}$$

$$\boxed{st = \phi(Re, Pr)}$$

Dimensional Analysis applied to Natural (or) Free Convection

Heat transfer :-

The heat transfer coefficient in case of natural convection, like forced convection heat transfer coefficient depends upon the variables ρ, μ, k, α and ΔT . Since the fluid circulation in free convection is owing to difference in density between the various fluid layers due to temperature gradient, velocity U is no longer an independent variable, but depends upon the following factors:

Variables	Symbol	Dimension
Fluid density	ρ	M/L^3
Fluid viscosity	μ	M/Lt
Fluid heat capacity	C_p	$L^2/t^2 T$
Fluid thermal conductivity	k	$\frac{ML}{t^3 T}$
Fluid coefficient of thermal expansion	β	$\frac{1}{T}$
Gravitational acceleration	g	$\frac{L}{t^2}$
Temperature difference	ΔT	T
Heat transfer coefficient	h	$\frac{M}{t^3 T}$
Significant length	L	L

No. of variables $n = 8$

Fundamental dimensions $m = 4$

No. of dimensionless π -terms $\Rightarrow n - m = 8 - 4 = 4$

$$f(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

We choose L, ρ, μ, k as core group,

$$\pi_1 = L^a \rho^b \mu^c k^d \Delta T$$

$$\pi_2 = L^a \rho^b \mu^c k^d \beta g$$

$$\pi_3 = L^a \rho^b \mu^c k^d C_p$$

$$\pi_4 = L^a \rho^b \mu^c k^d h$$

$$\pi_1 = L^a g^b \mu^c k^d \Delta T$$

$$= L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{L^2}\right)^c \left(\frac{ML}{L^3 T}\right)^d T$$

$$L : a - 3b - c + d = 0$$

$$M : b + c + d = 0$$

$$t : -c - 3d = 0$$

$$T : -d + 1 = 0$$

$$\boxed{d = 1}$$

$$-c - 3d = 0$$

$$-c - 3 = 0$$

$$\boxed{c = -3}$$

$$b + c + d = 0$$

$$b - 3 + 1 = 0$$

$$\boxed{b = 2}$$

$$a - 3b - c + d = 0$$

$$a + 9 - 6 + 1 = 0$$

$$\boxed{a = -4}$$

$$\pi_1 = L^{-4} g^2 \mu^{-3} k^1 \Delta T$$

$$\pi_1 = L^{-4} g^2 \mu^{-3} k^1 \Delta T$$

$$\boxed{\pi_1 = \frac{L^2 g^2 K \Delta T}{\mu^3}}$$

$$\pi_2 = L^a g^b \mu^c k^d \beta g$$

$$\pi_2 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{L^2}\right)^c \left(\frac{ML}{L^3 T}\right)^d \left(\frac{1}{T} \frac{L}{t^2}\right)$$

$$L : a - 3b - c + d + 1 = 0$$

$$M : b + c + d = 0$$

$$t : -c - 3d - 2 = 0$$

$$T : -d - 1 = 0$$

$$b + c + d = 0$$

$$b - 1 + 1 = 0$$

$$\boxed{b = 0}$$

$$a - 3b - c + d - 2 = 0$$

$$a - 0 + 1 + 1 - 2 = 0$$

$$\boxed{a = 0}$$

$$\pi_3 = L^a g^b \mu^c k^d c_p$$

$$= L^0 g^0 \mu^1 k^{-1} c_p$$

$$\boxed{\pi_3 = \frac{\mu c_p}{k}} \Rightarrow \text{Prandtl number (Pr)}$$

$$\pi_4 = L^a g^b \mu^c k^d h$$

$$\pi_4 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{L t}\right)^c \left(\frac{ML}{t^3 T}\right)^d \left(\frac{M}{t^3 T}\right)$$

$$L : a - 3b - c + d = 0$$

$$M : b + c + d + 1 = 0$$

$$t : -c - 3b - 3 = 0$$

$$T : -d - 1 = 0$$

$$\boxed{d = -1}$$

$$-c - 3b - 3 = 0$$

$$-c + 3 - 3 = 0$$

$$\boxed{c = 0}$$

$$b + c + d + 1 = 0$$

$$b + 0 - 1 + 1 = 0$$

$$\boxed{b = 0}$$

$$-d - 1 = 0$$

$$\boxed{d = -1}$$

$$-c - 3d - 2 = 0$$

$$-c + 3 - 2 = 0$$

$$\boxed{c = 1}$$

$$b + c + d = 0$$

$$b + 1 - 1 = 0$$

$$\boxed{b = 0}$$

$$a - 3b - c + d + 1 = 0$$

$$a - 0 - 1 - 1 + 1 = 0$$

$$\boxed{a = 1}$$

$$\pi_2 = L^a g^b \mu^c k^d \beta g$$

$$= L' g^0 \mu' k^{-1} \beta g$$

$$\boxed{\pi_2 = \frac{L \beta g \mu}{k}}$$

$$\pi_3 = L^a g^b \mu^c k^d c_p$$

$$\pi_3 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{ML}{t^3 T}\right)^d \left(\frac{L^2}{t^2 T}\right)$$

$$L : a - 3b - c + d + 2 = 0$$

$$M : b + c + d = 0$$

$$t : -c - 3d - 2 = 0$$

$$T : -d - 1 = 0$$

$$\boxed{d = -1}$$

$$-c - 3d - 2 = 0$$

$$-c + 3 - 2 = 0$$

$$\boxed{c = 1}$$

$$a - 3b - c + d = 0$$

$$a - 0 - 0 - 1 = 0$$

$$\boxed{a = 1}$$

$$\pi_4 = L^a g^b \mu^c k^d h$$

$$\pi_4 = L^1 g^0 \mu^0 k^{-1} h$$

$$\boxed{\pi_4 = \frac{hL}{k}} \Rightarrow \text{Nusselt number (Nu)}$$

The first two π terms together appears as a single dimensionless group. The parameter so formed is called Grashof number, Gr .

$$\boxed{Gr = \pi_1 \pi_2 = \frac{g^2 \beta g L^3 \Delta T}{\mu^2}}$$

According to buckingham π method :

$f(\pi_1, \pi_2, \pi_3, \pi_4)$ as

$$\pi_4 = \phi(\pi_1, \pi_2, \pi_3)$$

$$Nu = \phi(Gr, Pr)$$

$$\boxed{Nu = C Gr^m Pr^n}$$

In natural convection the flow is produced by buoyant effects from a temperature difference. These effects are included in Grashof number. This parameter replaces Reynolds number in case of forced convection.

Grashof number is as important as Reynolds number in forced convection. Stanton number has no significance in free convection.

Significance of dimensionless numbers :

and velocity strongly in

1. Reynold's number :

It is defined as the ratio of the inertia force to the viscous force

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho U^2 L^2}{\mu U L} = \frac{\rho U L}{\mu} = \frac{UL}{\nu}$$

- Reynolds number signifies the relative predominance of the inertia to the viscous force occurring in the flow system.
- The higher the value of Re the greater will be the relative contribution of inertia effect. The smaller the value of Re , the greater will be relative magnitude of viscous stress.
- Reynolds number is taken as an important criterion of kinematic and dynamic similarities in forced convection heat transfer.

2. Prandtl number (Pr) :

It is the ratio of kinematic viscosity (ν) to thermal diffusivity α .

$$Pr = \frac{\mu c_p}{k} = \frac{\rho \nu c_p}{k} = \frac{\nu}{(k/\rho c_p)} = \frac{\nu}{\alpha}$$

- Kinematic viscosity indicates impulse transport through molecular friction whereas thermal diffusivity indicates the heat energy transport by conduction process.
- Prandtl number provides a measure of the relative effectiveness of the momentum and energy transport by diffusion.

Prandtl number provides is a connection link between the velocity field and temperature field and its value strongly influences relative growth of velocity and thermal boundary layers.

3 Nusselt number (Nu):

It is a ratio of heat flow rate by convection process under temperature gradient to the heat flow rate by conduction process under a unit temperature gradient through a stationary thickness of L meters. Thus,

$$Nu = \frac{Q_{conv}}{Q_{cond}} = \frac{h}{k/L} = \frac{hL}{k}$$

Nusselt number is a measure of the convective heat transfer coefficient. For a given value of the Nusselt number the convective heat transfer coefficient is directly proportional to thermal conductivity of the fluid and inversely proportional to significant length.

4. Stanton number (St) :-

It is the ratio of heat transfer coefficient to the flow of heat per unit temperature rise due to the velocity of the fluid.

$$St = \frac{h}{\rho u c_p}$$

$$St = \frac{hL/k}{\left[\frac{\rho u L}{\mu} \right] \left[\frac{\mu c_p}{k} \right]} = \frac{Nu}{Re \times Pr}$$

Thus, Stanton number may also be defined as the ratio of Nusselt number and the product of Reynolds number and Prandtl number.

- Stanton number can be used only in correlating forced convection data.

5. Grashoff number (Gr) :-

Grashoff number, ~~corr~~ is related with natural convection heat transfer. It is defined as the ratio of the product of inertia force and buoyancy force to the square of viscous force.

$$Gr = \frac{(\text{Inertia force}) \times (\text{Buoyancy force})}{(\text{viscous force})^2}$$

$$= \frac{(\rho U^2 L^2) \times (\rho \beta g \Delta t L^3)}{(\mu U L)^2}$$

$$Gr = \frac{\beta g L^3 \rho^2 \Delta t}{\mu^2}$$

Free convection is usually suppressed at sufficiently small Gr , begins at some critical value of Gr depending upon the arrangement and then becomes more and more effective with increasing Gr .

:- (Gr) is dimensionless

Rayleigh's Method:

Forced convection:

Consider a fluid flowing through a pipe and its wall. Let us assume heat transfer coefficient h is a function of pipe diameter (D) fluid velocity (u) and fluid properties of density (ρ), viscosity (μ) thermal conductivity (k) and specific heat (C_p)

$$h = f(D, u, \rho, \mu, k, C_p)$$

$$h = B D^a u^b \rho^c \mu^d k^e C_p^f$$

Where B is constant.

Expressing the quantities in terms of fundamental dimensions M, L, t and T .

$$\frac{M}{t^3 T} = B L^a \left(\frac{L}{t}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{L t}\right)^d \left(\frac{ML}{t^3 T}\right)^e \left(\frac{L^2}{t^2 T}\right)^f$$

for dimensional homogeneity

$$M: 1 = c + d + e \quad \text{--- (1)}$$

$$L: 0 = a + b - 3c - d + e + 2f \quad \text{--- (2)}$$

$$t: -3 = -b - d - 3e - 2f \quad \text{--- (3)}$$

$$T: -1 = -e - f \quad \text{--- (4)}$$

from Equ (4): $e = 1 - f$

from Equ (1): $1 = c + d + 1 - f$

$$d = f - c$$

from Equ (3): $-3 = -b - f + c - 3 - 3f - 2f$

$$b = c$$

from (2) Equ: $0 = a + b - 3c - d + e + 2f$

$$0 = a + c - 3c - f + c + 1 - f + 2f$$

$$\boxed{a = c - 1}$$

Substitute a, b, c, d, e, f values in following Equation

$$h = B D^a u^b \rho^c \mu^d k^e C_p^f$$

$$h = B D^{c-1} u^c \rho^c \mu^{-c+f} k^{1-f} C_p^f$$

$$h = B \frac{D^c}{D} u^c \rho^c \frac{\mu^f}{\mu^c} \frac{k}{k^f} C_p^f$$

$$\frac{hD}{k} = B \left(\frac{\rho u D}{\mu} \right)^c \left(\frac{\mu C_p}{k} \right)^f$$

$$\boxed{Nu = B Re^c Pr^f}$$

i.e from Rayleigh method

$$\boxed{Nu = \phi(Re, Pr)}$$

Basic Equations:

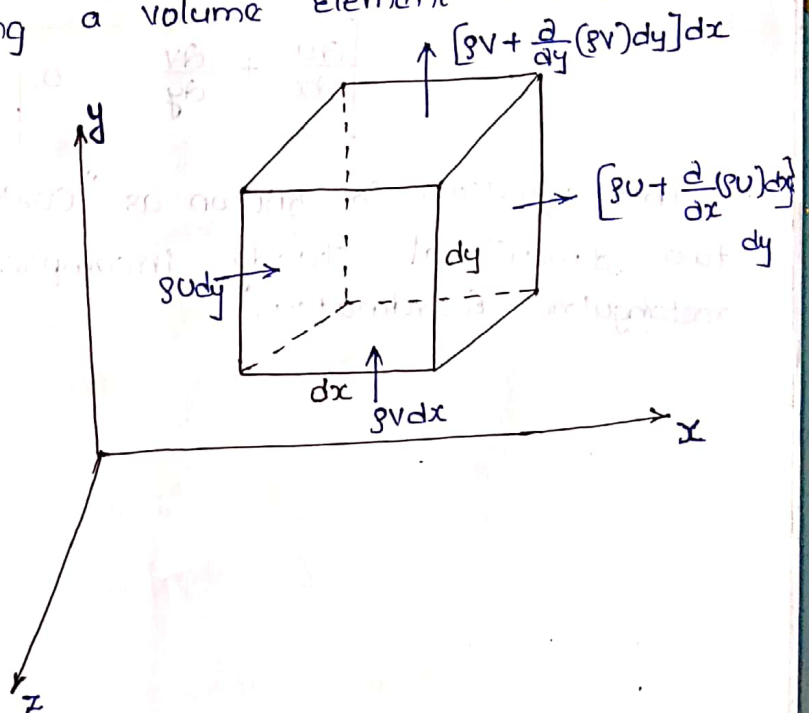
There are three basic Equations, they are continuity, momentum and energy Equations.

Continuity Equation (Mass conservation Equation):

The continuity Equation is essentially the Equation for the conservation of mass, that is, the matter may neither be created nor destroyed.

It is derived by taking a mass balance on the fluid entering and leaving a volume element in the flowfield.

consider flow through x - y plane with unit depth in z -direction. For 2-D flow, assume the velocity components to be u and v in x and y directions.



Volume of the Element is taken as $dx \cdot dy \cdot 1$

The rate of mass flow into the Element through surface perpendicular to x is $\rho u dy$.

The rate of mass flow that leaves the surface $x+dx$ may be expressed by an expression:

$$M_{x+dx} = \rho u dy + \frac{\partial}{\partial x}(\rho u) dx dy$$

The rate of mass flow into the Element through surface perpendicular to y is $\rho v dx$.

The rate of mass leave the surface $y+dy$ may be

by $M_{y+dy} = (\rho v) dx + \frac{\partial}{\partial y} (\rho v) dy dx$

By balancing the flow,

$$M_x + M_y = M_{x+dx} + M_{y+dy}$$

$$\cancel{\rho u dy} + \cancel{\rho v dx} = \cancel{\rho u dy} + \frac{\partial}{\partial x} \rho u dx dy + \cancel{\rho v dx} + \frac{\partial}{\partial y} \rho v dy dx$$

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0$$

If $\rho = \text{constant}$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

This equation is known as "Continuity Equation" for a two dimensional steady incompressible flow in the rectangular coordinates.

Dimensional Analysis

Dimensionless Numbers (or) Non dimension Numbers:-

(i) Nusselt Number:- It is the ratio of heat flow rate by convection process under a unit temp gradient to heat flow rate by conduction process under a unit temp gradient of stationary thickness 'l'. [Convenient method of convective heat transfer coefficient of (h)]

$$Nu = \frac{Q_{conv}}{Q_{cond}} = \frac{h \cdot A \cdot \Delta T}{k \cdot A \cdot \frac{\Delta T}{l}} = \frac{hL}{k}$$

(ii) Raynold's Number:- It is the ratio of Inertial force to Viscous force

$$Re = \frac{F_i}{F_v}$$

Note:- for a laminar flow Re value is less than 2000

for a turbulent flow Re value is 4000 and above.

$$= \frac{\rho v L}{\mu} = \frac{\rho v L}{\mu} = \frac{v L}{\nu} \quad \left(\because \frac{\rho}{\mu} = \nu \right)$$

(iii) Peclet Number:- It is the ratio of Kinematic viscosity to Thermal diffusivity

$$Pe = \frac{v}{\alpha} = \frac{v \cdot \rho \cdot c_p}{k} = \frac{\rho v c_p}{k} = \frac{\rho c_p}{k}$$

→ It is a measure of relative effectiveness of the momentum and energy transfer by diffusion

Assignment Questions

1) Determine the heat transfer rate from the rectangular fin of length 20 cm width 40 cm & thickness 2 cm. The tip of fin is not insulated and the fin has thermal conductivity of 150 W/mK. The base Temp is 100°C and the fluid is 20°C. The heat transfer coefficient b/w the fin and fluid is 30 W/m²°C.

2) A wall of furnace is made up of inside layer of silica brick 120 mm thickness provide with a layer of magnesite brick 20 mm thick. The Temp at the inside surface of a silica brick wall and outside surface of a magnesite brick wall are 725°C and 110°C. The contact thermal resistance b/w the two walls at the interface is 0.0035 °C per unit Watt. per unit wall area. If the thermal conductivity $k_s = 1.7 \text{ W/m}^\circ\text{C}$, $k_m = 5.8 \text{ W/m}^\circ\text{C}$, calculate rate of heat loss per unit area of wall. The Temp drop at interface

(iv) Stanton Number:-

It is the ratio of heat transfer coefficient to rate of heat flow per unit Temperature raise due to velocity of fluid.

$$St = \frac{h}{\rho v c_p} = \frac{hL}{k} = \frac{Nu}{Re \times Pr}$$

$$St = \frac{hL}{\rho v c_p \left(\frac{L}{k}\right)} = \frac{hL}{\rho v L c_p / \mu k}$$

$$St = \left(\frac{hL}{\mu}\right) \cdot \left(\frac{\mu c_p}{k}\right)$$

(v) Peclet Number :-

It is the ratio of mass heat flow rate by convection to mass heat flow rate by conduction for unit temp gradient for a thickness 'l'

$$(Pe) = \frac{Q_{\text{conv}}}{Q_{\text{cond}}} = \frac{\rho V C_p}{\frac{k}{L}} \quad (\because \alpha = \frac{k}{\rho C_p})$$

$$(Pe) = \frac{\rho V L}{k}$$

(vi) Graetz Number :- $Pe = Re \times Pr$

It is the ratio of heat capacity of the fluid flowing through the pipe per unit length of a pipe to the conductivity of the pipe

$$G = \frac{\frac{m \cdot C_p}{L}}{k} = \frac{\rho \times V \times C_p}{\frac{k}{L}} = \frac{\rho \times V \times C_p}{k}$$

$$G = \frac{\rho \times \frac{\pi}{4} D^2 \times V \times C_p}{kL}$$

$$G = \frac{\pi}{4} D^2 \cdot V \cdot \frac{\rho \cdot C_p}{kL} = \frac{\pi}{4} \rho V D \frac{C_p}{k} \cdot \frac{D}{L} \times \frac{M}{M}$$

$$G = \frac{\pi}{4} D^2 \times \frac{Pe}{L}$$

$$G = \frac{\pi}{4} \times Re \times Pr \times \frac{D}{L}$$

(viii) Grashoff number :-

→ Related with Natural convection, heat transfer

It is the ratio of product of inertia force and buoyant force to square of the viscous force

$$Gr = \frac{[\text{Inertia force}] [\text{Buoyant force}]}{[\text{Viscous force}]^2}$$

m/sec

$$= \frac{(\rho v^2 L) (\rho g \Delta T L^3)}{(\mu v L)^2}$$

$$\tau = \frac{F}{A}$$

$$= \mu \frac{du}{dy}$$

$$= \mu L \frac{v}{L}$$

$$= \mu v$$

$$F = ma$$

$$= \rho v \cdot \frac{L}{\text{sec}}$$

$$= \rho L \cdot \frac{L}{\text{sec}}$$

$$= \rho v L$$

$$= \frac{\rho^2 v^2 L^4 (g \Delta T)}{\mu^2 v^2}$$

Dimension Analysis :-

Dimension Analysis is an experimental investigation to solve simple engineering problems

length → m → L

Mass → kg → M

Time → sec → T

Temp → °C → θ

Density, Angular velocity, Area, Volume, Angular acceleration,

Acceleration, Discharge, Kinematic viscosity, Dynamic viscosity

Force, power, Work, specific weight, Thermal conductivity

Heat, specific heat, heat transfer coefficient,

$$\text{Work} = W = F \cdot S \Rightarrow MLT^{-2}L$$

$$= ML^2T^{-2}$$

$$\text{Specific volume} = \frac{\text{Weight}}{\text{Volume}} = \frac{N}{m^3} \Rightarrow \frac{MLT^{-2}L^{-3}}{ML^{-2}T^{-2}}$$

$$\text{Thermal Conductivity} - K - W/m^2C$$

$$= MLT^{-3}L^{-1}\theta^{-1}$$

$$= MLT^{-3}\theta^{-1}$$

$$\text{Heat} \rightarrow P.E / K.E.$$

$$\text{specific heat} \rightarrow Q = mc_p \Delta t \Rightarrow c_p = \frac{Q}{m \cdot \Delta t} = \frac{L^2T^{-2}\theta^{-1}}{ML^{-2}T^{-2}}$$

$$\text{heat transfer coefficient} \rightarrow h \rightarrow W/m^2C$$

$$h = \frac{Q}{A \cdot \Delta t}$$

$$= \frac{ML^2T^{-2}\theta^{-1}}{L^2\theta^{-1}}$$

$$= MLT^{-3}\theta^{-1}$$

$$= MLT^{-3}\theta^{-1}$$



$$= MT^{-2}\theta^{-1}$$

imp

Dimensional homogeneity

If the dimensions of the variables of equation on both sides of equation are equal then it said to be dimensional homogeneity

$$\text{Work} = W = F \cdot S \Rightarrow M L^2 T^{-2}$$

$$= M L^2 T^{-2}$$

$$\text{Specific volume} = \frac{\text{Weight}}{\text{Volume}} = \frac{N}{m^3} \Rightarrow \frac{M L^2 T^{-2}}{m^3} = M L^{-2} T^{-2}$$

$$\text{Thermal Conductivity} = K = W / m^2 \cdot C$$

$$= M L^2 T^{-3} L^{-1} \theta^{-1}$$

$$= M L T^{-3} \theta^{-1}$$

$$\text{Heat} \rightarrow P \cdot E / K \cdot E$$

$$\text{specific heat} \rightarrow Q = m \cdot c_p \Delta t \Rightarrow c_p = \frac{Q}{m \cdot \Delta t} = L^2 T^{-2} \theta^{-1}$$

$$\text{heat transfer coefficient} \rightarrow h \rightarrow W / m^2 \cdot C$$

$$h = \frac{Q}{A \cdot \Delta t}$$

$$= \frac{M L^2 T^{-2}}{L^2 \theta^{-1}}$$

$$= \frac{M L^2 T^{-2}}{L^2 \theta^{-1}}$$

$$= M T^{-2} \theta^{-1}$$



$$= M T^{-2} \theta^{-1}$$

Imp

Dimensional homogeneity

If the dimensions of the variables of equation on both sides of equation are equal then it said to be dimensional homogeneity

Applications :-

- It facilitates to determine the dimension of a physical quantity
- It helps to check whether the eqn is dimensionally homogeneous (or) not.
- It converts units from one system to another.

Dimensional Analysis with the help of :-

With the help of dimensional analysis the equation of physical phenomena can be developed in terms of dimensionless numbers or parameters. There are two methods.

→ Rayleigh's method

→ Buckingham's method.

Buckingham's method :-

This method designated / represented dimensionless group has π terms.

Definition - If there are 'n' variables [dependent / independent] in a dimensionally homogeneous equation, and if these variables contains 'm' fundamental dimensions [M, L, T] Then the variables are arranged in (n-m) dimensionless term. These dimensional terms are called as π terms.

Mathematically, if x_1 dependent on $x_2, x_3, x_4, \dots, x_n$. Then the functional eqn can be written as.

$$x_1 = f[x_2, x_3, x_4, \dots, x_n] \quad \text{--- (i)}$$

$$\Rightarrow f(x_1, x_2, x_3, \dots, x_n) = 0 \quad \rightarrow (ii)$$

If it is a dimensionally homogeneous equation containing 'n' variables
If there are 'm' fundamental dimensions then the According

Buckingham's theorem

$$f(\pi_1, \pi_2, \pi_3, \dots, \pi_{n-m}) = 0$$

Each dimensional free pie term is formed by combining 'm' variables
out of total 'n' variables with one of the remaining (n-1) terms

i.e. each pie term has (m+1) terms

These 'm' variable which appears repeatedly in a π term are called
as Repeated Variables.

Let $x_1, x_2, x_3, \dots, x_n$ are the variables in a homogeneous
equation Assume x_2, x_3, x_4 are the repeated variables and
fundamental dimensions are [M, L, T] which is equal to

Then each column is can be written as

$$\begin{aligned} \pi_1 &= x_2^{a_1} \cdot x_3^{b_1} \cdot x_4^{c_1} \cdot x_1 \\ \pi_2 &= x_2^{a_2} \cdot x_3^{b_2} \cdot x_4^{c_2} \cdot x_5 \\ \pi_{(n-m)} &= x_2^{a_{(n-m)}} \cdot x_3^{b_{(n-m)}} \cdot x_4^{c_{(n-m)}} \cdot \dots \cdot x_n \end{aligned}$$

Where a, b, c are constants that are determined by using
dimensional homogeneity

(i) These values sub in above eqns. Then by $\pi_1, \pi_2, \pi_3, \dots, \pi_{n-1}$ are
obtained.

the final general eqn for the phenomena by expressing any one of π Terms as the function of the other.

$$\therefore \pi_1 = f(\pi_2, \pi_3, \pi_4, \dots, \pi_{n-m})$$

$$\pi_2 = f(\pi_1, \pi_3, \pi_4, \dots, \pi_{n-m})$$

Selection of repeating variables :-

- m repeating variable must contain jointly all the fundamental dimensions involved in the phenomena.
 - The repeating variables must not form the non dimensional parameters among themselves.
 - As far as possible the dependent variable shouldn't be repeating variable.
 - No two repeating variables should have same dimensions.
 - The repeating variable should be chosen so that one variable contains geometry property and other variable contains flow property and third variable is a fluid property.
- the choice of repeating variables in most of the fluid mechanics problems may be (L, V, ρ) ; (D, V, ρ) ; $[L, V, \mu]$; $[D, V, \mu]$
- ① the resistance R are experienced by a partially submerged body depends upon velocity 'V' length of the body 'L' viscosity of fluid ' μ ' density of fluid ' ρ ' and gravitational acceleration 'g' obtain a dimensionless expression for R

Step 1:- 1) Resistance R

2) velocity V

3) length L

4) viscosity μ

5) density ρ

6) gravitational acceleration g

no. of variables $n=6$

$$R = f(V, L, \mu, \rho, g)$$

$$f[R, V, L, \mu, \rho, g] = 0$$

$$R = MLT^{-2}$$

$$V = LT^{-1}$$

$$L = L$$

$$\mu = ML^{-1}T^{-1}$$

$$\rho = ML^{-3}$$

$$g = LT^{-2}$$

$m = \text{fundamental terms} = 3 [M, L, T]$

$$\text{No. of dimension } (\pi \text{ terms}) (n-m) = 6-3 = 3$$

$(\pi \text{ terms are } \pi_1, \pi_2, \pi_3)$
 $f_1(\pi_1, \pi_2, \pi_3) = 0$

Step:- 2

Selection of repeating variables

$\therefore R$ is dependent variable
Variable

$\therefore$ velocity, length & density are selected as repeating

step 3:- $\pi_1 = l^{a_1} v^{b_1} f^{c_1} R$

$\pi_2 = l^{a_2} v^{b_2} f^{c_2} \mu$

$\pi_3 = l^{a_3} v^{b_3} f^{c_3} g$

step 4:- Each term is solved by principle of dimensional homogeneity

for M :- $\pi_1 = l^{a_1} v^{b_1} f^{c_1} R$

for $M \rightarrow M^0 L^0 T^0 = [L]^{a_1} [L T^{-1}]^{b_1} [M L^{-3}]^{c_1} [M L T^{-2}]$

for M , $0 = M^1 \cdot M^1$
 $= c_1 + 1$

for L , $L^0 = L^{a_1} L^{b_1} L^{-3c_1} L^1$
 $0 = a_1 + b_1 - 3c_1 + 1$

for T , $T^0 = T^{-b_1} T^{-2}$
 $0 = -b_1 - 2 \Rightarrow b_1 = -2$
 $0 = a_1 - 2 - 3(-1) + 1$

$a_1 = 2$

$$\pi_2 = l^2 \cdot v \cdot f \cdot M$$

$$\pi_3 = l^3 \cdot v \cdot f \cdot g$$

$$M^0 L^0 T^0 = L^{a_2} \cdot [LT^{-1}]^{b_2} \cdot [ML^{-3}]^{c_2} M^{-1} T^{-1}$$

$$M^0 L^0 T^0 = L^{a_3} \cdot [LT^{-1}]^{b_3} \cdot [ML^{-3}]^{c_3} LT^2$$

For M, $M^0 = M^{c_2} \cdot M^{-1}$

For M, $M^0 = M^{c_3}$

$$0 = c_2 + 1$$

$$\boxed{c_2 = -1}$$

$$\boxed{0 = c_3}$$

For L, $L^0 = L^{a_2} \cdot L^{b_2} \cdot L^{-3c_2} \cdot L^{-1}$

for T, $T^0 = T^{-b_3} \cdot T^{-2}$

$$0 = -b_3 - 2$$

$$\boxed{b_3 = -2}$$

$$0 = b_2 - 3c_2 - 1 + a_2$$

for T, $T^0 = T^{-b_2} \cdot T^{-1}$

for L, $L^0 = L^{a_3} \cdot L^{b_3} \cdot L^{-3c_3}$

$$\boxed{b_2 = -1}$$

$$0 = a_3 + b_3 - 3c_3 + 1$$

$$0 = -1 - 3(-1) - 1 + a_2$$

$$0 = a_3 - 2 - 3(0) + 1$$

$$\boxed{a_2 = -1}$$

$$\boxed{a_3 = 1}$$

$$\pi_1 = \frac{L^{-2} \cdot v^{-2} \cdot f^1 \cdot R}{L^2 v^2 f}$$

$$\pi_2 = \frac{L^{-1} \cdot v^{-1} \cdot f^{-1} \cdot M}{L v f}$$

$$\pi_3 = \frac{l^1 v^{-2} g}{v^2}$$

$$\pi_1 = \frac{R}{L^2 v^2 f}$$

$$\pi_2 = \frac{M}{L v f}$$

$$\pi_3 = \frac{lg}{v^2}$$

$$\pi_1 = \phi(\pi_2, \pi_3)$$

$$\pi_2 = \phi(\pi_1, \pi_3)$$

$$\pi_3 = \phi(\pi_1, \pi_2)$$

$$\frac{R}{L^2 v^2 f} = \phi \left[\frac{M}{L v f}, \frac{lg}{v^2} \right]$$

$$\frac{R}{\rho v^2 l} = f \left[\frac{\rho v l}{\mu}, \frac{v}{\sqrt{lg}} \right]$$

$$R = \rho v^2 l f(Re, Fr)$$

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

$$f_1 \left[\frac{R}{\rho v^2 l}, \frac{\mu}{\rho v l}, \frac{lg}{v^2} \right] = 0$$

Free Convection (Natural) :-

Convection is of two ways

(i) free Convection

(ii) forced Convection

(i) free Convection in which bulk motion of fluid is due to density difference of any buoyant force caused by heat transfer b/w fluids

Thermal boundary layer consider with hydro dynamic boundary layer

eg:- Thunderstorms, gliders, cooling fans in a motor, hot coffee in a cup.

(ii) forced Convection:- Bulk motion of fluid is due to pump

eg:- fluid flow in boilers flow over condensers heat exchangers -- etc.

Dimensional Analysis applied to force convection heat transfer

let us assume that heat transfer coefficient is a function of

$$\rho, D, V, \mu, k, c_p$$

$$h = f(\rho, D, \mu, k, c_p, V)$$

$$f_1[\rho, D, \mu, V, k, c_p] = 0$$

$$h = \frac{W}{m^2 \cdot ^\circ C} \Rightarrow \frac{MLT^{-2}T^{-1}}{L^2 \cdot \theta} = MT^{-3}\theta^{-1}$$

$$\rho = \frac{kg}{m^3} = \frac{ML^{-3}}{L^3} = ML^{-3}$$

$$D = L$$

$$V = m/sec \Rightarrow LT^{-1}$$

$$\mu = \frac{MLT^{-2}K}{L^2T^{-1}} = ML^{-1}T^{-1}$$

$$c_p = L^2T^{-2}\theta^{-1}$$

$$k = \frac{MT^{-2}L^2}{L^2\theta} = MLT^{-2}\theta^{-1}$$

$$m - m = 7 - 4 = 3$$

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

$$h, \rho, V, D$$

$$\pi_1 = h^{a_1} \cdot f^{b_1} \cdot D^{c_1} \cdot V^{d_1} \cdot M$$

$$\pi_2 = h^{a_2} \cdot f^{b_2} \cdot D^{c_2} \cdot V^{d_2} \cdot c_p$$

$$\pi_3 = h^{a_3} \cdot f^{b_3} \cdot D^{c_3} \cdot V^{d_3} \cdot k$$

$$\pi_1 = h^{a_1} \cdot f^{b_1} \cdot D^{c_1} \cdot V^{d_1} \cdot M$$

$$M^0 L^0 T^0 = [M^{-3} \theta^{-1}]^{a_1} [M^{-3}]^{b_1} L^{c_1} [L T^{-1}]^{d_1} M^{-1} T^{-1}$$

$$\text{for } M, \quad 0 = -3a_1 - 3b_1 - 1$$

$$0 = -3a_1 - 3b_1 - 1$$

$$\text{for } L, \quad 0 = -3b_1 + c_1 + d_1 - 1$$

$$0 = -3b_1 + c_1 + d_1 - 1$$

$$\text{for } T, \quad 0 = -3a_1 - d_1 - 1$$

$$0 = -3a_1 - d_1 - 1 \Rightarrow -3(0) - d_1 - 1 = 0$$

$$\text{for } \theta, \quad 0 = -a_1 \quad \boxed{d_1 = -1}$$

$$\boxed{a_1 = 0}, \quad \boxed{b_1 = -1}$$

$$\boxed{c_1 = -1}$$

$$\pi_1 = h^0 \cdot f^{-1} \cdot D^{-1} \cdot V^{-1} \cdot M$$

$$\pi_1 = \frac{M}{f V D}$$

$$\pi_2 = h^{a_2} \cdot f^{b_2} \cdot D^{c_2} \cdot V^{d_2} \cdot c_p$$

$$M^0 L^0 T^0 = [M^{-3} \theta^{-1}]^{a_2} [M^{-3}]^{b_2} L^{c_2} [L T^{-1}]^{d_2} M^{-2} T^{-1}$$

$$M: 0 = a_2 + b_2 \Rightarrow \boxed{b_2 = -1}$$

$$L: 0 = -3b_2 + c_2 + d_2 + 2; \boxed{c_2 = 0}$$

$$T: 0 = -3a_2 - d_2 - 2; \boxed{d_2 = -4}$$

$$\theta: 0 = -a_2 - 1; \boxed{a_2 = -1}$$

$$\pi_2 = h^{a_2} \cdot \rho^{b_2} \cdot D^{c_2} \cdot V^{d_2} \cdot c_p$$

$$\pi_2 = h^{-1} \cdot \rho^{-1} \cdot D^0 \cdot V^{-4} \cdot c_p = \boxed{\frac{c_p \cdot \rho \cdot V}{h \cdot D}}$$

$$\pi_3 = [ML^{-3}\theta^{-1}]^{a_3} \cdot [ML^{-3}]^{b_3} \cdot [L]^{c_3} \cdot [LT^{-1}]^{d_3} \cdot ML^{-3}\theta^{-1}$$

$$M: 0 = a_3 - 3b_3 + 1$$

$$L: 0 = -3b_3 + c_3 + d_3 + 1$$

$$T: 0 = -3a_3 - d_3 - 3$$

$$\theta: 0 = -a_3 - 1, \boxed{a_3 = -1}, \boxed{b_3 = 0}, \boxed{c_3 = -1}$$

$$\boxed{d_3 = 0}$$

$$\pi_3 = h^{-1} D^{-1} K$$

$$\boxed{\pi_3 = \frac{K}{hD}}$$

According to Buckingham's π Theorem, $\pi_3 = \phi(\pi_1, \pi_2)$

$$\frac{K}{hD} = \phi\left[\frac{\mu}{\rho V D}, \frac{\rho V c_p}{h}\right]$$

By introducing constants in place of ϕ and powers

$$\frac{K}{hD} = C \left[\frac{\mu}{\rho V D}\right]^m \left[\frac{\rho V c_p}{h}\right]^n$$

where C, m, n - constants

if $m > n$ then $\frac{K}{hD} = C \left[\frac{\mu}{\rho V D}\right]^m \left[\frac{\rho V c_p}{h}\right]^n \left[\frac{\rho \mu}{\rho V D}\right]^{m-n}$

① The pres. diff Δp in a pipe of diameter 'D' and length 'l' due to turbulent flow depends on the velocity 'V', viscosity ' μ ', density ' ρ ' and roughness 'k' using Buckingham's Theorem. Obtain an expression for Δp .

Sol. $\rightarrow$

$$\Delta p = f[D, l, V, \mu, \rho, k]$$

$$f_1[\Delta p, D, l, V, \mu, \rho, k] = 0$$

Since the dimension of h & $\frac{k}{D}$ are same.

$$n = 7$$

$$\Delta p - M L^{-1} T^{-2}$$

$$D - L$$

$$l - L$$

$$V - L T^{-1}$$

$$\mu - M L^{-1} T^{-1}$$

$$\rho - M L^{-3}$$

$$k - L$$

$$\therefore \pi_2 = \frac{\Delta p \cdot D}{\rho V^2 k}$$

$$\frac{k}{hD} = c \left[\frac{\mu}{\rho V D} \right]^m \left[\frac{\rho V^2 D}{k} \right]^n \left[\frac{\mu}{\rho V D} \right]^{m-n}$$

$$\frac{k}{hD} = c \left[\frac{\mu}{\rho V D} \right]^{m-n} \left[\frac{\rho V^2 D}{k} \right]^n$$

$$\frac{k}{hD} = c \left[\frac{\mu}{\rho V D} \right]^{m-n} \left[\frac{\rho V^2 D}{k} \right]^n$$

$$m = 3$$

$$\text{no. of } \pi \text{ terms } (n - m) = 7 - 3 = 4$$

$$f_1[\pi_1, \pi_2, \pi_3, \pi_4] = 0$$

repeating variables.

$$\pi_1 = D^a \rho^b V^c L$$

$$M^0 L^0 T^0 = L^a [M L^{-3}]^b [L T^{-1}]^c L$$

$$\text{for } M, \quad M^0 = M^b \quad \boxed{b_1 = 0}$$

$$Nu = c [Re]^{m-n} [Pr]^n$$

$$Nu = c [Re]^{m-n} [Pr]^n$$

For T , $T^0 = \frac{c_1}{T}$
 $\boxed{c_1 = 0}$

For L , $L^0 = L a_1 - 3b_1 + c_1 \cdot L$
 $0 = a_1 - 3b_1 + c_1 + 1$
 $a_1 - 3(0) + 0 + 1$
 $\boxed{a_1 = -1}$

$\pi_1 = \frac{1}{D} \int \frac{0}{V} L$
 $\boxed{\pi_1 = \frac{L}{D}}$

$\pi_2 = D a_2 \int b_2 \cdot v \cdot c_2 \mu$

$M^0 L^0 T^0 = L a_2 [ML^{-3}] b_2 [LT^{-1}] c_2 [ML^{-1} T^{-1}]$

For M , $M^0 = M b_2 \cdot M^1$
 $\Rightarrow b_2 + 1 = 0$
 $\boxed{b_2 = -1}$

For T , $T^0 = T^{-c_2} \cdot T^{-1}$
 $-c_2 - 1 = 0$ $\boxed{c_2 = -1}$

For L , $L^0 = L a_2 - 3b_2 + c_2 \cdot L$

$0 = a_2 - 3b_2 + c_2 - 1$

$a_2 - 3(-1) - 1 - 1$

$\boxed{a_2 = -1}$

$\pi_2 = D^{-1} \int^{-1} \cdot v^{-1} \mu$

$\boxed{\pi_2 = \frac{\mu}{D \rho v}}$

$\pi_3 = D a_3 \int b_3 \cdot v \cdot c_3 \cdot K$

$M^0 L^0 T^0 = L a_3 [ML^{-3}] b_3 [LT^{-1}] c_3 [ML^{-1} T^{-1}]$

for M , $M^0 = M^{b_3}$

$b_3 = 0$

for T , $T^0 = T^{-c_3}$

$c_3 = 0$

for $L \Rightarrow L^0 = L^{a_3 - 3b_3 + c_3 + 1}$

$0 = a_3 - 3b_3 + c_3 + 1$

$a_3 - 3(0) + (0) + 1$

$a_3 = -1$

$\pi_3 = D^1 \cdot \rho^0 \cdot v^0 \cdot k$

$\pi_3 = \frac{k}{D}$

$\pi_4 = D^{a_4} \cdot \rho^{b_4} \cdot v^{c_4} \cdot \Delta p$

$M^0 L^0 T^0 = L^{a_4} [ML^{-3}]^{b_4} [LT^{-1}]^{c_4} ML^{-1} T^{-2}$

for $M^0 = M^{b_4}$

$b_4 = -1$

$T^0 = T^{-c_4 - 2}$

$0 = -c_4 - 2 \Rightarrow c_4 = -2$

$L^0 = L^{a_4 - 3b_4 + c_4 - 1}$

$0 = a_4 - 3b_4 + c_4 - 1$

$a_4 - 3(-1) + (-2) - 1$

$a_4 = 0$

$\pi_4 = D^0 \cdot \rho^{-1} \cdot v^{-2} \cdot \Delta p$

$\pi_4 = \frac{\Delta p}{\rho v^2}$

Dimensional Analysis is applied to Natural/Free Convection:

The heat transfer coefficient depends upon $\nu, \beta, \mu, c_p, \Delta T$
 In case of free convection velocity dependent variable and
 Temperature independent

Since the motion of fluid is due to difference in density b/w
 fluid layers due to temp gradient.

$\therefore$ velocity depends on ΔT Temperature diff b/w heated
 surface and undisturbed fluid.

g acceleration due to gravity

Thus the heat transfer coefficient 'h' may be expressed as

$$h = f[\beta, L, \mu, c_p, k, \beta, g, \Delta T]$$

$$f_1(h, \beta, c_p, L, \mu, k, \beta, g, \Delta T) = 0$$

$$n = 7$$

$$h = \frac{Q}{A \cdot \Delta T} = \frac{MLT^{-2} \cdot LT^{-1}}{L^2 \theta} = MT^{-3} \theta^{-1}$$

$$\beta = ML^{-3}$$

$$L = L$$

$$\mu = \frac{MLT^{-2}}{L^2} \cdot \frac{L}{LT^{-1}} = ML^{-1}T^{-1}$$

$$c_p = \frac{Q}{m \cdot \Delta T} = \frac{MLT^{-2}}{ML \theta} = T^{-1}$$

$$\beta g \Delta T = LT^{-2}$$

$$k = \frac{MLT^{-2}}{L^2 \theta} = ML^{-1}T^{-2} \theta^{-1}$$

Fundamental dimensions
 $[M, L, \theta, T]$

$$m = 4$$

no. of π terms: $n-m = 7-4$

no. of π terms = 3

$$\therefore f_1[\pi_1, \pi_2, \pi_3] = 0.$$

repeating variables: $\pi_1 = \left[\rho, L, \mu, k, h \right]$

or

$$\pi_2 = \left[\rho, L, \mu, k, h, c_p \right]$$

$$\pi_3 = \left[\rho, L, \mu, k, h, \beta \right]$$

~~for ρ , μ~~

$$\theta M L T^0 = [M L^{-3}]^{a_1} [L]^{b_1} [M L^{-1} T^{-1}]^{c_1} [M L T^{-3} \theta^{-1}]^{d_1} M T^{-3} \theta^{-1}$$

for M , $M^0 = M^{a_1} M^{c_1} M^{d_1} M^1$

$$0 = a_1 + c_1 + d_1 + 1 \quad a_1 + 0 + (-1) + 1 = 0$$

for L , $L^0 = L^{-3a_1} L^{b_1} L^{-c_1} L^{d_1} \quad \boxed{a_1 = 0}$

$$0 = -3a_1 + b_1 - c_1 + d_1 \quad -3(0) + b_1 - 0 - 1$$

for T , $T^0 = T^{-c_1} T^{-3d_1} T^{-3} \quad \boxed{b_1 = 1}$

$$0 = -c_1 - 3d_1 - 3 \Rightarrow -c_1 - 3(-1) - 3 = 0$$

for θ , $\theta^0 = \theta^{-d_1} \theta^{-1} \quad \boxed{c_1 = 0}$

$$-d_1 - 1 = 0 \quad \boxed{d_1 = -1}$$

$$\pi_1 = \rho^0 L^1 \mu^0 k^{-1} h$$

$$\Rightarrow \boxed{\pi_1 = \frac{h L}{\mu k}}$$

$$\pi_2 = [M L^{-3}]^{a_2} L^{b_2} [M L^{-1} T^{-1}]^{c_2} [M L T^{-3} \theta^{-1}]^{d_2} L^{\gamma} T^{-3} \theta^{-1}$$

For M, $M^0 = M^{a_2} M^{c_2} M^{d_2}$

$$a_2 + 0 + -1 \quad \boxed{a_2 = 1}$$

$$0 = a_2 + c_2 + d_2$$

For L, $L^0 = L^{-3a_2} L^{b_2} L^{-c_2} L^{d_2} L^{\gamma}$

$$0 = -3a_2 + b_2 - c_2 + d_2 + \gamma \quad -3(1) + b_2 - 0 - 1 + \gamma$$

$$\boxed{b_2 = 2}$$

For T, $T^0 = T^{-c_2} T^{-3d_2} T^{-3}$

$$0 = -c_2 - 3d_2 - 3 \quad -c_2 + 3 - 3 = 0$$

$$\boxed{c_2 = 0}$$

For θ , $\theta^0 = \theta^{-d_2} \theta^{-1}$

$$0 = -d_2 - 1$$

$$\boxed{d_2 = -1}$$

$$\pi_2 = \int L^2 M^0 K^{-1} C_p$$

$$\pi_2 = \frac{A V C_p}{k}$$

$$\boxed{\pi_2 = \frac{C_p \int L^{\gamma}}{k}}$$

$$\pi_3 = [M L^{-3}]^{a_3} L^{b_3} [M L^{-1} T^{-1}]^{c_3} [M L T^{-3} \theta^{-1}]^{d_3} L T^{-2}$$

For M, $M^0 = M^{a_3} M^{c_3} M^{d_3}$

$$a_3 - 2 + 0 = 0$$

$$0 = a_3 + c_3 + d_3$$

$$\boxed{a_3 = 2}$$

For L, $L^0 = L^{-a_3} L^{b_3} L^{-c_3} L^{d_3} L$

$$0 = -a_3 + b_3 - c_3 + d_3 + 1 \quad -3(2) + b_3 - 2 + 0 + 1$$

$$\boxed{b_3 = 4}$$

For T, $T^0 = T^{-c_3} T^{-3d_3} T^{-2}$

$$0 = -c_3 - 3d_3 - 2 \quad -c_3 - 3(0) - 2 = 0$$

$$\boxed{c_3 = -2}$$

For θ , $\theta^0 = \theta^{-d_3}$

$$\boxed{d_3 = 0}$$

$$\pi_3 = \frac{\rho^2 \mu^2}{k^2} \frac{P_0 \Delta T}{\mu^2}$$

$$\boxed{\pi_3 = \frac{P_0 \Delta T}{\mu^2}}$$

$$\pi_1 = f(\pi_2, \pi_3)$$

$$\frac{hL}{k} = c \left[\frac{P_0 \Delta T}{\mu^2} \right]^m \left[\frac{\rho^2 \mu^2}{k^2} \frac{P_0 \Delta T}{\mu^2} \right]^n$$

$$\boxed{Nu = c \cdot (P_0)^m (\mu^2)^n}$$