UNIT - 2

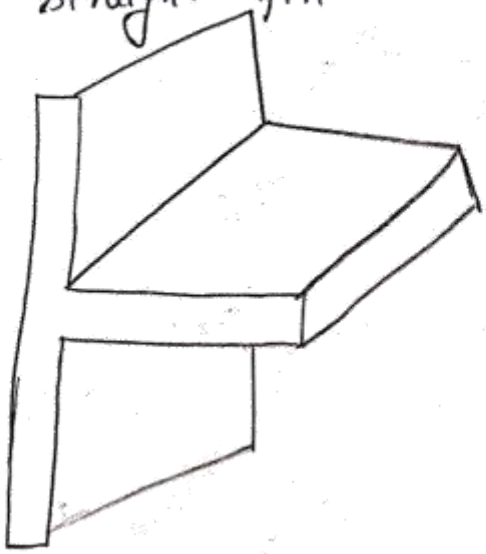
FINS

Fin: It is possible to increase the heat transfer rate by increasing the surface of heat transfer

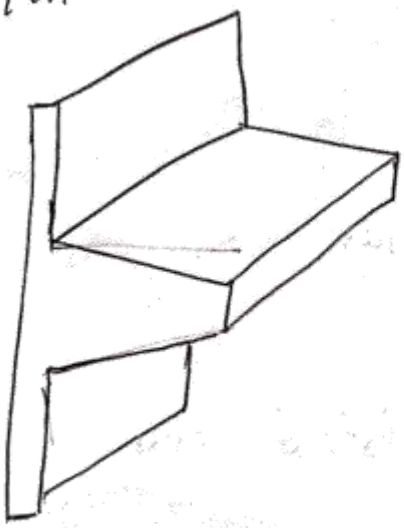
The surfaces used for increasing the heat transfer are called extended surfaces or fins.

Types of fins.

1. Uniform Straight fin



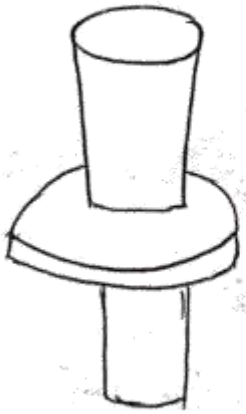
2. Tapered fin



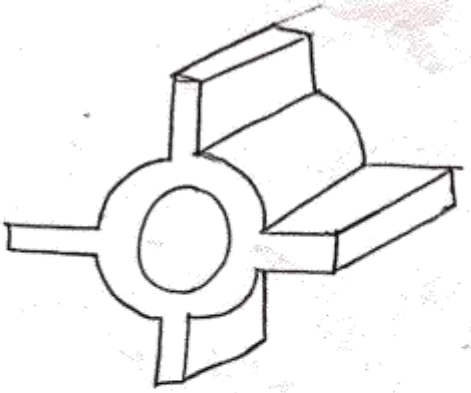
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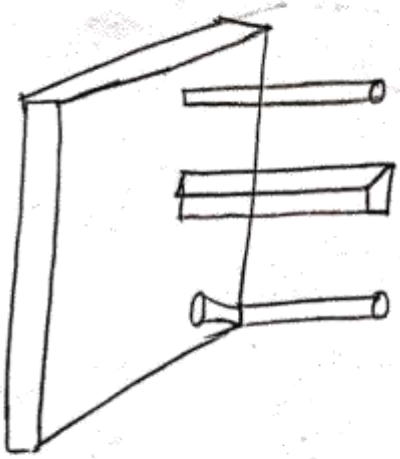
3. Annular fin



4. Spline shaft.



5. Pin fin



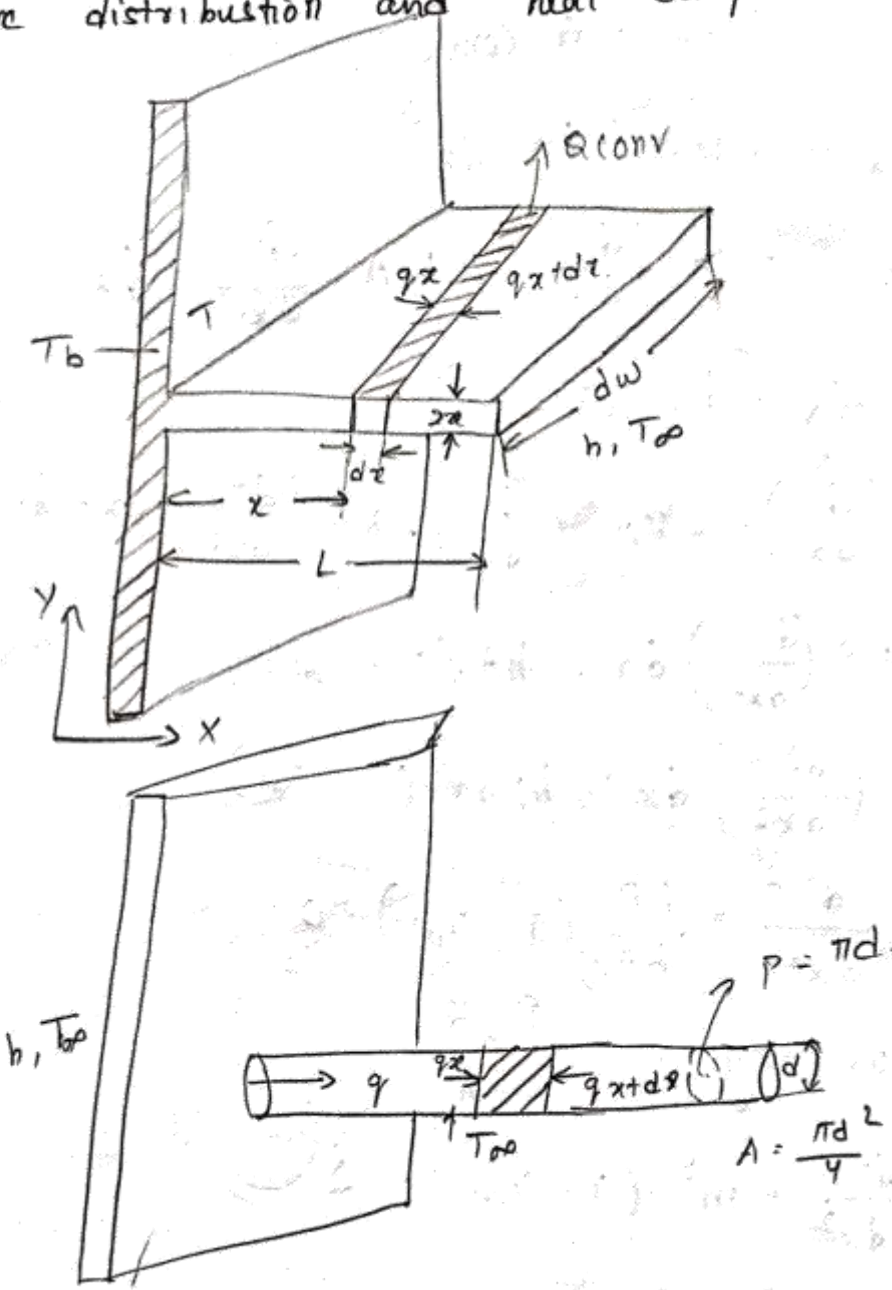
These are Commonly 3 types.

1. Infinitly long fin
2. Short fin with insulated end
3. Short fin with out insulated end

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Temperature distribution and heat desipation



1. Heat transfer occurce the rectangular fin and circular rod by Conduction.

2. from the surface of the fin heat transfer to the air by convection let as consider a small element of thickness dx which is at a distance of x from the base

By study state condition heat blancing equation :

Heat conducted in to element = heat conducted out of the element + heat convected to the surrounding

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$$Q_x = Q_{x+dx} + Q_{conv} \quad \text{—(1)}$$

$$Q_x = -KA\left(dT/dx\right)$$

$$Q_{x+dx} = -KA\left(\frac{dT}{dx}\right) - KA\left(\frac{d^2T}{dx^2}\right)dx$$

$$Q_{conv} = hA\,(T - T_\infty)$$

$$-KA\left(\frac{dT}{dx}\right) = -KA\left(\frac{dT}{dx}\right) - KA\left(\frac{d^2T}{dx^2}\right)dx + hA\,(T - T_\infty)$$

$$KA\left(\frac{d^2T}{dx^2}\right)dx = hA\,(T - T_\infty)$$

$$KA\left(\frac{d^2T}{dx^2}\right)dx = hP\,dx\,(T - T_\infty)$$

$$\frac{d^2T}{dx^2} - \frac{hP}{KA}(T - T_\infty) = 0$$

Let us say $m = \sqrt{\frac{hP}{KA}}$

$$\frac{d^2T}{dx^2} - m^2(T - T_\infty) = 0$$

$$\theta = T - T_\infty$$

$$\frac{d^2T}{dx^2} - m^2\theta = 0$$

it will show temperature is a function of x & m it is a function of second order linear differential equation. and the solution for the equation is

$$\theta = C_1 e^{-mx} + C_2 e^{mx}$$

This temperature distribution depends upon following conditions.

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Case - I :

Infinitly long fin

Boundary conditions at $x = 0$

$$T = T_b \;, \; x = \infty \;, \; T = T_\infty$$

we have

$$\theta = c_1 e^{-mx} + c_2 e^{mx}$$

$$T - T_\infty = c_1 e^{-mx} + c_2 e^{mx} \quad \text{— (1)}$$

Apply condition (1)

At $x = 0$, $T = T_b$

$$(T_b - T_\infty) = c_1 e^0 + c_2 e^0$$

$$T_b - T_\infty = c_1 + c_2 \quad \text{— (2)}$$

at $x = \infty$, $T = T_\infty$

$$(T_\infty - T_\infty) = c_1 e^{-m\infty} + c_2 e^{m\infty}$$

$$0 = c_1 e^{-m\infty} + c_2 e^{m\infty}$$

$$e^{m\infty} \neq 0$$

$$\therefore c_2 = 0$$

$$c_1 = T_b - T_\infty$$

we have c_1 & c_2

$$T_b - T_\infty = (T_b - T_\infty) e^{-mx} + 0$$

$$\therefore e^{-mx} = \frac{T - T_\infty}{T_b - T_\infty} \quad \text{— (3)}$$

T = Intermediate temperature

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T_∞ = Surrounding temperature

T_b = base temperature

x = distance.

$$m = \sqrt{\frac{hP}{kA}}$$

$h \longrightarrow$ Convective heat transfer coefficient

$P \longrightarrow$ perimeter

$k \longrightarrow$ Thermal Conductivity

$A \longrightarrow$ Area

for heat desipation from the fin would be obtain by integrating the heat lost by convection over the entire fin surface.

$$Q_{conv} = hA(T - T_\infty)$$

$$Q = \int_0^\infty h(P\,dx)(T - T_\infty)\,dx$$

from equation (3)

$$(T - T_\infty) = (T_b - T_\infty)\,e^{-mx}$$

$$= \int_0^\infty hP(T_b - T_\infty)\,e^{-mx}\,dx$$

$$= \left[hP(T_b - T_\infty) - \frac{1}{m}e^{-mx}\right]_0^\infty$$

$$= hP(T_b - T_\infty) - \frac{1}{m}(-1)$$

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$$\frac{hP}{m}(T_b - T_\infty)$$

$$= \frac{hP}{\sqrt{\frac{hP}{KA}}}(T_b - T_\infty)$$

$$= hP\sqrt{\frac{KA}{hP}}(T_b - T_\infty)$$

$$Q = \sqrt{hPKA}(T_b - T_\infty)$$

Short fin with insulated end:

Case-II:

Boundary conditions (for short fin)

at $x = 0$, $T = T_b$

at $x = L$, $T = T_\infty$, $\frac{dT}{dx} = 0$

We have.

$$\theta = C_1 e^{-mx} + C_2 e^{mx} \quad -(1)$$

$$(T - T_\infty) = C_1 e^{-mx} + C_2 e^{mx}$$

$$\frac{dT}{dx} = C_1 e^{-mx}(-m) + C_2 e^{mx} m$$

Apply boundary condition

$x = L$

$$\theta = C_1 e^{-mL}(-m) + C_2 e^{mL}(m)$$

$$C_1 = C_2 e^{2mL} \quad -(2)$$

at $x = 0$

$$(T_b - T_\infty) = C_1 e^0 + C_2 e^0$$

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$$(T_b - T_\infty) = C_1 + C_2$$

Substitute the equation (2)

$$C_2 e^{2mL} + C_2 = (T_b - T_\infty)$$

$$C_2 = \frac{T_b - T_\infty}{(C_2^{2mL} + 1)}$$

$$C_1 = C_2 e^{2mL}$$

$$= \left(\frac{T_b - T_\infty}{(e^{2mL} + 1)}\right) e^{2mL}$$

$$C_1 = \frac{T_b - T_\infty}{(e^{2mL} + 1)\, e^{-2mL}}$$

$$C_1 = \frac{T_b - T_\infty}{1 + e^{-2mL}}$$

Subtitute in equation (1)

$$T - T_\infty = \left(\frac{T_b - T_\infty}{1 + e^{-2mL}}\right) e^{-mx} + \frac{T_b - T_\infty}{(e^{2mL} + 1)}\, e^{mx}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = \left[\frac{e^{-mx}}{1 + e^{-2mL}} + \frac{e^{mx}}{1 + e^{2mL}}\right]$$

Multiply numerator and denominator by e^{mL} and e^{-mL}

$$\left(\frac{T - T_\infty}{T_b - T_\infty}\right) = \frac{e^{-mx}}{1 + e^{-2mL}} \times \frac{e^{mL}}{e^{mL}} + \frac{e^{mx}}{1 + e^{2mL}} \times \frac{e^{-mL}}{e^{-mL}}$$

$$= \frac{e^{-m(x-L)}}{e^{mL} + e^{-mL}} + \frac{e^{m(x-L)}}{e^{mL} + e^{-mL}}$$

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$$= \frac{e^{m(x-L)} + e^{-m(x-L)}}{e^{mL} + e^{-mL}}$$

$$= \frac{e^{m(L-x)} + e^{-m(L-x)}}{e^{Lm} + e^{-Lm}}$$

$$= \frac{\cosh(m(L-x))}{\cosh(mL)}$$

$$T - T_\infty = (T_b - T_\infty) \frac{\cosh m(L-x)}{\cosh(mL)}$$

$$\frac{dT}{dx} = (T_b - T_\infty)(-m) \frac{\sinh m(L-x)}{\cosh mL}$$

we have.

$$Q = -kA \frac{dT}{dx}$$

$$Q = -kA(T_b - T_\infty)(-m) \frac{\sinh m(L-x)}{\cosh mL}$$

$$= kAm(T_b - T_\infty) \frac{\sinh m(L-x)}{\cosh mL}$$

At $x = 0$

$$Q = kAm(T_b - T_\infty) \tanh mL$$

$$= kA\sqrt{hP/kA}(T_b - T_\infty) \tanh mL$$

$$= \sqrt{hPkA}(T_b - T_\infty) \tanh mL$$

Applications of fins:

The many applications are

1. Cooling the electronic components

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2. Cooling the motor cycle engines
3. Cooling the small capacity compressors
4. Cooling the transformers
5. Cooling the radiators and refrigerators.

fin efficiency:

It is the ratio of actual heat transfer in the fin maximum possible heat transfer

$$\gamma = \frac{Q_{fin}}{Q_{max}}$$

fin effectiveness:

It is the ratio of heat transfer with fin to t with out fin

$$\epsilon = \frac{Q_{with\ fin}}{Q_{without\ fin}}$$

Transient heat Conduction / un steady state heat Conduction

If the temperature of the body doesn't vary with time it is set to be steady state if there is an abrupt change in it surface temperature it attains steady state after some time during this period the temperature varys with the time and that body is said to be in an un steady state (or) transient state

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It occurs in cooling of I.C engines boiler Tubes heating and cooling of metal billets, rocket nozzles, electric irons etc.

It is divided into two types.

1, periodic heat flow

2. Non periodic heat flow

1. Periodic heat flow:-

Temperature varies an a regular basis

Ex: Surface of eirth during the period of 24 hrs

2, Non periodic heat flow:

Temperature at a any point with in the system varies non linearly with time.

Biot Number:

It is the ratio of internal heat conduction resistance to the surface heat convection resistance

$$Q_{conduction} = -kA\frac{dt}{L}$$

$$= \frac{dt}{L/kA}$$

$$Q_{convection} = hA\,dt$$

$$= \frac{dt}{1/hA}$$

$$= \frac{L/kA}{1/hA}$$

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$$= \frac{hA/L}{KA}$$

$$\text{Biot Number} = \frac{hL_c}{K}$$

$K \rightarrow$ Thermal Conductivity

$h \rightarrow$ heat transfer coefficient of convection

$L_c =$ Length (or) characteristic length

$$\frac{\text{volume}}{\text{Area}} = \frac{V}{A}$$

L for a slab:

$$L_c = \frac{V}{A} = \frac{A \times L}{2A}$$

$$L_c = L/2$$

$L =$ Thickness of the slab

Cylinder: $L_c = \frac{\pi R^2 L}{2\pi R L}$

$$L_c = R/2$$

for sphere:

$$\frac{4/3\ \pi R^3}{4\pi R^2}$$

$$= R/3$$

Cube:

$$= \frac{L^3}{6L^2}$$

$$= L/6$$

$L =$ side of the tube

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Lumped heat analysis:

The process in which the internal resistance is assumed as neglible in compared with the surface resistance is known as newton or newotonil heating on cooling process. In newotonil on cooling process the temperature is consider to be uniform at a given time such an analysis called lumped parameter analysis

Let as consider has a solid whose initial temperature is T_0, it is suddenly placed at ambient temperature T_∞ The transient response can be determine by following famula

Convective heat loss = Rate of change of internal energy

$$-hA(T-T_\infty) = \rho c_p V \frac{dT}{dt}$$

$$\frac{dT}{(T-T_\infty)} = \frac{-hA}{\rho c_p V} dt$$

Integrate both sides.

$$\int \frac{1}{(T-T_\infty)} = \frac{-hA}{\rho c_p V} \int dt$$

$$\ln(T-T_\infty) = \frac{-hA}{\rho c_p V} t + c_1 \quad \text{—①}$$

At $t=0$, $T=T_0$

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$$\ln(T_0 - T_\infty) = C_1$$

C_1 in substitute in equation (1)

$$\ln(T - T_\infty) = \frac{-hA}{\rho C_p V} t + \ln(T_0 - T_\infty)$$

$$\ln\left(\frac{T - T_\infty}{T_0 - T_\infty}\right) = \frac{-hA}{\rho C_p V} t$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left(\frac{-hAt}{\rho C_p V}\right)}$$

Where T_0 = initial temperature of the solid in K

T = intermediate temperature of the solid

T_∞ = Surface or final temperature of solid

A = Surface area of the body

ρ = density of the body

V = volume of the body

C_p = specific heat of the body

t = time.

Note: 1. In lumped parameter system Biot Number less than 0.1

$$Bi = \frac{hL_c}{k} V < 0.1$$

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