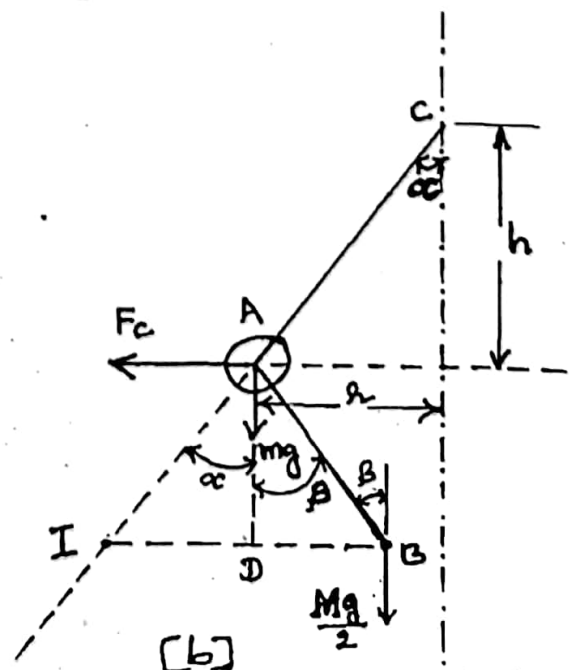
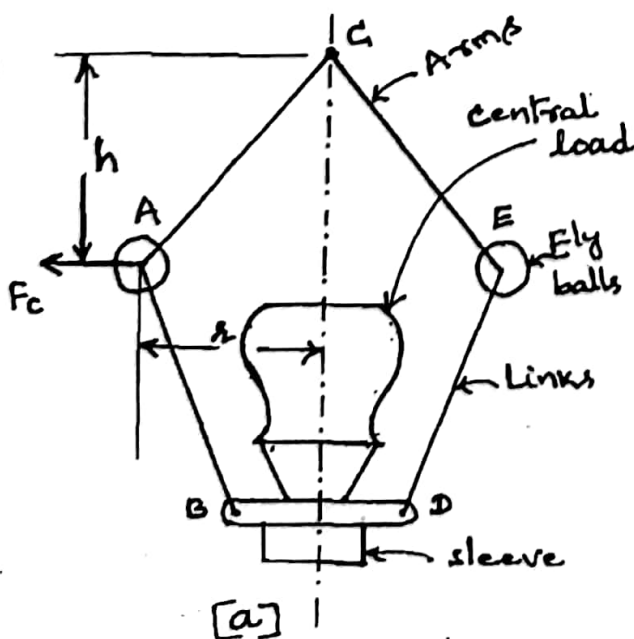


* What are the limitations of a simple watt governor? Why has this type of governor become obsolete?

(A) Height of watt governor $h = \frac{895}{N^2}$

From the above relation, it is seen that the height of governor is inversely proportional to the square of the speed. At very low speeds, the value of h obtained is very large and at very high speed, the value of h is very small, which is insufficient for the governor to operate the mechanism to give proper fuel supply. Due to this limitation, a simple watt governor is not in use now-a-days. It is only used for low speed applications.

Porter governor:- This is the modified shape of watt's governor with a central load attached to the sleeve as shown in figure. We can find expression for the height of the governor by instantaneous centre method. The instantaneous centre 'I' lies at the point of intersection of extension of line AC and perpendicular to spindle axis through B. AD is perpendicular to IB. It is assumed that the mass of links, arms etc is neglected in the analysis. For simplicity of mathematical expression half portion of the configuration is considered as shown in figure.



Let m = mass of each ball, kg
 M = mass of central load, kg
 r = Radius of rotation, metres
 h = Height of governor, metres
 N = Speed of balls, r.p.m
 $\omega = \frac{2\pi N}{60}$ rad/sec.

F_c = Centrifugal force acting on the balls, Newtons.

α = Angle of inclination of the upper arm to the vertical

β = Angle of inclination of the lower arm to the vertical

Taking moments about point I, we get

$$F_c \times AD = mg \times ID + \frac{Mg}{2} \times IB$$

$$F_c = mg \frac{ID}{AD} + \frac{Mg}{2} \cdot \frac{IB}{AD}$$

$$= mg \times \frac{ID}{AD} + \frac{Mg}{2} \left[\frac{ID}{AD} + \frac{DB}{AD} \right] \quad [\because IB = ID + DB]$$

$$= mg \times \tan \alpha + \frac{Mg}{2} [\tan \alpha + \tan \beta]$$

$$= \left[mg + \frac{Mg}{2} \left[1 + \frac{\tan \beta}{\tan \alpha} \right] \right] \tan \alpha \quad \left[\because \frac{ID}{AD} = \tan \alpha \right]$$

$$= \left[mg + \frac{Mg}{2} (1 + v) \right] \tan \alpha$$

$$\left[\frac{DB}{AD} = \tan \beta \right]$$

$$\frac{F_c}{\tan \alpha} = mg + \frac{Mg}{2} (1 + v)$$

$$\text{where } v = \frac{\tan \beta}{\tan \alpha}$$

$$(\because F_c = m\omega^2 r)$$

$$\frac{m\omega^2 r}{\frac{r}{h}} = mg + \frac{Mg}{2} (1 + v) \quad \left[\because \tan \alpha = \frac{r}{h} \right]$$

$$\frac{m\omega^2 r \times h}{r} = mg + \frac{Mg}{2} (1 + v)$$

$$\omega^2 = \frac{mg + \frac{Mg}{2} (1 + v)}{m} \times \frac{1}{h}$$

$$\omega^2 = \frac{m + \frac{M}{2} (1 + v)}{m} \left(\frac{g}{h} \right)$$

$$\left[\because \omega = \frac{2\pi N}{60} \right]$$

$$\left[\frac{2\pi N}{60} \right]^2 = \frac{m + \frac{M}{2} (1 + v)}{m} \left(\frac{g}{h} \right)$$

$$N^2 = \frac{m + \frac{M}{2} (1 + v)}{m} \frac{9.81 \times 60^2}{(2\pi)^2 \cdot h}$$

$$N^2 = \frac{m + \frac{M}{2} (1 + v)}{m} \times \frac{895}{h}$$

$$h = \frac{m + \frac{M}{2} (1 + v)}{m} \frac{895}{N^2}$$

—:6:—

When length of arms is equal to the length of links and points C and B lie on the same vertical line, then

$$\tan \alpha = \tan \beta$$

$$\phi = \frac{\tan \beta}{\tan \alpha} = 1$$

$$N^2 = \frac{m + \frac{M}{2}(1+1)}{m} \frac{895}{h} \Rightarrow \frac{m + \frac{M}{2}}{m} \left[\frac{895}{h} \right]$$

$$N^2 = \frac{m + M}{m} \frac{895}{h}$$

Since the sleeve moves up and down on the spindle, there will be some friction between them.

When the sleeve moves upwards, friction opposes its movement and acts in the downward direction and so on.

$$N^2 = \frac{mg + \left[\frac{Mg \pm F}{2} \right] (1+\phi) \frac{895}{h}}{m \cdot g} \quad (or) \quad N^2 = \frac{w + \left[\frac{W \pm F}{2} \right] (1+\phi)}{w}$$

($\because \phi = 1$)

$$N^2 = \frac{mg + \frac{Mg \pm F}{2}}{mg} \frac{895}{h}$$

$$= \frac{mg + (Mg \pm F)}{mg} \frac{895}{h}$$

where F = Frictional force, Newtons

+ve sign is used when the speed increases [Maximum speed]

-ve sign " " " " " " decreases [Minimum speed]

$$\text{Watt's governor } h = \frac{895}{N^2}$$

$$\text{Porter governor } h = \left[\frac{m+M}{m} \right] \frac{895}{N^2} \quad (\because \phi = 1)$$

Comparison of this equation with Watt's governor indicates that height of the governor is increased by $\left[\frac{m+M}{m} \right]$ times
or
 $\left[1 + \frac{M}{m} \right]$ times

① The lengths of the upper and lower arms of a porter governor are 200mm and 250mm respectively. Both the arms are pivoted on the axis of the rotation. The central load is 150N, the weight of each ball is 20N and the friction of the sleeve together with the resistance of the operating gear is equivalent to a force of 30N at the sleeve. At the limiting inclinations of the upper arms to the vertical are 30° and 40° . Determine the range of speed of the governor.

Given data:-

$$W = Mg = 150 \text{ N}$$

$$w = mg = 20 \text{ N}$$

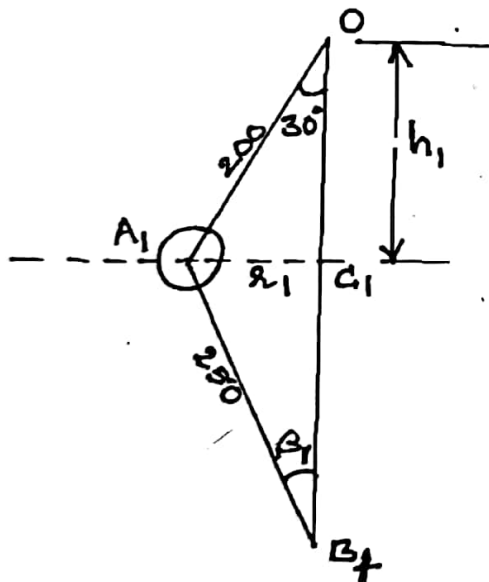
$$F = 30 \text{ N}$$

$$\alpha_1 = 30^\circ$$

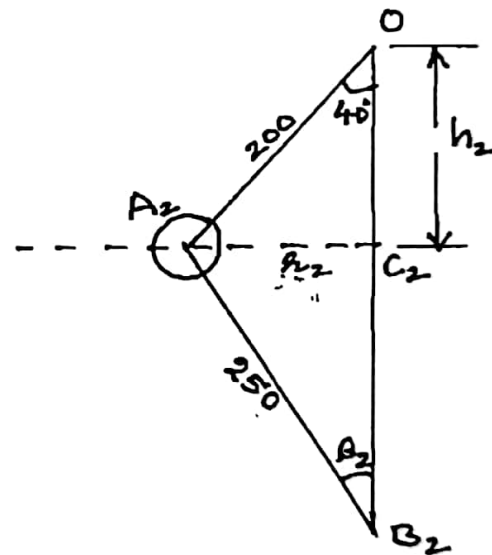
$$\alpha_2 = 40^\circ$$

$$\left[\begin{array}{l} m \rightarrow \text{kg} \\ w \rightarrow \text{newtons} \end{array} \right]$$

$$(\because 1 \text{ kg} = 9.81 \text{ N})$$



Minimum position



Maximum position

For minimum position:-

$$x_1 = A_1 O \sin 30^\circ = 200 \sin 30^\circ \Rightarrow 100 \text{ mm} \Rightarrow 0.1 \text{ m}$$

$$h_1 = A_1 O \cos 30^\circ = 200 \cos 30^\circ \Rightarrow 173.2 \Rightarrow 0.1732 \text{ m}$$

$$B_1 C_1 = \sqrt{250^2 - 100^2} \Rightarrow 229.12 \text{ mm} \Rightarrow 0.229 \text{ m}$$

$$\therefore \tan \beta_1 = \frac{0.1}{0.229} \Rightarrow 0.4366$$

$$\tan \alpha_1 = \tan 30^\circ \Rightarrow 0.5774$$

$$\therefore v_1 = \frac{\tan \beta_1}{\tan \alpha_1} \Rightarrow \frac{0.4366}{0.5774} \Rightarrow 0.756$$

$$\text{minimum speed is given by } N_1^2 = \frac{W + \left[\frac{W - F}{2} \right] (1 + v_1)}{w} \frac{895}{h_1}$$

$$N_1^v = \frac{20 + \left[\frac{150 - 30}{2} \right] [1 + 0.756]}{20} \times \frac{895}{0.1732}$$

$$N_1^v = 32389.49$$

$$N_1 = \sqrt{32389.49} \Rightarrow N_1 = 179.97 \Rightarrow N_1 \approx 180 \text{ rpm}$$

For maximum position:-

$$h_2 = A_2 O \sin 40^\circ \Rightarrow 200 \sin 40^\circ \Rightarrow 128.5 \text{ mm} \Rightarrow 0.1285 \text{ m}$$

$$h_2 = A_2 O \cos 40^\circ \Rightarrow 200 \cos 40^\circ \Rightarrow 153.2 \text{ mm} \Rightarrow 0.1532 \text{ m}$$

$$B_2 C_2 = \sqrt{250^2 - 128.5^2}$$

$$= 214.44 \text{ mm} \Rightarrow 0.2144 \text{ m}$$

$$\therefore \tan \beta_2 = \frac{0.1285}{0.2144} \Rightarrow 0.599$$

$$\tan \alpha_2 \Rightarrow \tan 40^\circ \Rightarrow 0.839$$

$$\epsilon_2 = \frac{\tan \beta_2}{\tan \alpha_2} \Rightarrow \frac{0.599}{0.839} \Rightarrow 0.714$$

Maximum speed is given by

$$N_2^v = \frac{\omega + \left[\frac{W + F}{2} \right] [1 + \epsilon_2]}{\omega} \times \frac{895}{h_2}$$

$$= \frac{20 + \left[\frac{150 + 30}{2} \right] [1 + 0.714]}{20} \times \frac{895}{0.1532}$$

$$N_2^v = 50901.66 \Rightarrow N_2 = \sqrt{50901.66} \Rightarrow N_2 = 225.61 \text{ rpm}$$

$$\therefore \text{Range of speed} = N_2 - N_1$$

$$= 225.61 - 180$$

$$= 45.61 \text{ rpm}$$

Q2) On a porter governor, the arms and links are each 25cm long and intersect on the main axis. Each ball weights 45N and the central load is 200N. The sleeve is in its lowest position when the arms are inclined at 30° to the axis. The lift of sleeve is 5cm. What is the force of friction at the sleeve, if the speed at ascent from the lowest position is equal to the speed at the beginning of descent from the highest position? What is then the range of speed, all other things remaining the same?

Given data:- length of arms of links $l = 25 \text{ cm}$
 $= 0.25 \text{ m}$

∴ q:-

$w = \text{Each ball weight} = 45 \text{ N}$

$W = \text{central load} = 200 \text{ N}$

$\alpha = 30^\circ$

Libt of sleeve = $5 \text{ cm} = 0.05 \text{ m}$

At the lowest position of the sleeve :-

$$\angle BAC = \angle BCA = \alpha = \beta = 30^\circ$$

$$\text{i.e. } \phi = \frac{\tan \beta}{\tan \alpha}$$
$$= \frac{\tan 30}{\tan 30}$$

$$\phi = 1$$

In $\triangle BAE$

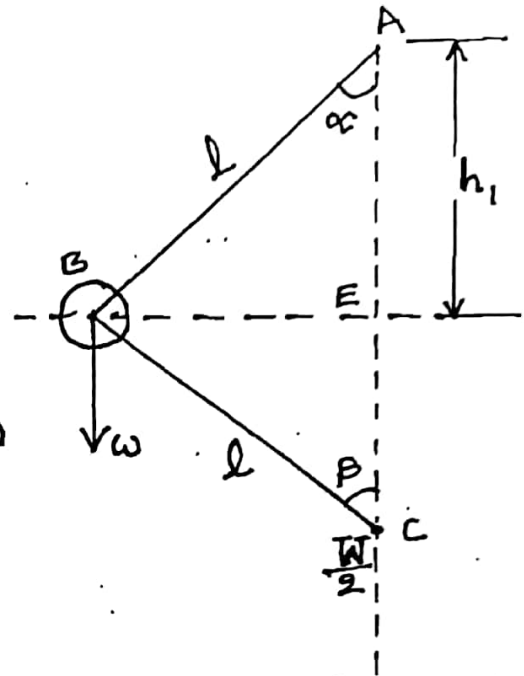
$$h_1 = AB \cos 30^\circ$$

$$= 0.25 \times \cos 30^\circ \Rightarrow 0.2165 \text{ m}$$

$$\text{Also } x = h_1 - h_2$$

$$0.05 = 0.2165 - h_2$$

$$[\text{Top position}] h_2 = 0.1665 \text{ m}$$



Speed of the governor at the instant the sleeve begins to rise i.e. due to lower speed.

$$N_1^2 = \frac{895}{h_1} \left[\frac{w + \left[\frac{W+F}{2} \right] (1+\phi)}{w} \right] \quad \text{--- (1)}$$

Speed of the governor at the instant the sleeve begins to fall for higher speed.

$$N_2^2 = \frac{895}{h_2} \left[\frac{w + \left[\frac{W-F}{2} \right] (1+\phi)}{w} \right] \quad \text{--- (2)}$$

Where $F = \text{Frictional force (in N)}$

But by given data

$$N_1 = N_2$$

$$\frac{895}{h_1} \left[\frac{w + \frac{W+F}{2} (1+\phi)}{w} \right] = \frac{895}{h_2} \left[\frac{w + \frac{W-F}{2} (1+\phi)}{w} \right]$$

$$w + \frac{W+F}{2} (1+\phi) = \frac{h_1}{h_2} \left[w + \frac{W-F}{2} (1+\phi) \right]$$

$$w + \frac{W+F}{2} (2) = \frac{h_1}{h_2} \left[w + \frac{W-F}{2} (2) \right]$$

$$\omega + W + F = \frac{h_1}{h_2} [\omega + W - F]$$

$$45 + 200 + F = \frac{0.2165}{0.1665} [45 + 200 - F]$$

$$245 + F = \frac{0.2165}{0.1665} [45 + 200 - F]$$

$$245 + F = 318.57 - 1.3F$$

$$F + 1.3F = 318.57 - 245$$

$$2.3F = 73.57$$

$$\text{Frictional force } F = 31.98 \text{ N}$$

$$\therefore \text{Maximum speed } N_2^r = \frac{895}{h_2} \left[\omega + \frac{W + F}{2} (1 + \mu) \right]$$

$$N_2^r = \frac{895}{0.1665} \left[45 + \frac{200 + 31.98}{2} (1 + 1) \right]$$

$$N_2 = 181.89 \text{ rpm}$$

$$\text{minimum speed } N_1^r = \frac{895}{0.2165} \left[45 + \frac{200 - 31.98}{2} (1 + 1) \right]$$

$$N_1^r = 19569.19$$

$$N_1 = \sqrt{19569.19}$$

$$N_1 = 139.88 \text{ rpm}$$

$$\text{Range of speed} = N_2 - N_1$$

$$= 181.89 - 139.88$$

$$= 42.01 \text{ rpm}$$

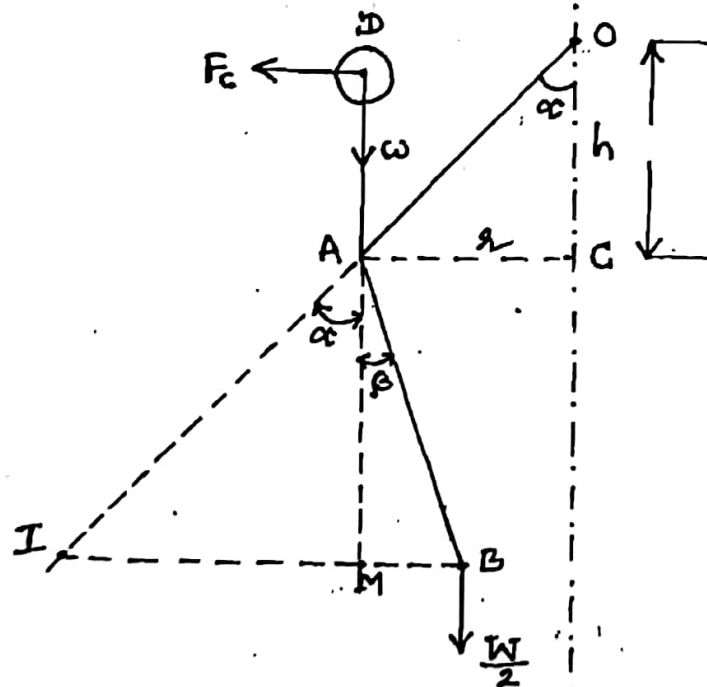
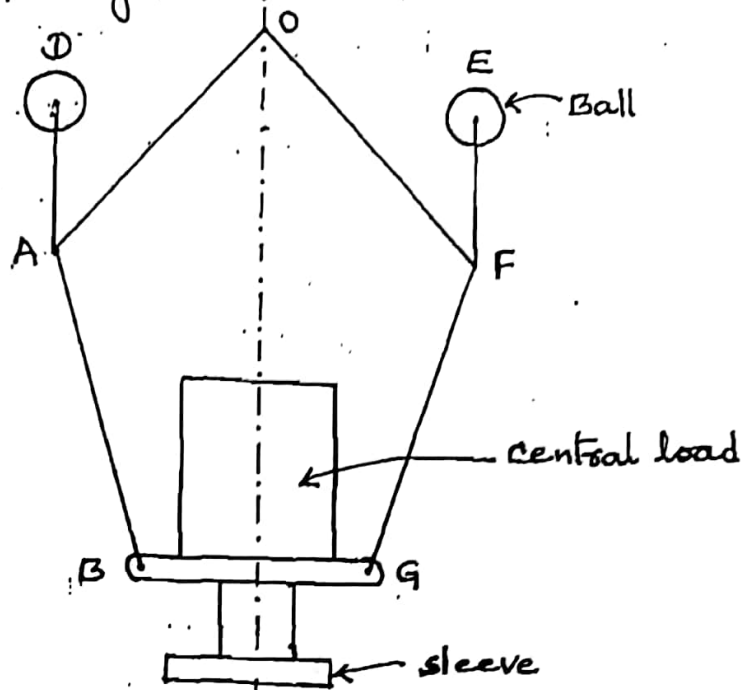
4. *** Proell governor:-**

Q3. *** Derive an expression for the height of proell governor?**

Line diagram of a proell governor as shown in figure, which is similar to porter governor having a heavy central load at the sleeve. But it differs from porter governor in the arrangement of balls. The balls are carried on the extension of the lower arms instead of at the junction of the upper and lower arms. An increase in the speed of rotation increases the radius of rotation and raise the sleeve, thus reducing the amount of energy supplied to the engine. Conversely a decrease in speed results in decrease in radius of

—:||:—

rotation, thus lowering the sleeve and increasing the amount of energy supplied to the engine.



Taking moment of all forces about I, we get

$$F_c \times DM = w \times IM + \frac{W}{2} \times IB$$

$$= mg \times IM + \frac{Mg}{2} \times IB$$

Multiplying and dividing both sides by AM

$$F_c = \frac{AM}{DM} \left[mg \times \frac{IM}{AM} + \frac{Mg}{2} \times \frac{IB}{AM} \right] \quad [\because IB = IM + MB]$$

$$= \frac{AM}{DM} \left[mg \times \frac{IM}{AM} + \frac{Mg}{2} \left[\frac{IM}{AM} + \frac{MB}{AM} \right] \right]$$

$$\left[\therefore \tan \alpha = \frac{IM}{AM} ; \tan \beta = \frac{BM}{AM} \right]$$

$$= \frac{AM}{DM} \left[mg + \frac{Mg}{2} (\tan \alpha + \tan \beta) \right]$$

$$F_c = \frac{AM}{DM} \tan \alpha \left[mg + \frac{Mg}{2} \left(1 + \frac{\tan \beta}{\tan \alpha} \right) \right]$$

$$\therefore F_c = m \omega^2 r$$

$$\Delta OAC, \tan \alpha = \frac{r}{h}$$

$$\frac{\tan \beta}{\tan \alpha} = v$$

$$m \omega^2 r = \frac{AM}{DM} \times \frac{r}{h} \left[mg + \frac{Mg}{2} (1+v) \right]$$

$$\omega^2 = \frac{AM}{DM} \times \frac{g}{h} \left[\frac{m + \frac{M}{2}(1+v)}{m} \right]$$

$$\left[\therefore \omega = \frac{2\pi N}{60} \right]$$

$$\left[\frac{2\pi N}{60} \right]^2 = \frac{AM}{DM} \times \frac{g}{h} \left[\frac{m + \frac{M}{2}(1+v)}{m} \right]$$

$$N^2 = \frac{AM}{DM} \left[\frac{m + \frac{M}{2}(1+v)}{m} \right] \frac{g}{h} \times \frac{60^2}{(2\pi)^2}$$

$$[\therefore g = 9.81]$$

$$N^2 = \frac{AM}{DM} \left[\frac{m + \frac{M}{2}(1+v)}{m} \right] \frac{895}{h}$$

21 ** what are the differences between porter and proell governor? why is the speed range of proell governor is less than that of a similar porter type?

A). In porter governor, the balls are fixed to the intersection point of lower and upper arm. where as in proell governor, the balls are fixed to the extension of the lower arm.

The upper arm for a porter governor is pivoted to the axis of governor where as the upper arms of the proell governor may be pivoted to the axis of governor & away from the axis.

$$\text{The speed of porter governor } N^2 = \frac{\left[m + \frac{M}{2} \right] (1+v)}{m} \frac{895}{h} \rightarrow \text{A}$$

The speed of proell governor $N^r = \frac{AM}{DM} \left[\frac{m + \frac{M}{2}(1+u)}{m} \right] \frac{895}{h} \rightarrow (2)$

By comparing equations (1) and (2) we see that the speed range of porter governor is more than that of the proell governor, thus, we can say that the speed range of proell governor is less than that of the porter governor.

- ① calculate the minimum speed of a proell governor, which has equal arms each 200mm and are pivoted on the axis of rotation. The mass of the each ball is 4kg and the central mass on the sleeve is 20kg. The extension arms of the lower links are each 60mm long and parallel to the axis when the minimum radius of the ball is 100mm.

Given data:- $OA = AB = 200\text{mm}$

$AD = 60\text{mm}$

$m = 4\text{kg}$

$M = 20\text{kg}$

$AC = 100\text{mm}$

From Figure:

$$h_1 = \sqrt{200^2 - 100^2}$$

$$= 173.2\text{mm}$$

$$h_1 = 0.1732\text{m}$$

Let $AM = CB$

$$= \sqrt{AB^2 - AC^2}$$

$$= \sqrt{200^2 - 100^2}$$

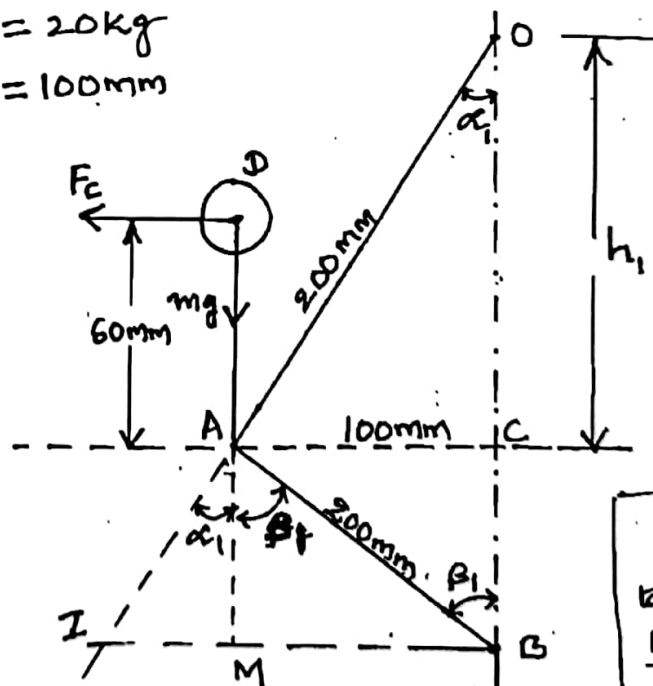
$$= 173.2\text{mm}$$

$$= 0.1732\text{m}$$

$$\therefore DM = 173.2 + 60$$

$$= 233.2\text{mm}$$

$$= 0.2332\text{m}$$



Since $AO = AB$
 $\tan \alpha_1 = \tan \beta_1$
 $\frac{\tan \beta_1}{\tan \alpha_1} = 1$
 $\beta = 1$

$\therefore$ minimum speed of the governor

$$N_1^r = \frac{AM}{DM} \left[\frac{m + \frac{M}{2}(1+u)}{m} \right] \frac{895}{h_1}$$

$$N_1^r = \frac{0.1732}{0.2332} \left[\frac{4 + \frac{20(1+1)}{2}}{4} \right] \frac{895}{0.1732}$$

$$N_1 = \sqrt{23027.44} \Rightarrow N_1 = 151.74\text{rpm}$$

— : 14 : —

Refer R.S. Khurmi
 Problems

501

$$AD = 150 - 40 \Rightarrow 110 \text{ mm}$$

$$\sin \alpha = \frac{AC}{AO}$$

$$\sin \alpha = \frac{150}{300}$$

$$\alpha = \sin^{-1} \left[\frac{150}{300} \right]$$

$$\alpha = 30^\circ$$

$$\sin \beta = \frac{AD}{AB}$$

$$= \frac{110}{250}$$

$$\beta = \sin^{-1} \left[\frac{110}{250} \right]$$

$$\beta \approx 26.1^\circ$$

$$AO^2 = AC^2 + CO^2$$

$$300^2 = 150^2 + CO^2$$

$$CO^2 = 300^2 - 150^2$$

$$CO = \sqrt{300^2 - 150^2}$$

$$h = 260 \text{ mm} = 0.26 \text{ m}$$

$$v = \frac{\tan \beta}{\tan \alpha}$$

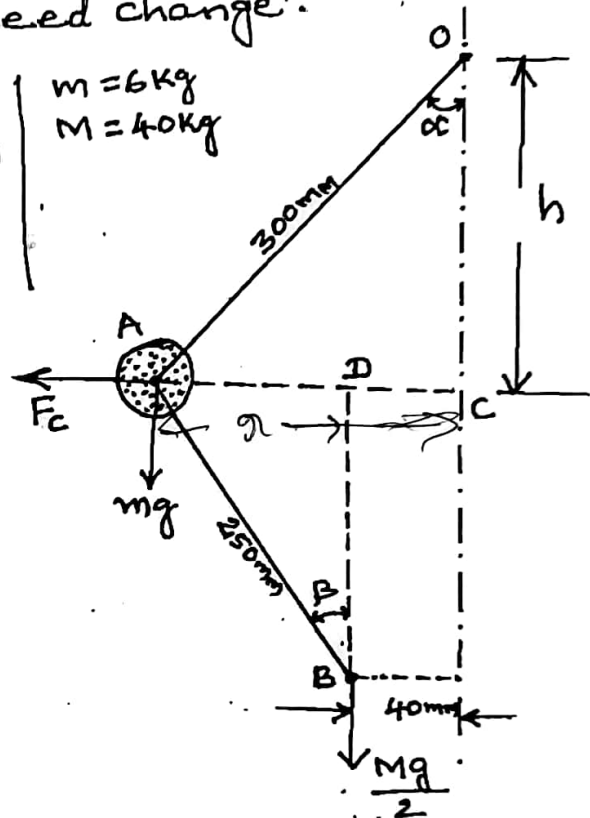
$$b = \frac{\tan 26.1^\circ}{\tan 30^\circ} \Rightarrow 0.849$$

$$\text{speed of the governor } N^2 = \frac{m + \frac{M}{2}(1+e)}{\tan 30^\circ} \left[\frac{895}{h} \right]$$

$$N^2 = \frac{6 + \frac{40}{2}(1 + 0.849)}{0.26} \left[\frac{895}{0.26} \right]^m \Rightarrow N = \sqrt{45874.5}$$

$$N = 214.2 \text{ rpm}$$

$$\Rightarrow: 14(1) = \dots$$



If the speed increased by 1%, the radius of rotation increases.

In order to maintain the same radius of rotation, the central mass M is to be increased to M' .

$$\begin{aligned}\text{Now speed } N' &= N \times 1.01 \\ &= 214.2 \times 1.01 \\ N' &= 216.34 \text{ rpm}\end{aligned}$$

$$\begin{aligned}&= 100 + 1\% \\ &= 100 + 0.01 \\ &= 1.01\end{aligned}$$

$$\text{Now } (N')^2 = \frac{m + \frac{M'}{2}(1+v)}{m} \left[\frac{895}{h} \right]$$

$$(216.34)^2 = \frac{6 + \frac{M'}{2}(1+0.849)}{6} \left[\frac{895}{0.26} \right]$$

$$46802.9 = \frac{6 + M'(0.9245)}{6} \left[\frac{895}{0.26} \right]$$

$$46802.9 = [6 + M'(0.9245)] [573.7]$$

$$46802.9 = 3442.3 + 530.4 M'$$

$$M' = \frac{46802.9 - 3442.3}{530.4}$$

$$M' = 81.75 \text{ kg}$$

$$\therefore \text{Additional mass required} = M' - M$$

$$= 81.75 - 40$$

$$(\text{or}) \text{ effort} = 41.75 \text{ kg}$$

$$\text{Total power} = \text{Total mass} \left[\frac{\text{Total effort}}{g} \right] \times h$$

$$= 81.75 \times 0.26$$

$$= 21.23 \text{ kg-m}$$

$$= 21.23 \times 9.81 \text{ N-m}$$

$$P = 208.5 \text{ N-m}$$

$$(\because 1 \text{ kg} = 9.81 \text{ N})$$

① The mass of each ball of a proell governor is 3 kg and the weight on the sleeve is 20 kg each arm is 220 mm long and the pivots of the upper and the lower arms are 20 mm from the axis. For the mid position of the sleeve the extension links of the lower arms are vertical, the height of the governor 180 mm and the speed 150 rpm. Determine the lengths of the extension links and the tension in the upper arms.

Given data:-

$$m = 3 \text{ kg}$$

$$M = 20 \text{ kg}$$

$$h = 180 \text{ mm} \Rightarrow 0.180 \text{ m}$$

~~$$h = 180 \text{ mm}$$~~

$$N = 150 \text{ rpm}$$

From ΔAOC

$$\cos \alpha = \frac{h}{220}$$

$$\cos \alpha = \frac{180}{220}$$

$$\alpha = \cos^{-1} \left(\frac{180}{220} \right)$$

$$\alpha = 35^\circ$$

$$\sin \alpha = \frac{r}{220}$$

$$r = 220 \sin(35^\circ)$$

$$r = 126 \text{ mm}$$

From ΔABC

$$\sin \beta = \frac{r}{220}$$

$$\beta = \sin^{-1} \left(\frac{126}{220} \right)$$

$$\beta = 35^\circ$$

$$v = \frac{\tan \beta}{\tan \alpha}$$

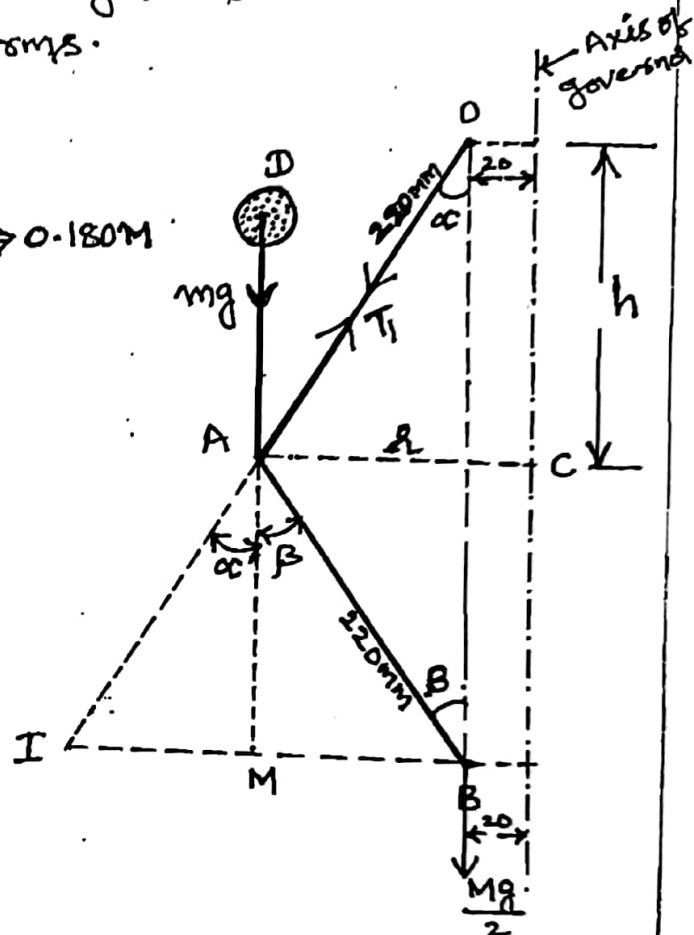
$$v = \frac{\tan 35^\circ}{\tan 35^\circ} \Rightarrow v = 1$$

from ΔMAB

$$\cos \beta = \frac{AM}{AB}$$

$$AM = 220 \cos 35^\circ$$

$$AM = 180.2 \text{ mm} \Rightarrow 0.1802 \text{ m}$$



speed of the proell governor

$$N^2 = \frac{AM}{DM} \left[\frac{m + \frac{M}{2}(1+e)}{m} \right] \left[\frac{895}{h} \right]$$

$$150^2 = \frac{0.1802}{DM} \left[\frac{3 + \frac{20}{2}(1+1)}{3} \right] \frac{895}{0.180}$$

$$DM = \frac{0.1802}{150^2} \left[\frac{3 + \frac{20(2)}{2}}{3} \right] \frac{895}{0.180}$$

$$DM = 0.3053 \text{ m}$$

$$DM = 305.3 \text{ mm}$$

from
figure

$$AD = DM - AM$$

$$= 305.3 - 180.2$$

$$\text{Length of the extension Link AD} = \underline{\underline{125.1 \text{ mm}}}$$

(b) Tension in upper Arm :-

$$\alpha = 35^\circ$$

Resolving the vertical forces

$$\Sigma f_v = 0$$

$$T \cos \alpha - mg - \frac{Mg}{2} = 0$$

$$T \cos 35^\circ = 3(9.81) + \frac{20(9.81)}{2}$$

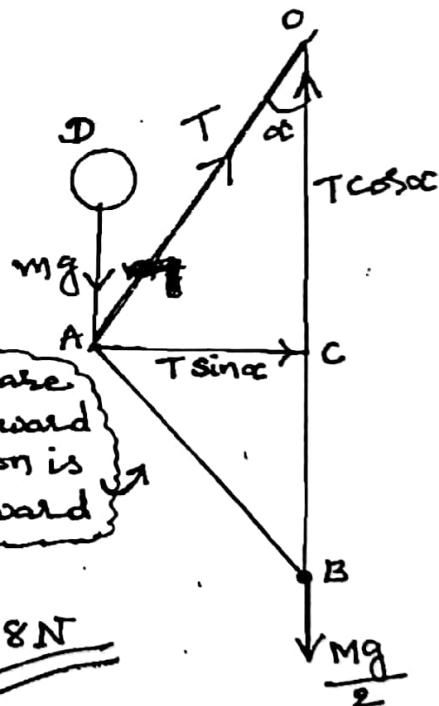
$$T \cos 35^\circ = 127.53$$

$$T = \frac{127.53}{\cos 35^\circ}$$

$$T = 155.68 \text{ N}$$

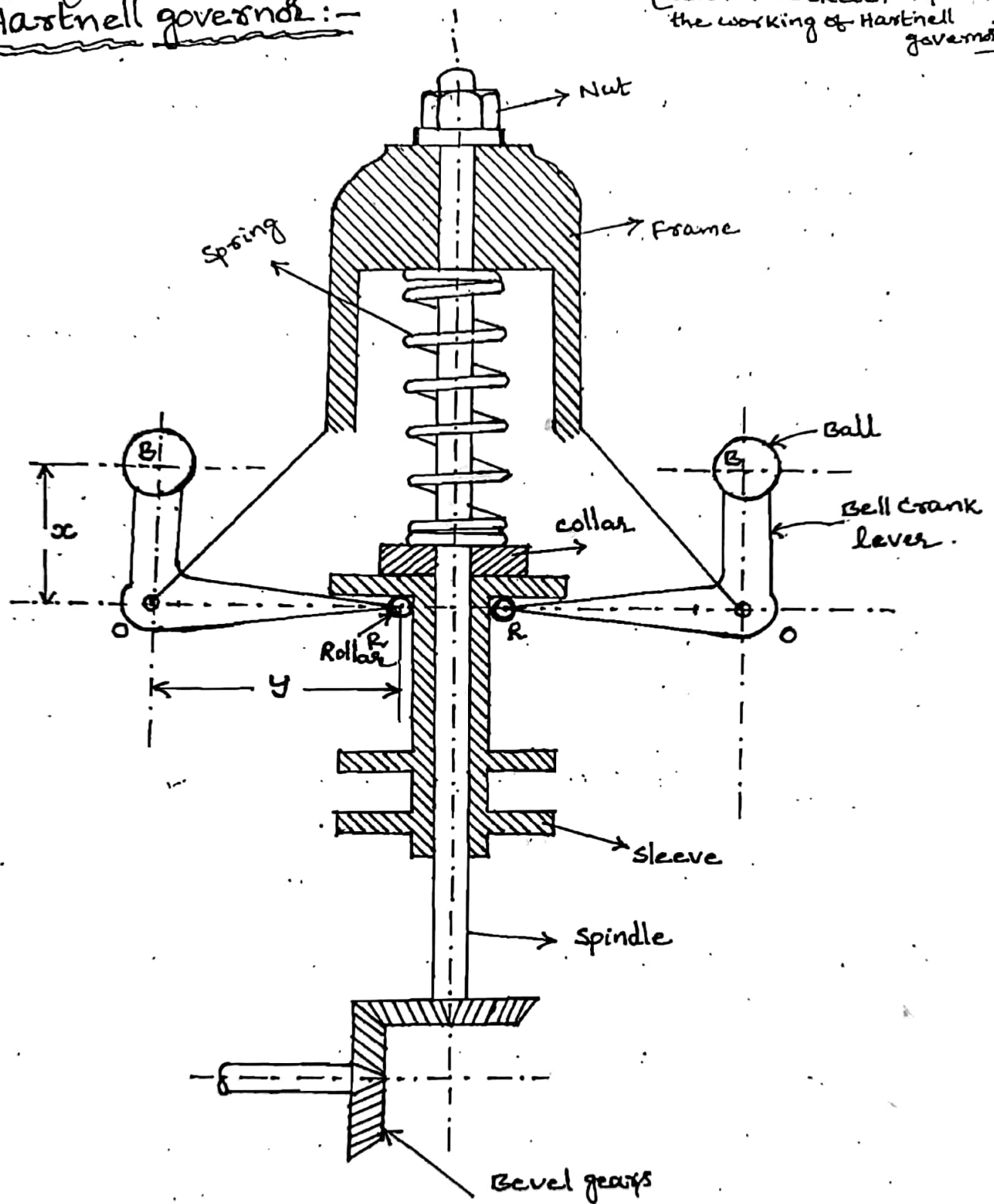
$$\text{Tension in upper Arm } T = \underline{\underline{155.68 \text{ N}}}$$

All loads are
acted downward
so the tension is
taken upward



Hartnell governor :-

With neat sketch explain the working of Hartnell governor



* (A hartnell governor is a spring loaded governor as shown in figure. It consists of two bell crank levers pivoted at the points O, O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal

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downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.

Let

m = mass of each ball in kg

M = mass of sleeve in kg

r_1 = minimum radius of rotation in metres,

ω_1 = Angular speed of the governor at minimum radius in rad/sec.

ω_2 = Angular speed of the governor at maximum radius in rad/sec.

S_1 = Spring force exerted on the sleeve at ω_1 in newtons

S_2 = Spring force exerted on the sleeve at ω_2 in newtons

F_{c1} = Centrifugal force at $\omega_1 = m r_1 \omega_1^2 \rightarrow$ newtons

$F_{c2} = \quad \quad \quad \omega_2 = m r_2 \omega_2^2 \rightarrow$ newtons

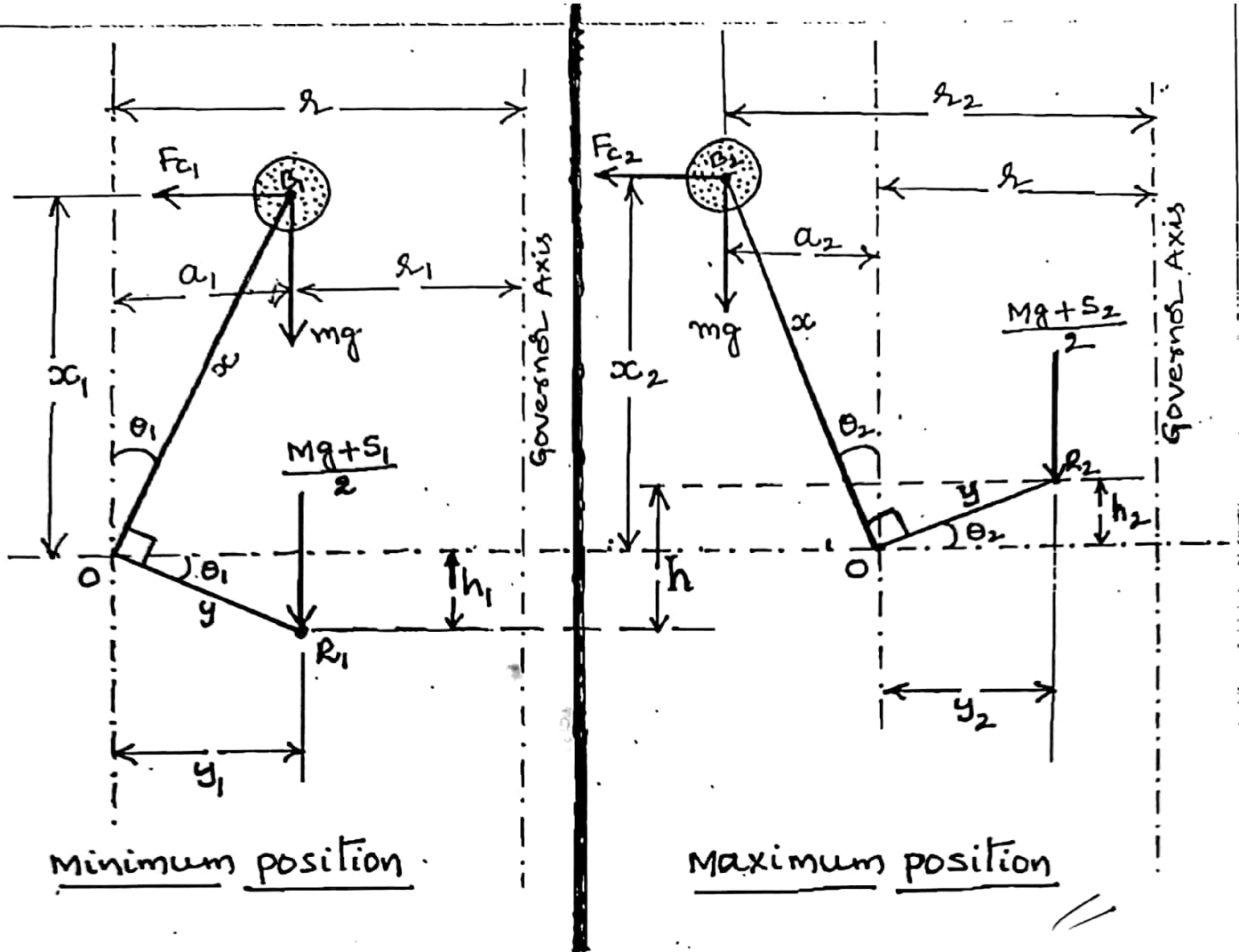
S = Stiffness of the spring or the force required to compress the spring by one mm.

x = Length of the vertical or ball arm of the lever in metres.

y = Length of the horizontal or sleeve arm of the lever in metres, and

h = Distance of fulcrum O from the governor axis [when mid-position]

Consider the forces acting at one bell crank lever. The minimum and maximum position as shown in figure. Let h be the compression of the spring when the radius of rotation changes from r_1 to r_2 .



For the minimum position :- When the radius of rotation changes from r to r_1 as shown in figure. The tensile of the spring & the down of sleeve h_1 is given by

$$\frac{h_1}{y} = \frac{a_1}{x}$$

$$\frac{h_1}{y} = \frac{(r - r_1)}{x} \quad \text{--- ①}$$

$\therefore \sin \theta_1 = \frac{h_1}{y}$ and $\sin \theta_1 = \frac{a_1}{x}$
From Figure

For the maximum position :- When the radius of rotation changes from r to r_2 as shown in figure. The compression of the spring & lift of sleeve h_2 is given by

$$\frac{h_2}{y} = \frac{a_2}{x}$$

$$\frac{h_2}{y} = \frac{r_2 - r}{x} \quad \text{--- ②}$$

$\therefore \sin \theta_2 = \frac{h_2}{y}$ and $\sin \theta_2 = \frac{a_2}{x}$
From Figure

Adding equation ① and ②

$$\frac{h_1 + h_2}{y} = \frac{r - r_1 + r_2 - r}{x}$$

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad [\because h = h_1 + h_2]$$

$$\frac{h}{y} = \frac{r_2 - r_1}{x}$$

Libt of the sleeve

$$h = r_2 - r_1 \left[\frac{y}{x} \right] \quad (3)$$

Now for minimum position, taking moments about point 'o' we get

$$\frac{Mg + S_1}{2} \times y_1 = F_{c1} \times x_1 - mg \times a_1$$

$$Mg + S_1 = \frac{2}{y_1} [F_{c1} \times x_1 - mg \times a_1] \quad (4)$$

Again for maximum position, taking moments about point 'o' we get

$$\frac{Mg + S_2}{2} \times y_2 = [F_{c2} \times x_2 + mg \times a_2]$$

$$Mg + S_2 = \frac{2}{y_2} [F_{c2} \times x_2 + mg \times a_2] \quad (5)$$

Subtracting equation (4) from equation (5)

$$S_2 - S_1 = \frac{2}{y_2} [F_{c2} \times x_2 + mg \times a_2] - \frac{2}{y_1} [F_{c1} \times x_1 - mg \times a_1] \quad (6)$$

We know that

$$S_2 - S_1 = h \cdot S$$

$$S = \frac{S_2 - S_1}{h}$$

$$[\because \text{from equation (3)}]$$

$$h = (r_2 - r_1) \left[\frac{y}{x} \right]$$

$$= \frac{S_2 - S_1}{(r_2 - r_1) \frac{y}{x}} \Rightarrow \frac{(S_2 - S_1)}{(r_2 - r_1)} \frac{x}{y}$$

$$S = \frac{(S_2 - S_1)}{(r_2 - r_1)} \frac{x}{y} \quad (7)$$

Neglecting the obliquity effect of the arms [i.e. $x_1 = x_2 = x$ and $y_1 = y_2 = y$] and the moment due to weight of the balls [i.e. mg]

From equation (6)

$$S_2 - S_1 = \frac{2}{y} [F_{c2} \times x + 0] - \frac{2}{y} [F_{c1} \times x - 0]$$

$$S_2 - S_1 = 2 [F_{c2} - F_{c1}] \frac{x}{y} \quad (8)$$

Substitute equation (8) in equation (7).

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$$S = \left[\frac{s_2 - s_1}{r_2 - r_1} \right] \frac{x}{y}$$

$$S = 2 \left[\frac{F_{c2} - F_{c1}}{r_2 - r_1} \right] \left[\frac{x}{y} \right]^2 \quad \text{--- (9)}$$

Stiffness for the Spring :-

For minimum and intermediate position

$$S = 2 \left[\frac{F_c - F_{c1}}{r - r_1} \right] \left[\frac{x}{y} \right]^2 \quad \text{--- (10)}$$

For intermediate and Maximum position

$$S = 2 \left[\frac{F_{c2} - F_c}{r_2 - r} \right] \left[\frac{x}{y} \right]^2 \quad \text{--- (11)}$$

∴ From equations (9), (10) & (11)

$$\frac{F_{c2} - F_{c1}}{r_2 - r_1} = \frac{F_c - F_{c1}}{r - r_1} = \frac{F_{c2} - F_c}{r_2 - r}$$

$$F_c = F_{c1} + \left[F_{c2} - F_{c1} \right] \left[\frac{r - r_1}{r_2 - r_1} \right]$$

$$F_c = F_{c2} - \left[F_{c2} - F_{c1} \right] \left[\frac{r_2 - r}{r_2 - r_1} \right]$$

Hartung governor :- A spring controlled governor of the hartung type as shown in figure. In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of the governor when the rollers at the horizontal arm press against the sleeve.

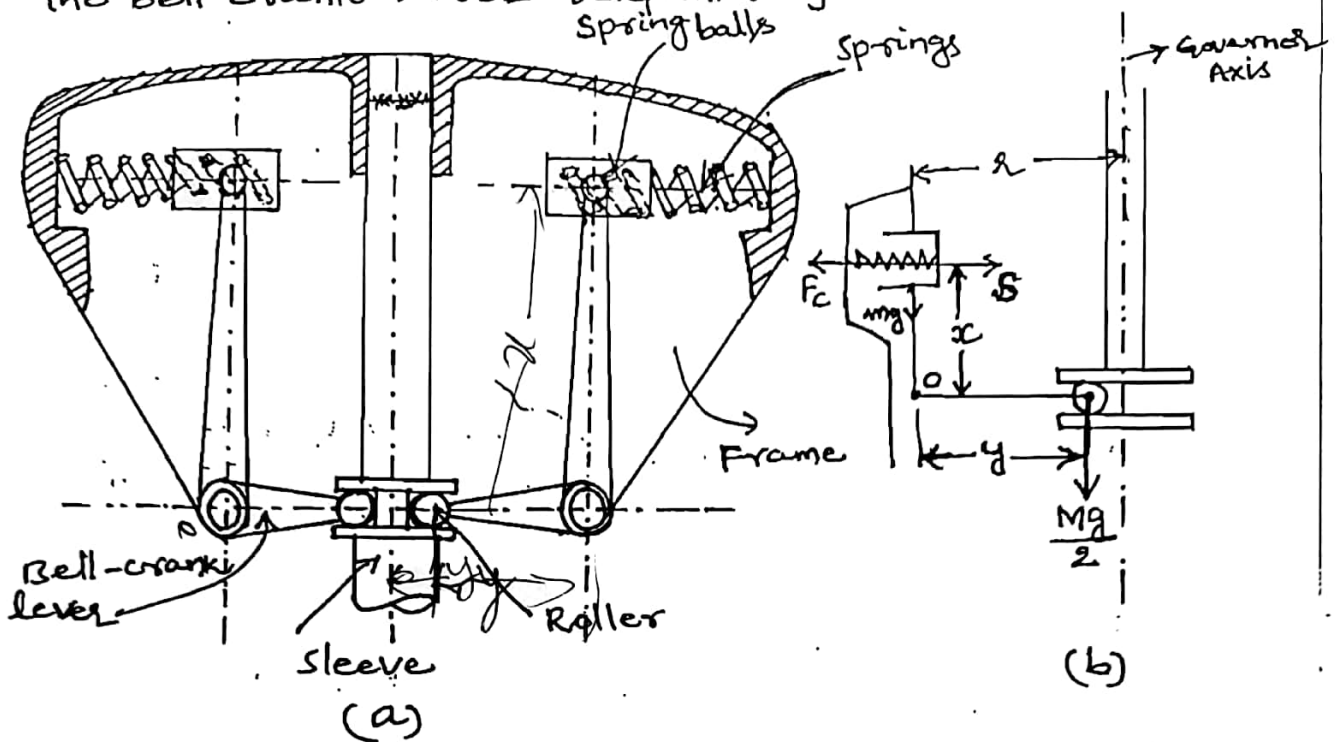
Let S = Spring force

F_c = centrifugal force

M = Mass on the sleeve, and

--- (19) ---

x and y = Length of the vertical and horizontal arm of the bell crank lever respectively.



Figures (a) & (b) show the governor in mid-position. Neglecting the effect of obliquity of the arms, Taking moments about the fulcrum 'O'

$$F_c \times x = S \times x + \frac{Mg}{2} \times y$$

$$F_c = S + \frac{Mg}{2} \cdot \frac{y}{x}$$

where S = Spring force = stiffness $\times$ Initial compression

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Hartnell governor

① On a spring loaded hartnell type governor, the extreme radii of rotation of the balls are 80mm and 120mm, the ball arm and the sleeve arm of the bell crank lever are equal in length. The mass of each ball is 2kg. If the speeds at the two extreme positions are 400 and 420 rpm. Find: 1. The initial compression of the central spring, and 2. the spring constant.

Solution:- $r_1 = 80\text{mm} = 0.08\text{m}$, $r_2 = 120\text{mm} = 0.12\text{m}$;

$$x = y \quad ; \quad m = 2\text{kg} \quad ; \quad N_1 = 400 \text{ r.p.m} \quad \& \quad \omega_1 = \frac{2\pi N_1}{60} \\ = \frac{2\pi(400)}{60} \\ = 41.9 \text{ rad/sec.}$$

$$N_2 = 420 \text{ rpm} \quad \& \quad \omega_2 = \frac{2\pi N_2}{60} \\ = \frac{2\pi(420)}{60} \Rightarrow 44 \text{ rad/sec.}$$

1. Initial compression of the central spring:-

$$\text{centrifugal force at the minimum speed, } F_{c1} = m \omega_1^2 \cdot r_1 \\ = 2 [41.9]^2 \cdot 0.08 \Rightarrow 281 \text{ N}$$

$$\text{centrifugal force at the maximum speed } F_{c2} = m \omega_2^2 \cdot r_2 \\ = 2 (44)^2 \cdot 0.12 \Rightarrow 465 \text{ N}$$

S_1 = spring force at the minimum speed

S_2 = " " " " maximum speed

We know that for minimum position

$$M \cdot g + S_1 = 2 F_{c1} \times \frac{x}{y}$$

$$S_1 = 2 F_{c1}$$

$$S_1 = 2 \times 281 \Rightarrow 562 \text{ N}$$

$$\text{Maximum position } \& \quad M g + S_2 = 2 F_{c2} \times \frac{x}{y}$$

$$S_2 = 2 F_{c2}$$

$$= 2 \times 465 \Rightarrow 930 \text{ N}$$

$$\text{Lift of the sleeve } h = (r_2 - r_1) \frac{y}{x} \quad (\because x = y)$$

$$r_2 - r_1 = h$$

$$120 - 80 = 40 \text{ mm}$$

$$\therefore \text{ stiffness of the spring } S = \frac{S_2 - S_1}{h} \Rightarrow \frac{930 - 562}{40} \Rightarrow 9.2 \text{ N/mm}$$

$$\text{Initial compression of the central spring} = \frac{S_1}{S} \Rightarrow \frac{562}{9.2}$$

$$= 61 \text{ mm}$$

2. Spring constant:-

$$\text{Spring constant} = \text{stiffness of spring } S = 9.2 \text{ N/mm}$$

—: 2:—

Q2) In a hartnell governor, the mass of each ball is 2 kg, length of vertical arm of bell crank lever is 150 mm and that of horizontal arm 75 mm. The distance of fulcrum of each bell crank lever from the axis of the governor is 80 mm. The extreme radii of rotation of balls are 75 mm and 112.5 mm. The maximum equilibrium speed is 5% greater than minimum equilibrium speed which is 360 rpm. Find initial compression of the spring and the equilibrium speed corresponding to the radius of rotation of 100 mm.

Sol $m = 2 \text{ kg}$, $x = 150 \text{ mm} = 0.15 \text{ m}$
 $y = 75 \text{ mm} = 0.075 \text{ m}$
 $R = 80 \text{ mm} = 0.08 \text{ m}$
 $R_2 = 112.5 \text{ mm} = 0.1125 \text{ m}$
 $R_1 = 75 \text{ mm} = 0.075 \text{ m}$

$\omega_2 = 1.05 \omega_1$

$N_1 = 360 \text{ rpm}$ & $\omega_1 = \frac{2\pi N_1}{60} \Rightarrow \frac{2\pi (360)}{60} \Rightarrow 37.7 \text{ rad/sec}$

Maximum equilibrium speed is 5% greater than the minimum equilibrium speed, so

Maximum equilibrium speed $\omega_2 = 1.05 \omega_1$
 $= 1.05 \times 37.7 \Rightarrow 39.6 \text{ rad/sec}$

$\frac{100 + 5\%}{100 + 0.05} = 1.05$

Centrifugal force at the maximum equilibrium speed

$F_{c2} = m \omega_2^2 R_2 \Rightarrow 2 (39.6)^2 \times 0.1125 \Rightarrow 352.84 \text{ N}$

Centrifugal force at the minimum equilibrium speed

$F_{c1} = m \omega_1^2 R_1 \Rightarrow 2 (37.7)^2 \times 0.075 \Rightarrow 213.2 \text{ N}$

For maximum equilibrium position:

$Mg + S_2 = 2 F_{c2} \frac{x}{y}$ ($\because M=0$)
 $S_2 = 2 (352.84) \frac{0.15}{0.075}$
 $S_2 = 1411.36 \text{ N}$

For minimum equilibrium position:

$Mg + S_1 = 2 F_{c1} \frac{x}{y}$ ($\because M=0$)
 $S_1 = 2 (213.2) \frac{0.15}{0.075}$
 $S_1 = 852.8 \text{ N}$

Where S_1 and S_2 are the spring forces corresponding to ω_1 and ω_2 respectively.

$$\begin{aligned}\text{Lift of the sleeve } h &= \frac{y}{x} (x_2 - x_1) \\ &= \frac{0.075}{0.15} [0.1125 - 0.075] \\ h &= 0.01875 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Spring stiffness } S &= \frac{S_2 - S_1}{h} \Rightarrow \frac{1411.36 - 852.8}{0.01875} \\ &= 29789.87 \text{ N/m}\end{aligned}$$

$$\begin{aligned}\text{Initial compression of spring} &= \frac{S_1}{S} \\ &= \frac{852.8}{29789.87} \\ &= 0.0286 \text{ m} \\ &= 28.6 \text{ mm}\end{aligned}$$

Equilibrium speed corresponding to radius of rotation $r = 100 \text{ mm} \Rightarrow 0.10 \text{ m}$

$$\begin{aligned}F_c &= F_{c_2} - (F_{c_2} - F_{c_1}) \frac{r_2 - r}{r_2 - r_1} \\ &= 352.84 - (352.84 - 213.2) \frac{0.1125 - 0.10}{0.1125 - 0.075} \Rightarrow 306.3 \text{ N}\end{aligned}$$

$$\begin{aligned}\text{Centrifugal force } F_c &= m \omega^2 r \\ 306.3 &= 2 \left[\frac{2\pi N}{60} \right]^2 \times 0.10 \\ N^2 &= 139651.69 \\ N &= \sqrt{139651.69} \\ N &= 373.7 \text{ rpm}\end{aligned}$$

③ On a spring controlled governor mass of each governor ball is 6.8 kg and moves radially under the action of a controlling force $F = a + br$, where ' r ' is the ball path radius. If the speed range is 42.5 to 44 radian per sec. and the corresponding values of ' r ' are 12.38 cm and 13.01 cm . Obtain the values of ' a ' and ' b ' and sketch the control force - radius curve. Find the equilibrium speed in rad/sec for $r = 12.7 \text{ cm}$.

Sol Given data:-

$$m = 6.8 \text{ kg}, \omega_1 = 42.5 \text{ rad/sec}, \omega_2 = 44 \text{ rad/sec}$$

$$r_1 = 12.38 \text{ cm} = 0.1238 \text{ m}; r_2 = 13.01 \text{ cm} = 0.1301 \text{ m}$$

$$\begin{aligned}\text{we know that } F_1 &= m \omega_1^2 r_1 \Rightarrow 6.8 \times [42.5]^2 \times 0.1238 \\ F_1 &= 1520.57 \text{ N}\end{aligned}$$

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$$F_2 = m\omega^2 \cdot r_2 \Rightarrow 6.8 \times (44)^2 \times 0.1301$$

$$F_2 = 1712.74 \text{ N}$$

$$F_1 = a + br_1 \text{ --- (given)}$$

$$1520.57 = a + b(0.1238) \text{ --- ①}$$

$$F_2 = a + br_2$$

$$1712.74 = a + b(0.1301) \text{ --- ②}$$

Solving equations ① & ② we get

$$a + 0.1301b = 1712.74$$

$$\text{subtracting } a + 0.1238b = 1520.57$$

$$b[0.13016 - 0.12386] = 1712.74 - 1520.57$$

$$b = 30503.17$$

'b' value is substitute in equation ①

$$1520.57 = a + 30503.17(0.1238)$$

$$a = -2255.72$$

Equilibrium speed when $r = 12.7 \text{ cm} \Rightarrow 0.127 \text{ m}$

Let $\omega =$ equilibrium speed

$$F = m\omega^2 r = a + br$$

$$6.8 \omega^2 \times 0.127 = -2255.72 + 30503.17 \times 0.127$$

$$\omega = 43.28 \text{ rad/sec.}$$

④ The fly balls of a spring loaded governor of Hartnell type running at 500 rpm. have a radius of rotation of 7 cm with sleeve in mid position and ball arms vertical. The ball arms length and sleeve arm length are equal. The maximum sleeve movement is 2 cm with $\pm 5\%$ variation in speed. The mass of the sleeve is 6 kg and friction may be assumed to be equivalent of an additional load of 25 N at the sleeve. The governor effort is sufficient to overcome the friction at the sleeve by 1% change of speed at mid point. Determine

- (i) mass of each ball,
- (ii) spring rate,
- (iii) Initial compression of spring,
- (iv) governor effort for 1% change of speed,
- (v) governor power.

Solution:- Given data : $M = 6 \text{ Kg}$, $F = 25 \text{ N}$

$$x = y, \quad h = 2 \text{ cm} \Rightarrow 0.02 \text{ m}$$

$$r = 7 \text{ cm} \Rightarrow 0.07 \text{ m}$$

(i) Mass of each ball :-

The speed at mid position. Assuming no friction

$$N = 500 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi \times 500}{60} \Rightarrow 52.36 \text{ rad/sec}$$

Minimum speed with friction $\omega_1 = \omega - \frac{5}{100} \omega$

$$= \frac{100\omega - 5\omega}{100}$$

$$= 0.95\omega \Rightarrow 0.95 \times 52.36$$

$$\omega_1 = 49.74 \text{ rad/s}$$

Similarly, maximum speed with friction $\omega_2 = \omega + \frac{5}{100} \omega$

$$= \frac{100\omega + 5\omega}{100}$$

$$= 1.05\omega \Rightarrow 1.05 \times 52.36$$

$$\omega_2 = 54.98 \text{ rad/sec}$$

The governor effort is to overcome the friction at the sleeve by $\pm 1\%$ change of speed at mid position.

So minimum speed at mid position

$$\omega_1' = \omega - \frac{1}{100} \omega = 0.99\omega \Rightarrow 0.99 \times 52.36 \Rightarrow 51.84 \text{ rad/sec}$$

and maximum speed at mid position $\omega_2' = \omega + \frac{1}{100} \omega$

$$= \frac{100\omega + \omega}{100}$$

$$= 1.01\omega \Rightarrow 1.01 \times 52.36$$

$$\omega_2' = 52.88 \text{ rad/sec}$$

Centrifugal force at the minimum speed,

$$F_{c1}' = m(\omega_1')^2 r \Rightarrow m(51.84)^2 \times 0.07 \Rightarrow 188.11 \text{ m} \rightarrow N$$

and centrifugal force at the maximum speed,

$$F_{c2}' = m(\omega_2')^2 r \Rightarrow m(52.88)^2 \times 0.07 \Rightarrow 195.74 \text{ m} \rightarrow N$$

we know that for maximum speed at mid position

$$\frac{S + Mg + F}{2} = F_{c2}' \cdot \frac{x}{y} \quad (\because x = y)$$

$$\frac{S + 6 \times 9.81 + 25}{2} = 195.74 \text{ m} \cdot \frac{x}{y}$$

$$S = 391.48 \text{ m} - 83.86 \quad \text{--- ①}$$

Similarly, For minimum speed at mid position

$$\frac{S + (Mg - F)}{2} = F_{c1}' \cdot \frac{x}{y} \quad (\because x = y)$$

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$$S + (6 \times 9.81 - 25) = 2 \times 188.11 \text{ m}$$

$$S = 376.22 \text{ m} - 33.88 \text{ ————— (2)}$$

from equations ① and ②, we get

$$\textcircled{1} = \textcircled{2}$$

$$391.48 \text{ m} - 83.86 = 376.22 \text{ m} - 33.88$$

$$15.26 \text{ m} = 49.98$$

$$m = 3.28 \text{ kg}$$

(ii) Spring rate :-

Let S = Spring rate

$$S = \frac{S_2 - S_1}{h}$$

We know that minimum radius of rotation considering friction

$$r_1 = r - h_1 \frac{w}{y} \Rightarrow 0.07 - \frac{0.02}{2} \quad [\because h_1 = \frac{h}{2} \text{ and } x = y]$$

$$r_1 = 0.06 \text{ m}$$

Similarly, maximum radius of rotation considering friction

$$r_2 = r + h_2 \frac{x}{y} \Rightarrow 0.07 + \frac{0.02}{2} \left(\frac{x}{y} \right) \quad [\because h_2 = \frac{h}{2}; x = y]$$

$$r_2 = 0.08 \text{ m}$$

centrifugal force at minimum speed

$$F_{c1} = m \omega_1^2 \cdot r_1 \Rightarrow 3.28 (49.74)^2 \times 0.06 \Rightarrow 486.89 \text{ N}$$

centrifugal force at maximum speed

$$F_{c2} = m \omega_2^2 \cdot r_2 \Rightarrow 3.28 (54.98)^2 \times 0.08 \Rightarrow 793.18 \text{ N}$$

For Maximum speed,

$$2 F_{c2} \cdot \frac{x}{y} = (W + F) + S_2 \text{ ————— } \textcircled{3} \quad (\because x = y)$$

For minimum speed

$$2 F_{c1} \cdot \frac{w}{y} = (W - F) + S_1 \text{ ————— } \textcircled{4} \quad (\because x = y)$$

so subtracting equation ④ and ③ we get

$$2 [F_{c2} - F_{c1}] = \cancel{W} + F + S_2 - \cancel{W} + F - S_1$$

$$2 [F_{c2} - F_{c1}] = 2F + S_2 - S_1$$

$$S_2 - S_1 = 2 [F_{c2} - F_{c1} - F]$$

$$= 2 [793.18 - 486.89 - 25]$$

$$S_2 - S_1 = 562.58 \text{ N}$$

$$\text{Spring rate } S = \frac{S_2 - S_1}{h} \Rightarrow \frac{562.58}{0.02} \Rightarrow 28129 \text{ N/m}$$

$$S = 28.129 \text{ N/mm}$$

From equation ④

$$S_1 = 2 F_{c1} - W + F$$

—: 26:—

$$\begin{aligned}
 &= 2 \times 486.89 - Mg + 25 \\
 S_1 &= 2 \times 486.89 - 6 \times 9.81 + 25 \Rightarrow 939.92 \text{ N} \\
 S_2 - S_1 &= 562.58 \\
 S_2 - 939.92 &= 562.58 \\
 S_2 &= 1502.5 \text{ N}
 \end{aligned}$$

(iii). Initial compression of the spring :

$$\begin{aligned}
 &= \frac{S_1}{S} \\
 &= \frac{939.92}{28129} \Rightarrow 0.0334 \text{ m}
 \end{aligned}$$

(iv) Governor effort for 1% change of speed :-

$$\begin{aligned}
 \text{Effort of governor} &= \text{percentage change of speed} \left[Mg + \frac{S_1 + S_2}{2} \right] \\
 &= 0.01 \left[6 \times 9.81 + \frac{939.9 + 1502.5}{2} \right]
 \end{aligned}$$

$$Q = 12.8 \text{ N}$$

(v) power of governor :-

$$P = Q \cdot h$$

$$= 12.8 \times 0.02$$

$$P = 0.256 \text{ N-m}$$

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Hartung Governor

- ① On a spring controlled governor of the Hartung type, the length of the ball and sleeve arms are 80 mm and 120 mm respectively. The total travel of the sleeve is 25 mm, In the mid position, each spring is compressed by 50 mm and the radius of rotation of the mass centres is 140 mm. Each ball has a mass of 4 kg and the spring has a stiffness of 10 kN/m of compression. The equivalent mass of the governor gear at the sleeve is 16 kg. Neglect the moment due to the revolving masses when the arms are inclined, determine the ratio of the range of speed to the mean speed of the governor. Find also speed in the mid position.

Solution:- $x = 80 \text{ mm} = 0.08 \text{ m}$ stiffness $= 10 \text{ kN/m} \Rightarrow 10 \times 10^3 \text{ N/m}$
 $y = 120 \text{ mm} = 0.12 \text{ m}$ $h_1 = h_2 = 25 \text{ mm} = 0.025 \text{ m}$
 $r = 140 \text{ mm} = 0.14 \text{ m}$
 $m = 4 \text{ kg}$
 $M = 16 \text{ kg}$
 Initial comp $= 50 \text{ mm} = 0.05 \text{ m}$

In mid-position :- Taking moment at point 'A'.

$$F_C(x) = S(x) + \frac{Mg(y)}{2}$$

$$m r \omega^2 x = S(x) + \frac{Mg(y)}{2}$$

$$4(40) \omega^2 \times 0.08 = 10 \times 10^3 (0.05 + 0.08) + \frac{16 \times 9.81 (0.12)}{2}$$

$$0.0448 \omega^2 = 40 + 9.42$$

$$\omega^2 = \frac{49.42}{0.0448}$$

$$\omega^2 = 1103.12$$

$$\omega = \sqrt{1103.12}$$

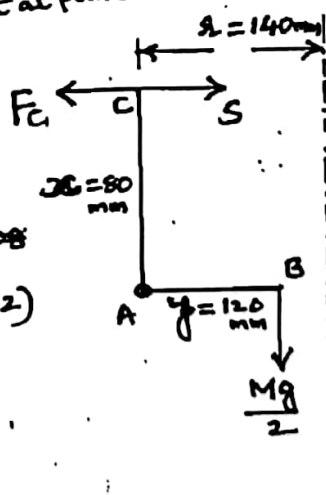
$$\omega = 33.21 \text{ rad/sec}$$

$$N = \frac{\omega \times 60}{2\pi}$$

$$= \frac{33.21 \times 60}{2\pi}$$

Mean speed $N = 317.13 \text{ R.P.M} \Rightarrow$ Speed in the mid position

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$S = \text{Spring force}$
 $= \text{stiffness} \times \text{Initial comp}$
 $= 10 \times 10^3 \times 0.05$
 $= 500 \text{ N}$

$$[\because \omega = \frac{2\pi N}{60}]$$

For the minimum speed, [Neglecting obliquity of arms]

$$\frac{x_2 - x_1}{x} = \frac{h}{y}$$

$$x_2 - x_1 = \frac{h x}{y}$$

$$x_1 = x_2 - \frac{h x}{y}$$

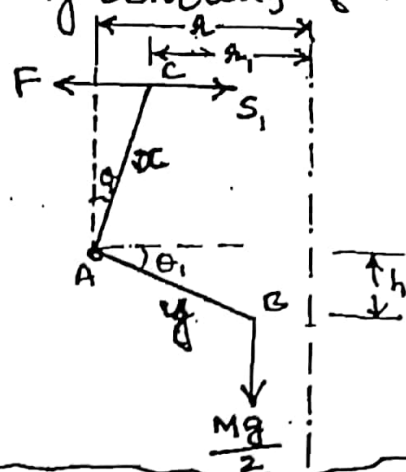
$$= 0.14 - \frac{0.025(0.08)}{0.12}$$

$$x_1 = 0.14 - 0.01667$$

$$= 0.1233 \text{ m}$$

$$\therefore \sin \theta_1 = \frac{x_2 - x_1}{a}$$

$$\text{and } \sin \theta_1 = \frac{h}{b}$$



$$\text{Spring force } S_1 = [\text{initial comp} - (x_2 - x_1)] \times \frac{\text{stiffness}}{10 \times 10^3}$$

$$= [0.05 - (0.14 - 0.1233)] \times 10 \times 10^3$$

$$= 333 \text{ N}$$

$$m x_1 \omega_1^2 x = S_1 (x_2 - x_1) + \frac{Mg}{2} \cdot \frac{y}{b}$$

$$4(0.1233) \omega_1^2 (0.08) = 333(0.08) + \frac{16 \times 9.81}{2} (0.12)$$

$$0.0395 \omega_1^2 = 26.64 + 9.417$$

$$\omega_1^2 = \frac{36.06}{0.0395} \Rightarrow \omega_1 = \sqrt{912.85}$$

$$\omega_1 = 30.21 \text{ rad/sec}$$

$$\omega_1 = \frac{2\pi N_1}{60}$$

$$N_1 = \frac{30.21 \times 60}{2\pi}$$

Thus minimum speed $N_1 = 288.5 \text{ rpm}$

For the maximum speed:- [Neglecting obliquity of arms]

$$\frac{x_2 - x}{x} = \frac{h}{y}$$

$$\frac{x_2 - 0.14}{0.08} = \frac{0.025}{0.12}$$

$$x_2 = 0.01667 + 0.14$$

$$x_2 = 0.15667 \text{ m}$$

$$S_2 = [0.05 + (0.15667 - 0.14)] \times 10 \times 10^3$$

$$S_2 = 666.7 \text{ N}$$

$$\sin \theta_2 = \frac{x_2 - x}{a}$$

$$\text{and } \sin \theta_2 = \frac{h}{b}$$

$$m x_2 \omega_2^2 x = S_2 (x_2 - x) + \frac{Mg}{2} \cdot \frac{y}{b}$$

$$4(0.15667) \omega_2^2 (0.08) = 666.7(0.08) + \frac{16 \times 9.81}{2} (0.12)$$

$$0.05 \omega_2^2 = 53.34 + 9.417$$

$$\omega_2^2 = 1255.15 \Rightarrow \omega_2 = \sqrt{1255.15} \Rightarrow \omega_2 = 35.43 \text{ rad/sec}$$

$$\omega_2 = \frac{2\pi N_2}{60} \Rightarrow N_2 = \frac{35.43 \times 60}{2\pi} \Rightarrow 338.3 \text{ rpm}$$

Thus maximum speed $N_2 = 338.3 \text{ rpm}$

$$\text{Range of Speed} = N_2 - N_1 \Rightarrow 338.3 - 288.5 \Rightarrow 49.8 \text{ rpm}$$

$$\text{Ratio of range of speed to mean speed} = \frac{49.8}{317.13}$$

$$= 0.157$$

∴ 29 :-

Definitions of governors:-

* Sensitiveness:- The ratio of the mean speed to the difference between the maximum and minimum speed.

$$\text{Sensitiveness} = \frac{\frac{N_1 + N_2}{2}}{N_1 - N_2} \quad \left[\because N = \frac{N_1 + N_2}{2} \right]$$
$$= \frac{N}{N_1 - N_2}$$

where N_1 = maximum speed
 N_2 = minimum speed
 N = Mean speed

For a given lift of sleeve, as the speed of the governor decreases, the sensitiveness increases. It can also be understood by considering two governors A and B running at the same speed. By altering this speed by a certain amount, the lift of the sleeve of governor 'A' is less than the lift of the sleeve of governor 'B'. This tells us that the governor B is more sensitive than governor A.

* Isochronism:- A governor with zero range of speed is called isochronism of governor.

$$\text{We know that Sensitiveness} = \frac{N}{N_1 - N_2} = \frac{N}{\delta} = \alpha$$

where $N_1 - N_2$ is known as the range of speed.

So Sensitiveness when $N_1 - N_2$ is zero becomes infinite. It can be stated that a governor is isochronous whose sensitiveness is infinite. So this leads to the practical difficulty in using the isochronous governor.

* Hunting:- When the speed in a governor fluctuates above and below the mean speed, it is called hunting.

* Stability:- A governor is said to be stable if there is only one radius of rotation for all equilibrium speeds of the balls within the working range.

Theory of machinery

Reference Books

1. R. S. Khurmi & J. K. Gupta.
2. S. S. Rattan.
3. V. P. Singh.