

TURNING MOMENTS DIAGRAMS

Jptendra

Flywheel:- "A flywheel acts as a energy reservoir for storing and releasing mechanical Energy of the engine".

During the working stroke in the engine, when the energy produced is more than the requirement the speed of the engine increases. This energy produced during working stroke is distributed among the idle strokes which results in the decrease of engine speed. That is not desirable. To overcome this fluctuation in speed, the function of flywheel comes into picture.

During the power & working stroke the flywheel absorbs the excess energy and during the idle stroke it gives away the same to the engine as per the load requirement.

✓ Co-efficient of fluctuation of speed:- It is defined as the ratio of maximum fluctuation of speed to the mean speed. It is denoted by C_s

$$\therefore C_s = \frac{N_1 - N_2}{N} \quad \left(\because N = \frac{N_1 + N_2}{2} \right)$$

$$= \frac{N_1 - N_2}{\frac{N_1 + N_2}{2}} \Rightarrow \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{2 \left[\frac{\omega_1 \times 60}{2\pi} - \frac{\omega_2 \times 60}{2\pi} \right]}{\frac{\omega_1 \times 60}{2\pi} + \frac{\omega_2 \times 60}{2\pi}}$$

$$= \frac{2 \times 60}{\frac{60}{2\pi}} \frac{(\omega_1 - \omega_2)}{(\omega_1 + \omega_2)} \Rightarrow C_s = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

$$\begin{aligned} \omega_1 &= \frac{2\pi N_1}{60} \\ N_1 &= \frac{\omega_1 \times 60}{2\pi} \\ N_2 &= \frac{\omega_2 \times 60}{2\pi} \end{aligned}$$

where

N_1 = Maximum speed ; N_2 = Minimum speed

N = mean speed = $\frac{N_1 + N_2}{2}$

ω_1 = maximum Angular velocity

ω_2 = minimum Angular velocity

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Co-efficient of fluctuation of energy:- "It may be defined as the ratio of maximum fluctuation of energy to work done per cycle". It is denoted by C_E

$$\therefore C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

Some Relations:-

- 1). work done per cycle = $\frac{P \times 60}{n}$
- 2). moment of Inertia of a solid flywheel disc $I = \frac{m D^2}{8}$
- 3). Mass of flywheel rim $m = A \cdot \pi D \cdot \rho$
- 4). Hoop stress $\sigma = \rho V^2 \Rightarrow \rho \left[\frac{\pi D N}{60} \right]^2$

$$A = b \times t$$

where

P = power transmitted Watts

n = Number of working strokes per minute
 $= \frac{N}{2}$ in case of four stroke I.C engine

A = cross sectional area of flywheel rim.

b = width of rim

t = thickness of rim.

σ = hoop stress N/m^2 due to centrifugal force

ρ = density of rim material Kg/m^3

D = Mean diameter of rim, ... m

V = linear velocity of flywheel m/sec

N = speed of flywheel in r.p.m.

$$P = \frac{2\pi NT}{60} \quad ; \quad \omega = \frac{2\pi N}{60}$$

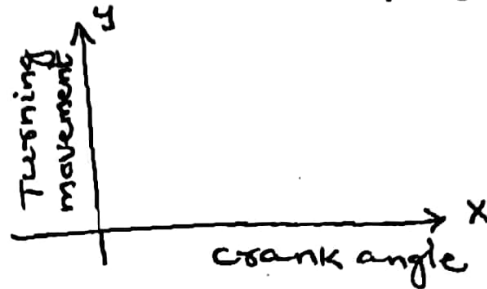
$$V = \frac{\pi D N}{60}$$

where P = power Watt
 ω = Angular velocity rad/sec
 V = Linear velocity m/s
 T = Torque N-m

Turning movement diagram . mention its uses?

Turning movement diagram is a diagram which gives relationship between the turning movement on crank for various position of the crank. It is also called T-m diagram & M-θ diagrams. This diagram is obtained by plotting turning movement on y-axis and crank angle on x-axis. This diagram is different for different engines. The variation of turning movement w.r.t crank angle can be determined with the help of these diagrams.

uses:-



- (i) Area of turning movement diagram per cycle gives the work done per cycle. When this is multiplied by number of cycles per minute power developed by the engine is obtained.
- (ii) The maximum height of the turning movement diagram gives the maximum torque acting on the crank shaft.
- (iii) When area of turning movement diagram is divided by length of the base, mean torque & mean turning movement is obtained.

Turning moment diagrams Refer page NO: 8 →

Q. 4 E D A M

Derive the equation $k = \frac{E}{2E}$ where k = co-efficient of fluctuation of speed, explain maximum fluctuation of energy and kinetic energy :-

Let

m = Mass of the flywheel in kg

k = Radius of gyration of flywheel in m

N_1 = Maximum speed

N_2 = Minimum speed

N = Mean speed = $\frac{N_1 + N_2}{2}$. & $\omega = \frac{\omega_1 + \omega_2}{2}$

$$= \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2$$

$$= \frac{1}{2} I [\omega_1^2 - \omega_2^2]$$

$$(\because a^2 - b^2 = (a+b)(a-b))$$

$$= \frac{1}{2} I (\omega_1 + \omega_2) [\omega_1 - \omega_2]$$

$$= I \left[\frac{\omega_1 + \omega_2}{2} \right] [\omega_1 - \omega_2]$$

$$(\because \frac{\omega_1 + \omega_2}{2} = \omega)$$

$$= I \cdot \omega \cdot [\omega_1 - \omega_2]$$

Multiplying and dividing both sides by ' ω ', we get

$$\Delta E = I \omega [\omega_1 - \omega_2] \frac{\omega}{\omega}$$

$$\Rightarrow I \omega^2 \left[\frac{\omega_1 - \omega_2}{\omega} \right]$$

$$= I \omega^2 \frac{C_s}{100}$$

$$\therefore C_s = \frac{\omega_1 - \omega_2}{\omega} \times 100$$

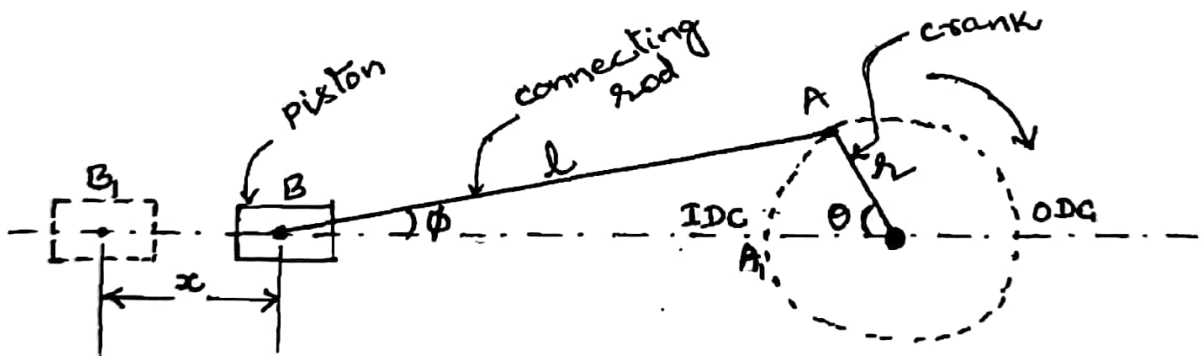
$$\frac{\omega_1 - \omega_2}{\omega} = \frac{C_s}{100}$$

$$= \frac{2 E C_s}{100}$$

$$\Delta E = 0.02 E C_s$$

$$(\because E = \frac{1}{2} I \omega^2)$$

$$I \omega^2 = 2 E$$



D. Torque on crank shaft $T = F_p \times r \left[\sin \theta + \frac{\sin 2\theta}{2[n^2 - \sin^2 \theta]} \right]$

2). Compression ratio = $\frac{V_1}{V_2}$
of piston

3). Displacement of the piston $x = r \left[(1 - \cos \theta) + \frac{\sin^2 \theta}{2n} \right]$
 $= r \left[(1 - \cos \theta) + \frac{(1 - \cos^2 \theta)}{2n} \right]$

4). Inertia forces on reciprocating parts

$$F_I = M_R \cdot \omega^2 \cdot r \left[\cos \theta + \frac{\cos 2\theta}{n} \right]$$

5). Piston effort $F_p = F_L - F_I + W_R$ ($\therefore W_R = m_R g$)

6). Net load on piston $F_L = \text{pressure} \times \text{Area}$
 $= p \times \frac{\pi d^2}{4}$

7). Crank pin effort $F_T = F_p \frac{\sin(\theta + \phi)}{\cos \phi}$

$$\left[\begin{aligned} \sin \phi &= \frac{\sin \theta}{n} \\ \therefore n &= \frac{l}{r} \end{aligned} \right]$$

8). Turning moment on the crank shaft $T = F_T \times r$

$$T = \left[F_p \frac{\sin(\theta + \phi)}{\cos \phi} \right] r$$

9). Turning moment = Accelerating torque + Resisting torque
 Acceleration torque = Turning moment - Resisting torque

where

$$\text{Acceleration torque} = I \cdot \alpha$$

F_p = piston effort

r = Radius of crank = $\frac{\text{stroke}}{2}$

l = length of connecting rod

n = Ratio of the connecting rod length and radius of crank = $\frac{l}{r}$

θ = Angle turned by the crank from inner dead centre

m_R = mass of the reciprocating parts

F_L = Net load on piston = pressure $\times$ Area
 $= p \times A$

W_R = Weight of the reciprocating parts = $m_R \cdot g$

or Problem

① A horizontal steam engine 20cm diameter by 40cm stroke, connecting rod 100cm makes 160 rpm. The mass of the reciprocating parts is 50kg. When the crank has turned through an angle of 30° , the steam pressure is 4.5 bar.

(a). calculate the turning moment on crank shaft.

(b) If the mean resistance torque is 30 N-m and the

mass of fly wheel is 50kg and the radius of gyration 70cm. calculate the acceleration of the flywheel.

Given data:-

$$d = 20\text{cm} = 0.2\text{m}$$

$$r = \frac{\text{stroke}}{2} \Rightarrow \frac{40}{2} \Rightarrow 20\text{cm} \Rightarrow 0.2\text{m}$$

$$l = 100\text{cm} = 1\text{m}$$

$$N = 160 \text{ rpm}$$

$$\omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi \times 160}{60} \Rightarrow 16.75 \text{ rad/s}$$

$$M_R = 50\text{kg} ; W_R = 50 \times 9.81 \text{ N} \quad [\because 1\text{kg} = 9.81\text{N}]$$

$$\theta = 30^\circ$$

$$p = 4.5 \text{ bar} \Rightarrow 4.5 \times 10^5 \text{ N/m}^2 \quad [\because 1 \text{ bar} = 10^5 \text{ N/m}^2]$$

$$n = \frac{l}{r} \Rightarrow \frac{1}{0.2} \Rightarrow 5$$

$$(i) \text{ Turning moment } T = F_p \times \frac{\sin(\theta + \phi)}{\cos \phi} \cdot r$$

$$F_p = F_L - F_I$$

$$F_L = p \times A \Rightarrow 4.5 \times 10^5 \times \frac{\pi (0.2)^2}{4} \Rightarrow 14137.1 \text{ N}$$

$$F_I = \text{Inertia force}$$

$$= M_R \cdot \omega^2 \cdot r \left[\cos \theta + \frac{\cos 2\theta}{n} \right]$$

$$= 50 \times (16.75)^2 \cdot 0.2 \left[\cos 30^\circ + \frac{\cos 2(30)}{5} \right]$$

$$= 2710 \text{ N}$$

$$\text{Effective piston effort } F_p = 14137.1 - 2710 \\ = 11427.2 \text{ N}$$

$$\sin \phi = \frac{\sin \theta}{n}$$

$$= \frac{\sin 30^\circ}{5} = 0.1$$

$$\phi = \sin^{-1}(0.1)$$

$$\phi = 5.74^\circ$$

$$\therefore T = F_p \frac{\sin(\theta + \phi)}{\cos \phi} \times r \Rightarrow 11427.2 \cdot \frac{\sin(30 + 5.74)}{\cos 5.74} \times 0.2$$

$$T = 1341.6 \text{ N-m}$$

$$(11) \text{ Resisting torque} = 30 \text{ N-m (given)}$$

$$\text{Turning moment} = 133.55 \text{ N-m}$$

$$\therefore \text{Acceleration torque} = T - M \cdot R \cdot T$$

$$I \cdot \alpha = 1341.6 - 30 \Rightarrow 1311.6 \text{ N}$$

$$I \alpha = 1311.6$$

$$\alpha = \frac{1311.6}{I}$$

$$\alpha = \frac{1311.6}{24.5}$$

$$\alpha = 53.53 \text{ rad/sec}^2$$

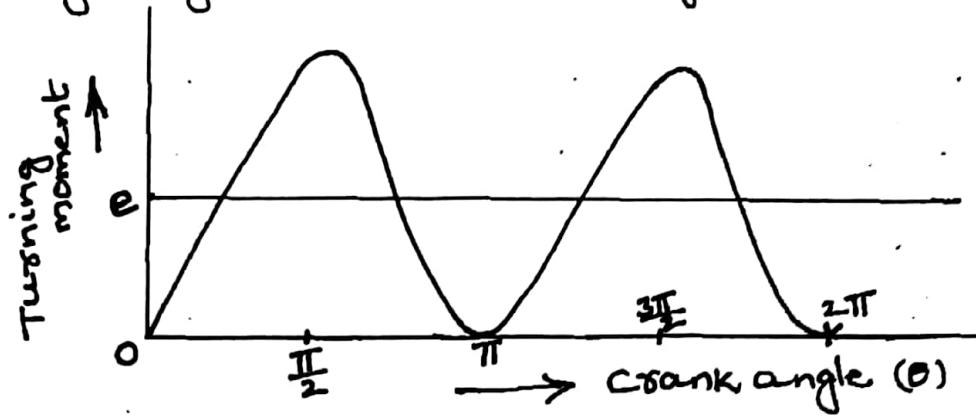
$$I = m k^2$$

$$= 50 \times 0.7^2$$

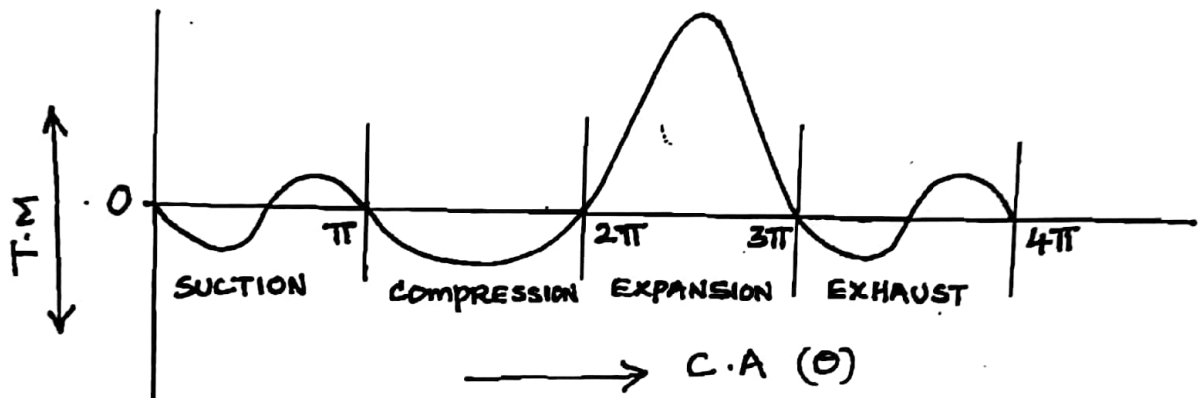
$$= 24.5 \text{ kg-m}^2$$

* Turning-moment diagram for :-

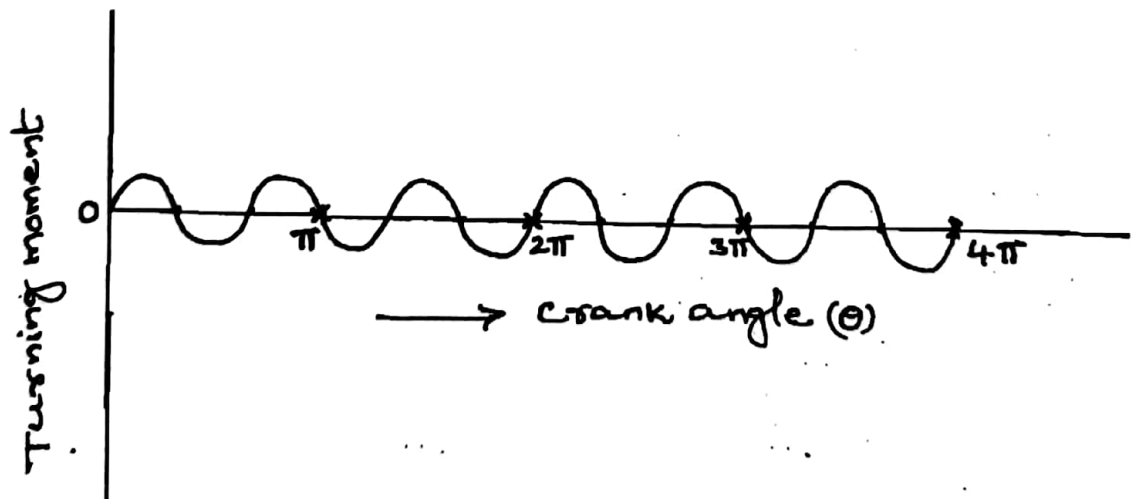
1) Single cylinders double-acting steam engine.



2) Single cylinders four-stroke engine.



3) Multi cylinders engine :-



Problems

- Mid
- ① The torque delivered by a two-stroke engine is represented by $T = [1000 + 300 \sin 2\theta - 500 \cos 2\theta]$ N-m. where θ is the angle turned by the crank from the inner-dead centre. The engine speed is 250 rpm. The mass of the flywheel is 400 kg and radius of gyration 400 mm. Determine,
- the power developed,
 - the total percentage fluctuation of speed,
 - the angular acceleration of flywheel, when the crank has rotated through an angle of 60° from the inner-dead centre.
 - the maximum angular acceleration and retardation of the flywheel.

Sol The expression for torque being a function of 2θ , the cycle is repeated every 180° of the crank rotation.

$$\begin{aligned} \text{(i) } T_{\text{mean}} &= \frac{1}{\pi} \int_0^\pi T \cdot d\theta \\ &= \frac{1}{\pi} \int_0^\pi [1000 + 300 \sin 2\theta - 500 \cos 2\theta] d\theta \\ &= \frac{1}{\pi} \left[1000\theta + 300 \left(-\frac{\cos 2\theta}{2} \right) - 500 \left(\frac{\sin 2\theta}{2} \right) \right]_0^\pi \\ &= \frac{1}{\pi} \left[1000[\pi] + 150 [-\cos(2\pi)] - 250 \sin(2\pi) \right] - [0 - 150 - 0] \end{aligned}$$

$$\frac{360}{2} = 180^\circ$$

$$\frac{d}{\pi \text{ rad}}$$

$$2\theta = \frac{360}{2} = 180^\circ$$

$$T_{\text{mean}} = 1000 \text{ N-m}$$

$$\text{Power } P = \frac{2\pi N T_{\text{mean}}}{60}$$

$$P = \frac{2\pi \times 250 \times 1000}{60} \Rightarrow 26179.9 \text{ W} \Rightarrow 26.18 \text{ kW}$$

(ii) At any instant $\Delta T = T - T_{\text{mean}}$

$$= [1000 + 300 \sin 2\theta - 500 \cos 2\theta] - 1000$$

$$\Delta T = 300 \sin 2\theta - 500 \cos 2\theta$$

ΔT is zero, when $300 \sin 2\theta - 500 \cos 2\theta = 0$

$$\text{or } 300 \sin 2\theta = 500 \cos 2\theta$$

$$\text{or } \frac{\sin 2\theta}{\cos 2\theta} = \frac{500}{300}$$

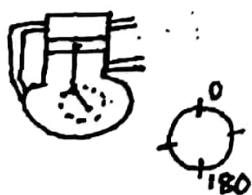
$$\Rightarrow \tan 2\theta = \frac{5}{3}$$

$$2\theta = \tan^{-1} \left(\frac{5}{3} \right)$$

$$2\theta = 59^\circ \text{ or } 59 + 180 = 239^\circ$$

$$\theta = 29.5^\circ \text{ or } 119.5^\circ$$

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$$\begin{aligned}\text{Maximum fluctuation of energy } [E_{\max}] &= \int_{29.5}^{119.5} \Delta T \cdot d\theta \\ &= \int_{29.5}^{119.5} [300 \sin 2\theta - 500 \cos 2\theta] d\theta \\ &= \left[-\frac{300 \cos 2\theta}{2} - \frac{500 \sin 2\theta}{2} \right]_{29.5}^{119.5}\end{aligned}$$

$$E_{\max} = 583.1 \text{ N-m}$$

$$\text{co-efficient of fluctuation of speed } K = \frac{E}{I \omega^2} \quad [\because I = mK^2]$$

$$= \frac{E}{mK^2 \omega^2}$$

$$= \frac{583.1}{400(0.4)^2 \times \left[\frac{2\pi(250)}{60} \right]^2}$$

$$\therefore \omega = \frac{2\pi N}{60}$$

$$\begin{aligned}K &= 400 \text{ mm} \\ &= 0.4 \text{ m} \\ N &= 250 \text{ rpm}\end{aligned}$$

$$K = 0.01329$$

$$= 1.329\%$$

(iii). The angular acceleration of flywheel, when the crank has rotated through an angle of 60° from the inner-dead centre.

$$\Delta T = 300 \sin 2\theta - 500 \cos 2\theta$$

$$\text{when } \theta = 60^\circ$$

$$\begin{aligned}\Delta T &= 300 \sin 2(60^\circ) - 500 \cos 2(60^\circ) \\ &= 509.8 \text{ N-m}\end{aligned}$$

$$\Delta T = I \cdot \alpha \Rightarrow mK^2 \cdot \alpha$$

$$509.8 = 400(0.4)^2 \cdot \alpha$$

$$\alpha = 7.966 \text{ rad/sec}^2$$

(iv). The maximum acceleration and retardation of the flywheel.

$$\frac{d}{d\theta} [\Delta T] = \frac{d}{d\theta} [300 \sin 2\theta - 500 \cos 2\theta] d\theta = 0$$

$$300 \cos 2\theta (2) - 500(\sin 2\theta)(2) = 0$$

$$600 \cos 2\theta + 1000 \sin 2\theta = 0$$

$$1000 \sin 2\theta = -600 \cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{-600}{1000}$$

$$\tan 2\theta = \frac{-6}{10}$$

$$2\theta = \tan^{-1} \left[\frac{-6}{10} \right]$$

$$2\theta = -30.96^\circ \text{ Next angle}$$

$$2\theta = 149.04^\circ \text{ and } 329.04^\circ$$

$$\begin{array}{r} -30.96 \\ + 180.00 \\ \hline 149.04 \end{array}$$

-ve so
Next angle
we can take

when $2\theta = 149.04^\circ$

$$\Delta T = 300 \sin 2\theta - 500 \cos 2\theta \Rightarrow 300 \sin [149.04^\circ] - 500 \cos [149.04^\circ]$$

$$\Delta T = 583.1 \text{ N-m}$$

when $2\theta = 329.04^\circ$

$$\Delta T = 300 \sin (329.04^\circ) - 500 \cos (329.04^\circ)$$

$$= -583.1 \text{ N-m}$$

As values of ΔT at maximum and minimum torque are same, maximum acceleration is equal to maximum retardation.

$$\Delta T = I \cdot \alpha$$

$$= (mk^2) \alpha$$

$$583.1 = 400 (0.4)^2 \cdot \alpha$$

$$\alpha = 9.11 \text{ rad/sec}^2$$

Maximum acceleration or retardation $\alpha = 9.11 \text{ rad/sec}^2$

② A machine is coupled to a two-stroke engine which produces a torque of $[800 + 180 \sin 3\theta] \text{ N-m}$, where θ is the crank angle. The mean engine speed is 400 rpm. The flywheel and the other rotating parts attached to the engine have a mass of 350 kg at a radius of gyration of 220 mm. Calculate

(i) The power of the engine

(ii) The total fluctuation of speed of the Flywheel when

(a) the resisting torque is constant.

(b) the resisting torque is $[800 + 80 \sin \theta] \text{ N-m}$.

Sol

$$m = 350 \text{ kg}$$

$$N = 400 \text{ rpm} \Rightarrow \omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi \times 400}{60} \Rightarrow 41.89 \text{ rad/s}$$

$$k = 220 \text{ mm}$$

The expression for torque being a function of 3θ , the cycle is repeated after every 120° of the crank rotation. $\frac{360}{3} = 120^\circ$

$$(i) T_{\text{mean}} = \frac{1}{\frac{2\pi}{3}} \int_0^{\frac{2\pi}{3}} T \cdot d\theta$$

$$= \frac{3}{2\pi} \int_0^{\frac{2\pi}{3}} [800 + 180 \sin 3\theta] d\theta$$

$$= \frac{3}{2\pi} \left[800\theta - 180 \frac{\cos 3\theta}{3} \right]_0^{\frac{2\pi}{3}} \Rightarrow 800 \text{ N-m}$$

$$\text{Power } P = \frac{2\pi NT}{60} \Rightarrow \frac{2\pi \times 400 \times 800}{60}$$

$$P = 33512 \text{ W} \Rightarrow 33.512 \text{ kW}$$

(ii) (a) At any instant, $\Delta T = T - T_{\text{mean}}$
 $= 800 + 180 \sin 3\theta - 800$
 $\Delta T = 180 \sin 3\theta$

$$\Delta T \text{ is zero, when } 180 \sin 3\theta = 0$$

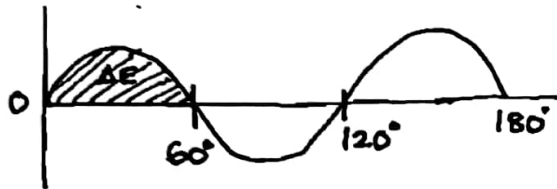
$$\sin 3\theta = 0$$

$$3\theta = \sin^{-1}(0)$$

$$3\theta = 0$$

$$3\theta = 0 \text{ or } 180^\circ$$

$$\theta = 0 \text{ or } 60^\circ$$



$$e_{\text{max}} = \text{max fluctuation of energy}$$

$$= \int_0^{60} \Delta T \cdot d\theta$$

$$= \int_0^{60} (180 \sin 3\theta) d\theta \Rightarrow \left[180 \left(-\frac{\cos 3\theta}{3} \right) \right]_0^{60}$$

$$= -\frac{180 \cos 3(60)}{3} - \left[-\frac{180 \cos 3(0)}{3} \right]$$

$$= 120 \text{ N-m}$$

$$\text{Co-efficient of fluctuation of speed } k = \frac{e}{I \omega^2} \quad (\because I = mk^2)$$

$$k = \frac{e}{mk^2 \cdot \omega^2} \Rightarrow \frac{120}{350 (0.22)^2 (41.89)^2}$$

$$k = 0.00404 \Rightarrow 0.404\%$$

(b) $\Delta T = T \text{ of engine} - T \text{ of machine}$

$$= 800 + 180 \sin 3\theta - [800 + 80 \sin \theta]$$

$$\Delta T = 180 \sin 3\theta - 80 \sin \theta$$

$$\Delta T \text{ is zero when } 180 \sin 3\theta - 80 \sin \theta = 0$$

$$180 \sin 3\theta = 80 \sin \theta$$

$$180 [3 \sin \theta - 4 \sin^3 \theta] = 80 \sin \theta$$

$$180 \sin \theta [3 - 4 \sin^2 \theta] = 80 \sin \theta$$

$$3 - 4 \sin^2 \theta = \frac{80 \sin \theta}{180 \sin \theta}$$

$$\sin^2 \theta = 3 - \frac{8}{18}$$

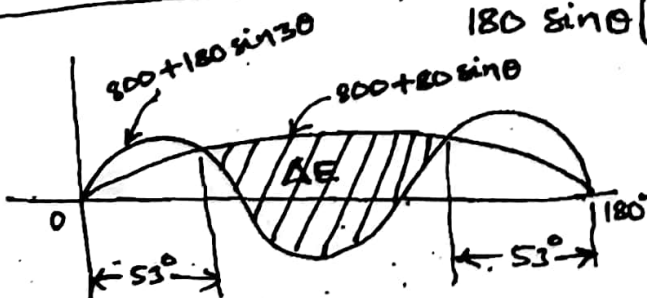
$$\sin^2 \theta = 0.639$$

$$\sin \theta = \pm 0.799$$

$$\theta = \pm 53^\circ \text{ and } \pm 127^\circ$$

$$180 - 53 = 127$$

$$\therefore \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$



$$\begin{aligned}
 e_{\max} &= \int_{53^\circ}^{127^\circ} \Delta T d\theta \Rightarrow \int_{53^\circ}^{127^\circ} [180 \sin 3\theta - 80 \sin \theta] d\theta \\
 &= \left[\frac{180(-\cos 3\theta)}{3} + 80 \cos \theta \right]_{53^\circ}^{127^\circ} \\
 &= -60 \cos 3(127^\circ) + 80 \cos 127^\circ - [-60 \cos 159^\circ + 80 \cos 53^\circ] \\
 &= -208.7 \text{ N-m}
 \end{aligned}$$

$$k = \frac{e}{I \omega^2} \Rightarrow \frac{e}{m k^2 \omega^2} \Rightarrow \frac{208.7}{350 \times (0.22)^2 \times [41.89]^2}$$

$$k = 0.00703$$

$$k = 0.703\%$$

Q3 The turning moment diagram for a multi cylinder engine has been drawn to a vertical scale of $1 \text{ mm} = 650 \text{ N-m}$ and a horizontal scale of $1 \text{ mm} = 4.5^\circ$. The areas above and below the mean torque line are $-28, +380, -260, +310, -300, +242, -380, +265$ and -229 mm^2 .

The fluctuation of speed is limited to $\pm 1.8\%$ of the mean speed which is 400 rpm . Density of the rim material is 7000 kg/m^3 and width of the rim is 4.5 times its thickness. The centrifugal stress [hoop stress] in the rim material is limited to 6 N/mm^2 . Neglecting the effect of the boss and arms, determine the diameter and cross-section of the flywheel rim.

Sol $\rho = 7000 \text{ kg/m}^3$; $\sigma = 6 \text{ N/mm}^2$ $\left[\because 1 \text{ mm}^2 = \frac{1}{10^6} \text{ m}^2 \right]$
 $N = 400 \text{ rpm}$ $= 6 \times 10^6 \text{ N/m}^2$

$$k = \pm 1.8\%$$

$$= 0.018$$

$$\frac{1.8}{100} = 0.018$$

$$b = 4.5t$$

$$\sigma = \rho v^2$$

$$6 \times 10^6 = 7000 v^2$$

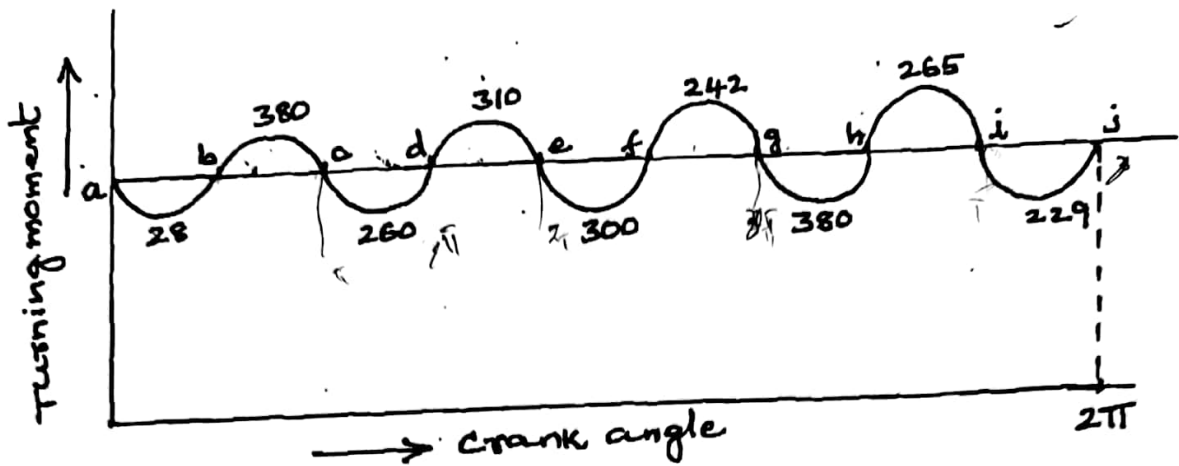
$$v = 29.28 \text{ m/sec}$$

$$v = \frac{\pi D N}{60}$$

$$29.28 = \frac{\pi D \times 400}{60}$$

$$D = 1.398 \text{ m}$$

$$\text{Radius of gyration } k = \frac{D}{2} \Rightarrow \frac{1.398}{2} \Rightarrow 0.699 \text{ m}$$



Let
flywheel kinetic energy at a = E

$$\text{at } b = E - 28$$

$$\text{at } c = E - 28 + 380 = E + 352$$

$$\text{at } d = E + 352 - 260 = E + 92$$

$$\text{at } e = E + 92 + 310 = E + 402 \checkmark$$

$$\text{at } f = E + 402 - 300 = E + 102$$

$$\text{at } g = E + 102 + 242 = E + 344$$

$$\text{at } h = E + 344 - 380 = E - 36 \checkmark$$

$$\text{at } i = E - 36 + 265 = E + 229$$

$$\text{at } j = E + 229 - 229 = E$$

$$\text{Maximum energy} = E + 402 \quad \text{--- [at point e]}$$

$$\text{Minimum energy} = E - 36 \quad \text{--- [at point h]}$$

Maximum fluctuation of energy

$$e_{\max} = [E + 402] - [E - 36] \times \text{Horizontal scale} \times \text{Vertical scale}$$

$$= 438 \times \left[4.5 \times \frac{\pi}{180} \right] \times 650$$

$$= 22360 \text{ N-m}$$

$$\begin{aligned} &= 438 \text{ mm}^2 \\ &= 438 \text{ mm} \times \text{mm} \\ &= 438 \times \left(4.5 \times \frac{\pi}{180} \right) \times 650 \end{aligned}$$

K = coefficient of fluctuation of speed

$$K = \frac{e_{\max}}{I \omega^2} \Rightarrow \frac{e_{\max}}{m K^2 \cdot \omega^2}$$

$$0.018 = \frac{22360}{m [0.699]^2 \left[\frac{2\pi \times 400}{60} \right]^2}$$

mass of the flywheel m = 5.5

$$1449 = 7000 (\pi \times 4.5 t \times t)$$

$$t = 0.1023 \text{ m} \Rightarrow 102.3 \text{ mm}$$

$$b = 4.5(t) \Rightarrow 4.5(102.3) \Rightarrow 460.5 \text{ mm}$$

$$\therefore \text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \pi d \times b \times t = \pi \times 4.5 t \times t$$

$$\Rightarrow m = 1449 \text{ kg}$$

- ④ On a machine, the intermittent operations demand the torque to be applied as follows.
- During the first half revolution, the torque increases uniformly from 800 N-m to 3000 N-m.
 - During next one revolution, the torque remains constant
 - During next one revolution, the torque decreases uniformly from 3000 N-m to 800 N-m.
 - During last $1\frac{1}{2}$ revolution, the torque remains constant

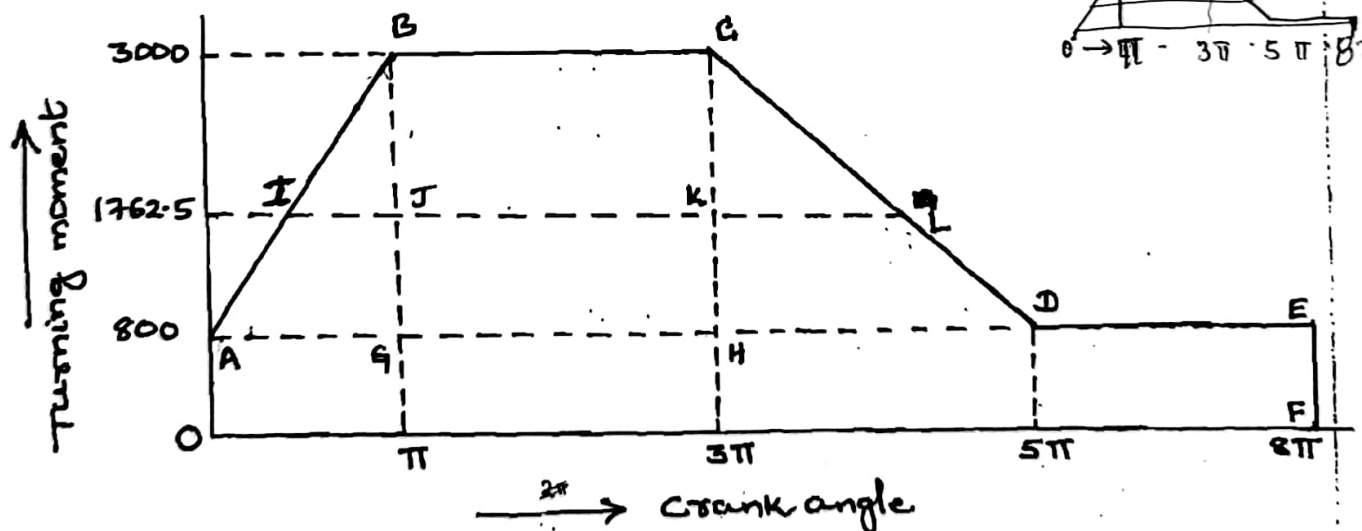
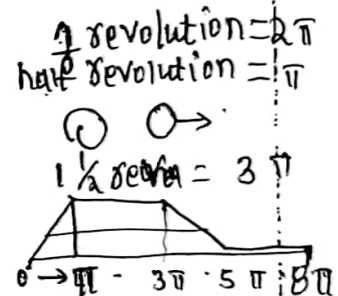
Thus a cycle is completed in 4 revolutions. The motor to which the machine is coupled exerts a constant torque at a mean speed of 250 rpm. A flywheel of mass 1800 kg and radius of gyration of 500 mm is fitted to the shaft. Determine (i) the power of the motor (ii) the total fluctuation of speed of the machine shaft.

Sol

$$m = 1800 \text{ kg}$$

$$k = 500 \text{ mm} = 0.5 \text{ m}$$

$$N = 250 \text{ rpm}$$



(i) from figure

Torque for one complete cycle $T = \text{Area of } OAB C D E F$

$$T = \text{Area } O A E F + \text{Area } A B G + \text{Area } B G C H + \text{Area } C H D$$

$$= 8\pi \times 800 + \frac{1}{2} \pi (3000 - 800) + 2\pi (3000 - 800) + \frac{1}{2} (2\pi) \frac{3000 - 800}{2}$$

$$= 44296 \text{ N-m}$$

$$T_{\text{mean}} = \frac{44296}{8\pi} \Rightarrow 1762.5 \text{ N-m}$$

$$\text{Power } p = \frac{2\pi N T_m}{60} \Rightarrow \frac{2\pi \times 250 \times 1762.5}{60}$$

$$P = 46142 \text{ watts}$$

$$P = 46.142 \text{ kW}$$

(ii) from $\triangle ABG$

$$\frac{IJ}{AG} = \frac{BJ}{BG}$$

$$IJ = \frac{BJ \times AG}{BG}$$

$$= \frac{(3000 - 1762.5) \pi}{(3000 - 800)} \Rightarrow IJ = 1.767$$

from $\triangle CHD$

$$\frac{KL}{HD} = \frac{CK}{CH}$$

$$KL = \frac{CK \times HD}{CH}$$

$$= \frac{(3000 - 1762.5) 2\pi}{(3000 - 800)} \Rightarrow 3.534$$

Fluctuation of energy is equal to the Area above the Mean torque line.

$$e = \text{Area of } IBCL$$

$$= \text{Area } IBJ + \text{Area } BJCK + \text{Area } CKL$$

$$= \left[\frac{1}{2} (1.767) \times (3000 - 1762.5) \right] + 2\pi [3000 - 1762.5] + \frac{1}{2} (3.534) [3000 - 1762.5]$$

$$e = 11055 \text{ N-m}$$

$$\text{Total fluctuation of speed } K = \frac{e}{I \omega^2}$$

$$= \frac{e}{m K^2 \omega^2} \quad [\because I = m K^2]$$

$$K = \frac{11055}{1800 (0.5)^2 \times \left[\frac{2\pi \times 250}{60} \right]^2} \quad [\because \omega = \frac{2\pi N}{60}]$$

$$K = 0.0358$$

$$K = 3.58\%$$

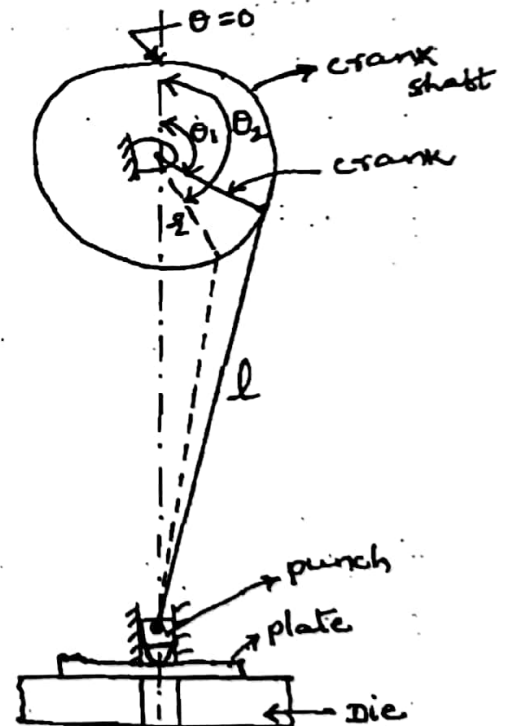
Flywheel in punching Press : - A punching press is driven by a motor which supplies constant torque and the punch is at the position of the sliders in a slider-crank mechanism. From figure, we see that the load acts only during the rotation of the crank from $\theta = \theta_1$ to $\theta = \theta_2$, when the actual punching takes place and the load is zero for the rest of the cycle. Unless a flywheel is used, the speed of the crankshaft will increase too much during the rotation of crank from $\theta = \theta_2$ to $\theta = 2\pi$ or $\theta = 0$ and again from $\theta = 0$ to $\theta = \theta_1$, because there is no load while input energy continues to be supplied. On the other hand, the drop in speed of the crankshaft is very large during the rotation of crank from $\theta = \theta_1$ to $\theta = \theta_2$ due to much load than the energy supplied.

Let E_1 be the energy required for punching a hole. This energy is determined by the size of the hole punched

d_1 = diameter of the hole punched

t_1 = thickness of the plate

τ_u = ultimate shear stress for the plate material.



Maximum shear force required for punching

$$F_s = \text{Area sheared} \times \text{ultimate shear stress} \\ = \pi d_1 t_1 \times \tau_u$$

$\therefore$ work done & energy required for punching a hole,

$$E_1 = \frac{1}{2} \times F_s \times t$$

$$\therefore E = \frac{1}{2} mgh \\ = \frac{1}{2} W \cdot h$$

Assuming one punching operation per revolution, the energy supplied to the shaft per revolution should also be equal to E_1 .