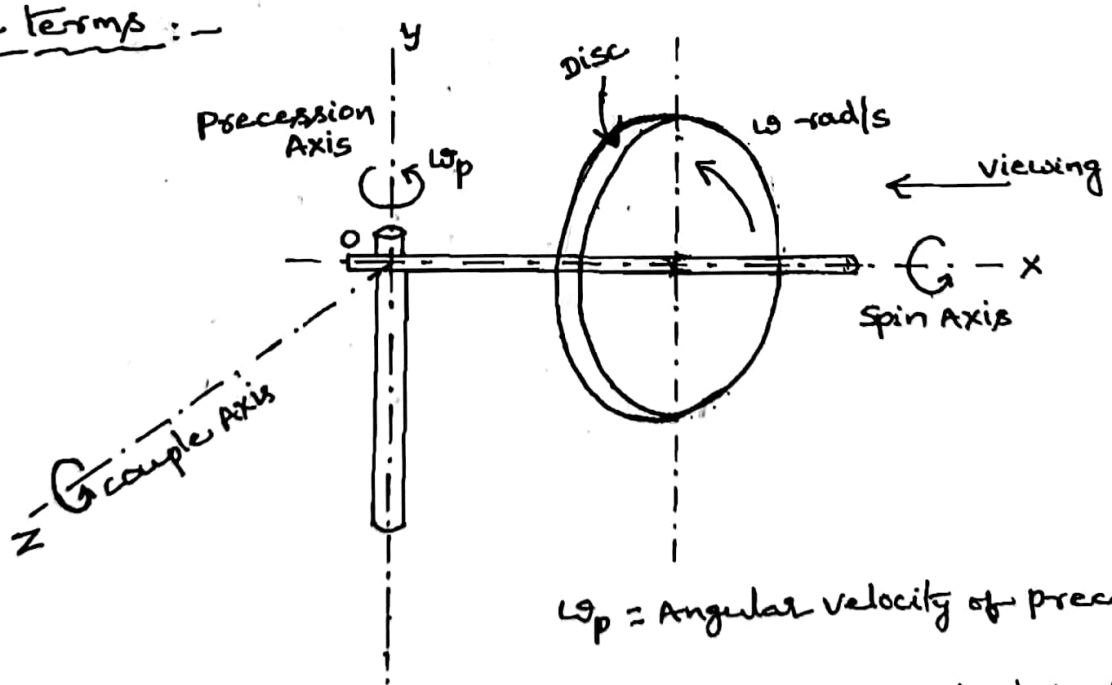
UNIT-I
GYROSCOPE

Introduction:- In the olden days the application of gyroscope was not so familiar. It was perhaps during world war-II when the chineses were in a position to know the use of gyroscope fully. It is just like a toy and free to move in all directions under the action of external forces. whenever a rotating body changes its axis of rotation, a couple is applied on the rotating body [shaft]. This couple is known as gyroscopic couple. The couple is applied on the bearings which support the rotating shaft. The reaction of this couple will be equal and opposite on each bearing. The couple makes a change in the direction of angular velocity, but it does not change the magnitude of angular velocity i.e it remains constant.

The application of gyroscope is seen in aeroplane while taking a turn, steering, pitching and rolling of ships, an automobile rounding a curve, etc

Basic terms:-

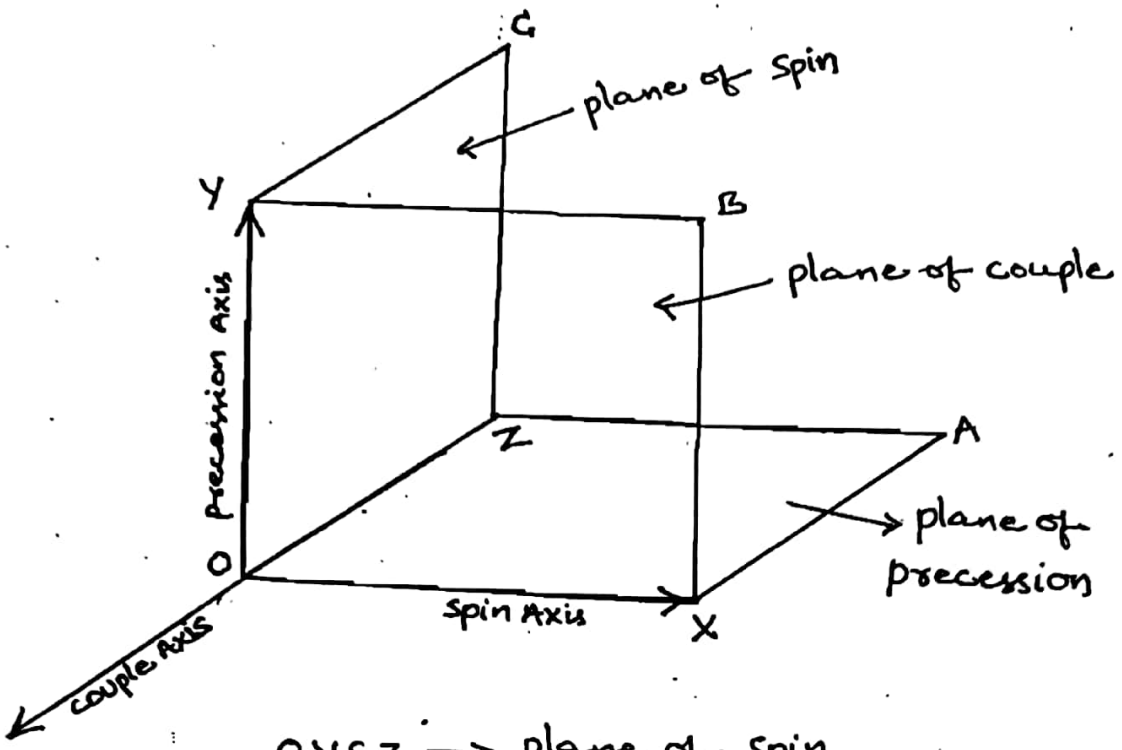


ω_p = Angular velocity of precession

Consider a disc of mass 'm' and radius of gyration 'k' rotating in anticlockwise direction when seen from right side as shown in figure. Let ω rad/sec be the angular velocity of the disc. OX is known as the axis of spin and
OY is the axis of precession
OZ is the axis of couple

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$OYCZ \rightarrow$ plane of Spin

$OXAZ \rightarrow$ plane of precession

$OXBY \rightarrow$ plane of couple

Mass moment of inertia I of the disc about OX-axis

$$I = mk^2 \text{ .... } kg\text{-}m^2$$

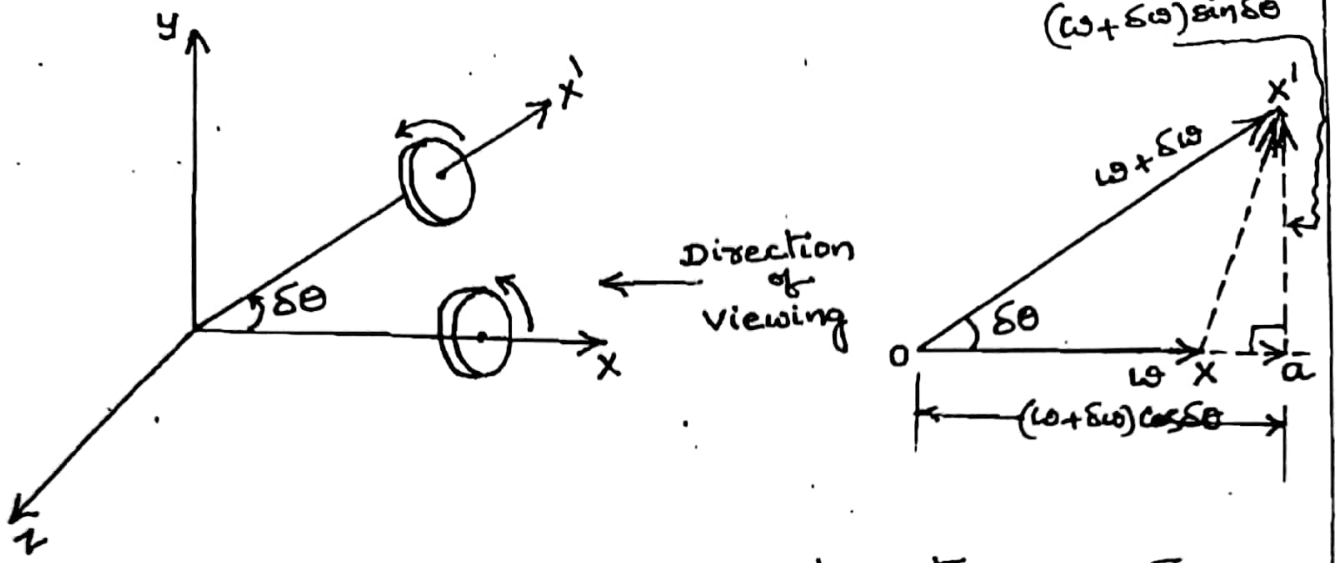
Angular Acceleration :- The disc of angular velocity ω rad/sec is shown rotating in anticlockwise direction. Let I be the mass moment of inertia of the disc about OX axis. OX, OY and OZ are the axes of spin, precession and couple respectively. Let the axis turned to new axis OX' in the plane of precession [XOZ] by an angle $\delta\theta$ in time δt. Angular acceleration is the rate of change of angular velocity and it is a vector quantity which is drawn with the help of "right hand screw rule" as shown in figure.

Right hand screw Rule :- "If a screw is revolving in a fixed nut in clockwise direction and an observer is looking along the axis of rotation, then the rotation can be represented by a vector, the sense of which is along the axis and away from the observer, the length of the vector represents the magnitude. So, vector OX is the angular velocity 'ω' and OX' is the new angular velocity. Say it is $\omega + \delta\omega$.

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Vector xx' represents the change in the angular velocity. It can be resolved into two components xa and ax'.



Change of angular velocity in x-direction, a_x = $(\omega+\delta\omega)\cos\delta\theta - \omega$

Rate of change of angular velocity = $\frac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t}$

Angular acceleration = $\lim_{\delta t \to 0} \frac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t}$ $(\because \cos\delta\theta = 1)$

$= \lim_{\delta t \to 0} \frac{(\omega+\delta\omega) - \omega}{\delta t}$

$$\boxed{\alpha_1 = \frac{d\omega}{dt}}$$

Change of angular velocity in y-direction, $a_{x'}$ = $(\omega+\delta\omega)\sin\delta\theta$

Rate of change of angular velocity = $\frac{(\omega+\delta\omega)\sin\delta\theta}{\delta t}$

Angular acceleration = $\lim_{\delta t \to 0} \frac{(\omega+\delta\omega)\sin\delta\theta}{\delta t}$ $(\because \sin\delta\theta = \delta\theta)$

$= \lim_{\delta t \to 0} \frac{(\omega+\delta\omega)\delta\theta}{\delta t}$

$= \lim_{\delta t \to 0} \frac{\omega.\delta\theta + \delta\omega.\delta\theta}{\delta t}$ $\{\because \delta\omega.\delta\theta$ is very small so, neglected$\}$

$= \lim_{\delta t \to 0} \frac{\omega.\delta\theta}{\delta t}$

$$\boxed{\alpha_2 = \frac{\omega.d\theta}{dt}}$$

Total angular acceleration $\alpha = \alpha_1 + \alpha_2$

$$\alpha = \frac{d\omega}{dt} + \omega.\frac{d\theta}{dt}$$

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when angular velocity 'ω' is constant

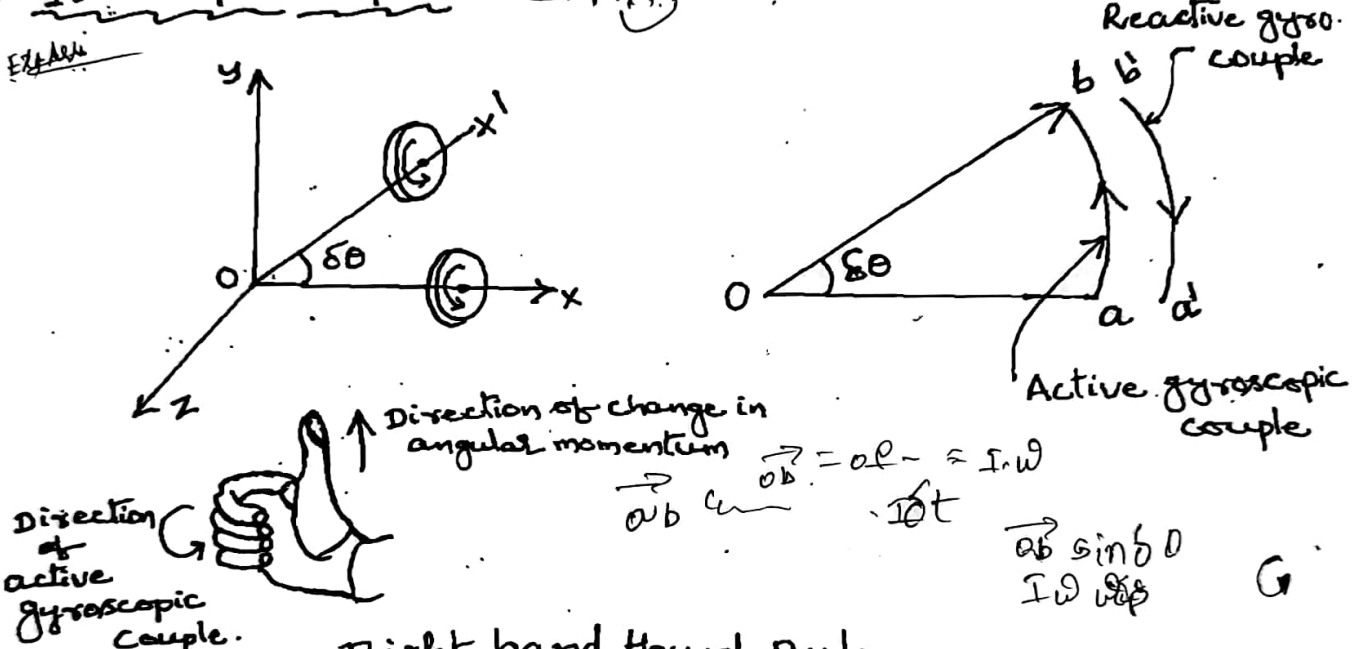
$\frac{d\omega}{dt} = 0$, so equation (i) can be written as

$$\alpha = \omega \cdot \frac{d\theta}{dt}$$

[∴ where $\frac{d\theta}{dt}$ is called the angular velocity of precission and denoted by ω_p.]

$$\boxed{\alpha = \omega \cdot \omega_p}$$

(7) Derive the Gyroscopic couple :-



$\vec{ob} = \vec{oa} = I \cdot \omega$

$\vec{ab} = \vec{ob} \cdot \delta\theta$

$\vec{ab} \sin\delta\theta$

$I\omega\omega_p$

G

Right hand thumb Rule

Let ox changes to ox' making an angle δθ in time δt. Angular velocity 'ω' remains constant and the couple changes the axis of spin of the rotor. Applying right hand thumb rule, the vector diagram of angular momentum as shown in figure.

$\vec{oa}$ = initial angular momentum

$\vec{ob}$ = angular momentum after time δt sec $= I \cdot \omega$

$\vec{ab}$ = change in angular momentum in time δt

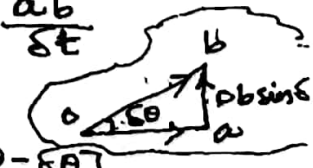
[∴ Angular momentum of the disc $= I \cdot \omega$]

Rate of change of angular momentum $= \frac{\vec{ab}}{\delta t}$

$$= \frac{\vec{ob} \sin\delta\theta}{\delta t}$$

$$= \frac{\vec{ob}\,\delta\theta}{\delta t}$$

[∴ $\sin\delta\theta = \delta\theta$]



[∴ $\delta t \to 0$]

$$C = I \cdot \omega \frac{d\theta}{dt}$$

[$\vec{ob} = I \cdot \omega$] ; [$\frac{d\theta}{dt} = \omega_p$]

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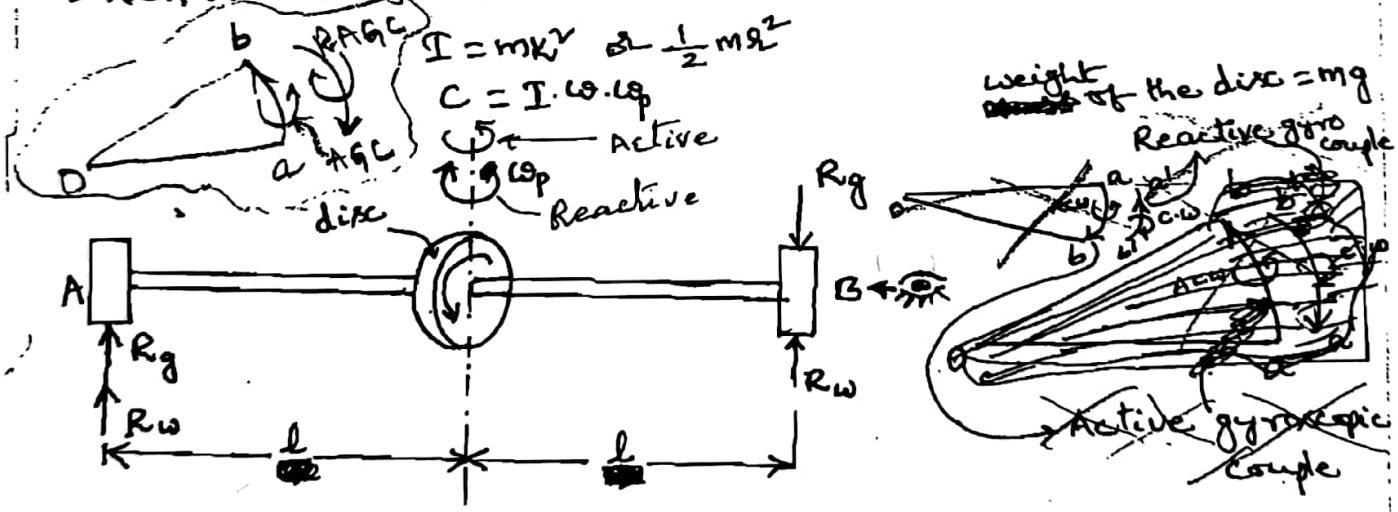
$C = I \cdot \omega \cdot \omega_p$

"Rate of change of angular momentum is known as couple This couple is known as gyroscopic couple.

Gyroscopic couple will be zero when $\omega_p = 0$.

A table fan will not experience a couple so long as its axis of spin is fixed. But as soon as the axis of spin of the fan changes it will generate gyroscopic couple.

Effect of gyroscopic couple on bearings :- consider a disc of mass 'm' and radius of gyration k mounted centrally on a horizontal axle of length 'l' between the bearings. Let the disc spin with angular speed 'ω' counter-clockwise when viewed from the right hand side bearing, as shown in figure. The axle precesses about a vertical axis at ω_p speed in anti clockwise direction when viewed from above. Then



The applied (active) and reactive couples are shown in figure - The reaction couple is clockwise when viewed from front and tend to raise the bearing A and lower the bearing B.

Force on bearing A due to gyro-couple $R_g = \frac{C}{l}$ [upwards]

Force on bearing B due to gyro-couple $R_g = \frac{C}{l}$ [downwards]

Force on each bearing due to self weight of disc $R_w = \frac{W}{2} = \frac{mg}{2}$ [upwards]

Reaction at bearing A = $R_g + R_w$ [upwards]

Reaction at bearing B = $R_g - R_w$ [downwards]

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① A uniform disc of diameter 30cm and weighting 5N is mounted on one end of an arm of length 60cm. The others end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m clock wise, looking from the front, with what speed will it precess about the vertical axis?

(A) Given data:- $d = 30cm \Rightarrow 0.3m$

$$r = \frac{d}{2} \Rightarrow \frac{0.3}{2} \Rightarrow 0.15m$$

$$W = mg = 5N$$

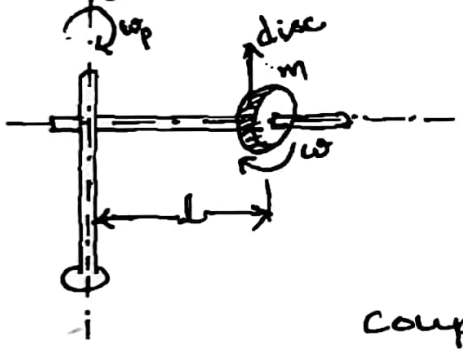
$$m = \frac{5}{g} \qquad [1kg = 9.81N]$$



$$= \frac{5}{9.81} kg \Rightarrow 0.509 kg$$

$$l = 0.6m, \quad N = 300 rpm$$

$$\therefore \omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi(300)}{60} \Rightarrow 31.42 rad/sec$$



we know that moment of inertia of a disc $I = mk^2$ ($k = \frac{1}{2}r$) $= \frac{1}{2}mr^2$

$$I = \frac{1}{2} 0.509 (0.15)^2$$

$$I = 5.726 \times 10^{-3} kg-m^2$$

couple due to weight of disc $C = \omega . l$

$$C = mg . l$$

$$= 0.509 \times 9.81 \times 0.6$$

$$C = 3 kg-m$$

Upward (+)
(-)

Gyroscopic couple $C = I . \omega . \omega_p$

$$3 = 5.726 \times 10^{-3} \times 31.42 \times \omega_p$$

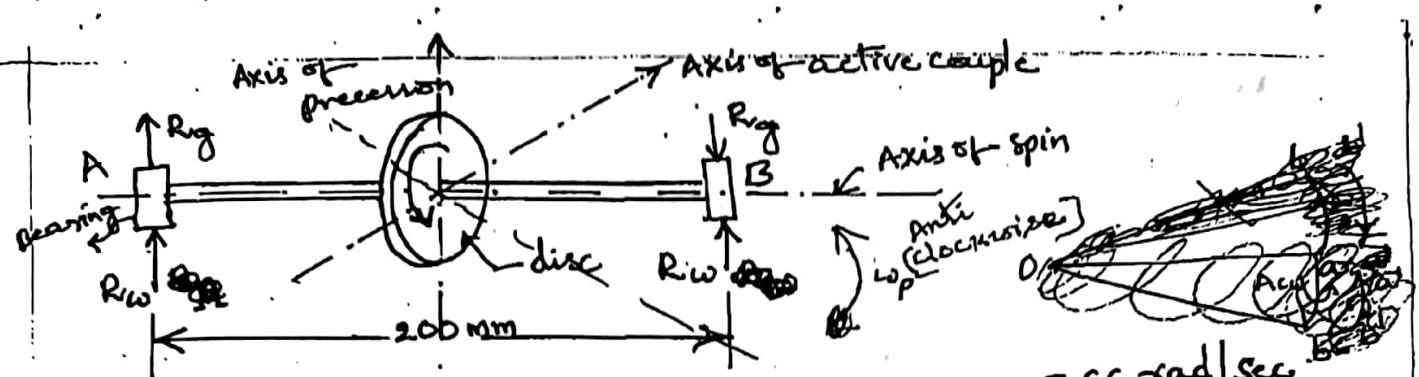
precission angular velocity, $\omega_p = 16.67 rad/sec$.

② A uniform disc of 100mm diameter and 5kg mass is mounted midway between bearings 100mm apart, which keeps it in a horizontal plane. The disc spins about its axis with a constant speed of 1200 rpm, anticlockwise, and precission angular speed of 50 rpm. Find the resultant reaction at each bearing due to the mass and gyroscopic effects.

(A) $d = 100mm \Rightarrow 0.1m$; $r = \frac{d}{2} \Rightarrow \frac{0.1}{2} = 0.05m$

$m = 5kg$

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$\omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi(200)}{60} \Rightarrow 125.66$ rad/sec

$\omega_p = \frac{2\pi N_p}{60} \Rightarrow \frac{2\pi(50)}{60} \Rightarrow 5.236$ rad/sec

$I = \frac{1}{2} m R^2 \Rightarrow \frac{1}{2} \times 5 \times 0.05^2 \Rightarrow 0.00625$ Kg-m^2

$C = I.\omega.\omega_p \Rightarrow 0.00625(125.66)(5.236) \Rightarrow 4.112$ N-m

Bearing reactions :-

(a) Due to self weight of the disc R_w at A & B $= \frac{5 \times 9.81}{2}$

$= 24.525$ N ↑

(b) Due to reaction gyroscopic couple,

$R_A = R_g = \frac{C}{l} \Rightarrow \frac{4.112}{0.1} \Rightarrow 41.12$ N ↑

$R_B = R_g = \frac{C}{l} = \frac{4.112}{0.1} \Rightarrow 41.12$ N ↓

Resultant bearing reactions :-

Anti ↺ Clockwise ↻

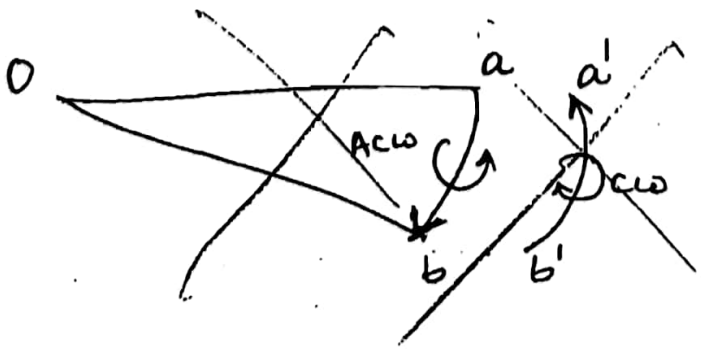
$R_A = R_w + R_g \Rightarrow 24.525 + 41.12$

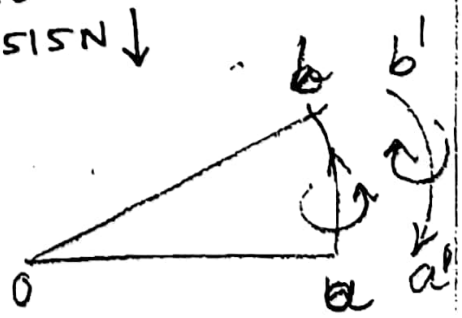
$\Rightarrow 65.645$ N ↑

$R_B = R_w - R_g \Rightarrow 24.525 - 41.12$

$\Rightarrow -16.515$ N

$= 16.515$ N ↓





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Aeroplane problem

① An aeroplane makes a complete half circle of 60m radius, to the left when flying at 200 kmph. The rotary engine and the propeller of the aeroplane weigh 4000N with a radius of gyration 30cm the engine runs at 2500 r.p.m clockwise, when viewed from rear. Find the gyroscopic couple on the aircraft and state its effect on it. Show gyroscopic effect by a sketch.

Given data:-

Radius of curvature (R) = 60m

Velocity of aeroplane (V) $= 200 \text{ kmph} = 200 \times \frac{1000}{3600}$ m/s $= 200 \times \frac{5}{18} = 55.56$ m/s

[$\because$ 1 km = 1000 m, 1 hour = 3600 sec]

Weight of rotating parts of engine (W) = 4000N

$\therefore m = \frac{W}{g} \Rightarrow \frac{4000}{9.81}$ Kgs

Radius of gyration (K) = 30 cm $\Rightarrow$ 0.3m [$\because$ 1m = 100cm]

Speed of engine (N) = 2500 rpm [CW]

Let,

I = Mass moment of inertia of rotating parts $= mK^2$

ω = Angular speed of rotating parts $= \frac{2\pi N}{60}$ rad/sec

ω_p = Angular speed of precession $= \frac{V}{R}$ rad/sec.

(i) Gyroscopic couple [i.e Applied couple]

$$C = I \cdot \omega \cdot \omega_p$$

$$= mK^2 \times \frac{2\pi N}{60} \times \frac{V}{R} \Rightarrow \left[\frac{W}{g}K^2\right] \times \left[\frac{2\pi N}{60}\right] \times \left[\frac{V}{R}\right]$$

$$= \left[\frac{4000}{9.81} \times 0.3^2\right] \times \left[\frac{2\pi \times 2500}{60}\right] \times \left[\frac{55.56}{60}\right]$$

$$= 8896.376 \text{ N-m}$$

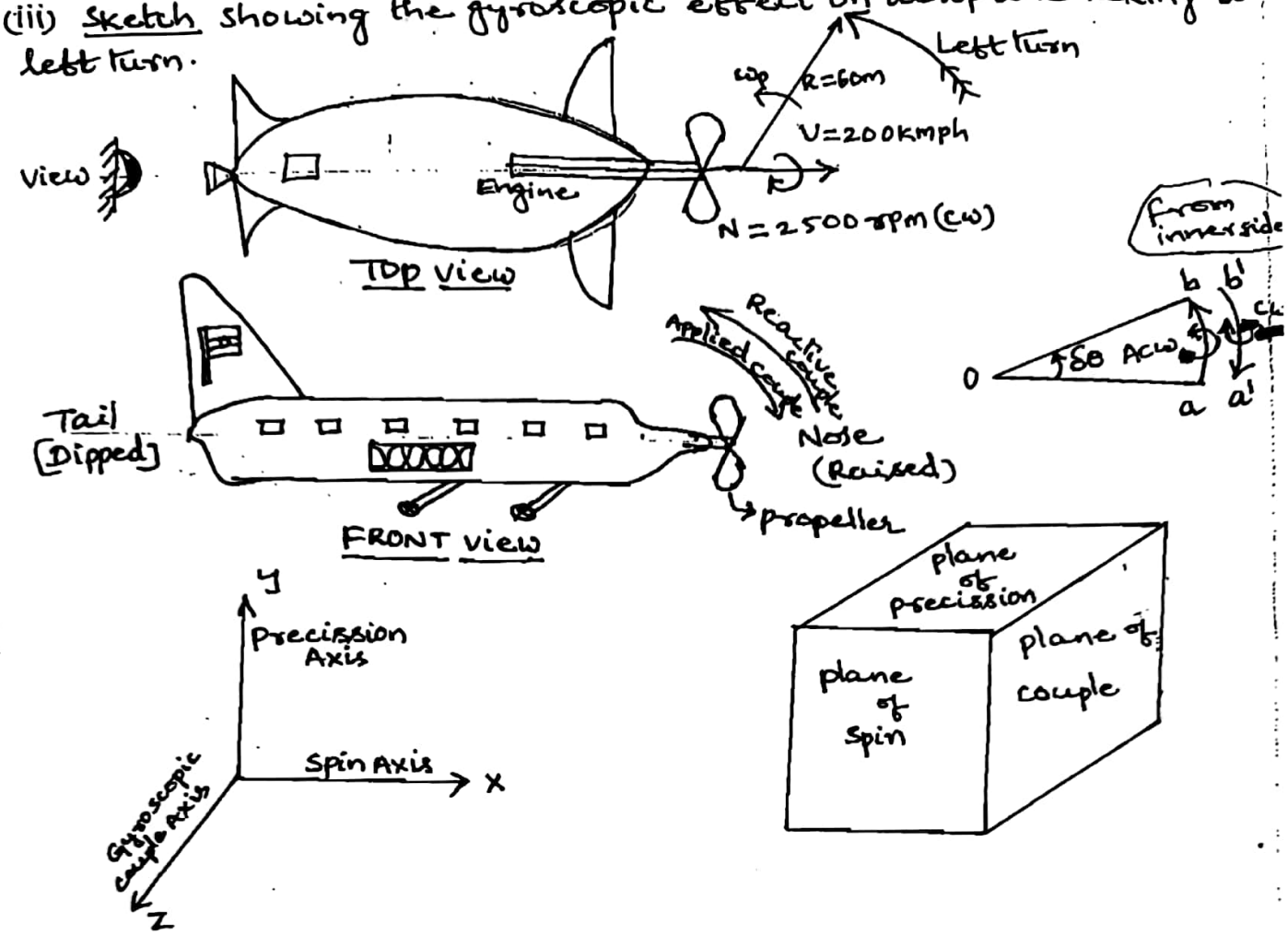
$$\approx 8.9 \text{ KN-m}$$

(ii). Effect:- When viewed from rear, engine and propeller rotating in clockwise direction and aeroplane taking left turn, then the effect of gyroscopic couple i.e reaction couple will raise the nose and dip the tail of aeroplane.

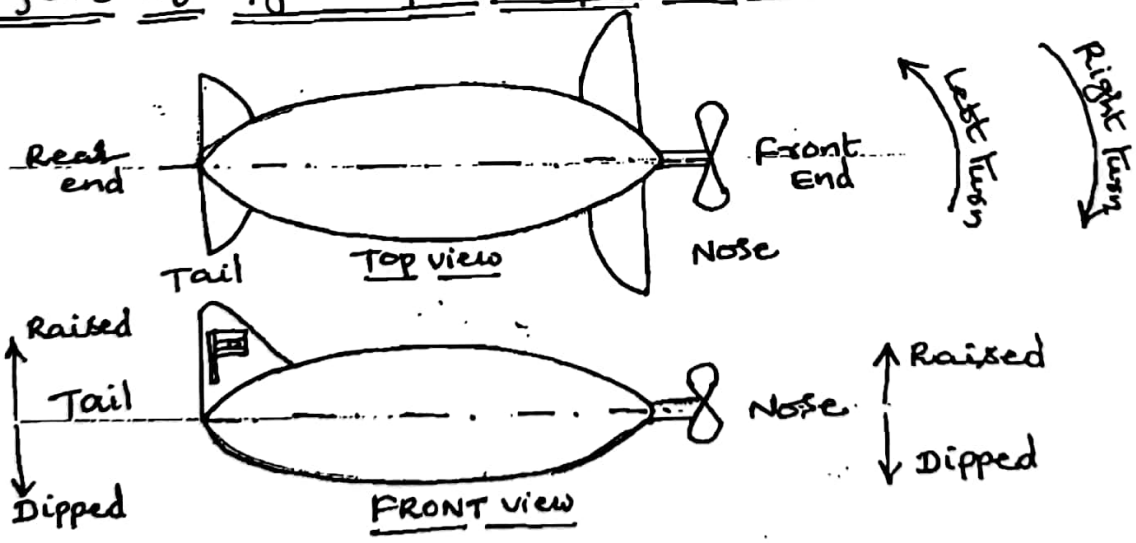
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(iii) Sketch showing the gyroscopic effect on aeroplane taking a left turn.



Effect of Gyroscopic couple on Aeroplane :-

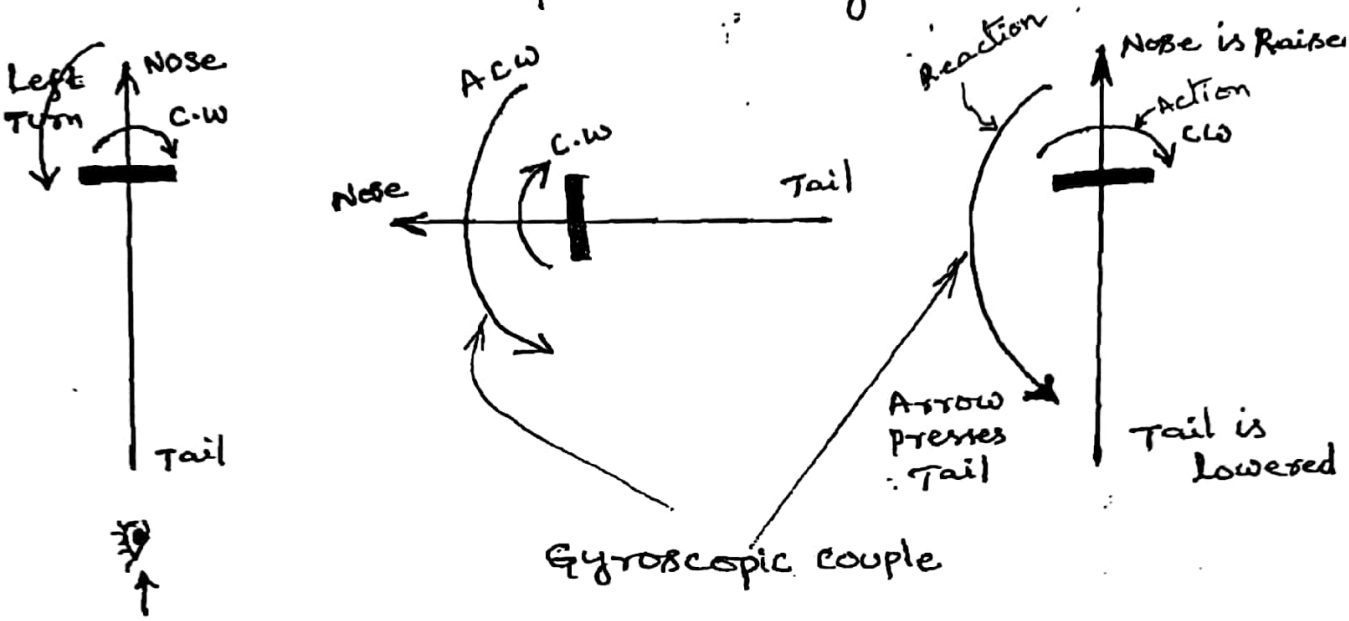


Case i :- When viewing from the rear, engine rotating clockwise and aeroplane turning to left.
Angular momentum diagram is shown in figure (ii) where 'oa' is the angular momentum vector before turning and ob is the angular momentum vector after turning left. vector a b is the active gyroscopic couple and b' a' is the reactive gyroscopic couple which is equal to gyroscopic couple but opposite.

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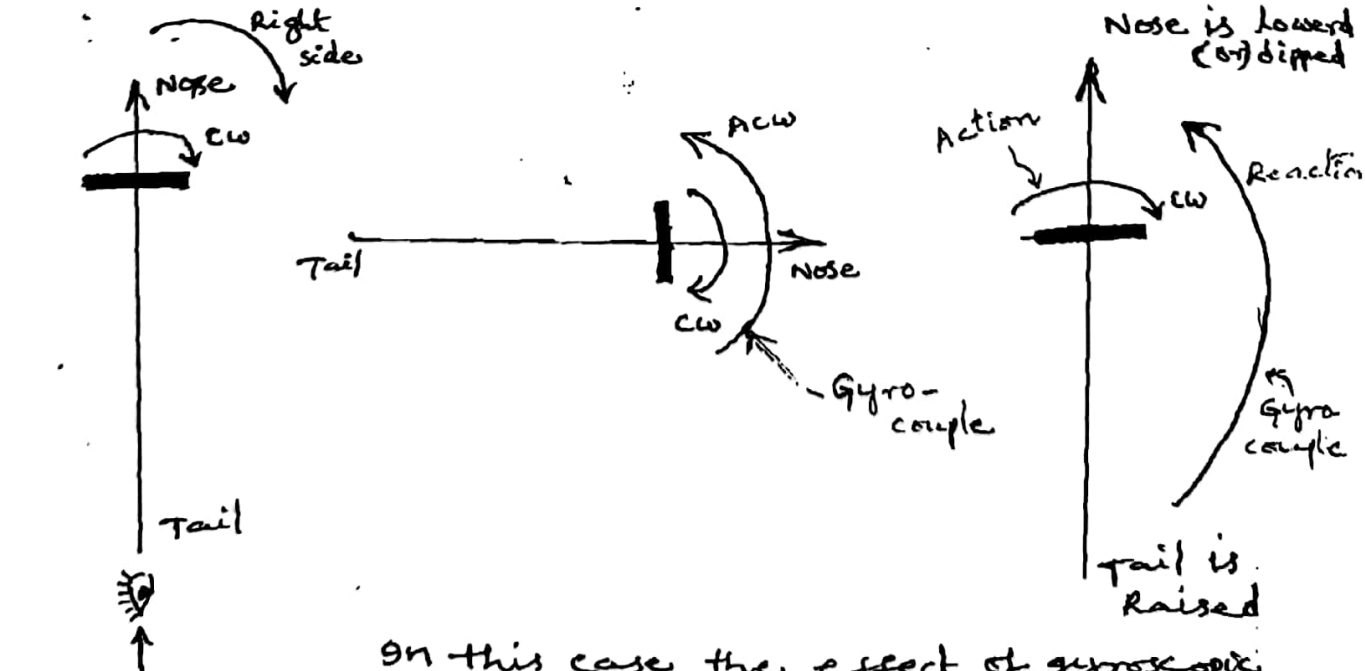
Effect of Gyroscopic couple on Aeroplane :-

case(1) :- When viewing from the rear, engine rotating clockwise and aeroplane turning to left.



On this case the effect of gyroscopic couple due to Raise the Nose and dipped the ~~nose~~ tail.

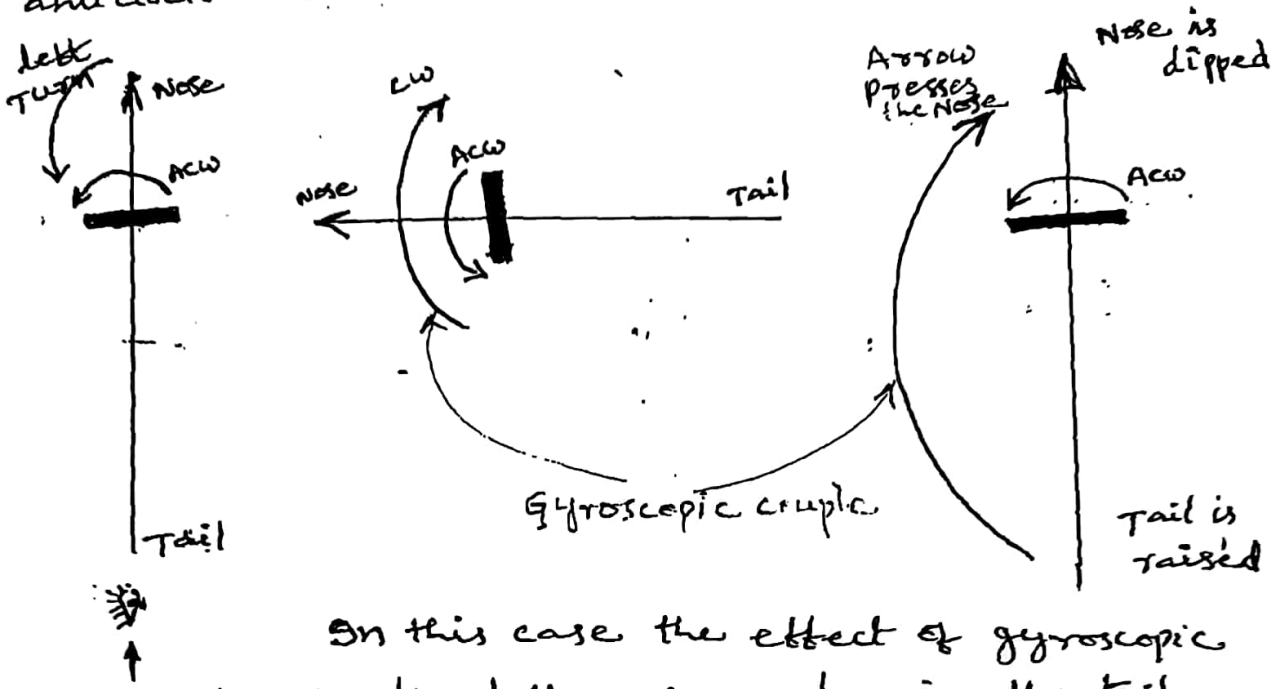
case (2) :- When viewing from the rear, engine rotating clockwise and aeroplane turning to Right.



On this case the effect of gyroscopic couple due to dipped the Nose and Raised the tail.

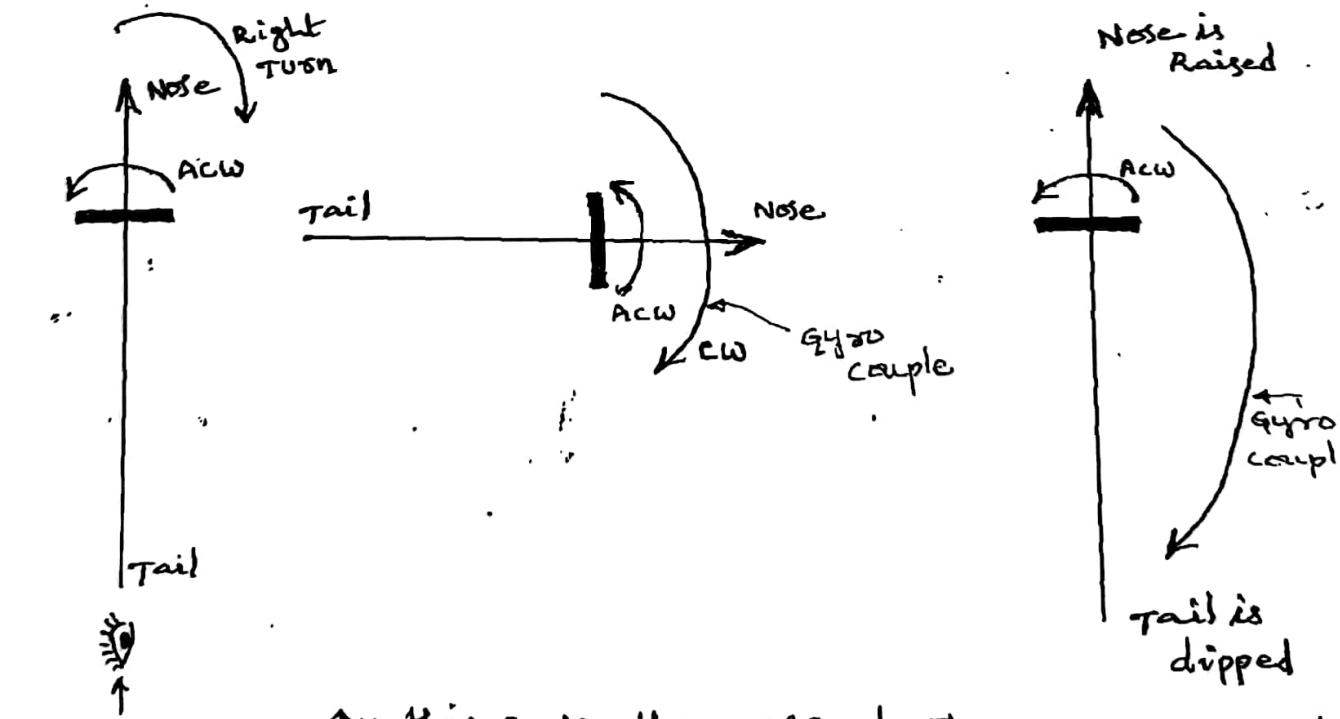
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Case(3) :- When viewing from the rear, Engine rotating anticlockwise and aeroplane turning to left.



On this case the effect of gyroscopic couple due to dipped the nose and raise the tail.

Case(4) :- When viewing from the rear, engine rotating anticlockwise and aeroplane turning to right side.



On this case the effect of gyroscopic couple due to raise the nose and dipped the tail.

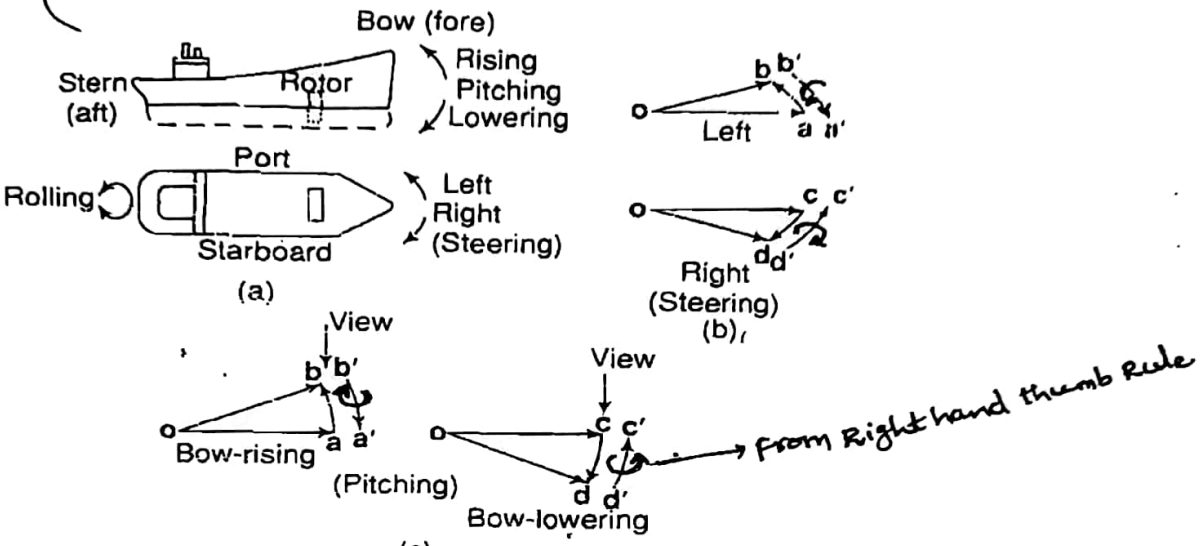
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* **Gyroscopic Effect on Naval ships :-** Some of the terms used in connection with the motion of naval ships or sea vessels are given below:

(i) **Bow** is the fore or the front end.

(ii) **Stern** or aft is the rear end.

(iii) **Starboard** is the right-hand side when looking from the stern.



[iv]. **Port** is the left-hand side when looking from the stern.

(v). **Steering** is the turning on the sides when viewing from the top.

(vi). **pitching** is limited angular motion of the ship about the transverse axis.

(vii) **Rolling** is limited angular motion of the ship about the longitudinal axis.

Let the plane of spin of the rotor and other rotating masses be horizontal and across the breadth of the ship. Assume 'ω' to be the angular velocity of the rotor in the clockwise direction when viewed from stern [rear end].

Gyroscopic effect during steering :- when the ship turns left, the angular momentum vector changes from oa to ob [fig (b)]. the reaction couple is found to be b'a' which tends to raise the bow and lower the stern. on turning right, the reaction couple is reversed so that bow is lowered and the stern is raised.

Gyroscopic effect on pitching :- pitching of usually considered to take place with simple harmonic motion. So

ω_p = Angular velocity of S.H.M = $\frac{2\pi}{\text{Time period}}$

$$\omega_p = \frac{2\pi}{T}$$

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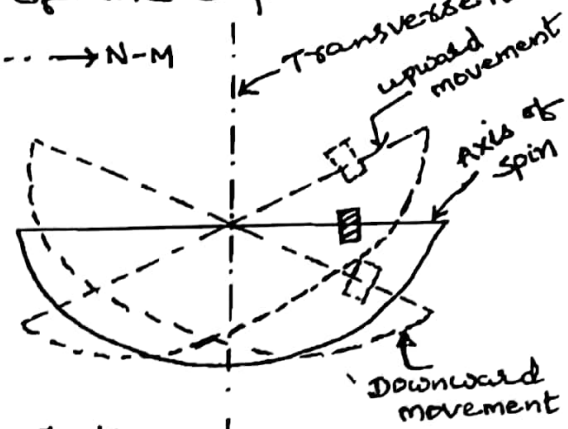
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D-MM-2
T-E-2, 3

precession angular velocity $\omega_p = \phi \omega$

where ϕ = pitch of the ship.

Gyroscopic couple $C = I \cdot \omega \cdot \omega_p$ ---→ N-M



Pitching of Ship

Gyroscopic effect on Rolling :–

As the axes of the rolling of the Ship and of the rotor are parallel, there is no precession of the axis of spin and thus there is no gyroscopic effect.

Problems:–

① An aeroplane makes a complete half circle of 80m radius, to the left when flying at 300 Kmph. The rotary engine and the propeller of the aeroplane weigh 4800N with a radius of gyration 30cm the engine runs at 3500 r.p.m clockwise when viewed from rear. Find the gyroscopic couple on the aircraft and state its effect on it. Show gyroscopic effect by a sketch.

Given data:– Home work.

① The rotor of the turbine of a ship has a mass of 2500 kg and rotates at a speed of 3200 rpm counter-clockwise when viewed from stern. The rotor has radius of gyration of 0.4 m. Determine the gyroscopic couple and its effect when,

(i) the ship steers to the left in a curve of 80-m radius at a speed of 15 knots [1 knot = 1860 m/h].

(ii) the ship pitches 5 degrees above and 5 degrees below the normal position and the bow is descending with its maximum velocity, the pitching motion is simple harmonic with a periodic time of 40 seconds.

(iii) the ship rolls and at the instant, its angular velocity is 0.04 rad/sec clockwise when viewed from stern. Also find the maximum angular acceleration during pitching.

Given data:- [∵ 1 knot = 1860 m/h]

$m = 2500$ kg ; $N = 3800$ rpm

$K = 0.4$ m ; $V = 15$ knots $\Rightarrow 15 \times 1860$ m/hour

$\Rightarrow 15 \times \frac{1860}{3600}$ m/sec [∵ 1 hour = 3600 sec]

$V \Rightarrow 7.75$ m/sec

$I = mK^2 \Rightarrow 2500 \times (0.4)^2 \Rightarrow 400$ kg-m^2

$\omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi[3800]}{60} \Rightarrow 335$ rad/sec.

(i) $R = 80$ m ; $\omega_p = \frac{V}{R} \Rightarrow \frac{7.75}{80} \Rightarrow \omega_p = 0.097$ rad/sec

Gyroscopic couple $C = I \cdot \omega \cdot \omega_p$

$= 400 \times 335 \times 0.097$

$C = 12981$ N-m

The effect is to lower the bow and raise the stern [Fig a & b]

(ii) $\phi = 5^\circ = 5 \times \frac{\pi}{180} \Rightarrow 0.0873$ rad

$T = 40$ sec

$\therefore \omega_0 = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{40} \Rightarrow 0.157$ rad/sec

$\omega_p = \phi \omega_0 \Rightarrow 0.0873 \times 0.157 \Rightarrow 0.0137$ rad/sec

$C = I \cdot \omega \cdot \omega_p \Rightarrow 400 \times 335 \times 0.0137 \Rightarrow 1837.5$ N-m

As the bow descends during pitching, the ship would turn towards right or starboard [Fig a & c].

(iii) $\omega_p = 0.04$ rad/sec

$C = 400 \times 335 \times 0.04 \Rightarrow 5360$ N-m

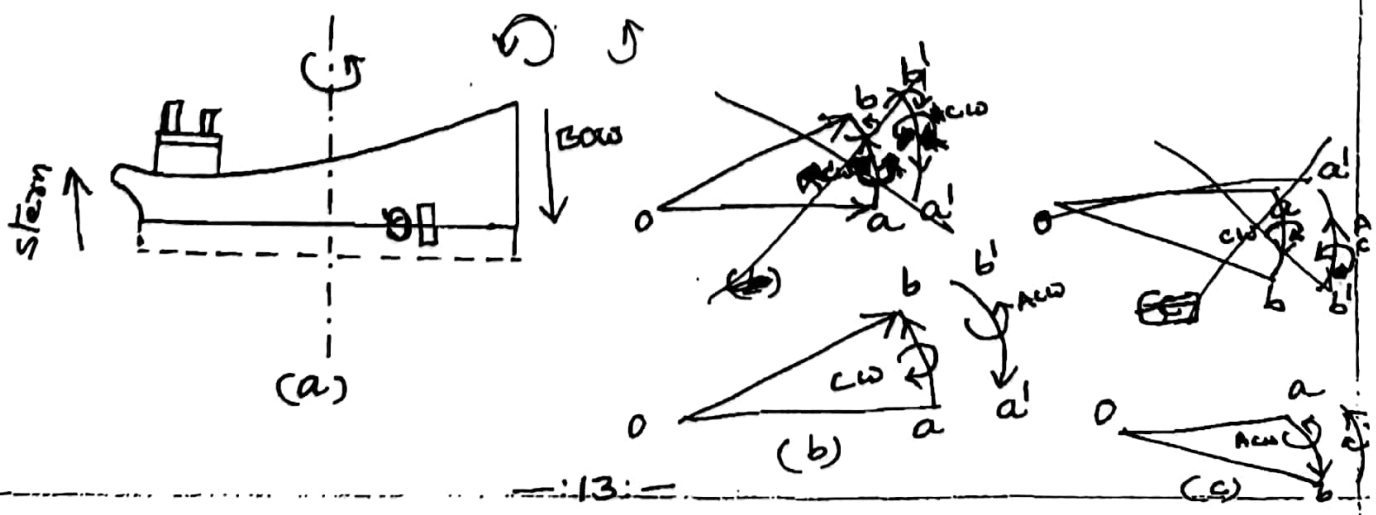
No gyroscopic effect [in Rolling]

$\omega_p = \checkmark$

Maximum angular acceleration during pitching.

$\alpha_{max} = \phi \omega_0^2$

$= 0.0873 \times [0.157]^2 \Rightarrow 0.00215$ rad/sec^2



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② The turbine rotor of a ship has a mass of 20 tones and a radius of gyration of 0.75 m. Its speed is 2000 rpm. The ship pitches 6° above and below the horizontal position. one complete oscillation takes 18 seconds and the motion is simple harmonic. Determine,

(a) the maximum couple tending to shear the holding down bolts of the turbine.

(b) the maximum angular acceleration of the ship during pitching and,

(c) the direction in which the bow will tend to turn while rising, if the rotation of the rotor is clockwise when looking from rear.

Given data:-

$m = 20$ tones $\Rightarrow 20 \times 10^3$ kg $\quad [\because$ tone $= 10^3$ kg$]$

$K = 0.75$ m

$N = 2000$ r.p.m $\Rightarrow \omega = \frac{2\pi N}{60} \Rightarrow \frac{2 \times \pi \times 2000}{60} \Rightarrow 209.52$ rad/sec

$\phi = 6^\circ \Rightarrow 6 \times \frac{\pi}{180} \Rightarrow 0.1047$ rad

$t_p = 18$ sec

moment of inertia of the rotor $I = mK^2$

$= 20{,}000 \times 0.75^2 \Rightarrow 11250$ Kg-m^2

Angular velocity of simple harmonic motion,

$\omega_0 = \frac{2\pi}{t_p} \Rightarrow \frac{2\pi}{18} \Rightarrow 0.349$ rad/sec

Maximum angular velocity of precession,

$\omega_{p\,max} = \phi \omega_0$

$= 0.1047 \times 0.349 \Rightarrow 0.0365$ rad/sec

(a) $\therefore$ Maximum couple $= I\omega \cdot \omega_{p\,max}$

$= 11250 \times 209.43 \times 0.0365$

$= 85997.19$ N-m $\Rightarrow 85.99$ KN-m //

(b) Maximum angular acceleration during pitching

$= \phi(\omega_0)^2 \Rightarrow 0.1047 \times [0.349]^2 \Rightarrow 0.0127$ rad/s

(c) "When the rotation of rotor is clockwise looking from the rear and when the bow is rising, then the reactive couple acts in the clockwise direction turning the bow towards right"

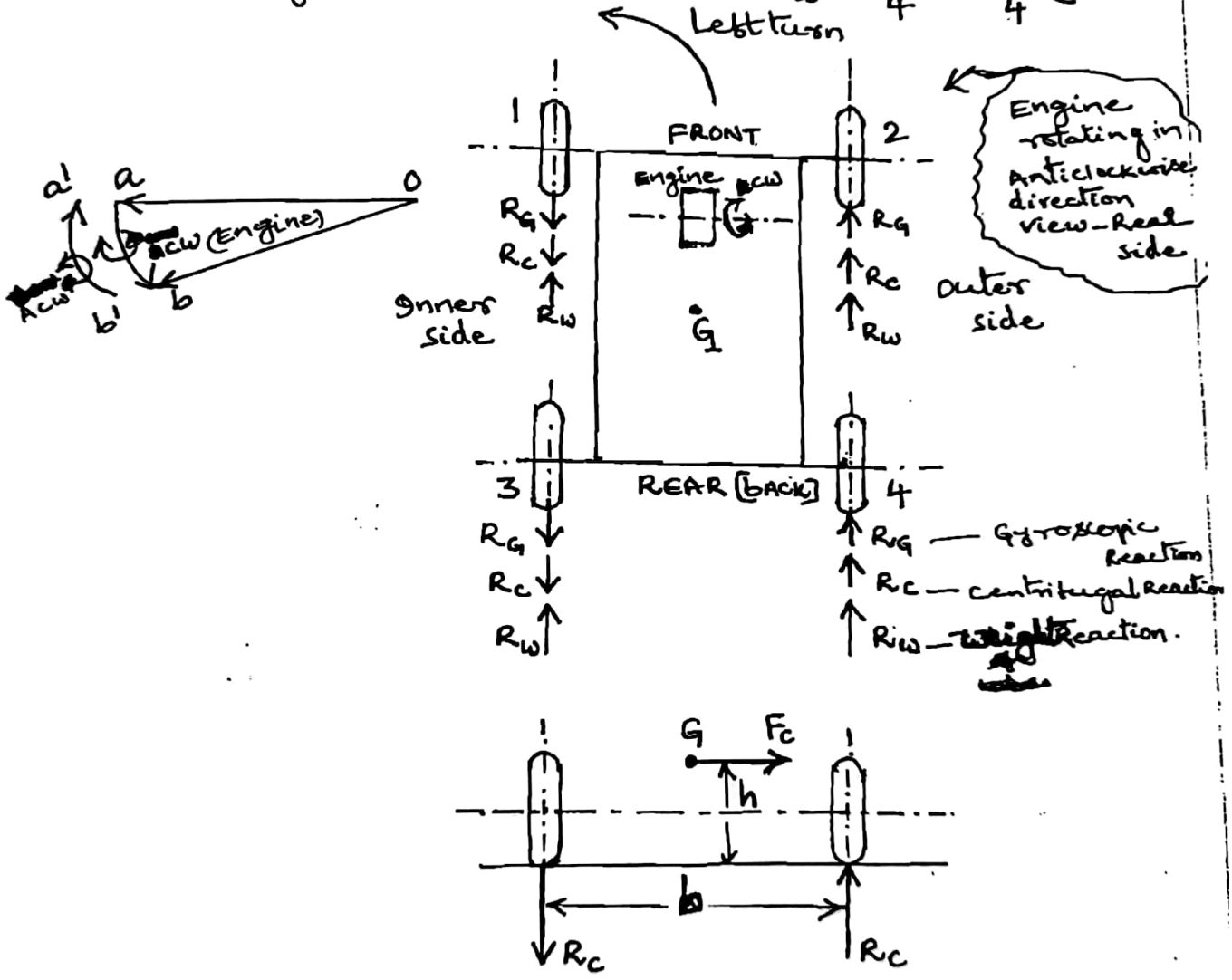
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* Stability of an Automobile :-

On case of a four-wheeled vehicle, it is essential that no wheel is lifted off the ground while the vehicle takes a turn. Figure shows a four-wheeled vehicle having a mass 'm'. Assuming that the weight is equally divided among the four wheels.

weight on each wheel $= \frac{W}{4} \Rightarrow \frac{mg}{4}$ [downwards]

Reaction of ground on each wheel $R_W = \frac{W}{4} \Rightarrow \frac{mg}{4}$ [upwards]



Effect of Gyroscopic couple :-

Gyroscopic couple due to four wheels $C_W = 4 I_W \omega_W \cdot \omega_P$

$$= 4 I_W \cdot \frac{V}{r} \times \frac{V}{R} \Rightarrow 4 I_W \frac{V^2}{r \cdot R}$$

where I_W = mass moment of inertia of each wheel

ω_W = angular velocity of wheels $= \frac{V}{r}$

ω_P = angular velocity of precession $= \frac{V}{R}$

V = linear velocity of the vehicle

R = Radius of curvature.

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Gyroscopic couple due to engine rotating parts $C_e = I_e \cdot \omega_e \cdot \omega_p$

$$= I_e \omega_e \cdot \omega_p \times \frac{\omega_w}{\omega_w}$$

$$= I_e \frac{\omega_e}{\omega_w} \cdot \omega_w \cdot \omega_p$$

$$C_e = I_e \cdot G \cdot \omega_w \cdot \omega_p \qquad \left[\because G = \frac{\omega_e}{\omega_w}\right]$$

where G = Gear ratio.

Total gyroscopic couple, $C_G = C_w + C_e$

positive sign is used when the engine parts rotate in the same direction as the wheel and the negative sign when they rotate in the opposite.

Assuming that C_G is positive and the vehicle takes a left turn, the reaction gyroscopic couple on it is clockwise when viewed from the rear of the vehicle. The reaction couple is provided by equal and opposite forces on the outer and the inner wheels of the vehicles.

Forces on the two outer wheels = $\frac{C_G}{b}$ [downwards]

Forces on the two inner wheels = $\frac{C_G}{b}$ [upwards]

Forces on each of the outer wheels = $\frac{C_G}{2b}$ [downwards]

Forces on each of the inner wheels = $\frac{C_G}{2b}$ [upwards]

Thus the force on each of the outer wheels is similar to the weight on the inner wheels it is in the opposite direction, thus,

Reaction of ground on each outer wheel $R_G = \frac{C_G}{2b}$ [upward]

Reaction of ground on each inner wheel $R_G = \frac{C_G}{2b}$ [downward]

Effect of centrifugal couple :- As the vehicle moves on a curved path, a centrifugal force also acts on the vehicle in the outward direction at the centre of mass of the vehicle.

Centrifugal force $= mR\omega_p^2$

$$= mR\left(\frac{V}{R}\right)^2$$

$$= \frac{mV^2}{R}$$

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This force would tend to overturn the vehicle outwards and the overturning couple will be

$$C_c = \frac{mv^2}{R} \times h$$

This is equivalent to a couple due to equal and opposite forces on outer and inner wheels.

Force on each outer wheel $= \frac{C_c}{2b}$ [downward]

Force on each inner wheel $= \frac{C_c}{2b}$ [upwards]

[∵ Force = Reaction]

Again, the force on each of the outer wheels is similar to the weight and on each of the inner wheels, it is opposite.

Reaction of ground on each outer wheel

$$R_c = \frac{C_c}{2b} \text{ [upwards]}$$

Reaction of ground on each inner wheel

$$R_c = \frac{C_c}{2b} \text{ [downwards]}$$

Vertical reaction on each outer wheel $= \frac{W}{4} + \frac{C_G}{2b} + \frac{C_c}{2b}$ [upwards]

Vertical reaction on each inner wheel $= \frac{W}{4} + \frac{C_G}{2b} + \frac{C_c}{2b}$ [upwards]

It can be observed that there are chances that the reaction of the ground on the inner wheels may not be upwards and thus the wheels are lifted from the ground. For positive reaction, the conditions will be

$$\frac{W}{4} - \frac{C_G}{2b} - \frac{C_c}{2b} \geq 0$$

$$\frac{W}{4} \geq \frac{C_G + C_c}{2b}$$

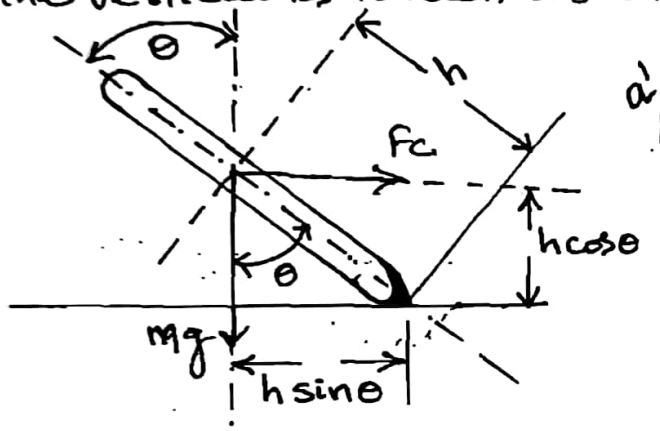
$$\boxed{R_W \geq R_G + R_C}$$

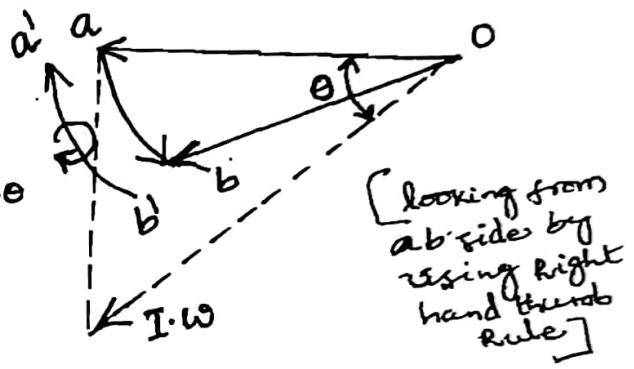
∴ where
b = ~~width~~ distance between the two wheels

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** Stability of a Two-wheel Vehicle :- The case of a two wheel vehicle can be taken in the same way as that of an automobile. However it is easier to tilt such a vehicle inwards to neutralise the overturning effect and the vehicle can stay in equilibrium while taking a turn.

Let a vehicle take a left turn as shown in figure. The vehicle is inclined to the vertical [inwards] for equilibrium. The angle of inclination of the vehicle to the vertical is known as the angle of heel.





Let
V = Linear velocity of vehicle on the track
r = Radius of the wheel
R = Radius of track
I_w = moment of inertia of each wheel
I_e = m.o.I of rotating parts of the engine
m = Total mass of the vehicle and the rider
ω_w = Angular velocity of the wheel
ω_e = Angular velocity of rotating parts of the engine
G = Gear ratio
h = height of centre of mass of the vehicle and the rider
θ = Inclination of vehicle to the vertical [angle of heel]

As the axis of spin is not horizontal but inclined to the vertical at an angle 'θ' and the axis of precession is vertical. It is necessary to take the horizontal component of the spin vector.

Spin vector [horizontal] $= I \cdot \omega \cos\theta$

$= [2 I_w \omega_w + I_e \cdot \omega_e] \cos\theta$

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and overturning couple due to Gyroscopic couple $= \left[2 I_w \omega_w + I_e \cdot G \cdot \omega_w\right] \cos\theta \times \omega_p$

$= \left[2 I_w + I_e \cdot G\right] \omega_w \times \omega_p \times \cos\theta$

$= \left[2 I_w + G I_e\right] \frac{V}{r} \cdot \frac{V}{R} \cos\theta$

$= \left[2 I_w + G I_e\right] \frac{V^2}{r \cdot R} \cos\theta$

$\left[\because \omega_w = \frac{V}{r}, \quad \omega_p = \frac{V}{R}\right]$

The reaction couple b'a' is clockwise when viewed from the rear (back) of the vehicle and tends to overturn it is the outward direction.

overturning couple due to centrifugal force $= \left[\frac{mV^2}{R}\right]\left[h\cos\theta\right]$

$\therefore$ total overturning couple $=$

$\left[2 I_w + G \cdot I_e\right] \frac{V^2}{r \cdot R} \cos\theta + \frac{mV^2}{R} h\cos\theta$

$= \frac{V^2}{R}\left[\frac{2 I_w + G \cdot I_e}{r} + mh\right] \cos\theta$ ——— (1)

Rightening [balancing] couple due to the weight of the vehicle $= mg\left[h \sin\theta\right]$ ——— (2)

For equilibrium, equating equations (1) & (2)

$\frac{V^2}{R}\left[\frac{2 I_w + G \cdot I_e}{r} + mh\right] \cos\theta = mgh \sin\theta$

$$\tan\theta = \frac{V^2}{mgh \cdot R}\left[\frac{2 I_w + G \cdot I_e}{r} + mh\right]$$

From this relation, the angle of heel 'θ' can be determined to avoid skidding of the vehicle.

problems

** Each wheel of a motor cycle is of 600 mm diameter and has a moment of inertia of 1.2 kg-m^2. The total mass of the motor cycle and rider is 180 Kg and the combined centre of mass is 580 mm above the ground level when the motor cycle is upright. The moment of inertia of the rotating parts of the engine is 0.2 kg-m^2. The engine

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speed is 5 times the speed of the wheels and is in the same sense. Determine the angle of heel necessary when the motor cycle takes a turn of 35m radius at a speed of 54 km/hr.

Given data:- $m = 180$ kg

$l = 0.3$ m

$R = 35$ m

$G = \frac{\omega_e}{\omega_w} = 5$

$I_w = 1.2$ kg-m^2

$I_e = 0.2$ kg-m^2

$V = 54$ kmph $= 54 \times \frac{1000}{3600}$

$V = 15$ m/sec

$h = 0.58$ m

Gyroscopic couple $C_G = [2I_w + G \cdot I_e]\frac{V^2}{l \cdot R}\cos\theta$

centrifugal couple $C_c = \frac{mV^2}{R} h\cos\theta$

total overturning couple $= G \cdot C + C \cdot C$ ——— (1)

Rightening couple $= mgh\sin\theta$ ——— (2)

Equating (1) & (2)

$$\tan\theta = \frac{V^2}{mgh \cdot R}\left[\frac{2I_w + G \cdot I_e}{l} + mh\right]$$

$$= \frac{15^2}{180 \times 9.81 [0.58][35]}\left[\frac{2(1.2) + 5(0.2)}{0.3} + 180[0.58]\right]$$

$\tan\theta = 0.727$

$\theta = \tan^{-1}[0.727]$

$\theta = 36^\circ$

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Four wheel vehicles

① The total mass of a four-wheeled trolley car is 1800Kg. The car runs on rails of 1.6m gauge and rounds a curve of 24m radius at 36 Km/hr. The track is banked at 10°. The external diameter of the wheels is 600mm and each pair with axle has a mass of 180Kg with radius of gyration of 240mm. The height of the centre of mass of the car above the wheel base is 950mm. Determine the pressure on each rail allowing for centrifugal force and gyroscopic couple actions.

Sol Given data :-

$M = 1800Kg$

b = width $= 1.6m$

$R = 24m$

pair, $m = 180Kg$

$\theta = 10°$

$h = 0.95m$

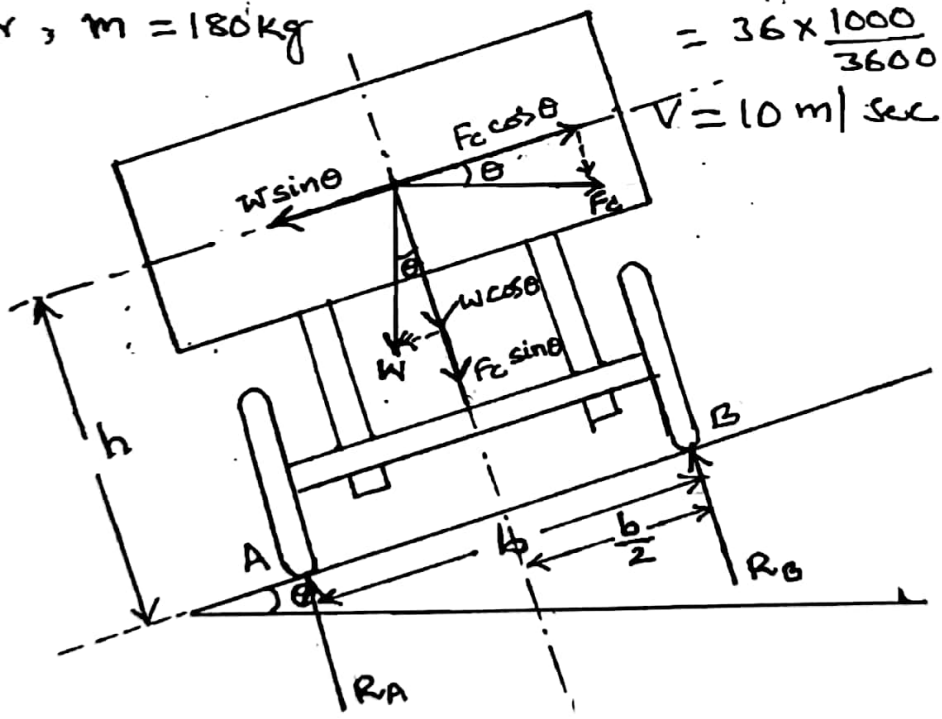
$k = 0.24m$

$d = 600mm = 0.6m$

$r = 0.3m$

$V = 36\ Km/hr = 36 \times \frac{1000}{3600}\ m/sec$

$V = 10\ m/sec$



First considering the effect of dead weight ($W = mg$) of the car and that of the centrifugal force on it, determine the reactions R_A and R_B at the wheels A and B. as shown in figure.

Taking moments about B, $\Sigma M_B = 0$

$$-R_A \times b + Fc \sin\theta \left(\frac{b}{2}\right) + W\cos\theta\left(\frac{b}{2}\right) + W \sin\theta[h] - Fc\cos\theta[h] = 0$$

$$-R_A \times b + \frac{MV^2}{R} \sin\theta\left[\frac{b}{2}\right] + mg\cos\theta\left[\frac{b}{2}\right] + mg \sin\theta[h] - \frac{mV^2}{R}\cos\theta[h] =$$

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$$-R_A \times 1.6 + \frac{1800(10)^2}{24} \sin 10^\circ \left[\frac{1.6}{2}\right] + 1800 \times 9.81 \cos 10^\circ \left[\frac{1.6}{2}\right] + 1800(9.81) \sin 10^\circ (0.95) - \frac{1800(10)^2}{24} \cos 10^\circ [0.95] = 0$$

$R_A = 6781N$

Upward forces = downward forces

$$R_A + R_B = F_c \sin\theta + W\cos\theta \qquad \left[\because F_c = \frac{MV^2}{R},\ W = mg\right]$$

$$= \frac{MV^2}{R}\sin\theta + Mg\cos\theta$$

$$R_B = \frac{1800(10)^2}{24} \sin 10^\circ + 1800(9.81)\cos 10^\circ - 6781$$

$$R_B = 11911N$$

gyroscopic couple of two outer wheels

$$C_W = 2 I_w \omega_w \omega_p \cos\theta$$

$$= 2mK^2 \frac{V}{r} \cdot \frac{V}{R} \cos\theta \qquad [m = 180kg]$$

$$= 2(180)(0.24)^2 \frac{10}{0.3} \times \frac{10}{24} \times \cos 10^\circ$$

$$= 283.6 N-M$$

force on two outer wheels $= \frac{C_W}{b}$

Reaction on each outer wheel $R_{Go} = \frac{C_W}{2b}$

$$= \frac{283.6}{2 \times 1.6}$$

$$= 88.6N \text{ [upwards]}$$

Reaction on each inner wheel $R_{Gi} = 88.6N$ [downwards]

Pressure on outer rails $= R_B + R_{Go}$

$= 11911 + 88.6 \Rightarrow 11999.6N$ [downward]

Pressure on inner rails $= R_A - R_{Gi}$

$= 6781 - 88.6$

$= 6692.4N$ [upwards]

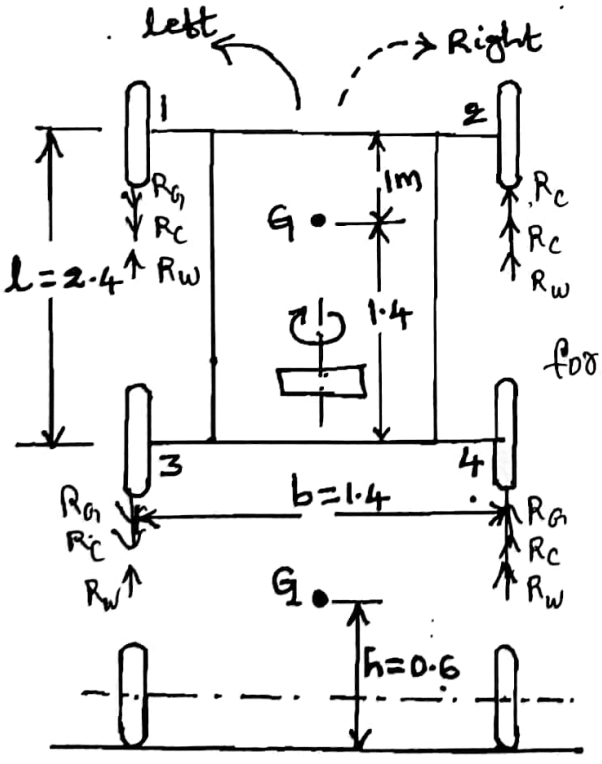
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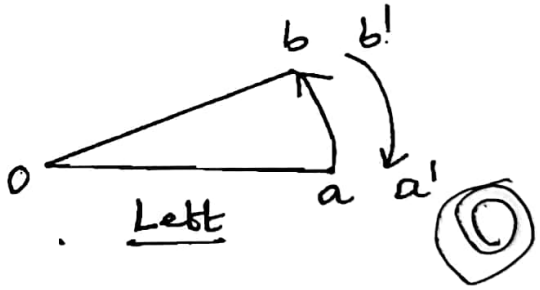
(2) A 2.2 tonne racing car has a wheel base of 2.4m and a track of 1.4m. The centre of mass of the car lies at 0.6m above the ground and 1.4m from the rear axle. Equivalent mass of engine parts is 140kg with radius of gyration of 150mm. The back axle ratio is 5. The engine shaft and flywheel rotate clockwise when viewed from front. Each wheel has a diameter of 0.8m and a moment of inertia of 0.7 kg-m². Determine the load distribution on the wheels when the car is rounding a curve of 100m radius at a speed of 72 km/hr to the (i) left (ii) right.

Sol

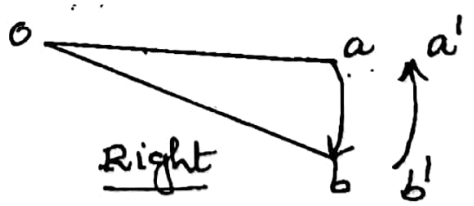
$M = 2200$ kg
$b = 1.4$m
$l = 2.4$m
$R = 100$m

$m = 140$ kg
$k = 0.15$m
$I_w = 0.7$ kg-m²
$V = 72 \times \frac{1000}{3600}$
$= 20$ m/sec.





for left turn



(i) car turning left as shown in figure.

(a) Reaction due to weight

Total weight $= 2200 \times 9.81 \Longrightarrow 21582$ N

$R_{W1,2} = \frac{Mg}{2} \times$ ratio $\longrightarrow \frac{21582}{2} \times \frac{1.4}{2.4} \Rightarrow 6295$ N [upwards]

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$R_{W\,3,4} = \frac{21582}{2} \times \frac{1}{2.4} \Longrightarrow 4496\,N$ [upward]

(b) Reaction due to gyroscopic couple $C_W = 4 I_w \omega_w \omega_p$

for wheels $= 4 I_w \frac{V}{\ell} \cdot \frac{V}{R}$

$= 4 \times 0.7 \times \frac{20}{0.4} \times \frac{20}{100}$

$= 28\,N\text{-}m$

For outer wheels $[R_{G\,2,4}]_{wheel} = \frac{C_W}{2b}$

$= \frac{28}{2 \times 1.4} \Longrightarrow 10N$ [upwards]

For inner wheels $[R_{G\,1,3}]_{wheel} = 10N$ [downwards]

Gyro couple for engine $C_e = I_e \cdot G \cdot \omega_w \cdot \omega_p$

$= mK^2 \cdot G \cdot \frac{V}{\ell} \cdot \frac{V}{R}$

$= 140(0.15)^2 \times 5 \times \frac{20}{0.4} \times \frac{20}{100}$

$= 157.5N$

For front wheels $[R_{G\,1,2}]_{engine} = \frac{C_e}{2\ell}$

$= \frac{157.5}{2 \times 2.4} \Longrightarrow 32.8N$ [downward]

For Rear wheels $[R_{G\,3,4}]_{engine} = 32.8N$ [upwards]

(c) Reaction due to centrifugal couple:-

$C_C = \frac{MV^2}{R} \cdot h \Longrightarrow 2200 \times \frac{20^2}{100} \times 0.6$

$= 5280\,N\text{-}m$

For outer wheels $[R_{C\,2,4}] = \frac{C_C}{2b}$

$= \frac{5280}{2(1.4)} \Rightarrow 1886N$ [upwards]

For inner wheels $[R_{C\,1,3}] = \frac{C_C}{2b} \Longrightarrow 1886N$ [downward]

∴ Reaction on wheels:

$R_1 = 6295 - 10 - 32.8 - 1886 = 4431.8N$

$R_2 = 6295 + 10 + 32.8 + 1886 = 8223.8N$

$R_3 = 4496 - 10 - 32.8 - 1886 = 2567.2N$

$R_4 = 4496 + 10 + 32.8 + 1886 = 6359.8N$

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(ii) car turning right :

All the reactions due to gyroscopic couples and centrifugal couple change signs.

$\therefore R_1 = 6295 + 10 + 32.8 + 1886 = 8158.2N$

$R_2 = 6295 - 10 - 32.8 - 1886 = 4366.8N$

$R_3 = 4496 + 10 + 32.8 + 1886 = 6424.8N$

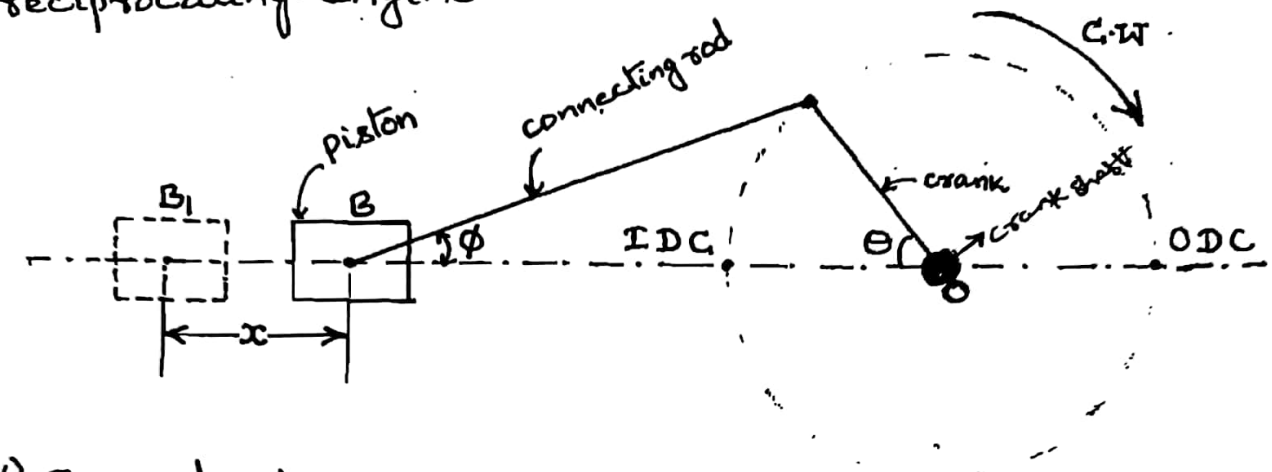
$R_4 = 4496 - 10 - 32.8 - 1886 = 2632.8N$

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Static and dynamic force Analysis of planar Mechanisms :-

Effect of inertia forces of reciprocating masses of a reciprocating engine :-



1). Force due to gas pressure on piston,

$$F_L = p \times \frac{\pi d^2}{4} \quad [\text{or}] \quad [P_1 - P_2]\frac{\pi d^2}{4}$$

2). $n = \frac{l}{r}$ where $r = \frac{L}{2}$

3). Inertia force on the piston $F_I = m_R \cdot \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

4). Thrust on piston, $I_p = F_L - F_I + m_R \cdot g$

5). Force on piston, $F_p = F_L - F_I$

6). $\sin\phi = \frac{\sin\theta}{n}$

7). Force on the crank shaft bearing

$$F_B = \frac{F_p \cos[\theta+\phi]}{\cos\phi} \quad \text{---- N}$$

8). Tangential force on the crank pin

$$F_T = \frac{F_p \sin[\theta+\phi]}{\cos\phi} \quad \text{---- N}$$

9). Torque acting on the frame $= F_T \times r$ ..... N-M

& Turning moment on the crank shaft

$$T_m = F_p \frac{\sin[\theta+\phi]}{\cos\phi} \times r \quad \text{..... N-M}$$

10). Distance moved by the piston $x = r\left[[1-\cos\theta] + \frac{\sin^2\theta}{2n}\right]$

$$x = r\left[[1-\cos\theta] + \frac{[1-\cos^2\theta]}{2n}\right] \quad [\text{or}] \quad \text{..... M}$$

Where
d = piston diameter
L = piston stroke
l = Length of the connecting rod.
m_R = mass of the reciprocating parts
r = crank radius
r_p = crank pin radius.

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